Wildland fire propagation modelling

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November 30, 2017

1 Introduction

Wildfire propagation modelling is a challenging problem due to its complex multi-scale multi-physics nature. This process can be described by a reaction-diffusion equation based on the energy balance principle, see e.g. [1]. Alternative technique is the so-called level-set method (LSM) [6] and it is used in wildfire modelling [4] as well as in many other fields. In present study a methodology for fire propagation modelling that reconciles these approaches is proposed. This methodology is distinguishable and significant from both academical and industrial point of view because of the inclusion of the random effects by preserving the existing algorithms and direct implementation as a post-processing numerical routine.

The random behaviour of the fire front is caused, for example, by the turbulence and the fire-spotting phenomenon. A probability density function (PDF) is employed in order to describe the random process. In earlier studies [5] it has been shown that new independent ignitions can increase the rate of spread (ROS) of fire and therefore should be carefully studied. In this respect,

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a physical parametrization of the fire-spotting distribution was proposed [3]. Special attention in the present study is paid to the atmospheric stability conditions. The parametrization proposed in Ref. [3] is completed by the multiple fire-spotting modelling. Afterwards special attention is paid to the study of uniqueness of the PDF and consistency with the energy balance equation. Numerical results and discussions complete the study.

2 Fire front model and non-uniqueness

Detailed explanation of the proposed method can be found in [2, 5]. Here we briefly describe the idea. Fire propagation is splitted into a drifting part and a fluctuating part. The drifting part is handled by a chosen existing method, while the fluctuating part that includes the random effects is found as a results of a comprehensive statistical description of the physical system. Thus, denoting the burning area by $\Omega(t)$, the evolution of the fire front can be defined by the following novel family of reaction-diffusion equations

$$\frac{\partial \phi_e}{\partial t} = \int_{\Omega(t)} \frac{\partial f}{\partial t} d\bar{\mathbf{x}} + \int_{\Omega(t)} \nabla_{\bar{\mathbf{x}}} \left[V(\bar{\mathbf{x}}, t) f(\mathbf{x}; t|\bar{\mathbf{x}}) \right] d\bar{\mathbf{x}}, \tag{1}$$

where $\phi_e(\mathbf{x}, t) = \int_{\Omega(t)} f(\mathbf{x}; t|\bar{\mathbf{x}}) d\bar{\mathbf{x}}$ is an effective indicator function, such that for some threshold value ϕ_e^* expression $\phi_e(\mathbf{x}, t) \geq \phi_e^*$ represents the burning area. $V(\mathbf{x}, t)$ is the ROS of the fire front, $f(\mathbf{x}; t|\bar{\mathbf{x}})$ is the PDF that accounts for turbulence and fire-spotting effects.

Wildfire model can be formulated as the balance equations for energy and fuel, see [1]. However, as it is stated in [5], the reaction-diffusion equation and the LSM "can indeed be considered complementary and can be reconciled" by the proposed technique when $\Omega(t)$ is the burned area estimated by the LSM approach. The connection between these two approaches is the connection between the indicator function ϕ_e and the temperature T, that can be expressed in an amount of heat:

$$\psi(\mathbf{x}, t) = \int_0^t \phi_e(\mathbf{x}, \eta) \frac{d\eta}{\tau} = \frac{T(\mathbf{x}, t) - T_a(\mathbf{x})}{T_{ign} - T_a(\mathbf{x})}, \quad T < T_{ign}, \tag{2}$$

where τ is the ignition delay. From (2) one can see, that $\psi(\mathbf{x}, t) = 1$ entails that $T(\mathbf{x}, t) = T_{ign}$ and the spacial point $\mathbf{x}$ at the moment t belongs to the burning area. Derivation of the energy balance equation can be found in [5], here we provide the result:

$$\frac{\partial T}{\partial t} = \epsilon T + \frac{T_{ign} - T_a}{\tau}\left(I_{\Omega_0}(\mathbf{x}) + W(\mathbf{x}, t)\right), \tag{3}$$

where T_{ign} is the ignition temperature, $T_a(\mathbf{x})$ is the ambient temperature, $I_{\Omega_0}(\mathbf{x}) = \phi_e(\mathbf{x}, 0)$ and $W(\mathbf{x}, t)$ is the rate of fuel consumed by the fire.

Equation (3) can be understood as the energy balance equation associated to the model. This preliminary result will be completed by the numerical comparison with existing models.

Denoting the wind velocity by U, the shape of the PDF is defined by the isotropic bi-variate Gaussian function (considering turbulence effects) $G(\mathbf{x} - \bar{\mathbf{x}}; t)$ and the firebrand landing distribution $q(l)$ as follows

$$f(\mathbf{x}; t|\bar{\mathbf{x}}) = \begin{cases} \int_0^\infty G(\mathbf{x} - \bar{\mathbf{x}} - l\hat{\mathbf{n}}; t) q(l) dl, & \text{downwind}, \\ G(\mathbf{x} - \bar{\mathbf{x}}; t), & \text{otherwise}, \end{cases} \tag{4}$$

where $q(t)$ is the lognormal distribution $q(l) = \frac{1}{\sqrt{2\pi}\sigma l} \exp \frac{-(\ln l/\mu)^2}{2\sigma^2}$. Parameter μ is the ratio between the square of the mean of landing distance l and its standard deviation, σ is the standard deviation of $\ln l/\mu$.

The physical parametrisation of the fire-spotting distribution is considered with the following parameters [3],

$$\mu = H\left(\frac{3\rho_a C_d}{2\rho_f r g}\right)^{1/2}, \quad \sigma = \frac{1}{2z_p} \ln\left(\frac{U^2}{rg}\right), \tag{5}$$

where, according to [7], the maximum loftable height is described in terms of atmospheric stability conditions, such that H_{ABL} is the height of the atmospheric boundary layer and N^2 is the Brünt-Väsäla frequency.

