Fast simulation of through-casing resistivity measurements using semi-analytical asymptotic models. Part 1: accuracy study

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INVENTEURS DU MONDE NUMÉRIQUE

UNIVERSITÉ DE PAU ET DES PAYS DE L'ADOUR

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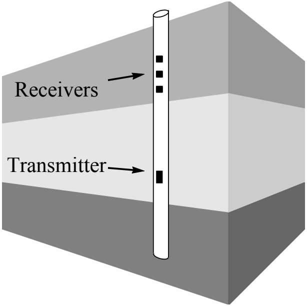
INTRODUCTION

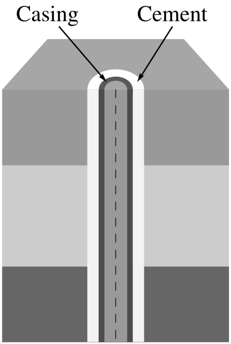
- **Main goal:** To obtain a better characterization of the Earth's subsurface

- **How:** Recording borehole resistivity measurements

- **Procedure:**
  - Well
  - Logging Instrument
  - Transmitters
  - Receivers



- **Practical Difficulties:**
  - It is not easy to drill a borehole
  - It may collapse
- **Practical Solutions:**
  - Use a metallic casing
  - Surround with a cement layer
- **Problem solved, but...**



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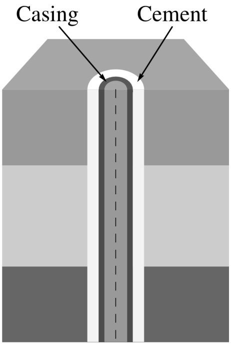
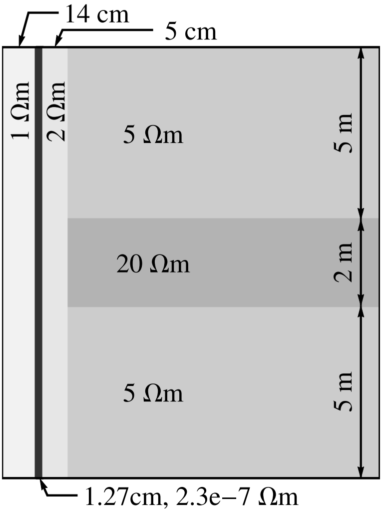


- **Problem solved, but... Numerical problems due to the high conductivity and thinness of the casing**

REALISTIC SCENARIO



- **Conductivity and casing width:**

$$\begin{cases} \delta & = 1.27\text{e} - 2 \text{ m} \\ \sigma_c & = 4.34\text{e}6 \;\; \Omega^{-1}\text{m}^{-1} \end{cases}$$

$$\Rightarrow \sigma_c \approx \delta^{-3}$$

- **First approach:**

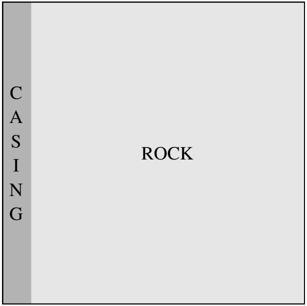
$$\sigma_c = \alpha \qquad\qquad \alpha \in \mathbb{R}$$

- **Case to be studied:**

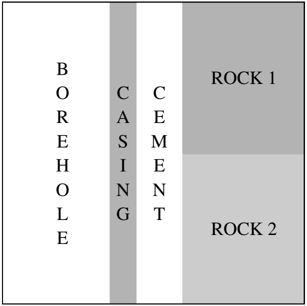
$$\sigma_c = \alpha\delta^{-3} \qquad\qquad \alpha \in \mathbb{R}$$

- **Develop:** Asymptotic method for avoiding the conflictive part of the domain (casing)
- **Scenarios:** As we are considering axisymmetric scenarios, we can work with two dimensional scenarios

Preliminary Scenario

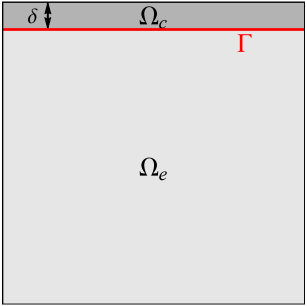


Target Scenario



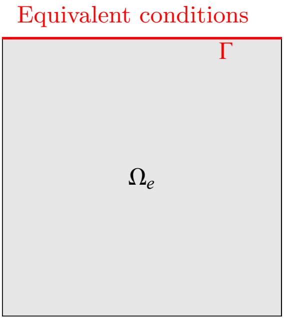
REFERENCE MODEL

Solution: u



ASYMPTOTIC MODEL

Solution: $u^{[n]}$



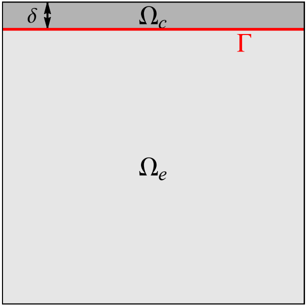
Definition: Let u be the reference solution. We say an asymptotic model is of **Order n+1**, if its solution $u^{[n]}$ satisfies

