$1/N$ corrections to F_1 and F_2 structure functions of vector mesons from holography

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The structure functions F_1 and F_2 of the hadronic tensor of vector mesons are obtained at order $1/N$ and strong coupling using the gauge/gravity duality. We find that the large N limit and the high energy one do not commute. Thus, by considering the high energy limit first, our results of the first moments of F_1 for the rho meson agree well with those from lattice QCD, with an important improvement of the accuracy with respect to the holographic dual calculation in the planar limit.

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I. INTRODUCTION

The idea of the present work is to investigate the leading $1/N$ corrections to the structure functions F_1 and F_2 of the hadronic tensor of unpolarized vector mesons at strong 't Hooft coupling λ, using the gauge/gravity duality. For this purpose we consider vector mesons from the D3D7-brane system in type IIB string theory [1].

We are interested in the electromagnetic deep inelastic scattering (DIS) of a charged lepton from a vector meson. The DIS cross section is given by the contraction of a leptonic tensor, $l^{\mu\nu}$, with a hadronic one, $W_{\mu\nu}$. The process involves an incoming charged lepton interacting with a hadron with momentum P through the exchange of a virtual photon with momentum q, with the condition $q^2 \gg -P^2$. We consider the definitions given in [2], however we use the mostly plus signature. Thus, the DIS differential cross section is given by

$$\frac{d^2\sigma}{dxdyd\phi} = \frac{e^4}{16\pi^2 q^4} y l^{\mu\nu} W_{\mu\nu}, \tag{1}$$

where y is the lepton fractional energy loss and e denotes the electron charge. The hadronic tensor depends on the hadron structure, where there are important contributions from soft QCD processes. For this reason the gauge/string theory duality becomes a suitable tool for the calculation of this tensor, and therefore the structure functions.

In this work we focus on the structure functions associated with unpolarized vector mesons.[1] The corresponding hadronic tensor has the form

$$W_{\mu\nu} = F_1(x, q^2)\eta_{\mu\nu} - \frac{F_2(x, q^2)}{P\cdot q} P_\mu P_\nu. \tag{2}$$

DIS is related to the forward Compton scattering (FCS) through the optical theorem, which is an special case of the Cutkosky rules, based on the fact that the S-matrix is unitary. It relates the imaginary part of the FCS amplitude to the DIS amplitude. Then, the tensor $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = i\langle P, \mathcal{Q}|\hat{T}(\tilde{J}_\mu(q)J_\nu(0))|P, \mathcal{Q}\rangle, \tag{3}$$

where J_μ and J_ν are the electromagnetic current operators, $\mathcal{Q}$ denotes the charge of the hadron, while $\hat{T}$ represents time-ordered product. Tildes indicate the Fourier transform. In terms of the optical theorem we can write

$$\mathrm{Im}\tilde{F}_j = 2\pi F_j, \tag{4}$$

being $\tilde{F}_j$ the jth structure function of the $T_{\mu\nu}$ tensor, while F_j is the one corresponding to the $W_{\mu\nu}$ tensor.

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[1]The polarized structure functions $b_{1,2,3,4}$ and $g_{1,2}$ [2] have been obtained at strong coupling and in the planar limit in [3,4] from supergravity and in [5] from superstring theory scattering amplitudes. The latter gives the relevant behavior for small values of the Bjorken parameter x.

At this point it is convenient to define the Bjorken parameter $x = -q^2/(2P \cdot q)$ for $q^2 > 0$, being its physical kinematic range $0 \leq x \leq 1$. On the other hand, in the unphysical region for $1 \ll x$ the product of the two electromagnetic currents in the hadron can be written as an operator product expansion (OPE), in terms of operators $\mathcal{O}_{n,k}$ multiplied by powers of $(\Lambda^2/q^2)^{\gamma_{n,k}/2}$, where n is the spin of $\mathcal{O}_{n,k}$, while $\delta_{n,k}$, $\gamma_{n,k}$, and $\Delta_{n,k} = \delta_{n,k} + \gamma_{n,k}$, represent the engineering, the anomalous and the total scaling dimensions of the operator, respectively [6]. Then, we can define the twist of each operator as $\tau_{n,k} = \Delta_{n,k} - n$. The relation with the physical parametric region $0 \leq x \leq 1$ is given by a contour argument, which allows to connect the OPE with the moments of the structure functions in the DIS process. Thus, the n-moment of the jth structure function can be expressed as the sum of three contributions[2]

$$M_n^j(q^2) \approx \frac{1}{4}\sum_k C^j_{n,k} A_{n,k}\left(\frac{\Lambda^2}{q^2}\right)^{\frac{1}{2}\tau_{n,k}-1} + \frac{1}{4}\sum_{\mathcal{Q}_p=\mathcal{Q}} C^j_{n,p} A_{n,p}\left(\frac{\Lambda^2}{q^2}\right)^{\tau_p-1} + \frac{1}{4}\frac{1}{N}\sum_{\mathcal{Q}_p\neq\mathcal{Q}} C^j_{n,p} a_{n,p}\left(\frac{\Lambda^2}{q^2}\right)^{\tau_p-1}, \qquad (5)$$

where the coefficients $C^j_{n,p}$ are dimensionless, while $A_{n,p}$ and $a_{n,p}$ depend on the matrix elements of the operators $\langle P, \mathcal{Q}|\mathcal{O}_{n,k}|P, \mathcal{Q}\rangle$ for a hadron state with four-momentum P and charge $\mathcal{Q}$.

Let us briefly explain how different contributions behave in Eq. (5). We can study this equation for the photon virtuality q to be large, intermediate or small, in comparison with the confinement scale Λ. At weak coupling the Feynman's parton model gives a suitable description of hadrons, thus the leading contribution comes from the first term. This contribution only sums over terms associated with operators with the lowest twist $\tau_{n,k} \approx 2$ at large q^2. In this case perturbative methods of quantum field theory are suitable. On the other hand, the second sum dominates at strong coupling and in the planar limit, i.e., $1 \ll \lambda \ll N$. In this case protected double-trace operators constructed from the protected single-trace ones have the smaller twist at strong coupling. Therefore, the calculations can be done by using the gauge/gravity duality, considering a forward Compton scattering with the exchange of a single on-shell particle between incoming and outgoing states. Within this regime exchange of more than one intermediate state is suppressed by $1/N$ powers. The third sum in Eq. (5) becomes the leading one when $q^2 \geq \Lambda^2 N^{1/(\tau_{\mathcal{Q}}-\tau_c)}$. In this case $\tau_{\mathcal{Q}}$ is the minimum twist of protected operators with charge $\mathcal{Q}$, while τ_c is the minimum twist of all electrically charged protected operators. The $1/N$ suppression of the third sum is expected for mesons, while for glueballs there is a $1/N^2$ suppression.

