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Wildland fire propagation modeling: fire-spotting parametrisation and energy balance

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Abstract

Present research concerns the physical background of a wild-fire propagation model based on the split of the front motion into two parts - drifting and fluctuating. The drifting part is solved by the level set method and the fluctuating part describes turbulence and fire-spotting. These phenomena have a random nature and can be modeled as a stochastic process with the appropriate probability density function. Thus, wildland fire propagation results to be described by a nonlinear partial differential equation (PDE) of the reaction-diffusion type. A numerical study of the effects of the atmospheric stability on wildfire propagation is performed through its effects on fire-spotting. Moreover, it is shown that the solution of the PDE as an indicator function allows to construct the energy balance equation in terms of the temperature.

Key words: fire propagation, fire-spotting, level set method, energy balance
MSC 2000: 00A59, 35K57, 65C20, 70H20

1 Introduction

In wildland fire propagation, fire-spotting phenomena cause isolated fire from the main fire. It is an important aspect because it affects the rate of spread of the fire and may cause dangerous effects. The present study deals with fire propagation modelling and its physical background.

In the proposed model, the front motion is splitted into two parts - drifting and fluctuating. Each of them can be solved by using an appropriate method. In the present study, the Eulerian Level Set Method (LSM) is chosen for the drifting part while the fluctuating part is the result of a comprehensive statistical description of the physics of the system and takes into account the randomness of the hot air turbulent transport and fire-spotting. Thus, in order to treat the fluctuating part, specific probability density function has to be taken into account [1].

Fire-spotting is a complicated physical process due to many factors, such as wind, fire intensity, fuel characteristics, atmospheric conditions, etc. The statistical formulation of fire-spotting has been proposed in [1] and completed by the physical parametrisation in [2]. This formulation does not depend on the method used for fire propagation. In the present study we extend this approach and include into account the change of wind direction and possibility of the several secondary fires appearance.

The main aim of the present study is to explain the physical background of the proposed model. The solution of the underlying PDE is connected to a temperature field. The transfer of the temperature due to the turbulent flows is then described by the energy balance equation. The rest of the paper is organized as follows. The wildland fire propagation model is proposed in the next section, including a brief description of the level-set method and physical parametrisation of the fire-spotting. Section 3 deals with the energy balance equation for the proposed model. Numerical examples for several test cases, such as different wind conditions, and merging of the secondary fires, are provided in Section 4.

2 Fire propagation model

2.1 Level-set method

LSM is widely used as effective method for the front-tracking. For some computational domain S the fire front contour is represented by a closed curve Γ. The region bounded by Γ is denoted by Ω and represents the burnt area. Let us introduce an indicator function $\phi(\mathbf{x}, t)$:

$$\phi(\mathbf{x}, t) = \begin{cases} 1, & \mathbf{x} \in \Omega(t), \\ 0, & \text{othervise}. \end{cases} \tag{1}$$

Let $\gamma : S \times [0, \infty) \to \mathbb{R}$ be a level-set function, such that for some fixed γ^* at the moment t the fire front can be described by $\Gamma(t) = \{\mathbf{x} \in S | \gamma(\mathbf{x}, t) = \gamma^*\}$. If $\gamma(\mathbf{x}, t) > \gamma^*$, then the ignition is observed at the point $\mathbf{x}$. The level-set function $\gamma(\mathbf{x}, t)$ evolves according to the following ordinary level-set equation

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$$\frac{\partial \gamma}{\partial t} = V(\mathbf{x}, t)||\nabla \gamma||, \tag{2}$$

where $V(\mathbf{x}, t)$ is a rate of spread (ROS) of the fire front. The ROS value depends on many elements, such as the intensity and direction of the wind, fuel conditions, etc.

2.2 Turbulence and fire-spotting modelling

Together with other factors, turbulence and fire-spotting cause the random motion of the fire-front. Thus, the random front contour can be defined by the effective indicator function [1]:

$$\phi_e(\mathbf{x}, t) = \int_{\Omega(t)} f(\mathbf{x}; t|\bar{\mathbf{x}}) d\bar{\mathbf{x}}, \tag{3}$$

where $f(\mathbf{x}; t|\bar{\mathbf{x}})$ is the probability density function (PDF) that accounts for turbulence and fire-spotting effects. Note that the point is labelled as burnt, if $\phi_e(\mathbf{x}, t)$ exceeds some threshold value ϕ_e^{th}.

