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Efficiency and shrinking in evolving networks

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Characterizing the spatio-temporal evolution of networks is a central topic in many disciplines. While network expansion has been studied thoroughly, less is known about how empirical networks behave when shrinking. For transportation networks, this is especially relevant on account of their connection with the socio-economical substrate, and we focus here on the evolution of the French railway network from its birth in 1840 to 2000, in relation to the country's demographic dynamics. The network evolved in parallel with technology (e.g. faster trains) and under strong constraints, such as preserving a good population coverage and balancing cost and efficiency. We show that the shrinking phase that started in 1930 decreased the total length of the network while preserving efficiency and population coverage: efficiency and robustness remained remarkably constant while the total length of the network shrank by 50% between 1930 and 2000, and the total travel time and time-diameter decreased by more than 75% during the same period. Moreover, shrinking the network did not affect the overall accessibility with an average travel time that decreases steadily since its formation. This evolution leads naturally to an increase in transportation multimodality (such as a massive use of cars) and shows the importance of considering together transportation modes acting at different spatial scales. More generally, our results suggest that shrinking is not necessarily associated with a decay in performance and functions but can be beneficial in terms of design goals and can be part of the natural evolution of an adaptive network.

1. Introduction

The evolution of networks has been the subject of numerous studies and books [1–4] and concerns different fields, ranging from biology to transportation engineering [5–7]. Many measures were defined and many models were proposed to describe the growth of these systems, but some important questions remain unanswered.

First, many networks interact with a substrate, and the question of the co-evolution of these components is still open. This interplay is especially relevant for transportation infrastructures, which are connected to the socio-economical conditions of the territory [8]. Indeed, these networks do not evolve in empty space, and the constraint of efficiency naturally imposes a coupling with the local population density. Railway networks are probably the best example of such a system, where the relation between network structure and the substrate is governed by complex feedbacks [5,9]. In the case of the French railway system, for instance, a recurrent debate revolves around the existence of a 'structuring effect', whereby investments in transportation infrastructures have positive effects on productivity, demography and the economy [10].

Second, almost all studies have been concerned with the expansion and growth of networks. However, networks can evolve by alternating periods of increase and decrease in the number of nodes and links, and very little is known about shrinking regimes. This is true from a theoretical point of view (with the notable exception of a simple model proposed in [11]), but even more so from the point of view of empirical studies. Shrinking dynamics has been partially explored in the case of natural transport networks. For example, in laboratory conditions, the Argentine ant builds globally optimized transport networks that connect spatially separated nests [12]. Such structures are achieved by initially creating several connections which later are either abandoned or amplified, causing the network to lose connections but not nodes. A similar pruning process is observed in the slime mould (*Physarum*), a unicellular organism often found in large multicellular aggregates that form a network of tubes that circulates nutrients and signals [6]. In both cases, the underlying mechanism is a self-organized positive feedback process where the passage of ants or nutrients reinforces pheromone traces [13] or widens the slime's tubes [14]. We note that shrinking dynamics could also be relevant for the important case of the brain organization [15].

While these are interesting examples, these networks probably evolve through very different processes compared to man-made infrastructures, such as roads, railways or pipelines, where planning is often centralized. For transport infrastructures, the main design goal is to obtain a high transport capacity at a reasonable cost: cost and efficiency appear naturally as critical parameters governing the formation and evolution of these systems, sometimes at the expense of resilience [3,4,6,7]. In the case of railway systems, in addition to the coupling with the population density, the network is also affected by technological advances that propose new and faster means which can be a cause of shrinking effects in these networks: older, slower lines can be abandoned as new faster lines appear, resulting in a global decrease in the total length of the network and its number of nodes. We thus apparently face here a trade-off problem: abandoning smaller lines and favouring faster lines while maintaining a reasonable level of population coverage. This is a particular illustration of the competition between global social optimum and individual comfort [16,17], and we could ask how the social optimum evolves during these various changes. More generally, one can ask how an evolutionary view could help to understand the development of transportation networks [18] by considering them as far from equilibrium processes which behave in an evolutionary fashion, implying, in particular, that the focus in planning should be on enhancing the resilience and adaptability of these systems.

These two fundamental questions are particularly relevant for a range of physical systems, from spatial networks such as transportation infrastructures (power grid, etc.), to other systems where nodes or links can disappear (e.g. in biology or in computer sciences). In this paper, we address these questions by empirically analysing the evolution of the French railway network, from its birth in 1840 to 2000, in correlation with the evolution of population growth in French communes. We will characterize and discuss in detail both the growing and the shrinking phase, and how these phases fit in a larger picture of network evolution. This crucial example of a country-wide transportation network will also allow us to address the problem of shrinking networks and their coupling with the substrate structure. In particular, we will analyse the relationship between railway accessibility and population change as well as the changing spatial relations among national, regional, and local scales.

