COMPDYN 2019
7th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Crete, Greece, 24–26 June 2019

ON THE EQUAL DISPLACEMENT APROXIMATION FOR MID-RISE REINFORCED CONCRETE BUILDINGS

Yeudy F. Vargas[1], Luís G. Pujades[1], José R. González-Drigo[1], Rodrigo E. Alva[1] and Luis A. Pinzón[1]

[1]Universitat Politècnica de Catalunya, Dep. of Civil and Environmental Engineering
C. Jordi Girona 1-3, Campus Nord UPC, Barcelona, Spain
{yeudy.felipe.vargas, lluis.pujades, jose.ramon.gonzalez, rodrigo.esteban.alva, luis.pinzon}@upc.edu

Abstract

The equal displacement approximation is a well-known procedure for estimating the non-linear behavior of structures subjected to earthquake ground motions. This procedure plays a significant role in current seismic design, since it constitutes the basic assumption for defining strength reduction factors. In this paper, calculation of the performance point based on this rule is used to estimate engineering demand parameters such as those obtained by advanced probabilistic non-linear dynamic analysis, NLDA. We present a modification to the classic approach, to improve the predictability of the equal displacement rule. Uncertainties in seismic action and structural properties are considered. Mid-rise reinforced concrete buildings will be used as a testbed. To obtain a representative sample of buildings for statistical analysis, we describe the development through implementation of a numerical tool for calculating probabilistic NLDA. This tool, which is expected to evolve into interoperable software for assessing the seismic risk of structures, is developed within the framework of the KaIROS project. The results presented in this paper could be used to estimate the seismic risk of structures in a very simplified manner.

Keywords: equal displacement approximation, probabilistic non-linear dynamic analysis, mid-rise reinforced concrete buildings, KaIROS project

1 INTRODUCTION

Disaster risk reduction is a major concern of world communities. The 2030 Agenda for Sustainable Development includes 17 goals to end poverty, fight inequality and injustice and tackle climate change. The eleventh goal is *sustainable cities and communities*, as half of humanity now lives in urban areas and almost 60 per cent will live in cities by 2030. Thus, an important target for 2030 is to ensure universal access to basic services and to adequate, safe, affordable housing. Achievement of this target is at risk if the negative impact of catastrophes is not reduced. The importance of resilient communities has been shown in past and recent earthquakes. However, the negative impact of these catastrophic events on mankind is increasing due to globalization. Sustainability, resilience and well-being are affected negatively at global level. Accordingly, a continuous effort to reduce seismic risk is fundamental to develop a stronger society.

Seismic loss mainly depends on the capacity of civil structures to withstand strong ground motions. Thus, the development of advanced numerical tools for assessing the seismic response of civil structures will contribute positively to the planning of optimal strategies for reducing seismic risk. Currently, several research projects are focused on reducing seismic risk. One is the Kairos project [1], which is aimed at maintaining and increasing the resilience and sustainability of communities against earthquakes. One research area in the Kairos project is the development of numerical tools to estimate the seismic risk of structures. In this paper, we present the development of one of the tools. To correctly estimate seismic risk scenarios, uncertainties in seismic hazard action and in the main features of the structures should be considered. The numerical tool presented herein takes into account uncertainties relating to the geometry of the structure, the mechanical properties of the materials and the seismic action, amongst many others. Moreover, the tool can be used to estimate several engineering demand parameters (EDP), which can be related to seismic damage of structures. The tool is developed through implementation of a hypothetical case study. To achieve this, the probabilistic seismic response of mid-rise reinforced concrete buildings is calculated via NLDA. A thousand numerical models were created and subjected to a set of earthquake records with different properties. The numerical tool, combined with information on variables characterizing the exposure of an urban environment, will help to estimate the seismic risk of the area precisely. However, the main target of the paper was to develop a procedure based on the equal approximation rule for estimating EDPs in a very simple way. Due to the amount of numerical data available, this simplified procedure could be developed easily based on a maximum correlation criterion.

2 PROBABILISTIC CONSIDERATIONS

The main factors affecting earthquake risk are hazard, exposure and vulnerability. Hazard refers to seismic actions and their occurrence probabilities. Probabilistic seismic hazard analysis [2] is estimated based on past seismicity, statistical models, ground motion prediction equations and site effects. Exposure concerns structures, facilities and properties in the stricken area. The quantification of exposure requires a considerable amount of data on facilities, population and built environment, including special and essential buildings and lifelines. Vulnerability is related to susceptibility to damage of exposed goods. It connects hazard and exposure to obtain risk, that is, expected damage and cost. Regarding exposure, it has been a common practice to classify structures with similar features within a structural class. For instance, buildings are often classified as low-rise (1-3 stories), mid-rise (4-7 stories) and high-rise (>7 stories). Of course, many other features are used to identify a structural class. Generally, the seismic behavior of buildings in the same structural class is represented using probabilistic functions. This classification within structural classes simplifies the characterization of exposure when the seismic risk is estimated at urban level. The finer the characterization of the exposure, the more precise is the quantification of the seismic risk. Thus, inventory is a critical issue in risk assessments, and geographical information systems (GIS) can be used to enhance characterization at urban scale. Thus, if precise inventory information is available, it should be included when exposure is modelled.

2.1 Characterizing exposure

It is important to distinguish between two types of random variables when exposure is characterized: those that refer to the population of buildings, for example, in a neighborhood, district or city, and those that refer to intrinsic properties of a building, which can be considered epistemic uncertainty. Consideration of these types of random variables depends on whether the intended application is the classes of structures considered in urban risk assessment or individual buildings. Modeling the random variation in building-to-building structural characteristics within a structural class is standard practice. This modelling must reflect the epistemic uncertainty and how many structural types are grouped together into a single class.

