Nature-inspired Heuristics for the Multiple-Vehicle Selective Pickup and Delivery Problem under Maximum Profit and Incentive Fairness Criteria

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***Abstract*—This work focuses on wide-scale freight transportation logistics motivated by the sharp increase of on-line shopping stores and the upsurge of Internet as the most frequently utilized selling channel during the last decade. This huge ecosystem of one-click-away catalogs has ultimately unleashed the need for efficient algorithms aimed at properly scheduling the underlying transportation resources in an efficient fashion, especially over the so-called *last mile* of the distribution chain. In this context the selective pickup and delivery problem focuses on determining the optimal subset of packets that should be picked from its origin city and delivered to their corresponding destination within a given time frame, often driven by the maximization of the total profit of the courier service company. This manuscript tackles a realistic variant of this problem where the transportation fleet is composed by more than one vehicle, which further complicates the selection of packets due to the subsequent need for coordinating the delivery service from the command center. In particular the addressed problem includes a second optimization metric aimed at reflecting a fair share of the net benefit among the company staff based on their driven distance. To efficiently solve this optimization problem, several nature-inspired meta-heuristic solvers are analyzed and statistically compared to each other under different parameters of the problem setup. Finally, results obtained over a realistic scenario over the province of Bizkaia (Spain) using emulated data will be explored so as to shed light on the practical applicability of the analyzed heuristics.**

I. Introduction

Technological advances on the security and privacy of electronic commerce and the subsequently better appreciation of the use of bank details in Internet have lately caused an unprecedented blossom of on-line stores and shopping websites, allowing users to remotely purchase almost any good and have it delivered at their place within deadlines of their convenience. The proliferation of hand devices such as tablets and smart-phones, along with the improvement of mobile Internet access has also favored the ubiquity of users when exploring and eventually utilizing such on-line stores. Indeed, according to the research report published in [1] the parcel delivery industry is projected to grow 9 % annually to more than 343 billion USD by 2020, with a 49 % penetration in mobile phones by 2017. Such figures are propelled by a deeper digitalization of this sector, which reduces costs and permits to aggregate auxiliary yet valuable services for the end user.

From the perspective of logistics the noted increase of the number and rate of on-line purchases calls for new methods aimed at efficiently allocating corporate transportation resources at different levels of the distribution chain [2]. At times where cost efficiency lies at the core of all processes within a service company, decision makers target at ensure that all their assets and tools are used efficiently, when needed and towards protecting the economical sustainability of the company by reducing costs to their minimum. This is the rationale why many problems in delivery logistics adopt economical profitability – computed as the difference between net incomes and the sum cost of the followed route(s) – as their main criteria for optimality [3], besides other objectives that impact on revenues as well (e.g. minimum completion time, inconvenience of the client or the number of vehicles).

When such economical terms involve a personal sacrifice of the transporter (e.g. long periods off his/her place), fairness should enter the picture as a criterion to be necessarily addressed in the scheduling of delivery resources. In ground logistics rewards per packet delivered in time are often the mechanism to ensure a fair share of the price for a certain service: the more preferential a packet is to be delivered, the higher the reward will be, which usually comes along with more stringent deadlines and/or distances to be driven for its successful delivery. This is often the case for autonomous transporters, whose salary comes in general as a proportional share of these rewards [4]. However, when dealing with relatively large transport companies this approach does not hold: the money paid for the transport service is collected by the company, from which base salaries and incentives for its personnel are retrieved. Such extra payments aim at fairly compensating employees depending on their performance when working for the company. Among the very diverse criteria adopted for establishing such primes (most depending on the individual price of the delivered good) the overall distance traveled by the transporter can be deemed as one of the most objective rewarding schemes. By taking into account fairness

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within the operations of a transport company decision makers may trade the maximization of the revenue of the overall company for the satisfaction of the staff with respect to the acknowledgments of their individual performance and results which, despite its relevance in many operational and relational aspects, is very rarely addressed in practice.

This being said, this manuscript gravitates on the selective pickup and delivery problem, a relaxed version of the static pickup and delivery problem [5] where the command control of the transport company must optimally allocate the corporate fleet to a set of pickup and delivery services, part of which is to be processed and completed within varying time frames. The optimization goal is to determine the subset and sequence of packets to be picked and delivered by each vehicle of the fleet so as to maximize a measure of fitness for the transport company. While most of the related literature focuses on different approximations of the same profit-maximizing statement of the problem [6], [7], [8], [9], [10], [11], the approach taken in this paper considers fairness as a second criteria postulated to clash with the overall profit of the company. In other words, if no objective criteria for a fair share of the profit were applied, the members of the staff could be granted with unfairly high commissions disregarding whether e.g. their traveled distances are comparable to those of other colleagues having delivered less profitable packets. While a number of contributions have hitherto gravitated on algorithmic derivations for other multi-objective formulations of the pickup and delivery problem, to the knowledge of the authors they all focus on operational aspects of the delivery service such as route length, response time, vehicle capacity and workload [12], [13], thus neglecting the social side of the company in regards to the fair distribution of its net profit.

This work focuses on mathematically modeling the above paradigm as a bi-objective optimization problem, for whose computationally efficient resolution a set of nature-inspired solvers will be proposed and compared to each other: a Non-Dominated Sorting Genetic Algorithm (NSGA-II, [14]) and multi-objective versions of Harmony Search [15], Firefly Algorithm [16] and Ant Colony Optimization [17]. All these algorithms will be compared to each other in several synthetic scenarios by resorting to multi-objective performance metrics and hypothesis tests. Finally, an emulated yet realistic scenario over the province of Bizkaia (Spain) will be designed and discussed to validate the practicality of the analyzed heuristics.

The rest of the manuscript is structured as follows: Section II introduces the mathematical notation of the problem and formulates it formally, whereas Section III describes in detail the proposed multi-objective algorithms and their modifications to tackle the problem. Next Section IV discusses the simulation benchmark and finally, Section V concludes the paper.