Taking into account the wind direction through the angle θ, σ defined in (5) becomes

$$\sigma = \frac{1}{2z_p} \ln\left(\frac{(U\cos\theta)^2}{rg}\right) \geq \sigma_0, \text{ with } \cos\theta \geq \frac{\sqrt{rg}}{U} \exp(z_p \sigma_0), \tag{6}$$

where σ_0 is minimum possible value of σ.

Thus, multiple fires due to the so-called fire-spotting can be modelled since secondary fires appear where (6) holds.

The uniqueness of the the effective indicator function ϕ_e requires a discussion. This analysis is based on the theorem that if an integral of a non-negative function is zero then the function is zero almost everywhere. The result agrees also with the Radon-Nikodym Theorem, that shows that there is no one-to-one correspondence between the PDF and the effective indicator function and the following statement can be formulated: Some burning area, described by the effective indicator function can be generated by the different PDFs.

This is an important issue since it allows to model a complex topology of the burning area by a simple suitable PDF. For instance, if the fuel inside the fire zone is completely burned out the combustion ceases, that leads to the crown or ring form of the burning area since the ignitions are observed only at the leading edge of the fire perimeter. However, fire propagation in such domain can be modelled by the PDF (4) as for the entire one just with the suitable parameters.

Due to parametrization (5)-(6) the dependence of the fire front upon the atmospheric stability is found. However, there are a lot of factors that can effect the process. Such that influence of the flame geometry, mainly flame length, is going to be studied in the future.

3 Numerical results and discussions

In this section some numerical examples are provided to show the results of parametrization and multiple fire-spotting effects. The choice of fire-spotting distribution parameters is crucial for the model since it manages the fire brand travel distance. As it is shown in Figure 1 (left), the merging effect can be observed for certain parameters μ and σ.

Inclusion of condition (6) into the numerical algorithm results in multiple secondary fires in some angle θ, as it is presented in Figure 1 (right).

The proposed numerical routine can also take into account a firebreak zones. From the real data it is known that when the fire front reaches the fire-break, it stops for a while, but then it crosses the zone and continues the propagation. Moreover, spot fires can overcome the firebreak, that can cause dangerous effects. Numerical simulations for a such case are presented in Figure 2. This is a very important issue for the fire-fighters management.

The proposed method reconciles existing wildfire propagation models improving operational codes by the significant post-processing numerical algorithm. The model includes effects of turbulence and fire-spotting that results in random fire front. It allows to model the fire that overcomes a fire-break zone and increase the rate of spread. This improvement has an important

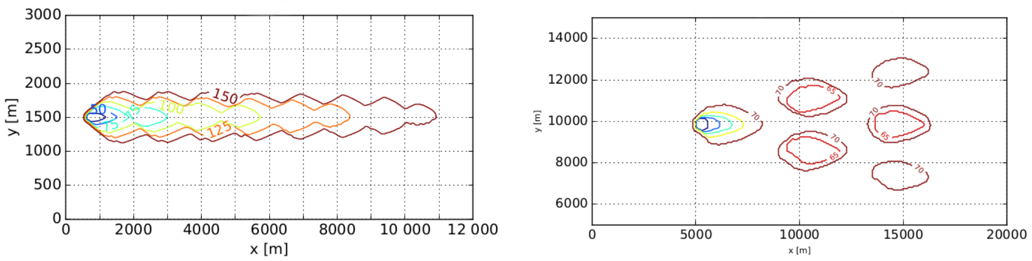

Figure 1: Fire-spotting effect: merging secondary fires with $\mu = 12$, $\sigma = 8$ (left) and multiple fire-spotting with $\mu = 9.75$ and $\sigma = 8.15$ and incorporation of (6) (right).

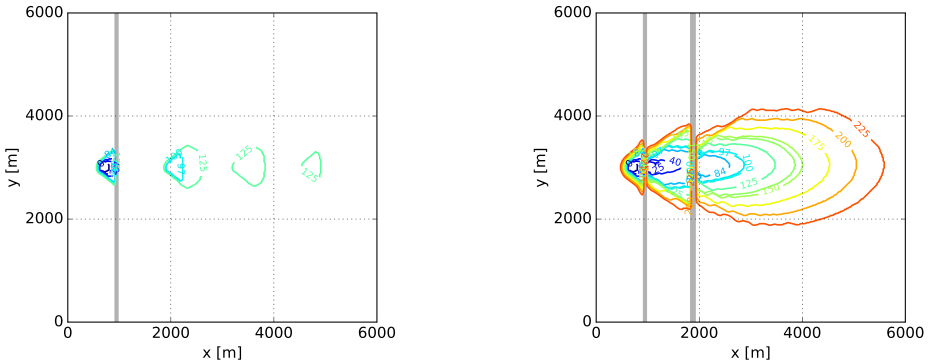

Figure 2: Wildfire propagation in the presence of the firebreak zone.

economical impact since it can be helpful for fire suppression and control. Parametrization of the fire-spotting distribution leads to the definition of the angle of the multiple fire-spotting and allows the future consideration of other factors, such as flame geometry.

The future intention is to incorporate data assimilation algorithms in order to adopt the methodology to the real topography, as well as to calibrate the model.

Acknowledgements

This research is supported by the Basque Government through the BERC 2014-2017 program and by Spanish Ministry of Economy and Competitiveness MINECO through BCAM Severo Ochoa excellence accreditation SEV-2013-0323 and through project MTM2016-76016-R "MIP" and by the PhD grant "La Caixa 2014".

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