

# Wildfire propagation modelling

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Wildfires are a concrete problem with a strong impact on human life, property and the environment, because they cause disruption and are an important source of pollutants. Climate change and the legacy of poor management are responsible for wildfires increasing in occurrence and in extension of the burned area. Wildfires are a challenging task for research, mainly because of their multi-scale and multi-disciplinary nature. Wildfire propagation is studied in the literature by two alternative approaches: the reaction-diffusion equation and the front tracking level-set method. The solution of the reaction-diffusion equation is a smooth function with an infinite domain, while the level-set method generates a sharp function that is not zero inside a compact domain.

However, these two approaches can indeed be considered complementary and reconciled. With this purpose we derive a method based on the idea to split the motion of the front into a drifting part and a fluctuating part. This splitting allows specific numerical and physical choices that can improve the models. In particular, the drifting part can be provided by chosen existing method (e.g. one based on the level-set method) and this permits the choice for the best drifting part. The fluctuating part is the result of a comprehensive statistical description of the physics of the system and includes the random effects, e.g., turbulent hot-air transport and fire-spotting. As a consequence, the fluctuating part can have a non-zero mean (for example, due to ember jump lengths), which means that the drifting part does not correspond to the average motion. This last fact distinguishes between the present splitting and the well-known Reynolds decomposition adopted in turbulence studies. Actually, the effective front emerges to be the weighted superposition of drifting fronts according to the probability density function of the fire-line displacement by random effects. The resulting effective process emerges to be governed by an evolution equation of the reaction-diffusion type. In this reconciled approach, the rate of spread of the fire keeps the same key and characterising role that is typical in the level-set approach.

Moreover, the model emerges to be suitable for simulating effects due to turbulent convection, such as fire flank and backing fire, the faster fire spread being because of the actions by hot-air pre-heating and by ember landing, and also due to the fire overcoming a fire-break zone, which is a case not resolved by models based on the level-set method. A physical parametrization of fire-spotting is also proposed and numerical simulations are shown.

## References

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## ABSTRACT

Wildfires are a challenging task for research, mainly because of their multi-scale and multi-disciplinary nature. Wildfire propagation is studied in the literature by two alternative approaches: the reaction-diffusion equation and the level-set method. These two approaches can indeed be considered complementary and reconciled. With this purpose we derive a method based on the idea to split the motion of the front into a drifting part and a fluctuating part. The drifting part can be provided by chosen existing method (e.g., one based on the level-set method) and this permits the choice for the best drifting part. The fluctuating part is the result of a comprehensive statistical description of the physics of the system and includes the random effects, e.g., turbulent hot-air transport and fire-spotting. In this reconciled approach, the rate of spread of the fire keeps the same key and characterising role that is typical in the level-set approach. Moreover, the model emerges to be suitable for simulating effects due to turbulent convection, such as fire flank and backing fire, the faster fire spread being because of the actions by hot-air pre-heating and by ember landing, and also due to the fire overcoming a fire-break zone, which is a case not resolved by standard models. A physical parametrization of fire-spotting is also proposed and numerical simulations are shown.

## 1. MODELLING OF WILDFIRE PROPAGATION

Wildfires are a concrete problem with a strong impact on human life, property and the environment, whose increasing in occurrence and burned area is due to the climate change and the legacy of poor management. In 2017, an unusual high fire levels occurred with extensive and severe fires in many parts of the world, including the Mediterranean area, the US and even Greenland. An extremely dangerous phenomenon is the so-called fire-spotting, i.e., the generation of secondary ignitions isolated from the primary fire zone leading to perilous situations and responsible for dangerous flare up in the wildfire. The aim of present study is fire-spotting modelling through the physical parametrization of the underlying processes. We propose a post-processing numerical routine that can be applied to any existing fire front propagation model including randomization of the fire front due to turbulence and fire-spotting. The main attainment of the proposed method is that the effects of random fluctuations are included in a way that preserves the existing structure of the operational and industrial codes and can be implemented directly.