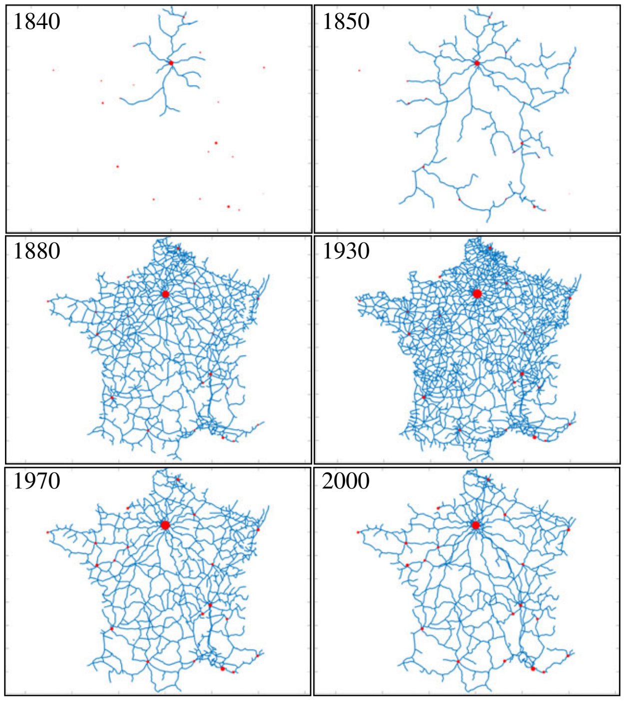

Figure 1. Stages of the evolution of the railway network for selected years. Red dots are the 20 largest communes in 2000, and the diameter of each circle is proportional to the population of the city itself divided by the population of Paris in 2000 (2 307 192 inhabitants). (Online version in colour.)

2. Evolution of the network

Between 1800 and 1900, the increasing industrialization caused a general trend in Europe of people moving from the countryside to cities. This urbanization process was relatively slow in France, and the rural population remained the majority until 1930. Concerning the French railway network, different periods marked its evolution [10,19] (figure 1): (i) first, between 1830 and 1860, the government started a national railway policy and assigned the construction of six radial lines departing from Paris to six monopolies in order to reinforce the capital's centrality through connections with important cities. These monopolies were private companies that did not interact or connect with each other. (ii) The second period (1860–1890) witnessed the creation of more lines between Paris and other regional cities, and the creation of 'lateral' lines connecting the initial six radial lines with each other. (iii) The third period (1890–1930) was mainly devoted to the creation of 'lines of local interest', with the main goal of using the railway system to connect and modernize smaller towns and rural areas. Thus, the network reached its maximal expansion in 1920, while in the fourth (iv) period (1930–1950), the network underwent a contraction due to the modernization of the equipment and the elimination of local 'narrow gauge lines', substituted by roads. (v) Finally, in the modern period (1980–2000), we observe the creation of high-speed lines (TGV) further reinforcing the use of main lines and the abandonment of smaller local lines.

In this study, we will use two different datasets: one concerns the evolution of the French railway network, and the other contains the historical records of the population size of French communes. Railway network data are constituted by stations (nodes), with their geographical position, and rail track segments (links) characterized by their length, travel time, etc. (for details about these datasets see the

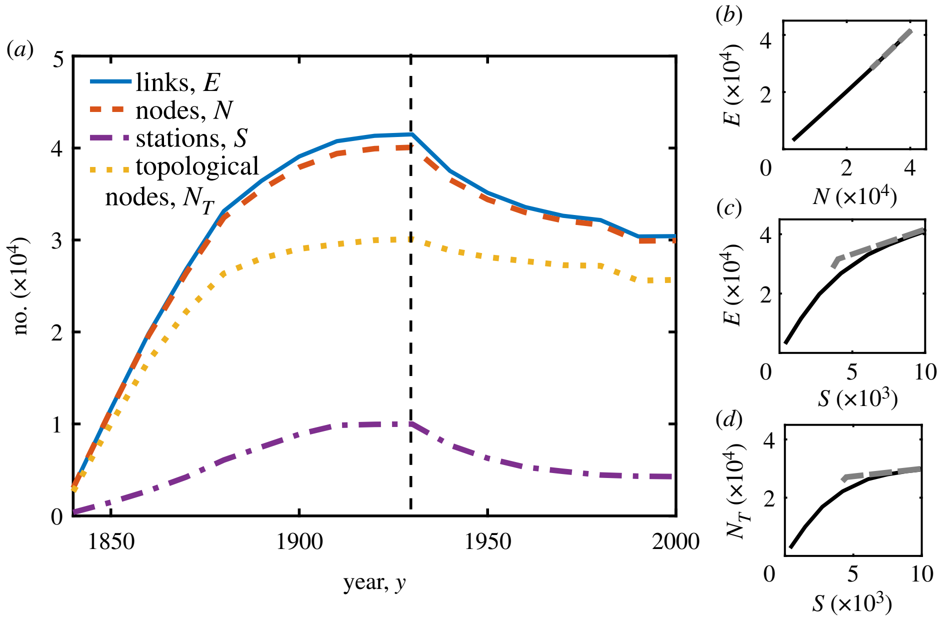

Figure 2. (*a*) Number of links, nodes, topological nodes and stations constituting the railway network versus time. The vertical dashed line indicates the beginning of the shrinking phase in 1930. Number of (*b*) links as a function of nodes. Number of (*c*) links and (*d*) nodes as a function of the number of stations. Full lines represent the growth phase, dashed lines the decreasing phase. (Online version in colour.)

