DNIPRO UNIVERSITY of TECHNOLOGY 1899

Mining of Mineral Deposits
ISSN 2415-3443 (Online) | ISSN 2415-3435 (Print)
Journal homepage http://mining.in.ua
Volume 12 (2018), Issue 4, pp. 37-45

UDC 622.023.23 https://doi.org/10.15407/mining12.04.037

NONLINEAR CRITERIA AND THE INFLUENCE OF THE AVERAGE PRINCIPAL STRESS ON ROCKS DESTRUCTION

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ABSTRACT

Purpose. Consider possible series methods of Mohr-Coulomb nonlinear criterion that takes into account stresses in the plane of fracture. Investigate the influence of the average principal stress on rocks destruction when cylinder modeling under axial and side load conditions. Determine the weakening effect on the internal friction angle in the rocks under study using the examples of sample testing simulation.

Methods. To study the processes in rock samples, we used a finite-element method DESTROCK-FE for modeling the deformation of rocks decomposes. The parameters of the deformation and fracture model are determined by comparing the deformation curves obtained by the modeling and experimental approaches.

Findings. Modeling of rocks deformation under various loading conditions with a nonlinear criterion of fracture variants, in which destructive stresses can grow limited or unlimited or even decrease with increasing pressure, is performed. The design charts of axial and side deformations of the samples with a good approximation coincide with the experimental ones. The model parameters, including those for triaxial load conditions, are determined with a mathematic method using simulation laboratory tests under axial compression. Modeling with an upgrade fracture criterion showed that the load-bearing strength of salt rock samples under side load is greater than with axial pressure. Internal friction angle reduction with loss of strength reaches 15%.

Originality. The Mohr-Coulomb series criterion is proposed, which makes it possible to set the nonlinearity in accordance with experimental data. Mechanical model parameters of rocks can be determined according to the results of single sample testing using the finite-element method DESTROCK-FE of decomposed rocks. The simulation was performed taking into account microdeformation, the influence of which is described by equations for a nonlinear-elastic condition.

Practical implications. Cylinder modeling showed that the load-bearing strength of samples under side load is 10% for sylvinite and 20% more for rock salt than at axial loading. The obtained results indicate the ability of the finite-element method DESTROCK-FE of rocks destruction modeling while geomechanics survey conducting. Using this method, based on single sample simulation results, tested at simple compression, the parameters of elasticity, plasticity, viscosity, crisping can be obtained, suitable for deformation and fracture processes studies under various loading conditions.

Keywords: *rock, mathematic simulation of rock deformation and fracture, microdeformation, finite-element method, fracture criteria, internal friction angle*

1. INTRODUCTION

In most cases, fracture criteria are developed based on the concept of interfacial fracture, orthographic to the plane of maximum and minimum stresses action. All fracture (plasticity) criteria for virgin formation describe the surface in the stress space that limits the conditions under which the rock is not destroyed. In addition to the need of curve and experimental data coincidence, verification of such criteria is performed on the demand of the boundary surface protuberance in the stress space.

An important factor influencing the homogeneous rocks destruction is the ratio of maximum and minimum stresses σ_1 and σ_3. According to the tests results of strong pyrogenic cubic rock samples *K*. Mogi found that increasing the average stress σ_2 from σ_3 to σ_1 leads to an increase in destructive pressure by 25 – 30%. In most destruction criteria (Drucker & Prager, 1952; Ruppeneit, 1954; Protod'yakonov, 1962; Ruppeneit & Liberman, 1965; Mogi, 1967; Wiebols & Cook, 1968; Bieniawski, 1974; Lade, 1977; Hoek & Braun, 1980; Matsuoka & Nakai, 1982; Matsuoka & Nakai, 1985; Ramamurthy, Rao, & Rao, 1985; Hoek, 1994; Sheorey, 1997; Shashenko & Pustovoitenko, 2001; Maïolino, 2005; Mogi, 2007; Benz, Schwab, Kauther, & Vermeer, 2008; Litvinskiy, 2008; You, 2009; Yu, Xia, & Kolupaev, 2009; You, 2013) the average stress is not taken into account or taken into account in the principal stresses space.

The scientific community is concerned that in most cases this influence is not based on the destruction criteria used in engineering calculations.

For the analysis of rock structures competence, the Mohr-Coulomb (M – C) and Hoek-Brown (H – B) failure criteria are popular, while the M – C criterion is linear, and the H – B criterion is nonlinear in nature. The influence of the average stress on the ultimate stress limit in these criteria is not taken into account in the experiments. Empirical criteria that take into account the influence of the average principal stress by including the third stress invariant and the Nadai-Lode parameter in the equations are not widely used. Fracture criteria formulated in terms of principal stresses cannot be applied to rocks with natural or developing weakening.

Mathematical models with a help of which the deformation and fracture processes in a material are simulated include, in addition to the fracture criterion, the link between stresses and the reversible and irreversible deformations, as well as patterns of plastic, viscous deformations and dilatancy development. It is obvious that the simulated weakening development processes and the ultimate stress limit magnitude depend not only on the fracture criterion, but also on the links between stresses and deformations.

