Mode-independent $\mathcal{H}_2$-control of a DC motor modeled as a Markov jump linear system

Ricardo C. L. F. Oliveira, Alessandro N. Vargas, João B. R. do Val,
and Pedro L. D. Peres

Abstract

The paper presents a control strategy for Markov jump linear systems (MJLS) with no access to the Markov state (or mode). The controller is assumed to be in the linear state-feedback format and the aim of the control problem is to design a static mode-independent gain that minimizes a bound to the corresponding $\mathcal{H}_2$-cost. This approach has a practical appeal since it is often difficult to measure or to estimate the actual operating mode. The proposed result is compared with a previous design, and its usefulness is illustrated by an application that considers the velocity control of a DC motor device subject to abrupt failures that is modeled as an MJLS.

I. INTRODUCTION

Let us consider the discrete-time Markov jump linear system (MJLS), defined on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_k, P)$, as follows:

$$
\mathcal{S} : \begin{cases} x_{k+1} = A_{\theta_k} x_k + B_{\theta_k} u_k + E_{\theta_k} w_k \\ y_k = C_{\theta_k} x_k + D_{\theta_k} u_k, \quad k \geq 0, \quad \theta_0 \sim \pi(0), \ x_0 \in \mathcal{R}^r, \end{cases}
$$

where the sequences $\{x_k\}$ on $\mathcal{R}^r$, $\{y_k\}$ on $\mathcal{R}^q$, $\{u_k\}$ on $\mathcal{R}^s$, $\{w_k\}$ on $\mathcal{R}^m$, represent the system state, output, control, and noise input, respectively. The Markov chain state is represented by $\{\theta_k\}$ and the underlying stochastic matrix is denoted by $\mathbb{P} = [p_{ij}]$, in that $i, j \in \mathcal{I} := \{1, \ldots, \sigma\}$, $\sigma > 1$. The initial distribution is denoted by $\pi(0)$. For each $k \geq 0$, the state process θ_k takes values in the set $\mathcal{I}$, in such a way that $\theta_k = i$ points out to a set of matrices $(A_i, B_i, C_i, D_i, E_i)$ that is assumed to be given for each $i \in \mathcal{I}$.

Many of the real systems that can be described within the MJLS setting face adversity when the design of a useful control is the primary concern. Actually, it is often difficult to determine the exact mode of the Markov chain due to physical limitations on sensors or nonexistence of realistic measurement instruments for that task. However, most of the results in the literature do not take this practical difficulty into account since they assume that the controllers have full access to the mode at each instant of time [1], [2], [3], [4], [5], [6], [7],[8], [9], [10], and [11].

Thus, from this standpoint, it is reasonable to consider the design of controllers that do not require access to the Markov mode at all. This subject represents the main line of investigation in this paper.

The discussion above stresses the need of simple control synthesis in real world implementations, and we choose the control law to be in the linear static state-feedback format with no mode observation, i.e.,

$$u_k = Gx_k, \quad \forall k \geq 0, \tag{1}$$

where G is a fixed matrix of dimensions $s \times r$ to be determined.

With the aid of simple calculations, the $\mathcal{H}_2$-control problem can be cast as that of finding G, the solution of the following optimization problem (Proposition 2.2):

$$\beta^* = \inf_G \sum_{i=1}^{\sigma} \text{tr} \left\{ \pi_i(0)E_i' \left(\sum_{j=1}^{\sigma} p_{ij}P_j \right) E_i \right\} \tag{2}$$

$$\text{s.t.} \quad (A_i + B_iG)' \left(\sum_{j=1}^{\sigma} p_{ij}P_j \right) (A_i + B_iG) + C_i'C_i - P_i = 0, \quad i = 1, \ldots, \sigma, \tag{3}$$

where $P_i = P_i' \geq 0, i = 1, \ldots, \sigma$, are matrices with dimensions $r \times r$. The problem of minimizing the $\mathcal{H}_2$-cost as in (2) and (3) is, to the best of the authors' knowledge, open. Our approach provides a step towards finding the solution for this problem because, although it does not calculate the optimal solution, our approach is capable to generate less conservative results

when compared with the previous method in [12]. This sets the main theoretical novelty of this paper.

