AMEM 2019 IOP Publishing
IOP Conf. Series: Materials Science and Engineering **731** (2020) 012003 doi:10.1088/1757-899X/731/1/012003

Algorithm for J-Integral Measurements by Digital Image Correlation

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Abstract. The work is devoted to the testing of the algorithm for calculating J-integral based on the construction of vector fields by digital image correlation (DIC) method. A comparative analysis of J-integral values calculated using DIC and instrumental data obtained in accordance with ASTM E 1820 “Standard Test Method for Measurement of Fracture Toughness” has made. It is shown that this approach can be used for cases when the standard technique for measuring the J-integral cannot be applied, or the standard technique does not allow achieving the required accuracy for the integral determination in local areas of the loaded material.

1. Introduction

The digital image correlation (DIC) method [1, 2] is becoming increasingly popular due to its versatility, ease of use, and the ability to obtain full-sized displacement/deformation fields when processing images of the surface of a loaded object. Based on the calculated fields, it becomes possible to determine the required parameters of the stress-strain state, for example, such as SIF, CTOD, J-integral and other characteristics adopted in fracture mechanics.

J-integral proposed by Cherepanov and Rice in 1967-68 [3, 4] is defined as a contour integral independent of the path which characterizes stresses and strains in the region of the crack tip in elastic or elastoplastic loaded materials. The main questions arising in determining the J-integral using direct integration over the contour that шincludes the crack tip with a gap on its edges are the choice of the integration contour, in particular, its shape, dimensions, position relative to the crack tip, and the effect of these parameters on the accuracy of final calculations. According to the definition of the J-integral it is independent on the integration path and this property is satisfied when working with model data [5]. However, in an experiment, due to errors in determining the displacements resulting from the use of the digital image correlation method, independence from the integration path is not fulfilled [6]. In the presented study, an algorithm for measuring the J-integral is described and tested with a help of data obtained using the digital image correlation method, and they are compared with the results obtained by the instrumental method (by means of COD gauge).

2. Algorithm for j-integral measurement

The process of measuring the J-integral (including the allocation of the plastic zone) when using the approach to calculating the optical flux during mechanical testing of materials can be divided into four stages:

1. calculation of displacement fields [7];
2. post-processing of the fields [8];
3. calculation of the components of strain and stresses [5];
4. calculation of the J-integral.

Since research on stages 1, 2 and 3 has published in the literature and describes the algorithms in sufficient detail, the results devoted only to aspects of the calculation of the J-integral are given below.

2.1. Calculation of the J-integral [5]

For known values of displacements, strains, and stresses, the J-integral [9] can be calculated by the well-known formula [3]:

$$J = \int_{\Gamma} \left(w\, dy - T \frac{\partial u}{\partial x} ds \right), \tag{1}$$

where w – strain energy density:

$$w = \int_0^{\varepsilon} \sigma\, d\varepsilon,$$

where D – plane stress stiffness matrix, E – elastic modulus, ν – Poisson's ratio. Thus, when the fields of the stress and strain tensor are known the J-integral can be estimated from the equation (1) as follows [10]:

$$J = \int_{\Gamma} \frac{1}{2}\left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + 2\tau_{xy}\gamma_{xy}\right)dy - \left[\sigma_{xx}n_x + \tau_{xy}n_y, \tau_{xy}n_x + \sigma_{yy}n_y\right] \cdot \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix} ds, \tag{2}$$

Γ – a contour covering the crack tip and traversing counter-clockwise; n_x, n_y – the components of the vector perpendicular to Γ; u, v – components of the displacement vector.

The contour used to calculate the J-integral should not only cover the crack tip but also lie outside the plastic zone. The shape of the plastic region was determined using a two-dimensional Mises which criterion for plane stress conditions as follows:

$$\sigma_{Mises} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau^2},$$

where σ_{Mesis} – stress at a certain point on the surface according to Mises criterion, σ_x – normal stress in the x-direction, σ_y – normal stress in the y-direction, τ – shear stress (σ_{xy}). When the Mises criterion is higher than the yield stress (σ_{yield}), it is assumed that the plastic deformation occurs:

$$\sigma_{Mises} \geq \sigma_{yield}$$

Figure 1 (c) shows the visualization of the plastic zone (according to the Mises) with excluded areas of the grips and the notch.

