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The Impact of Generator Market Power on the Electricity Hedge Market

M. R. Hesamzadeh, D. R. Biggar,* D. W. Bunn and E. Moiseeva

Abstract

The incentive of an electricity generating firm with market power to influence the market price depends strongly on the volume the firm has pre-sold in the forward, or hedge, markets. However, the choice of hedge level may be a strategic decision in itself. In the normal case where participants in the hedge market cannot observe the hedge position of dominant generators, we show that the optimal choice of hedging for a dominant generator facing a linear demand curve is an all-or-nothing decision and there is no equilibrium level of hedging in pure strategies. This outcome may explain an observed lack of hedge market liquidity in wholesale electricity markets where individual generators have substantial market power. We perform the analysis for the monopoly and oligopoly cases and extend it for realistic cost functions and various degrees of competitiveness in the market. These results contribute to the extensive body of research on the price formation and strategic behavior in electricity forward and spot markets, as well as providing implications for transparency initiatives in market design.

1 Introduction

The susceptibility of wholesale electricity markets to the exercise of market power has been a source of concern for regulators, competition authorities, and policy-makers ever since the lib-

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eralization of this sector first started to spread worldwide in the 1990s. As a consequence, there is an extensive body of theoretical and empirical research into market power in wholesale electricity markets. Twomey et al. (2005) provide a thorough review and in Section 2 we summarize some of the relevant background research on market power

In our view, whilst our understanding of spot market conduct has become very detailed, the implications of market power in the spot market for behaviour in the electricity forward or futures markets are only partially understood.

This is an important gap in our understanding. It is widely recognised that forward contract or hedge positions have a strong impact on the behaviour of generators in the spot market. A generator which is hedged to a high level has very little incentive to exercise market power, while an unhedged generator may have a significant influence on the spot market prices (Biggar and Hesamzadeh, 2014). But this raises the question: how does a generator with the potential to exercise market power in the spot market choose its forward contract position in the first place?

A common assumption in the literature on forward contracting is that the forward price for a commodity is equal to the expected future spot price (often referred to as a no arbitrage condition). But the expected future spot price itself depends on the level of hedge cover of the dominant generators. The level of hedge cover of a generator is typically a closely guarded commercial secret. This raises the question how arbitrageurs can ensure that the forward price is always equal to the expected future spot price without being able to observe the $< C3 >$aggregate forward position of the dominant generators. In this paper we relax the assumption that the forward price is always equal to the expected future spot price. Our key result is that, in the presence of generators with market power, a liquid forward market may cease to exist, and the generators with market power will adopt an all or nothing hedging decision.

This research can be seen as part of a wider literature on transparency in forward markets. Much of the analytical research in forward contracting, following Allaz (1992a), Allaz and Vila (1993a), Green (1999), Brown and Eckert (2017), presumes full observability of each company's hedge position. There are, however, a few exceptions. For example, Bagwell (1995) and Hughes and Kao (1997) suggest that unobservability leads only to a Cournot equilibrium without forward contracting, but this is predicated upon a no-arbitrage condition, with forward and spot prices being equal. However, this assumption is confronted both empirically by the widespread emergence of forward premia in the term structure of forward curves relative to the realized spot prices (Bunn and Chen, 2013) and analytically in the formulations that reflect forward contracting as manifesting equilibria between the hedging needs of heterogeneous risk averse market participants (Bessembinder and Lemmon, 2002). Further, from a theoretical perspective, with market power and the absence of risk aversion, it is difficult to see how the no arbitrage equilibrium condition can be imposed without participants knowing the forward positions of each other. Ferreira et al. (2006) addresses this obstacle in a pragmatic way by presuming that arbitrageurs can infer, from the forward prices as reported, the quantities traded forward by the physical players with market power. That is an open question, but presuming this inference is possible, competition between arbitrageurs then leads to forward and spot equality, but at equilibrium prices between the competitive outcome and that of Allaz and Vila under full observability. There is also a considerable empirical literature examining the relationship between forward prices and expected future spot prices (see, for example, the survey by Chow et al. (2000) and Jha and Wolak (2013) in the case of the electricity sector). Although there are methodological questions, those studies do not reject the hypothesis that forward prices, where they exist, are a good predictor of expected future spot prices. These studies are not inconsistent with the results in this paper. Our analysis shows that, where there is sufficient competition in the spot market, a forward market can emerge in which the forward price is equal to the expected future spot price. However, in the presence of market power, an equilibrium in the forward market may cease to exist. In other words, the liquidity in the forward market may dry up.

$< C6 >$ We analyze a stylized model of oligopoly with two stages. In the first stage the market participants choose their level of hedge cover by trading in hedge products. These hedge products are purely financial. Even if there is a single physical producer of electricity, and therefore a single participant in the spot market, there is no limit to the number of participants in the hedge market. Subsequently, the (residual) demand curve is realized and market participants can choose to exercise any market power in the spot market that they might have. We are interested in the question how the presence of market power in the spot market affects the hedging decisions and liquidity in the hedge market.

We assume that the forward contract positions of the dominant generators are unobservable. We look for a rational-expectations equilibrium in which the forward price and the resulting choice of hedge position is consistent with the subsequent expected future spot price. This case is compared to the case with the assumption of full observability in order to highlight the contributions of our analysis compared to the existing research. We assume linear demand, risk neutrality and constant marginal costs and show that there is no forward-spot equilibrium in pure strategies. This arises because the profit function of the generators in the choice of hedge level is U-shaped and the maxima lie at the extremes. We demonstrate that this result holds for both a single dominant firm[1] and a Cournot oligopoly. As we will see, with a larger number of firms there is a tendency towards the conventional competitive equilibrium in pure strategies. The same analysis also holds for quadratic cost functions for the generating firms.

We suggest that this absence of equilibrium in pure strategies result may explain certain outcomes in the South Australian region of the Australian National Electricity Market (NEM). In that market a dominant generator appears to have alternated between periods of exercising significant market power to raise prices, followed by periods with lower prices, coupled with a simultaneous drop in liquidity in the forward market. We motivate our theoretical analysis briefly by this case study, although we do not suggest that our analysis is a theoretical model of that particular behavior (other factors were also at play), but rather, the narrative adds to the plausibility of our stylized analysis and the conclusions that follow.

Finally, we extend the analysis in this paper to consider more general cases of asymmetric oligopolies using numerical examples. We show that the indications obtained from the closed-form solution of the symmetric oligopoly continue to hold. As a topical example, we provide

[1] $< C5 >$ That is, a firm with market power in a market with a 'competitive fringe' of price-taking firms.

numerical results for the effect of increased wind power penetration on the hedging decisions of generators in the context of market power. As in the analysis of Bessembinder and Lemmon (2002) without market power, we see that the skewness in price distribution also affects the optimal hedging decisions of the generators.

The paper proceeds as follows: In the next section, we provide a motivating example from the South Australian region of the Australian National Electricity Market. Section 4 introduces the theory and main results in the context of a single dominant generator and a symmetric Cournot oligopoly. Section 5 presents numerical results confirming the theoretical findings and extending them to more general cases. Section 5 extends the model to the case of high levels of wind penetration. Section 7 concludes.

2 Literature review on market power

$< C2 >$ Anderson and Hu (2008) model the forward and spot market interaction when market power exists on the generation side. The paper shows that the existence of hedge contracts between consumers and generators mitigates this market power. Allaz (1992b) shows that in a forward-spot market setup, forward transactions are effective tools in the hands of strategic producers. The effect of multiple forward markets on the electricity market efficiency is explored in Allaz and Vila (1993b). Using a model with two Cournot duopolists, Allaz and Vila (1993b) show that with increasing number of forward trading periods, the total output of a duopoly model becomes closer to a perfectly competitive market (and in the limit they are the same). The transmission-network related strategies for exercising market power are investigated in Cardell et al. (1997). The authors show that a generator might exercise market power by increasing its output in order to block transmission of its competing generation. The Conjectured Supply Function (CSF) approach is proposed in Day et al. (2002) to model imperfect competition between strategic generators. The authors discuss the strengths of the CSF model as compared with the other modeling approaches in the literature. A mathematical programming approach for finding the Nash equilibrium in oligopolistic power markets is proposed in Murphy et al. (1982). Harker (1984) presents the variational inequality modeling approach as an alternative to the approach proposed in Murphy et al. (1982) for finding the Nash equilibrium. A Nash-Cournot model is proposed in Metzler et al. (2003) for analyzing market power in the generation sector. The authors employ complementarity models for finding Nash-Cournot equilibrium. Hobbs et al. (2000) models the unilateral exercise of market power by a single dominant firm in an electricity market using a proposed Mathematical Program with Equilibrium Constraints (MPEC). The network-based linear programs are developed in Hobbs (1986) to find the equilibrium prices in network-constrained oligopolistic electricity markets. Two Cournot models of imperfect competition between generators are formulated as linear complementarity problems in Hobbs (2001). These models are solved using efficient algorithms for linear complementarity problems. Hu and Ralph (2007) proposes an equilibrium program with equilibrium constraints (EPEC) for modeling oligopolistic competition in electricity markets. Each generator is modeled as a mathematical program with equilibrium constraints (MPEC). The collection of MPEC models for all oligopolistic generators form the proposed EPEC model. The concepts of local Nash and Nash stationary equilibria are discussed as the possible solutions of the EPEC model. Kamat and Oren (2004) uses a duopoly model and shows that even with small probabilities of transmission congestion, forward trading may be reduced and this in turn might nullify the market power mitigating effect of forward markets. A linear asymmetric supply function equilibrium model is developed in Hui Niu et al. (2005) where the forward contracts of the electricity generating firms are explicitly considered. The developed model is then used to evaluate the effect of forward contracts on the Electric Reliability Council of Texas (ERCOT) market. Yu et al. (2010) develop an EPEC framework to model the strategic forward contracting in a two-stage forward-spot market setup. Cournot competition is assumed for the strategic forward market. A Cournot model of competition between electricity generating firms is analytically discussed in Ruiz et al. (2008). Price, quantities and profits are first derived and then analysed. The proposed model is used to explore the case of several identical Cournot players and the case of one dominant