M – C criterion is based on the physical idea of fracture under the action of tangential stresses at fault sites. The nonlinear nature of the crippling loadings on pressure dependence, observed in experiments, in the applied fracture criteria, including the H – B criterion, is described empirically as an approximation of the curve, compounding the Mohr limit circles, or the dependence of the maximum and minimum stress at failure. We note a special case of the nonlinear strength criterion proposed by Litvinskiy (2008), in which the nonlinear strength certificate is derived on the basis of physical ideas about the Coulomb friction effect on the strength of fault sites parts. In N. Barton's work (Barton, 2013) on the disturbed ground strength, the nonlinear nature of the strength certificate is explained by the variability of the internal friction angle. Generally, rocks fracture criterion is characterized with indefinite increasing pressure in the area of decomposed tangential stresses. Research (Barton, 1976; Singh, Raj, & Singh, 2011; Singh & Singh, 2012) has established that at high compressive stresses, decomposed tangential stresses can be limited. In soil mechanics, the fracture criteria of cap-models (Roscoe & Burland, 1968; Ulitskiy, Shashkin, & Shashkin, 2010) in which decomposed tangential stresses at high pressures can decrease to zero.

In the rock mass with natural weakening or with fracture processes started due to the mining influence with weakening surfaces formation, the description of fracture conditions with cambered planes in the stresses space is impossible. To simulate rocks fracture, methods are needed that take into account the processes of gradual development of fracture along existing and emerging surfaces of weakening strength. The finite-element method DESTROCK-FE of rocks destruction modeling possessed such capacities (Olovyannyy, 2012), in which the deformation and fracture is calculated taking into account the existing or developing of surface strength weakening (along sites).

Previously, this method was used with a linear condition of shear strength M – C and cleavages fracture (Olovyannyy, 2012). The problem of the average principal stress influence and the non-linearity of the strength certificate is considered in this work within the DESTROCK-FE model using the upgrade strength criterion M – C. The principle of method consists in the fact that the strength criterion is applied to surfaces with given directions. Destruction in separate parts can develop in one or several directions.

Now it is considered the possibility of construction a fracture criterion on the basis of the M – C one that will allow for a limited growth of destructive tangential stresses with an increase in all-round pressure. The series of the M – C criterion is that the linear variant includes not only tangential and normal stresses at the site, but also normal stresses in the fault sites. With regard to tight rocks, this criterion includes the effect of average principal stress. This effect is estimated by cylinder deformation modeling under axial and side load conditions.

The variability of the friction angle depending on the fault and normal pressure requires verification and is considered in this work when modeling the protoplasts deformation. The study was performed on the basis of sylvinite and rock salt protoplasts deformation modeling, taking into account the results of laboratory tests. The model para-meters of deformation and strength rocks properties are determined by the coincidence of the deformation curves in the model and those obtained from testing samples under simple compression.

2. CONCEPT OF FRACTURE CRITERION FORM

In the currently used criteria for rocks, including H – B, the ultimate stress limit can grow indefinitely with normal pressure. In Barton's work in 1976, based on the analysis of a large number of experimental data on rock testing, it was concluded that Mohr's compound curve $\sigma_1 \approx 3\,\sigma_3$ becomes parallel to axis σ (Fig. 1). Singh, Raj, & Singh (2011) taking into account his own tests and analysis of thousands registered experiments for the majority of rock types, it was established that the "critical state" occurs at $\sigma_{3(critical)} \approx \sigma_c$.

For contact rocks with zero tenacity, series of the M – C criterion are considered, in which the internal friction angle decreases with increasing pressure σ_n (Barton, 2013). In such models, the ultimate stress limit can be limited only by reducing the friction angle to zero. In the soil mechanics, the fracture criterion of cap-models (Roscoe & Burland, 1968; Ulitskiy, Shashkin, & Shashkin, 2010), in which decomposed tangential stresses at high pressures can decrease to zero.

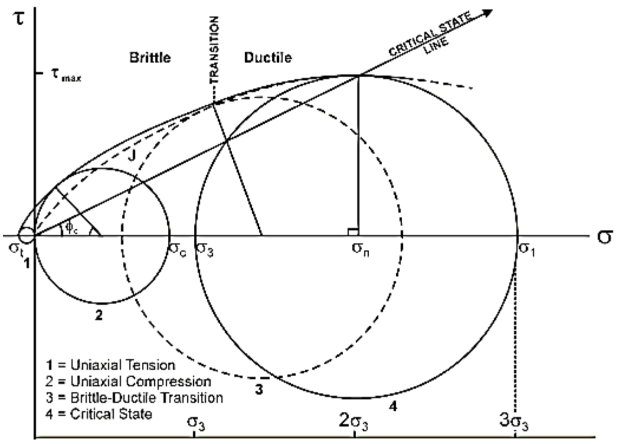

Figure 1. Mohr's compound curve according to the results of a triaxial strength test according to (Barton, 1976)

The failure criterion M – C and all subsequent ones are based on the idea that the effect of friction on ultimate stress limit depends only on the normal pressure at the fault site. In reality, fracture is realized in a certain flat zone having a thickness. It is natural to assume that not only normal and tangential stresses at the site, but also normal stresses acting in the shear plane affect shear fracture. Internal friction angle, as well as tenacity, can vary at the sites of growing shear depending on the degree of broken material condition. These effects must be considered when formulating the fracture criterion on shear surfaces.

