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Estimating jumps in volatility using realized-range measures ⋆

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Abstract

We introduce a generalization of the Heterogeneous Autoregressive (HAR) model for estimating the presence of jumps in volatility, using the realized-range measure as a volatility proxy. By focusing on a set of 36 NYSE stocks, we show that there is a positive probability of jumps in volatility.
Keywords: Volatility, Jumps in volatility, Realized range, HAR.

1 Introduction

Recent empirical studies indicate that diffusive stochastic volatility and jumps in returns are incapable of capturing the empirical features of equity index returns. Instead, it has been stressed that jumps in volatility can improve the overall fitting of stochastic volatility models. Eraker et al. (2003), for instance, report convincing evidence that volatility of financial returns is affected by rapid and large increments. We focus on the modeling and on the estimation of the volatility jump component in a discrete time setting. As a distinctive feature of our contribution, we use the realized range as non parametric ex-post measure of the daily integrated variance. Such a choice allows us to simplify the computational burden of estimating the jumps in

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volatility. In fact it circumvents the need to integrate out unobservable quantities. As suggested by Todorov (2009, 2011) we can make inference on the volatility jumps regardless of how complicated the model for the stochastic volatility is. Furthermore, recent theoretical findings by Christensen and Podolskij (2007, 2012) prove that realized range is a very efficient estimator of the quadratic variation of the returns. In our framework, efficiency of the integrated variance estimation is a crucial element, since the potential reduction in the measurement error obtained with realized-range measures can lead to more precise evaluations of the volatility jump component. In order to evaluate the contribution of jumps to the daily volatility dynamics we specify and estimate a parametric model in discrete time. In particular, given the well documented long-range dependence of the realized variance estimators, see Andersen et al. (2003) among others, and the persistent effects of jumps in volatility, we propose a conditional model that generalizes the HAR model, introduced by Corsi (2009). The Heterogeneous Autoregressive-Volatility-Jump (HAR-V-J) model includes an additive volatility jump term, which is modeled as a compound Poisson process allowing for multiple jumps per day, as in Chan and Maheu (2002) and Maheu and McCurdy (2004), whose intensity and magnitude parameters are varying over time according to an autoregressive specification. In this way, we are able to model and identify periods with higher volatility jump activity, that are also periods of high market stress. The empirical analysis focuses on 36 stocks quoted at the New York Stock Exchange, representing nine sectors of the U.S. economy: banks, insurance and financial services, oil gas and basic materials, food beverage and leisure, health care, industrial goods, retail and telecommunications, services, and technology. The estimation results point out that the jump activity is characterized by two different periods. The first one, from 2004 to 2007, of low jump activity, the second, from mid-2008 to mid-2009, of high jump activity. In particular, during the second period the jump component represents a relevant part of the estimated conditional volatility. Such a finding is perfectly in line with the known feature of equity data during the sample period we consider. Furthermore, we find an ex-post positive correlation between volatility and price jumps, in line with Bandi and Renò (2011).

2 A model for realized range with jumps

We choose to estimate the integrated variance by the realized range which produce considerable efficiency gains relative to a standard return-based estimator, even when the latter employs subsampling to exhaust the entire database (e.g. Zhang et al., 2005). We consider the bias-corrected realized range-based bipower variation, denoted as $RBV^{\Delta}_{m,BC}$, see Christensen et al. (2009). Such a quantity, which is a proxy of the integrated variance in presence of jumps in prices and microstructure noise, is evaluated at the daily level from stock prices sampled at 1 minute intervals. Here we focus on the extension of the HAR model proposed by Corsi (2009), adding a jump component to the conditional mean of the realized range sequence. Let $X_t = \log RBV^{\Delta}_{m,BC,t}$ be the daily logarithm of bias-corrected realized range-based bipower variation and I^{t-1} be the time $t-1$ information set, the HAR-Volatility Jump (HAR-V-J) model for X_t is given by,

$$X_t = \mu + \phi_D X_{t-1} + \phi_W X^W_{t-1} + \phi_M X^M_{t-1} + Z_t + \epsilon_t \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \tag{1}$$