In addition, we should comment on the different parametric regions in terms of x and the 't Hooft coupling. For $1/\sqrt{\lambda} \ll x \ll 1$ only supergravity states contribute since in this region the ten-dimensional s-channel Mandelstam variable satisfies $\tilde{s} \ll 1/\alpha'$, where α' is the string constant. When $\exp(-\sqrt{\lambda}) \ll x \ll 1/\sqrt{\lambda}$ excited strings are produced and their dynamics becomes important. The holographic dual calculation is derived from four-point superstring theory scattering amplitudes. Finally, for the exponentially small region the size of the excited string becomes comparable with the AdS radius. In this case dual Pomeron techniques are useful [7–17]. In previous papers we have calculated F_1 and F_2 by considering the FCS process with the exchange of a single intermediate state for scalar and vector mesons [3–5,18]. Then, we have also calculated these functions by considering the exchange of two intermediate states for glueballs [19] and for scalar mesons [20] in the D3D7-brane system of Ref. [1]. In both cases we found that the large N limit does not commute with the high energy one. By considering the high energy limit first, which corresponds to the physical situation, in the case of the pion we have obtained the first moments of the structure function F_2 and compared them with lattice QCD calculations [21–23], obtaining a substantial improvement of the accuracy, namely: from 10.8% for a single intermediate state [18] to 1.27% in the case of two intermediate states [20]. Then, a natural question is whether or not this effect also occurs in the case of vector mesons. The present work answers it positively as we shall explain in detail in the next sections.

For finite values of N we can expand the structure functions of mesons as follows

$$F_j = f_j^{(0)}\left(\frac{\Lambda^2}{q^2}\right)^{\tau_{\rm in}-1} + \frac{1}{N} f_j^{(1)}\left(\frac{\Lambda^2}{q^2}\right) + \frac{1}{N^2} f_j^{(2)}\left(\frac{\Lambda^2}{q^2}\right) + \cdots \qquad (6)$$

where $\tau_{\rm in}$ is the twist of the incident dual vector meson state in type IIB supergravity, $f_j^{(n)}$'s indicate the structure functions at order in $1/N^n$, with $j = 1, 2$ and $n = 1, \ldots$. Notice that in the definitions of $f_j^{(n)}$'s powers of Λ^2/q^2 have been factored out. The index n indicates the number of exchanged intermediate on-shell states of the FCS Feynman diagram. This corresponds to the number of hadrons in the final state of the DIS process. From expression (6) one can easily see that the high energy $(q^2 \gg \Lambda^2)$ and the large N limits do not commute. Moreover, by taking first the high energy limit, since the

[2] We use the notation of [6], in particular Eq. (5) is similar to Eq. (27) of that reference but for mesons, i.e., the second term has a factor $1/N$.

power of Λ^2/q^2 in the first term is larger than for the rest, it vanishes, and then in the $1/N$ expansion the second term dominates. We would like to emphasize that $1/N$ corrections to the F_1 and F_2 structure functions for scalar mesons were studied in [20], but not for vector mesons. Therefore, we consider this calculation to be important for the investigation of such limits beyond scalar hadrons, since there are also lattice QCD results of the first moments of F_1 for the rho meson to compare with [21].

The work is organized as follows. In Sec. II. we discuss generalities of DIS in the context of the D3D7-brane system at large N. In Sec. III we consider the $1/N$ expansion and obtain the relevant Feynman-Witten diagram in the bulk theory. Also in this section we develop the calculation of the F_1 and F_2 structure functions for vector mesons. In Sec. IV we carry out the analysis of our results. We focus on the calculation of the first moments of the structure function F_1 of the rho meson and compare them with the available results from lattice QCD.

II. DIS IN THE D3D7-BRANE SYSTEM

DIS processes of charged leptons from scalar and vector mesons in the D3D7-brane system have been studied in several papers [3–5], by considering the large N limit, which means that the final state has only a single hadron. A more realistic calculation for vector mesons must include $1/N$ corrections. This corresponds to final multiparticle states. In this work we consider $1/N$ corrections of DIS of charged leptons from unpolarized vector mesons.

Firstly, we give a brief description of the D3D7-brane system. Let us consider N coincident D3-branes in type IIB superstring theory. The corresponding near-horizon geometry is the $\mathrm{AdS}_5 \times S^5$ spacetime, with the metric

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}d\vec{Z}\cdot d\vec{Z}, \tag{7}$$

where $\vec{Z}$ are coordinates of the directions perpendicular to the D3-branes, being the radial coordinate $r = |\vec{Z}|$. The radius of AdS_5 is $R = (4\pi g_s N\alpha'^2)^{1/4}$, where g_s is the string coupling. Now, one can add a D7-brane in the probe approximation, at a distance $L = |\vec{Z}|$ in the (8,9) plane. The induced metric on the D7-brane is given by

$$ds^2 = \frac{\rho^2+L^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2+L^2}d\rho^2 + \frac{R^2\rho^2}{\rho^2+L^2}d\Omega_3^2, \tag{8}$$

where $\rho^2 = r^2 - L^2$ and the angles contained in Ω_3 span a three-sphere. For $L = 0$ Eq. (8) gives the $\mathrm{AdS}_5 \times S^3$ metric, otherwise the metric is only asymptotically $\mathrm{AdS}_5 \times S^3$. This is the situation where the conformal symmetry is preserved.

TABLE I. Some features of D7-brane fluctuations on the $\mathrm{AdS}_5 \times S^3$ background relevant to this work. The integer l indicates the $SO(4) \sim SU(2) \times SU(2)$ irreducible representation (irrep) and it defines the corresponding Kaluza-Klein mass. The relation between the scaling dimension of the associated operator Δ and l is also presented.

Field	Type of field in $5D$	Built from	$\Delta(l)$	$SU(2)\times SU(2)$ irrep
ϕ, χ	Scalars	ϕ, χ	$l+3$, $l\geq 0$	$(\frac{l}{2},\frac{l}{2})$
B_μ	Vector	B_μ^{II}	$l+3$, $l\geq 0$	$(\frac{l}{2},\frac{l}{2})$
ϕ_I^-	Scalar	B_i^I	$l+1$, $l\geq 1$	$(\frac{l+1}{2},\frac{l-1}{2})$
ϕ_I^+	Scalar	B_i^I	$l+5$, $l\geq 1$	$(\frac{l-1}{2},\frac{l+1}{2})$
ϕ_{III}	Scalar	$B_{i,z}^{III}$	$l+3$, $l\geq 1$	$(\frac{l}{2},\frac{l}{2})$

For $L > 0$ the 3-7 quarks become massive, and meson type excitations are energetically favored. Mesons correspond to excitations of open strings ending on the D7-brane. The dynamics of these fluctuations is described by the action

$$S_{D7} = -\mu_7\int d^8\xi\sqrt{-\det\left(P[g]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7\int P[C^{(4)}]\wedge F\wedge F, \tag{9}$$

where $\mu_7 = [(2\pi)^7 g_s\alpha'^4]^{-1}$ is the D7-brane tension, ξ^a denotes the D7-brane coordinates, g_{ab} stands for the metric (8), and P is the pullback of the background fields on the D7-brane. The second term is the Wess-Zumino term.