In accordance with the Reynold transport theorem, the evolution of the effective indicator function $\phi_e(\mathbf{x}, t)$ takes the form

$$\frac{\partial \phi_e}{\partial t} = \int_{\Omega(t)} \frac{\partial f}{\partial t} d\bar{\mathbf{x}} + \int_{\Omega(t)} \nabla_{\bar{\mathbf{x}}} \left[V(\bar{\mathbf{x}}, t) f(\mathbf{x}; t|\bar{\mathbf{x}}) \right] d\bar{\mathbf{x}}. \tag{4}$$

Evolution equation for the PDF $f(\mathbf{x}; t|\bar{\mathbf{x}})$ takes the form

$$\frac{\partial f}{\partial t} = \epsilon f, \tag{5}$$

where $\epsilon = \epsilon(\mathbf{x})$ is a generic evolution operator. Hence, (4) can be rewritten in the following form

$$\frac{\partial \phi_e}{\partial t} = \epsilon \phi_e + \int_{\Omega(t)} \nabla_{\bar{\mathbf{x}}} \left[V(\bar{\mathbf{x}}, t) f(\mathbf{x}; t|\bar{\mathbf{x}}) \right] d\bar{\mathbf{x}}. \tag{6}$$

In (6) the front-line velocity is controlled by the ROS, while random process, such as turbulence and fire-spotting, are modelled by modifying PDF. Thus, if $f(\mathbf{x}; t|\bar{\mathbf{x}}) = \delta(\mathbf{x} - \bar{\mathbf{x}})$, equation (6) reduces to the deterministic case described by (2).

Assuming that the downwind phenomenon of fire-spotting is independent of turbulence, the random process handled by $f(\mathbf{x}; t|\bar{\mathbf{x}})$ in (6) can be defined as follows

$$f(\mathbf{x}; t|\bar{\mathbf{x}}) = \begin{cases} \int_0^\infty G(\mathbf{x} - \bar{\mathbf{x}} - l\hat{\mathbf{n}}; t) q(l) dl, & \text{if downwind,} \\ G(\mathbf{x} - \bar{\mathbf{x}}; t), & \text{otherwise.} \end{cases} \tag{7}$$

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The shape of the PDF is defined by the isotropic bi-variate Gaussian function (considering turbulence effects) $G(\mathbf{x}-\bar{\mathbf{x}};t)$ and the firebrand landing distribution $q(l)$ is defined by a lognormal distribution as follows

$$q(l) = \frac{1}{\sqrt{2\pi}\sigma l} \exp \frac{-(\ln l/\mu)^2}{2\sigma^2}, \tag{8}$$

where μ is the ratio between the square of the mean of landing distance l and its standard deviation, σ is the standard deviation of $\ln l/\mu$.

In [3] authors complete the study of the fire-spotting proposed in [1] by describing this phenomenon in terms of the fire intensity, wind velocity and fuel characteristics. It includes only the vital ingredients, each firebrand is assumed to be spherical of the constant size. Then lognormal parameters μ and σ in (8) take the following form

$$\mu = H\left(\frac{3\rho_a C_d}{2\rho_f r g}\right)^{1/2}, \quad \sigma = \frac{1}{2z_p}\ln\left(\frac{U^2}{rg}\right), \tag{9}$$

where U is the wind velocity and H is the maximum loftable height, that is according to [4],

$$H = \alpha H_{ABL} + \beta\left(\frac{I}{dP_{f_0}}\right)^{\gamma} \exp\left(-\frac{\delta N_{FT}^2}{N_0^2}\right), \tag{10}$$

where all the parameters are defined in Table 1.

