Evolutionary algorithms to optimize low-thrust trajectory design in spacecraft orbital precession mission

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***Abstract*—In space environment, perturbations make the spacecraft lose its predefined orbit in space. One of these undesirable changes is the in-plane rotation of space orbit, denominated as orbital precession. To overcome this problem, one option is to correct the orbit direction by employing low-thrust trajectories. However, in addition to the orbital perturbation acting on the spacecraft, a number of parameters related to the spacecraft and its propulsion system must be optimized. This article lays out the trajectory optimization of orbital precession missions using Evolutionary Algorithms (EAs). In this research, the dynamics of spacecraft in the presence of orbital perturbation is modeled. The optimization approach is employed based on the parametrization of the problem according to the space mission. Numerous space mission cases have been studied in low and middle Earth orbits, where various types of orbital perturbations are acted on spacecraft. Consequently, several EAs are employed to solve the optimization problem. Results demonstrate the practicality of different EAs, along with comparing their convergence rates. With a unique trajectory model, EAs prove to be an efficient, reliable and versatile optimization solution, capable of being implemented in conceptual and preliminary design of spacecraft for orbital precession missions.**

I. INTRODUCTION

Spacecraft orbital precession refers to the rotation of the orbit major axis in space. The orbit major axis, hereinafter referred to as apse-line, has unfavorable changes in a perturbed space environment. One solution for this problem is using electric propulsion systems to correct the direction of the space orbit apse-line. The use of electric propulsion as a low-thrust solution for near-Earth application has become routine and its capabilities continues to grow [1]. Electric propulsion technology is widely used today, and multiple thrusters exist for primary electric propulsion application. NASA and the U.S. commercial market have developed several thrusters which are suitable for primary electric propulsion on full scale spacecraft [2].

Implementation of global optimization methods in the design process of low-thrust trajectories has received notable attention in the past years as it is becoming increasingly evident that such a framework introduces a high level of automation in a process that is otherwise still heavily reliant on expert aerospace engineering knowledge [3]. The systematic study of global optimization algorithms in relation to chemically propelled spacecraft [4] has proved that efficient computer algorithms are able to produce, for these types of spacecraft, competitive trajectory designs. Thus, the attention of communities not traditionally linked to aerospace engineering research [5] has increased, bringing a beneficial influx of new ideas and solutions, thus advancing the field considerably. While for problem formalizations, such as the multiple gravity assist [6] and the multiple gravity assist with deep space maneuver [7], the advantages of using global optimization algorithms have been proved, no convincing results have been produced so far in the case of the low-thrust orbital precession problem.

Orbital precession refers to the rotation of the space orbit in which the spacecraft motion is settled. This rotation is around the axis perpendicular to the orbit plane. As a simple description, the problem is to find the best thrust vector deviation and optimal initial point to start the orbit correction which results in the desired rotation of the space orbit in the presence of orbital perturbations. Regardless of the approach (direct [8] or indirect [9]) considered in this spacecraft trajectory optimization, the problem will typically culminate facing an optimization problem to be solved by an optimization method [10].

The aim of this paper is to deal with the trajectory optimization problem using EAs according to the following steps:

- Simulation of spacecraft dynamics in a perturbed space environment.
- Developing the optimization approach based on low-thrust trajectories.
- Specifying the objective function based on the space mission requirements.

- Transforming the problem to a black box with inputs and the objective function.
- Solving the problem using different evolutionary algorithms.

According to this process, this paper is organized as follows. Section 2 is devoted to the statement of the problem and mathematical modeling of spacecraft trajectory which includes the spacecraft dynamics along with the simulation of orbital perturbation. Section 3 presents the optimization approach, including the objective function and optimization algorithms. Section 4 discusses the computational results of the proposed approach in different space missions. Section 5 concludes this paper.

II. Trajectory model

Consider a spacecraft moving in initial orbit with semi-major axis of a and eccentricity of e as depicted in Fig. 1. Due to orbital perturbation [11], the argument of perigee is changed to ω with respect to the inertial coordinate system.