A general view of the failure criterion M – C over the fault sites is represented with a formula in which the influence of friction depends not only on the normal pressure on the site, but also on the stresses in its plane. The failure criterion for the shear surface, taking into account the presence of natural and growing weaknesses, can be represented as:

$$|\tau| = C\omega_n + \sigma_n Q_n(S) tg(\varphi(\omega_n)), \quad (1)$$

where:

σ_n and τ_n – normal and tangential stresses at the site;

C – tenacity;

φ – internal friction angle;

S – total principal stresses (tripled average pressure), which includes stresses in the weakening plane, $S = \sigma_1 + \sigma_2 + \sigma_3$;

ω_n – the parameter determining the weakening over the site (continuity), $0 \le \omega_n \le 1$.

In this criterion, the breaking shear stress depends on the tenacity taking into account the broken condition and normal pressure on the shear site, whose effect on friction depends on the avarage pressure including the stresses in the shear plane. With complete loss of strength, the tenacity in the shear plane becomes zero, and the internal friction angle may vary within certain limits.

Based on the possible idea of the finite quantity of breaking tangential stress, the formula $Q_n(S)$ will be represented by a continuous piecewise function:

$$Q_n(S) = \begin{cases} 1 & at\ S < q; \\ \left(\dfrac{q}{S}\right)^a & at\ S > q, \end{cases} \quad (2)$$

moreover, $q = \xi_1 E$.

The role of $Q_n(S)$ function is as follows:

– when $S < q$, the stresses in the shear plane do not affect the action of normal stresses and on the shear resistance. Here q is the value that determines the influence limits of predominantly normal pressure;

– as $S \to \infty$ $Q_n(S) \to 0$, the shear resistance decreases with increasing all-round pressure S; in the limit, shear resistance can only be determined with the tenacity.

The parameters a, ξ_1 and ξ_2 can be determined from the results of testing samples in the laboratory. The parameter a sets the nature of the ultimate stress limit dependence on the average pressure. With an increase in the average pressure, the value $\sigma_n Q_n(S)$ can grow continuously at $a < 1$, asymptotically tend to a constant value at $a = 1$, or approach zero at $a > 1$. As $a \to 0$, the criterion becomes the linear M – C criterion.

The threshold quantity q depends on the material deformation properties. The adopted dependence $q = \xi_1 E$ contains the rigidity modulus as a parameter of rock properties, but, for example, not the break-down point under simple compression, since the latter depends on the degree of broken condition and is zero in friable materials.

The upgrade nonlinear criterion M – C (1) is substantiated physically on the stress state destruction effect in the vicinity of the shear site.

3. FUNDAMENTAL POSITION OF THE FINITE-ELEMENT METHOD OF ROCK DESTRUCTION WITH UPGRADE MOHR-COULOMB CRITERION

Deformation and fracture samples processes modeling is performed using a finite-element method DESTROCK-FE of decomposed rocks (Olovyannyy, 2003; Olovyannyy, 2012). The main feature of this method is a method of fixing fractures in a limited number of directions. When calculating from an infinite number of possible orientations of cracks development in the elements, the fractures directions with increment of 45º are considered. In 3D space there are 13 such directions.

Under the conditions of axial symmetry, nine directions are distinguished (Fig. 2). In the figure, each of the systems (sites) is shown in three or four identical numbers: 1 – 1 – 1, 2 – 2 – 2, 3 – 3 – 3 – 3, 4 – 4 – 4, 5 – 5 – 5, 6 – 6 – 6 – 6, 6′ – 6′ – 6′, 7 – 7 – 7 – 7, 7′ – 7′ – 7′.

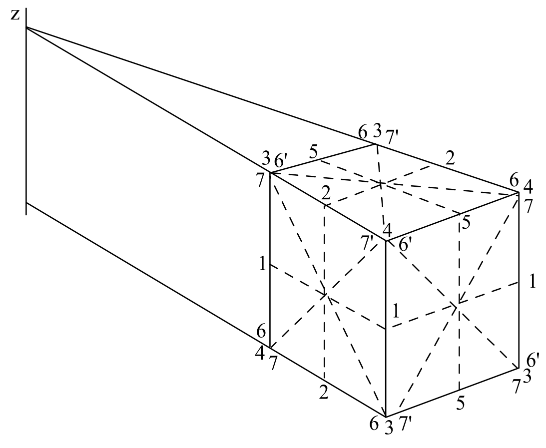

Figure 2. The scheme of fixed directions of possible weakening developing systems under axisymmetric deformation

When calculating at each allocated site, the possibility of shear and fracture is estimated. Previously, the linear criterion M – C was used to assess the possibility of shear failure. In this study, to take into account the nonlinearity of the of destructive stresses dependence on pressure, this method is applied with the upgraded criterion (1). In this version, the function $Q(S)$ is designed to take into account effect of pressure in the shear plane and normal pressure in the process of Coulomb friction. In this case, the coefficient of friction may depend on the degree of shear sites broken condition.

Taking into account the strength reduction over the sites, the fracture condition over the weakening surfaces (1) was adopted in the variant with a limited strength value (with $a = 1$):

$$|\tau_n| = C\omega_n + \sigma_n tg\left(\varphi\left(1-\xi_2\left(1-\omega_n\right)\right)\right),$$

if $\sigma_n > 0$; $S < \xi_1 E$;

$$|\tau_n| = C\omega_n + \sigma_n\left(\xi_1 \frac{E}{S}\right)tg\left(\varphi\left(1-\xi_2\left(1-\omega_n\right)\right)\right), \quad (3)$$

if $\sigma_n > 0$; $S > \xi_1 E$;

$\sigma_n = -\omega_n \sigma_t$, if $\sigma_n < 0$,

where:

σ_t – tensile strength;

ξ_1 and ξ_2 – the fracture criterion constants determined in the laboratory experiments simulation with samples.