There are two main contributions of this paper. First, a novel sufficient linear matrix inequality (LMI) relaxation is proposed for the computation of a mode independent state-feedback stabilizing gain with a guaranteed $\mathcal{H}_2$-cost, say $\beta > 0$, such that $\beta \geq \beta^*$.

The method is inspired by the two-step design procedure developed in the context of deterministic systems for output feedback control in [13], [14], [15], that in [16], [17] have been extended to incorporate polynomially parameter-dependent matrices. In this paper, the method is adapted to cope with MJLS control design as follows: first determine a mode-dependent stabilizing gain; then use this gain as an input parameter for an LMI based procedure (called second stage) that, if feasible, provides a mode-independent stabilizing gain associated to an $\mathcal{H}_2$ guaranteed cost.

As the second contribution, the theoretical design procedure proposed in the paper is validated in practice, by controlling the angular velocity of a DC motor apparatus in real-time. During the experiments, the DC motor was subject to failures that evolved according to a homogeneous Markov chain.

The paper is organized as follows. Section II quotes the basic notions of mean square stability and evaluation of the $\mathcal{H}_2$-norm of MJLS. Section III presents the main result based on the novel LMI method. Finally, Section IV illustrates the result via a workbench experiment involving a DC motor subject to failures. The method is applied to design a mode-independent control rule to control the shaft speed.

II. Notation, definitions, and basic results

The r-th dimensional Euclidean space is represented by $\mathcal{R}^r$ and $\|\cdot\|$ stands for the corresponding norm; $e_s \in \mathcal{R}^r$ represents the standard basis pointing in the direction of the s-th coordinate. The linear space made up by all $r \times s$ ($r \times r$) real matrices is denoted by $\mathcal{M}^{r,s}$ ($\mathcal{M}^r$). Let $\mathcal{I} := \{1, \ldots, \sigma\}$ be an index set, and let $\mathbb{M}^{r,s}$ denote the linear space formed by σ matrices belonging to $\mathcal{M}^{r,s}$, i.e., $\mathbb{M}^{r,s} = \{U = (U_1, \ldots, U_\sigma) : U_i \in \mathcal{M}^{r,s}, i \in \mathcal{I}\}$. Take, in particular, $\mathbb{M}^r \equiv \mathbb{M}^{r,r}$.

We employ the ordering $U > V$ ($U \geq V$) for elements of $\mathbb{M}^r$, meaning that $U_i - V_i$ is positive definite (semi-definite) for all $i \in \mathcal{I}$, and similarly for other mathematical relations. If

$V \in \mathcal{M}^{r,s}$ and $U \in \mathbb{M}^{s,r}$, then the multiplication VU results in $(VU_1, \ldots, VU_\sigma) \in \mathbb{M}^{r,r}$. The trace operators is denoted by $\mathrm{tr}\{\cdot\}$. The identity matrix is denoted by I.

The Markov chain $\{\theta_k\}$ is driven by the probability distribution $\pi_i(k) := \mathrm{Pr}(\theta_k = i)$, for all $k \geq 0$ and each $i \in \mathcal{I}$. Notice that $\pi(0) = \{\pi_1(0), \ldots, \pi_\sigma(0)\}$.

Let ℓ_2 denote the Hilbert space formed by the sequence $y = \{y_k\}$, a second order, real-valued stochastic processes that are $\{\mathcal{F}_k\}$-adapted and satisfies

$$\|y\|^2 := \sum_{k=0}^{\infty} \mathrm{E}[\|y_k\|^2] < \infty.$$

Assumption 2.1: The noise input $\{w_k\}$ in $\mathcal{S}$ belongs to the class ℓ_2.

A. Preliminary results for the system with no control

Next we recall the definition of mean square stability.