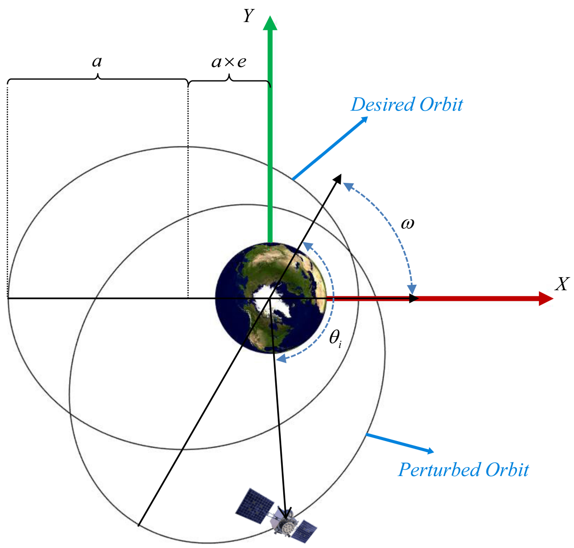

Fig. 1. Schematic view of space orbit apse-line rotation.

The objective is to rotate the spacecraft orbit back to its desired orientation within a specific time, starting at a point on the initial orbit, called the true anomaly of θ_i. According to this scenario, two different space missions can be defined. The first mission is to accomplish this task in less than a single period (i.e., mission time is less than the orbital period). This type of orbit transfer takes place using a continuous low-thrust transfer trajectory. Since the low-thrust propulsion system is employed, very little rotation can be made in this type of mission.

If large rotation is desired, the transfer should take place in several periods and more time steps instead of just one, since the thrust level is low and it is not possible to rotate the space orbit in less than one period. In this new concept, the space mission is the rotation of the orbit in multiple periods where the whole space rotation is divided into several transfer trajectories and each transfer takes place in less than one period. While the orbit rotation in the first concept is small, large space orbit rotation is expected in the second concept since this mission actually includes several orbit corrections in sequence.

Trajectory optimization of spacecraft in both of these types of missions requires adequate simulation of spacecraft motion, and forming an optimization problem as a black box and using a global optimization technique to solve it [12]. As a first step to this end, the dynamics of the spacecraft along with orbital perturbation should be adequately modeled.

The trajectory model which is used to convert low-thrust trajectory optimization into a non-linear programming problem (to be solved by global optimization methods) is crucial to the success of the overall algorithm one wants to produce [13]. Criteria to be accounted for include accuracy in the description of the spacecraft dynamics, computational efficiency in the objective function and constraints evaluation, problem dimension, and number of non-linear constraints produced. Bearing these issues in mind, in this article, the following dynamic model is used [14]:

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{\gamma}_p + \vec{\gamma}_F \tag{1}$$

where $\vec{r}$ is the radius vector of the spacecraft with the respect to inertial coordinate system, r is the magnitude of radius vector (i.e., $r = \sqrt{\vec{r}.\vec{r}}$), μ is the gravitational constant of the Earth, $\vec{\gamma}_p$ is the net perturbative acceleration from all sources other than the spherically symmetric gravitational attraction from the Earth and $\vec{\gamma}_F$ is the acceleration due to the external force generated by the propulsion system acting on the spacecraft within the orbit transfer.

For low-thrust propulsion system, we assume a constant specific impulse I_{sp} and a constant thrust $T(t) = T$. With these assumptions, the acceleration due to the propulsion system can be mathematically described as [15]:

$$\vec{\gamma}_F = \frac{\vec{T}}{m} \tag{2}$$

where $\vec{T}$ denotes the thrust vector and m represents the mass of the spacecraft. The decreasing rate is the following:

$$\dot{m} = -\frac{T}{I_{sp} g_0} \tag{3}$$

where g_0 is the gravitational acceleration at the surface of the Earth.

Four types of orbital perturbation are considered in this research including atmosphere drag, Earth oblateness, solar radiation pressure and lunar gravity. Therefore, the following model for perturbative acceleration is considered [16]:

$$\vec{\gamma}_p = \vec{\gamma}_{aero} + \vec{\gamma}_{oblate} + \vec{\gamma}_{lunar} + \vec{\gamma}_{solar} \tag{4}$$

In this equation, $\vec{\gamma_{aero}}$ is the perturbing acceleration due to the drag force. The commonly accepted altitude at which space begins is 100 km (60 miles). Although most of the Earth's atmosphere lies below 100 km, the air density at that altitude is nevertheless sufficient to exert drag and cause aerodynamic heating on objects moving at orbital speeds and to lower the speed and the height of a spacecraft [17]. The imperfection of the Earth and its mass distribution is the main cause of the Earth's oblateness, which produces $\vec{\gamma}_{oblate}$. This orbital perturbation acceleration is calculated based on the gravitational field of the Earth [18]. Detailed explanations are provided in [19]. Also, perturbing acceleration due to lunar gravity ($\vec{\gamma}_{lunar}$) can be modeled by modeling the position vector of the Moon relative to the Earth and the spacecraft. Details of this simulation are provided in [20]. Finally, for simulation of solar radiation pressure and its resulting perturbation acceleration ($\vec{\gamma}_{solar}$), the cannonball model [21] is adopted in this research.