Cournot. Ruiz et al. (2012) proposed an EPEC to model the strategic generators participating in a spot market. The strong stationary conditions of all MPECs in their proposed EPEC model are used to find the equilibria of the proposed EPEC model. Ruiz and Conejo (2009) models the strategic bidding of a producer in spot market as a bilevel program. Then it employs the duality theory and Karush-Kuhn-Tucker optimality conditions to transform the original bilevel model to a mixed-integer linear program. A methodology to solve Nash-Cournot games in electricity generation sector allowing some variables to be discrete is proposed in Gabriel et al. (2013). The proposed approach suggests a compromise between integrality and complementarity to avoid infeasible solutions and it allows for more realistic modeling. An electricity market with conjectural-variation electricity producing firms is analyzed in Ruiz et al. (2010). A MILP model for computing the extremal-Nash equilibria in a wholesale power market with transmission constraints is proposed in Hesamzadeh and Biggar (2012). Through the introduced concept of the extremal-Nash equilibria, the proposed MILP model can effectively locate all Nash equilibria. The MILP model in Hesamzadeh and Biggar (2012) is further studied in Hesamzadeh and Biggar (2013) for merger analysis in wholesale power markets. The New South Wales region of the Australian National Electricity Market is used as a real case study. An extended version of the MILP model in Hesamzadeh and Biggar (2012) is employed in Moiseeva et al. (2015) to study the exercise of market power on ramp-rates in wind-integrated power systems. Moiseeva and Hesamzadeh (2018b) and Moiseeva and Hesamzadeh (2018a) propose different decomposition algorithms to solve MPEC and EPEC models of imperfect competition in liberalized power markets.

3 The experience in South Australia

This study is partly motivated by anecdotal evidence of the behavior of one particular dominant generator in the South Australian region of Australia's National Electricity Market. Although the NEM market has historically been reasonably competitive at most times, problems can arise when transmission constraints limit flows into a specific region. The South Australian (SA) region of the NEM lies at the extreme western end of the NEM. Although the South Australian region is small (in terms of either energy consumption or generation capacity) relative to the rest of the NEM, transmission constraints into South Australia will occasionally bind, giving scope for certain generators in SA to exercise market power (see Biggar (2011), Hesamzadeh et al. (2011)). The largest generator in the South Australian region is the Torrens Island Power Station (TIPS). At times of high demand in South Australia, when transmission limits into South Australia are binding, the total South Australian demand cannot be met by the sum of other generating units in South Australia and the interconnector flows. At these times, Torrens Island Power Station is pivotal and may have material market power.

The acquisition of Torrens Island by AGL was approved by the Australian Competition and Consumer Commission in 2007. At that time there was no evidence that TIPS had been exercising market power and, in any case, it was argued that AGL as the new owner of TIPS would not have an incentive to exercise market power. This argument hinged on the observation that AGL was primarily a retailer, selling to downstream customers at a fixed price, and therefore would have no interest in increasing the wholesale price. This analysis turned out to be wrong. In each of the three subsequent summers, on very hot (high demand) days in South Australia, TIPS allegedly withheld capacity from the market, pushing the wholesale spot price in South Australia close to the wholesale price ceiling (which was, at the time, $10,000/MWh). This allegation was confirmed by AER in its annual report[2](AER, 2010).

As an example of the way in which TIPS apparently exercised market power, consider one particular episode. Figure 1 shows the offer curve for TIPS at three different time periods on

[2]In 2010 the Australian Energy Regulator (AER) made the following comments in its annual State of the Energy Market report: "Spot prices in South Australia rose by 20% to $82 per MWh in 2009/10, which was the second highest price for any region since the NEM commenced. This outcome reflects that around 50% of NEM prices above $5000 per MWh in 2009/10 occurred in South Australia [...]. Most of these events were associated with opportunistic bidding by AGL Energy, the region's largest electricity generator. AGL Energy owns the Torrens Island power station, which accounts for around 40% of South Australia's generation capacity. Transmission limits on importing electricity from Victoria mean AGL Energy can, on days of high electricity demand, bid a significant proportion of its capacity at prices around the market cap and drive up the spot price. It adopted this type of bidding strategy during many of South Australia's 47 extreme price events in 2009/10. The events typically occurred on days of extreme weather, which led to high electricity demand and a tight regional supply/demand balance."

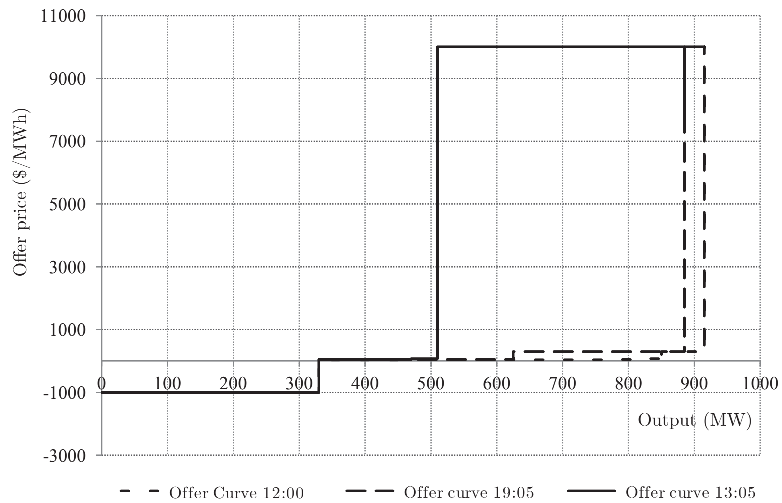

Figure 1: Offer curves of TIPS on 11 January 2011 illustrating possible use of market power to raise the wholesale spot price.

the 11th of January 2010. This was a hot day in South Australia, with temperatures exceeding 41 degrees Celsius. Throughout the morning, TIPS offered more than 900 MW of capacity at a price less than $300/MWh and the wholesale spot price remained in the range $60-80/MWh. Around 12:30 pm, import constraints in SA started to bind. TIPS responded by withdrawing around 400 MW of capacity from the market (by placing this capacity into very high-priced bands, close to the price ceiling of $10,000/MWh). The wholesale spot price rose rapidly to close to the price ceiling, and remained above $9000/MWh for most of the next six hours. By 7 pm in the evening, TIPS restored the withdrawn capacity to the market, offering close to 900 MW at below $300/MWh. The wholesale spot price returned to around $50/MWh for the remainder of the evening.

However, starting in 2011 the behavior of TIPS appeared to switch. Specifically, during 2011 and 2012 TIPS no longer withheld capacity on high-price days, but instead appeared to seek to lower the price at low-demand times. Figure 2 shows two scatter diagrams setting out the price-quantity combinations chosen by TIPS during each half-hour trading interval during the 2010 and 2011 calendar years. As can be seen, during 2010, at times of high prices, TIPS

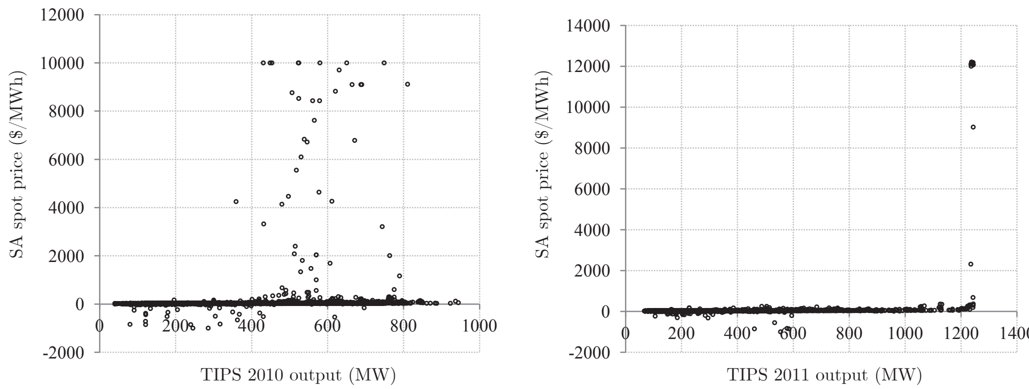

Figure 2: Scatter diagrams of the price-quantity combinations chosen by TIPS during 2010 and 2011.

was typically not producing more than around 800 MW and often was producing much less. In 2011, in contrast, at times of very high prices TIPS was consistently producing more than 1200 MW.

An example of exercising market power to *lower* the spot price occurred during the early hours of 28th June 2012. During June 2012, TIPS would normally reduce its production overnight to a low amount of around 120-160 MW. However, on this occasion at around 3:30 am TIPS increased the amount it offered at the market price floor ($-1000/MWh) from 160 MW up to 540 MW. This increased the production at TIPS up to 540 MW and reduced the wholesale spot price from around $20/MWh to less than $-900/MWh, where it remained for the next two hours. Around 6:30 am TIPS reduced the volume it offered at the price floor. Its output reduced to 220 MW and the wholesale spot price returned to a positive level. The offer curves for TIPS on this occasion at 3:05 am, 3:30 am, and 6:30 am are shown in Figure 3 (AER, 2012)[3].

This change in strategic spot market behaviour raises questions regarding the hedging activities of the company. According to industry sources, in 1999/2000 AGL was overwhelmingly dominant as a retailer in South Australia. The (previous) owner of the Torrens Island Power Sta-

[3]In its 2012 State of the Energy Market report the AER comments on this as follows: "... all instances of prices below $-100/MWh (including those near the $-1000/MWh market floor) were driven by AGL Energy bidding or rebidding large amounts of capacity to prices near the floor at times of low demand. On several occasions, this effectively shut down other generators (including wind generators). This type of disorderly market activity can have detrimental longer term consequences for market stability and investment."