2.2 Building-to-building variation

The main variables that characterize buildings in a structural class are random. In this section, we explain how building-to-building random variation will be considered within a structural class. Buildings belonging to the structural class 'reinforced concrete mid-rise buildings' are used as a testbed. Through implementation of this simulation, a numerical tool will be created that allows consideration of specific distributions within a structural class. Several variables that characterize a building are considered as input random variables. These variables will be the input of the software to be developed, once the simulation is achieved. In real cases, characterization of these random variables strongly depends on the information stored by local institutions. This information can be enhanced using GIS tools. For the purpose of the present study, the distribution of these variables will be assumed, and it will be mainly uniform or Gaussian. Thus, the number of stories, N_{st}, the number of spans, N_{sp}, the story height, H_{st} and the span length, S_l, are considered as the input random variables. N_{st} follows a uniform, discrete distribution in the interval (4, 7); N_v also follows a uniform, discrete distribution in the interval (2, 8). H_{st} is distributed uniformly in the interval (2.8, 3.2) m. S_l is distributed uniformly in the interval (4, 6) m. Functions based on the design of various hypothetical structures belonging to the structural class are used to assign the cross area to the structural elements. These models were designed by supposing that they are located in a moderate seismic area. Based on the results, several functions are developed. Thus, the width, W_c, and depth, D_c, of the columns of the first story will depend on the number of stories of the building and will be calculated using the following equation:

$$W_c \ or \ D_c = c_1 * \ln(N_{st} - c_2) + c_3 * \Phi_{1,0} + c_4 \tag{1}$$

where c_n are coefficients that could be adjusted depending on the data distribution of the analyzed area. For the study, $c_1 = 0.15$, $c_2 = 3$, $c_3 = 0.02$ and $c_4 = 0.35$. $\Phi_{1,0}$ is the standard normal distribution. Note that the columns are not necessarily square, that is, one random sample is generated for the width and one for the depth of the columns according to Equation 1. Moreover, the values generated are rounded to the nearest multiple of 5 cm. Figure 1a shows the size of the columns in the first story and Figure 1b shows the cross-sectional area. For the upper stories, the size of the columns will decrease systematically by 5 cm every two stories. To assign the steel percentage of the columns, ρ_c, a continuous Gaussian distribution is assumed. The mean value is 1.5% and the standard deviation is 0.15%. The width of the beams will also depend on the number of stories of the building model and will be calculated using the following equation:

$$W_b = b_1 * N_{st} + b_2 \tag{2}$$

where b_n are again coefficients that depend on the data distribution of the urban area. For the hypothetical case of study, $b_1 = 0.0053$ and $b_2 = 0.2947$. The depth of the beams will not only depend on the number of stories but also on the span length. The following equation has been used to calculate the depth of the beams:

$$D_b = g_1 * N_{st} + g_2 * S_l + g_3 \tag{3}$$

where g_1 = 0.0157, g_2 = 0.075 and g_3 = -0.0157. A random term was not considered for the beams. Either the coefficients, or the type of equation considered, can be modified so that they better represent the urban environment under consideration. Figure 2a shows the width of the beams as a function of the number of stories and Figure 2b depicts the depth of the beams as a function of the number of stories and the span length. To assign the steel percentage of the beams, ρ_b, a continuous Gaussian distribution is assumed whose mean value is 1% and standard deviation is 0.1%.

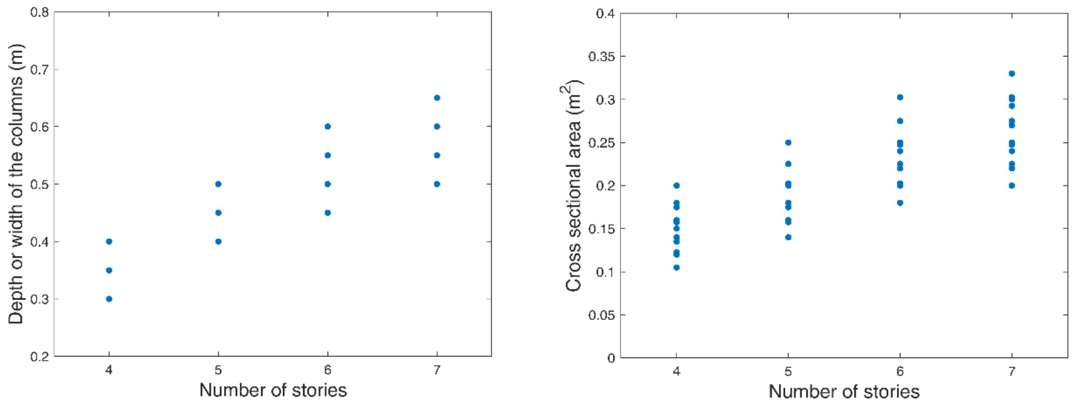

Figure 1 a) Depth and width of the columns of the first story and b) Cross-sectional area