III. Optimization approach

The optimization problem here can be defined as finding the optimal thrust vector which rotates the space orbit for a specific value ($\Delta\omega$). As a matter of fact, in the current constrained optimal control problem, the thrust magnitude is supposed to be constant and at its maximum value.

Considering the transfer as an in-plane maneuver in this research, semi-major axis (a), eccentricity (e) and argument of perigee (ω) are calculated in simulations. Derivation of these orbital elements from the state vectors are skipped due to lack of space. Details are provided in [15].

By knowing the initial condition ($\vec{r}_i$,$\dot{\vec{r}}_i$), thrust vector ($\vec{T}$) and the simulation time (t_s), and solving the presented differential equation of spacecraft dynamics (Eq. 1 to Eq. 3) along with described perturbations (Eq. 4), the motion of the spacecraft within the transfer trajectory will be revealed. This trajectory transfers the spacecraft from the initial orbit to another. Depending on the thrust variation and initial condition, the orbital parameters of the space orbit will change from a_i, e_i and ω_i to a_f, e_f and ω_f respectively.

Since the space mission is orbital precession, the shape of the orbit should remain unchanged ($a_f = a_i, e_f = e_i$). It is supposed that the initial orbit is known, however the starting position of the spacecraft in initial orbit where the transfer begins (θ_i) is unknown.

Since the space mission is an in-plane maneuver, the thrust vector ($\vec{T}$) can be stated with one direction angle (α) and thrust magnitude (T). The allowable variation of direction angle depends on the type of attitude control system of the spacecraft. As a common modeling, the direction angle is assumed to have linear variation with time in this research. Therefore α can be mathematically stated as an initial value (α_i) and a final value (α_f).

Knowing the initial orbit and the desired rotation, the required velocity change (Δv) can be calculated [15]. The ideal rocket equation shows the relation between the fuel mass and the required velocity change for each transfer as below.

$$\frac{m_f}{m_0} = 1 - e^{-\frac{\Delta v}{I_{sp} g_0}} \tag{5}$$

where m_f and m_0 are the fuel mass and the overall mass of the spacecraft respectively. It is a phenomenon that in spacecraft trajectory optimization, the transfer time and the thrust magnitude have a logical contradiction with each other in space travel as they can't be minimized at the same time. This fact is obvious regarding Eq. 3, which shows the relation of these two with the fuel mass and its variation. Using the fuel mass calculated by Eq. 3, the thrust magnitude can be calculated for any rotation.

This approach is suitable for single period transfers. However, for multi-period transfers, the mission needs to be divided into several single period transfers first. To this end, the overall maneuver is divided into N number of transfers. Each transfer occurs within less than one period and the space orbit is expected to have a rotation of $\delta\omega$ in each period. Therefore, the desired rotation in each step will be calculated as:

$$\delta\omega = \frac{\Delta\omega}{N} \tag{6}$$

Regarding this approach, the problem turns into N number of optimization problems to be solved in N steps in a sequence. In each step, the initial orbital parameters, along with the mass of the spacecraft, are gathered from the previous step. However, the initial true anomaly (θ_i), initial and final thrust direction (α_i and α_f) and the transfer time t_s remain unknown for the optimization algorithm to find. As a typical choice for the thrust magnitude, the dedicated thrust for each transfer is calculated based on having the one-step transfer time equal to one period. However, the optimization algorithms are forced to perform this transfer in less than one period. This makes the problem naturally complex to converge. The complexity can increase even more if the upper bound of the transfer time is limited to another value less than period time. It should also be noted that the complexity is more affected by the mission case rather than the boundaries of transfer time.

