Asymptotic analysis of RZF in large-scale MU-MIMO systems over Rician channels

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Abstract

In this work, we focus on the ergodic sum rate in the downlink of a single-cell large-scale multiuser MIMO system in which the base station employs N antennas to communicate with K single-antenna user equipments. A regularized zero-forcing (RZF) scheme is used for precoding under the assumption that each link forms a spatially correlated MIMO Rician fading channel. The analysis is conducted assuming that N and K grow large with a non trivial ratio and perfect channel state information is available at the base station. New results from random matrix theory and large system analysis are used to compute an asymptotic expression of the signal-to-interference-plus-noise ratio as a function of the system parameters, the spatial correlation matrix and the Rician factor. Numerical results are used to validate the accuracy of the asymptotic approximations in the finite system regime and to evaluate the performance under different operating conditions. It turns out that the provided asymptotic expressions provide accurate approximations even for relatively small values of N and K.

I. Introduction

Large-scale multiple-input multiple-output (MIMO) systems are considered as one of the most promising technology for next generation wireless communication systems [1]–[4] because of their considerable spatial multiplexing gains. The use of large-scale MIMO systems is beneficial not only in terms of coverage and spectral efficiency but also in terms of energy-saving [5]–[7].

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In this complex system model, a number of practical factors such as correlation effects and line-of-sight (LOS) components need to be included, which occur due to the space limitation of user equipments (UEs) and the densification of the antenna arrays resulting in a visible propagation path from the UEs, respectively. For typical systems of hundreds of antennas and tens of UEs, Monte Carlo simulations become challenging, thereby making performance analysis of large-scale MIMO systems an important subject of research.

A. *Major contributions*

In this work, we consider the downlink of a single-cell large-scale MIMO system in which the base station (BS), equipped with N antennas, makes use of regularized zero-forcing (RZF) precoding to communicate with K single-antenna UEs. In particular, we are interested in evaluating the ergodic sum rate of the system when a power constraint is imposed at the BS. The analysis is conducted assuming that N and K grow large with a non trivial ratio under the assumption that perfect channel state information is available at the BS. Differently from most of the existing literature [8]–[12], we consider a spatially correlated MIMO Rician fading model, which is more general and accurate to capture the fading variations when there is a LOS component. Compared to the Rayleigh fading channel, a Rician model makes the asymptotic analysis of large-scale MIMO systems much more involved. To overcome this issue, recent results from random matrix theory and large system analysis [12]–[14] are used to compute an asymptotic expression of the signal-to-interference-plus-noise ratio (SINR), which is eventually used to approximate the ergodic sum rate of the system. As shall be seen, the results are found to depend only on the system parameters, the spatial correlation matrix and the Rician factor. As a notable outcome of this work, the above analysis provides an analytical framework that can be used to evaluate the performance of the network under different settings without resorting to heavy Monte Carlo simulations and to eventually get insights on how the different parameters affect the performance. Further insights on the impact of the LOS components are obtained for simpler case studies.

B. *State-of-the-art*

The main literature related to this work is represented by [8], [11], [15]–[17]. Tools from random matrix theory are used in [8] to compute the ergodic sum rate in a single-cell setting with Rayleigh fading and different precoding schemes while the multicell case is analyzed in

[11]. In [15], the authors investigate a LOS-based conjugate beamforming transmission scheme and derive some expressions of the statistical SINR under the assumption that N grows large and K maintain fixed. In [16], the authors study the fluctuations of the mutual information of a cooperative small cell network operating over a Rician fading channel under the form of a central limit theorem and provide explicit expression of the asymptotic variance. In [17], a deterministic equivalent of the ergodic sum rate and an algorithm for evaluating the capacity achieving input covariance matrices for the uplink of a large-scale MIMO are proposed for spatially correlated MIMO channel with LOS components. The analysis of the uplink rate with both zero-forcing and maximum ratio combining receivers is performed in [18]. In [19], the authors derive tractable expressions for the achievable UL rate for ZF and MRC in the large-antenna limit, along with approximating results that hold for any finite number of antennas (N grows large and K is fixed). Based on these analytical results, the transmit power scaling law to meet a desirable quality of service is computed. A numerical analysis is used in [20] to show how LOS components may potentially improve the system performance and mitigate the pilot contamination problem. In [21], a full-duplex multicell Massive MIMO systems is analyzed. A deterministic approximation of the UL achievable rate with MRC is derived based on random matrix theory. In [22], a detailed achievable rate analysis of regular and large-scale single-user MIMO systems is presented under transceiver hardware impairments and Rician fading conditions.

C. *Outline and notation*

The remainder of this work is organized as follows. Next section introduces the system and channel models. The main results are provided in Section III, where the random matrix theory tools used in [13] are first extended and then use to compute deterministic approximations of the SINR under RZF precoding. A few case studies are also considered to get further insights into the effect of system parameters. Numerical results are used in Section IV to validate the theoretical analysis for systems of finite size and to investigate the performance of the network under different operating conditions. Finally, the major conclusions are drawn in Section V. All the technical proofs are presented in the Appendices.

The following notation is used throughout this work. The superscript ^{H} stands for the conjugate transpose operation. The operator $\mathrm{Tr}\mathbf{X}$ denotes trace of matrix $\mathbf{X}$ whereas $\mathbf{X}_{[k]}$ indicates that the kth column is removed from matrix $\mathbf{X}$. The Frobenius and spectral norms of a matrix $\mathbf{X}$ are denoted by $||\mathbf{X}||_F$ and $||\mathbf{X}||_2$, respectively. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$

whereas $\mathbf{X} = \mathrm{diag}\{x_1, \ldots, x_N\}$ is used to denote a $N \times N$ diagonal matrix of entries $\{x_n\}$. A random variable x is a standard complex Gaussian variable if $\sim \mathcal{CN}(0, 1)$. We use $a_n - b_n \to 0$ to denote $a_n - b_n \to_{n\to\infty} 0$ (almost surely (a.s.)) for two (random) sequences a_n, b_n.

II. System and channel models

We consider the DL of a network in which K UEs are served by a single BS equipped with N antennas. Denoting by $\mathbf{g}_i \in \mathbb{C}^N$ the precoding vector associated to UE i, the received signal y_k at a generic UE k takes the form [8]

$$y_k = \sqrt{\xi}\mathbf{h}_k^H \sum_{i=1}^{K} \mathbf{g}_i \varsigma_i + n_k \tag{1}$$

where $\varsigma_i \sim \mathcal{CN}(0, p_i)$ is the data symbol intended to UE i with variance $p_i \geq 0$, $\mathbf{h}_k \in \mathbb{C}^N$ is the random channel vector from the BS to UE k, and $n_k \sim \mathcal{CN}(0, \sigma^2)$ accounts for thermal noise. As shown next, the parameter ξ normalizes the average transmit power. For analytic tractability, we assume that the BS has perfect channel state information. The precoding matrix $\mathbf{G} = [\mathbf{g}_1 \ldots \mathbf{g}_K] \in \mathbb{C}^{N\times K}$ is designed according to the RZF scheme as follows [8]

$$\mathbf{G} = \left(\mathbf{H}\mathbf{H}^H + \lambda N \mathbf{I}_N\right)^{-1} \mathbf{H} \tag{2}$$

where $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \in \mathbb{C}^{N\times K}$ is the aggregate channel matrix, $\lambda > 0$ is the so-called regularization parameter and ξ is chosen to satisfy the following power constraint

$$\frac{\xi}{K}\mathrm{Tr}\mathbf{P}\mathbf{G}^H\mathbf{G} = P_T \tag{3}$$

where $\mathbf{P} = \mathrm{diag}\{p_1, \ldots, p_K\}$ and $P_T > 0$ denotes the total available transmit power. Plugging (2) into (3) yields

$$\xi = \frac{P_T}{\frac{1}{K}\mathrm{Tr}\mathbf{P}\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \lambda N \mathbf{I}_N\right)^{-2}\mathbf{H}}. \tag{4}$$

Under the assumption of Gaussian signaling, i.e., $\varsigma_k \sim \mathcal{CN}(0, p_k)$ and single-user detection with perfect channel state information at the receiver, the SINR γ_k of UE k takes the form

$$\gamma_k = \frac{p_k |\mathbf{h}_k^H \mathbf{g}_k|^2}{\sum\limits_{i=1, i\neq k}^{K} p_i \left|\mathbf{h}_k^H \mathbf{g}_i\right|^2 + \frac{\sigma^2}{\xi}}. \tag{5}$$

The rate r_k of UE k is given by $r_k = \log_2\left(1 + \gamma_k\right)$ whereas the ergodic sum rate is defined as

$$r_E = \sum_{k=1}^{K} \mathbb{E}\left[\log_2\left(1 + \gamma_k\right)\right] \tag{6}$$

where the expectation is taken over the random channel vectors $\{\mathbf{h}_k : k = 1, \ldots, K\}$. We assume that $\mathbf{h}_k = \sqrt{\beta_k}\mathbf{w}_k$ where β_k accounts for the large-scale channel fading gain of UE k and $\mathbf{w}_k \in \mathbb{C}^N$ is the small-scale fading channel component. The latter is modelled as

$$\mathbf{w}_k = \sqrt{\frac{1}{1+\rho}}\boldsymbol{\Theta}^{1/2}\mathbf{z}_k + \sqrt{\frac{\rho}{1+\rho}}\tilde{\mathbf{z}}_k \tag{7}$$

where $\mathbf{z}_k \in \mathbb{C}^N$ is assumed to be Gaussian with zero mean and unit covariance, i.e., $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{I}_N)$, and $\tilde{\mathbf{z}}_k \in \mathbb{C}^N$ is a deterministic vector. The scalar $\rho \geq 0$ is the Rician factor whereas the matrix $\boldsymbol{\Theta}^{1/2}$ is obtained from the Cholesky decomposition of $\boldsymbol{\Theta} \in \mathbb{C}^{N\times N}$, which accounts for the channel correlation matrix at the BS antennas. To make the problem analytically more tractable, we consider a system with a common UE channel correlation matrix [8], [23], [24]. Despite possible in principle, the extension to the case in which UEs have different channel correlation matrices is mathematically much more involved and it is left for future work. In practice, the considered scenario may arise in networks wherein the UEs are clustered on the basis of their covariance matrices such that UEs with different covariance matrices are put in different clusters [23], [25].

III. Main Results

We aim to exploit the statistical distribution of the channel matrix $\mathbf{H} = [\mathbf{h}_1 \ldots \mathbf{h}_K] \in \mathbb{C}^{N\times K}$ and the large dimensions of N and K to compute a deterministic approximation of γ_k, which will be eventually used to find an approximation of the ergodic sum rate. In doing so, we assume the following grow rate of system dimensions:

Assumption 1. *The dimensions N and K grow to infinity at the same pace, that is:*

$$1 \leq \liminf N/K \leq \limsup N/K < \infty. \tag{8}$$

A. Preliminaries

To begin with, we call

$$\tilde{\mathbf{D}} = \mathrm{diag}\left\{\frac{\beta_1}{1+\rho}, \ldots, \frac{\beta_K}{1+\rho}\right\} \tag{9}$$

and compute the eigendecomposition of $\boldsymbol{\Theta}^{1/2}$ to obtain $\boldsymbol{\Theta}^{1/2} = \mathbf{U}^H\mathbf{D}^{1/2}\mathbf{U}$ with $\mathbf{U} \in \mathbb{C}^{N\times N}$ being unitary. Then, we rewrite $\mathbf{H}$ as follows $\mathbf{H} = \sqrt{N}\mathbf{U}^H\boldsymbol{\Sigma}$ where $\boldsymbol{\Sigma}$ is defined as

$$\boldsymbol{\Sigma} = \frac{1}{\sqrt{N}}\mathbf{D}^{1/2}\mathbf{X}\tilde{\mathbf{D}}^{1/2} + \mathbf{A} \tag{10}$$

with $\mathbf{X} = \mathbf{U}\mathbf{Z}$, $\mathbf{A} = \mathbf{U}\tilde{\mathbf{Z}}$ and

$$\tilde{\mathbf{Z}} = \sqrt{\frac{1}{N}\frac{\rho}{1+\rho}}\left[\sqrt{\beta_1}\tilde{\mathbf{z}}_1, \ldots, \sqrt{\beta_K}\tilde{\mathbf{z}}_K\right]. \tag{11}$$

Plugging $\mathbf{H} = \sqrt{N}\mathbf{U}^H\mathbf{\Sigma}$ into (2) and (4) yields

$$\mathbf{G} = \frac{1}{\sqrt{N}}\mathbf{U}^H\left(\mathbf{\Sigma}\mathbf{\Sigma}^H + \lambda\mathbf{I}_N\right)^{-1}\mathbf{\Sigma} \tag{12}$$

and

$$\xi = \frac{NP_T}{\frac{1}{K}\mathrm{Tr}\mathbf{P}\mathbf{\Sigma}^H\left(\mathbf{\Sigma}\mathbf{\Sigma}^H + \lambda\mathbf{I}_N\right)^{-2}\mathbf{\Sigma}}. \tag{13}$$

To derive a deterministic equivalent of the SINR under RZF precoding, we require the following assumptions on the correlation matrix and the LOS component of the channel [8], [13], [14]. They are a common way to model that the array gathers more energy as N increases and also that this energy originates from many spatial dimensions.

Assumption 2. *As* $N, K \to \infty$, $\limsup_N ||\mathbf{\Theta}||_2 < \infty$ *and* $\inf_N \frac{1}{N}\mathrm{Tr}\mathbf{\Theta} > 0$.

Assumption 3. *As* $N, K \to \infty$, $\limsup_N ||\mathbf{A}||_2 < \infty$ *and* $\inf_N \frac{1}{N}\mathrm{Tr}\mathbf{A} > 0$.

Let us now introduce the fundamental equations that are needed to express a deterministic equivalent of γ_k. We start with the following set of equations:

$$\delta = \frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{T} \tag{14}$$

$$\tilde{\delta} = \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}\tilde{\mathbf{T}} \tag{15}$$

with

$$\mathbf{T} = \left(\lambda\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right) + \mathbf{A}\left(\mathbf{I} + \delta\tilde{\mathbf{D}}\right)^{-1}\mathbf{A}^H\right)^{-1} \tag{16}$$

$$\tilde{\mathbf{T}} = \left(\lambda\left(\mathbf{I}_K + \delta\tilde{\mathbf{D}}\right) + \mathbf{A}^H\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1}\mathbf{A}\right)^{-1} \tag{17}$$

which admits a unique positive solution in the class of Stieltjes transforms of non-negative measures with support $\mathbb{R}_+$ [13], [14]. The matrices $\mathbf{T}$ and $\tilde{\mathbf{T}}$ are approximations of the resolvent $\mathbf{Q} = (\mathbf{\Sigma}\mathbf{\Sigma}^H - z\mathbf{I}_N)^{-1}$ and the co-resolvent $\tilde{\mathbf{Q}} = (\mathbf{\Sigma}^H\mathbf{\Sigma} - z\mathbf{I}_K)^{-1}$ such that

$$\frac{1}{N}\mathrm{Tr}\mathbf{B}\mathbf{Q} - \frac{1}{N}\mathrm{Tr}\mathbf{B}\mathbf{T} \to 0 \tag{18}$$

$$\frac{1}{N}\mathrm{Tr}\tilde{\mathbf{B}}\tilde{\mathbf{Q}} - \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{B}}\tilde{\mathbf{T}} \to 0 \tag{19}$$

for any sequence of matrices $\mathbf{B} \in \mathbb{C}^{N\times N}$ and $\tilde{\mathbf{B}} \in \mathbb{C}^{K\times K}$ and

$$\mathbb{E}\left|\mathbf{u}^H\mathbf{Q}\mathbf{u} - \mathbf{u}^H\mathbf{T}\mathbf{u}\right|^p = \mathcal{O}(N^{-p/2}) \tag{20}$$

$$\mathbb{E}\left|\mathbf{u}^H\tilde{\mathbf{Q}}\mathbf{u} - \mathbf{u}^H\tilde{\mathbf{T}}\mathbf{u}\right|^p = \mathcal{O}(N^{-p/2}) \tag{21}$$

for any integer p and any sequence of vectors $\mathbf{u} \in \mathbb{C}^N$ with uniformly bounded norm. For later convenience, observe that

$$\mathbf{T}\mathbf{A}(\mathbf{I}_K + \delta\tilde{\mathbf{D}})^{-1} = (\mathbf{I}_N + \tilde{\delta}\mathbf{D})^{-1}\mathbf{A}\tilde{\mathbf{T}}. \tag{22}$$

Then, we proceed defining the following deterministic quantities (that will be extensively used in the remainder of this work):

$$\vartheta = \frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{T}\mathbf{D}\mathbf{T} \tag{23}$$

$$\tilde{\vartheta} = \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}\tilde{\mathbf{T}}\tilde{\mathbf{D}}\tilde{\mathbf{T}} \tag{24}$$

$$F = \frac{1}{N}\mathrm{Tr}\widetilde{\mathbf{D}}\widetilde{\mathbf{T}}\mathbf{A}^H\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1}\mathbf{D}\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1}\mathbf{A}\widetilde{\mathbf{T}} \tag{25}$$

$$\Delta = (1-F)^2 - \lambda^2\vartheta\tilde{\vartheta} \tag{26}$$

$$\alpha = \frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{T}^2 \tag{27}$$

$$V = \frac{1}{N}\mathrm{Tr}\widetilde{\mathbf{D}}\widetilde{\mathbf{T}}\mathbf{A}^H\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-2}\mathbf{A}\widetilde{\mathbf{T}} \tag{28}$$

$$\Upsilon(\mathbf{P}) = \frac{1}{N}\mathrm{Tr}\mathbf{D}\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1}\mathbf{A}\widetilde{\mathbf{T}}\mathbf{P}\widetilde{\mathbf{T}}\mathbf{A}^H\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1}. \tag{29}$$

Also, denote by $\mathbf{a}_k$ the kth column of matrix $\mathbf{A}$ and call $\tilde{t}_{kk}$, d_k and $\tilde{d}_k$ the kth diagonal element of $\widetilde{\mathbf{T}}$, $\mathbf{D}$ and $\tilde{\mathbf{D}}$, respectively.