3 Energy balance equation

Wildfire model can be formulated based on balance equations for energy and fuel, as it is proposed in [5,6]. In the present study we follow the level-set formulation with the stochastic process. However, there is a connection between these two formulations. In order to derive the energy balance equation, the physical laws are used, mainly, conservation of energy and fuel reaction. In present study we focus on the solution in the form of indicator function. Thus, it is important to show, that the found solution is connected to the temperature and the energy balance equation can be derived by using the indicator function.

An important part of study deals with the temperature field. Temperature is transferred due to the turbulent flows, and it can be modelled by the diffusion process. The heating-before-burning mechanism, that is accumulation in time of potential fire, can be associated with an amount of heat:

$$\psi(\mathbf{x},t) = \int_0^t \phi_e(\mathbf{x},\eta)\frac{d\eta}{\tau}, \tag{11}$$

where τ is the ignition delay, that can be understood as a resistance to the hot-air heating and firebrand landing in parallel.

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Notation	Description	
α	Part of ABL passed freely, $\alpha < 1$	0.24 [4]
β	$[m]$ Contribution of the fire intensity, $\beta > 0$	170 [4]
γ	Power-law dependence on FRP, $\gamma < 0.5$	0.35 [4]
δ	Dependence on stability of the FT, $\delta \geq 0$	0 [4]
H_{abl}	$[m]$ Height of the atmospheric boundary layer (ABL)	1200
d	$[m\ ]$ Unit depth of the combustion zone	1
P_{f_0}	$[MWm^{-2}]$ Ratio of reference fire power	1 [4]
N_{FT}^2	$[s^{-2}]$ Brunt-Väisälä frequency in the FT	$2.789 \cdot 1e-4$
N_0^2	$[s^{-2}]$ Brunt-Väisälä frequency	$2.5 \cdot 1e-4$
H	$[m]$ The maximum loftable height	
ρ_a	$[kg/m^3]$ Density of the ambient air	1.1
ρ_f	$[kg/m^3]$ Density of the wild-land fuels	542
C_d	Drag coefficient	0.45
z_p	p-th percentile	0.45
r	$[m]$ Brand radius	0.015
g	$[ms^{-2}]$ Acceleration due to gravity	9.81

Table 1: Physical parameters of the atmospheric boundary layer and fire-spotting.

The amount of heat is proportional to the increasing of the temperature $T(\mathbf{x},t)$. For the sake of simplicity, we can assume that

$$\psi(\mathbf{x},t) = \frac{T(\mathbf{x},t) - T_a(\mathbf{x})}{T_{ign} - T_a(\mathbf{x})}, \quad T < T_{ign}, \tag{12}$$

where ambient temperature is denoted by T_a, and T_{ign} stands for the ignition temperature. From (12) one can see, that $\psi(\mathbf{x},t) = 1$ entails that $T(\mathbf{x},t) = T_{ign}$ and the spacial point $\mathbf{x}$ at the moment t belongs to the burning area.

Temperature $T(\mathbf{x},t)$ can be found from (12) as follows

$$T(\mathbf{x},t) = T_a(\mathbf{x}) + \psi(\mathbf{x},t)\left(T_{ign} - T_a(\mathbf{x})\right), \tag{13}$$

that by the differentiation leads to

$$\frac{\partial T}{\partial t} = \frac{(T_{ign} - T_a(\mathbf{x}))}{\tau}\phi(\mathbf{x},t). \tag{14}$$

According to the heat-before-burning mechanism, the temperature field is described by the following reaction-diffusion type equation [1]

$$\frac{\partial T}{\partial t} = \epsilon T + \frac{T_{ign} - T_a}{\tau}\left(I_{\Omega_0}(\mathbf{x}) + W(\mathbf{x},t)\right), \tag{15}$$

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where T_{ign} is the ignition temperature, $T_a(\mathbf{x})$ is the ambient temperature, $I_{\Omega_0}(\mathbf{x}) = \phi_e(\mathbf{x}, 0)$ and

$$W(\mathbf{x}, t) = \int_0^t \left(\int_{\Omega(\theta)} \nabla_{\bar{x}} \cdot [\mathbf{V}(\bar{\mathbf{x}}, \theta) f(\mathbf{x}; \theta|\bar{\mathbf{x}})] \, d\bar{\mathbf{x}} \right) d\theta. \tag{16}$$