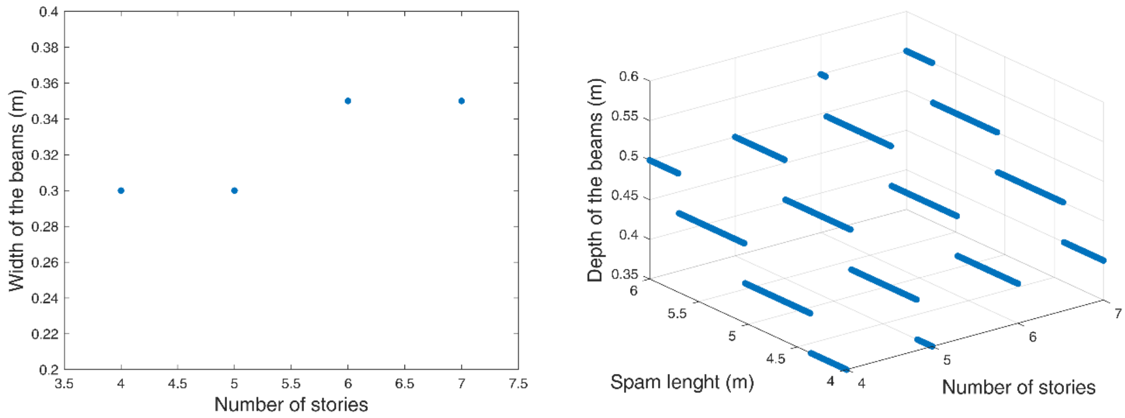

Figure 2 a) Width of the beams as a function of the number of stories and b) Depth of the beams as a function of the number of stories and the span length

2.3 Epistemic uncertainty

Several random variables should be considered when the behavior of a single building is modelled. Amongst many other variables, the mechanical properties of the materials, the loads acting on the structure and the participation of non-structural elements introduce epistemic uncertainty to the system. In this paper, the live loads, *LL*, the superimposed loads, *SL*, the compressive strength of the concrete, *fc*, the tensile strength of the steel, *fy*, the elastic modulus of the concrete, *Ec*, and the elastic modulus of the steel, *Es*, are considered random variables. A continuous Gaussian distribution is assumed for these variables. The mean values, μ, and standard deviations, σ, are summarized in Table 1.

Variable	μ	σ
LL (kPa)	1	0.15
SL (kPa)	2	0.3
fc (kPa)	2.1e4	2.1e3
fy (kPa)	4.2e6	4.2e5
Ec (kPa)	2e7	2e6
Es (kPa)	2e8	2e7

Table 1 Mean and standard deviation of the random variables

2.4 Monte Carlo simulation

The Monte Carlo method is a relatively modern technique that allows modelling of complex systems with a large number of random parameters. This method has been used to generate 1000 random samples of buildings, according to the established distributions described above. An algorithm was implemented with MATLAB to this end. This algorithm will be part of the source code of the software under development. Figure 3 shows ten building samples generated with this algorithm. In this figure, we can see how the geometrical properties of the models vary within the intervals considered.

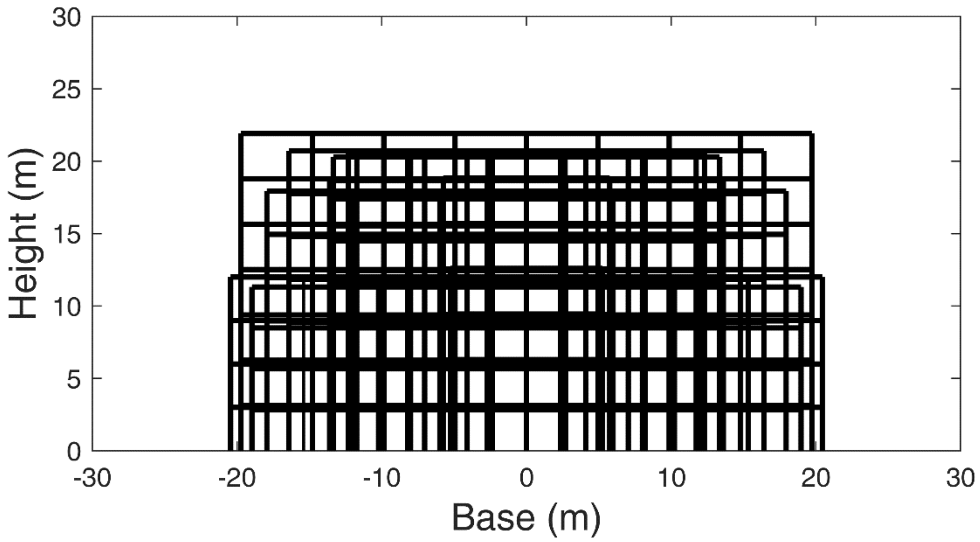

Figure 3 Building samples generated by the code

One question at this point relates to the validity of the models that are generated: do they properly describe the behavior of real structures? This question can be answered by comparing some properties of the models with a physical property of the real structures. For instance, Figure 4a shows a comparison between the fundamental period of the simulated models and those measured in real reinforced concrete buildings [3]. The fundamental periods of the buildings that are generated agree with those measured on real structures. Moreover, if the fundamental periods are tabulated into a histogram (Figure 4b) one can observe that the values are in agreement with those expected, according to the number of stories.

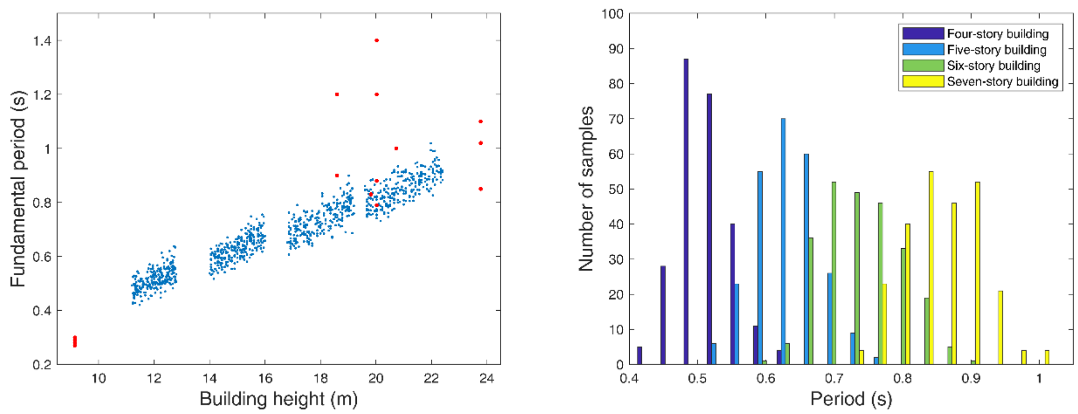

Figure 4 a) Variation in the fundamental period as a function of the building height and b) Histogram of the fundamental periods of the structural models that are generated