A. *Objective function*

According to the proposed approach, the objective of the optimization algorithm is to find the optimal values of θ_i (spacecraft position on its initial orbit), α_i, α_f (variation of) and t_s at each step in order to have a desired rotation in the space orbit while keeping the shape of the orbit unchanged. By selecting these four parameters, the initial condition of the problem will become known along with the transfer time. The objective is to have the spacecraft settled in the desired space orbit after it finishes its motion in transfer trajectory. Simulation of the spacecraft motion regarding the presented dynamic model in the previous section will give the final states of the spacecraft ($\vec{r}_i$,$\dot{\vec{r}}_i$). As stated previously, by converting the final states into the orbital parameters as described in [15], the final semi-major axis (a_f), final eccentricity (e_f) and final argument of perigee (ω_f) will be revealed. Following this, the objective function is calculated as follow:

$$J = (\frac{a_f - a_i}{\sigma_a})^2 + (\frac{e_f - e_i}{\sigma_e})^2 + (\frac{\omega_f - \omega_d}{\sigma_\omega})^2 \tag{7}$$

where σ_a, σ_e and σ_ω are weighting coefficients related to each parameter, which is specified based on desired accuracy according to mission objective [22]. ω_d is the desired argument of perigee in each step as shown in the following equation.

$$\omega_d = \omega_i + n\delta\omega \tag{8}$$

In this equation n represents the number of periods in which the transfer is supposed to be done ($0 < n < N$).

Regarding this approach, the constrained optimal control problem turns into a continuous, unconstrained, non-linear optimization problem. Since such a class of problem may present local minima that are not global, they are often unsolvable using only local optimization algorithms. Practical experience shows that this is usually the case with trajectory optimization problems, regardless of the propulsion type [14]. Thus, intelligent algorithms must be used as global optimization strategies in order to achieve solutions which are as close as possible to the optimal ones. In the following, the algorithms that have been tried on such problems are described.

B. Evolutionary Algorithms

By having the optimization problem as a black box, different evolutionary algorithms can be used to solve the problem. Four types of EAs are considered in this research.

One promising approach that has recently been applied to this type of design problem is Genetic algorithms (GAs). GAs are search algorithms based on a natural mechanism [23]. Versions of GAs have been used extensively in engineering design problems. The applications of GAs to some aerospace problems can be found in [24]. Although GAs have demonstrated better global convergence ability than the classical algorithms, there exist several critical disadvantages for applying GAs to solve practical optimization problems, of which premature and slow convergence rate are included. These problems are especially serious when the GAs are used to solve complex nonlinear constraint problems, such as trajectory design problems [25], with expensive computational costs.

Another type of EA, particle swarm optimization (PSO) [26], is also considered in this work. The PSO was first introduced by Kennedy and Eberhart [27] based on observation and simulation of the social behavior of flocks of birds or schools of fish. In this algorithm, the optimal solution is sought by moving a swarm of particles around in the search space according to simple mathematical rules. The movement of each particle is determined by its best known position and the best position achieved by the entire swarm. The algorithm is simple and can be implemented in a few lines of computer code. Examples of the applications of PSO in aerospace problems are described in [28], [29]

Besides the well-known evolutionary algorithms, a qualitatively different approach, long used for parameter optimization problems, is the use of Estimation of Distribution Algorithms (EDAs) [30]. Like most evolutionary algorithms, such as GA and PSO, EDAs use the principle of survival of the fittest applied to a population of individuals representing candidate solutions. But the major difference is that they produce the new population of individuals by sampling from a probability distribution, which is estimated from a database containing selected individuals from the previous generation [31]. In a detailed example of comparison between GA and EDAs, the well-known crossover and mutation process in GA is replaced by the sampling and learning process in EDAs. These methods are meta-heuristic optimizers that determine an optimal set of parameters that has been used to characterize the problem solution. In recent years the application of EDAs has become popular in different optimization problems [32], [33] including the orbital maneuvers based on impulsive transfers [34]. However, this optimization algorithm has never been used in spacecraft trajectory design regarding finite thrust analysis.

This article presents the application of EDAs and other optimization methods for trajectory optimization of space orbit apse-line rotation based on the employment of low-thrust propulsion systems. The simulation of orbital perturbations in space environment is also considered, which makes the behavior of spacecraft more realistic while moving in space. Regarding the optimization algorithm, the effect of clustering in EDAs [30] on the convergence rate is investigated. Moreover, the performance of the algorithm is compared with GA and PSO.

IV. Numerical results

A. Experimental Setting

In order to showcase the potential advantages of EDAs, multiple comparisons are presented in various space missions. Since the difficulty of the optimization problem is mainly affected by the mission characteristics, a database of enormous space missions is considered. The space missions are divided into two main categories as stated previously: single period transfers and multi-period transfers.

Single period transfers include the space missions in which the objective is to change the argument of perigee by a very small amount (typically less than 1 degree) within just one period. This category suits the space missions where the orbit will be corrected instantly in less than one period when the error of argument of perigee exceeds a specific value.