The parameter of current nominal strength (continuity) ω_n is calculated at all possible sites of destruction for each component of surroundings. Increase in strength at fissure and shear on the site $\bar{n}$ is associated with breaking strain, respectively:

$$\Delta\omega_n = M_\varepsilon \Delta\varepsilon_n; \quad (4)$$

$$\Delta\omega_n = -M_\gamma |\Delta\gamma_n|, \quad (5)$$

where:

$\Delta\varepsilon_n$ and $\Delta\gamma_n$ – increments of tensile and shear breaking strains on the site $\bar{n}$;

M_ε and M_γ – local decline module, respectively, in tension and shear, which describe the connection between the increments of breaking strains and decreasing nominal strength.

Plastic and breaking shear deformations are accompanied with tensile deformations normal to the shear site, which depend on the tangential and normal stresses on the site:

$$\Delta\varepsilon_n = -K_v \frac{|\tau_n|}{|\tau_n| + \xi_3 \sigma_n} |\Delta\gamma_n|, \quad (6)$$

where:

Kv – the parameter of volume change;

ξ_3 – a parameter that determines normal pressure effects on dilatancy.

Viscous properties of rocks are described with relations for a viscoelastic medium (Voigt solid). In this paper, the processes in rock samples are investigated during rapid tests, the effect of plastic flow is not considered.

Soft tenacious terrain has a porosity of up to 25% or more. Some of the porosity is represented with microdefects, when is deformed under the mechanical forces action, appears as a hazardous inelastic medium compared to the base material. This part of the mechanically active porosity will be called the microdefect medium. The microdefect medium is appeared in the initial sections of the deformation curves of compression samples.

To take into account the influence of microdefects, a phenomenological model of the medium deformation has been proposed (Olovyannyy, 2012). This medium is viscous, its manifestations depend substantially on the loading rate, the deformations are reversible, and the compressive deformations are limited.

The viscoelastic deformations rate of the microdefect medium in the directions of the principal deformations ($i = 1, 2, 3$):

$$d\varepsilon_i^{ve} = \left(e_i - \varepsilon_{md}^i\right)\left(1-\exp\left(-\eta_{md} dt\right)\right), \quad (7)$$

where:

e_i – the breaking microdefect strain along the i-th direction at a given stress level;

ε^i_{md} – the achieved level of deformation;

η_{md} – microdefect viscosity coefficient.

The magnitude of the breaking microdefects degree of compression is calculated as for a nonlinear elastic medium. At a given level of stress:

$$e_1 = \begin{cases} \left(\sigma_i - 0.5K_v\left(S-\sigma_i\right)\right)\dfrac{\left(1-\left(1-\dfrac{E_{md}}{E}\right)\dfrac{\varepsilon_{md}^i}{e_{md}}\right)}{E_{md}} & if\ \varepsilon_{md}^i > 0 \\ \dfrac{\sigma_i - 0.5K_v\left(S-\sigma_i\right)}{E_{md}} & if\ \varepsilon_{md}^i < 0, \end{cases} \quad (8)$$

where:

e_{md} – the largest possible degree of compression (effective power porosity);

E – the rigidity modulus;

E_{md} – the rigidity modulus of a microdefect medium under tension.

Formula (8) describes a microdefect medium as nonlinear-elastic. On the compressive strain branch, the rigidity modulus is changed from the modulus magnitude under tension to the rigidity modulus of the base material, which occurs when the breaking strain is reached. On the tension branch the rigidity modulus is minimal.

4. SIMULATION OF SAMPLE TESTING UNDER AXIAL AND TRIAXIAL COMPRESSION

The study was performed using the results of rock salt samples testing (height 100 mm, diameter 50 mm). The tests were carried out with axial compression of samples at a constant rate of 0.5 mm/min.

The cylinder deformation modeling was performed using the described method of DESTROCR-FE deformations and fractures calculating. The samples deformation under axial loading with complete tenacity at the contact of sample abuts with rigid sheet press is prohibited (radial displacements are forbidden, all contact points move along the axis with the same speed). The parameters of deformation and strength properties were obtained with their variation based on a comparison of calculated and experimental graphs (Fig. 3).

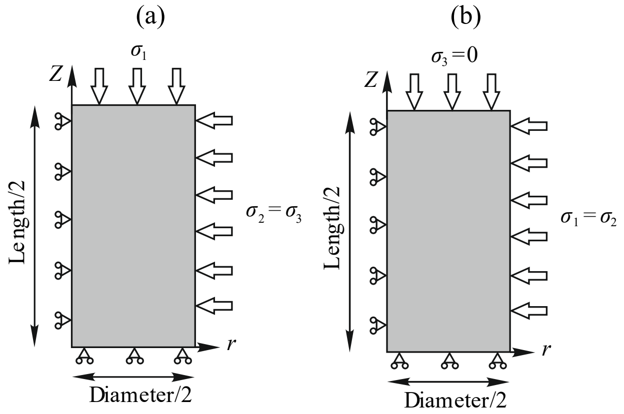

Figure 3. Cylinder schemes loading during modeling ($\sigma_3 \le \sigma_2 \le \sigma_1$): a – axis; b – side

The simulation was performed using the upgrade criterion M – C (3) with the following values of the parameters included in the strength criterion and the dilatancy function (6): $\xi_1 = 0.010$, $\xi_2 = 0.15$, $\xi_3 = 1.6$. Other model parameters that determine elastic, plastic, crisp, viscous properties are listed in the table. To assess the average principal stress influence on the destruction, samples were simulated under axis and side loading conditions (Fig. 3).