3 SEISMIC HAZARD

One of the main important sources of uncertainty in estimations of the seismic risk of structures is the random variability of the ground motion. There are several methodologies to properly select ground motion records that are consistent with the site-dependent spectral shape. However, the main objective of this paper is not to assess the seismic risk of an area, but to develop a simplified procedure for estimating EDPs, like those obtained with NLDAs. With this objective in mind, the most important requirement is to have earthquakes that demand the structural models at different intensity levels. To achieve this, based on the fundamental period range of the generated models (see Figure 4b), groups of earthquakes are selected whose mean spectral acceleration in the interval (0.4-1) s is between a stripe limited by two intensity levels. The intensity levels defining the upper and lower limits of each stripe range from 0 to 1.5 g at intervals of 0.15 g. The objective is to obtain 1000 records (as many as structural models generated) whereby 100 earthquake records per interval should be found. The database of Ambraseys et al. [4] is considered for the earthquake selection. Figure 5 shows 100 earthquake records (approximately 10 per interval) selected according to the procedure described.

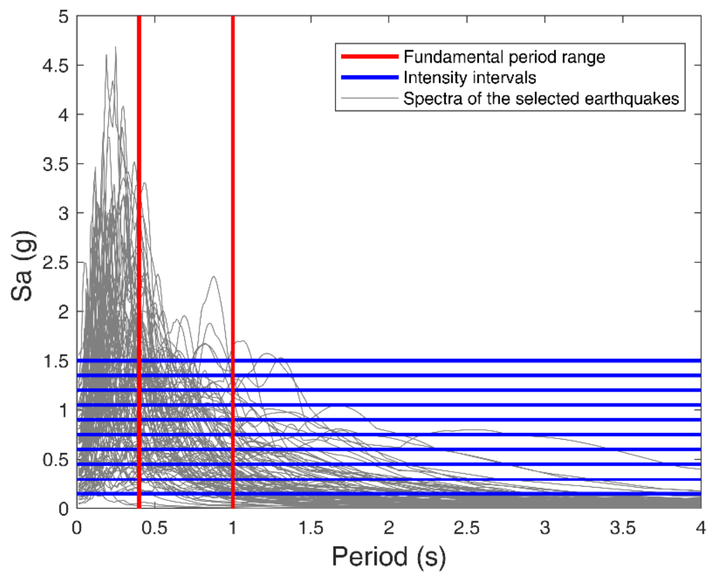

Figure 5 Response spectra of the selected earthquakes

However, because of the high number of earthquakes requested, several will be scaled versions of the ones that naturally fulfil the interval condition. Thus, if an interval does not contain 100 earthquakes records, because there are not enough records within the database, the number of missing records will be selected from the previous interval. This criterion avoids excessive scaling of the earthquake records.

4 NON-LINEAR DYNAMIC ANALYSIS

An NLDA allows simulation of the time history response of a structure, which enters or does not enter the nonlinear range when it is subjected to an earthquake. This analysis allows calculating EDPs such as the maximum displacement at the roof, δ_{roof}, the maximum interstory drift ratio (MIDR) and damage indices, among others. A key issue when an NLDA is performed is the hysteresis law assigned to the structural elements. The modified Takeda hysteresis law [5] has been used to perform the simulations. In-cycle strength degradation has also been considered. The yielding surfaces are defined by the bending moment-axial load interaction diagram for columns and bending moment-curvature for beams. Based on these modeling assumptions, the 1000 NLDA are performed. Ruaumoko software has been used to calculate the structural analyses [6]. Figures 6a and 6b show the histograms of δ_{roof} and MIDR, respectively. These EDP are commonly used in several methodologies to assess the seismic risk of structures. The maximum global drift ratio (MGDR), obtained as the ratio between the δ_{roof} and the height of the building, is also considered as EDP. Note that the MIDR is the envelope of the MGDR, as can be seen in Figure 7. Another important aspect of the relationship between these two EDP is their high correlation, which indicates that if one of them is known an accepted estimation of the other can be made. Pearson's linear correlation coefficient is also presented in Figure 7. This coefficient will always be used in this paper to measure the correlation between two variables. Although NLDA requires a high computational effort, it should be the reference procedure to correctly estimate the seismic risk of structures. Nevertheless, it would be of practical interest to have a simpler methodology to obtain similar results to those based on NLDA.