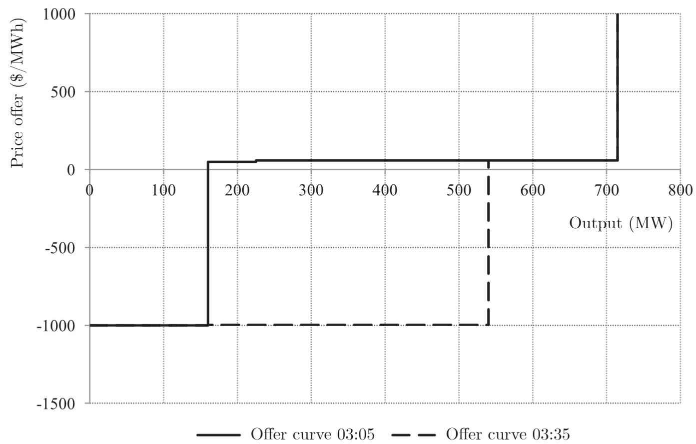

Figure 3: Offer curves of TIPS on 28 June 2012 illustrating possible use of market power to lower the wholesale spot price.

tion was able to hold out for a high price, high volume and medium term hedging contract with the local retailer (AGL). Under this contract Torrens Island was highly hedged and chose not to exercise market power for the subsequent five years. Average wholesale prices in South Australia during this period were moderate. By the time this contract expired around 2006, AGL's market share had eroded and its retail load was smaller. It found that it was able to cover its retail load with hedge contracts purchased from other generators and did not need to purchase hedges from Torrens Island. At this time, faced with low contract prices, the owners of TIPS chose a low hedge level. Following its purchase of TIPS, AGL apparently chose to maintain a largely unhedged position and to exploit opportunities to exercise market power when they arose over the next few years. But, by June 2009 forward prices for electricity in SA were significantly higher than in the other regions of the NEM AER (2009). In 2010 TIPS appears to have reversed its position and adopted a very high hedge level. Since 2011 TIPS does not appear to have withheld capacity at high-demand times and, indeed, on several occasions appears to have increased the volume it offers to the market at low-demand (and even negative price) times.

$< C8a >$ The impact on the prices for forward contracts in South Australia is illustrated in Figure 4. Figure 4 shows the forward price for a first-quarter (Q1) base swap in each year from 2006-2014, starting two years before the relevant quarter, and finishing at the end of the relevant quarter (by which time the forward price is equal to the out-turn average spot price for the quarter).

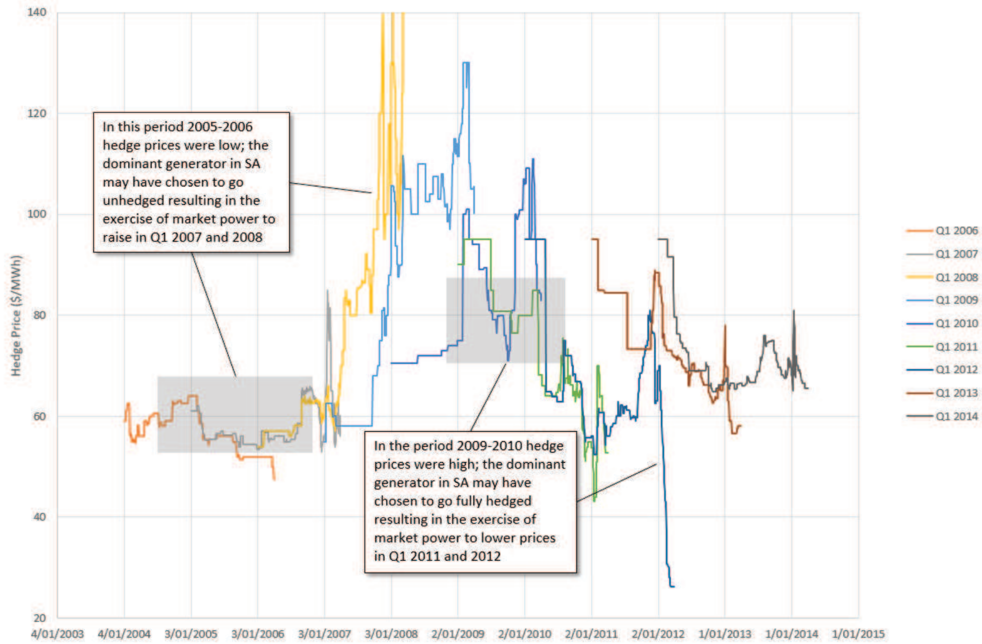

Figure 4: Q1 base forward contract prices in South Australia 2006-2014. Source: ASX Energy: This market outcome will be analyzed in Subsection 4.1.2

This exercise of market power may have had the effect of reducing liquidity in the hedge market. A 2009 report by the Australian Energy Regulator shows that the volume of exchange-based and over-the-counter in South Australian hedge contracts declined significantly after 2006 AER (2009). A survey of market participants in South Australia in mid-2010 found that the hedge market was illiquid ACIL (2010). In addition, the forward price for electricity in South Australia has been volatile. In mid-2009 the forward price for a calendar-year 2012 swap was around $70/MWh (AER, 2009). By mid-2010 the price for the same 2012 swap was closer to $45/MWh, AER (2010).

The above anecdotes suggest that the hedging choices of a dominant generator could be an all or nothing decision. That is, at times of potential market power a dominant generator will either choose to be hedged to a high level or will choose to be unhedged (or even a negative level of hedge cover). Motivated by this example, we revisit and extend some of the basic theory on strategic forward contracting. Under the assumption that the forward market is not transparent we indeed show that the optimal choice of hedging is an all-or-nothing decision and there is no equilibrium level of hedging in pure strategies. This is consistent with the observation of alternating high and low price regimes in the spot market.

4 < C4 >Theoretical results

In this section we derive a closed-form solution to the hedging-decision of a profit-maximizing generating firm in an electricity market. In Section 4.1 we focus on the hedging decision of a single firm. In Section 4.2 we extend the analysis by considering multiple generating firms and a symmetric oligopoly. We start the analysis by considering linear cost functions in Sections 4.1 and 4.2. We further extend the conclusion to the case of quadratic costs in Section 4.3. We will focus on the case where the hedge contract is a swap or contract-for-differences (i.e., a hedge contract with a fixed volume of hedge cover determined in advance). The only source of uncertainty facing each generator is the wholesale spot price (each generator faces no uncertainty in its input costs or its availability).

In the market each generating firm i makes a two-stage decision. In the first stage the firm chooses the level of hedge cover, with knowledge of the range of demand scenarios likely to arise in the future and the prevailing hedge price f. In the second stage the firm observes the actual demand reflected in the market demand curve $p(q)$ and makes a decision on its level of output q_i. Figure 5 shows this sequence[4].

[4] We do not model any constraints on the ability of generators to exercise market power in the spot market, as is the case in the Australian National Electricity Market. The model may not be directly relevant to wholesale electricity markets with strict controls on the exercise of market power, or to day-ahead markets in jurisdictions where there are strict limits on the ability of generators to change their production from their day-ahead position.

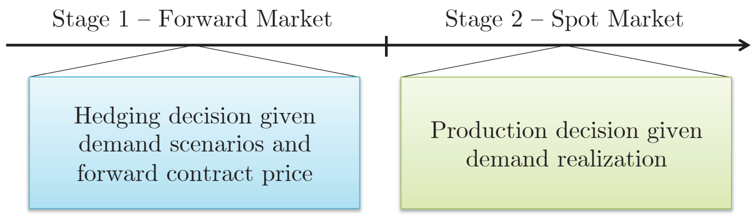

Figure 5: Decision-making of a generating firm.

The whole setup can be formulated as follows:

$$\forall i: \begin{cases} \underset{x_i}{\text{maximize}} & E[\pi_i(x_i|x_{-i})] = E[(p(q)-c_i)q_i] + (f - E[p(q)])x_i \quad \text{(1a)} \\ \text{subject to:} & q_i \in \arg\underset{q_i}{\text{maximize}}\ \pi_i(q_i|q_{-i},x_i) = (p(q)-c_i)q_i + (f-p(q))x_i \quad \text{(1b)} \end{cases}$$

Here q_i is the rate of production of firm i in MW, $q = \sum_i q_i$ is the total level of production (MW), x_i is the level of hedge cover of firm i (MW). Parameter f is the forward or hedge price ($/MW) and c_i is the marginal cost of the firm ($/MW). For tractability we will assume that the market price is determined by a linear demand curve:[5]

$$p(q) = \beta - \alpha q. \tag{2}$$

The parameters β and α are random variables which describe the intercept and slope of the residual demand curve respectively. These values are unknown at the time when the generator makes its hedging decision, but are realised before the firm makes its production decision. We place no ex ante constraints on the possible values of both β and α except for the following: Since we are focusing on the case of firms with market power, we will assume that the residual demand curve is downward sloping (i.e., $\alpha > 0$). In addition we will assume that it is always desirable to produce some output (i.e., $\beta > c$).

[5] $< C1 >$Ruiz et al. (2012) also model market power in a wholesale power market using a linear demand curve.

4.1 The hedging decision of a single dominant firm

We start by focusing on the hedging decisions of a single generating firm operating in a wholesale market with a degree of market power. $< C7 >$ There may be arbitrary many other generating firms in the market, however these are assumed to all be price-takers. Their behavior can be collectively summarized in the form of their supply curve. This supply curve can be subtracted off the market demand curve to yield the residual demand curve facing the dominant generator. Without loss of generality, therefore, we model the hedging decisions of a monopolist facing a residual demand curve.