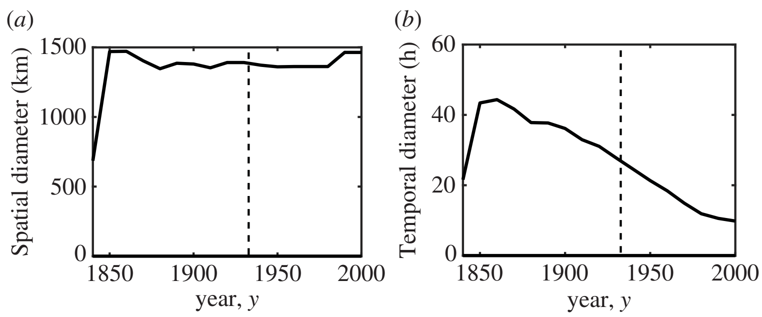

Figure 3. Diameter versus years computed for (*a*) the shortest path (in km) and (*b*) the quickest path (in h).

electronic supplementary material). More precisely, the network is composed by $N = N_T + S$ nodes, where S is the number of stations and N_T is the number of topological nodes (i.e. nodes that are not stations but are needed to indicate junctions or ramifications in the tracks). While for stations and track segments we have the explicit opening and closing dates, allowing us to measure the total number of nodes N and the number of stations S at a certain time t, the number of topological nodes N_T is in contrast obtained by subtraction. This information is available starting from 1840 every 10 years, allowing us to reconstruct the railway system and to analyse it (see figure 1).

As discussed also in [19], we observe that the growth of the number of links and nodes slows down around 1880, increases until 1930 and then decreases (figure 2*a*). The total number of both nodes and links displays a very similar temporal behaviour suggesting that, especially before 1930, the growth rule was to add a node and a link at a time. The number of nodes versus the number of links displays a linear behaviour which corresponds, as expected, to an average degree of order 2 (figure 2*b*). Taking S as a reference allows us to clearly distinguish the two phases of growth and decrease in the network (figure 2*c*,*d*). In particular, N_T and the total number of links E grow faster than S, and the decrease seems mostly linear in both cases.

Another macroscopic measure that characterizes this network is its diameter, defined as the length of the longest shortest path between two points (see for example [20]). For a transport network, we can compute shortest paths in terms of distance with the length of the tracks or travel time (so that the shortest path is the quickest path, measured in hours). We represent the evolution of the diameter for these two choices in figure 3.

We observe that, after an initial quick growth, the 'spatial diameter' is constant and of the order the maximum size of the country (∼1500 km). In contrast, the 'temporal diameter', based on quickest paths, displays a remarkable monotonous decrease. This is the first sign that the shrinking of the network is compensated in some way by technological advances (the increase in the speed of trains in this particular case). Other topological measures, such as the number of nodes of a given degree and the cyclomatic number, are reported in the electronic supplementary material (see electronic supplementary material, figures S2 and S3).

3. Cost, efficiency and robustness

A known challenge when designing transport networks is to balance between the network's cost, efficiency, and

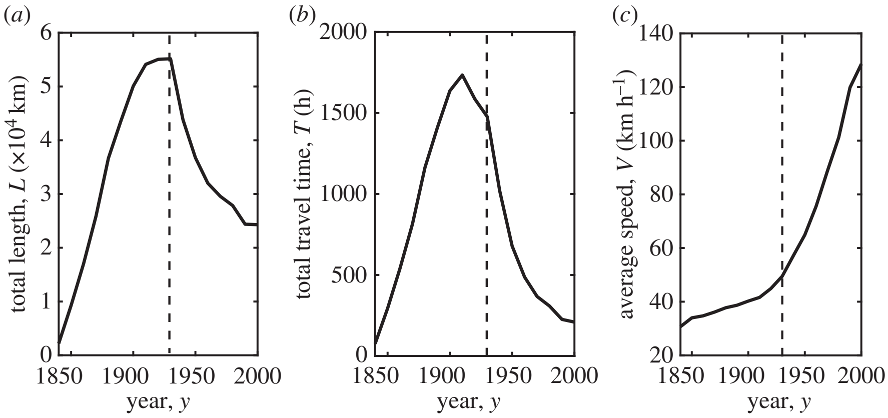

Figure 4. Evolution of the (*a*) total length of the network, (*b*) total travel time, (*c*) average train speed. The vertical dashed line indicates the beginning of the shrinking phase in 1930.