B. A useful and new result

The following theorem represents a major contribution of this work as it extends the results in [13]. This theorem is required to cope with the channel model in (7) and forms the mathematical basis of the subsequent large system analysis of RZF precoding. It provides deterministic equivalents of quadratic forms and normalized traces involving two occurrences of the resolvent matrix $\mathbf{Q}$. This problem was addressed in [8], [26] for various random matrix models associated with centered random matrices associated, and recently with non-centered random matrices [13]. This latter scenario is much more complicated and was only handled in [13] for specific functionals, as shown below.

Theorem 1. *Let* $\mathbf{C} \in \mathbb{C}^{N\times N}$ *be a sequence of deterministic matrices whose spectral norms are uniformly bounded in* N. *Let* $\mathbf{v} \in \mathbb{C}^{N}$ *be a sequence of deterministic vectors whose euclidean norms are uniformly bounded in* N. *Assume that* $\theta(\mathbf{v},\mathbf{C})$ *and* $\chi(\mathbf{C})$ *are defined as follows:*

$$\theta(\mathbf{v},\mathbf{C}) = \mathbb{E}\left[\mathbf{v}^H\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{30}$$

and

$$\chi(\mathbf{C}) = \mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}\mathbf{C}\mathbf{Q}\right]. \tag{31}$$

Then, we have that:

$$\begin{aligned}\theta(\mathbf{v},\mathbf{C}) = \mathbf{v}^H\mathbf{T}\mathbf{C}\mathbf{T}\mathbf{v} + \mathbf{v}^H\mathbf{T}\mathbf{A}\tilde{\mathbf{D}}\left(\mathbf{I}_K+\delta\tilde{\mathbf{D}}\right)^{-2}\mathbf{A}^H\mathbf{T}\mathbf{v}\left(\frac{\vartheta(\mathbf{C})(1-F)+\vartheta F(\mathbf{C})}{\Delta}\right)\\ + \mathbf{v}^H\mathbf{T}\mathbf{D}\mathbf{T}\mathbf{v}\left(\frac{F(\mathbf{C})(1-F)}{\Delta}+\frac{\lambda^2\vartheta(\mathbf{C})\tilde{\vartheta}}{\Delta}\right)+\mathcal{O}(N^{-\frac{1}{2}})\end{aligned} \tag{32}$$

and

$$\chi(\mathbf{C}) = \frac{(1-F)\vartheta(\mathbf{C})}{\Delta}+\frac{\vartheta F(\mathbf{C})}{\Delta}+\mathcal{O}(N^{-\frac{1}{2}}) \tag{33}$$

with $F(\mathbf{C})$ *and* $\vartheta(\mathbf{C})$ *given by*

$$F(\mathbf{C}) = \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}\left(\mathbf{I}_K+\delta\tilde{\mathbf{D}}\right)^{-2}\mathbf{A}^H\mathbf{T}\mathbf{C}\mathbf{T}\mathbf{A} \tag{34}$$

$$\vartheta(\mathbf{C}) = \frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{T}\mathbf{C}\mathbf{T}. \tag{35}$$

Proof. The proof is given in Appendix A and unfolds using the same approach in [13], which basically consists in replacing (by using the resolvent identity) one occurrence of the random matrix $\mathbf{Q}$ in (30) and (31) with $\mathbf{T}$. A sum of different random quantities is thus obtained. Each quantity is handled separately using random matrix theory tools. In doing so, however, we end up with expressions that are much more involved of those in [13] and obtained for the special case of $\mathbf{C}=\mathbf{D}$ and $\mathbf{v}=\mathbf{a}_\ell$. Such specific issues are addressed in the proof. □

Setting $\mathbf{C}=\mathbf{D}$ and $\mathbf{v}=\mathbf{a}_\ell$ leads to the results in [13] as shown in the following corollary.

Corollary 1. *If* $\mathbf{C}=\mathbf{D}$ *and* $\mathbf{v}=\mathbf{a}_\ell$ *for some* $\ell\in\{1,\ldots,K\}$, *then we have that* (32) *and* (33) *simplify to:*

$$\theta(\mathbf{a}_\ell,\mathbf{D}) = \frac{(1-F)}{\Delta}\mathbf{a}_\ell^H\mathbf{T}\mathbf{D}\mathbf{T}\mathbf{a}_\ell + \frac{\vartheta}{\Delta}\mathbf{a}_\ell^H\mathbf{T}\mathbf{A}\tilde{\mathbf{D}}\left(\mathbf{I}_K+\delta\tilde{\mathbf{D}}\right)^{-2}\mathbf{A}^H\mathbf{T}\mathbf{a}_\ell+\mathcal{O}(N^{-\frac{1}{2}}) \tag{36}$$

and

$$\chi(\mathbf{D}) = \frac{\vartheta}{\Delta} + \mathcal{O}(N^{-\frac{1}{2}}). \tag{37}$$

Proof. The proof follows by observing that if $\mathbf{C} = \mathbf{D}$ and $\mathbf{v} = \mathbf{a}_\ell$ then $\vartheta(\mathbf{C}) = \vartheta$ and $F(\mathbf{C}) = F$. □

C. Asymptotic Analysis

Theorem 1 plays a key role in proving the following theorem, which summarizes one of the major results of this work.

Theorem 2. *Let Assumptions 1 – 3 hold true. Then, we have that* $\max_k |\gamma_k - \overline{\gamma}_k| \to 0$ *with*

$$\overline{\gamma}_k = \frac{p_k \left(1 - \lambda \tilde{t}_{kk}\right)^2}{\overline{s}_k + \overline{\psi} \frac{\sigma^2}{N P_T}} \tag{38}$$

where $\overline{s}_k$ *is given by* $\overline{s}_k = \tilde{d}_k \overline{s}_{k,1} + \overline{s}_{k,2}$ *with* $\overline{s}_{k,1}$ *and* $\overline{s}_{k,2}$ *being defined as*

$$\overline{s}_{k,1} = \frac{(1-F)\lambda^2 \tilde{t}_{kk}^2}{\Delta N} \mathrm{Tr} \mathbf{P} \widetilde{\mathbf{T}} \mathbf{A}^H \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{D} \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{A} \widetilde{\mathbf{T}} + \frac{\vartheta}{\Delta} \frac{\lambda^4 \tilde{t}_{kk}^2}{N} \mathrm{Tr} \mathbf{P} \tilde{\mathbf{D}} \widetilde{\mathbf{T}}^2 \tag{39}$$

and

$$\begin{aligned}\overline{s}_{k,2} = &\sum_{\ell=1,\ell\neq k}^{K} \frac{\left|\left[\widetilde{\mathbf{T}} \mathbf{A}^H \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{a}_k\right]_\ell\right|^2}{(1+\tilde{d}_k \delta)^2} \left(p_\ell + \tilde{d}_\ell \left(\frac{\Upsilon(\mathbf{P})(1-F)}{\Delta} + \frac{\vartheta}{\Delta}\tilde{s}_{k,2}^\circ\right)\right) \\ &+ \left[\widetilde{\mathbf{T}} \mathbf{A}^H \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{D} \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{A} \widetilde{\mathbf{T}}\right]_{kk} \left(\frac{\lambda^2 \Upsilon(\mathbf{P}) \tilde{\vartheta}}{\Delta} + \frac{1-F}{\Delta} \tilde{s}_{k,2}^\circ\right)\end{aligned} \tag{40}$$

with

$$\tilde{s}_{k,2}^\circ = \lambda^2 \frac{1}{N} \mathrm{Tr} \mathbf{P} \tilde{\mathbf{D}} \widetilde{\mathbf{T}}^2. \tag{41}$$

Also, we have that

$$\begin{aligned}\overline{\psi} = &\left(\frac{1-F}{\Delta}\alpha + \frac{\vartheta}{\Delta} V\right) \frac{\lambda^2}{K} \mathrm{Tr} \mathbf{P} \tilde{\mathbf{D}} \widetilde{\mathbf{T}}^2 + \frac{1}{K} \mathrm{Tr} \mathbf{P} \widetilde{\mathbf{T}} \mathbf{A}^H \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-2} \mathbf{A} \widetilde{\mathbf{T}} \\ &+ \left(\frac{V(1-F)}{\Delta} + \frac{\lambda^2 \tilde{\vartheta} \alpha}{\Delta}\right) \frac{1}{K} \mathrm{Tr} \mathbf{P} \widetilde{\mathbf{T}} \mathbf{A}^H \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{D} \left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right)^{-1} \mathbf{A} \widetilde{\mathbf{T}}.\end{aligned} \tag{42}$$

Proof: The proof is very much involved and is given in Appendix B. It basically relies on results in random matrix theory [13], [14] as well as on those provided in Theorem 1. ■

We are ultimately interested in the individual rates $\{r_k\}$ and the ergodic sum rate r_E. Since the logarithm is a continuous function, by applying the continuous mapping theorem and the dominated convergence theorem, from the almost sure convergence results of Theorem 2 it follows that $r_k - \overline{r}_k \to 0$ almost surely with [8]

$$\overline{r}_k = \log_2\left(1 + \overline{\gamma}_k\right). \tag{43}$$

An approximation of r_E is obtained as follows [8]

$$\overline{r}_E = \sum_{k=1}^{K} \log_2\left(1 + \overline{\gamma}_k\right) \tag{44}$$

such that $\frac{1}{K}(r_E - \overline{r}_E) \to 0$ holds true almost surely.

The asymptotic expression provided in Theorem 2 will be shown to be very tight by means of numerical results in Section IV, even for systems with finite dimensions. This means that can be used for evaluating the performance of practical systems without the need for time-consuming Monte Carlo simulations. Despite being useful for this purpose, the expressions are quite involved and do not lend themselves to an easy interpretation of the interplay between different system parameters. Simplifications can be obtained under different operating conditions. We start looking into the limiting case in which $N \to \infty$ with $K/N \to 0$.

Corollary 2. *If $N \to \infty$ with $K/N \to 0$ and there exists i, k such that $i \neq k$ and $\mathbf{a}_k^H \mathbf{a}_i \not\to 0$, then, under Assumptions 2 and 3, we have that $\max_k |\gamma_k - \overline{\gamma}_k| \to 0$ with:*

$$\overline{\gamma}_k = \frac{p_k(1 - \lambda \tilde{t}_{kk})^2}{\overline{s}_k + \overline{\psi}\frac{\sigma^2}{NP_T}} \tag{45}$$

where

$$\overline{s}_k = \sum_{l=1, l\neq k} \frac{\left[\widetilde{\mathbf{T}}\mathbf{A}^H \mathbf{a}_k \mathbf{a}_k^H \mathbf{A}\widetilde{\mathbf{T}}\right]_{ll} p_l}{\left(1 + \frac{\tilde{d}_k}{\lambda}\frac{1}{N}\mathrm{Tr}\mathbf{D}\right)^2} \tag{46}$$

and

$$\overline{\psi} = \frac{1}{N}\mathrm{Tr}\mathbf{D}\frac{1}{K}\mathrm{Tr}\mathbf{P}\widetilde{\mathbf{D}}\widetilde{\mathbf{T}}^2 + \frac{1}{K}\mathrm{Tr}\mathbf{P}\widetilde{\mathbf{T}}\mathbf{A}^H\mathbf{A}\widetilde{\mathbf{T}} \tag{47}$$

with $\widetilde{\mathbf{T}}$ now given by

$$\widetilde{\mathbf{T}} = \left(\lambda \mathbf{I}_K + \left(\frac{1}{N}\mathrm{Tr}\mathbf{D}\right)\tilde{\mathbf{D}} + \mathbf{A}^H\mathbf{A}\right)^{-1}. \tag{48}$$

Proof: The proof follows from the results of Theorem 2 by noticing that if $K/N \to 0$, $\delta - \frac{1}{N\lambda}\mathrm{Tr}\mathbf{D} \to 0$, $\Upsilon(\mathbf{P}) \to 0$, $F \to 0$, $V \to 0$, $\tilde{\vartheta} \to 0$ and $\alpha - \frac{1}{N\lambda^2}\mathrm{Tr}\mathbf{D} \to 0$. ■

The above corollary shows that when N grows at a faster rate than K the asymptotic interference towards UE k does not necessarily vanish as it depends on the LOS components

$\{\mathbf{a}_i; i = 1, \ldots, K\}$. In particular, it depends on the inner products between the vector $\mathbf{a}_k$ and all other vectors $\mathbf{a}_i$ with $i \neq k$. Consider then a system in which $\mathbf{a}_k^H \mathbf{a}_i \to 0 \ \forall i \neq k$. This means that UEs are selected such that they satisfy asymptotically favorable propagation conditions [27]. Then, we have that:

Corollary 3. *If $N \to \infty$ with $K/N \to 0$ and $\mathbf{a}_k^H \mathbf{a}_i \to 0 \ \forall i \neq k$, then we have that*

$$\max_k \left| \sum_{i=1, i\neq k}^{K} p_i \left|\mathbf{h}_k^H \mathbf{g}_i\right|^2 \right| \to 0. \tag{49}$$

Proof: The proof easily follows from noticing that under favorable propagation conditions, i.e., $\mathbf{a}_k^H \mathbf{a}_i \to 0 \ \forall i \neq k$, the matrix $\widetilde{\mathbf{T}}$ becomes asymptotically diagonal and the quantities

$$\left[\widetilde{\mathbf{T}}\mathbf{A}^H \mathbf{a}_k \mathbf{a}_k^H \mathbf{A}\widetilde{\mathbf{T}}\right]_{ll} \to 0 \tag{50}$$

vanish asymptotically as $N \to \infty$ with $K/N \to 0$. ■

In agreement with [19], [27], the above corollary shows that if the MIMO Rician fading channel results in favorable propagations, then the interference vanishes as N grows unbounded, and consequently the ergodic sum rate grows unbounded. In practice, these asymptotic results can only be achieved if some UEs are dropped from service [28].

D. Case Studies

A few illustrative case studies are considered next to get further insights into the effect of system parameters. To begin with, we assume that the BS antennas are uncorrelated. This is equivalent to assuming that $\mathbf{\Theta} = \mathbf{I}_N$. Then, we have that:

Corollary 4. *Let Assumptions 1 and 3 hold true. If $\mathbf{\Theta} = \mathbf{I}_N$, then $\max_k |\gamma_k - \overline{\gamma}_k| \to 0$ with:*

$$\overline{\gamma}_k = \frac{p_k(1-\lambda \tilde{t}_{kk})^2}{\overline{s}_k + \overline{\psi}\frac{\sigma^2}{NP_T}} \tag{51}$$

where

$$\overline{\psi} = \frac{\vartheta}{\Delta}\frac{\lambda^2}{K}\mathrm{Tr}\mathbf{P}\tilde{\mathbf{D}}\widetilde{\mathbf{T}}^2 + \frac{1-F}{\Delta(1+\tilde{\delta})^2}\frac{1}{K}\mathrm{Tr}\mathbf{P}\widetilde{\mathbf{T}}\mathbf{A}^H\mathbf{A}\widetilde{\mathbf{T}} \tag{52}$$

and $\overline{s}_k = \tilde{d}_k \overline{s}_{k,1} + \overline{s}_{k,2}$ *with*

$$\overline{s}_{k,1} = \frac{(1-F)\lambda^2 \tilde{t}_{kk}^2}{\Delta(1+\tilde{\delta})^2} \frac{1}{N} \mathrm{Tr} \mathbf{P} \widetilde{\mathbf{T}} \mathbf{A}^H \mathbf{A} \widetilde{\mathbf{T}} + \frac{\vartheta}{\Delta} \frac{\lambda^4 \tilde{t}_{kk}^2}{N} \mathrm{Tr} \mathbf{P} \widetilde{\mathbf{D}} \widetilde{\mathbf{T}}^2 \tag{53}$$

$$\overline{s}_{k,2} = \sum_{\ell=1, \ell \neq k} \frac{\left| \left[\widetilde{\mathbf{T}} \mathbf{A}^H \mathbf{a}_k \right]_\ell \right|^2 \left(p_\ell + \tilde{d}_\ell \left(\frac{\Upsilon(\mathbf{P})(1-F)}{\Delta} + \frac{\vartheta}{\Delta} \tilde{s}_{k,2}^{\circ} \right) \right)}{(1+\tilde{d}_k \delta)^2 (1+\tilde{\delta})^2} + \frac{\left[\widetilde{\mathbf{T}} \mathbf{A}^H \mathbf{A} \widetilde{\mathbf{T}} \right]_{k,k}}{(1+\tilde{\delta})^2} \left(\frac{\lambda^2 \Upsilon(\mathbf{P}) \tilde{\vartheta}}{\Delta} + \frac{1-F}{\Delta} s_{k,2}^{\circ} \right) \tag{54}$$

and

$$\tilde{s}_{k,2}^{\circ} = \frac{\lambda^2}{N} \mathrm{Tr} \mathbf{P} \tilde{\mathbf{D}} \tilde{\mathbf{T}}^2. \tag{55}$$

If furthermore $\mathbf{a}_k^H \mathbf{a}_i = 0$ *for* $k \neq i$*, then* $\overline{s}_{k,2}$ *simplifies to*

$$\overline{s}_{k,2} = \frac{\left[\widetilde{\mathbf{T}} \mathbf{A}^H \mathbf{A} \widetilde{\mathbf{T}} \right]_{kk}}{(1+\tilde{\delta})^2} \left(\frac{\lambda^2 \Upsilon(\mathbf{P}) \tilde{\vartheta}}{\Delta} + \frac{1-F}{\Delta} \tilde{s}_{k,2}^{\circ} \right). \tag{56}$$

Proof: The proof follows by noting that when $\mathbf{D} = \mathbf{I}_N$, $\alpha = \vartheta$ and $V = F$. Replacing these quantities into the expressions of Theorem 2 yields (52), (53) and (54). ■