Definition 2.1: ([2], [18], [19]). MS-stability. We say the system $\mathcal{S}$ with $u_k \equiv 0$ is mean square stable (MS-stable) if

$$\mathrm{E}[\|x_k\|^2] \to 0 \quad \text{as} \quad k \to \infty, \tag{4}$$

for each $x_0 \in \mathbb{R}^r$ and each $\theta_0 \in \mathcal{I}$.

Proposition 2.1: ([2, Th. 3.9], [18, Th. 2]). The following assertions are equivalent.

(i) The system $\mathcal{S}$ with $u_k \equiv 0$ is MS-stable.

(ii) For some $V \in \mathbb{M}^{r,r}$, $V = V' > 0$, there holds

$$A_i' \left(\sum_{j=1}^{\sigma} p_{ij} V_j \right) A_i - V_i < 0, \quad i = 1, \ldots, \sigma. \tag{5}$$

Next we present the definition of the $\mathcal{H}_2$-norm for MJLS.

Definition 2.2: ([2, Ch. 4.4], [12]). The $\mathcal{H}_2$-norm associated with the system $\mathcal{S}$ is represented by the value

$$\|\mathcal{S}\|^2 = \sum_{s=1}^{m} \sum_{i=1}^{\sigma} \pi_i(0) \|y^{s,i}\|^2,$$

where $y^{s,i}$ denotes the output $y = \{y_k\}$ due to the specific input $w_0 = e_s \in \mathcal{R}^m$, $w_k = 0, \forall k \geq 1$, and initial condition $x_0 = 0$ and $\theta_0 = i$.

The next result presents an expression for the evaluation of the $\mathcal{H}_2$-norm.

Proposition 2.2: ([12, Append. A]). If $\mathcal{S}$ with $u_k \equiv 0$ is MS-stable and there exists $P \in \mathbb{M}^{r,r}$, $P = P' \geq 0$, the unique solution of the equation

$$A_i' \left(\sum_{j=1}^{\sigma} p_{ij} P_j \right) A_i + C_i' C_i - P_i = 0, \quad i = 1, \ldots, \sigma, \tag{6}$$

then $\|\mathcal{S}\|^2 = \sum_{i=1}^{\sigma} \sum_{j=1}^{\sigma} \pi_i(0) p_{ij} \mathrm{tr}\{E_i' P_j E_i\}$.

Remark 2.1: If the $\|\mathcal{S}\|^2$-norm is finite, then the system $\mathcal{S}$ is MS-stable. Indeed, this assertion comes from the fact that, with $\|\mathcal{S}\|^2$ finite, the equation (6) has a solution and implies that (5) holds true.

B. Control structure

The LMI method introduced in the sequel requires an input data, in the sense that the method should be initialized with a gain that depends on the Markov mode. To compute a gain that does not depend on the mode, i.e., some gain $G_{out} \in \mathcal{M}^{s,r}$, a mode dependent gain $G_{ini} \in \mathbb{M}^{s,r}$ is used in the design conditions. The single condition imposed on $G_{ini} \in \mathbb{M}^{s,r}$ is that it is MS-stabilizing, a basic concept as defined next.

Definition 2.3: ([20, p. 1283]). A gain $G \in \mathbb{M}^{s,r}$ is called MS-stabilizing if the resulting closed loop system in $\mathcal{S}$ (i.e., replacing A by $A + BG$) is MS-stable.

Remark 2.2: Recall that there exists a solution to the coupled Riccati equations if and only if the corresponding gain is MS-stabilizing [20, Prop. 2]. This stabilizing property of the Riccati gain will be used in the design control project of Section IV.

III. MAIN RESULT

This section is devoted to present and prove the main result based on an LMI strategy. An advantage of our LMI method is that it generates less conservative results than the existing ones (see Table II for a pragmatic comparison). Next we present the main result of this paper.