It is possible to induce excitations in the transverse directions to the D7-brane. These are two types of scalar excitations ϕ and χ, related to the Z^5 and Z^6 coordinates, respectively. On the other hand, it is also possible to perturb the gauge fields $F_{ab} = \partial_a B_b - \partial_b B_a$ on the Dirac-Born-Infeld (DBI) action. In this case, there are three types of solutions for the B_a modes, related to the expansion of the solutions in scalar or vector spherical harmonics on S^3. The three classes of solutions are [1]

$$\text{type I:}\; B_\mu = 0, \qquad B_\rho = 0, \qquad B_i = \phi_I^{\pm}(\rho)e^{ik\cdot x}Y_i^{l\pm}(\Omega), \tag{10}$$

$$\text{type II:}\; B_\mu = \epsilon_\mu\phi_{II}(\rho)e^{ik\cdot x}Y^l(\Omega), \qquad k\cdot\epsilon = 0, \qquad B_\rho = 0, \qquad B_i = 0, \tag{11}$$

$$\text{type III:}\; B_\mu = 0, \qquad B_\rho = \phi_{III}(\rho)e^{ik\cdot x}Y^l(\Omega), \qquad B_i = \tilde{\phi}_{III}(\rho)e^{ik\cdot x}\nabla_i Y^l(\Omega). \tag{12}$$

$Y^l(\Omega)$ and $Y_i^l(\Omega)$ are scalar and vector spherical harmonics, respectively. Some of their properties are described in the appendix and in references therein. Note that in this case type I and III are scalar modes, while type II modes represent vector fields from the (asymptotically) AdS perspective. The different modes of the scalar and vector perturbations are shown in Table I, together with their relevant quantum numbers.

Beyond the quadratic order, the interaction Lagrangian for these modes has been derived in reference [20].

Up to this point we have described the D3D7-brane system presented in [1], where the solutions were computed in terms of hypergeometric functions. In the context of DIS from mesons, one identifies the parameter that controls the separation between the D7 and the D3-branes in the (Z^5, Z^6) plane with the IR scale Λ introduced as a cutoff in the radial direction to induce confinement [6]. Thus, we take $L \sim \Lambda R^2$. Therefore, the relevant interactions take place at values of ρ considerably larger than L, and in this region the solutions are well approximated by the typical AdS_5 expressions in terms of Bessel functions, which we write in Sec. III. The AdS masses can only take discrete values. The presence of a small but nonzero value of L is important for the vertices we will need to consider.

A. One-particle exchange: The $N \to \infty$ limit

For unpolarized vector mesons we shall study only the contributions to the F_i structure functions. These functions can be written in terms of $W^{\mu\nu}$ and the vector $v^\mu = \frac{1}{q}(P^\mu + \frac{q^\mu}{2x})$ as

$$F_1(x, q^2) = \frac{1}{2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) W_{\mu\nu} + 2x^2 v^\mu v^\nu W_{\mu\nu}, \quad (13)$$

$$F_2(x, q^2) = x\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) W_{\mu\nu} + 12x^3 v^\mu v^\nu W_{\mu\nu}. \quad (14)$$

The FCS amplitude can be derived by using the gauge/string theory duality, by studying a four-point interaction with vector modes on the D7-brane and graviphotons related to current insertion on the boundary as external states. This gauge field arises from a particular decomposition of the graviton mode in ten dimensions:

$$\delta g_{mj} = A_m(\rho, x) v_j(\Omega_3), \quad (15)$$

where v_j are the Killing vectors on S^3, and $m = (\mu, \rho)$.

The structure functions have been calculated in this context by considering a single intermediate hadron state in [3,4], obtaining the following results at leading order

$$F_1(x, q^2) = A(x)\frac{1}{12x^3}(1-x), \quad (16)$$

$$F_2(x, q^2) = A(x)\frac{1}{6x^3}(1-x), \quad (17)$$

with

$$A(x) = A_0 \mathcal{Q}^2 \left(\frac{\mu_7^2(\alpha')^4}{\Lambda^8}\right)\left(\frac{\Lambda^2}{q^2}\right)^{l+1} x^{l+6}(1-x)^l, \quad (18)$$

and $A_0 = |c_i|^2 |c_X|^2 2^{6+2l}[\Gamma(3+l)]^2 \pi^5$ is a dimensionless constant. Also, c_i and c_X are the normalization constants of the incident and intermediate dual hadron states, respectively, while $\mathcal{Q}$ is the charge of the hadron, associated to a $U(1)$ subgroup of the S^3 isometries.

These results are valid for DIS from mesons considered in the context of the D3D7-brane system. However, it is important to keep in mind that [3,4] showed that completely analogous formulas hold in the context of different Dp-brane models, such as the D4D8$\overline{\mathrm{D8}}$-brane system [24] and the D4D6$\overline{\mathrm{D6}}$-brane system [25], both in type IIA superstring theory. These models are very different to each other, and each of them shares certain phenomenological features with large-N QCD. In consequence, it is reasonable to expect that the qualitative form of the structure functions we just described is universal, in the sense that it would hold in any holographic Dp-brane model for mesons at strong coupling and in the planar limit. Although in the present work we focus on the D3D7-model, we expect this universality to hold also for the leading one-loop correction, at least at the qualitative level.

III. DIS IN THE $1/N$ EXPANSION

The nonplanar $1/N$ corrections to F_1 and F_2 structure functions for scalar mesons were studied in [20], and also for the $\mathcal{N} = 4$ SYM theory glueball in Ref. [19]. From the point of view of DIS, they correspond to processes with multiparticle final states. The first nontrivial contribution comes from considering a two-hadron final state in the hadronic tensor $W_2^{\mu\nu}$, which can be related to a FCS process with two intermediate on-shell states, denoted as $T_2^{\mu\nu}$. Writing this tensor in terms of the $U(1)$ conserved current J^μ we obtain [19]

$$\begin{aligned}
\mathrm{Im}(T_2^{\mu\nu}) &= \pi \sum_{X_1, X_2} \langle P, \mathcal{Q}|\tilde{J}^\mu(q)|X_1, X_2\rangle\langle X_1, X_2|J^\nu(0)|P, \mathcal{Q}\rangle \\
&= \pi \sum_{M_2, M_3} \int \frac{d^3q'}{2E_{q'}(2\pi)^3}\frac{d^3p'}{2E_{p'}(2\pi)^3}\langle P, \mathcal{Q}|\tilde{J}^\mu(q)|X_1, X_2\rangle\langle X_1, X_2|J^\nu(0)|P, \mathcal{Q}\rangle \\
&= 4\pi^3 \sum_{M_2, M_3} \int \frac{d^4q'}{(2\pi)^4}\delta(M_2^2 - q'^2)\delta(M_3^2 - (P+q-q')^2) \\
&\quad \times \langle P, \mathcal{Q}|J^\mu(0)|X_1, X_2\rangle\langle X_1, X_2|J^\nu(0)|P, \mathcal{Q}\rangle, \quad (19)
\end{aligned}$$