II. System Model and Problem Formulation

In reference to Fig. 1 we assume a packet delivery company employing a staff of N different courier agents, which are in charge for picking up and delivering goods from their origin to their destination within an associated time window. Such a time window spans from the very moment the item at hand is made available for pickup in a certain location to a maximum delivery deadline. Mathematically we will define the P-sized set of available items as $\boldsymbol{\mathcal{P}} \doteq \{\mathbf{P}_p\}_{p=1}^{P}$, where each item compounding this set is given by

$$\mathbf{P}_p \doteq \{T_p^{\uparrow}, L_p^{\uparrow}, T_p^{\downarrow}, L_p^{\downarrow}, R_p\}, \tag{1}$$

with $T_p^{\uparrow}$ ($T_p^{\downarrow}$) denoting pickup (delivery) times; $L_p^{\uparrow}$ ($L_p^{\downarrow}$) pickup (delivery) locations; and $R_p \geq 0$ the reward associated to the successful delivery of the item to its destination in time. Locations are assumed to be drawn from a finite set of cities $\mathcal{L}$ with size $|\mathcal{L}|$. A route followed by agent $n \in \{1, \ldots, N\}$ can be casted as a vector of M_n integers $\mathbf{L}_n^{\triangleright} = \{L_n^{\triangleright,1}, L_n^{\triangleright,2}, \ldots, L_n^{\triangleright,M_n}\}$, with $L_n^{\triangleright,m} \in \mathcal{L}$ representing the city visited by courier agent n in m-th order within his/her M_n-length schedule. Depending on the time $T_n^{\bullet,m}$ at which agent n effectively arrives at the m-th city along the schedule he/she will be able to pick up items for their delivery provided that 1) they have not been already collected by any other courier agent; and 2) $T_n^{\bullet,m} \leq T_p^{\uparrow} \leq T_n^{\circ,m}$, i.e. the item is available within the time frame during which the courier stays in the city. It is assumed that when dealing with concurrent arrivals items are picked up by the courier that first reached the city at hand.

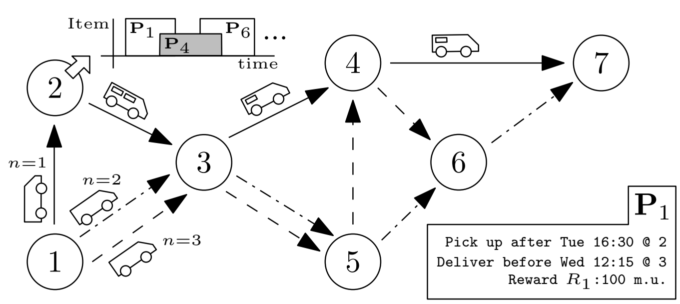

Fig. 1. Problem setup for $N = 3$ agents and $|\mathcal{L}| = 7$ cities, detailed information associated to item $\mathbf{P}_1 = \{\text{Tue 16:30}, 2, \text{Wed 12:15}, 3, 100\}$, and example of the schedule followed by agent $n = 1$, who does not pick up and deliver packet $\mathbf{P}_4$ due to the long travel time between cities 3 and 4.

The overall set of pickup and delivery routes followed by the N courier agents, denoted as $\{\mathbf{L}_n^{\triangleright}\}_{n=1}^{N} \doteq \boldsymbol{L}^{\triangleright}$, effectively delivers a subset $\boldsymbol{\mathcal{P}}(\boldsymbol{L}^{\triangleright}) \subseteq \boldsymbol{\mathcal{P}}$ of the overall set of available items $\boldsymbol{\mathcal{P}}$. This subset can be expressed as the union of the item sets processed by each agent deployed in the scenario at hand. Mathematically:

$$\boldsymbol{\mathcal{P}}(\boldsymbol{L}^{\triangleright}) \doteq \boldsymbol{\mathcal{P}}(\boldsymbol{L}_1^{\triangleright}) \cup \boldsymbol{\mathcal{P}}(\boldsymbol{L}_2^{\triangleright}) \cup \ldots \cup \boldsymbol{\mathcal{P}}(\boldsymbol{L}_N^{\triangleright}), \tag{2}$$

where

$$\boldsymbol{\mathcal{P}}(\boldsymbol{L}_n^{\triangleright}) \doteq \big\{p \in \{1, \ldots, P\} : \; L_p^{\uparrow} \in \boldsymbol{L}_n^{\triangleright}, \; L_p^{\downarrow} \in \boldsymbol{L}_n^{\triangleright}, \tag{3}$$

$$T_n^{\bullet,m} \leq T_p^{\uparrow} \text{ and } T_n^{\circ,m} \leq T_p^{\uparrow} \text{ if } T_n^{\bullet,m} = L_p^{\uparrow}, \tag{4}$$

$$T_n^{\bullet,m} < T_{n'}^{\bullet,m} \leq T_p^{\uparrow} \text{ for any } n' \neq n \big\}, \tag{5}$$

where the constraint imposed in (5) reflects that no other courier $n' \neq n$ should have arrived at the pickup location of the item at hand before courier n. With this notation in mind the total income received by the courier company as a reward for delivering the subset of items $\boldsymbol{\mathcal{P}}(\boldsymbol{L}^{\triangleright})$ will be given, in monetary units (hereafter m.u.), by

$$I(\boldsymbol{L}^{\triangleright}) \doteq \sum_{n=1}^{N}\sum_{p\in\boldsymbol{\mathcal{P}}} R_p \cdot \mathbb{I}\{p \in \boldsymbol{\mathcal{P}}(\boldsymbol{L}_n^{\triangleright})\} \text{ [m.u.]}, \quad (6)$$