Equation (15) can be understood as the energy balance equation associated to the model. In (15) $\epsilon = \epsilon(\mathbf{x})$ is a generic evolution operator, associated to the evolution of $f(\mathbf{x}; t|\bar{\mathbf{x}})$, and it models the turbulent heat transfer due to radiation. Moreover, the second term on the RHS corresponds to the convective heat lost to the atmosphere and the third to the rate of fuel consumed by the fire with Rate of Spread $\mathbf{V}(\bar{\mathbf{x}}, t)$ in the outward direction.

4 Numerical results

In this section some numerical examples are considered in order to study effects of the wind and the choice of underlying PDF on the rate of spread. It is natural that if there is no wind, for the homogeneous moisture the fire front would grow slower and in all directions, as it is shown in Figure 1. If there is wind (we consider $U = 6.7ms^{-1}$, the fire intensity $I = 20MW$), then the fire propagates in the downwind direction (see Figure 2).

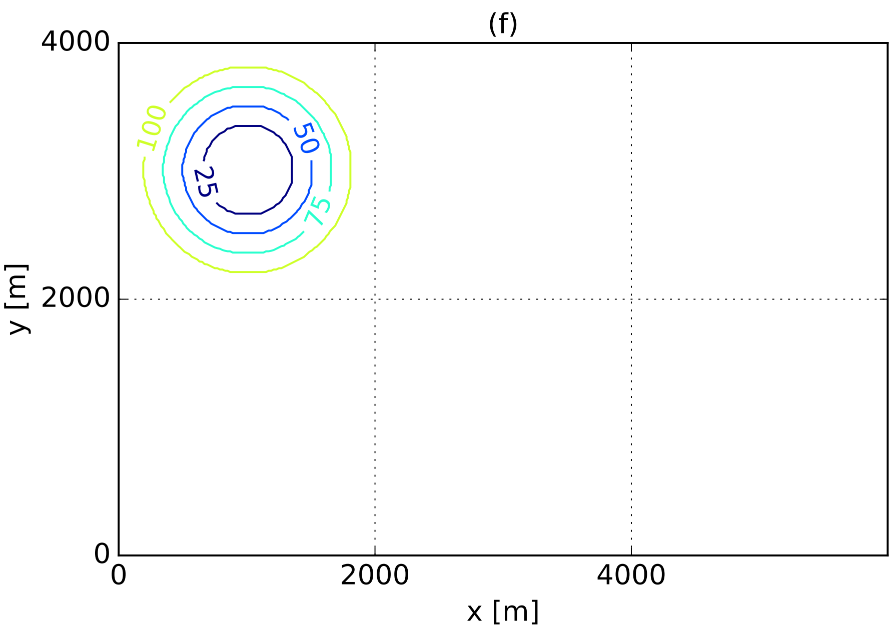

Figure 1: Fire propagation with zero wind.

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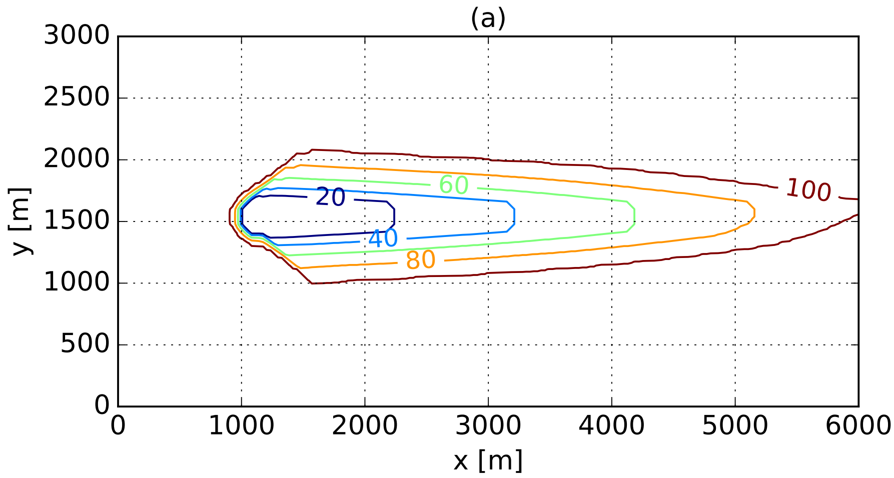

Figure 2: Fire propagation with mean wind $U = 6.7ms^{-1}$.