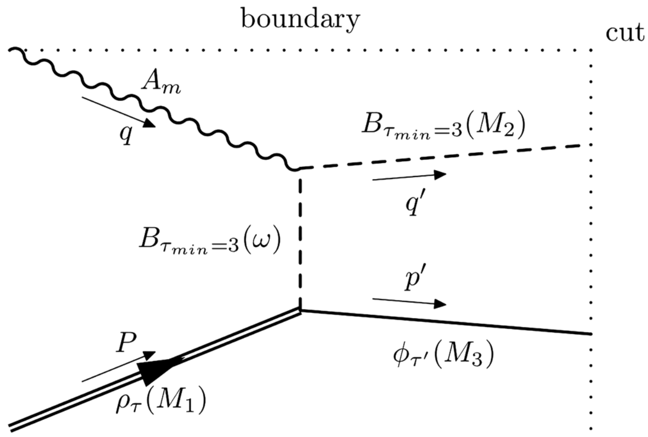

FIG. 1. Feynman-Witten diagram associated to the DIS process with two intermediate hadron states. The momentum and the twist of each field are indicated, while the solutions are given in section II. The incident hadron is represented by a double line, the intermediate spin-one modes are indicated by dashed lines, while the solid line denotes a scalar mode. The vertical dotted line represents the cut of the Cutkosky rule.

where X_1 and X_2 are the intermediate states with momenta p' and q' respectively, as shown in Fig. 1. The current J^μ matrix element is related to its Fourier transform as

$$\langle P, Q|\tilde{J}^\mu(q)|X_1, X_2\rangle = (2\pi)^4\delta^{(4)}(P + q - p' - q') \times \langle P, Q|J^\mu(0)|X_1, X_2\rangle. \quad (20)$$

Using the AdS/CFT duality, this current can be related to an specific field in the bulk theory. In [19], considering $1/N$ corrections to DIS from a scalar meson we have shown that the leading contribution to the DIS process with two hadron final states is given by a specific Feynman diagram where one of these two outgoing hadrons has the lowest twist. In the next subsection we will explain the amplitude we need to calculate in the case of a spin-1 meson.

A. Leading diagram for vector mesons at order $1/N$

We want to study the $1/N$ corrections to the DIS process from a vector meson (for instance a rho meson), associated to a type II vector mode B^{II}_μ on the D7-brane. Based on the results of [20], the leading diagram is the s-channel one, where the exchanged particle is the one with the lowest twist, $\tau = \Delta - n$. This can be done by looking at Table I, which gives the relevant quantum numbers of the different solutions.

Since the lowest τ is associated to the lowest Δ, the exchanged field should be the ϕ^-_I mode with $\tau = \Delta = 2$. This is what has been done in [20]. However, the interaction term between B^{II}_μ and ϕ^-_I modes given in [20]

$$L_3 = -\mu_7(2\pi\alpha')^3\sqrt{-g}\frac{L}{\rho^2 + L^2}\phi(F_{aJ}F^{aJ} - F_{a\mu}F^{a\mu}), \quad (21)$$

vanishes, due to the nature of the field solutions. It can be seen from Eqs. (10) and (11), that a type I scalar has only angular components, while type II vectors have only μ components. Therefore, the interaction between these two modes vanishes. The vector mode with $\tau = 2$ does not contribute to the DIS process either. This can be easily seen by analyzing the charge of this vector associated with the 3-sphere. For $\tau = 2$ we need to consider $\Delta = 3$, but this implies that $l = 0$, meaning that the vector mode has no charge over the 3-sphere. Thus, there is no interaction with the A_m photon. Another possible interaction could arise from the Wess-Zumino term in the low-energy action of the D7-brane. This is because the gauge field is actually a particular linear combination of the ten-dimensional graviton and RR 4-form perturbations. This is described in detail in [16,17], where this type of vertex has been used to study the antisymmetric contributions to the hadronic tensor for glueballs and spin-1/2 hadrons. However, it can be seen that the relevant angular integrals vanish in the present case.

The next step is to consider the exchange of $\tau = 3$ modes. There are two possibilities:

(1) ϕ, χ scalars, with $\Delta = 3$.
(2) B_μ vector, with $\Delta = 4$.

In the former case the perturbations have $l = 0$, thus they are not charged with respect to the $U(1)$ we are considering. Therefore, the only possibility is the exchange of a type II vector mode with $l = 1$.

In the IR region, the relevant interaction includes two type II modes (one associated to the $\tau = 3$ mode we just discussed and the other one with the incoming vector meson) and a scalar field, and from (21) we see that it is described by [1,20]

$$S_{\rm IR} \approx \mu_7(2\pi\alpha')^3\frac{L}{R^4}\int d^8\xi\sqrt{g}z^2\phi F_{\mu\nu}F^{\mu\nu}. \quad (22)$$

Note that we only keep the first term of the $L \ll \rho$ expansion, where $\rho \simeq R^2/z$, being z the Poincaré radial coordinate of AdS_5.

For the UV vertex we have to consider the interaction between the A_m gauge field and two B_μ modes. The standard interaction is [4]

$$S_{\rm UV} = -\mu_7(\pi\alpha')^2 iQ\int d^5x\sqrt{g}A_\mu(B^*_\nu F^{\mu\nu} - B_\nu(F^{\mu\nu})^*), \quad (23)$$

where we have already integrated over the S^3, being Q the charge of the vector mode, given by the eigenvalue equation $v^i\partial_i B_\mu = iQB_\mu$. Note that Q does not necessarily have to coincide with $\mathcal{Q}$, the charge of the original hadron. In fact, one has to sum over all possible intermediate states, and in particular all possible values of Q (see Appendix).

The diagram we need to calculate is shown in Fig. 1, where there is an incoming photon with momentum q_μ, which interacts with a massive B_μ vector with momentum $q'_\mu(q'^2 = -M_2^2)$ and $\tau_{\rm min} = 3$. This vector interacts with an incident rho meson of momentum P_μ, and with an outgoing scalar field with momentum $p'_m(p'^2 = -M_3^2)$ and conformal dimension τ'.