The graphs in Figure 4 indicate the curves proximity obtained under axial loading in the experiment and in modeling for both axial and side deformations.

Simulation showed that the maximum pressure on the sample under side pressure (model side curves) is greater than under axial loading (test axis and model axis curves) by 10% for sylvinite sample and by 20% for rock salt (Fig. 4). This result showed the appearance of a function describing a fracture criterion, including the action of normal stresses in fracture planes; however, the influence of deformation patterns describing microdefects, plasticity, crisp, viscosity, and oriented nature of developing weakening cannot be ruled out.

Table 1. Calculated parameters of the model

Parameters	Rock	
	Sylvinite	Rock Salt
Rigidity modulus, MPa	14300	9000
Poisson number	0.32	0.048
Compressive strength, MPa	35.2	17.8
Tensile strength, MPa	1.7	1.0
Internal friction angle	36.3	30.2
Lasting properties coefficient	0.40	0.61
Plasticity coefficient	13.2	1.0
The module of the local decline in shear M_y	34.9	47.0
The module of the local decline at fracture M_ε	50.0	107.0
Coefficient of volume change K_V	0.710	0.018
Largest possible degree of microdefects compression, %	0.22	0.19
Modulus of microdefects deformation, MPa	400.0	1100.0
Coefficient of microdefects viscosity, 1/h	11900	3200

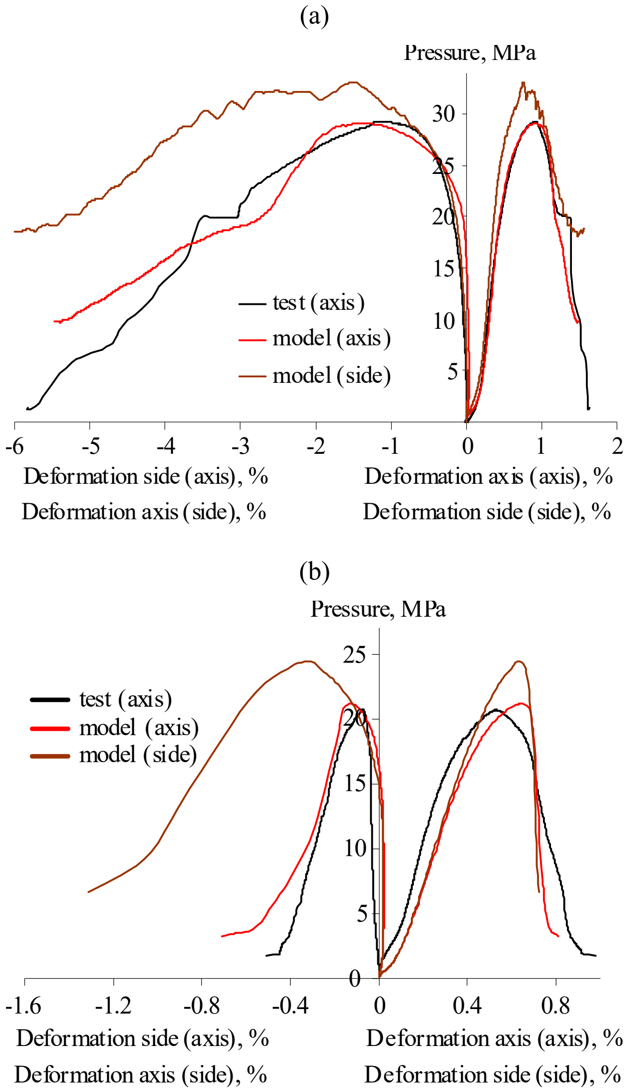

Figure 4. Cylinder deformation graphs, obtained at testing and mathematical modeling: (a) sylvinite; (b) rock salt

Variations of the internal friction angle showed that for the studied rocks salt, its decrease on surfaces with a complete loss of strength is not more than 15% ($\xi_2 = 0.15$). Destructive shear is accompanied with dilatancy. The tensile deformation along the normal to the shear site depends on the ratio of tangential and normal stresses. With the normal pressure growth, the volume change decreases. In formula (6) for volume change, $\xi_2 = 1.6$.

The component of microdefects deformations significantly affects the general nature of samples deformation and fracture.

5. STRENGTH CERTIFICATE ACCORDING TO THE TRIAXIAL LOAD SAMPLES RESULTS SIMULATION

The linear strength criterion M – C character corresponds to the experimental data in a limited range of loads. Virtually all non-linear criteria, including the H – B criterion, approximate experimental graphs of the maximum and minimum stresses connection at the load-bearing samples strength limit under triaxial compression with an infinite growth tendency.

The upgrage criterion M – C (3) with parameters $a = 1$ and $\xi_1 = 0.010$ describes a limited increase in ultimate stresses limit. Using DESTROCR-FE with an upgrage criterion M – C, we simulated the samples deformation under triaxial compression conditions $\sigma_1 > \sigma_2 = \sigma_3$ at a side pressure from 0 to 100 MPa. The calculations were performed for sylvinite and rock salt samples with parameters determined from the results of axial loading of samples. The simulation was performed for tests carried out according to the following scheme: at the first stage, the sample is loaded with side and axial pressure with the same rate of growth to a given value ($\sigma_1 = \sigma_2 = \sigma_3$); then the axial deformation continues to grow at a constant rate until fracture. Figure 5 shows the test graphs. The graphs show that the initial growth points of the graphs are related from origin of coordinates on the deformation axis due to the triaxial compression and closure of microdefects and with an increase in confined pressure.