Theorem 3.1: Let $K = \{K_1, \ldots, K_\sigma\} \in \mathbb{M}^{s,r}$ be a given MS-stabilizing gain. If there exist a set of matrices $P \in \mathbb{M}^{r,r}$, $P = P' > 0$, $W \in \mathbb{M}^{m,m}$, $F \in \mathbb{M}^{r,r}$, $H \in \mathbb{M}^{q,q}$, matrices $R \in \mathcal{M}^{s,s}$, $L \in \mathcal{M}^{s,r}$, and a scalar β such that the following LMIs hold:

$$\beta > \sum_{i=1}^{\sigma} \pi_i(0)\mathrm{tr}(W_i), \tag{7}$$

$$W_i - E_i'\left(\sum_{j=1}^{\sigma} p_{ij}P_j\right)E_i > 0, \;\; i = 1, \ldots, \sigma; \tag{8}$$

$$\begin{bmatrix} -P_i & (A_i' + K_i'B_i')F_i & (C_i' + K_i'D_i')H_i & L' - K_i'R' \\ \star & \left(\sum_{j=1}^{\sigma} p_{ij}P_j\right) - F_i - F_i' & 0 & F_i'B_i \\ \star & \star & I - H_i - H_i' & H_i'D_i \\ \star & \star & \star & -R - R' \end{bmatrix} < 0, \quad i = 1, \ldots, \sigma; \tag{9}$$

then $G = R^{-1}L$ is a mode-independent MS-stabilizing gain and $\sqrt{\beta}$ is an upper bound (guaranteed cost) for the $\mathcal{H}_2$-norm of the system $\mathcal{S}$.

Proof: By multiplying (9) on the left by T and on the right by T', with

$$T = \begin{bmatrix} I & 0 & 0 & S' \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, \quad S = R^{-1}L - K,$$

we obtain

$$\begin{bmatrix} -P_i & A_{cl_i}'F_i & C_{cl_i}'H_i \\ \star & \left(\sum_{j=1}^{\sigma} p_{ij}P_j\right) - F_i - F_i' & 0 \\ \star & \star & I - H_i - H_i' \end{bmatrix} < 0, \tag{10}$$

with $A_{cl_i} \equiv A_i + B_iR^{-1}L$ and $C_{cl_i} \equiv C_i + D_iR^{-1}L$. Now, multiply (10) on the left by $\begin{bmatrix} I & A_{cl_i}' & C_{cl_i}' \end{bmatrix}$ and on the right by its transpose, and as a result, we get that

$$A_{cl_i}'\left(\sum_{j=1}^{\sigma} p_{ij}P_j\right)A_{cl_i} - P_i + C_{cl_i}'C_{cl_i} < 0, \quad \forall i \in \mathcal{I}. \tag{11}$$

The inequalities in (7), (8), and (11) assure that the gain matrix $G = R^{-1}L$ is MS-stabilizing (Remark 2.1) and that $\sqrt{\beta}$ is a guaranteed cost for the $\mathcal{H}_2$-norm of the system $\mathcal{S}$. $\blacksquare$

Remark 3.1: Notice that Theorem 3.1 is a two-steps procedure, i.e., it requires as input data a previous calculated MS-stabilizable gain (with full mode observation), and in the second step the set of LMIs in (7)–(9), if feasible, generates a corresponding MS-stabilizing gain (with no mode observation). This is the main novelty of our design method. As illustrated in the next section, the numerical evaluation confirms that the result can be less conservative than the one
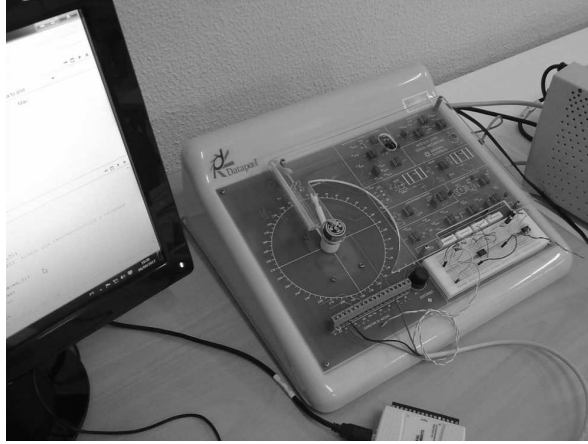
Fig. 1. Laboratory testbed used in the practical experiments of Section IV.

available in the literature [12]. To the best of the authors' knowledge, the result in [12] for the design of mode-independent controllers presents the less conservative result described so far in the literature in the context of the present work.