In order to calculate the diagram we need the AdS$_5$ solutions of the fields. In the axial gauge the graviphoton solution is

$$A_\mu(x,z) = n_\mu e^{iq\cdot x}(qz)K_1(qz), \qquad A_z(x,z) = 0, \tag{24}$$

while the fields on the probe D7-brane are given at small L by the following approximate expressions [4]

$$B^*_\mu(x,z) = \frac{1}{\sqrt{N}} c^*_B \zeta_\mu e^{-iq'\cdot x}\sqrt{\frac{\Lambda}{M_2}} z J_2(M_2 z),$$
$$\rho_\mu(x',z') = \frac{1}{\sqrt{N}} c_\rho \epsilon_\mu e^{iP\cdot x'}\sqrt{\frac{\Lambda}{M_1}} z J_{\tau-1}(M_1 z'),$$
$$\phi^*(x',z') = \frac{1}{\sqrt{N}} c^*_\phi e^{-ip'\cdot x'}\sqrt{\Lambda M_3} z J_{\tau'-1}(M_3 z'), \tag{25}$$

where c's are numerical constants, and we have also included the polarization vectors. We have only written the AdS$_5$ solution, the full ten-dimensional solution includes the 3-sphere contribution that only have the product of the scalar spherical harmonics $Y^l(\Omega_3)$. On the other hand, the propagator of the type II vector mode is given by [26]

$$\begin{aligned} G_{\mu\nu}(x,x',z,z') &= -\frac{i}{N}\int \frac{d^4k}{(2\pi)^4} G^{(k)}(z,z')\mathcal{T}_{\mu\nu} e^{ik\cdot(x-x')} \\ &= -\frac{i}{N}\int \frac{d^4k}{(2\pi)^4}\frac{d\omega^2}{2}\frac{(zz')J_{\tau-1}(\omega z)J_{\tau-1}(\omega z')}{k^2+\omega^2-i\epsilon} \\ &\quad \times \mathcal{T}_{\mu\nu} e^{ik\cdot(x-x')}, \end{aligned} \tag{26}$$

where $\mathcal{T}_{\mu\nu} = \eta_{\mu\nu} + \frac{k_\mu k_\nu}{\omega^2}$. Recall that in our case of interest the exchanged mode has conformal dimension $\tau_{\min} = 3$.

Finally, we redefine the D7-brane fields as $\Phi \to \frac{\Phi}{\sqrt{N}}$, such that they are canonically normalized in terms of N. The $1/N$-power counting shows that the interaction terms now scale as

$$S_{\mathrm{UV}} \to \frac{1}{N} S_{\mathrm{UV}}, \qquad S_{\mathrm{IR}} \to \frac{1}{N^{3/2}} S_{\mathrm{IR}}. \tag{27}$$

B. Structure functions for vector mesons

We now derive the FCS amplitude related to the one-point function $n_\mu\langle X_1, X_2|J^\mu(0)|P,\mathcal{Q}\rangle$. Looking at the diagram of Fig. 1, the associated amplitude is

$$\begin{aligned} \mathcal{A} = &-8i\mu_7^2(\pi\alpha')^5 Q\frac{L}{R^4}(2\pi)^4\delta^{(4)}(P+q-p'-q')\mathcal{I} \\ &\times \int dzdz'\sqrt{g}\sqrt{g'}z'^2\partial'^{[\alpha}\rho^{\beta]}(z')\phi^*(z') \\ &\times [B^{*\nu}(z)(\partial'_\alpha\partial_\mu G_{\nu\beta}(z,z') - \partial'_\alpha\partial_\nu G_{\mu\beta}(z,z')) \\ &- \partial'_\alpha G_{\nu\beta}(z,z')(\partial_\mu B^{*\nu}(z) - \partial^\nu B^*_\mu(z))]A^\mu(z), \end{aligned} \tag{28}$$

where ∂'_α and ∂'_β are derivatives with respect to the primed coordinates. Also, we have already performed the integrals in the variables x and x', which lead to the momentum conservation relations

$$k_\mu = q'_\mu - q_\mu = P_\mu - p'_\mu. \tag{29}$$

In Eq. (28) $\mathcal{I}$ represents the integral of the scalar spherical harmonics over the 3-sphere, which is given in the Appendix. Replacing the solutions (24), (25) and (26) in the amplitude, and using the relations (13), (14) and (19), the structure functions F_i ($i = 1, 2, L$) can be written as

$$\begin{aligned} F_i(q^2,x) = &|\tilde{c}|^2 Q^2\mu_7^4 L^2\alpha'^{10}R^8 \\ &\times \sum_{M_2,M_3}\int \frac{d^3q'}{2E_{q'}(2\pi)^3}\frac{d^3p'}{2E_{p'}(2\pi)^3}\delta^{(4)} \\ &\times (P+q-q'-p') \\ &\times \Lambda^3\frac{M_3}{M_1M_2}q^2|C_t|^2 F_i^T(x,P,q,q'), \end{aligned} \tag{30}$$

where $|\tilde{c}|^2 = |c_\phi|^2|c_B|^2|c_\rho|^2$, C_t contains the radial coordinate integrals, given by

$$\begin{aligned} C_t = &\int dzdz'\int d\omega\frac{\omega}{(q-q')^2+\omega^2} \\ &\times [z'^4 J_{\tau-1}(M_1z')J_{\tau'-1}(M_3z')J_2(\omega z')] \\ &\times [z^2K_1(qz)J_2(M_2z)J_2(\omega z)]. \end{aligned} \tag{31}$$

In order to obtain F_i^T, the factor in (30) which depends only of the tensor contractions, we need to calculate $J^T_\mu J^{T*}_\nu$, with

$$J^T_\mu = [\zeta^{*\nu}(k_\mu\mathcal{T}_{\nu\beta} - k_\nu\mathcal{T}_{\mu\beta}) + \mathcal{T}_{\nu\beta}(q'_\mu\zeta^{*\nu} - q'^\nu\zeta^*_\mu)]k_\alpha P^{[\alpha}\epsilon^{\beta]}. \tag{32}$$

Recall that B^*_μ is an intermediate state, thus we need to sum over the outgoing vector polarizations ζ_μ

$$\sum_\lambda \zeta_\mu\zeta^*_\nu = -q'^2\eta_{\mu\nu} + q'_\mu q'_\nu. \tag{33}$$

Since we are only interested in the unpolarized structure functions, we also average over the polarization vector of the incoming hadron

$$\overline{\epsilon_\mu\epsilon^*_\nu} = \frac{1}{3}(-P^2\eta_{\mu\nu} + P_\mu P_\nu). \tag{34}$$