where $\mathbb{I}(\cdot)$ is an auxiliary function taking value 1 if the condition in its argument is true and 0 otherwise. The net income achieved by the courier company results from subtracting the travel costs incurred by each agent when driving from one location to another within his/her route. If the distance from location L_m to $L_{m'}$ is denoted as $d_{m,m'}$ [km] and costs are compiled as a generic quantity per kilometer C_{km} [m.u.] (aggregating e.g. fuel consumption and tire wear costs), the net income is thus expressed as

$$I_{\aleph}(\boldsymbol{L}^{\triangleright}) = I(\boldsymbol{L}^{\triangleright}) - \sum_{n=1}^{N}\sum_{m=2}^{M_n} C_{km} d_{L_n^{\triangleright,m\text{-}1}, L_n^{\triangleright,m}}, \quad (7)$$

which embodies the first objective considered in our problem formulation. This net income is collected by the courier company, from which incentives to the agents are computed proportionally depending on the contribution made by each agent to the corporate profit. The ratio $\zeta_n \in \mathbb{R}[0,1]$ given by

$$\zeta_n \doteq \frac{\sum_{p\in\boldsymbol{\mathcal{P}}} R_p \mathbb{I}\{p \in \boldsymbol{\mathcal{P}}(\boldsymbol{L}_n^{\triangleright})\} - \sum_{m=2}^{M_n} C_{km} d_{L_n^{\triangleright,m\text{-}1}, L_n^{\triangleright,m}}}{I_{\aleph}(\boldsymbol{L}^{\triangleright})},$$

provides a quantitative estimation of the aforementioned contribution of agent $n \in \{1,\ldots,N\}$ to the net profit of the company, such that the incentive to this staff member might be computed proportionally as $\propto \zeta_n I_{\aleph}(\boldsymbol{L}^{\triangleright})$ [m.u.]. However, this ratio is uncoupled with respect to several factors that should be taken into account by the company in order to avoid unfairly distributing incentives among the personnel. In particular we focus on the distance driven by the agent to collect items along his/her route, which might not be related at all to the reward R_p associated to such goods. This issue is particularly involved in those situations where traveled distances are long enough to require overnight work shifts. In this hypothesized scenario it is deemed mandatory to enforce an incentive policy that balances between the business profitability of the company and a fair share of the profit among its employees.

To numerically assess the difference between the distance-aware distribution of the profit and the incentive policy given by factors $\{\zeta_n\}_{n=1}^N$, a set of distance-aware ratios $\{\lambda_n\}_{n=1}^N$ can be computed based on the total distance traveled by agent n when following route $\mathbf{L}_n^{\triangleright}$, yielding

$$\lambda_n \doteq \frac{\sum_{m=2}^{M_n} d_{L_n^{\triangleright,m\text{-}1}, L_n^{\triangleright,m}}}{\sum_{n'=1}^{N}\sum_{m=2}^{M'_n} d_{L_n^{\triangleright,m\text{-}1}, L_{n'}^{\triangleright,m}}} \in \mathbb{R}[0,1], \quad (8)$$

from which the difference between policies enforced by $\{\zeta_n\}_{n=1}^N$ and $\{\lambda_n\}_{n=1}^N$ can be quantified as the relative mean error (RME) between both vectors, given by

$$\text{RME}(\boldsymbol{L}^{\triangleright}) \doteq \frac{100}{N}\sum_{n=1}^{N}\frac{|\zeta_n - \lambda_n|}{|\zeta_n| + |\lambda_n|} \text{ [\%]}, \quad (9)$$

where the case $\zeta_n = \lambda_n$ is forced to yield a value of 0. The problem addressed in this paper seeks a group of route sets $\{\boldsymbol{\mathcal{L}}^{\triangleright,*,k}\}_{k=1}^K$ for the agents of the courier company different albeit optimally balancing – in the Pareto sense – the trade-off between the maximization of the net income of the company and the fairness in the distribution of incentives among its staff members. In mathematical notation,

$$\{\boldsymbol{L}^{\triangleright,*,k}\}_{k=1}^K = \arg_{\boldsymbol{L}^{\triangleright}}\left[\max I_{\aleph}(\boldsymbol{L}^{\triangleright}),\ \min \text{RME}(\boldsymbol{L}^{\triangleright})\right], \quad (10)$$

subject to a maximum time horizon $T_n^{\bullet,m}|_{m=L_n^{\triangleright,M_n}} \leq T_{max}$ for all agents $n \in \{1,\ldots,N\}$. It should be clear that in the above problem formulation an interesting interaction between both objectives arise in regards to the sequence of cities visited by every courier agent. It is not a priori straightforward to decide whether a service should be processed and hence resources – namely, an agent – allocated to pick up and deliver the item in question: not only the associated reward R_p should be evaluated, but also the time frame within which the service should be accomplished, the commitments and relative distance of agents to the locations involved in the service and the fairness of their assigned incentives with respect to their already traveled distances. Also important is to observe that not all items may be delivered to their destination as a result of the service selectiveness assumed for the problem (as in e.g. [7] and references included therein).

Despite the relaxation with respect to its non-selective counterpart, the complexity of the above problem requires the adoption of approximate heuristic solvers that sacrifice optimality of their produced solutions for an enhanced efficiency of their search procedure. In this regard 4 different nature-inspired solvers will be next described and compared to each other, all relying on observed behaviors in Nature.

III. Bio-inspired Multi-Objective Solvers

In what follows different algorithms for multi-objective optimization problems will be thoroughly described. They all share a similar solution encoding approach to represent the sequence of cities visited by each courier, with times taken to traverse from one location to another resolved during the fitness calculation. In particular the k-th solution $\boldsymbol{L}^{\triangleright}(k)$ to the problem is represented by integers, which are in turn the index of every visited city in the set $\mathcal{L}$. Therefore, the solution vector can be unfolded as

$$\boldsymbol{L}^{\triangleright}(k) = \{\boldsymbol{L}_1^{\triangleright}(k), \boldsymbol{L}_2^{\triangleright}(k), \ldots, \boldsymbol{L}_N^{\triangleright}(k)\}, \quad (11)$$

namely, as the concatenation of the labels of the cities visited by each user along his/her proposed schedule. When needed, the heuristic operators governing the search process of the considered heuristics will be modified with respect to their nominal definition so as to account for the particular structure of the encoding strategy.