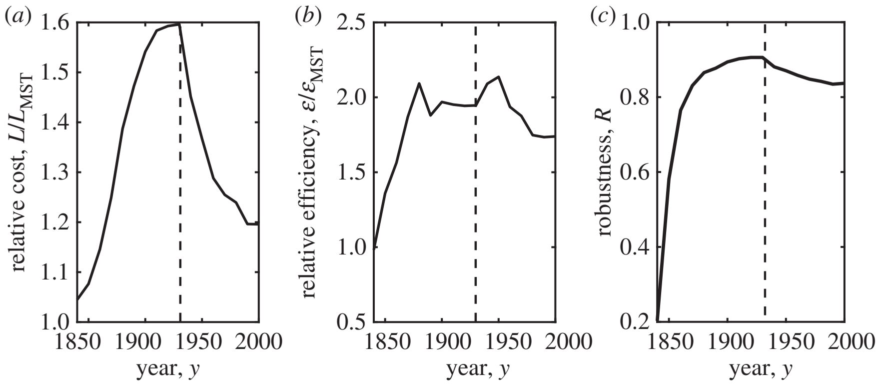

Figure 5. Evolution of the (*a*) cost of the train network divided by the cost of the MST (constructed over the same set of nodes), (*b*) efficiency of the train network divided by the efficiency of the MST, (*c*) robustness. The vertical dashed line indicates the beginning of the shrinking phase in 1930.

robustness [5,6,21]. The cost of a spatial network G built on N nodes is usually estimated by its total length

$$L(G) = \sum_{e \in E(G)} \ell_e, \tag{3.1}$$

where ℓ_e is the Euclidean length of a link e belonging to the set of links of G. We can also compute the total travel time $T(G)$ on the network (given by the sum of all travel times over all rail segments), or the average speed $V(G) = \sum_e v(e)/E$ where $v(e)$ is the speed on link e. We plot these quantities versus years in figure 4. These figures demonstrate that both the total travel time and the total length have a peak at the beginning of 1900 and then decrease until nowadays. For the total length, the decrease is mainly due to the elimination of 'local narrow gauge lines', replaced by roads after 1930 (figure 4*a*). The decrease in the total travel time starts slightly earlier due to the modernization of locomotives and to the systematic electrification of railway lines after 1920, and is further enhanced by the elimination of slow lines after 1930 and by the construction of TGV lines in 1980 (figure 4*b*). These technological advances are well summarized by the evolution of the average speed (figure 4*c*), which displays a constant increase, in particular, after 1930 (more details about the properties of appearing and disappearing links can be found in the electronic supplementary material, in particular, see figure S9).

In order to understand the order of magnitude of the network's cost, expressed by the total length, we can compare it to the most economical network that connects all the stations and topological nodes present at a certain time. This is the minimum spanning tree (MST, see for example [22]), which represents an excellent benchmark for spatial networks (see for example [6,9,23] and references therein). We can then construct the relative cost L/L_{MST} for connecting the same set of nodes and see how this ratio varies with time (figure 5*a*). We observe that the relative cost has a peak around 1930, indicating that the total length of the actual railway network is 1.6 times the minimum length needed to connect all stations. After 1930, this ratio decreases to 1.2 in 2000, showing the large reduction in costs that governed this period.

While the total length is generally accepted as a good proxy for the cost of the network, several definitions of transport efficiency can be found in the literature [9,24–26]. Efficiency is often regarded as one of the main design goals in planning and building transportation infrastructures [27,28]. Here, we follow [24] and define it as

$$\mathcal{E}(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{d_{\text{E}}(i, j)}{d_{\text{N}}(i, j)}, \tag{3.2}$$

where i and j are nodes in G, $d_{\text{E}}(i, j)$ is their Euclidean distance, and $d_{\text{N}}(i, j)$ is the length of the shortest path connecting them on the network G. With this definition, efficiency takes values between 0 and 1, and quantifies, on average, how much the shortest paths on the network are close to straight lines. We compare it to its value for the MST which is explicitly built by prioritizing cost reduction over efficiency, and we observe that, despite the strong reduction in nodes and links, the French railway network remains twice as efficient as the cheapest network connecting all the nodes (figure 5*b*, efficiency alone is shown in electronic supplementary material, figure S4). We note that this measure does not take into account the speed of trains, but simply the ability of the network to transport passengers between communes along the straightest possible path. Another relevant measure for transport networks is robustness, or fault tolerance, computed as the probability of *not* disconnecting the network by removal of a random link [6]. In the literature, robustness is used to measure the resilience of the network against the breakdown of its components (links in this case), and is remarkably high here (see figure 5*c*; we recall that the MST is a tree thus its robustness is zero).