Since there is only one firm we will drop the subscript i for the rest of this subsection. The problem as given in (1) can be solved by backward induction. The optimal solution to the lower-level optimization problem (1b) is found by taking the first-order conditions with respect to the firm's rate of production q. The corresponding optimal lower-level rate of production parameterized by hedge level x is as follows:

$$q(x) = \frac{1}{2\alpha}(\beta - c + \alpha x). \tag{3}$$

Combining (2) and (3) we get an expression for price:

$$p(x) = \frac{1}{2}(\beta + c - \alpha x). \tag{4}$$

We can express this result another way: given linear demand and constant marginal cost, the profit-maximizing quantity lies exactly halfway between the hedge level x and the point of intersection of the demand and the marginal cost curves, denoted $q_c = (\beta - c)/\alpha$. The profit-maximizing price lies exactly halfway between the marginal cost c and the price corresponding to the hedge level x, denoted $p_x = \beta - \alpha x$. In other words:

$$q(x) = \frac{1}{2}(x + q_c), \quad p(x) = \frac{1}{2}(p_x + c) = \frac{\alpha}{2}(q_c - x) + c \tag{5}$$

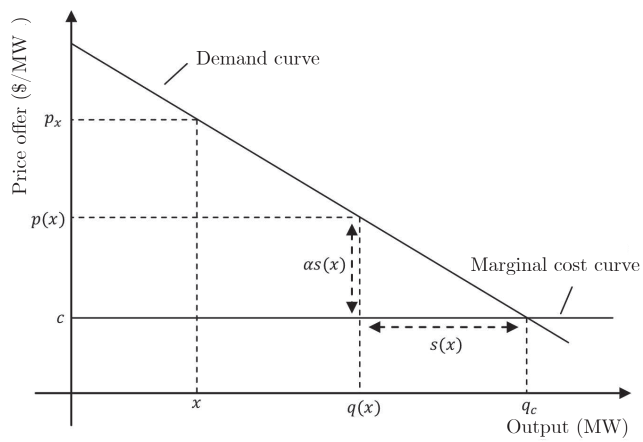

Figure 6: The profit-maximizing price-quantity combination as a function of the hedge level.

This is illustrated in Figure 6 where in this figure $s(x) = (q_c - x)/2$.

Inserting expressions (5) into the upper-level profit function (1a) yields the following expected profit as a function of the hedge level:

$$E[\pi(x)] = E[\frac{1}{\alpha}(p(x) - c)^2] + (f - c)x = E[\frac{\alpha}{4}(q_c - x)^2] + (f - c)x \quad (6)$$

In Sections 4.1.1 and 4.1.3 we distinguish two cases regarding the forward contracts' price f, depending whether the hedge level of the firm is observable to traders in the hedge market.

4.1.1 Hedge price independent of firm's hedge level

The hedge level of a firm is typically commercial-in-confidence and not available to traders in the hedge market. Also the traded volumes in the hedge market are typically much higher than those of the spot market. To proceed, therefore, we make the assumption that, in the first instance, the price at which hedge contracts can be traded is invariant to changes in the hedge level. We will look for a rational expectations equilibrium in which trader's forecasts of the hedge level are confirmed ex post.

Since the coefficient α is positive, it follows that the expected profit given in (6) is U-shaped in the hedge level (i.e., the second derivative of the expected profit with respect to the hedge level is positive) and accordingly the choice of hedge level x, which maximizes the expected profit, will be an all-or-nothing decision. The firm will either choose to be hedged to a very high level or will choose to be hedged to a very low (and possibly negative) level.

Let's suppose that we are given any two different possible hedge levels x_1 and x_2 (with $x_2 > x_1$). We make use of the following observation: Whether or not the dominant firm has an incentive to choose hedge level x_1 over x_2 depends on whether or not the price of hedges is above or below the average of the expected market price when the firm chooses hedge level x_1 and when the firm chooses hedge level x_2. Using equation 6:

$$\begin{aligned} E[\pi(x_2)] - E[\pi(x_1)] &= E[\frac{\alpha}{4}(q_c - x_2)^2 + f x_2] - E[\frac{\alpha}{4}(q_c - x_1)^2 + f x_1] \\ &= (f - \frac{E[p(x_2)] + E[p(x_1)]}{2})(x_2 - x_1) \end{aligned} \tag{7}$$

Let's suppose that there is some permissible range $[x^{min}, x^{max}]$ for the hedge level where $x^{max} > x^{min}$. This permissible range represents a range of feasible practicality for the dominant generator based on its physical operating limits and market constraints.[6] Using equation 7 we can conclude that the firm will choose the maximum level of hedging x^{max} if and only if the forward price is greater than the average of the future expected spot price when the hedging is at each end of the range:

$$E[\pi(x^{max})] > E[\pi(x^{min})] \iff f > \frac{p^{max} + p^{min}}{2}. \tag{8}$$

Here $p^{max} = E[p(x^{max})] = \frac{1}{2}(\overline{\beta} + c - \overline{\alpha} x^{max})$ and $p^{min} = E[p(x^{min})] = \frac{1}{2}(\overline{\beta} + c - \overline{\alpha} x^{min})$ are

[6] According to equation (5) there is a direct link between hedge level and the choice of production of the dominant firm and the wholesale spot price. Limits on production or spot price will imply limits on the feasible range of hedge levels. For example, the higher the hedge level, the higher the level of output of the dominant firm and the lower the spot price. In practice the dominant generator may come up against capacity constraints, limiting its ability to expand output further. Alternatively the spot price may be pushed down to the price floor ($-1000/MWh in Australia). Conversely, for a low enough (negative) hedge level, the output of the dominant generator would eventually have to turn negative. Unless the generator is integrated with a load this may prove impractical. Alternatively, the wholesale spot price may exceed the market price cap (currently around $14,500/MWh). In either case there are practical upper and lower limits on the hedge level of the dominant generator.

the expected price outcomes when the firm chooses the maximum and minimum hedging levels, respectively.

Let's define a rational expectations equilibrium to be a pair of hedge level and hedge price (x, f) such that, given the hedge price f, x is the profit-maximizing hedge level for the generator and, simultaneously, given the hedge level x, the hedge price is equal to the expected future spot price (i.e., $f = E[p(x)]$). We can now formulate the following proposition:

Proposition 1. *Under the assumption of linear demand and constant marginal cost, with a single dominant generator in the market, there is no rational expectations equilibrium in pure strategies in the level of hedge cover of the dominant firm.*

Proof. Assume that traders in the hedge market expect the dominant firm to choose the minimum level of hedging x^{min}. The corresponding expected price outcome is $E[p(x^{min})] = p^{min}$. Assuming effective competition between traders in the hedge market, the hedge price will be forced to the level $f = E[p(x^{min})] = p^{min}$. But, by condition (8), given this hedge price, the profit-maximizing level of hedge cover is x^{max}, contradicting the original assumption. This proves that there cannot be an equilibrium in which traders expect the firm to choose the minimum level of hedging. Similar arguments show that there cannot be an equilibrium in which traders expect the firm to choose the maximum level of hedging. There is no equilibrium level of hedging in pure strategies. ■

Although there is no equilibrium in pure strategies, there is a mixed-strategy equilibrium. Assume traders expect that the generating firm chooses the minimum level of hedge cover with probability 0.5 and the maximum level of hedge cover with probability 0.5. In this case, the hedge traders will set the hedge price equal to the expected future price outcome: $f = \frac{1}{2}(p^{max} + p^{min})$. Given this hedge price, the generating firm will be indifferent between choosing the maximum or minimum level of hedge cover, which is consistent with the initial hypothesis.

There is a graph of a typical profit function of a single dominant generator as a function of its hedging decision and the price of forward contracts in section 5.1 below.

$< C8b >$

4.1.2 A realistic simulation based on Figure 4 and the proposed economic model in the current paper

As an illustration of how this might work in practice, we apply this framework to a stylized model of the wholesale electricity market in the South Australia (SA) region of the NEM during the time period 2005-2014, as illustrated in Figure 4 above. As noted above, during this period Torrens Island Power Station held a dominant position in the SA market and, at the times when the interconnector to the neighbouring state of Victoria was experiencing congestion, Torrens Island Power Station likely had a degree of market power. We model this market as a dominant generator with a variable cost of $40/MWh and a capacity of 1000 MW, facing a stochastic demand curve $p(q) = \beta - \alpha q$. There are four possible states of the market, with the parameters β and α chosen with the probabilities in each of the four states set out in Table 1. The upper limit of hedging is assumed to be the level that induces full production $x^{max} = 1000$. We make the additional assumption that due to physical limits this generator cannot easily reduce its output below around 350 MW, so the lower feasible limit on hedging is $x^{min} = 0$.

Table 1: Stochastic demand curve probabilities

State	Probability	β	α
A	0.20%	$20,000	20.00
B	1.50%	$3,000	5.96
C	6.00%	$220	0.37
D	92.30%	$42	0.008

The expected outcomes for quantity, price, and profit for two different hedge prices ($f = \$60$ and $f = \$80$) are shown in Table 2. Since the profit function is convex, to find the optimal choice of hedge level x we need only consider the outcomes at the extremes $x = x^{min}$ and $x = x^{max}$, which yields the expected future prices p^{min} and p^{max}.

From equation 8, the critical or threshold value of the forward price is $(p^{max} + p^{min})/2$, which from Table 2 is equal to $78.51. As set out in Figure 4, during the period 2005 and 2006,

Table 2: Modelled outcomes for the simple example

Hedge Price f	Hedge Level x	Expected Quantity	Expected Price	Expected Profit
\$60	0	384.2	\$118	\$38,629
\$60	1000	884.2	\$39	\$20,117
\$80	0	384.2	\$118	\$38,629
\$80	1000	884.2	\$39	\$40,117

Q1 base swaps were trading at around \$60/MWh. Since this forward price is below the critical value, the optimal response of the generator is to go unhedged ($x = x^{min} = 0$), which results in it exercising market power, especially at peak times. The resulting expected future spot price in this model is then \$118.23/MWh (c.f. out-turn average prices in the range \$100-\$140 in Figure 4). Conversely, during the period 2009 and the first half of 2010, forward prices were in the range \$70-\$100/MWh. At the forward price of \$80/MWh, since this is above the critical value, the optimal response of the generator is to be fully hedged ($x = x^{max} = 1000$) - by selling 1000 MW in the forward market. The resulting expected future spot price is then \$38.74/MWh (c.f. out-turn average prices in the range \$30-\$50 in Q1 2011 and 2012 in Figure 4).