$$||u - u^{[n]}||_{L^2} \leq C\delta^{n+1}$$

EQUATIONS FOR THE ELECTRIC POTENTIAL

$$\mathrm{div}\left[(\sigma - i\delta\omega)\nabla u\right] = -\mathrm{div} j$$

PRELIMINARY SCENARIO ($\omega = 0$)



$$\Omega = \Omega_e \cup \Omega_c \cup \Gamma$$

$$\begin{cases} \sigma_e \Delta u_e = f & \text{in} \quad \Omega_e \\ \sigma_c \Delta u_c = 0 & \text{in} \quad \Omega_c \\ u_e = u_c & \text{on} \quad \Gamma \\ \sigma_c \partial_n u_c = \sigma_e \partial_n u_e & \text{on} \quad \Gamma \\ u_c = 0 & \text{on} \quad \partial\Omega \end{cases}$$

Where the solution is expressed as

$$u = \begin{cases} u_e & \text{in } \Omega_e \\ u_c & \text{in } \Omega_c \end{cases}$$

and σ_e, σ_c, f are known data

Asymptotic expansion of the solution:

- In the casing: $u_c(x,y) = \sum_{n\in\mathbb{N}} \delta^n U_c^n\left(x, \frac{y}{\delta}\right)$
- Outside the casing: $u_e(x,y) = \sum_{n\in\mathbb{N}} \delta^n u_e^n(x,y)$

Where δ is the width of the casing and we use the scaling

$$Y = \frac{y}{\delta} \in (0,1) \qquad \text{when} \qquad y \in (0,\delta)$$

IMPEDANCE CONDITIONS

One identifies simpler problems satisfied by truncated expansions outside the casing (up to residual terms)

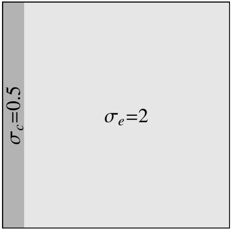
- **Order 1:** $$\longrightarrow \begin{cases} \sigma_e \Delta u &= f \quad \text{in} \quad \Omega_e \\ u &= 0 \quad \text{on} \quad \Gamma \end{cases}$$

- **Order 3:** $$\longrightarrow \begin{cases} \sigma_e \Delta u &= f \quad \text{in} \quad \Omega_e \\ u + \delta \frac{\sigma_e}{\sigma_c} \partial_n u &= 0 \quad \text{on} \quad \Gamma \end{cases}$$

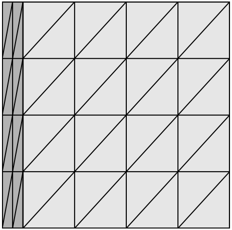
Remark: Second model has already order 3 of convergence due to the flat configuration of the layer

- **FINITE ELEMENT METHOD** (Matlab Code)
  - Straight triangular elements
  - Lagrange shape functions of any degree

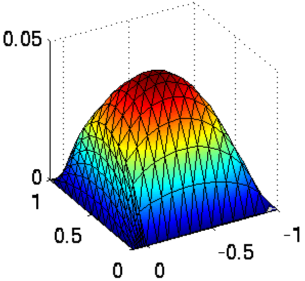
Domain



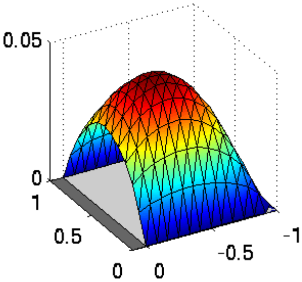
Mesh



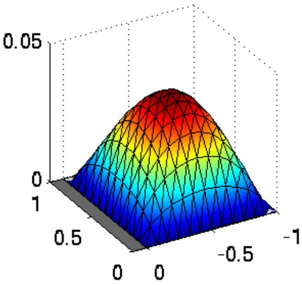
Reference Solution



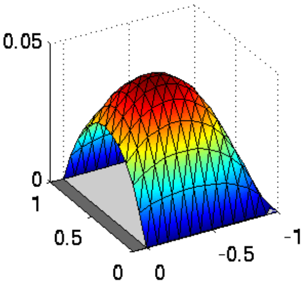
Reference Solution (in Ω_e)



Order 1 Model



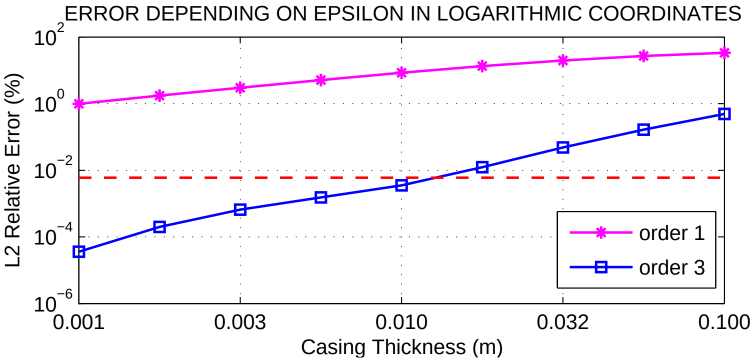
Order 3 Model



Definition: We define the relative error between the reference solution u and the asymptotic solution $u^{[n]}$, as