The latter category includes the space missions where the objective is to rotate the space orbit to have a large variation in argument of perigee within several periods. This category is a more general form of the previous category in which the orbit correction is conducted several times in a consecutive order. The entire transfer is divided into several periods and in each period the objective is to change the specific amount of argument of perigee. This consecutive process ends when the argument of perigee reaches zero (or alternatively the desired final value). Unlike the previous category, the optimization algorithm is used iteratively in a sequence.

In order to cover different complexities of missions, three cases of orbit transfers are considered for each category. The first and second cases consist of orbits in low Earth orbits (LEO) and middle Earth orbits (MEO). The third case includes the orbits with perigee radius in LEO and apogee radius in MEO region. Ten instances are considered in each case.

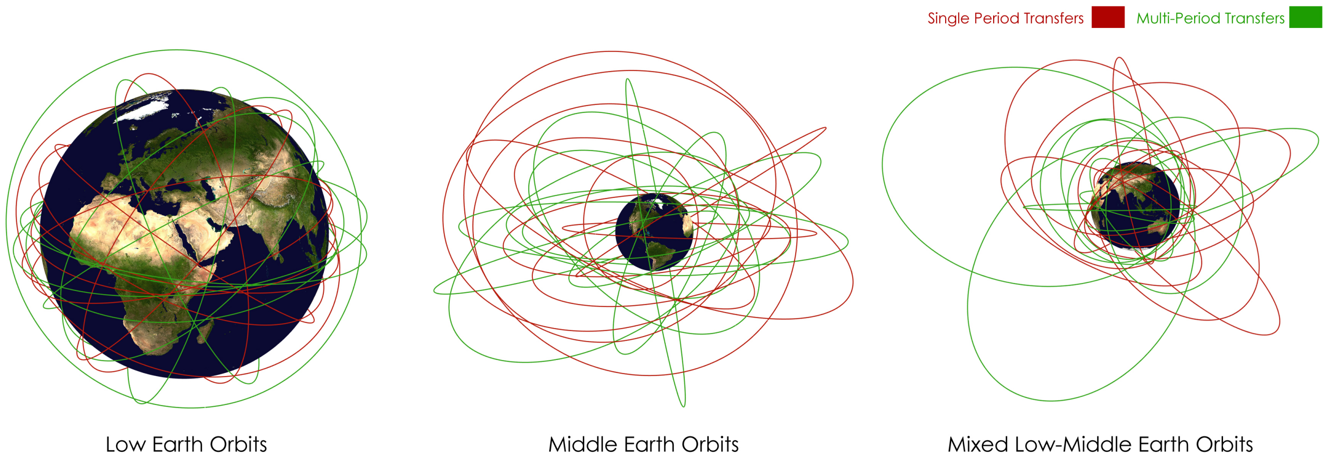

Fig. 2. Instances of space orbits.

Therefore, a total number of 60 instances is considered in this research (30 instances for each category). These cases are illustrated in Fig. 2. This high number of instances is considered in this research to make sure that the optimization approach faces all kinds of complexities within this kind of problem, since the complexity depends on the type of mission.

Although all of the missions are considered as an in-plane maneuver (2D problem), different inclinations and right ascension of ascending node are considered for each instance in order to have a fully covered benchmark of transfers in various perturbed environments. Also, the rest of the parameters, such as initial mass, specific impulse, drag coefficient and other physical characteristics of the spacecraft, are considered as random values within a realistic range, so that no two instances have the same physical parameters.

Four optimization techniques have been employed in the problems: Estimation of distribution algorithm using full multivariate Gaussian model (FMG-EDA) [30], Estimation of distribution algorithm using mixture of multivariate Gaussian model (MMG-EDA) [31], Particle Swarm Optimization (PSO) [8] and Genetic Algorithm (GA) [23]. All algorithms are employed with a population size of 100 and maximum generations of 50. Since it is not the aim of this paper to find the best combination of parameters to optimize the introduced problem, the parameters have been set without performing any previous experimentation. The upper bound of space mission transfer time is set to 0.9 of the orbital period, forcing the algorithms to search within the low transfer times and making the problems relatively tough to deal with. The acceptable accuracies for semi-major axis σ_a, eccentricity σ_e and the argument of perigee σ_ω are set to 1 km, 10^{-3} and 10^{-2} deg. respectively. The upper and lower bounds of initial direction, final direction and the starting true anomaly are set to -180 to +180 degrees. In order to avoid the possible accidental results which may occur during the optimization processes, each algorithm has been conducted 10 times and the results are gathered based on the summation of outputs. The stopping criterion for all optimization algorithms is considered as $J < 1$. The reason for this selection regarding Eq. 7 is that all of the terms are rescaled regarding the selected weighting coefficients. If an algorithm doesn't converge based on this stopping criterion before reaching the maximum generation of 50, the optimization process will be considered as a failure.