Consider now the case for which only the channel of the UE of interest, i.e, UE k, is characterized by a Rician fading channel, whereas a Rayleigh fading channel model is valid for all other UEs with indexes $i \neq k$. Assume also that $\|\mathbf{a}_k\|^2 = \beta_k \frac{\rho}{1+\rho}$. Then, we have that $[\tilde{\mathbf{D}}]_{k,k} = \frac{\beta_k}{1+\rho}$ and $[\tilde{\mathbf{D}}]_{i,i} = \beta_i$ for $i \neq k$, which is slightly different from the original setting described by (9). Under the above circumstances, the following result is found:

Corollary 5. *Let Assumptions 1 and 3 hold true. If* $\mathbf{\Theta} = \mathbf{I}_N$, $\|\mathbf{a}_k\|^2 = \frac{\beta_k \rho}{\rho+1}$ *and no LOS component is present for all other UEs with indexes* $i \neq k$*, then* $\max_k |\gamma_k - \overline{\gamma}_k| \to 0$ *with:*

$$\overline{\gamma}_k = \frac{p_k \left(1 - \frac{1}{1+\frac{\beta_k}{\lambda(1+\tilde{\delta})}} \right)^2 \left(1 - \frac{1}{N} \sum_{l=1}^{K} \frac{\beta_l^2}{\left(\lambda(1+\tilde{\delta})+\beta_l\right)^2} \right)}{\frac{1}{N} \sum_{l=1}^{K} p_l \beta_l \frac{\left(\lambda(1+\tilde{\delta})+\beta_k\right)^{-2} + \frac{\sigma^2 \lambda^2}{P_T K}(1+\tilde{\delta})^{-2}}{\left(1+\frac{\beta_l}{\lambda(1+\tilde{\delta})}\right)^2}} \tag{57}$$

where $\tilde{\delta}$ *is the unique solution of the following equation:*

$$\tilde{\delta} = \frac{1}{N} \sum_{k=1}^{K} \frac{\beta_k (1+\tilde{\delta})}{\lambda(1+\tilde{\delta}) + \beta_k}. \tag{58}$$

Proof: By substituting the expressions of $\mathbf{A}$ and $\tilde{\mathbf{D}}$ into (16) and (17), from (14) and (15) (using the fact that $\mathbf{A}$ has rank 1) we get (58) and

$$\delta = \frac{1}{\lambda(1+\tilde{\delta})}. \tag{59}$$

Also, it is found that $\tilde{t}_{kk} = \frac{1}{\lambda(1+\frac{\beta_k \delta}{1+\rho}) + \frac{\beta_k \rho}{(1+\rho)(1+\tilde{\delta})}} = \frac{1}{\lambda + \frac{\beta_k}{1+\tilde{\delta}}}$ and $\tilde{t}_{ii} = \frac{1}{\lambda + \frac{\beta_k}{1+\tilde{\delta}}}$ for $i \neq k$. Since $\mathbf{A}$ has finite rank, $\Upsilon(\mathbf{P}) \to 0$, $F \to 0$ and thus $\Delta = 1 - \lambda^2 \vartheta \tilde{\vartheta}$. Moreover, it follows that $\mathbf{D} = \mathbf{I}_N$ and $\vartheta = \delta^2$. Using all the above results, we can simplify $\overline{s}_{k,1}$ and $\overline{s}_{k,2}$ in (53) and (54) as:

$$\overline{s}_{k,1} = \frac{\lambda^2 \tilde{t}_{kk}^2}{\left(1 - \lambda^2 \vartheta \tilde{\vartheta}\right)\left(1 + \tilde{\delta}\right)^2} \frac{1}{N} \sum_{l=1}^{K} \frac{p_l \beta_l}{\left(\lambda + \frac{\beta_l}{1+\tilde{\delta}}\right)^2} \tag{60}$$

$$\overline{s}_{k,2} = \frac{\beta_k \rho}{1+\rho} \frac{\lambda^2 \tilde{t}_{kk}^2}{\left(1 - \lambda^2 \vartheta \tilde{\vartheta}\right)\left(1 + \tilde{\delta}\right)^2} \frac{1}{N} \sum_{l=1}^{K} \frac{p_l \beta_l}{\left(\lambda + \frac{\beta_l}{1+\tilde{\delta}}\right)^2}. \tag{61}$$

Hence, $\overline{s}_k = \frac{\beta_k}{1+\rho} \overline{s}_{k,1} + \overline{s}_{k,2}$ reduces to:

$$\overline{s}_k = \frac{\beta_k \tilde{t}_{kk}^2}{\left(1 - \lambda^2 \vartheta \tilde{\vartheta}\right)\left(1 + \tilde{\delta}\right)^2} \frac{1}{N} \sum_{l=1}^{K} \frac{p_l \beta_l}{\left(1 + \frac{\beta_l}{\lambda(1+\tilde{\delta}})\right)^2}. \tag{62}$$

By using the results above, we can also prove that

$$\overline{\psi} = \frac{\lambda^2}{\left(1 - \lambda^2 \vartheta \tilde{\vartheta}\right)\left(1 + \tilde{\delta}\right)^2} \frac{N}{K} \frac{1}{N} \sum_{l=1}^{K} \frac{p_l \beta_l}{\left(1 + \frac{\beta_l}{\lambda(1+\tilde{\delta})}\right)^2}. \tag{63}$$

By substituting (62) and (63) into (51) yields (57), after some standard calculus. ■

The results of Corollary 5 coincide with those obtained for the case in which all UEs experience Rayleigh fading propagations. This means that the asymptotic expression of the SINR for a given UE k is independent from its own channel model if all the other UEs are affected by Rayleigh fading propagations.

To gain further insights into the impact of the Rician factor ρ, assume that the channel can be simply modelled as $\mathbf{\Theta} = \mathbf{I}_N$, $\tilde{\mathbf{D}} = \frac{\beta}{1+\rho}\mathbf{I}_K$ and $\mathbf{A}^H\mathbf{A} = \frac{\rho\beta}{1+\rho}\mathbf{I}_K$. In this case, the following result holds true:

Corollary 6. *Let Assumption 1 holds true. Assume furthermore that* $\tilde{\mathbf{D}} = \frac{\beta}{1+\rho}\mathbf{I}_K$, $\mathbf{A}^H\mathbf{A} = \frac{\rho\beta}{1+\rho}\mathbf{I}_K$ *and* $\mathbf{\Theta} = \mathbf{I}_N$. *Let* x *be the unique positive solution of the following third order equation:*

$$-\rho\beta x^3 + \beta x^2(\rho - 1) + x\left(\frac{\beta(N-K)}{N} - \lambda(1+\rho)\right) + \lambda(1+\rho) = 0. \tag{64}$$

Define ϑ *as:*

$$\vartheta = \frac{1}{\lambda\beta}\left(1+\rho+\frac{(N-K)\beta}{N\lambda}x\right)\left(1-\frac{\rho N}{K}(1-x)^2\right)-\frac{1}{\lambda\beta}x(1+\rho)$$

and

$$\Delta = \left(1-\frac{N\rho}{K}(1-x)^2\right)^2 - \lambda^2\vartheta\frac{N}{K}\left(\frac{1}{x}-1\right)^2. \tag{65}$$

Then, $\max_k |\gamma_k - \overline{\gamma}_k| \to 0$ *with:*

$$\overline{\gamma}_k = \frac{p_k\left(1-\lambda\left(\frac{1}{x}-1\right)\frac{N}{K}\left(\frac{1+\rho}{\beta}\right)\right)^2}{\overline{s}_k + \overline{\psi}\frac{\sigma^2}{NP_T}} \tag{66}$$

where

$$\overline{s}_k = \frac{\lambda^2}{N}\mathrm{Tr}\mathbf{P}\left(\frac{1+\rho}{\beta}\right)^2\left(\frac{N}{K}\right)^3\left(\frac{1}{x}-1\right)^2\left(-1+\frac{1}{\Delta}\right) \tag{67}$$

and

$$\overline{\psi} = \left(\frac{1-\frac{N\rho}{K}(1-x)^2}{\Delta}-1\right)\frac{1+\rho}{\beta}\frac{N}{K}\frac{1}{K}\mathrm{Tr}\mathbf{P}. \tag{68}$$

Proof: The proof relies on working with the variable $x = \frac{1}{1+\tilde{\delta}}$ and proving that it satisfies the third order equation given in (64). Towards this aim, we first note that in this specific case, $\tilde{\delta}$ simplifies to

$$\tilde{\delta} = \frac{\beta}{N(1+\rho)}\mathrm{Tr}\left(\lambda(1+\delta\frac{\beta}{1+\rho})\mathbf{I}_K + \frac{\rho\beta}{(1+\rho)(1+\tilde{\delta})}\mathbf{I}_K\right)^{-1} \tag{69}$$

$$= \frac{\frac{K}{N}\frac{\beta}{1+\rho}}{\lambda+\lambda\delta\frac{\beta}{1+\rho}+\frac{\rho\beta}{(1+\rho)(1+\tilde{\delta})}}. \tag{70}$$

The right-hand-side of (69) can be rewritten as:

$$\tilde{\delta} = \left(1+\frac{\delta\beta}{1+\rho}\right)^{-1}(1+\tilde{\delta})\frac{\beta}{1+\rho}\left[\frac{1}{N}\mathrm{Tr}\left(\lambda(1+\tilde{\delta})\mathbf{I}_N+\frac{\mathbf{A}\mathbf{A}^H}{1+\frac{\delta\beta}{1+\rho}}\right)^{-1}-\frac{N-K}{N\lambda(1+\tilde{\delta})}\right] \tag{71}$$

$$= \frac{(1+\tilde{\delta})\beta}{1+\rho+\delta\beta}\left[\delta-\frac{N-K}{N\lambda(1+\tilde{\delta})}\right] \tag{72}$$

from which we obtain

$$\beta\delta = (1+\rho)\tilde{\delta}+\frac{\beta}{\lambda}\frac{N-K}{N}. \tag{73}$$

By plugging (73) into (70) and replacing $\tilde{\delta}+1$ with $\frac{1}{x}$, we obtain (64) after some standard calculus. Once the equation involving x is established, the expressions of the interference and the power terms $\overline{s}_k$ and $\overline{\psi}$ follow after tedious but straightforward calculations. To this end, we need to compute the asymptotic expressions of Δ and ϑ as a function β, $\frac{N}{K}$ and ρ. Let us start with simplifying F. Since $\tilde{\mathbf{T}}$ is proportional to identity, and using the fact that $\tilde{\delta} = \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}\tilde{\mathbf{T}} = \frac{\beta}{1+\rho}\frac{1}{N}\mathrm{Tr}\tilde{\mathbf{T}}$, all elements of $\tilde{T}$ are equal to $\tilde{t}_{kk} = \frac{N}{K}\frac{1+\rho}{\beta}\tilde{\delta}$. Therefore, F simplifies to

$$F = \frac{N\rho}{K}(1-x)^2. \tag{74}$$

Following the same arguments, $\tilde{\vartheta}$ can be rewritten as:

$$\tilde{\vartheta} = \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}\tilde{\mathbf{T}}\tilde{\mathbf{D}}\tilde{\mathbf{T}} = \frac{N}{K}\left(\frac{1}{x}-1\right)^2. \tag{75}$$

We are now left with the asymptotic expression of ϑ. To begin with, we start from the following relation

$$\mathbf{T}\left(\frac{\lambda}{x}\mathbf{I}_N + \mathbf{A}\left(\mathbf{I}_K + \delta\tilde{\mathbf{D}}\right)^{-1}\mathbf{A}^H\right) = \mathbf{I}. \tag{76}$$

Multiplying the left and right hand sides of the above relation with $\mathbf{T}$, we obtain:

$$\frac{\lambda}{x}\mathbf{T}^2 + \mathbf{T}\mathbf{A}\left(\mathbf{I}_K + \delta\tilde{\mathbf{D}}\right)^{-1}\mathbf{A}^H\mathbf{T} = \mathbf{T} \tag{77}$$

Using the relation $\mathbf{T}\mathbf{A}\left(\mathbf{I}_K + \delta\tilde{\mathbf{D}}\right)^{-1} = \left(\mathbf{I}_N + \delta\tilde{\mathbf{D}}\right)^{-1}\mathbf{A}\tilde{\mathbf{T}}$, we obtain:

$$\frac{1}{N}\frac{\lambda}{x}\mathrm{Tr}\mathbf{T}^2 + x^2\frac{\beta\rho}{1+\rho}\left(1+\frac{\delta\beta}{1+\rho}\right)\frac{1}{N}\mathrm{Tr}\tilde{\mathbf{T}}^2 = \frac{1}{N}\mathrm{Tr}\mathbf{T}. \tag{78}$$

Hence,

$$\begin{aligned}
\frac{1}{N}\mathrm{Tr}\mathbf{T}^2 &= \frac{x}{\lambda}\left[\frac{1}{N}\mathrm{Tr}\mathbf{T} - \frac{\rho(1+\rho+\delta\beta)}{\beta}\frac{N}{K}(1-x)^2\right] \\
&= \frac{x}{\lambda\beta}\left[\delta\beta(1-\rho\frac{N}{K}(1-x)^2) - \rho(1+\rho)\frac{N}{K}(1-x)^2\right] \\
&= \frac{x}{\lambda\beta}\left[\left((\frac{1}{x}-1)(1+\rho) + \frac{N-K}{N\lambda}\beta\right)\left(1-\frac{\rho N}{K}(1-x)^2\right) - \rho(1+\rho)\frac{N}{K}(1-x)^2\right] \\
&= \frac{1}{\lambda\beta}\left[\left[(1-x)(1+\rho) + \frac{(N-K)\beta}{N\lambda}x\right]\left[1-\frac{\rho N}{K}(1-x)^2\right] - x\rho(1+\rho)\frac{N}{K}(1-x)^2\right] \\
&= \frac{1}{\lambda\beta}\left[\left[1+\rho+\frac{(N-K)\beta}{N\lambda}x\right]\left[1-\frac{\rho N}{K}(1-x)^2\right] - x(1+\rho)\right].
\end{aligned} \tag{79}$$

Plugging (74), (75) and (79) into (26) yields (65). With these expressions at hand, we are ready to simplify $\overline{\psi}$ and $\overline{s}_k$. Since $\mathbf{\Theta} = \mathbf{I}_N$, $\overline{\psi}$ writes as:

$$\begin{aligned}
\overline{\psi} &= \frac{\vartheta}{\Delta}\frac{\lambda^2}{K}\mathrm{Tr}\mathbf{P}\tilde{\mathbf{D}}\tilde{\mathbf{T}}^2 + \frac{1-F}{\Delta(1+\tilde{\delta})^2}\frac{1}{K}\mathrm{Tr}\mathbf{P}\tilde{\mathbf{T}}\mathbf{A}^H\mathbf{A}\tilde{\mathbf{T}} \\
&= \frac{\vartheta}{\Delta}\frac{\lambda^2}{K}\mathrm{Tr}\mathbf{P}\frac{N}{K}\left(\frac{1+\rho}{\beta}\right)\frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}^2\tilde{\mathbf{T}}^2 + \frac{(1-F)F}{\Delta}\frac{1+\rho}{\beta}\frac{N}{K}\frac{1}{K}\mathrm{Tr}\mathbf{P} \\
&= \frac{1}{K}\mathrm{Tr}\mathbf{P}\frac{N}{K}\frac{1+\rho}{\beta}\left(-1+\frac{1-F}{\Delta}\right).
\end{aligned} \tag{80}$$

Plugging (74) into the above equation yields (68). We are now left with simplifing the expression of the interference term $\overline{s}_k$. From Corollary 4, $\overline{s}_k = \tilde{d}_k \overline{s}_{k,1} + \overline{s}_{k,2}$ where $\overline{s}_{k,1}$ and $\overline{s}_{k,2}$ are given by (53) and (56). Notice that

$$\Upsilon(\mathbf{P}) = \frac{1}{N}\mathrm{Tr}\mathbf{P}F\frac{N}{K}\frac{1+\rho}{\beta} \tag{81}$$

and

$$\tilde{s}_{k,2}^{\circ} = \lambda^2 \frac{1}{N}\mathrm{Tr}\mathbf{P}\frac{1+\rho}{\beta}\frac{N}{K}\tilde{\vartheta}. \tag{82}$$

Hence, we have that

$$\frac{\lambda^2 \Upsilon(\mathbf{P})\tilde{\vartheta}}{\Delta} + \frac{1-F}{\Delta}\tilde{s}_{k,2}^{\circ} = \frac{\lambda^2}{N\Delta}\mathrm{Tr}\mathbf{P}\frac{1+\rho}{\beta}\frac{N}{K}\tilde{\vartheta} \tag{83}$$

from which it follows that:

$$\overline{s}_{k,2} = \left(\frac{N}{K}\right)^4 \rho \left(\frac{1+\rho}{\beta}\right)^2 \frac{\lambda^2}{N\Delta}\mathrm{Tr}\mathbf{P}\left(\frac{1}{x}-1\right)^2 (1-x)^2. \tag{84}$$

Using similar arguments as above, we can prove that:

$$\begin{aligned}\overline{s}_{k,1} &= \frac{F(1-F)}{\Delta}\lambda^2 \tilde{t}_{kk}^2 \frac{1}{N}\mathrm{Tr}\mathbf{P}\frac{N}{K}\frac{1+\rho}{\beta} + \frac{\vartheta\tilde{\vartheta}}{\Delta}\lambda^4 \tilde{t}_{kk}^2 \frac{1}{N}\mathrm{Tr}\mathbf{P}\frac{1+\rho}{\beta} & (85)\\ &= \lambda^2 \tilde{t}_{kk}^2 \frac{1}{N}\mathrm{Tr}\mathbf{P}\frac{N}{K}\frac{1+\rho}{\beta}\left(-1+\frac{1-F}{\Delta}\right). & (86)\end{aligned}$$

Since $\overline{s}_k = \tilde{d}_k \overline{s}_{k,1} + \overline{s}_{k,2}$, we obtain:

$$\begin{aligned}\overline{s}_k &= \lambda^2 \tilde{t}_{kk}^2 \frac{1}{N}\mathrm{Tr}\mathbf{P}\frac{N}{K}\left(-1+\frac{1-F}{\Delta}\right) + \left(\frac{N}{K}\right)^4 \rho \left(\frac{1+\rho}{\beta}\right)^2 \frac{\lambda^2}{\Delta N}\mathrm{Tr}\mathbf{P}\frac{N}{K}\left(-1+\frac{1-F}{\Delta}\right)\\ &= \lambda^2 \left(\frac{N}{K}\right)^3 \left(\frac{1+\rho}{\beta}\right)^2 \frac{(1-x)^2}{x^2}\frac{1}{N}\mathrm{Tr}\mathbf{P}\left(-1+\frac{1}{\Delta}\right). & (87)\end{aligned}$$

This completes the proof. ■

From Corollary 6, it follows that the asymptotic sum rate for RZF takes the form:

$$\overline{r}_E = \sum_{k=1}^{K} \log\left(1 + p_k \mu\right) \tag{88}$$

where

$$\mu = \frac{\overline{\gamma}_k}{p_k} = \frac{\left(1 - \lambda\left(\frac{1}{x}-1\right)\frac{N}{K}\left(\frac{1+\rho}{\beta}\right)\right)^2}{\overline{s}_k + \overline{\psi}\frac{\sigma^2}{NP_T}} \tag{89}$$

does not depend on k. Hence, the power allocation policy that maximize the sum rate in (88) subject to $\frac{1}{K}\mathrm{Tr}\mathbf{P} = P$ with $p_k \geq 0$ is the uniform power allocation $p_k^* = \frac{1}{K}\mathrm{Tr}\mathbf{P}$ for all $k \in \{1, \ldots, K\}$.