During the radial and the capillarization expansion phases of the network, we observe a large increase in all these quantities. It is remarkable to observe how the decreasing phase after 1930 has greatly reduced costs while keeping efficiency and robustness almost unchanged. This is a probable consequence of the strong centralization of the national railway system, where the addition and deletion of lines, and thus network optimality, were planned ahead at the governmental level. We also note that one can visualize the evolution of the trade-off between these design goals by plotting these quantities one against each other as in [6] which supports the fact that during the shrinking phase, costs are reduced but the robustness and efficiency are almost unchanged (see electronic supplementary material, figure S4).

Another indicator (non-trivially) related to efficiency, and which allows a clear characterization of the structural changes at various scales, is given by the detour profile ϕ [26], which is defined as follows

$$\phi(d) = \frac{1}{N(d)} \sum_{i,j \text{ s.t. } d_{\mathrm{E}}(i,j)=d} \frac{d_{\mathrm{N}}(i,j)}{d_{\mathrm{E}}(i,j)} \tag{3.3}$$

and is a function of the distance d and where i and j are communes with a station, d_{E} is their Euclidean distance, d_{N} is their distance on the network, and $N(d)$ is the number of pairs of communes that are at distance $d_{\mathrm{E}}(i,\ j) = d$. ϕ is larger than 1 and indicates the average deviation from a straight line needed to travel on the transport network between any two communes at distance d. This measure is suitable for understanding the focus distance of the operations on the transport networks (expansion, pruning) through time. Moreover, it highlights which distances between communes were typically favoured during the different phases of evolution of the network. For the French railway system, we observe a strong decrease in the detour profile at large distances, due to the construction of the main radial lines after 1860, which remains constantly low through network evolution (figure 6). This also has the effect of reducing the detour at shorter distances (<200 km), although a peak remains at short–intermediate distances of order 20–100 km. This is the range of distances that was targeted in the following capillarization phase, which implied a strong reduction in the detour index above 30 km, but not below, as probably this was a reasonable distance to cover by walking or riding. The pruning phase starting in 1930 determines a gradual but significant increase in the detour index in the same distance range (less than 100 km), which seems to correlate with the increasing speed of other transport means. For example, in 1980, it was already possible to cover 100 km by car in about an hour. We also observe that pruning at a local scale also slightly affects intermediate distances (100–600 km).

Finally, the detour profile can be averaged over distances and compared to the detour index for the MST constructed over the same set of nodes, allowing us to monitor the time evolution of the network (figure 6*b*,*c*) through a measure that is complementary to efficiency (equation (3.2) and figure 5*b*). As the network expands until 1920, we observe a decrease in the average detour index: in 1910, there is on average a 25% difference between the trip on the rail network and the Euclidean distance. After 1920, in the shrinking phase, the average detour index increases due to the removal of narrow gauge lines. However, when compared with the MST, which is the most economical network but known to have a high detour profile, the relative detour profile remains roughly constant, indicating that the efficiency of the network is preserved as the cost is decreased.

4. Evolution of the population and coverage properties

4.1. Population and the network

The railway network co-evolves with the population distribution, and it is therefore important to characterize quantitatively the correlation between the network's extension and the population density at both a global and a local level. First, we consider the evolution of the number of communes with and without a station (figure 7*a*). As expected, we observe a peak around 1920 for the number of communes with a station, while the total number of communes is roughly constant. It is interesting to observe that, while the total population grows constantly, during the expansion phase of the network, the growth of the population appears to occur mainly in communes with a station, while it is concentrated in communes without a station during the shrinking phase (figure 7*b*). Although this is expected in the growing phase, it is a surprise to observe that the majority of the population is growing in communes without a station. This is consistent with the fact that the average population of communes with a train station displays a minimum around 1930, when many small communes were directly connected to the network as a result of the government's policy of reaching small countryside towns (figure 7*c*; for the full distributions of population sizes, see electronic supplementary material, figure S6). After 1930, we then observe an increase in the average size of communes connected to the network, which is consistent with the fact that the average population of communes where the station closes is increasing (see electronic supplementary material, figure S9). Overall, it is interesting to observe that, while the fraction of communes with a station is always lower than 0.1, in the moment of maximum expansion of the network almost half of the French population lived in a commune served by a station (figure 7*d*).

4.2. Accessibility

An important aspect of the relation between a transport network and its substrate is the network's accessibility. Several quantitative ways of estimating accessibility have been proposed in the literature, and the different approaches are reviewed in [29,30]. For instance in our case, a rough but