A. Non-Dominated Sorting Genetic Algorithm (NSGA)

The first heuristic approach considered in our benchmark is the Nondominated Sorting Genetic Algorithm (NSGA) widely utilized for tackling multi-objective problems stemming from different disciplines. In essence this evolutionary solver hinges on a selection strategy that jointly considers both the Pareto optimality and spread of produced solutions along iterations. To this end candidate vectors are ranked according to whether they are dominated by other individuals from the pool of available solutions. Diversity among non-dominated solutions is ensured by a second criterion that allows sorting individuals within a given dominance rank in decreasing order of the distance to their closest neighbors along each of the objectives (crowding distance). If we denote as $\Re(k) \in \mathbb{N}$ and $\text{CD}(k)$ the non-dominance rank and crowding distance of solution $\boldsymbol{L}^{\triangleright}(k)$, we will prefer solution $\boldsymbol{L}^{\triangleright}(k)$ to $\boldsymbol{L}^{\triangleright}(k')$ if 1) $\Re(k) < \Re(k')$; or 2) if $\Re(k) = \Re(k')$ and $\text{CD}(k) > \text{CD}(k')$. This case is often expressed as $k \prec k'$.

Algorithm 1: `NSGA` approach.

Input : Population size P, number of iterations $\mathcal{I}$, mutation probability P_c, selection, crossover, mutation and replacement operators.

Output: Approximated Pareto set $\{\boldsymbol{L}^{\triangleright}(k)\}_{k=1}^{K_{\text{NSGA}}}$

1 Generate initial population of P candidate solutions
2 Set iteration counter $it = 1$
3 **while** $it \leq \mathcal{I}$ **do**
4 Generate a new offspring population from the pool of individuals using the selected selection strategy and crossover operator with probability P_c
5 Apply mutation operator with probability P_m to each produced offspring
6 Evaluate $I_{\aleph}(\cdot)$ and $\text{RME}(\cdot)$ for the offspring
7 Combine newly produced individuals and the previous population
8 Identify ranks $\{\Re(k)\}$ and compute crowding distances $\{\text{CD}(k)\}$ over the combined population
9 Sort and choose P individuals based on non-dominance ranking and crowding distance ($\prec$)
10 **end**
11 The estimated Pareto front $\{\boldsymbol{L}^{\triangleright}(k)\}_{k=1}^{K_{\text{NSGA}}}$ is given by the individuals with $\Re(k) = 1$ in the population

Genetic operators for crossover and mutation abound in the related literature. In our case we opt for uniform crossover, by which each produced offspring is the result of a uniformly random, component-wise mating between two selected parents. It should be noted that as opposed to other mating strategies such as N-point crossover, this operator can be applied in its naïve form at no risk of mixing genotype corresponding to schedules of different courier agents. Likewise, uniform mutation is chosen so as to imprint genetic differences between mating individuals and their descendants. Parents are selected by means of a binary tournament procedure where pairwise matches are resolved based on Pareto dominance.

B. Non-Dominated Sorting Harmony Search (`NSHS`)

The second multi-objective solver considered in this study is similar to `NSGA` except for the heuristic operators that control its search procedure. In this case genetic crossover and mutation operators (lines 4 and 5 in Algorithm 1) are replaced with those of Harmony Search [15], a population-based meta-heuristic optimization algorithm that inspires from the music composition process observed in music bands to solve complex optimization problems in diverse application scenarios [18]. Despite its apparent similarity to other evolutionary schemes such as Evolution Strategies, there are subtle differences in the definition of the HS operators aimed at emulating the behavior of musicians when improvising new harmonies under an aesthetic measure of musical quality:

- The Harmony Memory Considering Rate is controlled by a parameter HMCR $\in \mathbb{R}[0, 1]$ that establishes the probability that the new value for a variable $L_n^{\triangleright,m}(k)$ is drawn uniformly at random from the discrete set

$$\{L_n^{\triangleright,m}(1), \ldots, L_n^{\triangleright,m}(k-1), L_n^{\triangleright,m}(k+1), \ldots, L_n^{\triangleright,m}(\text{HM})\}$$

  i.e. from the values taken by the same variable in the rest of candidate solutions in the HM-sized population (often referred to as Harmony Memory). This can be indeed regarded as a probabilistically driven uniform crossover operator with polygamy which, as claimed in [19] and rebutted in [20], [21], resembles an instance of $(\mu + \lambda)$ Evolution Strategies.
- The Pitch Adjustment Rate is again driven by a probabilistic parameter PAR $\in \mathbb{R}[0, 1]$, which sets the probability that the value of any given optimization variable $L_n^{\triangleright,m}(k)$ is replaced by any of its neighboring values in the variable alphabet, with equal probability. While this operator could be deemed as a sort of random mutation with restricted support, it is in the implicit definition of neighborhood where the novelty of this local search strategy resides: values along the alphabet should be sorted depending on their expected influence of the fitness function to be optimized. This, however, comes along with a penalty in regards to the capability of the algorithm to explore globally the search space at hand, which is often circumvented by adding a third operator (coined as Random Selection Rate [22]).

Considering these differences a multi-objective approach of the HS algorithm can be built in a straightforward manner by replacing the standard `plus` replacement method of the original algorithm (i.e. survival of the best HM individuals among the newly produced ones and those remaining from the previous iterations) with the `NSGA` criterion based on non-dominance ranking and crowding distance explained in the previous section. The resulting algorithm, hereafter labeled as `NSHS`, is completed by adapting the concept of neighborhood of any city label in $\mathcal{L}$ as the labels corresponding to the closest cities to the one to be mutated. Finally, the proposed `NSHS` approach can be described as in Algorithm 1, replacing lines 4 and 5 with the aforementioned HMCR and PAR operators.