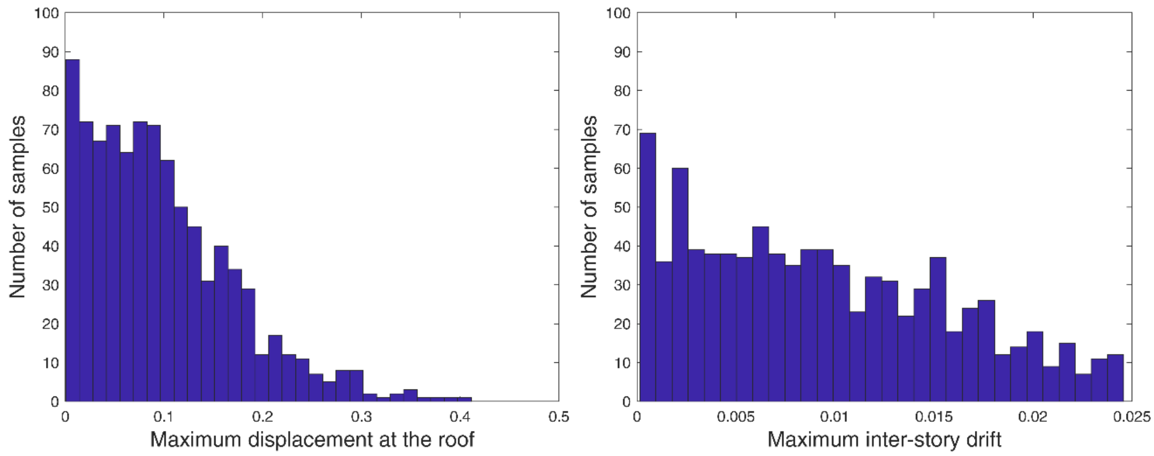

Figure 6 a) Histogram of $\boldsymbol{\delta_{roof}}$ and b) histogram of the MIDR (NLDA)

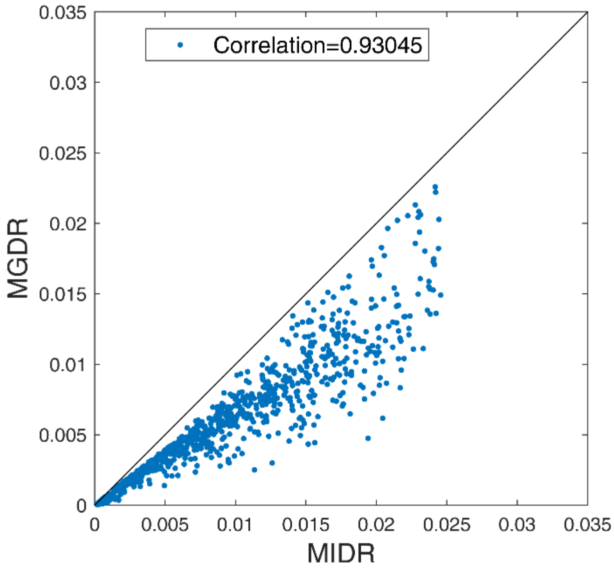

Figure 7 MGDR vs. MIDR

5 EDP ESTIMATION BASED ON THE EQUAL DISPLACEMENT APPROXIMATION

The most simplified assumption, and one of the most commonly used in practice, is to estimate the expected seismic response of the structure based on the equal displacement approximation, EDA. EDA is a well-known empirical rule for the assessment of non-linear behavior of structures subjected to earthquake ground motions. This procedure states that the predicted inelastic displacement response of oscillators is often very similar to the predicted displacement response of elastic oscillators with the same period. Noticeably, oscillators with short periods of less than approximately 0.5 seconds are often significantly larger than the predicted response of elastic structures of the same period. Accordingly, most of the structures analyzed in this paper meet the requirements for a good prediction based on this simplified rule. The displacement calculated using EDA, commonly known as a performance point, corresponds to a single-degree-of-freedom (SDOF) approximation of the structure; if the δ_{roof} of the represented multi-degree-of-freedom system (MDOF) is the target, the spectral displacement should be multiplied by the participation load factor [7], *PF1*, as shown in Equation 4:

$$\delta_{roof,EDA} = PF1 * Sd\left(T_f\right) \tag{4}$$

where T_f is the fundamental period of the structure. In this way, after applying the EDA rule to the structural models described above, and factoring the results by *PF1*, the δ_{roof} based on EDA is obtained. Figure 8a shows the comparison between the δ_{roof} obtained using NLDA and EDA methodologies. The dispersion increases as the displacement rises. The EDA rule allows the δ_{roof} of a building to be estimated, but the MIDR cannot be directly estimated based on this approximation. However, an MGDR based on EDA can be calculated by dividing the δ_{roof} by the height of the building, *H*. According to Figure **7**, there is a high correlation between MIDR and MGDR. Consequently, it is expected that this correlation also exists between these variables when EDA is applied. Thus, the following equation can be used to estimate the MIDR from the $\delta_{roof,EDA}$:

$$MIDR_{EDA} = \alpha * \frac{\delta_{roof,EDA}}{H} \quad (5)$$

where α is a coefficient intended to minimize the mean square error between the MIDR drift based on NLDA and the MIDR based on EDA. It was found that α=1.3675. Figure **8**b shows this comparison. Noticeably, the correlation between these variables is similar to that observed when δ_{roof} (Figure 8a) is analyzed. Nevertheless, the EDA rule does not consider, explicitly, either the higher mode response or the structural period elongation because of the accumulation of damage.