3. Algorithm testing and results

To test the proposed method for calculating the J-integral the obtained results were compared with the data from Table 1 calculated on instrumental measurements according to the standard [11]. The optical method and the J-integral calculation method according to the standard were applied to the results of uniaxial tension of a compact specimen (10 mm thick with a 21.8 mm long fatigue crack, figure 1, a) from AA2024 aluminum alloy (figure 1, b) (the elastic modulus of the alloy is 71 000 MPa, Poisson's ratio 0.33, yield strength 286.7 MPa).

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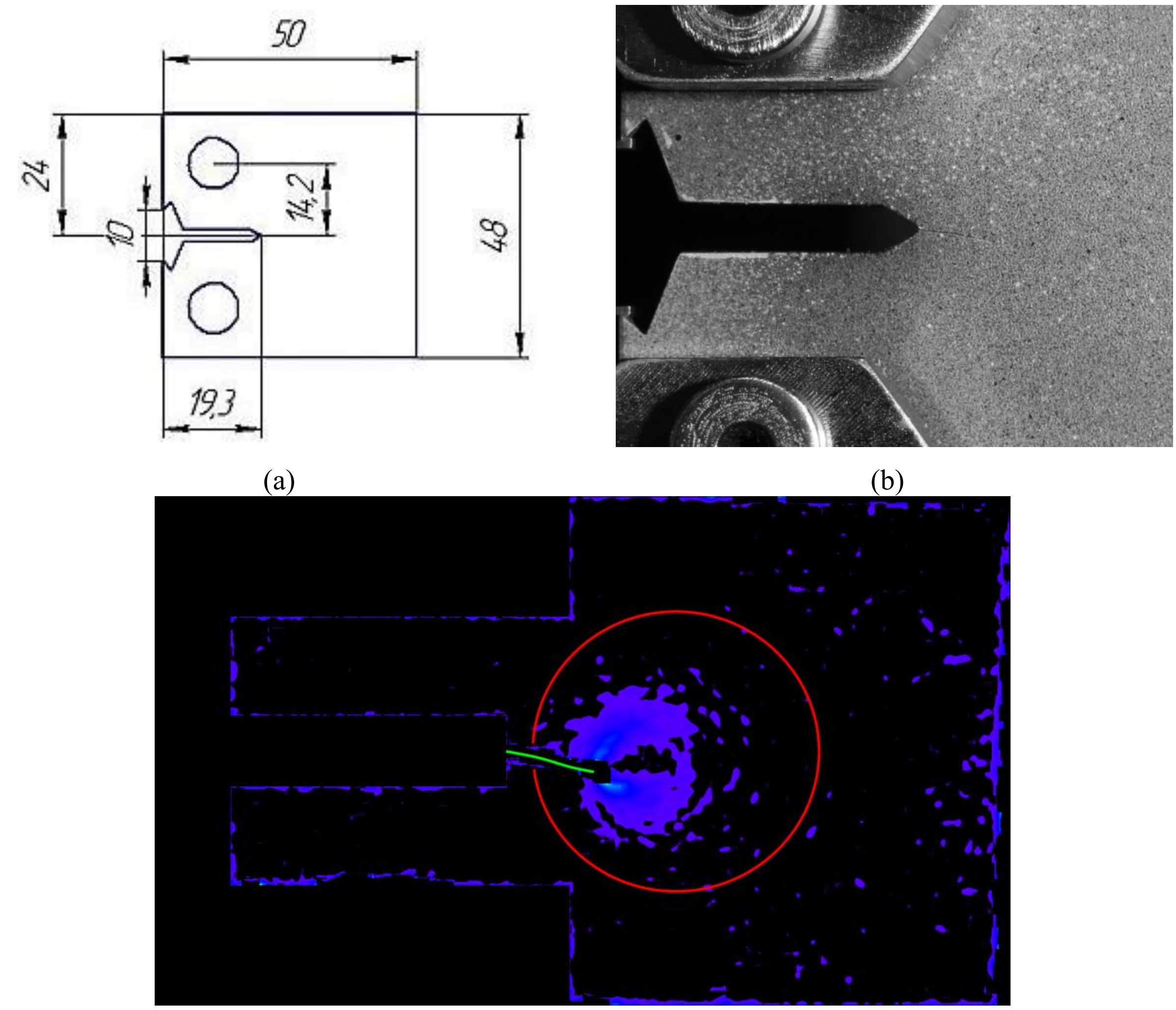
3.1. Estimation of the J-integral deviation