where $X^W_t = \frac{1}{5}\sum_{j=0}^{4} X_{t-j}$ and $X^M_t = \frac{1}{22}\sum_{j=0}^{21} X_{t-j}$ represent the weekly and monthly volatility components, respectively, see also Corsi (2009). This model for X_t implies that the $RBV^{\Delta}_{m,BC}$ is given by a multiplicative structure such as $RBV^{\Delta}_{m,BC,t} = \exp\{\bar{X}_{t-1}\}\exp\{Z_t\}\exp\{\epsilon_t\}$ where $\bar{X}_{t-1} = \mu + \phi_D X_{t-1} + \phi_W X^W_{t-1} + \phi_M X^M_{t-1}$. Hence, the jump term $J_t = \exp\{Z_t\}$ acts as a multiplicative term in the volatility process, such that it can be considered as a *burst* factor of the volatility dynamics. In case of no jumps $J_t = 1$, and the volatility follows a HAR process. In the period t, the jump term, Z_t, is given by $Z_t = \sum_{k=1}^{N_{\sigma,t}} Y_{t,k}$ where the jump size is $Y_{t,k} \sim i.i.d.N\left(\Theta_{\sigma,t}, \Delta_{\sigma,t}\right)$, and ϵ_t and $Y_{t,k}$ are assumed to be independent. Following Chan and Maheu (2002), $\Theta_{\sigma,t}$ and $\Delta_{\sigma,t}$ are modeled as a function of past log-volatility, namely

$$\Theta_{\sigma,t} = \zeta_0 + \zeta_1 X_{t-1} \tag{2}$$

and

$$\Delta_{\sigma,t} = \eta_0 + \eta_1 X^2_{t-1}. \tag{3}$$

The jump component has a compound Poisson structure where the number of jumps arriving between $t-1$ and t, $N_{\sigma,t}$, is a Poisson counting process with intensity parameter $\Lambda_{\sigma,t} > 0$ and density

$$P\left(N_{\sigma,t} = j | I^{t-1}\right) = \frac{e^{-\Lambda_{\sigma,t}}\Lambda^j_{\sigma,t}}{j!}, \qquad j = 0, 1, 2, ...$$

This implies that $\mathrm{E}\left[N_{\sigma,t}|I^{t-1}\right] = \mathrm{Var}\left[N_{\sigma,t}|I^{t-1}\right] = \Lambda_{\sigma,t}$ so that the conditional density of Z_t given $N_{\sigma,t}$ and I^{t-1} is

$$Z_t | N_{\sigma,t} = j, I^{t-1} \sim N\left(j\Theta_{\sigma,t}, j\Delta_{\sigma,t}\right). \tag{4}$$

Since $\mathrm{E}\left[Z_t|N_{\sigma,t} = j, I^{t-1}\right] = j\Theta_{\sigma,t}$, the conditional expected value of the jump component is

$$\mathrm{E}\left[Z_t|I^{t-1}\right] = \Theta_{\sigma,t}\Lambda_{\sigma,t} \tag{5}$$

where $\Theta_{\sigma,t}$ is assumed to be measurable with respect to I^{t-1}, as in (2). Given the conditional density of Z_t in (4), the conditional variance of the jump component is

$$\mathrm{Var}\left[Z_t|I^{t-1}\right] = \left(\Delta_{\sigma,t} + \Theta^2_{\sigma,t}\right)\Lambda_{\sigma,t}, \tag{6}$$

where $\Delta_{\sigma,t}$ is assumed to be measurable with respect to I^{t-1}, see (3). Whereas, as in Chan and Maheu (2002), the unobserved log-volatility jump intensity is assumed to follow an autoregressive specification

$$\Lambda_{\sigma,t} = \Lambda_0 + \lambda_1 \Lambda_{\sigma,t-1} + \psi \xi_{t-1}. \tag{7}$$

As a result, the conditional jump intensity in period t depends on its own lag and on the lag of the innovation term ξ_t, which represents the measurable shock constructed ex-post. This shock, or jump intensity residual, is defined as $\xi_t = \mathrm{E}\left[N_{\sigma,t}|I^t\right] - \Lambda_{\sigma,t}$. Therefore, ξ_t depends on the expected number of jumps measured with respect to the information set including the contemporaneous information, i.e. at time t. It follows that the jump intensity equation can be rewritten as

$$\Lambda_{\sigma,t} = \Lambda_0 + (\lambda_1 - \psi)\,\Lambda_{\sigma,t-1} + \psi\,\mathrm{E}\left[N_{\sigma,t-1}|I^{t-1}\right]$$

with

$$\mathrm{E}\left[N_{\sigma,t}|I^t\right] = \sum_{j=0}^{\infty} j P\left(N_{\sigma,t} = j|I^t\right). \tag{8}$$