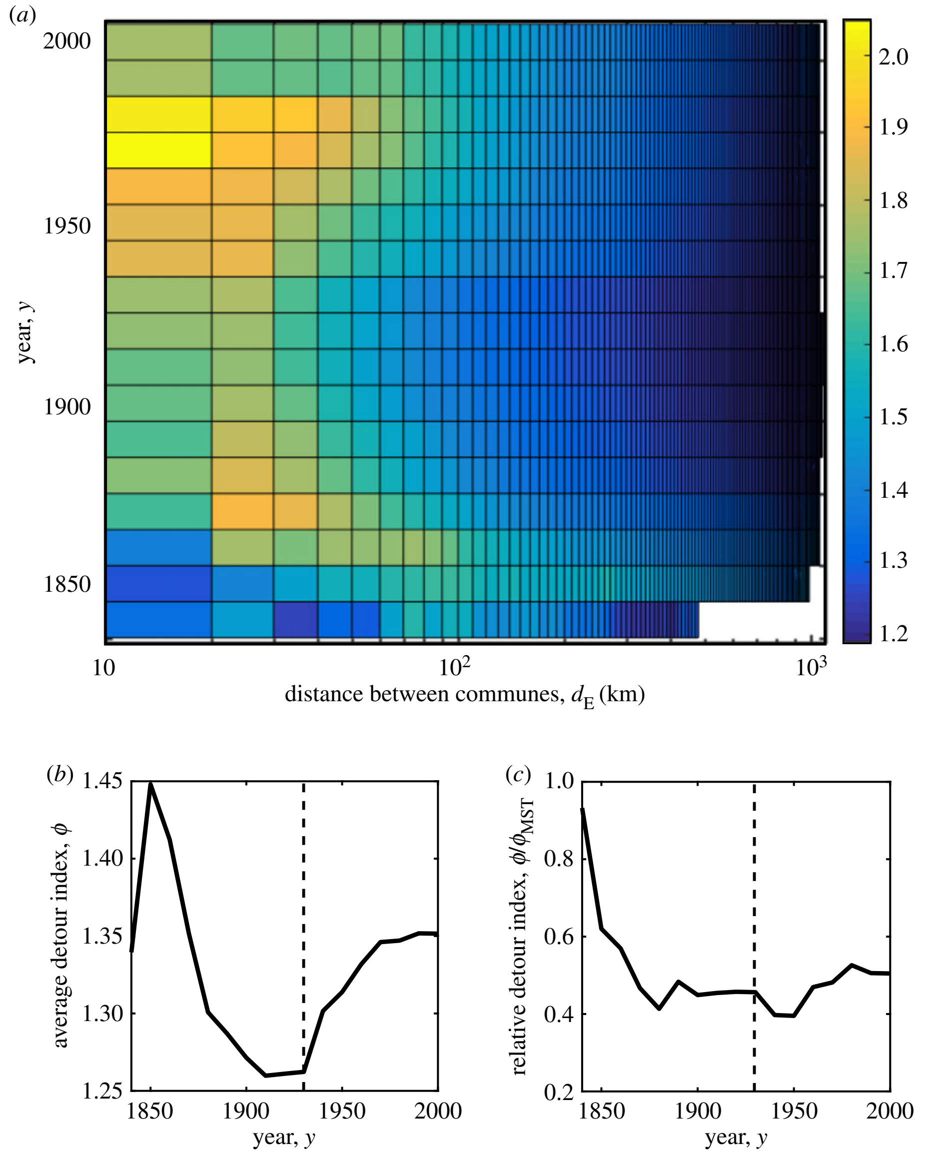

Figure 6. (*a*) Heat map (in logarithmic scale) showing the evolution of the detour index at different Euclidean distances between communes (the brighter the map, the higher the detour index). (*b*) Average detour index for the French railway network and (*c*) relative detour index (average detour index of the train network divided by the average detour index of the corresponding MST) versus time. The vertical dashed line indicates the beginning of the shrinking phase in 1930. (Online version in colour.)

straightforward way to estimate the railway's accessibility is to measure its *pedestrian accessibility* [19], defined as the Euclidean distance d_E between a commune c and its nearest train station s: $A_c = d_E(c, s)$ (for a commune s' that has a station we then have $A_{s'} = d_E(s', s') = 0$). By averaging over all communes (with or without a train station) and by weighting with a commune's population P_c, we obtain the network's average pedestrian accessibility

$$\langle A \rangle = \frac{\sum_c P_c A_c}{\sum_c P_c}. \qquad (4.1)$$

This quantity depends on how the network extends in the territory with respect to the local population density and measures the typical distance needed by a random individual to reach the nearest station. Note that the definition equation (4.1) implies that the larger the railway network coverage the lower its accessibility. Figure 8*a* shows that from the birth of the railway network to the moment of maximum expansion and capillarization in 1930, the average travel distance per person to the closest train station dropped from 25 km to less than 5 km. After 1930, the removal of the smallest lines increased this average distance which, however, remained bounded (slightly above 5 km).