B. Single period transfers

As for single period transfers, all space missions are investigated and instances have been optimized. The main focus in extracting the results is to evaluate the number of objective function evaluations for each algorithm along with the total number of failures. These results are shown in Fig. 3.

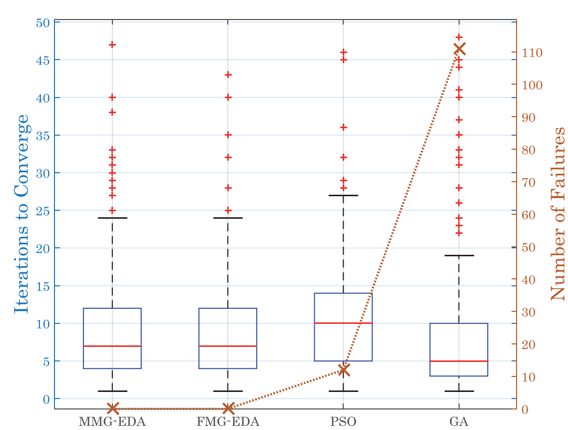

Fig. 3. Performance of algorithms for single period transfers.

The bars in Fig. 3 display the average number of objective function evaluations for each algorithm. The number of failures are excluded from the bars, and are shown separately for each method. Details of results, including the overall number of iterations (ξ) (including the failed processes) and the number of failures (ζ) for all instances in each case, are tabulated in Table I.

According to Table I, the overall number of failures for PSO and GA are 12 and 111 respectively. However, none of the

TABLE I
OVERALL NUMBER OF ITERATIONS (ξ) AND THE NUMBER OF FAILURES (ζ) FOR SINGLE PERIOD TRANSFERS

	LEO		MEO		Mixed	
	ξ	ζ	ξ	ζ	ξ	ζ
MMG-EDA	**789**	**0**	**1025**	**0**	**990**	**0**
FMG-EDA	**809**	**0**	**966**	**0**	**926**	**0**
PSO	1015	1	1108	2	1455	9
GA	2307	30	2676	44	2296	37

EDAs faced any failures in any space missions. By excluding the number of failures and comparing the algorithms, it can be concluded that EDAs have a fair performance as does PSO. It confirms that the EDAs have shown to be competitive algorithms alongside PSO. Nevertheless, the number of failures supports the reliability of EDAs over PSO and GA.

C. Multi-period transfers

For multi-period transfers, the results have been generated similarly. However, the evaluation process is different. As it stated previously, the selected optimization algorithm will be used several times during the transfer (separated by each period), and the total number of periods varies from case to case. As a matter of fact, the optimization algorithm will be used in the sequences of space orbit rotations iteratively. Results for this category are illustrated in Fig. 4.

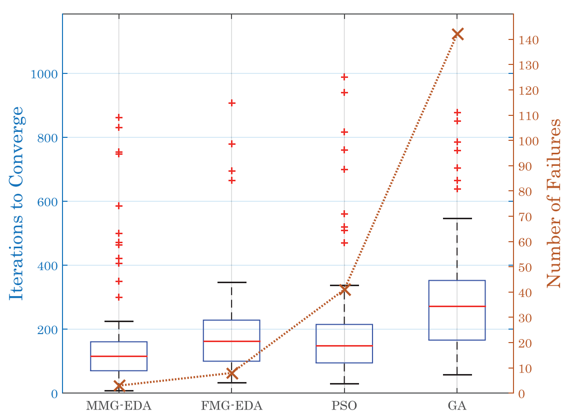

Fig. 4. Performance of algorithms for multi-period transfers.

As can be seen, similar to single period transfers, EDAs perform much better than PSO and GA. However, the failures for EDAs are not zero this time. Nevertheless, they are still more reliable in comparison to PSO and GA regarding the average number of iterations for convergence.

It should be noted that, in this category, the two sequential orbits within the rotation are very similar to each other since the rotation of space orbit is less than one degree. Thus, one slight advantage that can be considered in these transfers is that the optimal solution in every period is very similar to the previous one. So, the best answer found at each step can be used as the initial guess for the next step. Applying this idea can significantly lower the amount of computation for convergence regardless of the algorithms. Therefore, the initial generation for each algorithm has been constructed based on the best solution found at the previous step in this category. Bearing this in mind, the details of results for this category are provided in Table II.