As noted by Chan and Maheu (2002), other functional forms that include nonlinearity also may be very useful. For example, in Bandi and Renò (2011), the intensities of the jumps are nonlinear functions of the underlying variance level. The filtered probabilities $P\left(N_{\sigma,t} = j|I^t\right)$ are obtained by means of the Bayes' law

$$P\left(N_{\sigma,t} = j|I^t\right) = \frac{P\left(X_t|N_{\sigma,t} = j, I^{t-1}\right) P\left(N_{\sigma,t} = j|I^{t-1}\right)}{P\left(X_t|I^{t-1}\right)}, \quad j = 0, 1, 2, \ldots \tag{9}$$

where

$$P\left(X_t|I^{t-1}\right) = \sum_{j=0}^{\infty} P\left(X_t|N_{\sigma,t} = j, I^{t-1}\right) P\left(N_{\sigma,t} = j|I^{t-1}\right)$$

and $P\left(X_t|N_{\sigma,t} = j, I^{t-1}\right)$ is given by the density of ϵ_t. Analogously, we can compute the conditional probability of tail events, such as $P(X_t > u|I^{t-1})$. This allows us to compare the probability of extreme events implied by the HAR-V-J model, with those implied by the Gaussian HAR. Model estimation is performed by maximum likelihood methods introducing a truncation equal to 20 when integrating with respect to the number of jumps.

	HAR				Size		Variance		Intensity			LR	Q_ϵ^5	Q_ϵ^{20}
	μ	ϕ_D	ϕ_W	ϕ_M	ζ_0	ζ_1	η_0	η_1	λ_0	λ_1	ψ			
BA	-0.79^a	0.24^a	0.39^a	0.21^a	0.56^a	0.08^a	0.02^b	0.01^b	0.00	0.99^a	0.08^b	0.00	0.0102	0.06
IBM	-1.03^a	0.26^a	0.45^a	0.10^a	0.73^a	0.11^a	0.00	0.01	0.00	0.99^a	0.12^b	0.00	0.00	0.00
JPM	-0.56	0.35^a	0.32^a	0.22	0.42	0.04	0.00	0.00	0.03	0.95^b	0.40	0.00	0.19	0.14
UPS	-0.41^a	0.25^a	0.50^a	0.17^a	0.22	-0.01	0.29^a	0.00	0.00	0.99^a	0.11	0.00	0.02	0.06

Table 1: Estimated parameters of the HAR-V-J model using price data from January 2, 2004 to December 31, 2009 for selected companies. Q_ϵ^5 and Q_ϵ^{20} are the p-values of the Ljung-Box test on the residuals, with 5 and 20 lags, respectively. LR is p-value of the likelihood-ratio test for the null hypothesis $Z_t = 0$.

3 Volatility jumps in the US stock market

Our empirical analysis is based on the intradaily returns of 36 equities of the S&P 500 index. Prices are sampled at one minute frequency, from January 2, 2004 to December 31, 2009, for a total of 1510 trading days. The companies details are available upon request. We compute the $RBV^{\Delta}_{m,BC,t}$, for each stock, using one-minute returns. Figure **??** plots the dynamic behavior of the volatility of Boeing (BA), IBM, JP Morgan (JPM), and UPS. The volatility is characterized by two dominant regimes. A long period of low volatility, approximately from 2004 to 2007, which is followed by a period of high volatility in correspondence of the financial crisis. It is interesting to note that the first part of the sample is not characterized by large jumps, while the period in correspondence of the recent financial crisis has many large spikes. As expected, this suggests that during financial crises, the probability and the magnitude of the jumps could be higher. We estimate the model in (1) and report in Table 1 the estimated parameters of the HAR-V-J for the 4 stocks mentioned before. Introducing the jump component in the model induces a better in-sample fit. The likelihood-ratio test strongly rejects the null hypothesis, i.e. the HAR model with no jumps, in all cases. Thus, when $\Lambda_{\sigma,t}$, $\Theta_{\sigma,t}$ and $\Delta_{\sigma,t}$ are allowed to vary over time, we obtain an improvement over the more traditional HAR model without jumps. Looking at the Ljung-Box test on the model residuals, see the last columns of Table 1 for an example, for some series the HAR-V-J model is not able to completely capture the dynamics of the log-volatility series. This is due to the peculiar autoregressive lag structure of the HAR-RV model which appears too restrictive for many series under exam, thus leaving some autocorrelation in the residuals. It is important to stress that the autocorrelation in the residuals is not due to the inclusion of the jump term in the HAR-V-J model.

Focusing on the jump size mean, $\Theta_{\sigma,t}$, the estimates of ζ_0 in (2) are significant in 18 out of 36 cases, while those of ζ_1 are generally positive but not statistically different from zero. However, this does not imply that jumps do not affect the conditional moments of log-volatility.