4.1.3 Hedge price dependent on firm's hedge level

Let's now take a look at the case where the dominant firm's hedge position is observable. We will assume that, for any level of hedging chosen by the dominant firm, arbitrage in the hedge market pushes the hedge price to be equal to the expected future spot price, so that $f = E[p(x)]$. The key difference from the previous case is that, in this case, the dominant firm takes into account the impact of its hedge decision on the forward price, whereas in the previous case the dominant firm chose its level of hedging, taking the hedge price as given.

As we have already noted, we do not consider this case to be a satisfactory model of the hedge market as the hedge position of each generator is a commercial secret and hedge-market traders do not have access to the hedge position of the generators in the market at the time they are making bids and offers. We include this case to highlight the differences in implications and for consistency with the previous research literature. We rewrite expression (1a) using $f = E[p(x)]$ (compare this expression with (6)):

$$E[\pi(x)] = E[(p(x) - c)q(x)] \overset{(5)}{=} E[\frac{\alpha}{4}(q_c - x)(q_c + x)]$$
$$= E[\frac{\alpha}{4}(q_c^2 - x^2)] \tag{9a}$$

This expression has a unique interior maximum at $x = 0$. In other words, a risk-neutral dominant firm facing a no-arbitrage condition, has a unique interior profit-maximizing choice of hedge level, which is to be completely unhedged. This result is consistent with the conclusions in Allaz and Vila (1993a) and Newbery (2008).

4.2 Hedging in a two-stage oligopoly

Proposition 1 makes the stark claim that, in the case of a single dominant generator, the forward market may cease to exist entirely. But we know from casual observation that forward markets do exist in a wide range of commodities. Presumably these markets exist where there is sufficient competition in the spot market. But how much competition is required? Is there a 'tipping point' at which further consolidation might lead to the 'drying up' of the forward market?

To answer these questions we extend the previous analysis to a Cournot oligopoly. We assume a set of n generators. The ith generator is assumed to produce the output q_i. Capacity constraints on these generators are assumed to be not binding. The total output in the wholesale spot market is therefore $q = \sum_{i=1}^{n} q_i$. Each generating firm solves optimization problem (1). The first order optimality conditions of the lower-level problem in this case of oligopoly are as following:

$$\frac{\partial \pi_i(q_i|x_i)}{\partial q_i} = -\alpha q_i - \alpha q + \beta - c_i + \alpha x_i = 0, \quad \forall i. \tag{10}$$

Summing equation (10) over i yields expressions for the total price and quantity as a function of the total hedge level (compare with equation (5)):

$$q(X) = \frac{1}{n+1}(nq_{\bar{c}} + X), \quad p(X) = \frac{1}{n+1}(\beta - \alpha X + n\bar{c}) = \frac{\alpha}{n+1}(q_{\bar{c}} - X) + \bar{c} \tag{11}$$

Here we have used the following notation:

$$X = \sum_{i \in N} x_i, \quad \bar{c} = \sum_{i \in N} c_i/n, \quad q_{\bar{c}} = (\beta - \bar{c})/\alpha \tag{12}$$

In addition, from equation (10) we have:

$$q_i(x_i|X) = q_{c_i} - q(X) + x_i \tag{13}$$

Returning to expression (1a), the profit of generator i in the first stage of the game is therefore (compare with equation (6)):

$$\begin{aligned}
\pi_i(x_i|X) &= (p(X) - c_i)q_i(x_i|X) + (f - p(X))x_i = (p(X) - c_i)(q_i(x_i|X) - x_i) + (f - c_i)x_i \\
&= \frac{1}{\alpha}(p(X) - c_i)^2 + (f - c_i)x_i \\
&= \frac{\alpha}{(n+1)^2}(q_{c_i} - X + \frac{n}{\alpha}(\bar{c} - c_i))^2 + (f - c_i)x_i
\end{aligned} \tag{14}$$

As before, the expected profit is U-shaped in x_i. The profit-maximizing choice of x_i is not in the interior, but at the extremes.

4.2.1 Symmetric Cournot oligopoly

Now let's focus on the specific case of a symmetric oligopoly, where $c_i = \bar{c} = c$. Let's look at the hedging decision of firm i. The expected profit of the ith generator when all the other generators are hedged to $X_{-i} = \sum_{j \neq i} x_j$ is as follows:

$$E[\pi_i(x_i|X_{-i})] = E[\frac{\alpha}{(n+1)^2}(q_c - X_{-i} - x_i)^2] + (f - c)x_i \tag{15}$$

As before, we will assume there are minimum and maximum level of hedging for firm i which we denote $x_i^{min}(X_{-i})$ and $x_i^{max}(X_{-i})$. Firm i will choose the maximum level of hedging if and only if $E[\pi_i(x_i^{max}|X_{-i})] > E[\pi_i(x_i^{min}|X_{-i})]$. Now, we can write the difference in the expected profit in the two cases as follows:

$$E[\pi_i(x_i^{max}|X_{-i})] - E[\pi_i(x_i^{min}|X_{-i})]$$

$$\stackrel{(15)}{=} E[\frac{\alpha}{(n+1)^2}((x_i^{max} + x_i^{min}) - 2(q_c - X_{-i}))(x_i^{max} - x_i^{min})] + (f-c)(x_i^{max} - x_i^{min})$$

$$\stackrel{(11)}{=} \big((f-c) - \frac{1}{n+1}((p(x_i^{max} + X_{-i}) - c) + (p(x_i^{min} + X_{-i}) - c))\big)(x_i^{max} - x_i^{min}) \qquad (16)$$

We can conclude that each firm in the symmetric oligopoly will choose the maximum level of hedging if and only if there is sufficient margin from selling hedges relative to the potential gains from going unhedged (compare with equation (8)):

$$f - c > \frac{(p(x_i^{max} + X_{-i}) - c) + (p(x_i^{min} + X_i) - c)}{n+1}. \qquad (17)$$

As before, we distinguish two cases: where the hedge price depends on firms' hedge level and where it does not:

4.2.2 Symmetric oligopoly: hedge price independent of firms' hedge level

As in Section 4.1.1, let's assume that each generator's hedge position is unobservable so the hedge price in the hedge market (if one exists) is independent of each generator's hedge position. As before we will look for a rational expectations equilibrium in pure strategies. We can prove the following proposition:

Proposition 2. *Under the assumption of linear demand and constant marginal cost, given a set of n identical generators, and assuming that the firms play a Cournot game in quantities in the spot market and play a Cournot game in hedge levels in the hedge market, then there is no* *rational expectations equilibrium in pure strategies in the hedge market unless the number of players in the market n is large enough to satisfy the following condition:*

$$n > \frac{p^{min|max} - c}{p^{max|max} - c}$$

Here $p^{min|max}$ and $p^{max|max}$ are the expected future spot prices when one of the generators chooses the minimum and maximum practical levels of hedging, respectively, and all the other generators choose the maximum practical level of hedging: $p^{min|max} = p(x^{min} + (n-1)x^{max})$ and $p^{max|max} = p(x^{max} + (n-1)x^{max}) = p(nx^{max})$

Proof. We have already established that for each generator the profit-maximising choice of x_i is at the extremes. Let's first look for a rational expectations equilibrium where all generators hedge to the $x^{min|min}$ level (here the $x^{min|min}$ level is the minimum practical hedge level for any one generator when all the other generators also hedge to the minimum practical level). In this case the hedge price must be equal to the resulting expected future spot price, so $f = E[p^{min|min}]$. Let's suppose that all but one of the generators hedges to this minimum level. The remaining generator faces a choice between choosing the same level $x^{min|min}$ or hedging to the maximum practical level (given the minimum hedge position of the other generators) $x^{max|min}$. Let the corresponding expected spot price be $E[p^{min|min}]$ and $E[p^{max|min}]$, respectively[7]. Since the demand curve is downward sloping, $p^{max|min} < p^{min|min}$ so equation (17) is satisfied, so the generator would prefer to hedge to the $x^{max|min}$ level. This contradicts the assumption that hedging to $x^{min|min}$ was an equilibrium.

So let's look for a rational expectations equilibrium where all generators hedge to the maximum practical level $x^{max|max}$ when all the other generators also hedge to the maximum practical level. With this expectation, the forward price would be $f = E[p^{max|max}]$. Using (17), we see that, with this level, each generator has an incentive to choose to hedge to the $x^{max|max}$ level if and only if:

[7] Here $p^{min|min}$ and $p^{max|min}$ refer to the future spot price outcomes when all generators but one hedge to the x^{min} level and the remaining generator hedges to the minimum or maximum level, respectively.

$$E[p^{max|max}]-c > \frac{(E[p^{max|max}]-c)+(E[p^{min|max}]-c)}{n+1}$$
$$\iff n(E[p^{max|max}]-c) > (E[p^{min|max}]-c)$$
$$\iff n > \frac{E[p^{min|max}]-c}{E[p^{max|max}]-c}$$

This expression is more likely to be satisfied the more players there are in the market. ■

We can now answer the question asked at the outset. As expected the greater the level of competition in the market, the more likely it is that the forward market can be sustained. The number of players that are required in order to sustain a forward market equilibrium depends on the degree of market power of the individual generators. If an individual generator is able to increase the expected future spot price margin by up to, say, four times by choosing the minimum practical level of hedging, rather than the maximum practical level, then in the symmetric Cournot equilibrium at least four players in the market are required in order for a forward market equilibrium to exist. A numerical example which shows the number of players required to ensure the forward market exists in a specific context is set out in section 5.

Another corollary of Proposition 2 is both interesting and striking: As long as $p^{min|max}-c$ is positive, a forward equilibrium is less likely to arise the smaller is the expected wholesale price margin $E[p^{max|max}]-c$. If the oligopoly generators are able to collectively push the expected future spot price down close to their common marginal cost, it is very unlikely that a forward market equilibrium will arise. In fact if the expected wholesale price margin $E[p^{max|max}]-c$ is zero or negative, no forward market equilibrium can exist regardless of the level of competition in the market.