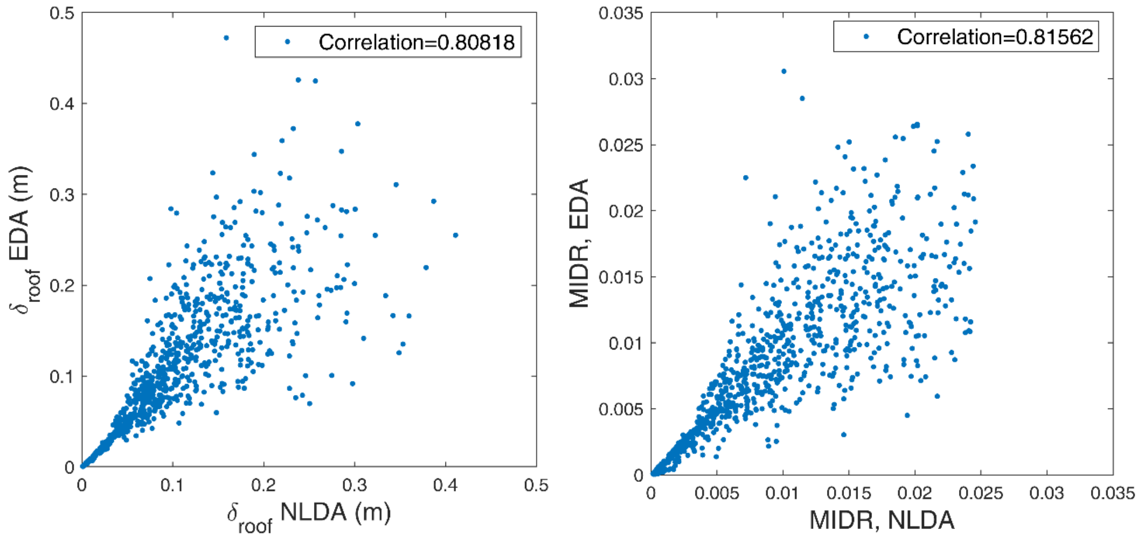

Figure 8 a) Comparison between the $\boldsymbol{\delta_{roof}}$ via NLDA and EDA and b) MIDR comparison obtained via NLDA and EDA

5.1 Modification to the EDA rule

When a structure undergoes inelastic deformation due to a dynamic action, a change of in its vibrational elastic properties is expected, whereby the participation of periods that are longer than the elastic ones becomes important. Moreover, depending on the mass participation factor of higher modes of the structure and the frequency content of the dynamic action, the contribution of these modes can become significantly high, even in low-rise buildings. Neither of these effects are considered explicitly by the EDA rule. More advanced methodologies, also based on the principles of the capacity spectrum method, e.g. FEMA 440 [8], allow such effects to be considered. Nevertheless, these methodologies are intended for spectral shapes that are smooth, and they can provide multiple solutions when real earthquakes are considered. If earthquakes are selected to fit a smooth spectral shape, such as a design or a uniform hazard spectrum, these methodologies provide very good results [9]. The most important aspect of the EDA, at least in our opinion, is its ease of use. Thus, a modification of the EDA rule, aimed at maintaining its simplicity, is presented. The new performance point will be calculated not as the response of the oscillator related to the fundamental period but as the average spectral displacement of several SDOF systems. Therefore, the term *Sd* from Equation 4 becomes the average spectral displacement of several oscillators as follows:

$$\delta_{roof,MEDA} = PF1 * \frac{\sum_{i=1}^{n} Sd(T_i)}{n} \quad (6)$$

Of course, the periods of these oscillators should consider softening of the structure due to the accumulation of damage and the participation of higher modes. These periods will be equally spaced, with an interval of 0.01 s, in the $\beta_{inf}\cdot T_f$ to $\beta_{sup}\cdot T_f$ range. T_f represents the fundamental period of the structure. Moreover, it is expected that β_{inf} <=1 and β_{sup}>=1. We designed an algorithm based on Monte Carlo to find the coefficients β_{inf} and β_{sup} that minimize the mean square error between the δ_{roof} obtained via NLDA and the modified EDA. It has been found that β_{inf}= 0.1 and β_{sup}= 1.8 are the coefficients that fulfil the minimization condition. Figure 9a shows the comparison between the δ_{roof} obtained using NLDA and the modified EDA. A significant increase in the correlation can be seen after the applied modification. Then, to calculate the MIDR based on the modified EDA, the MGDR based on EDA are recalculated and a new α coefficient is obtained. In this case, it has been found that α=1.6976. Figure 9b shows the comparison between the MIDR using NLDA and the modified EDA. Again, the correlation between these variables is similar to that obtained for the δ_{roof} comparison (Figure 9a).

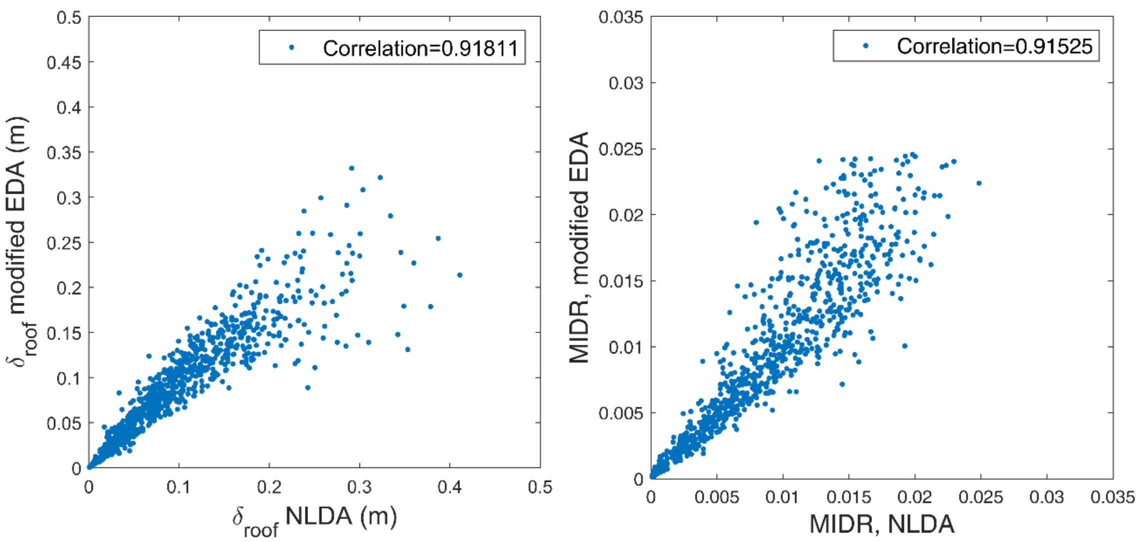

Figure 9 a) Comparison between the maxima $\boldsymbol{\delta_{roof}}$ via NLDA and modified EDA and b) MIDR comparison obtained via NLDA and modified EDA