Figure 1 reports some illustrative plots for the JPM company, where we note that the estimated jump size increases as the level of volatility increases, such as during the last two years.

The persistence parameter in the jump intensity, λ_1, is strongly significant and greater than 0.9 in 24 out of 36 cases. This result confirms the evidence in Eraker et al. (2003) and Duffie et al. (2000), where the jump arrivals in volatility are highly persistent, producing clusters in jumps. The close-to-unit-root behavior of the jumps intensity could stem from a change of regime in the number of jumps arrivals during the financial crisis. Interestingly, the expected number of jumps (see Figure 1 for an example) suggest the presence of three regimes in the jumps intensity. The first period, from 2004 to the beginning of 2007, is characterized by an absence of jumps in volatility (the number of jumps is on average one in twenty days). In the second period, the estimated jump arrivals sharply increase to a daily average of 0.5-0.6, while between mid-2008 and mid-2009, the average number of jump arrivals dramatically increases, implying approximately one jump per day. This result is in line with the findings of Todorov and Tauchen (2011), where the high number of jumps in volatility is attributed to the pure jump nature of the volatility process.

The sample averages of the estimated $\Lambda_{\sigma,t}$ turn out to be, in a few cases, larger than one. However, on average across all assets, the probability of a volatility jump is around 0.5. This result is in line with the findings in Dotsis et al. (2007), who estimate a constant probability of jumps equal to 0.4 for a set of implied volatility series, during the period 1997-2004. On the other hand, our results suggest that time variation in the jumps intensity is not negligible, so that the assumption of constant jump arrival probability turns out to be unrealistic, especially during periods of financial turmoil. This is also confirmed by looking at the parameter ψ in the intensity equation which is always positive, and significant in most of the cases. As a consequence the unobserved past innovation has always a positive and significant impact on the jump intensity.

Finally, the estimated expected exponential jumps, $\hat{J}_{t|t-1}$ is reported in Figure 1 for JPM. The expected exponential jumps increase during the period 2008-2009 for all stock, while for JPM the increase starts even in 2007.

4 Concluding remarks

This paper studies the contribution of volatility jumps to the evolution of volatility. Differently from some earlier contribution we propose a modified version of the HAR, the HAR-V-J, for modeling the realized range instead of relying on continuous-time stochastic volatility specification.

We model the corrected bipower realized range, a consistent estimator of the integrated variance in presence of jumps in prices and microstructure noise, with a HAR-V-J model that allows for the presence of jumps in volatility. The inference on the parameters of the model is carried out utilizing maximum likelihood estimation, after having specified the dynamics of the jumps sizes and intensities.

The estimation results of the HAR-V-J model with high-frequency data from 36 NYSE stocks suggest that jumps in volatility are more likely to happen during the financial crises, i.e., when the level of volatility is high, and are positively correlated with jumps in prices.

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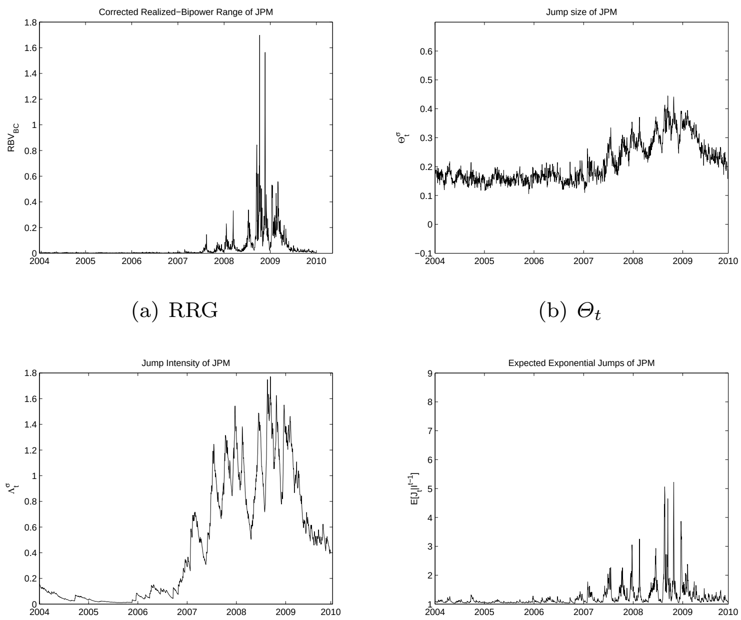

(a) RRG (b) Θ_t

(c) λ_t (d) Exp. Jumps

Fig. 1: Illustrative plots for JPM