IV. Numerical results

MC simulations are now used to validate the above asymptotic analysis for a network with finite dimensions.[1] We consider a cell of radius $R = 250$ m. We use the standard correlation model $[\mathbf{\Theta}]_{i,j} = \eta^{|i-j|}$ and assume that the large scale coefficients β_k are obtained as [23], [29]

$$\beta_k = 2L_{\overline{x}} \left(1 + \frac{x_k^{\kappa}}{\overline{x}^{\kappa}}\right)^{-1}. \tag{90}$$

The parameter $\kappa > 2$ stands for the pathloss exponent, x_k denotes the distance of UE k from the BS, $\overline{x} > 0$ is a cut-off parameter where $L_{\overline{x}}$ is a constant that regulates the attenuation at distance $\overline{x}$. We assume $\kappa = 3.5, L_{\overline{x}} = -86.5$ dB and $\overline{x} = 25$ m. The results are obtained for 5000 channel realizations assuming that the UEs are randomly distributed in the circular region between $\overline{x}$ m and $R = 250$ m. We assume that the BS is outfitted with a uniform linear array, such that the nth entry of $\tilde{\mathbf{z}}_k$ is given by

$$[\tilde{\mathbf{z}}_k]_n = \exp(-\imath(n-1)\pi \sin\theta_k) \tag{91}$$

where θ_k is the angle of arrival associated with UE k and chosen such that $\sin\theta_k$ is uniformly distributed between $[-\sigma_A, \sigma_A]$ with σ_A being a constant controlling the angular spread of the LOS components at the BS. The transmit power P_T is fixed to 10 Watt and the regularization parameter λ is computed as $\lambda = \sigma^2 \mathbb{E}\left[\beta_k^{-1}\right]/P_T$, where the expectation is taken over the distribution of the UEs' positions.

Fig. 1 illustrates the average rate per UE when N grows large and K is kept fixed to 20. The correlation factor is set to $\eta = 0.5$ and σ_A takes the following values $\{0.4, 0.1, 0.05\}$ while the Rician factor is $\rho = 1$. The red curves are obtained using MC simulations whereas the black ones are obtained using the closed-form approximation of Theorem 2. As seen, the approximation matches perfectly with the MC simulations for any N. Therefore, we may conclude that the large system analysis is accurate even for networks of finite size. As seen, the average rate per UE improves as the angular spread increases (e.g., large values of σ_A). This is because the spatial separation or diversity of the LOS components of the different UEs increases as σ_A grows. A relatively good performance is achieved even for quite small values of σ_A. This means that a tiny spatial separation of the LOS vectors is not substantially detrimental for interference mitigation when $\rho = 1$ (which corresponds to the case in which the non-LOS and LOS components have the same strength).

[1] The code is available online at https://github.com/lucasanguinetti/ for testing different network configurations.

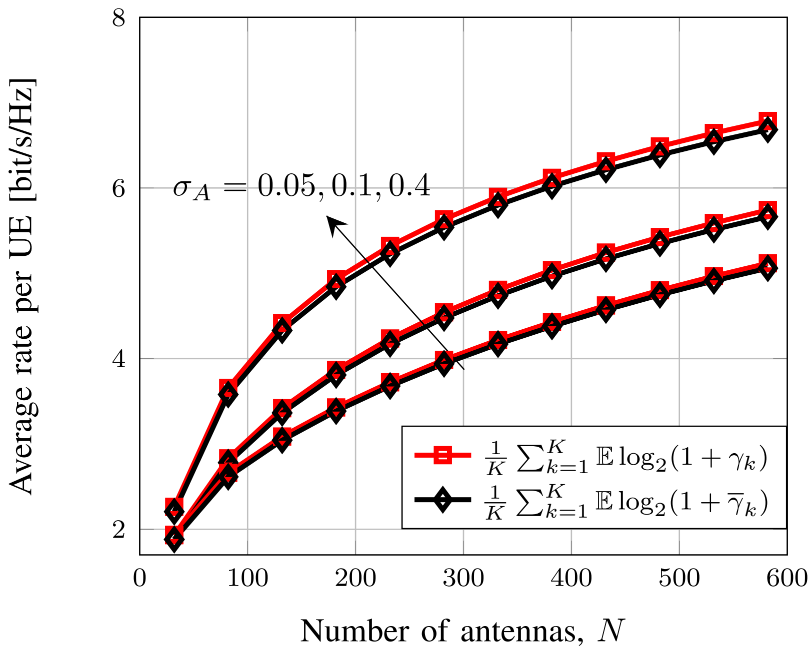

Fig. 1. Average rate per UE versur the number of antennas N when $K = 20$ and the Rician factor $\rho = 1$.

The impact of the Rician coefficient ρ is investigated in Figs. 2 and 3 for $N = 250$ and $K = 50$ with different values of σ_A. As before, the approximation matches perfectly with the MC simulations for any ρ. Fig. 2 shows that, if σ_A is relatively small, the average rate reduces with ρ since the LOS vectors tend to be closely aligned in space, the average rate decreases as ρ grows. This is a consequence of the high inter-user interference. However, as σ_A increases, Fig. 3 shows that an increase in the Rician coefficient has a positive effect on the rate performance. These results are in line with those reported in [28], wherein it is shown that large-scale MIMO systems benefit from LOS contributions when these are relatively different among the UEs.

V. Conclusions

This work analyzed the ergodic sum rate in the downlink of a single-cell large scale multi-user MIMO system operating over correllated Rician fading channels. A regularized zero-forcing precoding is employed under the assumption of perfect channel state information. Stating and proving new results from large-scale random matrix theory allowed us to provide accurate approximations of the SINRs of different UEs, from which the average rate expressions were obtained. Such approximations were shown to depend only on the system parameters (number of

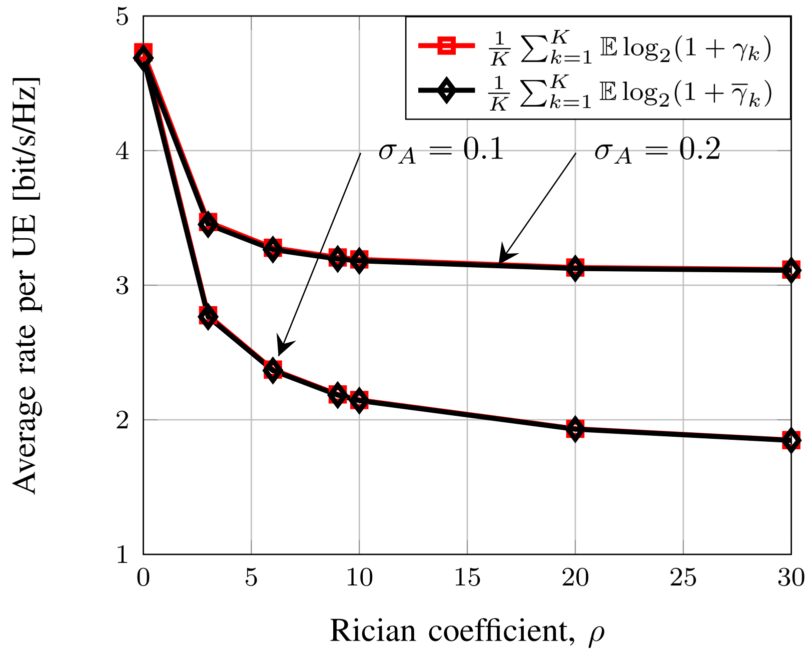

Fig. 2. Average rate per UE for low angular spreads when $N = 250$ and $K = 50$

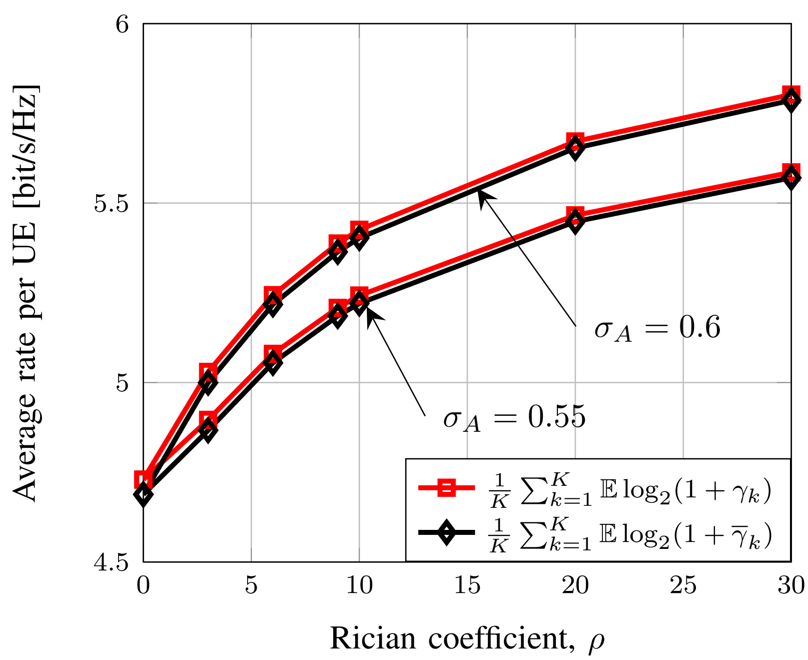

Fig. 3. Average rate per UE for high angular spreads when $N = 250$ and $K = 50$

antennas and UE) as well as on the long-term channel statistics; that is, the Rician factor and the spatial correlation matrix of UEs. Interesting insights were obtained and analytically characterized through a few illustrative examples. A set of Monte Carlo simulations were presented in order to illustrate the accuracy of the provided approximations and to show how they can lead to important insights into how the different parameters affect the network performance. To allow further investigation, the code used for Monte Carlo simulations was provided.

APPENDIX A
PRELIMINARIES AND USEFUL RESULTS

We start defining $\tilde{\mathbf{Q}} = \left(\boldsymbol{\Sigma}^H\boldsymbol{\Sigma} + \lambda\mathbf{I}_N\right)^{-1}$ and $\mathbf{Q} = \left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^H + \lambda\mathbf{I}_N\right)^{-1}$. Matrix $\mathbf{T}$ and $\widetilde{\mathbf{T}}$ are approximations of the resolvent matrices $\mathbf{Q}$ and $\tilde{\mathbf{Q}}$ in that:

$$\frac{1}{N}\mathrm{Tr}\mathbf{B}\mathbf{Q} - \frac{1}{N}\mathrm{Tr}\mathbf{B}\mathbf{T} \xrightarrow{\text{a.s.}} 0 \tag{92}$$

and

$$\frac{1}{N}\mathrm{Tr}\tilde{\mathbf{B}}\tilde{\mathbf{Q}} - \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{B}}\tilde{\mathbf{T}} \xrightarrow{\text{a.s.}} 0 \tag{93}$$

for any sequence of matrices $\mathbf{B}$ and $\tilde{\mathbf{B}}$ of bounded spectral norm. Moreover, we also have

$$\mathbb{E}\left|\mathbf{u}^H\mathbf{Q}\mathbf{u} - \mathbf{u}^H\mathbf{T}\mathbf{u}\right|^p = \mathcal{O}(N^{-p/2}) \tag{94}$$

$$\mathbb{E}\left|\mathbf{u}^H\tilde{\mathbf{Q}}\mathbf{u} - \mathbf{u}^H\tilde{\mathbf{T}}\mathbf{u}\right|^p = \mathcal{O}(N^{-p/2}) \tag{95}$$

for any integer p and any sequence of vectors $\mathbf{u} \in \mathbb{C}^N$ with uniformly bounded norm.

$\mathbf{Q}_k = \left(\boldsymbol{\Sigma}_{[k]}\boldsymbol{\Sigma}_{[k]}^H + \lambda\mathbf{I}_N\right)^{-1}$ where $\boldsymbol{\Sigma}_{[k]}$ is obtained after removing column k from $\boldsymbol{\Sigma}$. Denote by $\tilde{q}_{kk}$ the k-th diagonal element of $\tilde{\mathbf{Q}}$. Then, the following relations hold true:

$$\tilde{q}_{kk} = \frac{1}{\lambda(1 + \boldsymbol{\eta}_k^H\mathbf{Q}\boldsymbol{\eta}_k)} \tag{96}$$

$$\mathbf{Q} = \mathbf{Q}_k - \lambda\tilde{q}_k\mathbf{Q}_k\boldsymbol{\eta}_k\boldsymbol{\eta}_k^H\mathbf{Q}_k \tag{97}$$

$$\mathbf{Q}\boldsymbol{\eta}_k = \frac{\mathbf{Q}_k\boldsymbol{\eta}_k}{1 + \boldsymbol{\eta}_k^H\mathbf{Q}_k\boldsymbol{\eta}_k} \tag{98}$$

Let δ_k and $\tilde{\delta}_k$ be the solutions to the following set of equations:

$$\delta_k = \frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{T}_k \tag{99}$$

$$\tilde{\delta}_k = \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{D}}\tilde{\mathbf{T}}_k \tag{100}$$

with

$$\mathbf{T}_k = \left(\lambda\left(\mathbf{I}_N + \tilde{\delta}_k\mathbf{D}\right) + \mathbf{A}_{[k]}\left(\mathbf{I}_{K-1} + \delta_k\tilde{\mathbf{D}}_{[k]}\right)^{-1}\mathbf{A}_{[k]}^H\right)^{-1} \tag{101}$$

$$\tilde{\mathbf{T}}_k = \left(\lambda\left(\mathbf{I}_{K-1} + \delta_k\tilde{\mathbf{D}}\right) + \mathbf{A}_{[k]}^H\left(\mathbf{I}_N + \tilde{\delta}_k\mathbf{D}_{[k]}\right)^{-1}\mathbf{A}_{[k]}\right)^{-1}. \tag{102}$$

where $\mathbf{A}_k$ is obtained by removing the k-th column of matrix $\mathbf{A}$. The matrices $\mathbf{T}_k$ and $\tilde{\mathbf{T}}_k$ are respectively approximations of the resolvent matrices $\mathbf{Q}_k$ and $\tilde{\mathbf{Q}}_k = \left(\mathbf{\Sigma}_{[k]}^H\mathbf{\Sigma}_{[k]} - z\mathbf{I}_{K-1}\right)^{-1}$ such that

$$\frac{1}{N}\mathrm{Tr}\mathbf{B}\mathbf{Q}_k - \frac{1}{N}\mathrm{Tr}\mathbf{B}\mathbf{T}_k \to 0 \tag{103}$$

$$\frac{1}{N}\mathrm{Tr}\tilde{\mathbf{B}}\tilde{\mathbf{Q}}_k - \frac{1}{N}\mathrm{Tr}\tilde{\mathbf{B}}\tilde{\mathbf{T}}_k \to 0 \tag{104}$$

for any sequence of matrices $\mathbf{B} \in \mathbb{C}^{N\times N}$ and $\tilde{\mathbf{B}} \in \mathbb{C}^{K-1\times K-1}$ and

$$\mathbb{E}\left|\mathbf{u}^H\mathbf{Q}_k\mathbf{u} - \mathbf{u}^H\mathbf{T}_k\mathbf{u}\right|^p = \mathcal{O}(N^{-p/2}) \tag{105}$$

$$\mathbb{E}\left|\mathbf{u}^H\tilde{\mathbf{Q}}_k\mathbf{u} - \mathbf{u}^H\tilde{\mathbf{T}}_k\mathbf{u}\right|^p = \mathcal{O}(N^{-p/2}) \tag{106}$$

for any integer p and any sequence of vectors $\mathbf{u} \in \mathbb{C}^N$ with uniformly bounded norm.