Corollary 1. *As long as the expected future spot price margin when an individual generator chooses the low level of hedging $E[p^{min|max}]-c$ is positive, a forward market equilibrium is less likely to arise the smaller the expected future spot price margin when all generators choose the* *high level of hedging $E[p^{max|max}]-c$. If $E[p^{max|max}]-c$ is zero or negative no forward market equilibrium can exist no matter how many competitors compete in the market.*

4.2.3 Symmetric oligopoly: Hedge price dependent on firms' hedge level

We now consider the final case in which the traders are assumed to be able to observe the hedging level of all the generators with market power, so that the hedge price can perfectly track the expected future spot price. Assuming $f = E[p]$ we can write the expected profit of the ith generator as follows:

$$\begin{aligned} E[\pi_i(x_i|X_{-i})] &= E[(p(X)-c)q_i(X,x_i)] \\ &\overset{(5)}{=} E[\frac{\alpha}{(n+1)^2}(q_c-X)(q_c-X+(n+1)x_i)] \end{aligned} \tag{18}$$

Proposition 3. *Suppose we have a symmetric Cournot oligopoly of n capacity-unconstrained firms, facing a capacity-constrained fringe of firms and a linear and uncertain demand, with the property that profit-maximising choice of output is independent of the realisation of demand. Each firm will choose to be hedged the same proportion of a weighted-average of its expected output: $1-1/n$.*

Proof. The expected profit expression (18) has a unique interior maximum since the expected profit has a negative second-order derivative with respect to x_i . The first order condition for x_i can be formulated as follows:

$$\frac{\partial E[\pi_i(x_i|X_{-i})]}{\partial x_i} = E[\frac{\alpha}{(n+1)^2}((q_c-X)n-(q_c-X+(n+1)x_i)] = 0 \tag{19}$$

Which implies:

$$x_i(X) = (n-1)\frac{q_c-X}{n+1} \tag{20}$$

Using expression (13) we have that:

$$q_i(X) = n\frac{q_c - X}{n+1} \tag{21}$$

Combining equations (20) and (21), it follows that in the Nash equilibrium, each firm chooses to be hedged a fixed proportion of a weighted average of its output, with the proportion increasing as the number of firms in the market increases (consistent with Newbery (2008)).

$$x_i(X) = \big(1 - \frac{1}{n}\big)E[q_i(X)]$$

■

4.3 Quadratic cost functions

The derivations in Sections 4.1 and 4.2 have assumed linear cost functions and, therefore, constant marginal costs. However, it is commonly recognized that the cost function in electricity markets is a steep polynomial function, sometimes approximated as quadratic. In this section we extend the previous results for the case of quadratic cost function $C(q_i) = a_i q_i^2 + c_i q_i + b_i$. A decision-making problem of profit-maximizing generating company with quadratic costs can be formulated as follows:

$$\forall i : \begin{cases} \underset{x_i}{\text{maximize}} \quad E[\pi_i(x_i|x_{-i},q)] = E[p(q)q_i - a_i q_i^2 - c_i q_i] - b_i \\ \qquad\qquad + (f - E[p(q)])x_i & \text{(22a)} \\ \text{subject to:} \quad q_i \in \arg \underset{q_i}{\text{maximize}}\ \pi_i(q_i|q_{-i},x) = p(q)q_i - a_i q_i^2 - c_i q_i \\ \qquad\qquad - b_i + (f - p(q))x_i & \text{(22b)} \end{cases}$$

4.3.1 Single firm

Assuming single dominant firm with a quadratic cost function, the optimal lower-level output and the corresponding price, as a function of the hedge level x_i are as follows (compare with equations (3) and (4)):

$$q(x) = \frac{1}{2(\alpha + a)}(\beta - c + \alpha x), \tag{23a}$$

$$p(x) = \frac{\alpha}{2(\alpha + a)}(\beta + c - \alpha x + \frac{2a\beta}{\alpha}) \tag{23b}$$

In the following corollaries we show that the results obtained for the case with linear costs hold in the case of quadratic costs.

Corollary 2. *A single dominant generating firm with quadratic cost function has no pure equilibrium hedging strategy.*

Proof. The firm will choose to hedge to the maximum level if $E[\pi_i(x^{max})] > E[\pi(x^{min})]$. In Subsections 4.4 and 4.5 the expression (24) is simplified to (7). Therefore, Proposition 1 holds for the case of quadratic costs and there is no equilibrium in pure strategies.

4.4 Independent hedge price

Given two hedge levels x_1 and x_2, a single dominant firm with quadratic cost function will choose to hedge to level x_2, iff $E[\pi(x_2)] > E[\pi(x_1)]$. Now:

$$
\begin{aligned}
&\pi(x_2) - \pi(x_1) \\
&= (p_2 - c)(\frac{\beta - p_2}{\alpha}) - a(\frac{\beta - p_2}{\alpha})^2 + (f - p_2)x_2 - (p_1 - c)(\frac{\beta - p_1}{\alpha}) + a(\frac{\beta - p_1}{\alpha})^2 \\
&- (f - p_1)x_1 \\
&= \frac{1}{\alpha}(p_2\beta - p_2^2 - c\beta + cp_2 - p_1\beta + p_1^2 + c\beta - cp_1) + f(x_2 - c_1) - p_2x_2 - p_1x_1 + \\
&a((\frac{\beta - p_1}{\alpha})^2 - (\frac{\beta - p_2}{\alpha})^2) \\
&= \frac{1}{\alpha}(p_2(\beta - p_2 - \alpha x_2 + c) - p_1(\beta - p_1 - \alpha x_1 + c)) + f(x_2 - x_1) + \\
&\frac{a}{\alpha^2}(\beta^2 - 2\beta p_1 + p_1^2 - \beta^2 + 2\beta p_2 + p_2^2) \\
&= \frac{1}{\alpha^2}(p_2(\alpha p_2 + ap_2) - p_1(\alpha p_1 + ap_1)) + f(x_2 - x_1) \\
&= \frac{\alpha + a}{\alpha^2}(p_2 - p_1)(p_2 + p_1) + f(x_2 - x_1) \\
&= \frac{1}{2}(x_2 - x_1)(p_2 + p_1) + f(x_2 - x_1).
\end{aligned} \tag{24}
$$

The final expression is equivalent to the result of (7), therefore Proposition 1 holds for the case of a single dominant firm and quadratic costs.

4.5 Hedge price equals the expected spot price

In the case the hedge price is exactly equal to the expected spot price, profit can be expressed from (22a) using (23a) and (23b):

$$
\begin{aligned}
\pi_i(x) =& p(x)q_i(x) - a_iq_i^2 - c_iq_i - b_i = \frac{1}{4(a_i + \alpha)}(-\alpha^2x_i^2 - 4b_i\alpha + b_i^2 - 2c_i\beta + \beta^2 - 4a_ib_i) = \\
&\frac{1}{4(a_i + \alpha)}(-\alpha^2x_i^2 - 4b_i(a_i + \alpha) + (c_i - \beta)^2).
\end{aligned} \tag{25}
$$

The unique maximum of this expression is, when $x_i = 0$. ■

Corollary 3. *A single dominant firm with quadratic cost function and hedge price exactly equal to the expected spot price has a unique optimal strategy to choose zero hedge cover.*

Proof. The profit expression for the case when hedge price is equal to the spot price is demonstrated in expression (25). It has a unique maximum at $x_i = 0$, which agrees with the previous result for linear cost function. ■

4.5.1 Oligopoly

For completeness we consider the case of quadratic cost for the oligopolistic situation. There is a set of capacity-constrained generators producing the total output K and N unconstrained generators. The first order optimality conditions of the lower-level optimization problem in case of oligopoly are as follows:

$$\frac{\partial \pi_i(q_i|x_i)}{\partial q_i} = -\alpha q_i - \alpha \sum_{i \in N} q_i + \beta - b_i - 2a_i q_i + \alpha x_i = 0, \quad \forall i. \tag{26}$$

First we consider the case, when hedge cover is not observable to the traders and therefore the hedge price f does not depend on the hedge level x_i. Solving a system of these expressions for all firms and deriving the optimal profit expression provides us with the following proposition:

Proposition 4. *In an asymmetric oligopoly with N unconstrained generating firms and unobservable hedge cover, there may exist an equilibrium at which the firms take an all-or-nothing hedging decision.*

Proof. The oligopoly case with quadratic costs presents a more complex case than the one with linear cost functions. We show that the expression corresponding to the coefficient of the quadratic hedging term in the profit formulation of any strategic unconstrained firm i may become negative for sufficiently large coefficients of quadratic cost functions a_i and a high number of generating firms. This means that depending on the value of parameters and the number of competing frms, the equilibrium strategy may be an all-or-nothing solution. The derivation is provided in Subsection 4.6.