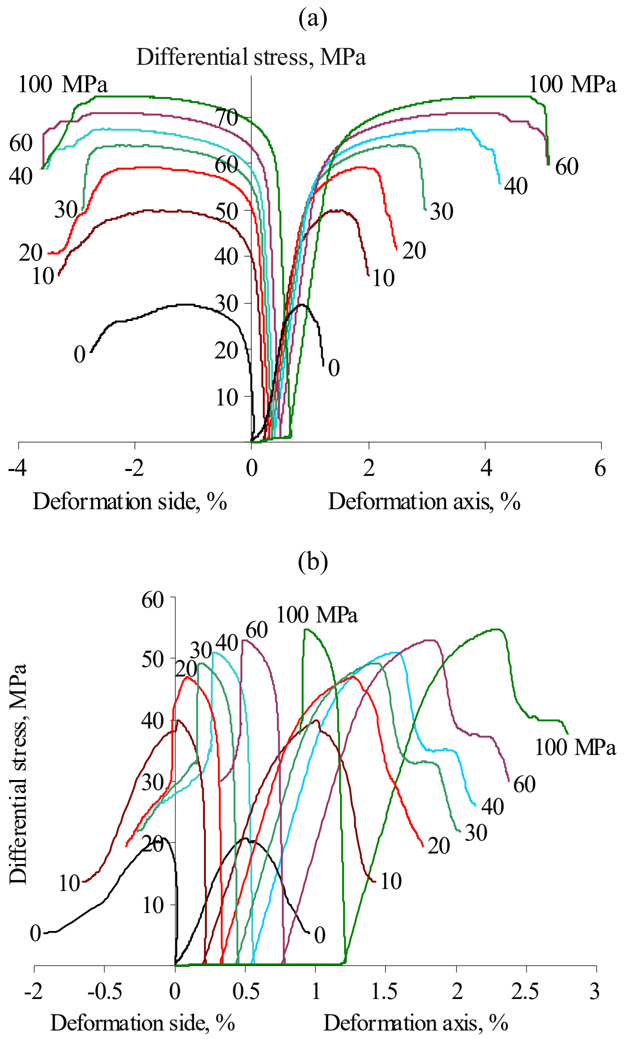

Figure 5. Measurement charts of axis and side deformations in sylvinite samples (a) and rock salt samples (b) at axial loading with required side pressure 0, 10, 20, …, 100 MPa

Figure 6 shows the measurement charts of the maximum difference between axial and side stresses versus side pressure. These charts represent the fracture criteria for rocks salt with the upgraded M – C criterion with a limited amount of ultimate stress limit.

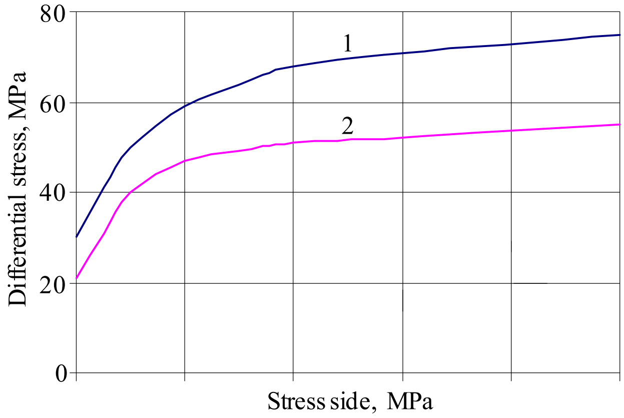

Figure 6. Measurement chart of maximum difference $\sigma_1 - \sigma_3$ versus side pressure σ_3: 1 – sylvinite; 2 – rock salt

Figure 7 shows the cross cylinder sections with troubles at the moment of complete loss of load-bearing strength, illustrating the results of modeling with the DESTROCK-FE method.

6. CONCLUSIONS

The Mohr-Coulomb (M – C) criterion is based on the concept of shear destruction sites. The problem of the non-linearity of the strength certificate is solved using the upgrade M – C fracture criterion in terms of the friction effect on the rupture resistance. The upgraded criterion takes into account the effect of normal stresses in the plane of surface weakening. The proposed version of the function describing the fracture conditions upon shear makes it possible to take into account different ideas about the mechanisms of ultimate stress limit and increasing pressure connections. Depending on the adopted parameters, ultimate stress limit with increasing average pressure can grow continuously, asymptotically tend to a constant value, or approach zero. The H – B criterion, the Barton and Singh models and the cap-models can serve as analogues of such models, respectively.

The study was conducted in a mathematic simulation of samples testing using finite-element method of DESTROCK-FE rock destruction. As a fracture criterion, a version of the upgrade Mohr-Coulomb criterion with limited growth of ultimate stress limit was used. To determine the model parameters, the results of laboratory tests of sylvinite and rock salt cylinder were used.

At different stages of samples loading accumulated weaknesses, microdefects, elasticity, plasticity, viscosity, crisp, scarification, etc. appear in different parts of the samples. The models parameters describing these processes are determined by varying them, based on minimizing the deviation of the deformation curves in the experiment and in the simulation, using a variant of the criterion with limited ultimate stress limit. In the processes of deformation and fracture natural microdefects play an important role. The microdefect component of deformations is described as for a nonlinear-elastic non-dispersive medium.