Let us also introduce the following results and technical identities that will be extensively used in the remainder. Let $\tilde{\mathbf{q}} \in \mathbb{C}^{K-1}$ be the vector of diaognal element of $\tilde{\mathbf{Q}}_k$ and $\tilde{\mathbf{q}}_0 \in \mathbb{C}^{K\times 1}$ be $\tilde{\mathbf{q}}$ where 0 is added at the kth position. Denote by $\tilde{q}_{ii}^k$ the ith diagonal element of $\tilde{\mathbf{q}}_0$ and call

$$\mathbf{Q}_{ki} = \left(\mathbf{\Sigma}_{[ki]}\mathbf{\Sigma}_{[ki]}^H - \lambda\mathbf{I}_N\right)^{-1}. \tag{107}$$

Then, for $i \neq k$ we have that:

$$\tilde{q}_{ii}^k = \frac{1}{\lambda\left(1 + \boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right)} \tag{108}$$

$$\mathbf{Q}_k = \mathbf{Q}_{ki} - \lambda\tilde{q}_{ii}^k\mathbf{Q}_{ki}\boldsymbol{\eta}_i\boldsymbol{\eta}_i^H\mathbf{Q}_{ki} \tag{109}$$

$$\mathbf{Q}_k\boldsymbol{\eta}_i = \frac{\mathbf{Q}_{ki}\boldsymbol{\eta}_i}{1 + \boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i} = \lambda\tilde{q}_{ii}^k\mathbf{Q}_{ki}\boldsymbol{\eta}_i. \tag{110}$$

Moreover, let $\mathbf{u} \in \mathbb{C}^N$ be a sequence of deterministic vectors with bounded Euclidean norm. Then, for any integer p and $i \neq k$, we have:

$$\mathbb{E}\left|\mathbf{u}^H\left(\mathbf{Q}_{ki} - \boldsymbol{\mathcal{T}}_{ki}\right)\mathbf{u}\right|^{2p} = \mathcal{O}(N^{-p}) \tag{111}$$

with $\boldsymbol{\mathcal{T}}_{ki}$ being given by:

$$\boldsymbol{\mathcal{T}}_{ki} = \left(\lambda\left(\mathbf{I}_N + \tilde{\delta}_k\mathbf{D}\right) + \mathbf{A}_{[ki]}\left(\mathbf{I}_{K-2} + \delta_k\tilde{\mathbf{D}}_k\right)^{-1}\mathbf{A}_{[ki]}^H\right)^{-1}.$$

where $\mathbf{A}_{[ki]}$ is matrix $\mathbf{A}$ with the k-th and i-th columns removed. Define similarly $\boldsymbol{\mathcal{T}}_k$ as:

$$\boldsymbol{\mathcal{T}}_k = \left(\lambda\left(\mathbf{I}_N + \tilde{\delta}\mathbf{D}\right) + \mathbf{A}_{[k]}\left(\mathbf{I}_K + \delta_k\tilde{\mathbf{D}}_k\right)^{-1}\mathbf{A}_{[k]}^H\right)^{-1}$$

Denote by $\tilde{\mathbf{t}}^k \in \mathbb{C}^{K-1}$ the vector of diagonal elements of $\tilde{\mathbf{T}}_k$ and call $\tilde{\mathbf{t}}_0 \in \mathbb{C}^K$ the vectors obtained from $\tilde{\mathbf{t}}_k$ after adding a 0 to its kth position. Denote by $\tilde{t}_{ii}^k$ the ith element of $\tilde{\mathbf{t}}_0$. Then, for $i \neq k$:

$$\tilde{t}_{ii}^k = \frac{1}{\lambda\left(1 + \mathbf{a}_i^H\boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_i + \tilde{d}_i\delta_k\right)} \tag{112}$$

whereas $\tilde{t}_{ii}^k = 0$ if $i = k$. Also, we have that

$$\lambda\tilde{t}_{ii}^k\mathbf{a}_i^H\boldsymbol{\mathcal{T}}_{ki}\mathbf{u} = \frac{\mathbf{a}_i^H\mathbf{T}_k\mathbf{u}}{1 + \tilde{d}_i\delta_k} \tag{113}$$

with $\mathbf{u} \in \mathbb{C}^N$ being a deterministic vector. The identities in (112) and (113) are needed to retrieve $\mathbf{T}_k$ from expressions involving $\boldsymbol{\mathcal{T}}_{ki}$. In a similar way to (112) for $k = 1, \ldots, K$, we have that

$$\tilde{t}_{kk} = \frac{1}{\lambda\left(1 + \mathbf{a}_k^{\mathrm{H}}\boldsymbol{\mathcal{T}}_k\mathbf{a}_k + \tilde{d}_k\delta_k\right)} = \frac{1}{\lambda\left(1 + \mathbf{a}_k^{\mathrm{H}}\mathbf{T}_k\mathbf{a}_k + \tilde{d}_k\delta_k\right)} + O(N^{-1}). \tag{114}$$

Notice that $\tilde{t}_{kk}$ denotes the diagonal elements of matrix $\tilde{\mathbf{T}}$. They are exactly equal to $\frac{1}{\lambda\left(1+\mathbf{a}_k^{\mathrm{H}}\boldsymbol{\mathcal{T}}_k\mathbf{a}_k+\tilde{d}_k\delta_k\right)}$. The replacement of $\boldsymbol{\mathcal{T}}_k$ by $\mathbf{T}_k$ is at the cost of an error of order $O(N^{-1})$. Finally, the following result will be frequently used in the appendices:

$$\mathbb{E}\left|\tilde{q}_{ii}^k - \tilde{t}_{ii}^k\right|^p = \mathcal{O}(N^{-p/2}). \tag{115}$$

APPENDIX B
DETERMINISTIC EQUIVALENT FOR $\theta(\mathbf{v}, \mathbf{C})$ AND $\chi(\mathbf{C})$

A. *Deterministic equivalent for* $\theta(\mathbf{v}, \mathbf{C})$

We begin by observing that

$$\mathbf{Q} - \mathbf{T} \stackrel{(a)}{=} \mathbf{T}\left(\mathbf{T}^{-1} - \mathbf{Q}^{-1}\right)\mathbf{Q} \tag{116}$$

$$\stackrel{(b)}{=} \mathbf{T}\left(\lambda\tilde{\delta}\mathbf{D} + \mathbf{A}\left(\mathbf{I}_K + \delta\tilde{\mathbf{D}}\right)^{-1}\mathbf{A}^H - \boldsymbol{\Sigma}\boldsymbol{\Sigma}^H\right)\mathbf{Q} \tag{117}$$

where (a) follows from applying the resolvent identity[2] and (b) is obtained by using (16) and $\mathbf{Q}^{-1} = \mathbf{\Sigma}\mathbf{\Sigma}^H - \lambda \mathbf{I}_N$. Plugging the above expression into (30) leads to $\theta(\mathbf{v}, \mathbf{C}) = Z_1 + Z_2 + Z_3 - Z_4$ with

$$Z_1 = \mathbb{E}\left[\mathbf{v}^H \mathbf{T}\mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{118}$$

$$Z_2 = \lambda\tilde{\delta}\mathbb{E}\left[\mathbf{v}^H \mathbf{T}\mathbf{D}\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{119}$$

$$Z_3 = \mathbb{E}\left[\mathbf{v}^H \mathbf{T}\mathbf{A}\left(\mathbf{I}_K + \delta\tilde{\mathbf{D}}\right)^{-1} \mathbf{A}^H \mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{120}$$

$$Z_4 = \mathbb{E}\left[\mathbf{v}^H \mathbf{T}\mathbf{\Sigma}\mathbf{\Sigma}^H \mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{v}\right]. \tag{121}$$

Notice that Z_2 will be compensated by some terms in Z_4. This can be taken as a rule of thumb whenever the resolvent identity is used for computing deterministic equivalents (see [13]). Therefore, we restrict ourselves to the development of Z_1, Z_3 and Z_4.

Let's start with Z_1. From (94), it follows that:

$$Z_1 = \mathbf{v}^H \mathbf{T}\mathbf{C}\mathbf{T}\mathbf{v} + \mathcal{O}(N^{-\frac{1}{2}}). \tag{122}$$

By using (97), we may write $Z_3 = Z_{3,1} + Z_{3,2}$ with

$$Z_{3,1} = \sum_{\ell=1}^{K} \mathbb{E}\left[\frac{\mathbf{v}^H \mathbf{T}\mathbf{a}_\ell \mathbf{a}_\ell^H}{1 + \delta\tilde{d}_\ell}\mathbf{Q}_\ell \mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{123}$$

$$Z_{3,2} = -\sum_{\ell=1}^{K} \frac{\mathbf{v}^H \mathbf{T}\mathbf{a}_\ell \mathbb{E}\left[\lambda\tilde{q}_{\ell\ell}\mathbf{a}_\ell^H \mathbf{Q}_\ell \boldsymbol{\eta}_\ell \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell \mathbf{C}\mathbf{Q}\mathbf{v}\right]}{1 + \delta\tilde{d}_\ell} \tag{124}$$

where $\{\tilde{q}_{\ell\ell}\}$ are the diagonal elements of $\tilde{\mathbf{Q}}$. We can prove that at the cost of an error $O(N^{-\frac{1}{2}})$, we can replace in $Z_{3,2}$, $\tilde{q}_{\ell\ell}$ by $\tilde{t}_{\ell\ell}$ and $\mathbf{a}_\ell^H \mathbf{Q}_\ell \boldsymbol{\eta}_\ell$ by $\mathbf{a}_\ell^H \boldsymbol{\mathcal{T}}_\ell \mathbf{a}_\ell$. Since the treatment of these terms is alike, we provide only the details concerning the replacement of $\tilde{q}_{\ell\ell}$ by $\tilde{t}_{\ell\ell}$. Let ϵ be given by:

$$\epsilon = -\sum_{\ell=1}^{K} \frac{\mathbf{v}^H \mathbf{T}\mathbf{a}_\ell \mathbb{E}\left[\lambda(\tilde{q}_{\ell\ell} - \tilde{t}_{\ell\ell})\mathbf{a}_\ell^H \mathbf{Q}_\ell \boldsymbol{\eta}_\ell \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell \mathbf{C}\mathbf{Q}\mathbf{v}\right]}{1 + \delta\tilde{d}_\ell}$$

Using Cauchy-Schwartz inequality:

$$|\epsilon| \leq \frac{1}{1 + \delta\tilde{d}_\ell}\sqrt{\sum_{l=1}^{K} |\mathbf{v}^H \mathbf{T}\mathbf{a}_\ell|^2 \mathbb{E}|\tilde{q}_{\ell\ell} - \tilde{t}_{\ell\ell}|^2 (1 + \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell \boldsymbol{\eta}_\ell)^2 (\mathbf{a}_\ell^H \mathbf{Q}_\ell \boldsymbol{\eta}_\ell)^2}\sqrt{\mathbb{E}|\boldsymbol{\eta}_\ell^H \mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{v}|^2}$$

[2] $\mathbf{A} - \mathbf{B} = \mathbf{B}(\mathbf{B}^{-1} - \mathbf{A}^{-1})\mathbf{A}$ [30].

From (95), $\mathbb{E}|\tilde{q}_{\ell\ell}-\tilde{t}_{\ell\ell}|^2(1+\boldsymbol{\eta}_\ell^H\mathbf{Q}_\ell\boldsymbol{\eta}_\ell)^2(\mathbf{a}_\ell^H\mathbf{Q}_\ell\boldsymbol{\eta}_\ell)^2=O(N^{-\frac{1}{2}}$ and hence, $\epsilon=O(N^{-\frac{1}{2}})$. Hence, we obtain that $Z_{3,2}=U_1+U_2+O(N^{-\frac{1}{2}})$ with

$$U_1=-\sum_{\ell=1}^K\lambda\tilde{t}_{\ell\ell}\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\frac{\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{1+\delta\tilde{d}_\ell}\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{125}$$

$$U_2=-\sum_{\ell=1}^K\lambda\tilde{t}_{\ell\ell}\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\frac{\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{1+\delta\tilde{d}_\ell}\mathbb{E}\left[\mathbf{y}_\ell^H\mathbf{Q}_\ell\mathbf{C}\mathbf{Q}\mathbf{v}\right]. \tag{126}$$

By using (97) and replacing $\tilde{q}_{\ell\ell}$ with $\tilde{t}_{\ell\ell}$, U_2 can be further developed as:

$$U_2=\sum_{\ell=1}^K\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\frac{\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{1+\delta\tilde{d}_\ell}\mathbb{E}\left[\mathbf{y}_\ell^H\mathbf{Q}_\ell\mathbf{C}\mathbf{Q}_\ell\boldsymbol{\eta}_\ell\boldsymbol{\eta}_\ell^H\mathbf{Q}_\ell\mathbf{v}\right]+\mathcal{O}(N^{-1/2}). \tag{127}$$

By writing $\boldsymbol{\eta}_\ell\boldsymbol{\eta}_\ell^H=\mathbf{a}_\ell\mathbf{a}_\ell^H+\mathbf{a}_\ell\mathbf{y}_\ell^H+\mathbf{y}_\ell\mathbf{y}_\ell^H+\mathbf{y}_\ell\mathbf{a}_\ell^H$, it turns out that only $\mathbf{a}_\ell\mathbf{y}_\ell^H$ produces non-vanishing terms. Therefore, computing the expectation over $\mathbf{y}_\ell$ and using rank-one perturbation arguments [26, Lemma 7] we have:

$$U_2=\sum_{\ell=1}^K\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\frac{\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{1+\delta\tilde{d}_\ell}\tilde{d}_\ell\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\left(\mathbf{D}\mathbf{Q}\mathbf{C}\mathbf{Q}\right)\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{v}\right]+\mathcal{O}(N^{-1/2}). \tag{128}$$

By using Lemma 5.2 in [13], we finally get:

$$U_2=\sum_{\ell=1}^K\lambda\tilde{t}_{\ell\ell}\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\frac{\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{(1+\delta\tilde{d}_\ell)^2}\tilde{d}_\ell\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}\mathbf{C}\mathbf{Q}\right]\mathbf{a}_\ell^H\mathbf{T}\mathbf{v}+\mathcal{O}(N^{-1/2}). \tag{129}$$

By collecting the above results and using the relation $\lambda\tilde{t}_{\ell\ell}=\frac{1-\lambda\tilde{t}_{\ell\ell}\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{1+\tilde{d}_\ell\delta}$, we obtain:

$$\begin{aligned}Z_3=&\sum_{\ell=1}^K\lambda\tilde{t}_{\ell\ell}\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{C}\mathbf{Q}\mathbf{v}\right]\\&+\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}\mathbf{C}\mathbf{Q}\right]\sum_{\ell=1}^K\lambda\tilde{t}_{\ell\ell}\tilde{d}_\ell\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\frac{\mathbf{a}_\ell^H\boldsymbol{\mathcal{T}}_\ell\mathbf{a}_\ell}{(1+\delta\tilde{d}_\ell)^2}\mathbf{a}_\ell^H\mathbf{T}\mathbf{v}+\mathcal{O}(N^{-\frac{1}{2}}).\end{aligned} \tag{130}$$

The first term in the above expression will be compensated by some terms in Z_4 and as such will not be worked out further. The second term contains $\chi(\mathbf{C})$ given by (31) unveiling the interplay between $\theta(\mathbf{v},\mathbf{C})$ and $\chi(\mathbf{C})$. Let's now move to the computation of Z_4. By using $\mathbf{Q}\boldsymbol{\eta}_\ell=\lambda\tilde{q}_{\ell\ell}\mathbf{Q}_\ell\boldsymbol{\eta}_\ell$, we obtain:

$$Z_4=\sum_{\ell=1}^K\mathbf{v}^H\mathbf{T}\mathbb{E}\left[\lambda\tilde{q}_{\ell\ell}\boldsymbol{\eta}_\ell\boldsymbol{\eta}_\ell^H\mathbf{Q}_\ell\mathbf{C}\mathbf{Q}\mathbf{v}\right] \tag{131}$$

$$=\sum_{\ell=1}^K\mathbf{v}^H\mathbf{T}\mathbb{E}\left[\lambda\tilde{t}_{\ell\ell}\boldsymbol{\eta}_\ell\boldsymbol{\eta}_\ell^H\mathbf{Q}_\ell\mathbf{C}\mathbf{Q}\mathbf{v}\right]+\mathcal{O}(N^{-\frac{1}{2}}) \tag{132}$$

$$=Z_{41}+Z_{42}+Z_{43}+Z_{44}+\mathcal{O}(N^{-\frac{1}{2}}) \tag{133}$$

with

$$Z_{41} = \sum_{\ell=1}^{K} \lambda \tilde{t}_{\ell\ell} \mathbf{v}^H \mathbf{T} \mathbb{E}\left[\mathbf{a}_\ell \mathbf{a}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q} \mathbf{v}\right] \tag{134}$$

$$Z_{42} = \sum_{\ell=1}^{K} \lambda \tilde{t}_{\ell\ell} \mathbf{v}^H \mathbf{T} \mathbb{E}\left[\mathbf{y}_\ell \mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q} \mathbf{v}\right] \tag{135}$$

$$Z_{43} = \sum_{\ell=1}^{K} \lambda \tilde{t}_{\ell\ell} \mathbf{v}^H \mathbf{T} \mathbb{E}\left[\mathbf{y}_\ell \mathbf{a}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q} \mathbf{v}\right] \tag{136}$$

$$Z_{44} = \sum_{\ell=1}^{K} \lambda \tilde{t}_{\ell\ell} \mathbf{v}^H \mathbf{T} \mathbb{E}\left[\mathbf{a}_\ell \mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q} \mathbf{v}\right]. \tag{137}$$

Since Z_{41} will be compensated by the first term in Z_3, only the other terms will be considered. Let's start with Z_{42}. Using as usual the key decomposition in (97) along with the same techniques allowing the replacement of $\tilde{q}_{\ell\ell}$ by $\tilde{t}_{\ell\ell}$, we obtain :

$$\begin{aligned} Z_{42} &= \sum_{\ell=1,}^{K} \lambda \tilde{t}_{\ell\ell} \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{y}_\ell \mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{v}\right] \\ &- \sum_{\ell=1}^{K} \lambda^2 \left(\tilde{t}_{\ell\ell}^k\right)^2 \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{y}_\ell \mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \boldsymbol{\eta}_\ell \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell \mathbf{v}\right] + \mathcal{O}(N^{-\frac{1}{2}}) \qquad (138) \\ &= \frac{1}{N} \sum_{\ell=1}^{K} \lambda \tilde{t}_{\ell\ell} \tilde{d}_\ell \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{D} \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{v}\right] - \sum_{\ell=1}^{K} \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{y}_\ell \mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{y}_\ell \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell \mathbf{v}\right] \\ &- \sum_{\ell=1}^{K} \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{y}_\ell \mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{a}_\ell \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell \mathbf{v}\right] + \mathcal{O}(N^{-\frac{1}{2}}) \qquad (139) \end{aligned}$$