C. Non-Dominated Sorting Firefly Algorithm (`NSFA`)

Similarly to how the HS heuristic is adapted to deal with multi-objective problem, a non-dominated sorting strategy can be also applied to the Firefly Algorithm (FA), a solver from Swarm Intelligence that emulates the flashing behavior, attraction and relative movement between these insects [16]. Attractiveness is the result of the brightness (fitness) perceived by every firefly from the rest of individuals in the swarm, which in turn is subject to their mutual distance. It has been widely applied to problems in many sectors [23].

Algorithm 2: NSFA algorithm.

Input : Swarm size S, P_α, maximum iterations $\mathcal{I}$.
Output: Approximated Pareto set $\{\boldsymbol{L}^{\triangleright}(k)\}_{k=1}^{K_{\text{NSFA}}}$

Generate initial swarm of S fireflies (candidate solutions)
Set iteration counter $it = 1$
while $it \leq \mathcal{I}$ **do**
  for k=1:S **do**
    for k'=k+1:S **do**
      Compute $D_H(k,k')$ in Expression (13)
      if $k \prec k'$ **then**
        Move firefly k' towards k as per (12)
      else
        Move firefly k towards k' as per (12)
      end
      Reevaluate objectives $I_{\aleph}(\cdot)$ and RME$(\cdot)$ for the evolved firefly (either k or k')
    end
  end
end
The Pareto front estimation $\{\boldsymbol{L}^{\triangleright}(k)\}_{k=1}^{K_{\text{NSFA}}}$ is given by the fireflies with $\Re(k) = 1$ remaining in the swarm

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At this point it is worth mentioning the work in [24], which deals with the application of FA to a vehicle routing problem with time windows that differs from our approach in several aspects: 1) it does not consider selectivity of clients (items) in the design of routes; and 2) it aims at minimizing the number of routes and traveled distance by blending both objectives together in a single metric definition. However, our proposed non-dominated FA scheme shares algorithmic similarities with the FA in the above reference, with slight differences:

- A proper definition of distance suited to the problem at hand must be defined in order to measure the phenotypical differences between two candidate solutions (fireflies). Attractiveness varies exponentially with such a distance, and so does the amount by which the attracted firefly moves towards its attracting peer. In this regard we embrace the Hamming Distance – namely, the number of disagreements between two sets – proposed in [24] to measure the distance between two fireflies k and k' within the swarm. However, differences are computed in terms of the successfully delivered goods associated to the schedules represented by the fireflies; if $\boldsymbol{\mathcal{P}}_{\checkmark}(k) \subseteq \boldsymbol{\mathcal{P}}$ and $\boldsymbol{\mathcal{P}}_{\checkmark}(k') \subseteq \boldsymbol{\mathcal{P}}$ denote such sets of delivered items for solutions k and k' in the swarm, the movement from k to k' will be given by

$$L_n^{\triangleright,m}(k') = \begin{cases} L_n^{\triangleright,m}(k) & \text{with prob. } 1\text{-}D_H(k,k'), \\ L_n^{\triangleright,m}(k') & \text{with prob. } (1\text{-}P_\alpha)D_H(k,k'), \\ \text{rand}(1,|\mathcal{L}|) & \text{with prob. } P_\alpha D_H(k,k'), \end{cases} \quad (12)$$

where $P_\alpha \in \mathbb{R}[0,1]$ serves as a mutation probability aimed at simulating the random movement of fireflies; rand(a,b) is the realization of a discrete random variable with support $\mathbb{N}[a,b]$ (with $a < b$); and $D_H(k,k')$ is defined as

$$D_H(k,k') \doteq \frac{\sum_{p \in \boldsymbol{\mathcal{P}}_{k,k'}^{\checkmark}} \mathbb{I}(p \in \boldsymbol{\mathcal{P}}_{\checkmark}(k))\mathbb{I}(p \in \boldsymbol{\mathcal{P}}_{\checkmark}(k'))}{|\boldsymbol{\mathcal{P}}_{k,k'}^{\checkmark}|}. \quad (13)$$

where $\boldsymbol{\mathcal{P}}_{k,k'}^{\checkmark} \doteq \boldsymbol{\mathcal{P}}_{\checkmark}(k) \cup \boldsymbol{\mathcal{P}}_{\checkmark}(k)$.
- Since we deal with a multi-objective problem, light intensity is measured in terms of non-dominance rank and crowding distance, as in the NSGA and NSHS approaches.

Algorithm 2 summarizes the NSFA solver resulting from the above modifications. It should be noted that while for NSGA and NSHS the number of fitness evaluations is given by the product of $\mathcal{I}$ times the size of the pool of candidate solutions (P and HM, respectively), in NSFA this number increases up to $\mathcal{I}S(S-1)/2$ due to the reevaluations done for every pairwise comparison in the inner loop of every iteration, which is of utmost relevance for a fair comparison between heuristics.

D. Multi-objective Ant Colony Optimization (MACO)

The last nature-inspired solver considered in this study is an adapted version of the well-known Ant Colony Optimization algorithm [17], a solver from the field of Swarm Intelligence that imitates the foraging behavior of ant colonies when seeking sources of food to search good paths over graphs defining an optimization problem. The movements of ants through the graph emulates the stigmergy mechanism observed in ant colonies by which pheromone trails are deposited by those members of the colony that found good paths to food sources. Deposited pheromone evaporates along time provided that it is not reinforced by successive ants traversing the path.

As shown in Figure 2, N graphs of dimensions $M_n \times |\mathcal{L}|$ nodes are constructed to model the space of possible trip schedules that every courier agent can follow. Every iteration a colony of K ants traverse every such graph, each tracing a possible sequence of cities visited by the agent at hand. While colonies operate independently of each other over uncoupled solution graphs, it is in the computation of the overall fitness given by $I_{\aleph}(\cdot)$ and RME$(\cdot)$ where paths found by all ant colonies are assembled and their joint quality is evaluated.