IV. PRACTICAL APPLICATION: CONTROL OF VELOCITY IN A DC MOTOR DEVICE

This section describes a practical experiment of controlling the angular velocity of a DC motor device subject to abrupt failures. These failures change the equipment behavior from the normal mode of operation to the failure modes, and vice versa, in which a computer is responsible to make these triggers to occur. To control the velocity of the DC motor in this scenario, we implement the control strategy suggested by Theorem 3.1 and the control design method from [12] for sake of comparison. As we will see in the sequence, the control strategy from Theorem 3.1 produces a better response for the DC motor device under abrupt failures.

The laboratory testbed used in this section is composed of the DC motor Module 2208, Datapool Eletronica Ltda, Brazil (Fig. 1), linked with a National Instruments USB-6008 data card and a computer running an instance of Matlab. This laboratory setup was used previously by some of the authors in a time-varying feedback experiment, see [21].

We design a controller based on Theorem 3.1, considering the regulator problem, and then implement it in the DC motor testbed with a nonzero constant plus a sinusoid reference. The idea of the experiment is to adjust the DC motor to track this sinusoid reference with null steady-state error. To accomplish this goal, we implemented the proportional-integrative (PI) scheme

as described in [22, Sec. 1.8.2, p. 56] to cope with the steady-state error and it is included in the synthesis by augmenting the system state. Let us set the system state x_k as follows: take $x_k \equiv [v_k \ i_k \ x_{3,k}]'$, where v_k and i_k represent the angular velocity of the rotor and the electrical current consumed by the motor, and the third element denotes the integrator term from the PI scheme.

The failures occur on the power delivered to the shaft, which are triggered by a computer. They are based on three distinct operation modes, i.e., the normal ($\theta_k = 1$), low ($\theta_k = 2$), and medium ($\theta_k = 3$) power modes. In fact, the DC motor Module 2208 has a round potentiometer, coupled with an electronic driver, that allows us to the change abruptly the power delivered to the shaft, but respecting certain limits for the power shaft. We arranged the circuitry to bypass the potentiometer and commanded its functionality by a computer, so that the jumps were chosen so as to correspond to the three levels: (i) 0% of rotary (normal mode); (ii) $+20\%$ of rotary for improving the power (low mode); (iii) and -40% of rotary for decreasing the power (medium mode).

These elements allow us to represent the DC motor device with abrupt failures as the Markov jump linear system:

$$x_{k+1} = A_{\theta_k} x_k + B_{\theta_k} u_k + E_{\theta_k} w_k + \Gamma_{\theta_k} r_k,$$

$$y_k = C_{\theta_k} x_k + D_{\theta_k} u_k, \quad x_0 \in \mathcal{R}^3, \tag{12}$$

$$A_i = \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} & 0 \\ a_{21}^{(i)} & a_{22}^{(i)} & 0 \\ a_{31}^{(i)} & 0 & a_{33}^{(i)} \end{bmatrix}, \ B_i = \begin{bmatrix} b_1^{(i)} \\ b_2^{(i)} \\ 0 \end{bmatrix}, \Gamma_i = \begin{bmatrix} 0 \\ 0 \\ \gamma^{(i)} \end{bmatrix}, \ E_i \equiv 0.1I, \quad i = 1, 2, 3.$$

where the parameters are given in Table I ([21, Sec. 4]). To design the mode-independent controller, let us define

$$C_i \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 \\ 0 & 0 & 2.5 & 0 & 0 \end{bmatrix}', \ D_i \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}'.$$