To obtain a quantitative assessment of the accuracy and noise immunity of the J-integral calculation, it is proposed to use the following value:

$$D = \left|\frac{J_{ASTM} - J_{DIC}}{J_{ASTM_{max}}}\right| * 100\%,$$

where J_{ASTM} –J- integral obtained according to ASTM standard; J_{DIC} – the value of the J-integral determined using DIC, $J_{ASTM_{max}}$ – the maximum value of the J-integral obtained according to [11].



(a) (b)

(c)

Figure 1. (a) The dimensions of a compact specimen with a thickness of 10 mm and fatigue crack of 21.8 mm long; (b) The original photo of the specimen with the speckle pattern on the surface for DIC evaluation; (c) An appearance of the plastic zone (according to the Mises) near the crack tip. The red line denotes integration contour. Crack wake is shown by the green line.

According to the standard, the values of the J-integral were calculated (Table 1, figure 2). The deviation between the J-integral value calculated using the DIC method relative to that determined one found using the COD gauge vary from 0.695 % to 4.088 %. Moreover, the average error for the analyzed range of load is 1.753 %.

Table 1. Values of the J-integral and their deviations from the instrumentally measured ones.

Value	Load, kN										
	1.102	1.906	2.704	3.649	4.304	5.1	5.9	6.701	7.499	8.301	9.096
J_{ASTM}, N/m	208	624	1256	2303	3184	4471	5983	7675	9815	12736	17185
J_{DIC}, N/m	328	953	1500	2214	2844	3865	5281	7205	9742	13032	17139
D, %	0.695	1.911	1.418	0.519	1.975	3.526	4.088	2.735	0.421	1.719	0.268

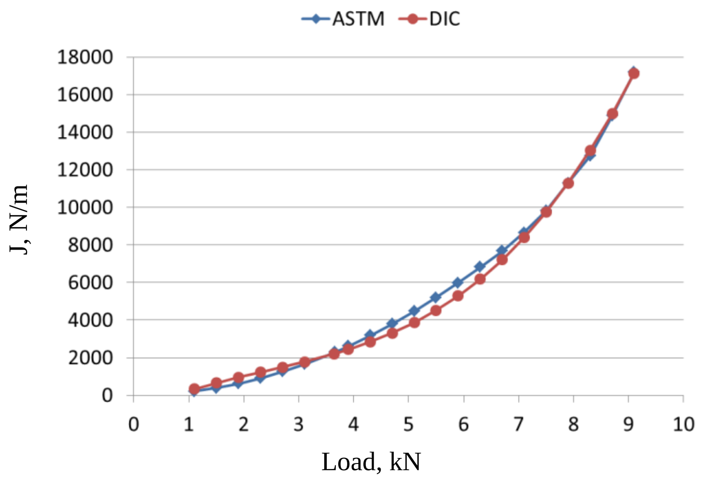


Figure 2. The change in the value of the J-integral with increasing applied load: (1) calculated according to ASTM standard, and (2) according to the proposed method.

4. Conclusion

Application of the J-integral measurement algorithm using the DIC method which consists in constructing a field of displacement vectors, calculating deformations and stresses, and then calculating the value of the specified parameter of fracture mechanics has been considered. As a result of testing the algorithm it was found that the deviation of the calculated values of the J-integral from the values instrumentally measured using the COD gauge (according to ASTM) averages 1.753 % at a minimum and maximum deviation 0.695 % and 4.088%, respectively. Thus, this approach can be used in cases when the standard technique for measuring the J-integral cannot be applied, or the integral nature of the measurements does not allow to achieve the required accuracy for determining the loaded material in local areas.

Acknowledgement

The work was performed in the framework of fundamental research Program of the Russian State Academies of Sciences for 2013–2020, line of research III.23. RF President Council Grants i) for the support of young researchers No. MK-6762.2018.9, ii) leading research schools No. NSh-5875.2018.8 is acknowledged. Authors also appreciate partial support of RFBR Grants No. 18-47-700006.

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