By comparing $J^T_\mu J^{T*}_\nu$ with Eqs. (13) and (14) we obtain the following expressions

$$F_1^T = \frac{1}{24q^2x^2}P^2[128x^4(q')^2(P\cdot q')^4 + 128x^3(q')^2 q\cdot q'(P\cdot q')^3 + 8q^6 x(q')^2 P\cdot q' \\ +40q^4x^2(q')^2(P\cdot q')^2 - 8q^4x^2(q\cdot q')(P\cdot q')^2 + 32q^4x(q')^4 P\cdot q' \\ -16q^4x(q\cdot q')^2 P\cdot q' + 28q^4x(q')^2(q\cdot q')(P\cdot q') - 16q^2x^4(P\cdot q')^4 \\ +128q^2x^3(q')^2(P\cdot q')^3 - 48q^2x^3(q\cdot q')(P\cdot q')^3 + 32q^2x^2(q')^4(P\cdot q')^2 \\ -40q^2x^2(q\cdot q')^2(P\cdot q')^2 + 128q^2x^2(q')^2(q\cdot q')(P\cdot q')^2 + 2q^8(q')^2 \\ +9q^6(q')^4 - 2q^6(q\cdot q')^2 - 2q^6(q')^2 q\cdot q'] \tag{35}$$

$$F_2^T = \frac{1}{12q^2x}P^2[384x^4(q')^2(P\cdot q')^4 + 384x^3(q')^2(q\cdot q')(P\cdot q')^3 \\ +64x^2(q')^2(q\cdot q')^2(P\cdot q')^2 + 8q^6x(q')^2 P\cdot q' + 96q^4x^2(q')^2(P\cdot q')^2 \\ -8q^4x^2(q\cdot q')(P\cdot q')^2 + 32q^4x(q')^4(P\cdot q') - 16q^4x(q\cdot q')^2(P\cdot q') \\ +76q^4x(q')^2(q\cdot q')(P\cdot q') - 48q^2x^4(P\cdot q')^4 + 384q^2x^3(q')^2(P\cdot q')^3 \\ -144q^2x^3(q\cdot q')(P\cdot q')^3 + 32q^2x^2(q')^4(P\cdot q')^2 - 136q^2x^2(q\cdot q')^2(P\cdot q')^2 \\ +384q^2x^2(q')^2(q\cdot q')(P\cdot q')^2 - 32q^2x(q\cdot q')^3(P\cdot q') + 64q^2x(q')^2(q\cdot q')^2(P\cdot q') \\ +2q^8(q')^2 + 9q^6(q')^4 - 2q^6(q\cdot q')^2 - 2q^6(q')^2(q\cdot q') + 8q^4(q')^2(q\cdot q')^2]. \tag{36}$$

In order to calculate the integrals in (31) we need to use a few reasonable approximations, in a similar way as in [19,20]. The main assumption is that Λ and the masses of the hadrons are small in comparison with the momentum of the virtual photon. The IR integral selects the mass of the exchanged field as follows [19]

$$\int_0^{\Lambda^{-1}} dz' z'^4 J_{\tau-1}(M_1 z') J_{\tau'-1}(M_3 z') J_2(\omega z') \\ \approx \frac{1}{\Lambda^3}\frac{1}{\sqrt{M_1M_3}}[(-1)^\alpha \delta(\omega-(M_1+M_3)) \\ + (-1)^\beta \delta(\omega-(M_1-M_3))], \tag{37}$$

for some integers α and β. The integral leads to $\omega = |M_1 \pm M_3|$. Then, the UV integral can be obtained by expanding $J_2(\omega z) \approx \omega^2 z^2/8$ for $\omega \ll q$, and taking the upper limit as infinity since K_1 decays quickly in the bulk. We obtain

$$\int_0^\infty dz \frac{\omega^2}{8} z^4 K_1(qz) J_2(M_2 z) \approx \frac{6M_2^2 q\omega^2}{(M_2^2+q^2)^4}. \tag{38}$$

With these two equations we can obtain C_t, after noticing that the leading term comes from $\omega = M_1 - M_3$ [20], we obtain

$$|C_t|^2 = \frac{q^2 36 M_2^4}{(M_2^2+q^2)^8}\frac{1}{\Lambda^6}\frac{(M_1-M_3)^6}{M_1M_3}\frac{1}{((q-q')^2+(M_1-M_3)^2)^2}. \tag{39}$$

The next step is to integrate over the on-shell momenta $\vec{p}'$ and $\vec{q}'$, and sum over the corresponding masses.[3] The final results for the structure functions are

$$F_1(x,q^2) = \frac{1}{\lambda N}C\left(\frac{M_1}{\Lambda}\right)^6 \frac{\Lambda^2}{q^2}\frac{1}{2}x^3(1-x)^3 \tag{40}$$

$$F_2(x,q^2) = \frac{1}{\lambda N}C\left(\frac{M_1}{\Lambda}\right)^6 \frac{\Lambda^2}{q^2}x^3(1-x)^3(4-3x) \tag{41}$$

$$F_L(x,q^2) = \frac{1}{\lambda N}C\left(\frac{M_1}{\Lambda}\right)^6 \frac{\Lambda^2}{q^2}4x^3(1-x)^4, \tag{42}$$

where C is a numerical constant.

We expect qualitatively similar results to hold in the context of different Dp-brane models.

IV. DISCUSSION AND CONCLUSIONS

We have obtained the $1/N$ corrections to the F_1, F_2 and F_L structure functions corresponding to vector mesons, using the gauge/gravity duality. Motivated by previous work for $\mathcal{N}=4$ SYM theory glueballs, and particularly for scalar mesons in the D3D7-brane system, the idea is to investigate how two very different limits behave, namely: the large N limit in comparison with the high energy limit ($\Lambda^2/q^2 \to 0$). Our first result is that they do not commute for the vector mesons. Then, since the physical way to

[3] For more details we refer the reader to Ref. [20].

consider these limits implies to take first the high energy one, followed by the large N limit, we find that in this situation the third term in the expansion of Eq. (5) dominates the moments of the structure functions. This is a very interesting result which says that at strong coupling this third term becomes the leading one for $q^2 \geq \Lambda^2 N^{1/(\tau_Q - \tau_c)}$, where τ_Q and τ_c are the minimum twist of protected operators with charge $\mathcal{Q}$ and the minimum twist of all electrically charged protected operators, respectively.

This is similar to what happens for the glueball in the IR deformed version of $\mathcal{N} = 4$ SYM [6,19], where the process is given in terms of closed string modes and at strong coupling one finds that the $1/N$ result dominates for $q^2 \geq \Lambda^2 N^{2/(\tau_Q - \tau_c)}$. However, note that in the present case, i.e., for mesons, the correction is proportional to the $1/N$ instead of $1/N^2$, as expected. Thus, at large N the critical value for the photon virtuality q where this happens is much smaller. The same occurs for the results presented in [20] for the case of scalar mesons.

The physical implication of this result is that, at strong coupling, for the above energy range DIS is dominated by a two-hadron final state. From the viewpoint of the FCS process it corresponds, through the optical theorem, to a situation where there are two intermediate hadron states. The structure functions F_1 and F_2 behave as $(1-x)^3$ as x approaches 1. We have also considered the longitudinal structure function F_L which behaves as $(1-x)^4$ as x approaches 1. We should notice that, although the states as well as the interactions for the vector and scalar mesons are different, all these structure functions have the same dependence on Λ^2/q^2, $1/N$, $1/\lambda$ and M_1/Λ as in the case of scalar mesons in the D3D7-brane system.

It is important to consider the moments of the structure functions defined as

$$M_n[F_i] = \int_0^1 dx x^{n-1} F_i(x, q^2), \tag{43}$$

where F_i can be F_1 and F_2 in this case.