Studies of the average principal stress influence on rocks destruction, performed on the example of cylinder rocks salt tests modeling with upgrade criterion M – C, showed that with side loading $\sigma_2 = \sigma_1$ ultimate stress limit is more than with axial loading $\sigma_2 = \sigma_3 = 0$ with 10% in sylvinite and 20% in rock salt.

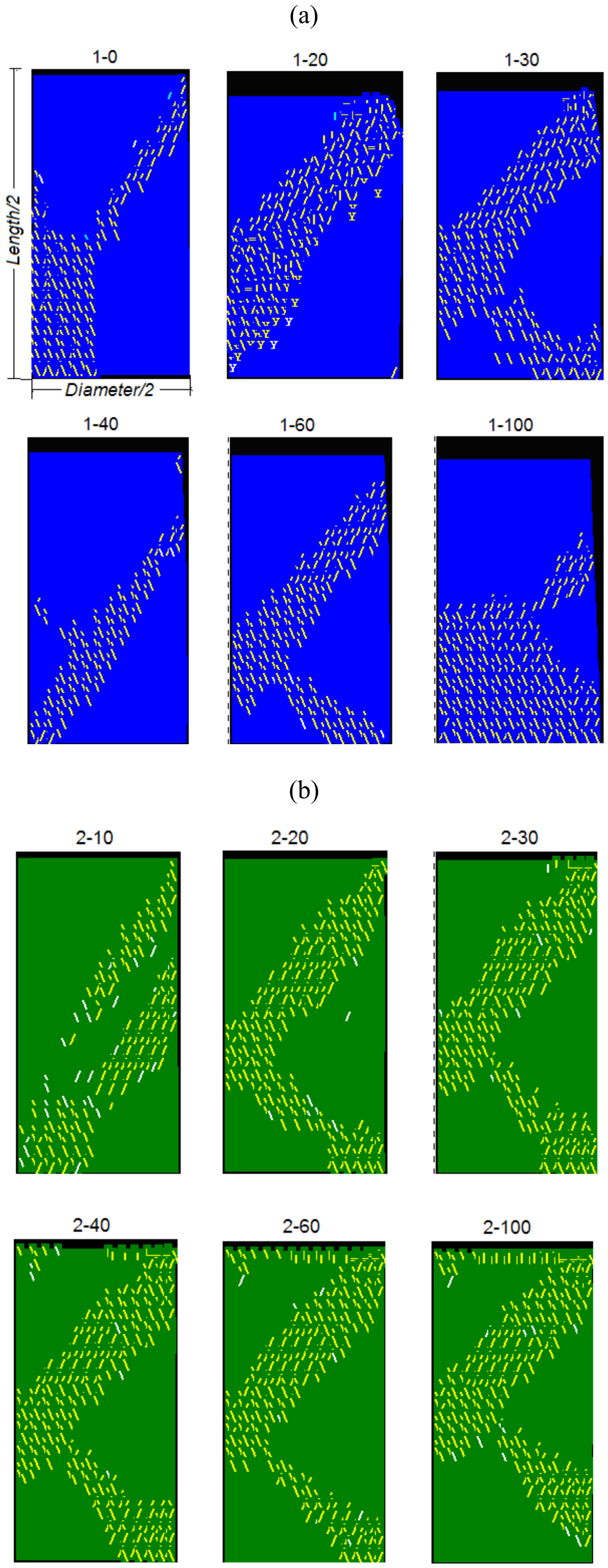

***Figure 7.** Cross sections with troubles at loading with required side pressure 10, 20, …, 100 MPa: (a) sylvinite samples (1 – 10, 1 – 20, …); (b) rock salt samples (2 – 10, 2 – 20, …)*

It was established that for the considered rocks salt the internal friction angle on the fracture surfaces decreases slightly, by no more than 15%. The finite-element method of rock destruction (DESTROCK-FE) was refined in terms of taking into account the stresses effect in the surfaces weakening zone and the variability of the internal friction angle, scarification and microdefects function.

The finite-element method of rock destruction (DESTROCK-FE) implementation allowed us to obtain simulation results comparable to laboratory studies. The proposed upgraded Mohr-Coulomb criterion allows one to describe fracture conditions corresponding to different ideas about decomposed tangential and normal stresses connection.

ACKNOWLEDGEMENTS

This study was supported by the Institute of Environmental Geoecology of the Russian Academy of Sciences. Materials with the laboratory rocks tests results were provided by the Research Laboratory of Physico-Mechanical Properties and Rocks Destruction at Saint Petersburg Mining University (Candidate of Technical Sciences M. Il'inov, Candidate of Technical Sciences V. Korshunov, Candidate of Technical Sciences D. Petrov).

These studies were made possible by the help of Candidate of Physical and Mathematical Sciences V. Chantsev in the implementation of the author's ideas in computer programs.

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НЕЛІНІЙНІ КРИТЕРІЇ ТА ВПЛИВ СЕРЕДНЬОГО ГОЛОВНОГО НАПРУЖЕННЯ НА РУЙНУВАННЯ ГІРСЬКИХ ПОРІД

А. Олов'яний

Мета. Розглянути можливі варіанти модифікації нелінійного критерію Кулона-Мора, що враховує напруження в площині руйнування. Дослідити вплив середнього головного напруження на руйнування гірських порід при моделюванні циліндричного зразка в умовах осьового та бічного навантаження. Оцінити вплив ослаблення міцності в досліджуваних гірських породах по поверхнях на кут внутрішнього тертя на прикладах моделювання випробування зразків.