Level set method (LSM) is one of the widely used and successful tools for studying evolving interfaces. Let  $\Gamma$  be a simple closed curve representing the fire-front interface in two dimensions, and let  $\Omega$  be the region bounded by the fire-front  $\Gamma$ .

Let  $\varphi: \mathcal{S} \times [0, +\infty[ \rightarrow \mathbb{R}$  be a function defined on the domain of interest  $\mathcal{S} \subseteq \mathbb{R}^2$  such that the level set  $\varphi(x, t) = c$  coincides with the evolving front.

Apart from the statistical effects to be described later, motion of the fire-front is given by the well known Level Set Equation:

$$\frac{\partial \varphi(x, t)}{\partial t} = u(x, t) \|\nabla \varphi(x, t)\|, \quad (1)$$

where  $u(x, t)$  is the rate of spread (ROS) of the fire-front. The subsets of the domain  $\mathcal{S}$  corresponding to the burning region  $\Omega$  may be conveniently described by the following indicator function  $\mathcal{I}_\Omega: \mathcal{S} \times [0, +\infty[ \rightarrow \{0, 1\}$  defined as follows:

$$\mathcal{I}_\Omega(x, t) = \begin{cases} 1, & \text{if } \varphi(x, t) \leq c, \\ 0, & \text{elsewhere.} \end{cases} \quad (2)$$

Let the motion of each active flame holder be random, e.g due to turbulence and fire-spotting effects. Let  $X^\omega(t, \hat{x})$  be the  $\omega$ -realization of the trajectory of a active flame holder, with the same initial condition  $X^\omega(0, \hat{x}) = \hat{x}_0$ , and characterised by average position  $\hat{x}$ . By using statistical mechanics formalism, the trajectory of a single active flame holder is described by the one-particle probability density function (PDF)  $p_\Omega^\omega(x; t) = \delta(x - X^\omega(t, \hat{x}_0))$ , where  $\delta(x)$  is the Dirac  $\delta$ -function. By using the sifting property of the  $\delta$ -function and averaging the indicator function including fluctuations we obtain an effective indicator of the region surrounded by a random front,  $\varphi_e(x, t): \mathcal{S} \times [0, +\infty[ \rightarrow [0, 1]$ , as follows [1, 2]:

$$\begin{aligned} \varphi_e(x, t) &= \left\langle \int_{\mathcal{S}} \mathcal{I}_\Omega(\hat{x}, t) \delta(x - X^\omega(t, \hat{x}_0)) d\hat{x} \right\rangle \\ &= \int_{\mathcal{S}} \mathcal{I}_\Omega(\hat{x}, t) \langle \delta(x - X^\omega(t, \hat{x}_0)) \rangle d\hat{x} \\ &= \int_{\mathcal{S}} \mathcal{I}_\Omega(\hat{x}, t) p(x; t | \hat{x}) d\hat{x} \\ &= \int_{\Omega(t)} p(x; t | \hat{x}) d\hat{x}. \end{aligned} \quad (3)$$

The PDF  $p(x; t | \hat{x}) = \langle \delta(x - X^\omega(t, \hat{x})) \rangle$  accounts two independent random variables representing turbulence and fire-spotting respectively. Turbulence is assumed to be isotropic and modelled by a bi-variate Gaussian PDF  $G(x - \hat{x}; t)$ . The fire-spotting is assumed to be an independent downwind phenomenon. Thus, the PDF of the random processes can be defined as

$$p(x; t | \hat{x}) = \begin{cases} \int_0^{+\infty} G(x - \hat{x} - \ell \hat{n}_U; t) q(\ell; t) d\ell, & \text{downwind,} \\ G(x - \hat{x}; t), & \text{upwind,} \end{cases} \quad (4)$$

where  $\hat{n}_U$  is the unit vector aligned with the mean wind direction,  $q(\ell; t)$  is the downwind landing distribution of the firebrands.