The sequences $\{u_k\}$ on $\mathcal{R}$ and $\{w_k\}$ on $\mathcal{R}^2$ obey the definitions of the previous sections, whereas $\{r_k\}$ on $\mathcal{R}$ represents a reference tracking input. The Markov chain has the initial

distribution $\pi_0 = [1\ 0\ 0]'$ and the transition probability matrix $\mathbb{P}$ is defined as

$$\mathbb{P} = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.36 & 0.6 & 0.04 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

The controller applied to the DC motor device is in the static state-feedback format

$$u_k = Gx_k, \quad \forall k \geq 0, \tag{13}$$

where the value of the gain $G \in \mathcal{M}^{1,3}$ is determined according to Theorem 3.1 (taking (12) with $r_k \equiv 0$). Indeed, we use the Riccati gains as the first step, as described in Remark 2.2. In the second step, we solve the LMIs in Theorem 3.1 with the Riccati gains to obtain the $\mathcal{H}_2$-cost and the corresponding admissible gain G, which stabilizes the system and attains the $\mathcal{H}_2$-cost in Table II. Note that the $\mathcal{H}_2$-cost we obtain represents a good improvement when compared with the one obtained with the method in [12], see Table II.

In practice, the failures tend to deviate the velocity of the DC motor device from its nominal path. To illustrate how the designed controller deals with these fluctuations, we set the apparatus to run with the sinusoid reference signal

$$r_k = 1 + 0.2\sin(k/70), \quad \forall k \geq 0.$$

TABLE I

THE DISCRETE-TIME MJLS THAT MODELS THE DC MOTOR DEVICE.

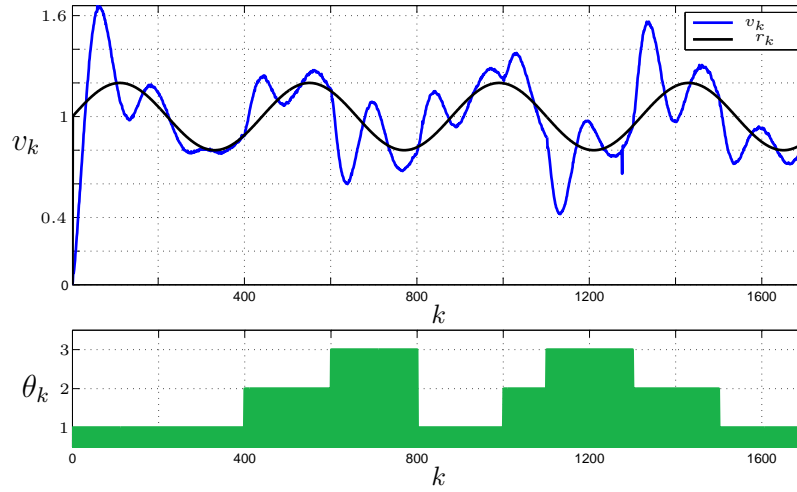| Parameters	$i = 1$	$i = 2$	$i = 3$
$a_{11}^{(i)}$	-0.4799	-1.6026	0.6346
$a_{12}^{(i)}$	5.1546	9.1632	0.9178
$a_{21}^{(i)}$	-3.8162	-0.5918	-0.5056
$a_{22}^{(i)}$	14.4723	3.0317	2.4811
$a_{31}^{(i)}$	0.1399	0.0740	0.3865
$a_{33}^{(i)}$	-0.9255	-0.4338	0.0982
$b_1^{(i)}$	5.8705	10.2851	0.7874
$b_2^{(i)}$	15.5010	2.2282	1.5302
$\gamma^{(i)}$	0.1176	-0.1328	0.1632

Fig. 2. Upper: The curve in blue represents the velocity (rad/s) of the DC motor measured in practice and the black one represents the reference. Lower: The states of the Markov chain evaluated in the experiment.

Although the failures produce a significant deviation of the system from its desired path in the moment of their occurrence, we observed in practice that the control rule (13) tends to restore the equilibrium of the system (Fig. 2).