6 DISCUSSION AND CONCLUSIONS

In this paper two main issues were considered. The first is related to the development of a numerical tool that can be used to estimate the seismic risk of structures through the implementation of a hypothetical case study. This tool considers several sources of uncertainties such as those related to geometry, the mechanical properties of the materials and the seismic hazard. Currently, the source code used to obtain the results presented herein is being reviewed and enhanced. It is expected that an interoperable version for potential users will be created by the end of the KaIROS project [1]. This tool could be used to develop detailed vulnerability models as the one presented in [10]. The second is a statistical analysis of the results aimed at developing a simplified procedure for estimating commonly used EDPs. The EDA rule principle was employed as the basis of a modified version that can be used to calculate similar results to those obtained via NLDA. This simplified procedure can be used to estimate, for instance, fragility curves via cloud analysis [11] without the need to perform NLDAs. Thus, the fragility curves for several damage state thresholds, related to MIDR 0.005, 0.01, 0.015 and 0.02, for the reinforced concrete six-story building shown in Figure 10a will be obtained based on the NLDA and the modified EDA. The fundamental period of this structure is 0.84 s. Table 2 summarizes the details of the building. The epistemic uncertainties are considered according to the values presented in Table 1.

Story	W_c (m)	B_c (m)	W_b (m)	D_b (m)	ρ_c (%)	ρ_b (%)
1	0.7	0.7	0.35	0.4	1.2	0.8
2	0.7	0.7	0.35	0.4	1.2	0.8
3	0.65	0.65	0.3	0.35	1.1	0.8
4	0.65	0.65	0.3	0.35	1.1	0.8
5	0.6	0.6	0.3	0.3	1	0.8
6	0.6	0.6	0.3	0.3	1	0.8

Table 2 Characteristic of the building analyzed (Figure 10a)

Fifty earthquake records are selected in similar way to the procedure presented in Section 3, but in this case the spectral acceleration is related to the fundamental period of the structure. The values defining the upper and lower limit of each stripe range from 0 to 1 g, at intervals of 0.1 g (Figure 10b).

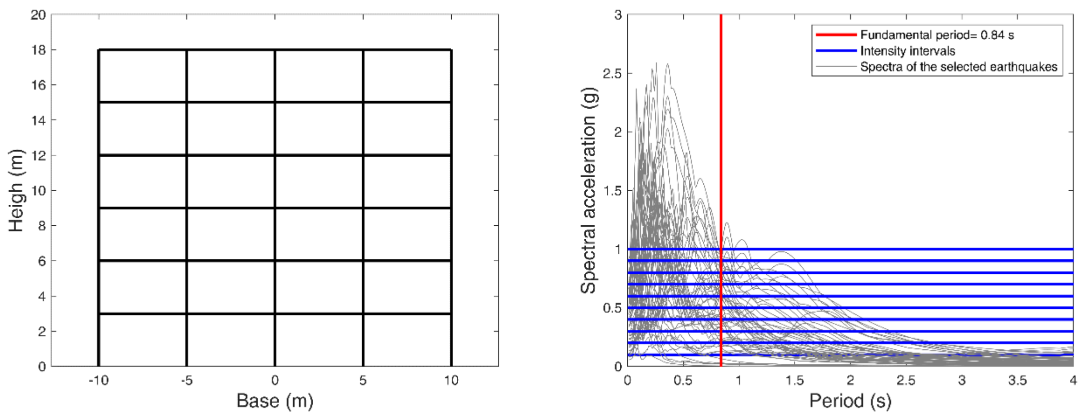

Figure 10 a) Six-story building and b) Spectra of the selected earthquakes

The fragility curves are then obtained using the cloud analysis via NLDA and the modified EDA. Figure 11a shows the comparison between the fragility curves obtained with both approaches. Significant differences can be seen between both curves. Nevertheless, these curves have been calculated for α=1.6976, which is a value that minimizes the error of all the structural models analyzed in the paper. If one analyzes the evolution of α depending on the number of stories, we will find that α increases with the number of stories. This makes sense, because the higher the building the higher the participation of superior modes of vibration. Figure 12 shows the evolution of α as a function of the number of stories. Then, after performing the calculations using α=1.85, obtained from the regression analysis presented in Figure 12, the fragility curves shown in Figure 11b are obtained. Noticeably, a better fit is achieved. This example proves the ability of the proposed procedure to estimate EDP in a very simplified way. Of course, the building that is analyzed should be in the domain of the simulation performed in this paper.