4.6 Independent hedge price

Expression (26) can be rewritten in order to express the production level parametrized by the hedging decision of other generators:

$$\alpha q_i + 2a_i q_i + \alpha \sum_{i\in N} q_i = \beta - c_i + \alpha x_i,$$

$$q_i + \frac{\alpha}{2a_i+\alpha}\sum_{i\in N} q_i = \frac{1}{2a_i+\alpha}(\beta - c_i + \alpha x_i),$$

$$\sum_{i\in N} q_i + \sum_{i\in N}\Big(\frac{\alpha}{2a_i+\alpha}\sum_{i\in N} q_i\Big) = \sum_{i\in N}\Big(\frac{1}{2a_i+\alpha}(\beta - c_i + \alpha x_i)\Big),$$

$$\sum_{i\in N} q_i = \frac{\sum_{i\in N}\left(\frac{1}{2a_i+\alpha}(\beta - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)}$$

$$q_i = \frac{1}{2a_i+\alpha}(\beta - c_i + \alpha x_i) - \frac{\alpha}{2a_i+\alpha}\frac{\sum_{i\in N}\left(\frac{1}{2a_i+\alpha}(\beta - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)},$$

$$q_i = \frac{1}{2a_i+\alpha}\Big(\beta - c_i + \alpha x_i - \frac{\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}(\beta - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)}\Big). \tag{27}$$

The corresponding price is:

$$p = \beta - \alpha(\sum_{i\in N} q_i) = \beta - \frac{\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}(\beta - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)}. \tag{28}$$

Combining expressions (27) and (28) we can also write that:

$$q_i = \frac{1}{2a_i+\alpha}(p - c_i + \alpha x_i). \tag{29}$$

Using expressions (28) and (29) in the profit formulation (22a) we find out that the profit

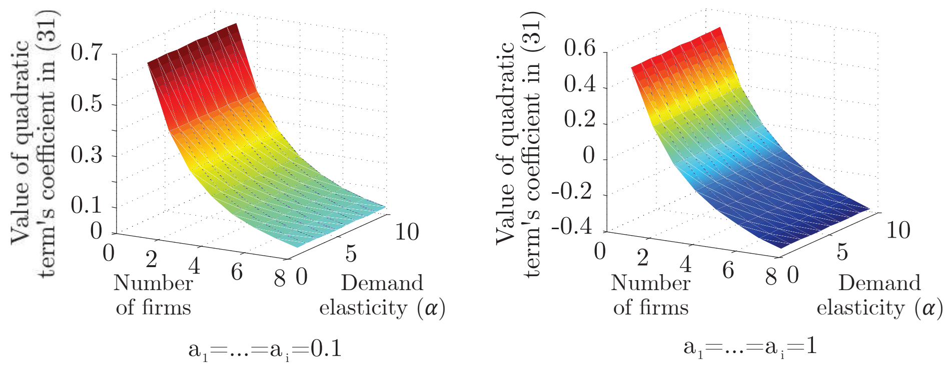

Figure 7: The value of the quadratic term's coefficient in the profit formulation (30) related to a firm in the symmetric oligopoly case

expression is a quadratic function of the hedging decision x_i:

$$
\begin{aligned}
\pi_i =& (p-c_i)q_i - a_i q_i^2 - b_i + f x_i - p x_i = \\
& \frac{p-c_i}{2a_i+\alpha}(p-c_i+\alpha x_i) - \frac{a_i}{(2a_i+\alpha)^2}(p-c_i+\alpha x_i)^2 - b_i + f x_i - p x_i = \\
& \frac{a_i+\alpha}{(2a_i+\alpha)^2}(p-c_i)^2 + \frac{\alpha}{(2a_i+\alpha)^2}(p-c_i)\alpha x_i - \frac{a_i}{(2a_i+\alpha)^2}(\alpha x_i)^2 - b_i + f x_i - p x_i. \quad (30)
\end{aligned}
$$

Some of the coefficients corresponding to the quadratic terms are positive while the sign of some other coefficients is indeterminate.

This means that two situations are possible: (1) the profit function is ∪-shaped and optimal solutions lie on the extremes of the feasible range and (2) the profit function is ∩-shaped, which means that there is a single solution, which might not be all-or-nothing decision. Numerical results shown in Figure 7 confirm that there exist cases, when the coefficient corresponding to the quadratic term in (30) is positive, meaning that market may become illiquid in certain situations.

■

4.7 Hedge price equals the expected spot price

We also consider the case of perfectly observable hedge cover. In this case the forward price f will be exactly equal to the expected market price $E[p]$. We obtain the following proposition:

Proposition 5. *In an asymmetric oligopoly with N unconstrained generating firms and observable hedge cover there may be an equilibrium at which the generating firms will prefer an all-or-nothing solution.*

Proof. We show that profit function is quadratic. The coefficients corresponding to the quadratic terms may or may not be positive, depending on the value of parameters. We conclude that there might exist an equilibrium at which generating firms may choose an all-or-nothing hedging decision.

The profit expression can be rewritten:

$$\pi_i = (p-c_i)q_i - a_i q_i^2 - b_i = \frac{p-c_i}{2a_i+\alpha}(p-c_i+\alpha x_i) - \frac{a_i}{(2a_i+\alpha)^2}(p-c_i+\alpha x_i)^2 - b_i =$$
$$= \frac{1}{2a_i+\alpha}(p-c_i)^2 + \frac{1}{2a_i+\alpha}(p-c_i)\alpha x_i - \frac{a_i}{(2a_i+\alpha)^2}((p-c_i)^2 + 2(p-c_i)(\alpha x_i) +$$
$$(\alpha x_i)^2) - b_i = \frac{a_i+\alpha}{(2a_i+\alpha)^2}(p-c_i)^2 + \frac{\alpha}{(2a_i+\alpha)^2}(p-c_i)(\alpha x_i) - \frac{a_i}{(2a_i+\alpha)^2}(\alpha x_i)^2 - b_i.$$

As before the profit function is quadratic in the hedging decision. We again encounter the situation, where there might be all-or-nothing solution, depending on the value of the coefficients. ∎

5 Numerical results

In this section we illustrate the closed-form results obtained in Section 4 using numerical simulations.

5.1 Hedging decision of a single dominant generating company

We have seen in the previous section that profit-maximizing company faces an all-or-nothing hedging decision. Assume a company with marginal costs $c = 5$ \$/MW, facing a decision to hedge with the forward price $f = 5$ \$/MW. The demand is characterized by elasticity $\alpha = 1$ \$/MW^2 and demand intercept $\beta = 10$ \$/MW.

The expected profit of the generator with respect to the hedging decision (from equation (6)) is presented in Figure 8(a). Assuming that there is a range of available hedging decisions in $[x^{min}, x^{max}] = [0, 10]$ interval[8], the generator is facing a symmetric all-or-nothing decision. The values maximizing the expected profit lie on the borders of the available range of forward contracts $[x^{min}, x^{max}]$.

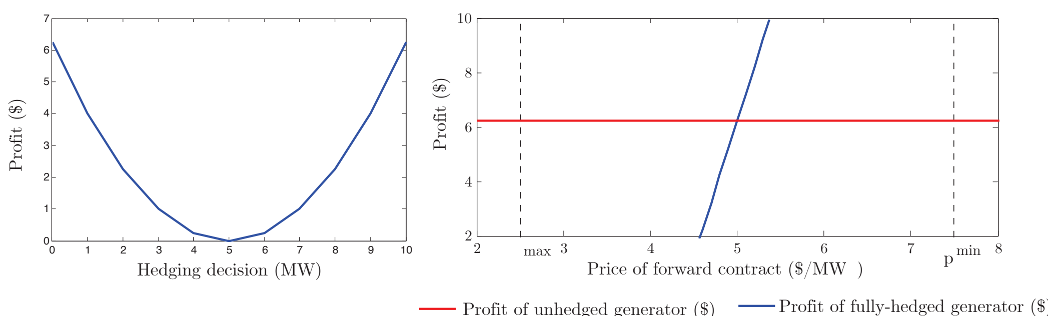

Figure 8: The expected profit of the generating firm as a function of its hedging decision: hedge cover is confidential information.

In Figure 8(b) we plot the expected profit of the generator as it chooses all-or-nothing hedging decisions for different prices of forward contracts. We can directly observe that the all-hedging decision becomes more profitable when the price of forward contracts exceeds the average of the prices in case of minimum and maximum hedging (p^{min} and p^{max}) i.e. the mid-point of the graph. The expected profit in the no-hedging case is independent of the forward price f.

For completeness of the discussion in Figure 9, we plot the expected profit of the same dominant company for the situation where the generator's choice of hedge cover is publicly-

[8] This corresponds to production in the range 2.5-7.5 MW, reflecting the assumed physical minimum and maximum production of this generator.

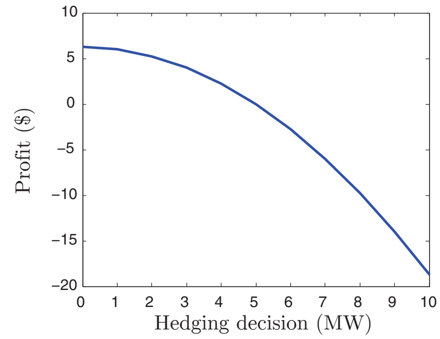

Figure 9: The expected profit of the generating firm as a function of its hedging decision: hedge cover is public information.

available information. In that case the price of the forward contract is set equal to the expected spot price. We see that in contrast with Figure 8 there is only one profit-maximising choice of hedge level, corresponding to the case where the hedge cover is zero.

5.2 Hedging decision in a symmetric oligopoly

Now we look at a case of symmetric oligopoly with two companies with marginal costs $c = 5$ \$/MWh, facing the same demand function as in Section 5.1. As before we will look for a rational-expectations equilibrium.

Let's start by assuming that there is a symmetric rational-expectations equilibrium in which both firms are hedged to a symmetric minimum level. If, as above, we assume that each generator has a feasible range of output $[2.5, 7.5]$ MW, it follows that the symmetric minimum hedge level is at $x_1 = x_2 = 2.5$ MW . Let's therefore assume that firm 2 is hedged to $x_2 = 2.5$ MW. The feasible practical hedge range for firm 1 is then $[2.5, 10]$ MW.

We know from equation (15) that the optimal choice of firm 1 is a hedge level at the ends of the practical range. From equation (16) we know that firm 1 will choose the minimum level of hedging if the hedge price is low enough. In this case, given that firm 2 is hedged to 2.5, the expected price if firm 1 hedges to the minimum $x_1 = 2.5$ MW is $p^{min} = 3.3$ \$/MW and the expected price if firm 1 is hedged to the maximum $x_1 = 10.0$ MW is $p^{max} = 0.8$ \$/MW, so by equation (16), firm 1 will choose to be hedged to the minimum level if and only if the forward price is below 3.06 \$/MW. But if the forward price is below 3.06 \$/MW, firm 1 chooses to be hedged to the minimum level $x_1 = 2.5$ MW so the expected future spot price is 3.3 \$/MW, contradicting the assumption that there is a rational-expectations equilibrium.