Another way to look at accessibility is through the average travel time required to reach the closest train station $\langle \tau \rangle = \langle A \rangle / v_p$, where v_p is the typical speed of complementary transport means. At the beginning of the century, we can assume the main transport mode was walking ($v_p \approx 5\ \mathrm{km\ h^{-1}}$), and that, later on, coaches ($v_p \approx 30\ \mathrm{km\ h^{-1}}$) and cars ($v_p \approx 50\ \mathrm{km\ h^{-1}}$), together with better road infrastructures, increased this velocity. In figure 8*b*, we compare the average travel time for different transportation speeds. We observe that, due to the capillarization of the network in the territory, from 1900 to 1950, it was possible, on average, to walk to the closest station within 1 h. While this may have been a completely reasonable option at the beginning of the century, the current lifestyle and needs require either public transportation, when available, or transport via a car to the station, reducing the average travel time to about 10–15 min. By taking into account the average road transport speed typical of each decade, we observe a decrease

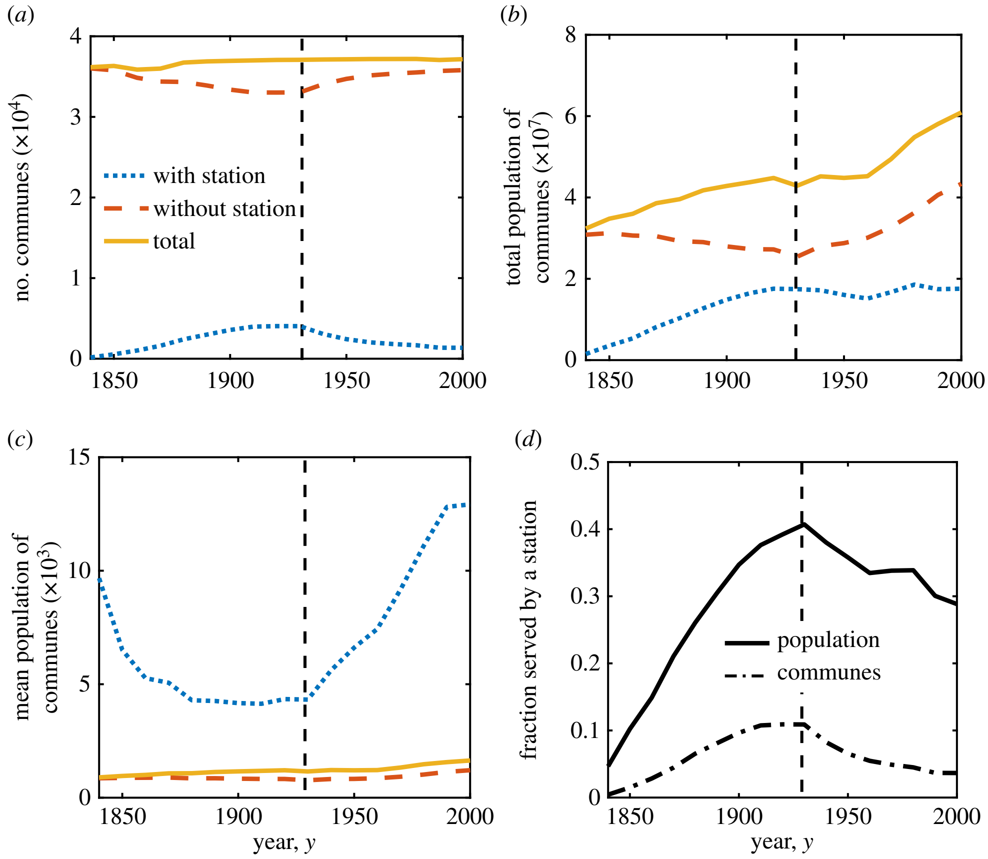

Figure 7. Evolution of the (*a*) number of communes, (*b*) total population, (*c*) mean population, (*d*) fraction of communes and population directly served by a station. The vertical dashed line indicates the beginning of the shrinking phase in 1930. (Online version in colour.)