Since we deal with a multi-objective optimization problem, pheromones are deposited exclusively by those ants whose trajectories compose a Pareto-dominant set of schedules in the iteration at hand. The pheromone update process is driven by

$$\tau_{l,l'}^{m,m',n} \longleftarrow (1-\rho)\tau_{l,l'}^{m,m',n} + \rho\sum_{k=1}^{K}\text{CD}(k)\mathbb{I}(L_n^{\triangleright,m}(k)=l)\mathbb{I}(L_n^{\triangleright,m'}(k)=l')\mathbb{I}(\Re(k)=1), \quad (14)$$

where the pheromone deposited in the transition from city l at sequence index m to city l' at m' for agent n is reinforced – at a evaporation rate $\rho \in \mathbb{R}[0,1]$ – by the sum of the crowding distance of those Pareto-dominant solutions containing such a transition in the tour followed by courier n. This updated pheromone is then used to probabilistically guide ants in subsequent iterations of the MACO solver, using the inverse of the distance between cities l and l' (i.e. $1/d_{l,l'}$) as the heuristic contribution to the probabilistic transition rule among nodes. Two exponential parameters $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}[1,\infty)$ permit to tune the trade-off between the heuristic value and the trail intensity in the probabilistic transition between graph nodes.

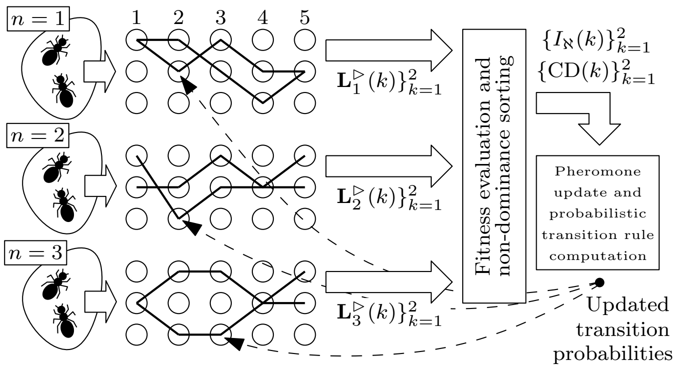

Fig. 2. Schematic diagram of the considered MACO heuristic with $K=2$ ants per colony for a problem with $N=3$ agents visiting $M_n=5$ cities.

IV. Experimental Results

In order to shed light on the comparative performance of the considered multi-objective algorithms, computer experiments have been carried out over synthetically generated problem instances involving different combinations of $|\mathcal{L}|$ (number of cities) and N (number of agents). The scope is placed on verifying how the designed algorithms behave when the complexity of the search space characterizing the problem increases in terms of 1) the cardinality of the involved optimization variables; and 2) the interactions and conflicts derived from coordinating a higher number of vehicles/agents deployed over the scenario. For this reason the simulated instances are labeled as $\{N,|\mathcal{L}|\}$, spanning the set of cases given by the Cartesian product $\{5,10,20\}\times\{20,40,80\}$. In all cases a maximum time T_{max} equal to one weeks (168 hours) is imposed with goods being continuously generated over this time frame to avoid biasing the obtained results with a potential shortage of items with respect to the value of N under consideration. In this regard the generation of items follows an exponential distribution with average inter-arrival time equal to 20 minutes, pickup and delivery locations $L_p^\uparrow$ and $L_p^\downarrow$ drawn at random from $\mathcal{L}$, and delivery deadlines calculated as

$$T_p^\downarrow = T_p^\uparrow + \xi_p \cdot \frac{d_{L_p^\uparrow,L_p^\downarrow}}{V_p}, \qquad (15)$$

where V_p denotes the average speed of the vehicle in the route from $L_p^\uparrow$ to $L_p^\downarrow$; and $\xi_p \in \mathbb{R}[\xi_H,\xi_L]$ (with $\xi_H < \xi_L$) denotes a priority factor: the delivery of items with $\xi_H \ll \xi_p \approx \xi_L$ is less urgent than that of services with $\xi_H \approx \xi_p \ll \xi_L$, as for the latter no flexibility is granted to accommodate other deliveries between the pickup and delivery of the p-th item. The priority factors ξ_p are assumed to follow an uniform distribution over its allowed support, whose values are drawn independently of their value of $d_{L_p^\uparrow,L_p^\downarrow}$. By forcing a reward inversely proportional to ξ_p we emulate the realistic situation where rewards and traveled distance are statistically independent from each other, while maintaining a tight coupling between the priority of a service and its associated reward R_p. In particular we will model the reward as $100\cdot(\xi_L+\xi_H-\xi_p)$ [m.u.], with $\xi_H=1$ and $\xi_L=5$, from which agents receive their incentive depending on the distribution policy adopted by the company. For simplicity and without loss of generality V_p will be set fixed to 80 km/h, with cities distributed uniformly at random over a square area of 1000×1000 km. In all simulated cases M_n is set to 30 $\forall n \in \{1,\ldots,N\}$. Fuel costs of 0.1 m.u. per km are also included in the cost model.

Comparison between multi-objective heuristics will be done based on two different quality indicators:

- Effective number of points in the estimated Pareto front by each algorithm, which are given by those archived solutions with $\Re(k)=1$ once the multi-objective solver at hand has finished iterating. In the algorithmic explanations of Section III this indicator has been labeled as K_θ, with $\theta \in \{\texttt{NSGA},\texttt{NSHS},\texttt{NSFA},\texttt{MACO}\}$.
- Hypervolume indicator, which quantifies the relative Pareto volume (area in problems with two objectives) between an estimated Pareto front and a reference point $\{I_\aleph^\odot,\mathrm{RME}^\odot\}$. In our particular $\max-\min$ optimization problem the hypervolume will be computed as the ratio between the area enclosed by the estimated Pareto front with respect to reference point $\{0,100\%\}$ and the area enclosed between $\{0,0\}$ and $\{R_{max},100\%\}$, with R_{max} given by the highest reward attained over all algorithms and experiments.