The experiment using this sinusoid signal was repeated for 700 distinct realizations of the Markov chain, running the controllers proposed by Theorem 3.1 and by the method in [12]. The corresponding path density in Fig. 3 suggests that the velocity v_k and electrical current i_k have finite values for mean and standard deviation for all $k \geq 0$ for both methods. This fact indicates that the two controllers for the DC motor device are uniformly second moment stable [23].

In addition, Fig. 3 shows that the controller from Theorem 3.1 produces an outcome with

TABLE II

MODE-INDEPENDENT $\mathcal{H}_2$ COSTS PROVIDED BY THE METHODS FROM THEOREM 3.1 AND [12, P. 347, $G_i = G$].

Method	$\mathcal{H}_2$-cost	Gain
[12, p. 347]	3.8119	$G = [0.1889 \ -0.8817 \ -3.4384 \times 10^{-3}]$
Theorem 3.1	0.7963	$G = [0.2192 \ -0.9492 \ -2.3497 \times 10^{-3}]$
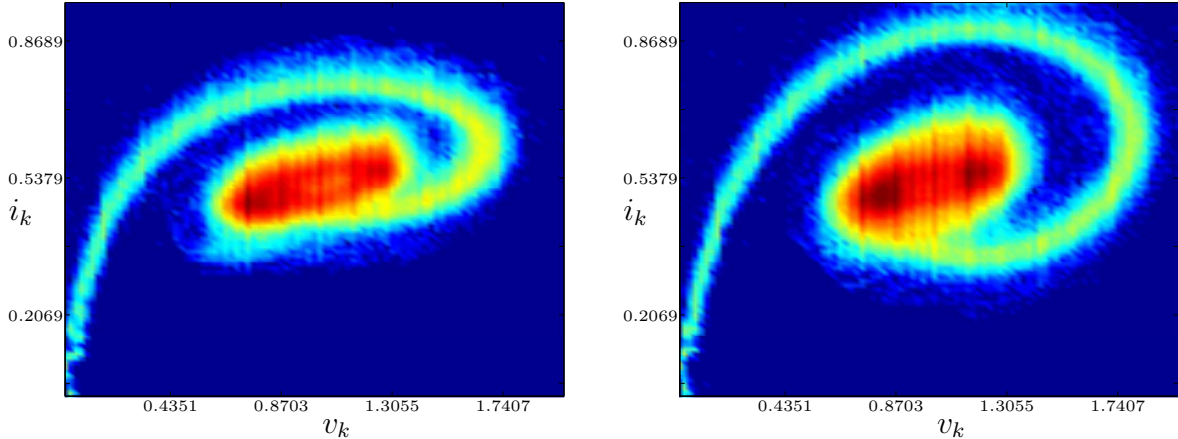
(a) Response due to the mode-independent controller of Theorem 3.1.

(b) Response due to the mode-independent controller of [12, p. 347, $G_i = G$].

Fig. 3. Phase portraits representing the electrical current i_k (A) and velocity v_k (rad/s) measured from the DC motor device, corresponding to 700 distinct realizations. The shading of the colors represents the statistical dispersion of the variables produced by the power failures. Their statistical means follow an spiral path, and reach an ellipsoidal limit cycle (colored in red). A comparison between the two responses indicates that the result of Theorem 3.1 generates an improved, less dispersive phase portrait.

less statistical dispersion when compared with the one from [12]. In fact, the better behaviour attained by the controller obtained from Theorem 3.1 is related to the $\mathcal{H}_2$ bounds, as shown in Table II. In practice, the controller of Theorem 3.1 induces a fast transient to the system, with less oscillations around the tracking signal, when failures occur.

The experiments described in this section emphasize the applicability of the derived approach to design controllers for stochastic systems subject to real-time failures.

V. CONCLUDING REMARKS

This note presented an LMI formulation to design mode-independent controllers for Markov jump linear systems. For the corresponding control problem associated with the $\mathcal{H}_2$-cost, our LMI approach improves a previous result from the literature [12, mode-independent case] since it is able to produce lower $\mathcal{H}_2$-costs, as illustrated by the application of Section IV. Our approach proves to be useful in the control of real-time processes, as verified by the experiments shown in the paper.

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