The model treats the fire-spotting. In order to model this phenomenon the lognormal PDF (8), as it is shown in Figure 3. The point of ignition of the secondary fire also depends on the wind direction. If at some moment $t = t^*$ the wind changes the direction, then new secondary fires appear in new direction, as it is shown in Figure 4. For the sake of simplicity, the initial conditions for new ignitions are chosen the same as for the main fire zone, that causes the same form of the secondary fires.

However, not for any set of the parameters the sire-spotting can be observable. In some cases the fire intensity is not high enough to let the firebrands jump from the main column and produce new independent fire. Moreover, it can occur the situation when the firebrand jumps not that far, so it merges to the main fire changing the curvature of the fire front (see Figure 5).

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References

[1] G. Pagnini and A. Mentrelli, "Modelling wildland fire propagation by tracking

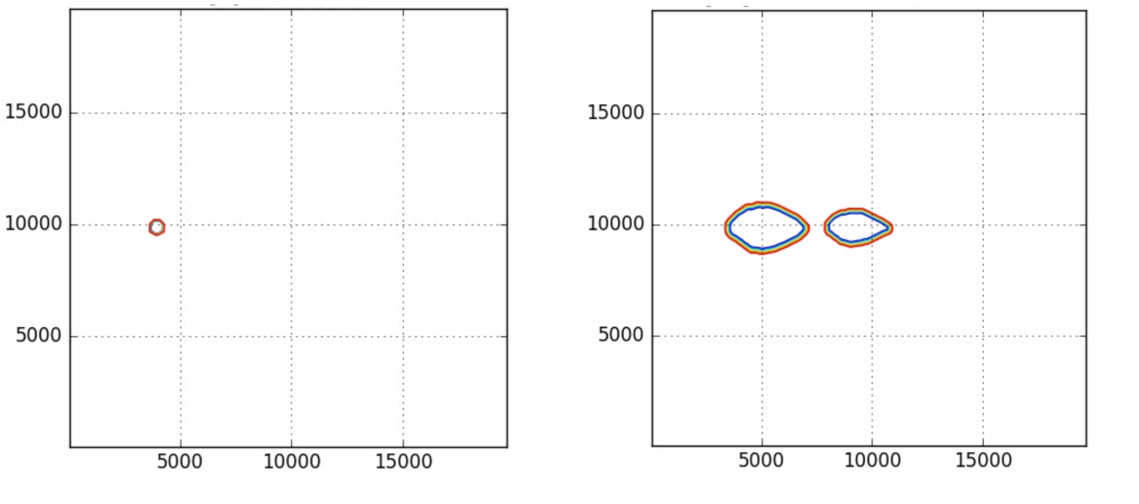

Figure 3: Fire-spotting effect (initial moment $t = 0$ and $t = 74$ min).

random fronts," *Natural Hazards and Earth System Sciences*, vol. 14, no. 8, pp. 2249–2263, 2014.

[2] I. Kaur, A. Mentrelli, F. Bosseur, J.-B. Filippi, and G. Pagnini, "Turbulence and fire-spotting effects into wild-land fire simulators," *Communications in Nonlinear Science and Numerical Simulation*, vol. 39, pp. 300 – 320, 2016.

[3] I. Kaur and G. Pagnini, "Fire-spotting modelling and parametrisation for wild-land fires," *International Congress on Environmental Modelling and Software*, Paper 55, 2016.

[4] M. Sofiev, T. Ermakova, and R. Vankevich, "Evaluation of the smoke-injection height from wild-land fires using remote-sensing data," *Atmospheric Chemistry and Physics*, vol. 12, no. 4, pp. 1995–2006, 2012.