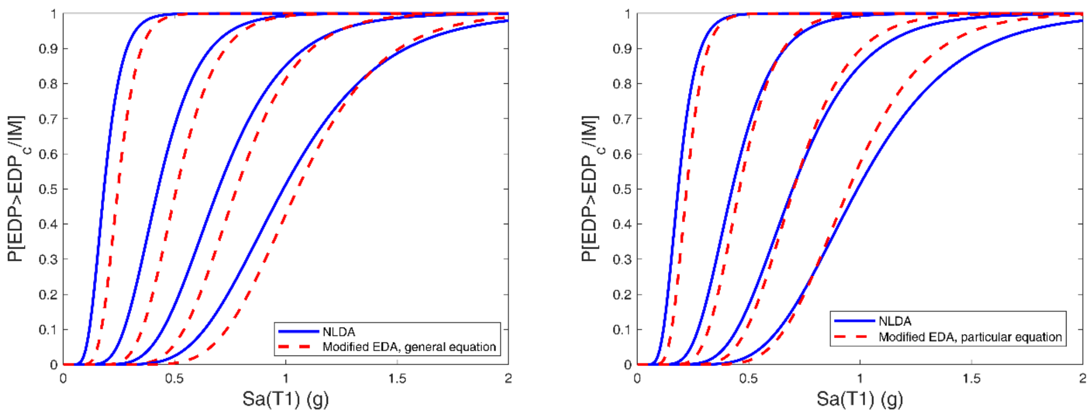

Figure 11 a) Fragility curves via cloud analysis with α=1.6976 and b) α=1.85

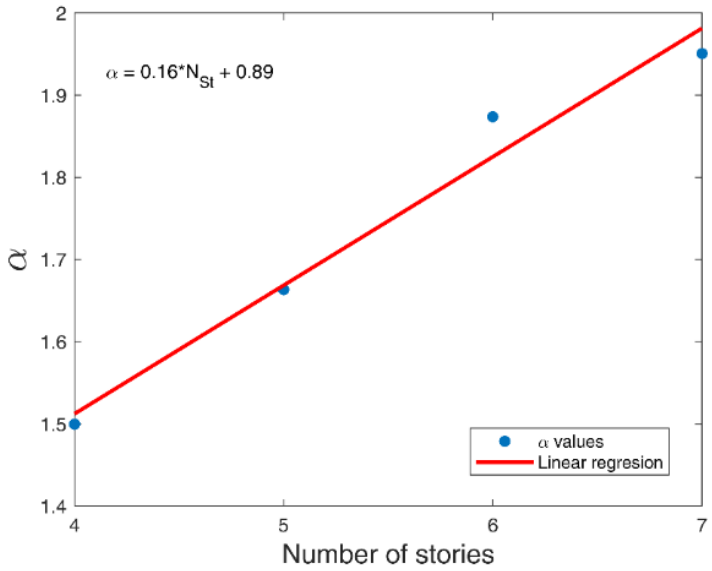

Figure 12 Evolution of α as a function of the number of stories

Finally, the key to reducing seismic risk is to decrease the vulnerability of existing structures and provide new insights to improve the design of new structures and protect them against seismic events. The current capacity of computers, combined with the versatility offered by probabilistic numerical methods, helps face this fundamental challenge in current society.

Acknowledgments

We thank Lucille Banham for her assistance in the preparation of the English manuscript. Yeudy F. Vargas-Alzate has been granted an Individual Fellowship (IF) in the research grant program of the Marie Sklodowska-Curie Actions (MSCA), European Union/European (H2020-MSCA-IF-2017) No 799553. Rodrigo E. Alva holds a PhD fellowship from the Centro Nacional de Ciencia y Tecnología (CONACyT) in Mexico. The authors acknowledge the support of these institutions. This research was partially funded by the Spanish Government's Ministry of Economy and Competitiveness (MINECO) and by the European Regional Development Fund (ERDF) of the European Union (EU) through projects with references CGL2015-65913 -P (MINECO/ERDF, EU) and EFA158/16/POCRISC (INTERREG/POCTEFA. EU).

REFERENCES

[1] KaIROS Project. Keeping and Increasing Resilience Opportunities and Sustainability of communities against earthquakes. https://cordis.europa.eu/project/rcn/215743/factsheet/en

[2] Baker J.W., An Introduction to Probabilistic Seismic Hazard Analysis (PSHA). White Paper, Version 1.3, 72 pp. Available at: http://web.stanford.edu/~bakerjw/publications.html.

[3] Goel R., Chopra A., Period Formulas for Moment-Resisting Frame Buildings. *Journal of Structural Engineering*, **123**, 1454-1461, 1997.

[4] Ambraseys N., Smit, p., Sigbjornsson r., Suhadolc P., Margaris b., Internet-Site for European Strong-Motion Data, European Commission, Research-Directorate General, Environment and Climate Programme. http://www.isesd.hi.is/ESD_Local/frameset.htm (Last visited on March 12, 2019)

[5] Otani S. Inelastic analysis of RC frame structures, *Journal of Structural Division, ASCE*. **100**, 1974.

[6] Carr, A.J. *Ruaumoko-Inelastic Dynamic Analysis Program*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand 2000.

[7] ATC-40. *Seismic evaluation and retrofit of concrete buildings. ATC*-40, Redwood City, California, 1996.

[8] FEMA-440, Improvement of Nonlinear Static Seismic Procedures, ATC-55, Redwood City, 2005.

[9] Vargas-Alzate Y.F., Pujades L.G., Barbat A.H., Hurtado J.E, An efficient methodology to estimate probabilistic seismic damage curves. *Journal of Structural Engineering, ASCE*. **145**, 2019.

[10] Silva V., Crowley H., Varum H., Pinho R., Sousa L., Investigation of the characteristics of Portuguese regular moment-frame RC buildings and development of a vulnerability model. *Bulletin of earthquake engineering*, **13**, 1455-1490, 2015.

[11] Bakalis K., Vamvatsikos D., Seismic fragility functions via nonlinear dynamic methods. ASCE *Journal of Structural Engineering, ASCE*, **144**, 2018.