Now let's assume that there is a symmetric rational-expectations equilibrium in which both firms are fully hedged. The symmetric equilibrium in which both firms are fully hedged is where both firms choose the hedge level $x_1 = x_2 = 17.5$ MW. Let's suppose firm 2 chooses to hedge to $x_2 = 17.5$ MW. The feasible hedge range for firm 1 is $[10, 17.5]$ MW. If firm 1 chooses the lower end of this range, the expected price is $p^{min} = -4.2$ \$/MW. If firm 1 chooses the upper end of this range the expected price is $p^{max} = -6.7$ \$/MW. From equation (16), firm 1 will choose to be hedged to the maximum level if and only if the forward price is above -1.94 \$/MW. But if both firms hedge to the maximum level, the expected future spot price is -6.7 \$/MW, contradicting the assumption that there is a rational-expectations equilibrium. So, there is no equilibrium in the forward market in this Cournot duopoly. We could ask how much competition is required in the market before the forward can be sustained?

5.3 How much competition is required for the forward market to exist?

In this example, we assume a set of symmetric generators which are participating in the hedge and spot market. The demand curve in this example is held fixed, with an an intercept of $\beta = 3000$ \$ and slope of $\alpha = -2$ $\$/MW^2$. We carry out the following thought experiment: We start with one (monopoly) generator with a marginal cost of 20 $\$/MW$, capacity of 1450 MW and minimum production level of 500 MW. In the subsequent steps we divide this monopoly generator into a larger and larger number of smaller competing generators. In the case where there are n generators, the capacity and minimum production level is the proportion $\frac{1}{n}$ of the original monopoly generator (with a capacity of 1450 MW and minimum production of 500 MW). This approach allows us to focus exclusively on the impact of increasing the level of competition in the market, without otherwise affecting supply and demand.

In this problem, the common maximum level of hedging in each case is the level of hedging which induces each generator to produce at its total capacity; the common minimum level of hedging is the level which induces each generator to produce at its minimum level of production. We know from the previous analysis that there is no rational expectations equilibrium in which all generators hedge to the common minimum level, so let's look for an equilibrium in which each generator hedges to the maximum level of hedging. If every generator hedges to the maximum level, each generator produces at its total output and the total production in the market is 1450 MW (by definition), and the wholesale spot price is $\$100/MW$. If every generator but one hedges to the maximum level, the remaining generator can choose a hedge level $x^{min|max}$ which induces it to hedge to its own minimum production level. This results in a total level of output in the market equal to $(1450(n-1)+500)/n$ and a corresponding spot price of $100+3800/n$.

From equation (17) we know that there is an equilibrium in which every generator hedges to the maximum level if and only if the expected future spot price when all generators hedge to the maximum level is above a critical level given by:

$$f^{crit} = c + \frac{(E[p^{max|max}] - c) + (E[p^{min|max}] - c)}{n+1}$$

Figure 10 shows (a) the expected price when all generators hedge to the maximum level $E[p^{max|max}]$, (b) the expected price when one generator hedges to the minimum level and all other generators hedge to the maximum level $E[p^{min|max}]$, and (c) the critical value for the forward price f^{crit}. A rational expectations equilibrium in the forward market can only exist when the expected future spot price $E[p^{max|max}]$ is below the critical value f^{crit} which, in this case, requires six or more competing generators in the market.

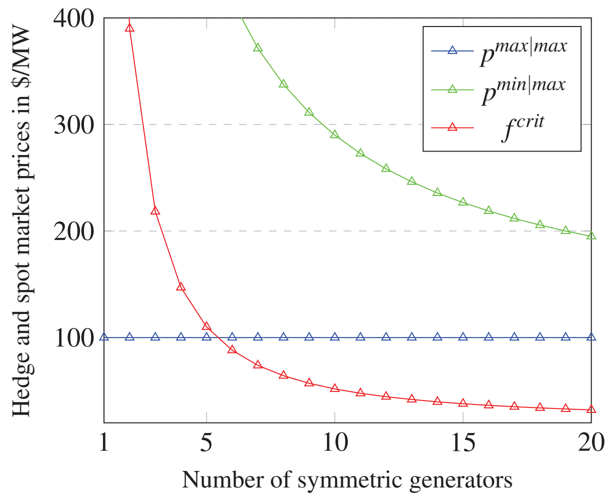

Figure 10: Illustration of the number of competing generators required to sustain equilibrium in the forward market

6 Future work: hedging decision of a single firm with wind power uncertainty

In Sections 4 and 5 we have looked at a case of symmetric probability distribution function of demand parameters, when the exact realization is not affecting the optimal hedging decision. However, power systems are increasingly being affected by the integration of renewable energy sources, wind power in particular, with a substantial part being active on the demand-side as embedded generation. The share of demand covered by wind energy in European Union has reached 10.4% in 2016 (Nghiem and Mbistrova, 2017). Since demand has a direct relationship with market price (Coulon et al., 2013), it is important to account for the statistical characteristics of wind power, when devising an optimal hedging strategy. It is also suggested in Bunn and Chen (2013) that wind power brings radical market structure changes and, therefore, has to be accounted for in the hedging decision analysis.

In this section we indicate the direction for extending the work proposed in this paper to the case of wind-integrated power systems. The authors in Bessembinder and Lemmon (2002) demonstrate how the optimal hedging decision of a generating firm depends on forecast output and on the skewness of power demand. The authors consider high order polynomial cost functions. This implies that marginal production costs increase very fast in demand, reflecting the fact that industry employs an array of production technologies and fuel sources from hydro to natural gas. In this section we consider the skewness of the demand, originating from the wind power integration.

We assume that the electricity market has high share of wind generation with the mean value covering 20% of the demand. Wind power generation can be represented by beta distribution (Morales et al., 2013) with mean value $\beta^{wind} = 2.5$ \$/MW subtracted from the net demand (Biggar and Hesamzadeh, 2014). The profit calculation is presented in Figure 11. System demand is partially covered by wind generation, having a beta distribution. Accordingly, net demand also has beta-distribution as shown in the top plot in Figure 11. This makes the profit-maximizing hedging decision of generator asymmetric. In the bottom plot in Figure 11 we observe that with high wind integration, the generator obtains higher profits by going to the spot market without hedge cover. This effect can be attributed to the shape of the beta distribution function setting higher probability to lower production outcomes of wind generation.

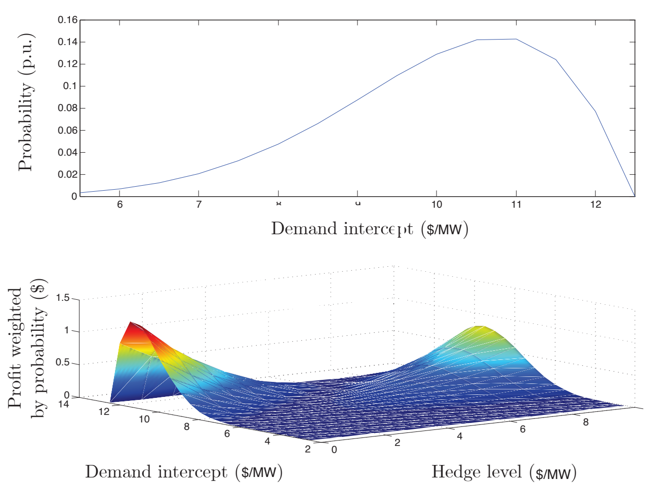

Figure 11: The expected profit of the generating firm as a function of its hedging decision and demand intercept (demand intercept has beta distribution).

7 Conclusion

The decision of the appropriate level of hedging for a generator with market power is a crucial operational decision. The existing research on the equilibrium level of hedging in an electricity market with risk neutral players has assumed that hedge prices are equal to expected future spot prices. We argue that this requires detailed knowledge of the hedge position of dominant firms, and that this does not generally hold in the forward markets. This transparency of information on contract positions is unlikely to be available to hedge traders. As a consequence, we argue that it is more appropriate and important to assume that hedge prices are independent of hedging decisions. We show that under this assumption, in a market dominated by a firm with a high degree of market power there may be *no* equilibrium hedge position in pure strategies. Instead, the hedging decision of a dominant generator will be all-or-nothing, depending on the price at which the dominant generator can sell its hedge contracts. We therefore suggest that electricity wholesale markets featuring a dominant generator are prone to a lack of liquidity in the hedge market and potentially volatile wholesale spot and forward prices. This intuition reflects the pattern of outcomes observed in the South Australian region of the Australian National Electricity Market.

The model also shows that as the number of firms in the market increases, there arises an equilibrium in which all the firms choose to be fully hedged. Our results hold when we consider the cases of linear and quadratic cost functions. We study the more general case of asymmetric oligopolies numerically. We arrive at similar conclusions as for the symmetric case (i.e. the generators may choose to prefer hedging, if the price of the forward contract is sufficiently high). An extension includes the effect of market structure on competition, which we model using conjectured price response. A further extension, indicated as a future work, includes the specific characteristics of uncertainty, e.g. wind power, on hedging decision of the generator. Our preliminary results show that the skewness in the probability distribution function of wind power production makes the hedging decision of generators tend towards zero hedge cover.

In this research we assumed the risk neutrality of market participants. We have looked at hedging from a strategic perspective, and not as a consequence of risk aversion. Evidently, in many corporate contexts, risk management requirements will direct adequate hedge cover according to Value-at-Risk or other compliance metrics. Also, the need to report adequate hedging to investors at periodic intervals is often an important part of managing investor relations in large corporations. All of these risk-induced motivations for hedging will moderate the conclusions of our research. Nevertheless, the analysis presented here involves a new consideration of the effect of a lack of transparency in the analysis of strategic forward trading, which would appear to be very relevant in practice. It provides new insights on how particular companies with market power may operate their supply chains with and without hedges and what this may mean for market prices. In general, it raises a policy and regulatory question about greater transparency in forward markets and how that might be achieved.

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