TABLE I
Parameter setting for the algorithms in the benchmark

Description	Algorithm	Parameter	Value
Replacement strategy	ALL	Plus replacement	
Population size	NSGA, NSHS, MACO	P, HM, K	40
	NSFA	S	20
Crossover probability	NSGA	P_c	0.7
	NSHS	HMCR	0.6
Mutation probability	NSGA	P_m	0.1
	NSHS	PAR	0.15
	NSFA	P_α	0.1
Number of iterations	NSGA, NSHS, MACO	$\mathcal{I}$	300
	NSFA	$\mathcal{I}$	65

Since all algorithms in the benchmark are controlled by stochastic processes, the above performance measures are computed for 20 independent Monte Carlo experiments for every $\{N,|\mathcal{L}|,\texttt{algorithm}\}$ combination, after which a non-parametric Wilcoxon rank sum test will be run between every pair of algorithms to verify the statistical significance between their performance differences. To further ensure a fair comparison, the number of iterations $\mathcal{I}$ of each heuristic is adjusted so as to yield the same number of fitness function evaluations. Table I summarizes the parameters used for the solvers, which have been optimized off-line over a value grid.

We start our discussion by Table II, which shows the statistics (mean $\pm$ standard deviation) computed for the hypervolume indicator and the number of points estimated by

TABLE II

MEAN ± STD STATISTICS OF THE PERFORMANCE INDICATORS COMPUTED OVER 20 MONTE CARLOS FOR EVERY ALGORITHM AND SCENARIO

	$\{N, \lvert\mathcal{L}\rvert\}$	NSGA		NSHS		NSFA		MACO	
		$K_{\texttt{NSGA}}$	$\text{HV}_{\texttt{NSGA}}$	$K_{\texttt{NSHS}}$	$\text{HV}_{\texttt{NSHS}}$	$K_{\texttt{NSFA}}$	$\text{HV}_{\texttt{NSFA}}$	$K_{\texttt{MACO}}$	$\text{HV}_{\texttt{MACO}}$
Synthetic problems	{5, 20}	20.1 ± 1.2	85.5 ± 1.1	22.4 ± 4.8	86.8 ± 3.8	12.2 ± 1.4	84.4 ± 2.5	20.2 ± 5.2	86.5 ± 2.7
	{5, 40}	21.4 ± 2.5	79.3 ± 2.1	24.4 ± 3.5	81.8 ± 3.5	13.5 ± 2.4	82.1 ± 3.2	22.6 ± 4.3	84.3 ± 1.8
	{5, 80}	23.5 ± 2.4	64.2 ± 3.3	23.2 ± 3.4	65.8 ± 5.1	14.1 ± 4.1	68.5 ± 2.1	24.3 ± 2.9	70.7 ± 1.5
	{10, 20}	22.8 ± 3.3	80.1 ± 1.9	30.5 ± 3.3	81.3 ± 4.4	17.5 ± 1.6	82.4 ± 4.6	28.8 ± 3.2	83.5 ± 1.1
	{10, 40}	27.3 ± 2.2	76.5 ± 3.1	32.5 ± 2.7	79.8 ± 4.1	16.8 ± 1.9	80.5 ± 5.2	29.2 ± 3.4	82.5 ± 2.1
	{10, 80}	33.8 ± 1.4	55.6 ± 4.3	35.6 ± 2.3	57.6 ± 2.4	16.1 ± 2.4	59.7 ± 3.1	37.2 ± 1.3	61.5 ± 4.2
	{20, 20}	30.2 ± 4.7	74.7 ± 4.1	32.2 ± 4.2	75.9 ± 2.2	18.8 ± 2.5	77.5 ± 5.3	34.8 ± 2.1	79.5 ± 2.2
	{20, 40}	36.4 ± 3.2	58.3 ± 5.2	34.6 ± 4.1	61.1 ± 3.4	17.5 ± 3.1	63.5 ± 4.1	35.2 ± 3.6	68.8 ± 5.4
	{20, 80}	38.5 ± 1.1	50.1 ± 8.2	35.1 ± 1.9	58.8 ± 5.6	15.5 ± 1.9	62.5 ± 4.5	35.9 ± 3.0	65.5 ± 3.3
Bizkaia	{5, 70}	22.3 ± 2.2	76.6 ± 4.0	19.1 ± 4.3	74.8 ± 3.2	15.2 ± 1.8	72.5 ± 5.2	28.9 ± 3.1	74.0 ± 4.3
	{10, 70}	27.5 ± 2.6	50.3 ± 4.3	21.4 ± 3.2	52.2 ± 5.2	13.1 ± 2.7	53.5 ± 4.9	29.4 ± 3.8	57.9 ± 3.9
	{20, 70}	31.8 ± 2.8	46.5 ± 2.8	20.3 ± 3.1	49.7 ± 2.8	12.1 ± 2.4	50.3 ± 5.6	31.3 ± 2.9	53.1 ± 4.0

every algorithm averaged over 20 independent experiments run over each simulated scenario. Several patterns can be found in this table: to begin with, the cardinality of the inferred Pareto optimal front and the hypervolume improve with the number of agents (N) and cities ($\mathcal{L}$) for all algorithms. The reason behind this unexpected behavior is the fixed size of the square grid over which cities are randomly located: as $|\mathcal{L}|$ increases the relative distance between cities decreases, and so does the transit time of the courier agents. Furthermore, the availability of more agents allows for a higher flexibility in their scheduling along the commit time T_{max}, which ultimately yields better performance figures in regards to hypervolume and the cardinality of the estimated front.