## 2. PHYSICAL PARAMETRIZATION

The firebrand landing distribution  $q(\ell)$  is defined by a lognormal distribution as follows [1, 2],

$$q(\ell) = \frac{1}{\sqrt{2\pi}\sigma\ell} \exp\left(-\frac{(\ln \ell / \mu)^2}{2\sigma^2}\right), \quad (5)$$

where  $\mu$  is the ratio between the square of the mean of landing distance  $\ell$  and its standard deviation,  $\sigma$  is the standard deviation of  $\ln \ell / \mu$ .

Assuming the shape of the firebrands to be spherical and combining experimental and theoretical approaches to characterize the maximum landing distance of the firebrands, the maximum travel distance  $\ell_{\max}$  for firebrands from a vertical convective column in terms of maximum loftable height  $H$ , the mean wind  $U$  and radius of the firebrands  $r$  is:

$$\ell_{\max} = H \left( \beta \tan \bar{\alpha}_f + U \left( \frac{3\rho C_d}{2\rho_f r g} \right)^{1/2} \right), \quad (6)$$

$\rho$  and  $\rho_f$  represent the density of the ambient air and of the wildland fuel respectively,  $C_d$  is the drag coefficient,  $g$  is the acceleration due to the gravity,  $\bar{\alpha}_f$  is the angle of flame to a vertical plane and  $\beta = 0.7$  is a correction factor. Then, by choosing the value  $z_p$  corresponding to the  $p^{\text{th}}$  percentile of the lognormal distribution, the shape parameters  $\mu$  and  $\sigma$  can be physically parametrized as follows:

$$\mu = H \left( \frac{3\rho C_d}{2\rho_f} \right)^{1/2}, \quad \sigma = \frac{1}{z_p} \ln \left( \beta \left( \frac{2\rho_f U^2}{3\rho C_d g L_f} \right)^{1/2} + \frac{U}{\sqrt{r g}} \right), \quad (7)$$

such that  $\ell_{\max} = \mu \exp(z_p \sigma)$ . Here  $L_f$  represents the flame length – the distance between the ground midway of the combustion zone and the tip of the flame, that is defined here by the following formula derived on the basis of the energy conservation principle and the energy flow rate in the convection column above the fireline:

$$L_f = \frac{1}{\cos \alpha_f} \left[ \frac{1}{2g(\rho_c \rho_a T_a)^2} \right]^{1/3} I_f^{2/3}, \quad (8)$$

where  $\alpha_f$  is so-called flame angle,  $I_f$  is the fireline intensity,  $c_p$  is the specific heat of fuel and the ambient temperature is denoted by  $T_a$ .

The flame length is crucial for the fire-spotting modelling due to its effects to the parameters of the lognormal distribution. The flame geometry changes the travel distance of the firebrand and, consequently, the parameter  $\sigma$  of the lognormal fire-spotting distribution, such that the following situation can be observed:

1. No fire-spotting: when the travel distance is not enough to produce an independent secondary fire. It is characterized by the small value of  $\sigma$ .
2. Merging of secondary fires: when the main fire front propagates faster than new fires appear. This is a limit case such that the fire-spotting is observed but it almost does not affect to the propagation speed.
3. Fire-spotting: if  $\sigma$  is big enough to generate new independent fires. Each secondary fire later produces new secondary fires, thus the fire front propagation speed increases extremely. The growth of  $\sigma$  leads to the increasing travel distance, hence larger area is covered and the risk to overcome obstacles and fire safety zones is higher.

Moreover, according to a heating-before-burning mechanism, we introduce now the integral field

$$\Psi(x, t) = \int_0^t \frac{1}{\tau(x, \varepsilon)} \varphi_e(x, \varepsilon) d\varepsilon \quad (9)$$

that can be understood as an accumulation of the heat that results in an increase of the fuel temperature. This feedback mechanism between the two fields  $\Psi$  and  $\varphi$  is given by the following relation:

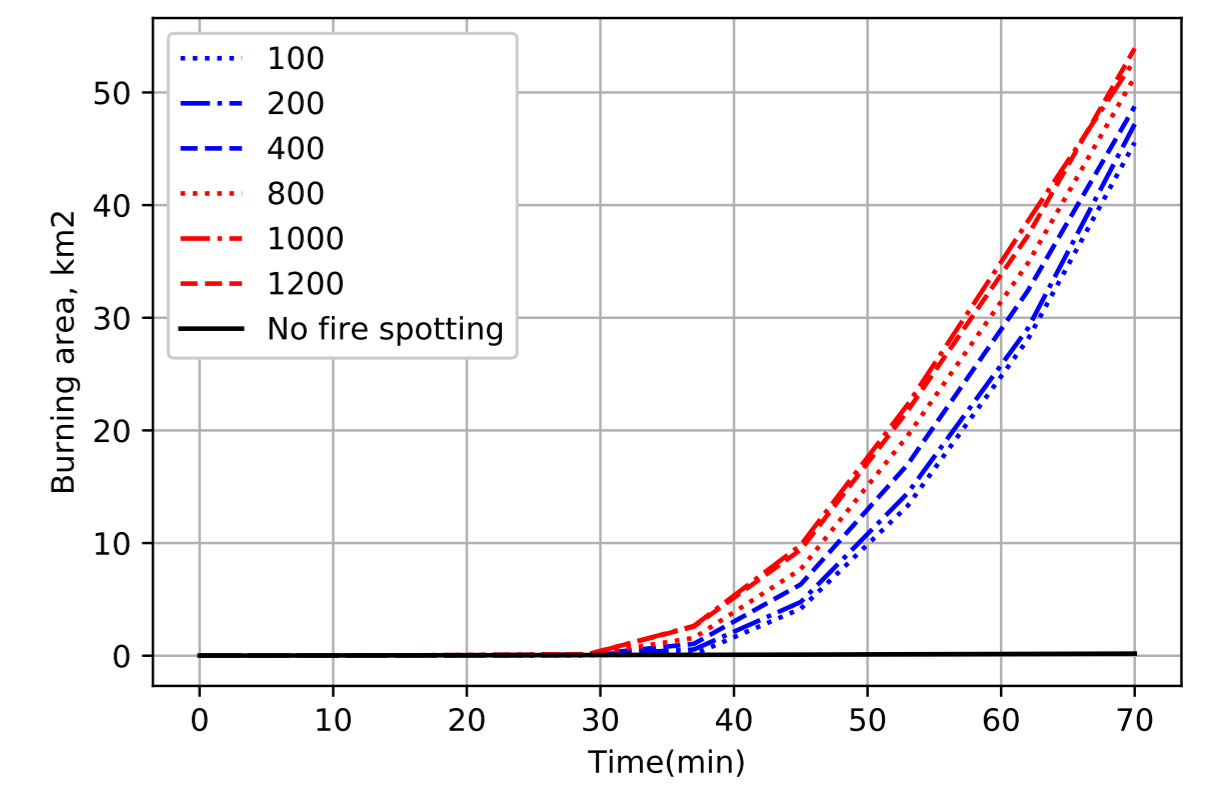
$$\Psi(x, t) \geq 1 \rightarrow \mathcal{I}_\Omega(x, t) = 1, \quad (10)$$

that is, when a certain amount of heat have been accumulated in the considered spot and have endured a certain amount of time that accounts for the environmental availability of resources, it forms a new ignition.

## 3. SIMULATIONS

The proposed physical parametrization allows to study the role of atmospheric conditions on fire-spotting, since  $\mu$  linearly depends on the height of the atmosphere boundary layer  $H_{ABL}$ . Thus, the model is adopted to any daytime taking into account stable (during the night) and unstable (during the day) atmosphere.