TABLE II
OVERALL NUMBER OF ITERATIONS (ξ) AND THE NUMBER OF FAILURES (ζ) FOR MULTI-PERIOD TRANSFERS

	LEO		MEO		Mixed	
	ξ	ζ	ξ	ζ	ξ	ζ
MMG-EDA	**14829**	**1**	**12129**	**0**	**11758**	**2**
FMG-EDA	**18407**	**2**	**16185**	**1**	**16554**	**5**
PSO	17324	5	16800	17	15536	19
GA	26505	30	29206	49	25844	63

Regarding Table II, again the overall number of failures for EDAs are significantly less than those for PSO and GA. Besides, the number of iterations to converge indicates the better performance of EDAs. The major difference which should be considered in these cases is that the failures in multi-period transfers are calculated differently. Since the problem is sequential in this category, the algorithm should succeed in all periods to accomplish the space mission. Therefore, if the selected algorithm fails to converge in any period, it will be considered as a total failure, regardless of how many successful periods have passed before. Therefore, even the number of failures can be divided in two groups, the failures at the beginning of the space missions (when no initial guess is employed) and at the middle of space missions (when initial guess exists from the previous period). This data is shown in Table III.

TABLE III
FAILURES IN MULTI-PERIOD TRANSFERS

	MMG-EDA	FMG-EDA	PSO	GA
Start of missions	3	5	15	96
Middle of missions	0	3	26	46
Total	**3**	**8**	41	142

Table III shows the fact that EDA using a mixture of multivariate Gaussian model (MMG-EDA) has the minimum number of failures and has an advantage over EDA based on full multivariate Gaussian model (FMG-EDA). It is a promising result that MMG-EDA never fails in the middle of space missions. This shows the high level of reliability of this algorithm in space orbit transfer mission.

Another question arises about the distribution of failures with the transfers. In response to this query, one should be reminded that each instance in this research has different argument of perigee and reduction rate. So, the number of periods varies from one instance to another. In order to have a fair illustration of failure distribution, all transfer periods have been scaled from 0 to 1 and the times when the algorithms failed are spotted within this time scale. By excluding the failures at the start of space missions, the results are provided in Fig. 5.

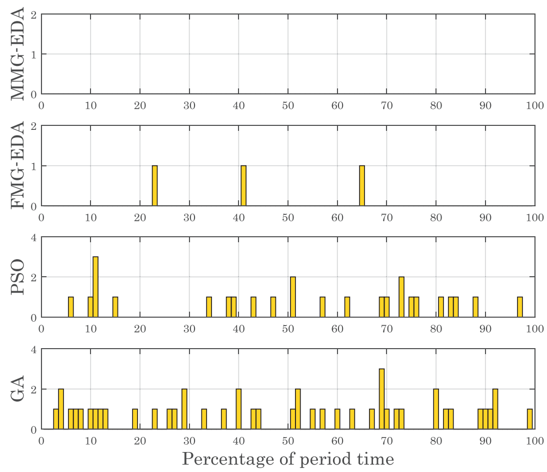

Fig. 5. Failures of optimization algorithms in the middle of space missions

Fig 5 confirms that, with a high possibility, no clear dependencies exist about the time of failures. Since the distribution of failures is approximately uniform, it can be concluded that the complexity of the problem is almost unique in an orbital precession mission.

Besides the number of failures, the average number of iterations for convergence in each algorithm can also be plotted as functions of time as in Fig. 6.

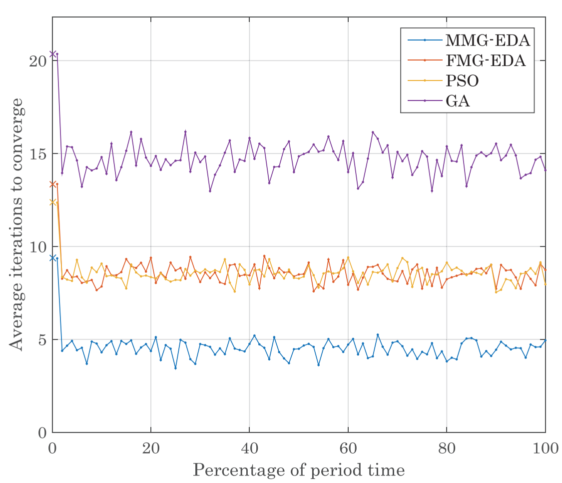

Fig. 6. Time histories of average iterations to converge

Fig. 6 shows the effect of initialization in each algorithm while marching through the end of space mission. Obviously, the failed processes are excluded from these results. According to this figure, MMG-EDA benefits from the lowest average number of iterations for convergence. FMG-EDA and PSO are competitive. However, EDA benefits from a higher level of improvement from the initial guess, leading to the conclusion that EDAs are relatively more intelligent in taking advantage of initial guesses.