The third term in the above right-hand side is of order $\mathcal{O}(N^{-\frac{1}{2}})$. For the same reasons, we can substitute in the second term $\mathbf{y}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{y}_\ell$ by $\tilde{d}_\ell \frac{1}{N} \mathrm{Tr} \mathbb{E} \mathbf{D} \mathbf{Q} \mathbf{C} \mathbf{Q}$. Moreover, in the first term $\mathbf{Q}_\ell$ can be replaced by $\mathbf{Q}$, which is in passing can be proven by decomposing $\mathbf{Q}_\ell$ as $\mathbf{Q}_\ell = \mathbf{Q}_\ell - \mathbf{Q} + \mathbf{Q}$ and using (97) along with the arguments developed so far. We thus obtain:

$$\begin{aligned} Z_{42} &= \sum_{\ell=1}^{K} \lambda \tilde{t}_{\ell\ell} \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{D} \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{v}\right] \\ &- \frac{1}{N} \sum_{\ell=1}^{K} \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \tilde{d}_\ell^2 \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{D} \mathbf{Q}_\ell \mathbf{v}\right] \frac{1}{N} \mathrm{Tr} \mathbb{E}\left[\mathbf{D} \mathbf{Q} \mathbf{C} \mathbf{Q}\right] + \mathcal{O}(N^{-\frac{1}{2}}) \qquad (140) \\ &= \lambda \tilde{\delta} \mathbb{E}\left[\mathbf{v}^H \mathbf{T} \mathbf{D} \mathbf{Q} \mathbf{C} \mathbf{Q} \mathbf{v}\right] \\ &- \mathbf{v}^H \mathbf{T} \mathbf{D} \mathbf{T} \mathbf{v} \mathbb{E}\left[\frac{1}{N} \mathrm{Tr} \mathbf{D} \mathbf{Q} \mathbf{C} \mathbf{Q}\right] \frac{1}{N} \sum_{\ell=1}^{K} \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \tilde{d}_\ell^2 + \mathcal{O}(N^{-\frac{1}{2}}) \qquad (141) \end{aligned}$$

We look now to Z_{43} and Z_{44}. Using the same technicalities, we can prove that:

$$Z_{43} = -\mathbf{v}^H\mathbf{TDTv}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]+\mathcal{O}(N^{-\frac{1}{2}}) \tag{142}$$

and

$$Z_{44} = -\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]\sum_{\ell=1}^{K}\lambda\tilde{t}_{\ell\ell}\frac{\mathbf{v}^H\mathbf{Ta}_\ell\mathbf{a}_\ell^H\mathbf{Tv}}{1+\tilde{d}_\ell\delta}+\mathcal{O}(N^{-\frac{1}{2}}). \tag{143}$$

Gathering all these results together, we end up with the following expression for Z_4:

$$\begin{aligned}
Z_4 = &\sum_{\ell=1}^{K}\lambda\tilde{t}_{\ell\ell}\mathbf{v}^H\mathbf{Ta}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQv}\right]+\lambda\tilde{\delta}\mathbb{E}\left[\mathbf{v}^H\mathbf{TDQCQv}\right]\\
&-\mathbf{v}^H\mathbf{TDTv}\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell^2\\
&-\mathbf{v}^H\mathbf{TDTv}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]\\
&-\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]\sum_{\ell=1}^{K}\lambda\tilde{t}_{\ell\ell}\frac{\mathbf{v}^H\mathbf{Ta}_\ell\mathbf{a}_\ell^H\mathbf{Tv}}{1+\tilde{d}_\ell\delta}+\mathcal{O}(N^{-\frac{1}{2}})
\end{aligned} \tag{144}$$

The first two terms in Z_4 will be cancelled with Z_2 and some terms in Z_3. We finally obtain:

$$\begin{aligned}
\theta(\mathbf{v},\mathbf{C}) = &\mathbf{v}^H\mathbf{TCTv}+\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]\left[\sum_{\ell=1}^{K}\tilde{d}_\ell\frac{\mathbf{v}^H\mathbf{Ta}_\ell\mathbf{a}_\ell^H\mathbf{Tv}}{(1+\tilde{d}_\ell\delta)^2}+\mathbf{v}^H\mathbf{TDTv}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell^2\right]\\
&+\mathbf{v}^H\mathbf{TDTv}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]+\mathcal{O}(N^{-\frac{1}{2}}).
\end{aligned} \tag{145}$$

Equation (145) establishes a relation between $\theta(\mathbf{v},\mathbf{C})$ and the unknown quantities $\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]$ and $\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]$. Quantity $\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]$ can be related to $\theta(\mathbf{a}_\ell,\mathbf{C})$ as:

$$\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{QCQa}_\ell\right]=\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2(1+\tilde{d}_\ell\delta)^2\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right] \tag{146}$$

$$+\tilde{d}_\ell\left(\frac{\mathbf{a}_\ell^H\mathbf{Ta}_\ell}{1+\tilde{d}_\ell\delta}\right)^2\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]+\mathcal{O}(N^{-\frac{1}{2}}). \tag{147}$$

This can be proven by substituting in $\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{QCQa}_\ell\right]$, matrix $\mathbf{Q}$ by $\mathbf{Q}_\ell-\lambda\tilde{q}_\ell\mathbf{Q}_\ell\boldsymbol{\eta}_l\boldsymbol{\eta}_\ell^H\mathbf{Q}_\ell$ as per (97), and controlling the vanishing terms. To determine $\theta(\mathbf{v},\mathbf{C})$, it is easy to see that we need a second equation between $\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{QCQa}_\ell\right]$ and $\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]$. This is the objective of the subsequent section.

B. Deterministic equivalent for χ_C

By using the resolvent identity and then handling each term separately (derivations are similar to before, and hence details are omitted), after some calculations we obtain

$$\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right] = \frac{1}{N}\mathrm{Tr}\mathbf{DTCT} \tag{148}$$

$$+\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQDQ}\right]\left[\frac{1}{N}\sum_{\ell=1}^{K}\frac{\tilde{d}_\ell \mathbf{a}_\ell^H\mathbf{TCTa}_\ell}{(1+\delta\tilde{d}_\ell)^2}+\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\tilde{d}_\ell^2\left(\tilde{t}_{\ell\ell}\right)^2\frac{1}{N}\mathrm{Tr}\mathbf{DTCT}\right]$$

$$+\frac{1}{N}\mathrm{Tr}\mathbf{DTCT}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{DQ}_\ell\mathbf{a}_\ell\right]+\mathcal{O}(N^{-\frac{1}{2}}) \tag{149}$$

Now let $m \in \{1, K\}$. Then, from (145),

$$\mathbb{E}\left[\mathbf{a}_m^H\mathbf{QDQa}_m\right] = \mathbf{a}_m^H\mathbf{TCTa}_m + \mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]\left[\sum_{\ell=1}^{K}\frac{\tilde{d}_\ell\mathbf{a}_m^H\mathbf{Ta}_\ell\mathbf{a}_\ell^H\mathbf{Ta}_m}{(1+\tilde{d}_\ell\delta)^2}+\mathbf{a}_m^H\mathbf{TDTa}_m\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2(\tilde{t}_{\ell\ell}^2)\tilde{d}_\ell^2\right]$$

$$+\mathbf{a}_m^H\mathbf{TDTa}_m\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2(\tilde{t}_{\ell\ell})^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]+O(N^{-\frac{1}{2}})$$

Multiplying both sides by $\frac{\tilde{d}_m}{(1+\tilde{d}_m\delta)^2}$ and summing over m, we obtain:

$$\frac{1}{N}\sum_{m=1}^{K}\frac{\tilde{d}_m\mathbb{E}\left[\mathbf{a}_m^H\mathbf{QDQa}_m\right]}{(1+\tilde{d}_m\delta)^2} = \frac{1}{N}\sum_{m=1}^{K}\frac{\tilde{d}_m\mathbf{a}_m^H\mathbf{TCTa}_m}{(1+\tilde{d}_m\delta)^2}$$

$$+\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right]\left[\frac{1}{N}\sum_{\ell=1}^{K}\sum_{m=1}^{K}\frac{\tilde{d}_\ell\tilde{d}_m\mathbf{a}_m^H\mathbf{Ta}_\ell\mathbf{a}_\ell^H\mathbf{Ta}_m}{(1+\tilde{d}_\ell\delta)^2(1+\tilde{d}_m\delta)^2}+\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2(\tilde{t}_{\ell\ell})^2\tilde{d}_\ell^2\frac{1}{N}\sum_{m=1}^{K}\frac{\mathbf{a}_m^H\mathbf{TDTa}_m\tilde{d}_m}{(1+\tilde{d}_m\delta)^2}\right]$$

$$+\frac{1}{N}\sum_{m=1}^{K}\frac{\tilde{d}_m\mathbf{a}_m^H\mathbf{TDTa}_m}{(1+\tilde{d}_m\delta)^2}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2(\tilde{t}_{\ell\ell})^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]+O(N^{-\frac{1}{2}})$$

from which, we obtain using (147)

$$\frac{1}{N}\sum_{\ell=1}^{K}\tilde{d}_\ell\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right] = \frac{1}{N}\sum_{m=1}^{K}\frac{\tilde{d}_m\mathbf{a}_m^H\mathbf{TCTa}_m}{(1+\tilde{d}_m\delta)^2}+\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{DQCQ}\right] \tag{150}$$

$$\times\left[\frac{1}{N}\sum_{m=1}^{K}\sum_{\ell=1}^{K}\tilde{d}_\ell\tilde{d}_m\frac{\mathbf{a}_m^H\mathbf{Ta}_\ell\mathbf{a}_\ell^H\mathbf{a}_m}{(1+\tilde{d}_\ell\delta)^2(1+\tilde{d}_m\delta)^2}-\frac{1}{N}\sum_{\ell=1}^{K}\tilde{d}_\ell^2\frac{(\mathbf{a}_\ell^H\mathbf{Ta}_\ell)^2}{(1+\tilde{d}_\ell\delta)^4}+\frac{1}{N}\sum_{m=1}^{K}\frac{\tilde{d}_m\mathbf{a}_m^H\mathbf{TDTa}_m}{(1+\tilde{d}_m\delta)^2}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell^2\right] \tag{151}$$

$$+\frac{1}{N}\sum_{m=1}^{K}\tilde{d}_m\frac{\mathbf{a}_m^H\mathbf{TDTa}_m}{(1+\tilde{d}_m\delta)^2}\frac{1}{N}\sum_{\ell=1}^{K}\lambda^2\left(\tilde{t}_{\ell\ell}\right)^2\tilde{d}_\ell\mathbb{E}\left[\mathbf{a}_\ell^H\mathbf{Q}_\ell\mathbf{CQ}_\ell\mathbf{a}_\ell\right]+\mathcal{O}(N^{-\frac{1}{2}}) \tag{152}$$

By defining the following quantities

$$\varphi(\mathbf{C}) = \frac{1}{N}\sum_{\ell=1}^{K} \tilde{d}_\ell \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \mathbb{E}\left[\mathbf{a}_\ell^H \mathbf{Q}_\ell \mathbf{C} \mathbf{Q}_\ell \mathbf{a}_\ell\right] \tag{153}$$

$$F(\mathbf{C}) = \frac{1}{N}\sum_{m=1}^{K} \frac{\tilde{d}_m \mathbf{a}_m^H \mathbf{TCT} \mathbf{a}_m}{(1+\tilde{d}_m\delta)^2} \tag{154}$$

$$G = \frac{1}{N}\sum_{m=1}^{K}\sum_{\ell=1}^{K} \tilde{d}_\ell \tilde{d}_m \frac{\mathbf{a}_m^H \mathbf{T}\mathbf{a}_\ell \mathbf{a}_\ell^H \mathbf{a}_m}{(1+\tilde{d}_\ell\delta)^2(1+\tilde{d}_m\delta)^2} - \frac{1}{N}\sum_{\ell=1}^{K} \tilde{d}_\ell^2 \frac{(\mathbf{a}_\ell^H \mathbf{T}\mathbf{a}_\ell)^2}{(1+\tilde{d}_\ell\delta)^4} \tag{155}$$

$$M = \frac{1}{N}\sum_{\ell=1}^{k} \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \tilde{d}_\ell^2. \tag{156}$$

we can write (152) as:

$$(1-F)\varphi(\mathbf{C}) = F(\mathbf{C}) + \chi(\mathbf{C})\left(G + MF\right) + \mathcal{O}(N^{-\frac{1}{2}}) \tag{157}$$

Let $\vartheta(\mathbf{C}) = \frac{1}{N}\mathrm{Tr}\mathbf{DTCT}$. We also write (149) as:

$$\chi(\mathbf{C}) = \vartheta(\mathbf{C}) + \chi(\mathbf{D})\left(F(\mathbf{C}) + M\vartheta(\mathbf{C})\right) + \vartheta(\mathbf{C})\varphi(\mathbf{D}) + \mathcal{O}(N^{-\frac{1}{2}}) \tag{158}$$

Let $\Delta = (1-F)^2 - M\vartheta - \vartheta G$. From Lemma 5.3 in [13], Δ satisfies also $\Delta = (1-F)^2 - \lambda^2\vartheta\tilde{\vartheta}$ which merely results from the fact that $G + M = \lambda^2\tilde{\vartheta}$. Setting $\mathbf{C}$ to $\mathbf{D}$ in (157) and (158) we thus obtain: $\chi(\mathbf{D}) = \frac{\vartheta}{\Delta} + \mathcal{O}(N^{-\frac{1}{2}})$ and $\varphi(\mathbf{D}) = -1 + \frac{1-F}{\Delta} - \frac{M\vartheta}{\Delta} + \mathcal{O}(N^{-\frac{1}{2}})$ We thus get:

$$\chi(\mathbf{C}) = \frac{\vartheta(\mathbf{C})(1-F)}{\Delta} + \frac{\vartheta F(\mathbf{C})}{\Delta} + \mathcal{O}(N^{-\frac{1}{2}}). \tag{159}$$

Plugging the above expression into (157), we thus obtain after some calculations:

$$\varphi(\mathbf{C}) = \vartheta(\mathbf{C})\left(\frac{M(F-1)}{\Delta} + \frac{\lambda^2\tilde{\vartheta}}{\Delta}\right) + F(\mathbf{C})\left(\frac{1-F}{\Delta} - \frac{M\vartheta}{\Delta}\right) + \mathcal{O}(N^{-\frac{1}{2}}) \tag{160}$$

and

$$\begin{aligned}\theta(\mathbf{v},\mathbf{C}) = \mathbf{v}^H\mathbf{TCT}\mathbf{v} &+ \sum_{\ell=1}^{K}\tilde{d}_\ell \frac{\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\mathbf{a}_\ell^H\mathbf{T}\mathbf{v}}{\left(1+\tilde{d}_\ell\delta\right)^2}\left(\frac{\vartheta(\mathbf{C})(1-F)}{\Delta} + \frac{\vartheta F(\mathbf{C})}{\Delta}\right) \\ &+ \mathbf{v}^H\mathbf{TDT}\mathbf{v}\left(\frac{F(\mathbf{C})(1-F)}{\Delta} + \frac{\lambda^2\tilde{\vartheta}\vartheta(\mathbf{C})}{\Delta}\right) + \mathcal{O}(N^{-\frac{1}{2}})\end{aligned} \tag{161}$$

In particular if $\mathbf{C} = \mathbf{D}$, we obtain:

$$\theta(\mathbf{v},\mathbf{D}) = \frac{1-F}{\Delta}\mathbf{v}^H\mathbf{TDT}\mathbf{v} + \frac{\vartheta}{\Delta}\sum_{\ell=1}^{K}\tilde{d}_\ell \frac{\mathbf{v}^H\mathbf{T}\mathbf{a}_\ell\mathbf{a}_\ell^H\mathbf{T}\mathbf{v}}{\left(1+\tilde{d}_\ell\delta\right)^2}. \tag{162}$$

APPENDIX C
PROOF OF THEOREM 2

Let $\boldsymbol{\eta}_k$ denote the kth column of $\boldsymbol{\Sigma}$. Then, the SINR at UE k can be rewritten as:

$$\gamma_k = \frac{p_k \left|\boldsymbol{\eta}_k^H \mathbf{Q} \boldsymbol{\eta}_k\right|^2}{\sum_{i=1, i\neq k}^{K} p_i \left|\boldsymbol{\eta}_k^H \mathbf{Q} \boldsymbol{\eta}_i\right|^2 + \frac{\sigma^2}{NP_T}\frac{1}{K}\mathrm{Tr}\mathbf{P}\boldsymbol{\Sigma}^H\mathbf{Q}^2\boldsymbol{\Sigma}} \tag{163}$$

from which it follows that γ_k can be expressed as functionals of the resolvent matrix $\mathbf{Q}$. By applying (97), we obtain

$$\mathbf{Q}\boldsymbol{\eta}_k = \frac{\mathbf{Q}_k\boldsymbol{\eta}_k}{1+\boldsymbol{\eta}_k^H\mathbf{Q}_k\boldsymbol{\eta}_k} = \lambda\tilde{q}_{kk}\mathbf{Q}_k\boldsymbol{\eta}_k \tag{164}$$

where $\mathbf{Q}_k = \left(\boldsymbol{\Sigma}_{[k]}\boldsymbol{\Sigma}_{[k]}^H + \lambda\mathbf{I}_N\right)^{-1}$. Plugging (164) into (163) yields:

$$\gamma_k = \frac{p_k(1-\lambda\tilde{q}_{kk})^2}{s_k + \frac{\sigma^2}{NP_T}\psi} \tag{165}$$

where we have defined

$$s_k = \lambda^2\tilde{q}_{kk}^2\boldsymbol{\eta}_k^H\mathbf{Q}_k\boldsymbol{\Sigma}_{[k]}\mathbf{P}_{[k]}\boldsymbol{\Sigma}_{[k]}^H\mathbf{Q}_k\boldsymbol{\eta}_k \tag{166}$$

$$\psi = \frac{1}{K}\mathrm{Tr}\mathbf{P}\boldsymbol{\Sigma}^H\mathbf{Q}^2\boldsymbol{\Sigma}. \tag{167}$$

Therefore, computing an asymptotic approximation for γ_k requires to determine asymptotic approximations for $\tilde{q}_{kk}$, s_k and ψ. From (106), it easily follows that $\tilde{q}_{kk} - \tilde{t}_{kk} \xrightarrow{\text{a.s.}} 0$. The computation of the asymptotic approximations for s_k and ψ is more involved and is addressed in the following.