[5] M. I. Asensio and L. Ferragut, "On a wildland fire model with radiation," *InternationalL Journal for Nuerical Methods in Engineering*, vol. 54, pp. 137-157,2002.

[6] J. Mandel, L. S. Bennethuma, J. D. Beezley, J. L. Coenb, C. C. Douglas, M. Kima, and A. Vodacek, "A wildland fire model with data assimilation," *Mathematics and Computers in Simulation*, vol. 79, pp. 584-606,2008.

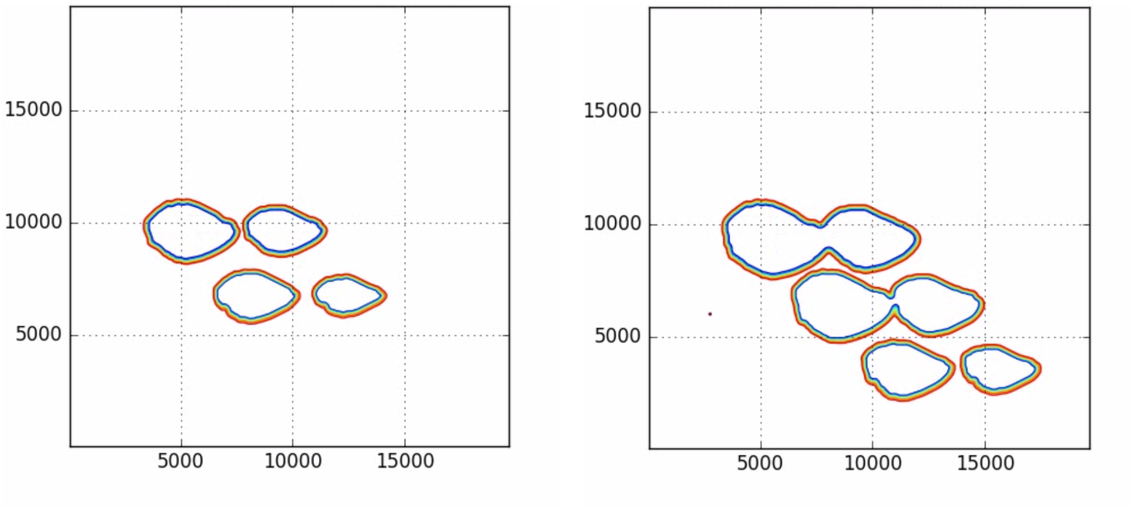

Figure 4: Fire-spotting effect with wind changing ($t = 90$ and $t = 110$ min).

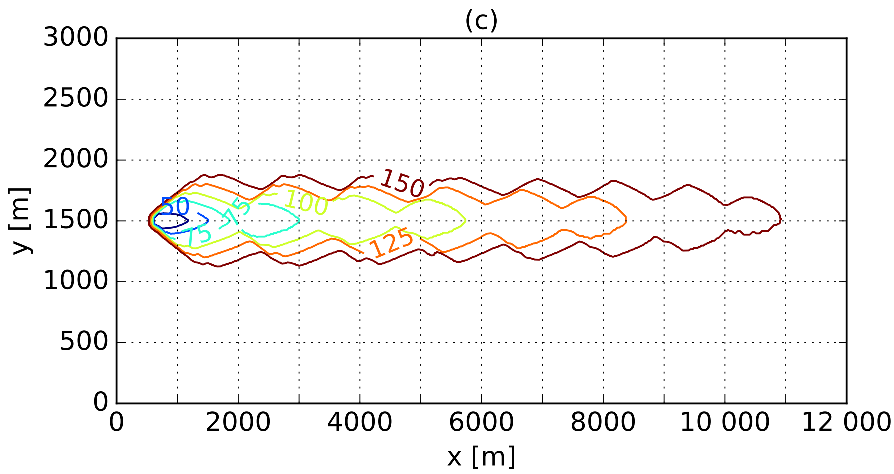

Figure 5: Secondary fire merges to the main fire.

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