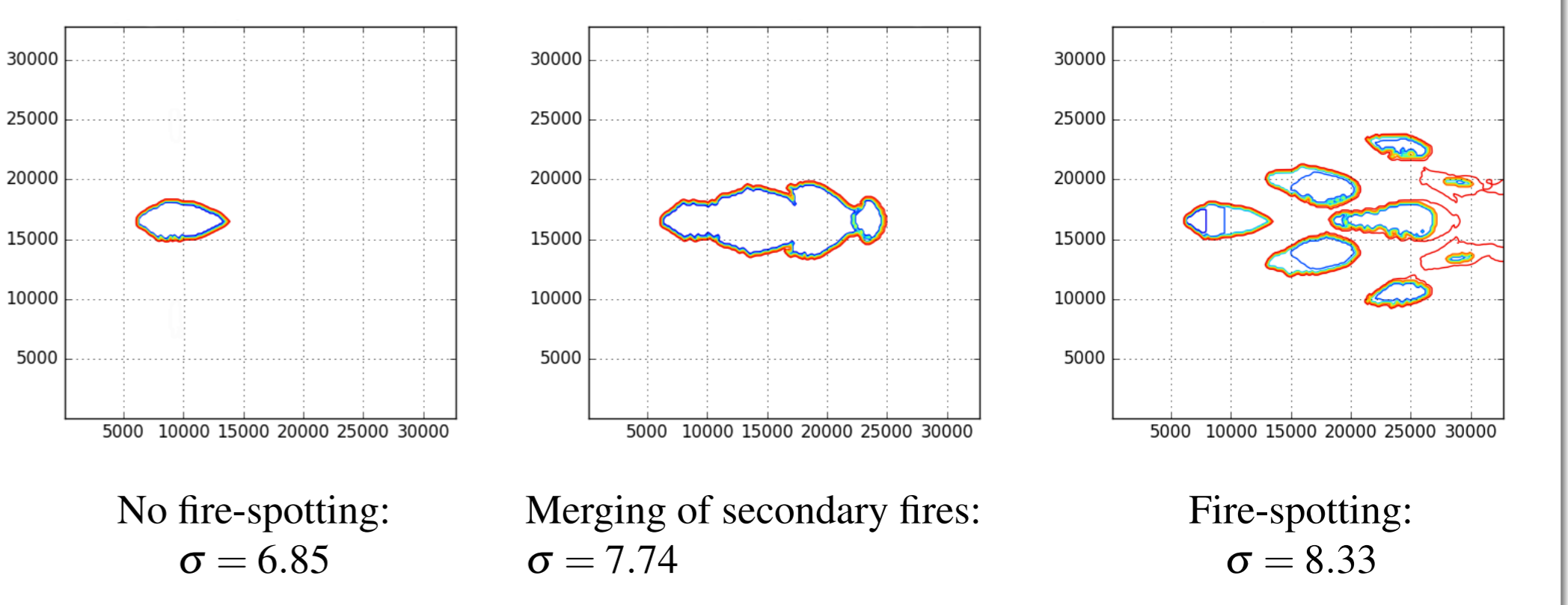
$H_{ABL}$ [m]	$\mu$ [m]
<i>Stable Atmosphere</i>	
100	3.733
200	4.436
400	5.843
<i>Unstable Atmosphere</i>	
800	8.656
1000	9.739
1200	11.469



Values of parameter  $\mu$  for different atmospheric boundary layer depth  $H_{ABL}$ .

A comparison of the burned area in different time moments in stable (blue) and unstable (red) atmospheric boundary conditions for various  $H_{ABL}$  [m].

The flame geometry changes the travel distance of the firebrand and, consequently, the parameter  $\sigma$  of the lognormal fire-spotting distribution, such that the following situation can be observed:



## 5. CONCLUSIONS

Fire-spotting is one of the most vexing problems associated with wildland fires. Thus, improvement of the fire propagation modelling by introducing fire-spotting effects becomes more and more urgent. We propose a methodology whose breakthrough is the inclusion of the random effects, such as turbulent convection and fire-spotting, it preserves the structure of the existing operational and industrial codes and can be implemented directly as a post-processing routine. It has been tested on several fire simulators demonstrating its applicability and effectiveness. The proposed physical parametrization takes into account the macroscale phenomena, as atmosphere stability conditions, and mesoscale phenomena, such as flame characteristics, in a concurrent multi-scaling approach. Moreover, proposed physical parametrization admits deeper studying of environmental factors or extension to further randomness, as vegetation inhomogeneity, the terrain slope or fuel load and type.

## 6. REFERENCES

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