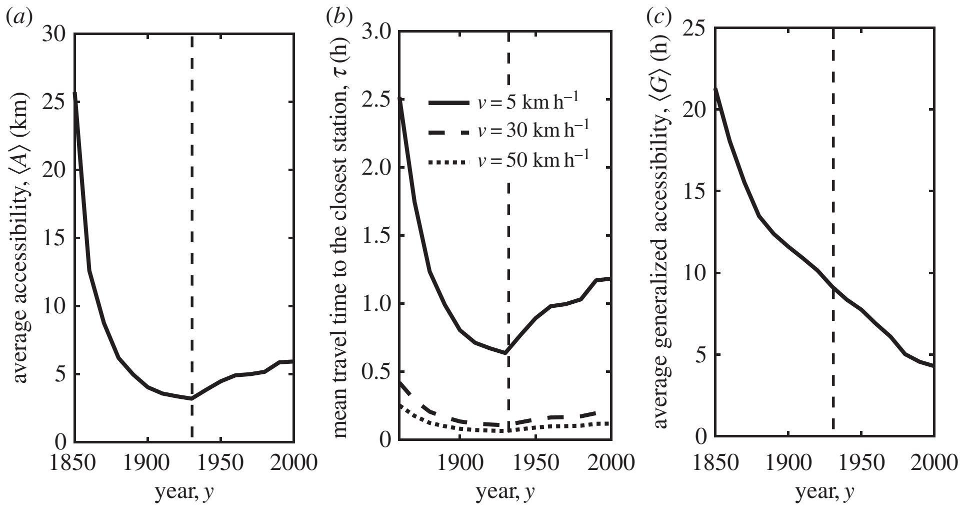

Figure 8. (*a*) Average pedestrian accessibility. (*b*) Average travel time to reach the closest station for a random French citizen at different values of the transport velocity v_p. (*c*) Average generalized accessibility versus time computed with $v_p = 5$ km h^{-1}. Data are shown starting from 1850 to better appreciate the variations in recent times ($\langle A\rangle(1840) = 162$ km, $\langle \tau\rangle(1840) = 32$ h). The vertical dashed line indicates the beginning of the shrinking phase in 1930.

in $\langle \tau\rangle$, despite the decreasing amount of communes being directly served by a station (as shown in figure 7*a*,*d*). Therefore, even if the network is shrinking after 1930, the stations that remain open seem to have been chosen strategically such that pedestrian accessibility is kept roughly constant, in particular, if we take into account the possibility of other transport modes.

So far, we have characterized how effectively the transport network extends in the territory with respect to the spatial distribution of communes and population. In order to quantitatively assess how well it connects two random communes in the French territory, we also need to take into account the average travel time on the network. We thus weight each link in the network with its travel time, which, in contrast to the track's length, changes in time with technological improvements. This leads us to define the generalized accessibility G_c as

$$G_c = v_p^{-1} A_c + \frac{1}{S-1}\sum_{s'} t_N(s, s'), \tag{4.2}$$

where t_N is the shortest time-weighted path between c's closest station (s) and any other station s' (S is the total number of stations at time t). G_c is thus the average time needed to travel from a commune c to any other point in the network. We then average over all communes weighted with their population size and obtain the average generalized accessibility $\langle G \rangle = \sum_c P_c G_c / P$, where P is the total population in France (at a given time), which represents the average time needed by French citizens to reach the railway network and travel to any other station. Figure 8*c* shows that $\langle G \rangle$ computed with fixed $v_p = 5\,\text{km h}^{-1}$ decreases from 22 h in 1850 to less than 5 h in the year 2000 (see also electronic supplementary material, figures S7 and S8 for the geographical distribution of the accessibility measures).

5. Discussion

The shrinking of the network characterized by a decrease in the number of stations and lines, started in the 1930s and was reinforced in the 1980s by the appearance of high-speed trains, which led to a further trimming of the network's smallest and slowest lines. Removing links and nodes from the network may affect negatively the general transport performances of the railway, potentially reducing the efficiency of the network and increasing the travel times for a large sector of the population. However, we find that efficiency indicators are not negatively affected by the country-wide re-organization of the railway network. At a topological level, efficiency and robustness remain remarkably constant while the total length of the network shrinks by 50% between 1930 and 2000. At an efficiency level, thanks to technological improvements, the total travel time and time-diameter decreased by more than 75% during the same period. Moreover, shrinking the network did not affect the overall accessibility when considering the distribution of the population across the territory. Indeed, the average travel time decreased steadily since its formation. All these results seem to point to one conclusion: even if pruning the network and closing stations and lines may initially appear as purely cost-driven governance, it seems that this evolution is natural and beneficial in terms of design goals. In contrast to naive intuition, taking advantage of new technologies in both railway and road transportation further improved the average network performances for covering the territory.

Our analysis shows the importance of considering the evolution of transportation infrastructure in conjunction with the socio-technological substrate and technological improvement. The increasing quality of roads and mass availability of cars decreased the access time to train stations and favoured the re-organization of the French railway system. In this sense, removing smaller local lines was concomitant with an increase of multimodality in the transportation system. With an eye to the current debate on global warming and sustainable transportation, it seems necessary to scrutinize decisions such as substituting local electrified train lines with roads. Overall, our quantitative analysis suggests that the French railway system provides an efficient and sustainable large-scale transport infrastructure, which could be better exploited by strategically planning other public transportation means, acting at a smaller spatial scale. Relatively slower, but collective, transport means (e.g. electric or biogas buses) could provide a better trade-off between transportation efficiency and environmental impact. At a more fundamental level, our results promote a unified framework where network and substrate evolution are considered jointly, and where mutual influences are taken into account. In the case studied here, our measures suggest that the transformation of the French railway during the last two centuries is associated with a profound scale-dependent transport mode diversification, and that shrinking is not necessarily associated with a decrease in efficiency but can be a part of the natural co-evolution of this system.

Data accessibility. The train data that support the findings of this study are available from Thomas Thevenin (Thomas.Thevenin@u-bourgogne.fr) on reasonable request. Population data are available at www.geohistoricaldata.org.

Authors' contributions. A.B., M.G. and M.B. designed the study and A.B. performed the numerical analysis of the data. A.B. and M.B. wrote the paper.

Competing interests. We declare we have no competing interests.

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