Методика. Для дослідження процесів у зразках гірських порід використаний скінчено-елементний метод моделювання деформування гірських порід, що руйнуються, DESTROCK-FE. Параметри моделі деформування і руйнування визначаються при порівнянні кривих деформування, отриманих модельним та експериментальним шляхом.

Результати. Виконано моделювання деформування зразків гірських порід у різних умовах навантаження з нелінійним варіантом критерію руйнування, в якому руйнуючі напруження можуть зростати обмежено або необмежено та навіть знижуватися з ростом тиску. Розрахункові графіки осьових і бічних деформацій зразків з достатньою достовірністю збігаються з експериментальними. Параметри моделі, в тому числі для умов об'ємного навантаження, визначаються чисельним методом за допомогою моделювання лабораторних випробувань при осьовому стисненні. Моделювання циліндричних зразків показало, що несуча здатність зразків в умовах бічного навантаження на 10% для сильвініту і на 20% для кам'яної солі більше, ніж при осьовому навантаженні. Зниження кута внутрішнього тертя при втраті міцності по площинах зсуву і відриву для розглянутих соляних порід сягає 15%.

Наукова новизна. Запропоновано модифікований критерій Кулона-Мора з урахуванням впливу міцності проміжного головного напруження, що дозволяє задавати нелінійність відповідно до експериментальних даних. Параметри механічної моделі гірських порід можуть визначатися за результатами випробування одного зразка із застосуванням скінчено-елементної моделі гірських порід, що руйнуються, DESTROCK-FE. Моделювання виконано з урахуванням мікродефектів, вплив яких описується рівняннями для нелінійно-пружного середовища.

Практична значимість. Отримані результати свідчать про здатність скінчено-елементного методу моделювання руйнування гірських порід DESTROCK-FE вирішувати геомеханічні завдання. За допомогою цього методу за результатами моделювання одного зразка, випробуваного при одноосьовому стисненні, можуть бути отримані параметри пружності, пластичності, в'язкості, крихкості, придатні для застосування у дослідженнях процесів деформування і руйнування в різних умовах навантаження.

Ключові слова: *гірська порода, математичне моделювання, деформування і руйнування гірських порід, мікродефектні деформації, метод скінчених елементів, критерії руйнування, кут внутрішнього тертя*

НЕЛИНЕЙНЫЕ КРИТЕРИИ И ВЛИЯНИЕ СРЕДНЕГО ГЛАВНОГО НАПРЯЖЕНИЯ НА РАЗРУШЕНИЕ ГОРНЫХ ПОРОД

А. Оловянный

Цель. Рассмотреть возможные варианты модификации нелинейного критерия Кулона-Мора, учитывающего напряжения в плоскости разрушения. Исследовать влияние среднего главного напряжения на разрушение горных пород при моделировании цилиндрического образца в условиях осевого и бокового нагружения. Оценить влияние ослабления прочности в исследуемых горных породах по поверхностям на угол внутреннего трения на примерах моделирования испытания образцов.

Методика. Для исследования процессов в образцах горных пород использован конечно-элементный метод моделирования деформирования разрушающихся горных пород DESTROCK-FE. Параметры модели деформирования и разрушения определяются при сравнении кривых деформирования, полученных модельным и экспериментальным путем.

Результаты. Выполнено моделирование деформирования образцов горных пород в различных условиях нагружения с нелинейным вариантом критерия разрушения, в котором разрушающие напряжения могут расти ограниченно или неограниченно, или даже снижаться с ростом давления. Расчетные графики осевых и боковых деформаций образцов с достаточной достоверностью совпадают с экспериментальными. Параметры модели, в том числе для условий объемного нагружения, определяются численным методом с помощью моделирования лабораторных испытаний при осевом сжатии. Моделирование цилиндрических образцов показало, что несущая способность образцов в условиях бокового нагружения на 10% для сильвинита и на 20% для каменной соли больше, чем при осевом нагружении. Снижение угла внутреннего трения при потере прочности по площадкам сдвига и отрыва для рассмотренных соляных пород достигает 15%.

Научная новизна. Предложен модифицированный критерий Кулона-Мора с учетом влияния прочности промежуточного главного напряжения, позволяющий задавать нелинейность в соответствии с экспериментальными данными. Параметры механической модели горных пород могут определяться по результатам испытания одного образца с применением конечно-элементной модели разрушающих горных пород DESTROCK-FE. Моделирование выполнено с учетом микродефектов, влияние которых описывается уравнениями для нелинейно-упругой среды.

Практическая значимость. Полученные результаты свидетельствуют о способности конечно-элементного метода моделирования разрушения горных пород DESTROCK-FE решать геомеханические задачи. С помощью этого метода по результатам моделирования одного образца, испытанного при одноосном сжатии, могут быть получены параметры упругости, пластичности, вязкости, хрупкости, пригодные для применения в исследованиях процессов деформирования и разрушения в различных условиях нагружения.

Ключевые слова: *горная порода, математическое моделирование, деформирование и разрушение горных пород, микродефектные деформации, метод конечных элементов, критерии разрушения, угол внутреннего трения*

ARTICLE INFO

Received: 7 February 2018
Accepted: 24 October 2018
Available online: 6 November 2018

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