Several moments of these structure functions have been calculated in Ref. [18] in the large N limit, i.e., by considering a single intermediate hadron state in the FCS process. This has been done for the first moments of the structure function F_2 in the case of the pion as well as for F_1 of the rho meson. In [18] we have compared these results with lattice QCD data from Refs. [21–23] for the pion, associated with the lightest pseudoscalar mode. In addition, in the case of the rho meson associated to the $l = 2$ spin-1 mode of the type II solutions, the comparison has been made with respect to results from lattice QCD of [21]. The best fit for the case of the pion leads to results with an accuracy of 10.8% or better, while in the case of the rho meson the accuracy is of 18.5% or better,[4] for the D3D7-brane system. In [18] also the Sakai-Sugimoto model of the D4D8$\overline{\text{D8}}$-brane system and the D4D6$\overline{\text{D6}}$-brane system, both in type IIA string theory, have been considered for FCS with a single intermediate exchanged state. The next step has been done in [20] where we have considered the leading $1/N$ corrections to the structure functions. The accuracy is notoriously enhanced to 1.27% for the scalar mesons in the D3D7-brane system in this case. It leads to a natural question which is whether for vector mesons the accuracy of the fit can also be substantially improved by considering $1/N$ corrections.

In order to investigate this point we have carried out the best fit of the structure F_1 including $1/N$ corrections in comparison with lattice QCD data from [21]. Recall that the results of the present work have been obtained in the type IIB supergravity regime, i.e., where $1/\sqrt{\lambda} \ll x < 1$, which means that for the calculation of the moments we have integrated our result for the functions between $x = 0.1$ and $x = 1$. On the other hand, we also need to carry out the integration over the range $\exp(-\sqrt{\lambda}) \ll x \ll 1/\sqrt{\lambda}$, where we assume that the behavior of the structure functions is similar to the behavior shown in [5] and used in [18], i.e., $F_L^{small-x} \propto x^{-1}$. We support this assumption on the fact that, in the $1/\sqrt{\lambda} \ll x < 1$ range the difference between the large-N calculation (where there is only a single on-shell hadron state exchanged in the FCS process) and the $1/N$ calculation (where the leading Feynman-Witten diagram has two on-shell hadron states exchanged) is that in the former the dependence with the photon virtuality and the Bjorken parameter is given by τ_{in} corresponding to the incident meson, while in the later the q^2 and x dependence is determined by τ_{min}. This corresponds to one of the lowest conformal dimension from the supergravity excitations. However, for $\exp(-\sqrt{\lambda}) \ll x \ll 1/\sqrt{\lambda}$ things are different, namely: the calculation from the two-open and two-closed strings scattering amplitudes is independent of τ_{in}. Thus, we assume that in this low-x regime the genus-zero result from type IIB superstring theory should not be very different with respect to a much more complicated calculation on the torus. Then, for this string theory regime of the holographic dual calculation we use the expressions for the structure functions at tree level from [18]. For our numerical calculation at low-x we consider that the integration for the moments is performed between $x = 0.0001$ and $x = 0.1$ as before [18,20]. Then, we split each structure function in two parts, each of one having a dimensionless constant to be fixed by fitting with respect to lattice QCD data [21]. There is a constant C_1 multiplying the low-x F_1 function. In addition, there is a second constant C_2 on the large-x F_1 function.

Results of the first moments of F_1 of the rho meson are presented in Table II. The values we obtain for the constants are $C_1 = 0.0087$ and $C_2 = 32.1939$. Note that they are of the same order as the ones found in our previous work [18] in the large N limit, for which the constants associated with

[4]There is a mistake in Table 5 of Ref. [18] that we have corrected here. The original errors presented in that reference were overestimated.

TABLE II. Comparison of our results for the first moments of the structure function F_1 of the lightest vector meson for a suitable choice (best fitting) of the normalization constants with respect to the results of the lattice QCD simulations in [21] and in comparison with previous results presented in [18]. Uncertainties in the lattice QCD computations are omitted.

Model/Moment	$M_2(F_1)$	$M_3(F_1)$	$M_4(F_1)$
Lattice QCD	0.1743	0.074	0.035
D3D7 ($N \to \infty$)	0.1753	0.060	0.039
Percentage error	−0.6	18.5	−12.8
D3D7 ($1/N$)	0.1750	0.065	0.038
Percentage error	−0.39	12.5	−9.6

the small-x F_1 and with the large-x F_1 of the rho meson are 0.012 and 78.07, respectively.

Figure 2 shows the structure function F_1 as a function of x. The blue bell-shaped curve indicates the $1/N$ calculation of this work. The black dashed bell-shaped line corresponds to the case obtained in Ref. [18] for the large N limit. For small-x values we use the result from [5], leading to the monotonically decreasing curves. The difference between the two curves at small values of the Bjorken parameter comes from the slightly different constants which correspond to the best fit developed in each situation. There is an important improvement with the inclusion of the leading $1/N$ correction. As it happened in the scalar case, the location of the maximum is shifted to the left with respect to the results obtained in the planar limit, matching better the phenomenological expectations.[5]

From Table II we can appreciate the enhancement of the accuracy of the moments of F_1 for the case of the rho meson which goes from 18.5% for one particle exchange in the FCS, down to 12.5% in the leading $1/N$ contribution, i.e., for the exchange of two intermediate on-shell states. This is very important because it confirms the trend found previously for glueballs in $\mathcal{N} = 4$ SYM theory [19] and for the scalar mesons of the $\mathcal{N} = 2$ SYM theory from the D3D7-brane system [20]. Thus, it indicates that in order to infer realistic conclusions for physical systems it is crucial to consider the $1/N$ expansion of the observables, and consider first the large momentum transfer limit. Possibly, this behavior can be identified in other high energy physical processes.

In addition, there are two very important aspects related to $1/N$ corrections that we should briefly comment, namely: unitarization and saturation. These issues are crucial for the low-x physics, both at weak and strong coupling regimes. There is a number of very important references using holographic dual descriptions at low x and strong coupling, focusing on glueball DIS and other hadronic processes [7,9–12,29–34]. Following [29] the saturation line for glueball DIS in $\mathcal{N} = 4$ SYM theory at

[5]See for example Refs. [27,28].