As for the comparison between solvers it can be observed that NSGA, NSHS and NSFA perform similarly, with subtle performance gains that lack statistical significance as concluded by a pairwise Wilcoxon rank sum run offline with a 95 % confidence interval. Interesting is to highlight the fact that even when configured with lower number of iterations and swarm size to ensure fairness in the comparison, the NSFA approach performs competitively in terms of hypervolume when compared to NSGA and NSHS. However, MACO is found to dominate the rest of solvers over the whole benchmark, with certifiable gaps not only by the visual inspection of the mean and standard deviation statistics, but also by running a Wilcoxon test between algorithm pairs, with p-value below 0.05 in all simulated cases. It can be thus concluded that result samples follow distributions with different medians at a 95 % confidence interval and, hence, MACO renders the best performance results with statistical significance.

A. A Practical Use Case over Bizkaia (Spain)

To further buttress the practical applicability of the compared heuristics a realistic use case has been designed and tested over the province of Bizkaia (Basque Country, northern Spain). The simulated scenario is built upon the time and distance information provided by Google Maps API over the totality of $|\mathcal{L}| = 70$ municipalities of this region with a population above 2000 inhabitants (Figure 3). By repeatedly querying this service expected travel time matrices and distances between cities can be constructed for every municipality pair, which are then fed to the simulation benchmark for a realistic quantization of costs and transit times of the agents.

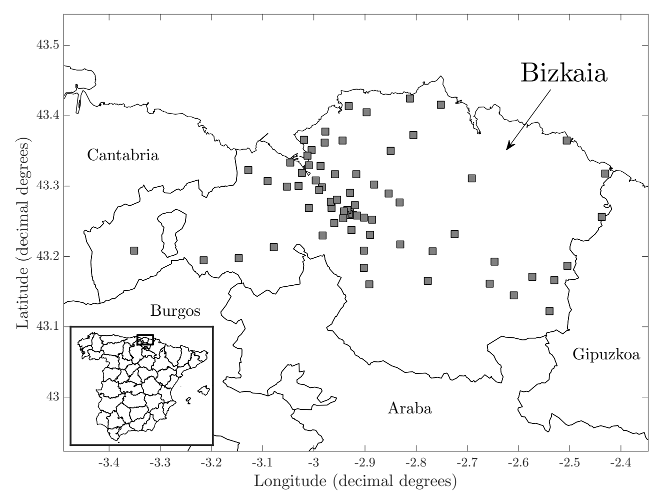

Fig. 3. Map of the province of Bizkaia (Spain) and its municipalities (■).

Results for this practical scenario with $N \in \{5, 10, 20\}$ are listed in the last rows of Table II, with a behavioral trend aligned with those identified for the synthetic scenarios: the higher N is, the more flexibly agents can be scheduled over the scenario to pick up and deliver goods, hence spanning a broader set of possible Pareto schedules. Again, MACO was found to dominate over the rest of heuristics with statistical significance, from which we conclude that ant-based models excel at tackling the problem formulated in this work. Finally, we exemplify the output produced by the designed algorithms in Figure 4, where it can be noted that for this scenario, the MACO solver and $N = 10$ revenues are limited to 1228 m.u. when incentive fairness is ensured (RME$(\cdot) = 0$ %). By relaxing this fairness revenues of the courier company might increase up to 5554 m.u. at the risk of decoupling significantly (up to RME$(\cdot) = 37.62$ %) the distance traveled by each agent and the proportionality of his/her received incentives.

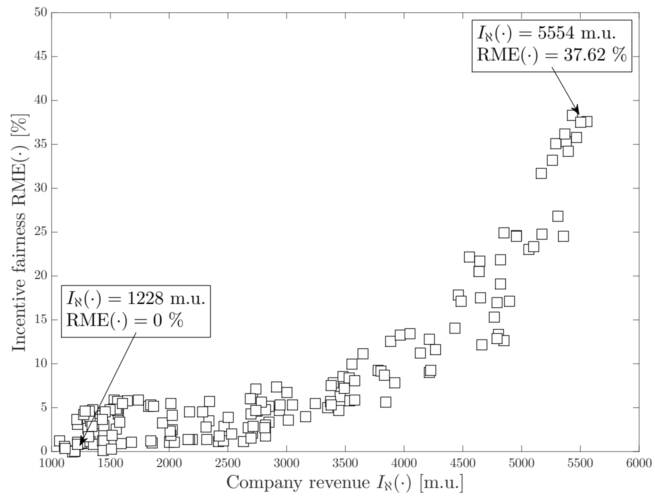

Fig. 4. Example of an estimated Pareto front by MACO for the realistic simulation scenario over Bizkaia with $N = 10$ agents and 5 Monte Carlos.

V. Conclusions and Future Research Lines

This manuscript has elaborated on a multi-objective formulation of the selective pickup and delivery problem comprising several novel ingredients with respect to the related state of the art: 1) the inclusion of several courier agents that concurrently pickup and deliver goods over the same given geographical area; 2) the consideration and mathematical modeling of the fairness when providing incentives to the courier agents of the company in addition to the usual maximum revenue criterion; and 3) the overall formulation of this trade-off as a bi-objective optimization problem, which calls for the adoption of nature-inspired algorithms to efficiently solve for suboptimal solutions. To this purpose four multi-objective optimization methods have been designed and implemented, each relying on avant-garde heuristics with adhoc modifications suited to deal with the particularities of the problem at hand. Computer simulations run over synthetically generated scenarios of varying dimensionality and a realistic setup modeled over the province of Bizkaia (Spain) have unveiled performance gaps between the compared heuristics, with a slight dominance of the MACO solver over all simulated cases.

Future research efforts will be conducted towards including further real aspects of this problem into the problem formulation, such as the transfer of goods between courier agents at intermediate points of their routes or the multi-modality of the transportation chain (along with the modeling of costs and charges between courier companies derived therefrom). Furthermore, other heuristic solvers with innovative operators will be also investigated (e.g. Coral Reefs Optimization [25]).

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