A.1) Deterministic equivalent for s_k

The computation of a deterministic equivalent for the interference term s_k is much more involved. In short, the approach taken in the remainder basically relies on showing that the interference term s_k is asymptotically given by the weighted sum of two terms, whose approximations depend on quantities in the same form of those of Theorem 1.

As done for u_k, we begin by applying Lemma 1 in Appendix C to $\breve{s}_k \triangleq \boldsymbol{\eta}_k^H\mathbf{Q}_k\boldsymbol{\Sigma}_{[k]}\mathbf{P}_{[k]}\boldsymbol{\Sigma}_{[k]}^H\mathbf{Q}_k\boldsymbol{\eta}_k$ and then use Lemma 2 to obtain $\max_k |\breve{s}_k - \tilde{d}_k\breve{s}_{k,1} - \breve{s}_{k,2}| \xrightarrow{\text{a.s.}} 0$ with

$$\breve{s}_{k,1} = \mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}_k\boldsymbol{\Sigma}_{[k]}\mathbf{P}_{[k]}\boldsymbol{\Sigma}_{[k]}^H\mathbf{Q}_k\right] \tag{168}$$

and

$$\breve{s}_{k,2} = \mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_k\boldsymbol{\Sigma}_{[k]}\mathbf{P}_{[k]}\boldsymbol{\Sigma}_{[k]}^H\mathbf{Q}_k\mathbf{a}_k\right]. \tag{169}$$

From the convergence result of $\tilde{q}_{kk}$ established above, we have:

$$\max_k \left| \tilde{s}_k - \tilde{d}_k \lambda^2 \tilde{t}_{kk}^2 \breve{s}_{k,1} - \lambda^2 \tilde{t}_{kk}^2 \breve{s}_{k,2} \right| \to 0. \tag{170}$$

The above result states that a deterministic equivalent for $\breve{s}_k$ can be computed by looking for approximations of $\breve{s}_{k,1}$ and $\breve{s}_{k,2}$ in which the expectation largely simplifies the derivations as it allows to discard terms with zero mean (though they do not converge to zero in the almost sure sense).

A.2.1) Deterministic equivalent for $\breve{s}_{k,1}$: To begin with, observe that:

$$\breve{s}_{k,1} \overset{(a)}{=} \frac{1}{N} \sum_{i=1, i\neq k}^{K} p_i \mathbb{E}\left[\lambda^2 \left(\tilde{q}_{ii}^k\right)^2 \boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \boldsymbol{\eta}_i \right] \tag{171}$$

$$\overset{(b)}{=} \frac{\lambda^2}{N} \sum_{i=1, i\neq k}^{K} p_i \left(\tilde{t}_{ii}^k\right)^2 \mathbb{E}\left[\boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \boldsymbol{\eta}_i \right] + \mathcal{O}(N^{-1/2}) \tag{172}$$

$$= \frac{\lambda^2}{N} \sum_{i=1}^{K} p_i \left(\tilde{t}_{ii}^k\right)^2 \mathbb{E}\left[\boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \boldsymbol{\eta}_i \right] + \mathcal{O}(N^{-1/2}) \tag{173}$$

where (a) and (b) follows from (110) and (115), respectively. As seen, the replacement of $\tilde{q}_{ii}^k$ with $\tilde{t}_{ii}^k$ allows to express $\breve{s}_{k,1}$ as a classical quadratic form - up to a vanishing term $\mathcal{O}(N^{-1/2})$. By taking the expectation with respect to $\boldsymbol{\eta}_i$, we obtain:

$$\mathbb{E}\left[\boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \boldsymbol{\eta}_i \right] = \mathbb{E}\left[\frac{1}{N} \mathrm{Tr} \mathbf{D} \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \right] + \mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \mathbf{a}_i \right]. \tag{174}$$

To proceed further, we rewrite the two terms above as a function of $\mathbf{Q}_k$ (rather than of $\mathbf{Q}_{ki}$). By applying twice Lemma 3.1 in [13], the first term is such that:

$$\left| \frac{1}{N} \mathrm{Tr} \mathbf{D} \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} - \frac{1}{N} \mathrm{Tr} \mathbf{D} \mathbf{Q}_k \mathbf{D} \mathbf{Q}_k \right| \leq \frac{2\|\mathbf{D}\|^2}{N\lambda^2}. \tag{175}$$

The second term in (174) is a quadratic form (not a normalized trace) and thus it cannot be treated in the same way. We need to develop $\mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_k \mathbf{D} \mathbf{Q}_k \mathbf{a}_i \right]$. By using (109) we obtain:

$$\mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_k \mathbf{D} \mathbf{Q}_k \mathbf{a}_i \right] = \mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \mathbf{a}_i \right] + \lambda^2 \mathbb{E}\left[\left(\tilde{q}_{ii}^k\right)^2 \boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \boldsymbol{\eta}_i |\boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{a}_i|^2 \right]$$
$$- \lambda \mathbb{E}\left[\tilde{q}_{ii}^k \mathbf{a}_i^H \mathbf{Q}_{ki} \boldsymbol{\eta}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \mathbf{a}_i \right] - \lambda \mathbb{E}\left[\tilde{q}_{ii}^k \mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \boldsymbol{\eta}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki} \mathbf{a}_i \right] \tag{176}$$

By using (115) and taking the expectation with respect to $\boldsymbol{\eta}_i$, we obtain:

$$\mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_k \mathbf{D} \mathbf{Q}_k \mathbf{a}_i \right] = \mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \mathbf{a}_i \right] - 2\lambda \tilde{t}_{ii}^k \mathbb{E}\left[\mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \mathbf{a}_i \right]$$
$$+ \lambda^2 \left(\tilde{t}_{ii}^k\right)^2 \mathbb{E}\left[\left(\mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{a}_i \right)^2 \mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \mathbf{a}_i \right]$$
$$+ \lambda^2 \left(\tilde{t}_{ii}^k\right)^2 \mathbb{E}\left[\left(\mathbf{a}_i^H \mathbf{Q}_{ki} \mathbf{a}_i \right)^2 \tilde{d}_i \frac{1}{N} \mathrm{Tr} \mathbf{D} \mathbf{Q}_{ki} \mathbf{D} \mathbf{Q}_{ki} \right] + \mathcal{O}(N^{-1/2}) \tag{177}$$

from which, by using (111), we have

$$\mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_i\right] = \mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_{ki}\mathbf{D}\mathbf{Q}_{ki}\mathbf{a}_i\right]\left(1-\lambda\tilde{t}_{ii}^k\mathbf{a}_i^H\boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_i\right)^2 + \lambda^2\left(\tilde{t}_{ii}^k\right)^2\left(\mathbf{a}_i^H\boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_i\right)^2\tilde{d}_i\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}_{ki}\mathbf{D}\mathbf{Q}_{ki}\right] + \mathcal{O}(N^{-1/2}). \tag{178}$$

From the results in (112) and (113), the above expression can be equivalently rewritten as a function of $\mathbf{T}_k$ as follows:

$$\mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_i\right] = \mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_{ki}\mathbf{D}\mathbf{Q}_{ki}\mathbf{a}_i\right]\lambda^2\left(\tilde{t}_{ii}^k\right)^2\left(1+\tilde{d}_i\delta_k\right)^2 + \left(\frac{\mathbf{a}_i^H\mathbf{T}_k\mathbf{a}_i}{1+\tilde{d}_i\delta_k}\right)^2\tilde{d}_i\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}\mathbf{D}\mathbf{Q}\right] + \mathcal{O}(N^{-1/2}) \tag{179}$$

By plugging the above results into (174) yields

$$\breve{s}_{k,1} = \frac{1}{N}\sum_{i=1}^{K}\frac{p_i}{\left(1+\tilde{d}_i\delta_k\right)^2}\mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_i\right] + \mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\right]\left(\frac{1}{N}\sum_{i=1}^{K}p_i\lambda^2\left(\tilde{t}_{ii}^k\right)^2 - \frac{1}{N}\sum_{i=1}^{K}\frac{p_i\tilde{d}_i(\mathbf{a}_i^H\mathbf{T}_k\mathbf{a}_i)^2}{(1+\tilde{d}_i\delta_k)^4}\right) + \mathcal{O}\left(N^{-\frac{1}{2}}\right) \tag{180}$$

which turns out to be only a function of $\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}\mathbf{D}\mathbf{Q}\right]$ and $\mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_i\right]$. Theorem 1 is now used to get deterministic equivalents for both terms. Setting $\mathbf{C}=\mathbf{D}$ yields:

$$\mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}\mathbf{D}\mathbf{Q}\right] = \frac{\vartheta}{\Delta} + \mathcal{O}(N^{-\frac{1}{2}}) \tag{181}$$

and

$$\mathbb{E}\left[\mathbf{a}_i^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_i\right] = \frac{1-F}{\Delta}\mathbf{a}_k^H\mathbf{T}_k\mathbf{D}\mathbf{T}_k\mathbf{a}_k + \frac{\vartheta}{\Delta}\sum_{\ell=1,\ell\neq k}^{K}\tilde{d}_\ell\frac{\mathbf{a}_k^H\mathbf{T}_k\mathbf{a}_\ell\mathbf{a}_\ell^H\mathbf{T}_k\mathbf{a}_k}{(1+\tilde{d}_\ell\delta_k)^2}. \tag{182}$$

where F and ϑ are expressed in terms of $\mathbf{T}$ instead of $\mathbf{T}_k$ thanks to one-rank perturbation arguments. Plugging the above results into (180) yields

$$\max_k\left|\breve{s}_{k,1} - \tilde{s}_{k,11} - \frac{\vartheta}{\Delta}\tilde{s}_{k,12}\right| \xrightarrow{\text{a.s.}} 0 \tag{183}$$

with

$$\tilde{s}_{k,11} = \frac{1-F}{\Delta N}\sum_{i=1}^{K}\frac{p_i}{(1+\tilde{d}_i\delta_k)^2}\mathbf{a}_i^H\mathbf{T}_k\mathbf{D}\mathbf{T}_k\mathbf{a}_i \tag{184}$$

and

$$\tilde{s}_{k,12} = \frac{1}{N}\sum_{i=1}^{K}\frac{p_i}{(1+\tilde{d}_i\delta_k)^2}\left(\sum_{\ell=1,\ell\neq k,i}^{K}\tilde{d}_\ell\frac{\mathbf{a}_i^H\mathbf{T}_k\mathbf{a}_\ell\mathbf{a}_\ell^H\mathbf{T}_k\mathbf{a}_i}{(1+\tilde{d}_\ell\delta_k)^2}\right) + \frac{1}{N}\sum_{i=1}^{K}p_i\lambda^2\left(\tilde{t}_{ii}^k\right)^2. \tag{185}$$

Using the same steps as those employed in the proof of Lemma 5.3 in [13], we can prove that $\tilde{s}_{k,12} = \frac{\lambda^2}{N}\mathrm{Tr}\mathbf{P}_k\tilde{\mathbf{D}}_k\tilde{\mathbf{T}}_k^2$. By using rank-one perturbation arguments, we may replace in $\tilde{s}_{k,11}$ and $\tilde{s}_{k,12}$, $\mathbf{T}_k$ with $\mathbf{T}$, $\tilde{\mathbf{T}}_k$ with $\tilde{\mathbf{T}}$ and $\tilde{\delta}_k$ with δ to obtain:

$$\max_k \left| \breve{s}_{k,1} - \tilde{s}_{11} - \frac{\vartheta}{\Delta}\frac{\lambda^2}{N}\mathrm{Tr}\mathbf{P}\tilde{\mathbf{D}}\tilde{\mathbf{T}}^2 \right| \xrightarrow{\text{a.s.}} 0 \tag{186}$$

with

$$\tilde{s}_{k,11} = \frac{1-F}{\Delta N}\sum_{i=1}^{K}\frac{p_i}{(1+\tilde{d}_i\delta)^2}\mathbf{a}_i^H\mathbf{T}\mathbf{D}\mathbf{T}\mathbf{a}_i + O(N^{-1}). \tag{187}$$

By using the identity (188) resulting from the well known matrix identity $(\mathbf{I}+\mathbf{U}\mathbf{V})^{-1}\mathbf{U} = \mathbf{U}(\mathbf{I}+\mathbf{V}\mathbf{U})^{-1}$ [13],

$$\mathbf{T}\mathbf{A}(\mathbf{I}_K + \delta\tilde{\mathbf{D}})^{-1} = (\mathbf{I}_N + \tilde{\delta}\mathbf{D})^{-1}\mathbf{A}\tilde{\mathbf{T}} \tag{188}$$

we ultimately get $\max_k \left| \breve{s}_{k,1} - \bar{s}_{k,1}/(\lambda^2\tilde{t}_{kk}^2) \right| \xrightarrow{\text{a.s.}} 0$ with $\bar{s}_{k,1}$ given by (39).

A.1.2) Deterministic equivalent for $\breve{s}_{k,2}$*:* We start observing that $\mathbf{\Sigma}_{[k]}\mathbf{P}_{[k]}\mathbf{\Sigma}_{[k]}^H = \sum_{i\neq k} p_i\boldsymbol{\eta}_i\boldsymbol{\eta}_i^H$ from which using (110) yields:

$$\breve{s}_{k,2} = \mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_k\mathbf{\Sigma}_{[k]}\mathbf{P}_{[k]}\mathbf{\Sigma}_{[k]}^H\mathbf{Q}_k\mathbf{a}_k\right] = \sum_{i\neq k} p_i\mathbb{E}\left[\lambda^2(\tilde{q}_{ii}^k)^2\boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\mathbf{a}_k\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right]. \tag{189}$$

Due to the absence of the normalization factor $1/N$, we cannot directly replace $\tilde{q}_{ii}^k$ by $\tilde{t}_{ii}^k$. Therefore, we proceed as follows. Rewrite $\breve{s}_{k,2}$ in (189) as:

$$\breve{s}_{k,2} = \sum_{i\neq k} p_i\lambda^2\left(\tilde{t}_{ii}^k\right)^2\mathbb{E}\left[\boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\mathbf{a}_k\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right] + \varepsilon \tag{190}$$

where:

$$\varepsilon = \sum_{i\neq k} p_i\lambda^2\mathbb{E}\left[\left(\left(\tilde{q}_{ii}^k\right)^2 - \left(\tilde{t}_{ii}^k\right)^2\right)|\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i|^2\right]. \tag{191}$$

Since $\max(\tilde{q}_{ii}^k, \tilde{t}_{ii}^k) \leq \lambda^{-1}$, we have:

$$|\epsilon| \leq \sum_{i\neq k}\frac{p_i\lambda}{2}\mathbb{E}\left[\left|\tilde{q}_{ii}^k - \tilde{t}_{ii}^k\right|\boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\mathbf{a}_k\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right] = |\varepsilon_1| + |\varepsilon_2|. \tag{192}$$

with

$$\varepsilon_1 = \sum_{i\neq k}\frac{p_i\lambda}{2}\mathbb{E}\left[\left|\tilde{q}_{ii}^k - \tilde{t}_{ii}^k\right|\mathbf{y}_i^H\mathbf{Q}_{ki}\mathbf{a}_k\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right] \tag{193}$$

$$\varepsilon_2 = \sum_{i\neq k}\frac{p_i\lambda}{2}\mathbb{E}\left[\left|\tilde{q}_{ii}^k - \tilde{t}_{ii}^k\right|\mathbf{a}_i^H\mathbf{Q}_{ki}\mathbf{a}_k\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right]. \tag{194}$$

To control $|\epsilon_1|$, we make use of the Jensen and Cauchy-Schwartz inequalities such that:

$$|\varepsilon_1| \leq \sqrt{\sum_{i\neq k} \frac{p_i^2\lambda^2}{4}\mathbb{E}\left[\left|\mathbf{y}_i^H\mathbf{Q}_{ki}\mathbf{a}_k\right|^2\left|\tilde{q}_{ii}^k-\tilde{t}_{ii}^k\right|^2\right]}\sqrt{\sum_{i\neq k}\mathbb{E}\left[\left|\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right|^2\right]} = \mathcal{O}(N^{-1/2}). \tag{195}$$

One can easily convince himself that the same arguments used for ε_1 allow to replace $\mathbf{a}_i^H\mathbf{Q}_{ki}\mathbf{a}_k$ by $\mathbf{a}_i^H\boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_k$ in ε_2 with an inducing error of order $\mathcal{O}(N^{-\frac{1}{2}})$. This is because the difference $\mathbf{a}_i^H\mathbf{Q}_{ki}\mathbf{a}_k - \mathbf{a}_i^H\boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_k$ would behave like $\mathbf{y}_i^H\mathbf{Q}_{ki}\mathbf{a}_k$. We thus obtain the following inequality

$$\begin{aligned}
|\varepsilon_2| &= \sum_{i\neq k}\frac{p_i\lambda}{2}\mathbb{E}\left[\left|\tilde{q}_{ii}^k-\tilde{t}_{ii}^k\right|\mathbf{a}_i^{\mathrm{H}}\boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_k\mathbf{a}_k^{\mathrm{H}}\mathbf{Q}_k\boldsymbol{\eta}_i\left(1+\boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i\right)\right] + \mathcal{O}(N^{-1/2}) && (196)\\
&= \frac{\lambda}{2}\mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_k\boldsymbol{\Sigma}_k\mathrm{diag}\left\{\frac{\left|\tilde{q}_{ii}^k-\tilde{t}_{ii}^k\right|}{1+\tilde{d}_i\delta_k}(1+\boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i)\right\}_{i\neq k}\mathbf{A}_k^H\mathbf{T}_k\mathbf{a}_k\right] + \mathcal{O}(N^{-1/2}) && (197)\\
&\leq \frac{\lambda}{2}\mathbb{E}\left[\|\mathbf{a}_k\|\|\mathbf{Q}_k\boldsymbol{\Sigma}_k\|\sqrt{\sum_{\ell\leq k}\left|\tilde{q}_{ii}^k-\tilde{t}_{ii}^k\right|^2\left|\mathbf{a}_\ell^H\mathbf{T}_k\mathbf{a}_k\right|^2(1+\boldsymbol{\eta}_i^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i)^2}\right] + \mathcal{O}(N^{-1/2}). && (198)
\end{aligned}$$