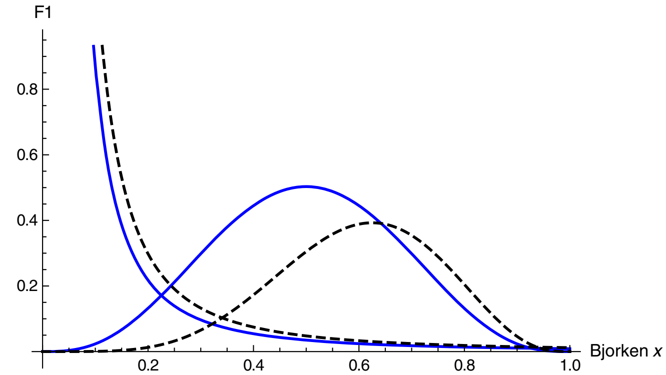

FIG. 2. F_1 as a function of x. We display the leading results at low and moderate ($0.1 < x \leq 1$) values of x. Dashed curves represent the F_1 computed for $N \to \infty$, while the blue ones correspond to the $1/N$ corrections. The constants C_1 and C_2 are those which give the moments of F_1 shown in Table II.

strong coupling corresponds to a region of high virtual momentum transfer (q) while the Bjorken parameter $x \sim 1/N^2$, being N large but finite. On the other hand, let us recall that the supergravity calculation we have done in the present work holds in the range $1/\sqrt{\lambda} \ll x < 1$. This describes a region parametrically distant from the saturation line (in fact the region which we have explored is near the horizontal axis in the $(\log(q^2/\Lambda^2), \log(1/x))$-plane, far below the saturation line depicted in Fig. 3 of [29]). While at low x it has been commented on the possibility of a sort of a "partonic" description at strong coupling [29], in the region $1/\sqrt{\lambda} \ll x < 1$ the supergravity calculation including Feynman diagrams with two or more intermediate states propagating in the s-channel only allows one to explore the scattering of a charged lepton off an entire hadron in the cloud of wee hadrons surrounding the incident one (see for instance [6,19,20]).

In order to properly account for unitarity corrections in the holographic string theoretical dual picture one should be able to calculate a string worldsheet genus expansion of the four-point superstring theory scattering amplitude, which is at present an extremely difficult task. The string theory genus expansion of the scattering amplitude contains the sum of all massive string theory states propagating in the t-channel at all orders (which obviously contains all possible multi-graviton exchanges), but also it can be seen as the sum of all the s-channel multi-loop diagrams including all massive string theory states. Both the s- and t-channel series are dual to each other by virtue of the s/t-duality. However, the result of the genus expansion is unknown, and only limited effective n-loop supergravity calculations can be done. Therefore, in terms of the s-channel two-intermediate states supergravity calculation we have done in the present work (as well as in [19,20]) it is not possible to contribute to the discussion of the saturation physics.

On the other hand, fortunately at low x we can explore the saturation line in terms of the proposal made in [29].

The idea is to obtain the saturation line which separates the weak-scattering and the strong-scattering regions in the $(\log(q^2/\Lambda^2), \log(1/x))$-plane. This has been done for glueball DIS in [29]. In that paper the authors found a critical value for the momentum transfer proportional to $\exp[4\log N^2/\sqrt{\lambda}]$ below which the DIS amplitude is dominated by a single Pomeron exchange, but in order to saturate the unitarity bound it is necessary to go beyond that value and therefore the dominant contribution becomes the multigraviton exchange. Considering our previous paper [5] where we have calculated the structure functions at low x for scalar and vector mesons, in principle we can draw analogous conclusions as [29] for the saturation line and the phase diagram in the kinematic plane $(\log(q^2/\Lambda^2), \log(1/x))$. A detailed study of these issues will be reported elsewhere.

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APPENDIX: ANGULAR INTEGRALS

Scalar spherical harmonics on the 3-sphere belong to the $(l/2, l/2)$ representation of $SU(2)\times SU(2)\equiv SO(4)$, where l is a non-negative integer, while $-l/2\le m, n\le l/2$. They form a basis of eigenfunctions of the Laplace operator on the sphere,

$$\nabla^2 Y_l^{m,n} = -l(l+2)Y_l^{m,n}, \tag{A1}$$

and satisfy the orthogonality relation

$$\int_{S^3} (Y_l^{m,n})^* Y_{l'}^{m',n'} = \delta_{ll'}\delta_{mm'}\delta_{nn'}. \tag{A2}$$

where

$$(Y_l^{m,n})^* = (-1)^{m+n} Y_l^{-m,-n}. \tag{A3}$$

The integral we want to calculate has three scalar spherical harmonics. An analytic expression for this type of integrals can be found in [35,36], and it reads

$$\int_{S^3} Y_l^{m,n} Y_{l'}^{m',n'} Y_{l''}^{m'',n''} = R_1(l,l',l'')\begin{pmatrix} \frac{l}{2} & \frac{l'}{2} & \frac{l''}{2} \\ m & m' & m'' \end{pmatrix} \times \begin{pmatrix} \frac{l}{2} & \frac{l'}{2} & \frac{l''}{2} \\ n & n' & n'' \end{pmatrix}, \tag{A4}$$

where we have included the 3j-symbols, and the function R_1 is defined as

$$R_1(x,y,z) = \frac{(-1)^\sigma}{\pi}\sqrt{\frac{(x+1)(y+1)(z+1)}{2}}, \quad \text{with} \quad \sigma = \frac{(x+y+z)}{2} \tag{A5}$$

and must satisfy the triangle inequality $|x-z|\le y\le x+z$ in order to be nonvanishing. Also, the 3j-symbols must satisfy the physical condition: $m+m'+m'' = n+n'+n'' = 0$.

In our case of interest, i.e., the UV vertex of the diagram shown in Fig. 1, the incoming state is a rho meson with quantum numbers (l,m,n), the exchanged particle is a vector mode with (l',m',n'), and the outgoing state is the scalar ϕ with (l'',m'',n''). The leading diagram is the one where $\Delta' = 4$, fixing $l' = 1$ and $-1/2\le m', n'\le 1/2$. The rho meson is a hadron which can have positive, null or negative charge. In the D3D7-brane system, the charge is given by the R-symmetry associated to the R sector of the $SU(2)_L\times SU(2)_R$. This implies that $l = 2$, while m can take three values 1, 0, -1 for a positive, null or negative charged meson respectively. Therefore, we have the following product of 3j-symbols

$$R_1(2,1,l'')\begin{pmatrix} 1 & \frac{1}{2} & \frac{l''}{2} \\ m & \pm\frac{1}{2} & -m\mp\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & \frac{1}{2} & \frac{l''}{2} \\ n & \pm\frac{1}{2} & -n\mp\frac{1}{2} \end{pmatrix}, \tag{A6}$$

where m can be 1, 0, -1. For any of these values, Eq. (A6) does not vanish only if $l'' = 1, 2, 3$. We want to sum over all possible exchanged and outgoing states, this is a sum over $m', n' = \pm 1/2$, and over l''. The final result is the same for the three values of m, and it reads

$$\sum_{m',n'=-1/2}^{1/2}\sum_{l''=1}^{3}\left[R_1(2,1,l'')\begin{pmatrix} 1 & \frac{1}{2} & \frac{l''}{2} \\ m & m' & -m-m' \end{pmatrix} \times \begin{pmatrix} 1 & \frac{1}{2} & \frac{l''}{2} \\ n & n' & -n-n' \end{pmatrix}\right]^2 = \frac{6}{\pi^2}, \tag{A7}$$

which is the value of $\mathcal{I}$ present in Eq. (28).

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