Based on the proposed approach, a three-dimensional trajectory design framework is developed utilizing EDAs. The developed approach is applied successfully on all of the space missions with different low-thrust three-dimensional orbital precession problems from LEO, MEO and the mixed transfers. One of the realistic three dimensional representations is illustrated in Fig. 7.

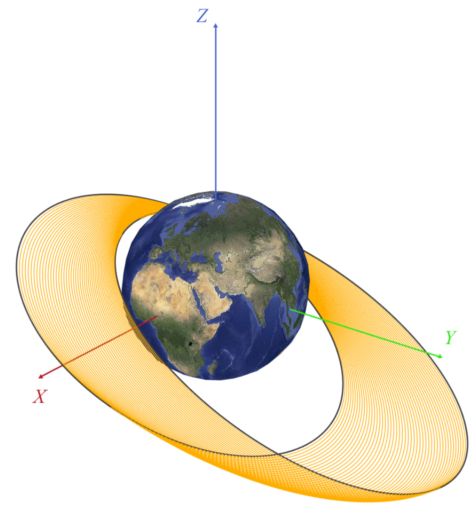

Fig. 7. Sample low-thrust transfer trajectory

Fig. 7 depicts the uniform rotation of the space orbit with semi-major axis of 14000 km, eccentricity of 0.5 and argument of perigee of 80 degrees. The orbital precession is accomplished in 100 periods with the rate of 0.8 degree per revolution using 200 mN thrust with specific impulse of 3000 s. This sample is one of the instances with a successful optimization process in all periods using MMG-EDA. The average working time of propulsion system is less than 3.6 hours per revolution. It demonstrates that, by using EDAs based on mixture of full Gaussian model, the proposed technique can quickly generate three dimensional low-thrust trajectories feasible with respect to the desired orbit rotation. What can be inferred from Fig. 7 is the practicality of the developed framework in the design and simulation of spacecraft orbit transfer missions.

V. Conclusion

Design and accurate optimization of low-thrust orbital precession trajectories, in combination with various EDA methods, are considered as the key techniques and prerequisites of future low-thrust missions. In this paper, a general framework for design and optimization of low-thrust trajectories is developed, which can be accommodated to various optimization techniques without performing significant modification. In this general framework, numerous cases of orbit transfers in different perturbative environments in space are considered.

In all cases, EDAs prove to be more reliable algorithms in comparison to other meta-heuristics such as PSO and GA, since they showed a much lower number of divergence. The performance of the technique can be enhanced to handle more complicated problems by increasing the number of populations at the expense of slowing down the technique.

It is also shown that clustering provides enough flexibility to find various feasible transfer trajectory profiles and even rapid convergence in reasonable times. Furthermore, this approach is flexible in generating various feasible solutions rather than learning based on full multivariate Gaussian model, which is quite favorable in the preliminary phases of mission trajectory design. Finally, the suitability of using the optimal solution found at each revolution as an initial guess of the next period for high-fidelity direct optimization techniques in multi-period transfers is demonstrated successfully.

Besides the investigation of optimization algorithms, the practical output of this research is the development of a framework based on EDAs and other optimization techniques for optimal low-thrust orbit transfer trajectory design. This is surely a matter of interest for generating optimal trajectories in conceptual and preliminary design of spacecraft in space engineering.

Future research may include the analysis of other types of EDAs in orbital precession mission or other spacecraft trajectory optimization problems. Also, it is an open question to investigate whether better solutions can be obtained by tuning the parameters of EDAs (for example number of clusters) or changing them dynamically while the spacecraft moves in its transfer trajectory and marches through the end of its space mission.

ACKNOWLEDGMENTS

This research is supported by the Basque Government through the BERC 2014-2017 and Research Groups IT-609-13 2013-2018 programs, Spanish Ministry of Economy and Competitiveness MINECO: BCAM Severo Ochoa excellence accreditation SEV-2013-0323 and TIN2016-78365-R Spanish Ministry of Economy, Industry and Competitiveness.

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