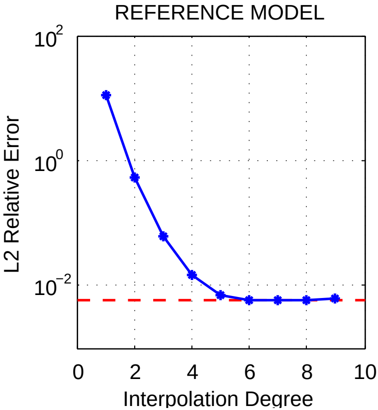
$$\frac{||u - u^{[n]}||_{L^2}}{||u||_{L^2}}$$

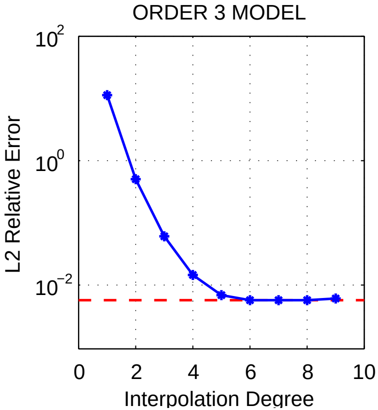
ERROR ANALYSIS



Casing Thickness	0.001	0.002	0.004	0.007	0.013	0.024	0.043	0.078
Order 1 Slopes	0.975	0.956	0.925	0.873	0.792	0.677	0.534	0.382
Order 3 Slopes	2.990	2.074	1.468	1.440	2.188	2.362	2.148	1.872

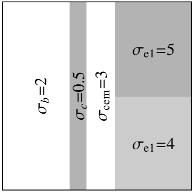
Relative error between a solution of degree 10 and solutions of lower degrees





CONCLUSION: Error analysis is not relevant once we reach a relative error of 10^{-2}

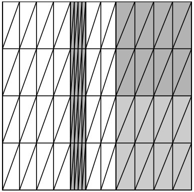
Domain



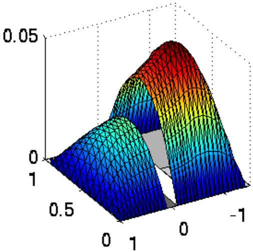
TARGET SCENARIO

Number of elements = 112
Degree of polynomials = 3

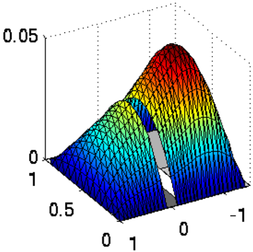
Mesh



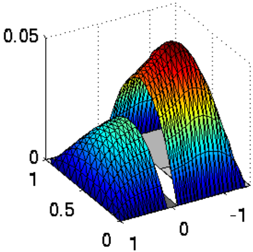
Reference Solution outside Ω_c



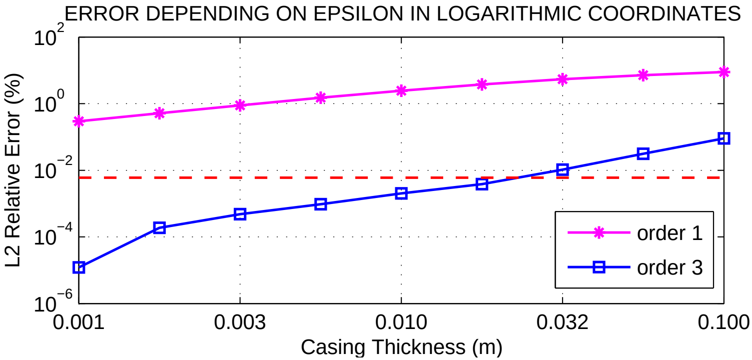
Order 1 Model



Order 3 Model



ERROR ANALYSIS



Casing Thickness	0.001	0.002	0.004	0.007	0.013	0.024	0.043	0.078
Order 1 Slopes	0.967	0.944	0.904	0.843	0.751	0.631	0.493	0.365
Order 3 Slopes	4.764	1.637	1.187	1.291	1.116	1.742	1.911	1.846

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- M. Duruflé, V. Péron and C. Poignard. Thin Layer Models for Electromagnetism.*Communications in Computational Physics* 16(1):213-238, 2014.

Perspectives

- Asymptotic models with $\sigma_c = \alpha\delta^{-3} \quad \alpha \in \mathbb{R}$
- Physically more realistic models
- 3D electromagnetic models

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THANK YOU FOR YOUR ATTENTION