Noting that $\|\mathbf{Q}_k\boldsymbol{\Sigma}_k\|$ is bounded yields $\epsilon_2 = \mathcal{O}(N^{-1/2})$. All this proves that:

$$\breve{s}_{k,2} = \sum_{i\neq k}\lambda^2\left(\tilde{t}_{ii}^k\right)^2 p_i\mathbb{E}\left[|\mathbf{a}_k^H\mathbf{Q}_{ki}\boldsymbol{\eta}_i|^2\right] + \mathcal{O}(N^{-1/2}). \tag{199}$$

Taking the expectation with respect to $\boldsymbol{\eta}_i$ produces $\breve{s}_{k,2} = \breve{s}_{k,2,1} + \breve{s}_{k,2,2} + \mathcal{O}(N^{-1/2})$ with

$$\breve{s}_{k,2,1} = \frac{1}{N}\sum_{i=1,i\neq k}^{K}\lambda^2\left(\tilde{t}_{ii}^k\right)^2 p_i\tilde{d}_i\mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_{ki}\mathbf{D}\mathbf{Q}_{ki}\mathbf{a}_k\right] \tag{200}$$

$$\breve{s}_{k,2,2} = \sum_{i=1,i\neq k}^{K}\lambda^2\left(\tilde{t}_{ii}^k\right)^2 p_i\mathbb{E}\left[|\mathbf{a}_k^H\mathbf{Q}_{ki}\mathbf{a}_i|^2\right]. \tag{201}$$

Similarly to $\breve{s}_{k,1}$, we rewrite $\breve{s}_{k,2}$ as function of $\mathbf{Q}_k$ rather than of $\mathbf{Q}_{ki}$. To this end, let

$$\breve{\breve{s}}_{k,2,1} = \frac{1}{N}\sum_{i=1,i\neq k}^{K}\lambda^2\left(\tilde{t}_{ii}^k\right)^2 p_i\tilde{d}_i\mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_k\right] \tag{202}$$

and observe that (details are omitted for space limitations)

$$\breve{s}_{k,2,1} = \breve{\breve{s}}_{k,2,1} + \mathcal{O}(N^{-1}). \tag{203}$$

As for $\breve{s}_{k,2,2}$, we proceed studying $\breve{\breve{s}}_{k,2,2}$ defined as:

$$\breve{\breve{s}}_{k,2,2} = \sum_{i=1,i\neq k}^{K}\lambda^2\left(\tilde{t}_{ii}^k\right)^2 p_i\alpha_i\mathbb{E}\left[|\mathbf{a}_k^H\mathbf{Q}_k\mathbf{a}_i|^2\right] \tag{204}$$

where $\{\alpha_i\}_{i=1}^K$ is some deterministic bounded sequence. Using (109), we may write:

$$\breve{\breve{s}}_{k,2,2} = \sum_{i\neq k} \lambda^2 \alpha_i \left(\tilde{t}_{ii}^k\right)^2 p_i \mathbb{E}\left[|\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{a}_i|^2\right]$$
$$- \sum_{i\neq k} \lambda^2 \left(\tilde{t}_{ii}^k\right)^2 p_i\alpha_i \mathbb{E}\left[\lambda \tilde{q}_{ii}^k \mathbf{a}_k^H \mathbf{Q}_{ki}\boldsymbol{\eta}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_{ki}\mathbf{a}_k\right]$$
$$- \sum_{i\neq k} \lambda^2 \left(\tilde{t}_{ii}^k\right)^2 p_i\alpha_i \mathbb{E}\left[\lambda \tilde{q}_{ii}^k \mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_{ki}\boldsymbol{\eta}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_k\right]$$
$$+ \sum_{i\neq k} \lambda^2 \left(\tilde{t}_{ii}^k\right)^2 p_i\alpha_i \mathbb{E}\left[\lambda^2 \left(\tilde{q}_{ii}^k\right)^2 |\mathbf{a}_k^H \mathbf{Q}_{ki}\boldsymbol{\eta}_i|^2 |\boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_i|^2\right]. \quad (205)$$

As before, the quantities $\mathbf{a}_k^H \mathbf{Q}_{ki}\boldsymbol{\eta}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_k$ and $\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{a}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_k$ ensure the boundedness of the sum, while the terms $\tilde{q}_{ii}^k$ and $\boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_i$ can be replaced by their deterministic equivalents with an error of order $\mathcal{O}(N^{-1/2})$. Therefore, we obtain:

$$\breve{\breve{s}}_{k,2,2} = \sum_{i\neq k} \lambda^2 \left(\tilde{t}_{ii}^k\right)^2 p_i\alpha_i \mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_{ki}\mathbf{a}_k\right]$$
$$- \sum_{i\neq k} \lambda^3 \left(\tilde{t}_{ii}^k\right)^3 p_i\alpha_i \mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_{ki}\boldsymbol{\eta}_i \mathbf{a}_i^H \boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_{ki}\mathbf{a}_k\right]$$
$$- \sum_{i\neq k} \lambda^3 \left(\tilde{t}_{ii}^k\right)^3 p_i\alpha_i \mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{a}_i \mathbf{a}_i^H \boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_i \boldsymbol{\eta}_i^H \mathbf{Q}_{ki}\mathbf{a}_k\right]$$
$$+ \sum_{i\neq k} \lambda^4 \left(\tilde{t}_{ii}^k\right)^4 p_i\alpha_i \mathbb{E}\left[|\mathbf{a}_k^H \mathbf{Q}_{ki}\boldsymbol{\eta}_i|^2 \left(\mathbf{a}_i^H \boldsymbol{\mathcal{T}}_{ki}\mathbf{a}_i\right)^2\right] + \mathcal{O}(N^{-1/2}) \quad (206)$$

from which, taking the expectation with respect $\boldsymbol{\eta}_i$, we finally obtain:

$$\breve{\breve{s}}_{k,2,2} = \sum_{i\neq k} p_i\alpha_i\lambda^4 \left(\tilde{t}_{ii}^k\right)^4 \mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_{ki}\mathbf{a}_k\right]\left(1+\tilde{d}_i\delta_k\right)^2$$
$$+ \frac{1}{N}\sum_{i=1}^K p_i\alpha_i\tilde{d}_i\lambda^2 \left(\tilde{t}_{ii}^k\right)^2 \frac{\left(\mathbf{a}_i^H \mathbf{T}_k \mathbf{a}_i\right)^2}{(1+\tilde{d}_i\delta_k)^2}\mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{D}\mathbf{Q}_{ki}\mathbf{a}_k\right] + \mathcal{O}(N^{-1/2}). \quad (207)$$

Taking $\alpha_i = \frac{1}{\lambda^2\left(\tilde{t}_{ii}^k\right)^2(1+\tilde{d}_i\delta_k)^2}$ leads to

$$\breve{s}_{k,2,2} = \sum_{i\neq k} \frac{p_i}{(1+\tilde{d}_i\delta_k)^2}\mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_k\mathbf{a}_i \mathbf{a}_i^H \mathbf{Q}_k\mathbf{a}_k\right]$$
$$- \frac{1}{N}\sum_{i=1}^K p_i\tilde{d}_i \frac{\left(\mathbf{a}_i^H \mathbf{T}_k \mathbf{a}_i\right)^2}{(1+\tilde{d}_i\delta_k)^4}\mathbb{E}\left[\mathbf{a}_k^H \mathbf{Q}_{ki}\mathbf{D}\mathbf{Q}_{ki}\mathbf{a}_k\right] + \mathcal{O}(N^{-1/2}). \quad (208)$$

As done before, the factor $1/N$ allows to replace $\mathbf{Q}_{ki}$ by $\mathbf{Q}_k$ – up to an error of $\mathcal{O}(N^{-1})$. Plugging (208) and (203) into (201), we ultimately obtain

$$\breve{s}_{k,2} = \mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_k\mathbf{A}_k\mathbf{P}_k\left(\mathbf{I}_{K-1}+\delta_k\tilde{\mathbf{D}}_k\right)^{-2}\mathbf{A}_k^H\mathbf{Q}_k\mathbf{a}_k\right] - \frac{1}{N}\sum_{i=1,i\neq k}^{K}\left(\frac{p_i\tilde{d}_i\left(\mathbf{a}_i^H\mathbf{T}_k\mathbf{a}_i\right)^2}{(1+\tilde{d}_i\delta_k)^4} - \lambda^2\left(\tilde{t}_{ii}^k\right)^2\tilde{d}_ip_i\right)\mathbb{E}\left[\mathbf{a}_k^H\mathbf{Q}_k\mathbf{D}\mathbf{Q}_k\mathbf{a}_k\right] + \mathcal{O}(N^{-1/2}). \tag{209}$$

By applying Theorem 1 with

$$\mathbf{C} = \mathbf{A}_k\mathbf{P}_k\left(\mathbf{I}_{K-1}+\delta_k\tilde{\mathbf{D}}_k\right)^{-2}\mathbf{A}_k^H \tag{210}$$

we obtain

$$\max_k\left|\breve{s}_{k,2} - \mathbf{a}_k^H\mathbf{T}_k\mathbf{C}\mathbf{T}_k\mathbf{a}_k - \sum_{\ell\neq k}\frac{\tilde{d}_\ell\left|\mathbf{a}_\ell^H\mathbf{T}_k\mathbf{a}_k\right|^2}{(1+\tilde{d}_\ell\delta_k)^2}\left(\frac{\vartheta(\mathbf{C})(1-F)}{\Delta}+\frac{\vartheta}{\Delta}\breve{s}_{k,2}^{\circ}\right) - \mathbf{a}_k^H\mathbf{T}_k\mathbf{D}\mathbf{T}_k\mathbf{a}_k\left(\frac{\lambda^2\vartheta(\mathbf{C})\tilde{\vartheta}}{\Delta}+\frac{1-F}{\Delta}\breve{s}_{k,2}^{\circ}\right)\right| \xrightarrow{\text{a.s.}} 0. \tag{211}$$

with

$$\breve{s}_{k,2}^{\circ} = F(\mathbf{C}) - \frac{1}{N}\sum_{i=1}^{K}\frac{p_i\tilde{d}_i(\mathbf{a}_i^H\mathbf{T}_k\mathbf{a}_i)^2}{(1+\tilde{d}_i\delta_k)^4} + \frac{1}{N}\sum_{i=1}^{K}\lambda^2\left(\tilde{t}_{ii}^k\right)^2\tilde{d}_ip_i. \tag{212}$$

Similar to $\breve{s}_{k,1}^{\circ}$, it can be proved that $\breve{s}_{k,2}^{\circ} = \frac{\lambda^2}{N}\mathrm{Tr}\mathbf{P}\tilde{\mathbf{D}}\tilde{\mathbf{T}}^2 + O(N^{-1})$. For the same arguments used in (113), we have:

$$\mathbf{a}_k^H\mathbf{T}_k\mathbf{a}_\ell = \frac{\mathbf{a}_k^H\mathbf{T}\mathbf{a}_\ell}{\lambda\tilde{t}_{kk}(1+\delta\tilde{d}_l)} + O(N^{-1}). \tag{213}$$

Using the above result along with the identity (188), we obtain $\max_k|\breve{s}_{k,2}-\tilde{s}_{k,2}| \xrightarrow{\text{a.s.}} 0$ with $\tilde{s}_{k,2}$ given by (40). This completes the proof for s_k.

A.2) Deterministic equivalent for ψ

To begin with, we apply Lemma 2 in Appendix C to $\psi = \frac{1}{K}\mathrm{Tr}\mathbf{P}\boldsymbol{\Sigma}^H\mathbf{Q}^2\boldsymbol{\Sigma}$. In doing so, we obtain:

$$\psi - \frac{1}{K}\sum_{k=1}^{K}p_k\mathbb{E}\left[\boldsymbol{\eta}_k^H\mathbf{Q}^2\boldsymbol{\eta}_k\right] \xrightarrow{\text{a.s.}} 0. \tag{214}$$

Then, observe that (using the same calculus for s_k):

$$\frac{1}{K}\sum_{k=1}^{K} p_k \mathbb{E}\left[\boldsymbol{\eta}_k^H \mathbf{Q}^2 \boldsymbol{\eta}_k\right] \tag{215}$$

$$= \frac{1}{K}\sum_{\ell=1}^{K} p_\ell \lambda^2 \left(\tilde{t}_{\ell\ell}\right)^2 \boldsymbol{\eta}_\ell^H \mathbf{Q}_\ell^2 \boldsymbol{\eta}_k + \mathcal{O}(N^{-\frac{1}{2}}) \tag{216}$$

$$= \frac{1}{K}\sum_{\ell=1}^{K} p_\ell \lambda^2 \tilde{t}_{\ell\ell}^2 \left[\tilde{d}_\ell \frac{1}{N}\mathrm{Tr}\mathbb{E}\mathbf{D}\mathbf{Q}^2 + \mathbb{E}\mathbf{a}_\ell^H \mathbf{Q}_\ell^2 \mathbf{a}_\ell\right] + \mathcal{O}(N^{-\frac{1}{2}}) \tag{217}$$

$$= \frac{1}{K}\sum_{\ell=1}^{K} \left[p_\ell \lambda^2 \tilde{t}_{\ell\ell}^2 \tilde{d}_\ell - p_\ell \tilde{d}_\ell \frac{\left(\mathbf{a}_\ell^H \mathbf{T}\mathbf{a}_\ell\right)^2}{\left(1+\tilde{d}_\ell \delta\right)^4}\right] \mathbb{E}\left[\frac{1}{N}\mathrm{Tr}\mathbf{D}\mathbf{Q}^2\right]$$

$$+ \frac{1}{K}\sum_{\ell=1}^{K} \frac{p_\ell}{(1+\tilde{d}_\ell \delta)^2} \mathbb{E}\left[\mathbf{a}_\ell^H \mathbf{Q}^2 \mathbf{a}_\ell\right] + \mathcal{O}(N^{-\frac{1}{2}}). \tag{218}$$

Using similar arguments as those adopted above for s_k (omitted due to space limitations), we get:

$$\frac{1}{K}\sum_{k=1}^{K} p_k \mathbb{E}\left[\boldsymbol{\eta}_k^H \mathbf{Q}^2 \boldsymbol{\eta}_k\right] = \left(\vartheta_I \frac{1-F}{\Delta} + \frac{\vartheta F_I}{\Delta}\right) \frac{\lambda^2}{K} \mathrm{Tr}\mathbf{P}\tilde{\mathbf{D}}\tilde{\mathbf{T}}^2 + \frac{1}{K}\sum_{\ell=1}^{K} \frac{p_\ell}{(1+\tilde{d}_\ell \delta)^2} \mathbf{a}_\ell^H \mathbf{T}^2 \mathbf{a}_\ell$$

$$+ \frac{1}{K}\sum_{\ell=1}^{K} \frac{p_\ell \mathbf{a}_\ell^H \mathbf{T}\mathbf{D}\mathbf{T}\mathbf{a}_\ell}{(1+\tilde{d}_\ell \delta)^2} \left[\frac{F_I(1-F)}{\Delta} + \frac{\lambda^2 \tilde{\vartheta} \vartheta_I}{\Delta}\right] + \mathcal{O}(N^{-\frac{1}{2}}). \tag{219}$$

By using (188), the expression in (42) is obtained.

APPENDIX C
SOME ADDITIONAL USEFUL RESULTS

Lemma 1 (Trace Lemma)**.** *Let* $\mathbf{z} \in \mathbb{C}^N$ *be a complex Gaussian vector with mean* $\sqrt{N}\boldsymbol{\mu}$ *and covariance matrix* $\mathbf{I}_N$*. Let* $\mathbf{M}$ *be a complex matrix independent of* $\mathbf{z}$*. Then, there exists a constant* K_p *such that:*

$$\mathbb{E}_{\mathbf{z}}\left|\frac{1}{N}\mathbf{z}^H \mathbf{M}\mathbf{z} - \frac{1}{N}\mathrm{Tr}\mathbf{M} - \boldsymbol{\mu}^H \mathbf{M}\boldsymbol{\mu}\right|^p \leq \frac{K_p}{N^p}\left(\left(\mathrm{Tr}\mathbf{M}\mathbf{M}^H\right)^{\frac{p}{2}} + N^{\frac{p}{2}}\left|\boldsymbol{\mu}^H \mathbf{M}^2 \boldsymbol{\mu}\right|^{\frac{p}{2}}\right). \tag{220}$$

Lemma 2. *If* $\mathrm{B} \in \mathbb{C}^{N\times N}$ *is a sequence of deterministic matrices with uniformly bounded spectral norm, then*

$$\mathbb{E}\left|\frac{1}{K}\mathrm{Tr}\mathbf{B}\mathbf{Q}_k \boldsymbol{\Sigma}_k \boldsymbol{\Sigma}_k^H \mathbf{Q}_k - \mathbb{E}[\frac{1}{K}\mathrm{Tr}\mathbf{B}\mathbf{Q}_k \boldsymbol{\Sigma}_k \boldsymbol{\Sigma}_k^H \mathbf{Q}_k]\right|^4 = \mathcal{O}(N^{-4}). \tag{221}$$

If $\mathbf{u}$ *is a sequence of deterministic vectors with bounded norm, then:*

$$\mathbb{E}\left|\mathbf{u}^H\mathbf{Q}_k\mathbf{\Sigma}_k\mathbf{\Sigma}_k^H\mathbf{Q}_k\mathbf{u}-\mathbb{E}[\mathbf{u}\mathbf{Q}_k\mathbf{\Sigma}_k\mathbf{\Sigma}_k^H\mathbf{Q}_k\mathbf{u}]\right|^6=\mathcal{O}(N^{-3}). \tag{222}$$

Proof. The proof follows from the Nash-Poincaré inequality. Calculations are similar to the ones performed to prove Lemma 5 in [31]. Details are thus omitted. □

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