

A methodology for the calculation of the noise radiated by the rails and the tunnel structure in railway tunnels

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ABSTRACT

In this paper, a robust and fast numerical methodology to compute re-radiated noise in underground railway tunnels is proposed. In this study, the noise analysis does not account for the noise radiation from the train wheels, the rest of the rolling stock structure and the aerodynamic noise. The method is based on decoupled approach, where the acoustic and elastodynamic problems are solved separately on the assumption of weak coupling between the two subdomains. Two-and-a-half dimension (2.5D) finite element boundary element (FEM-BEM) is used to analyse the elastodynamic problem. The computation of the re-radiated noise from the vibration of the structure is done with a 2.5D acoustic boundary element method (BEM). The acoustic as well as elastodynamic BEM used in this analysis is based on globally regularized integrals based on singularity subtraction.

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1. INTRODUCTION

Noise generated from these underground railway infrastructures affects the comfort of the passengers inside the vehicle as well as the commuters on the train station [1–3]. A major component of the noise generated inside railway tunnels is the noise radiated by the tunnel structure and the track structural components on account of train pass-by. Therefore, developing a method for the re-radiated noise induced by the tunnel and the track becomes highly important in order to assess the acoustic field inside underground railway tunnels.

Methodologies based on 2.5D formulation are being used nowadays in modelling the problems related to acoustics and soil-structure interactions [4–8]. The methodology is applied to 3D cases, where the geometry is invariant (or assumed to be) in one direction. It involves taking Fourier transform of the coordinate of invariant direction and solving a set of 2D problems which are then Fourier anti-transformed to obtain the 3D solution. The advantage of this methodology is the reduction of the mesh dimensions by one.

Coupled formulation based on FEM and BEM in 2.5D was proposed by Sheng et al. [9, 10] to study soil/structure interactions. François et al. [7] proposed by using globally regularized integrals based on singularity subtraction, thereby avoiding the explicit computation of singular terms. The BEM described in this work are based on the formulation of François et al. [7].

Many alternative methods have been used to model the soil-structure interaction in conjunction with FEM. Perfectly matched layers (PML) [11] in conjunction with 2.5D FEM was used to study the vibration induced by railway traffic to buildings. The method of fundamental solutions (MFS) was used by [12] to model the soil in tunnel/soil interaction problems. Yaseri et al. [13] proposed FEM coupled with scaled boundary finite elements (SBFEM) for the analysis of ground-borne vibrations induced by train traffic, where the soil was modelled with SBFEM.

In the context of noise radiation, 3D FEM-BEM methodology was used by Fiala et al. [14, 15] to model the re-radiated noise in buildings arising from ground-borne vibrations induced by underground railway traffic. Colaço et al. [16] proposed a decoupled approach for the computation of the re-radiated noise in buildings generated by railway-induced ground-borne vibration that used MFS to study the interior acoustics problem. A decoupled BEM approach was used by Sheng et al. [17] in the analysis of noise radiated in at-grade railway tracks.

In this paper, a robust and fast numerical methodology to compute re-radiated noise in underground railway tunnels is proposed. In this study, the noise analysis does not account for the noise radiation from the train wheels, the rest of the rolling stock structure and the aerodynamic noise. The method is based on a decoupled approach, where the acoustic and elastodynamic problems are solved separately on the assumption of weak coupling between the two subdomains. 2.5D FEM-BEM is used to analyse the elastodynamic problem. The computation of the re-radiated noise from the vibration of the structure is done with a 2.5D acoustic BEM. The acoustic, as well as elastodynamic BEM used in this analysis, is based on globally regularised integrals based on singularity subtraction.

2. NUMERICAL METHOD

The study aims to assess the re-radiated noise generated in underground railway tunnels due to train traffic. The computational scheme consists of three different models: the train/track interaction, the track/tunnel/soil system and the model for the interior acoustics. In this scheme, coupled train/track/tunnel/soil system is solved in the two initial steps. In the first one, the track response is obtained from the 2.5D elastodynamic FEM-BEM model, and the vehicle response is used to obtain the wheel/rail interaction forces. In the second step, these forces are applied to the 2.5D elastodynamic FEM-BEM model of the track/tunnel/soil system to obtain the structural response. Then, the re-radiated noise generated by this structural response is computed by means a 2.5D acoustic BEM. The following subsections provide a summary of the models used.

2.2.1. Train/track model

In the train/track interaction, two components of loads are considered, the static loads which result from the movement static components (the weight) and the dynamic loads which are a result of the dynamic interaction between the train and the track. A compliance approach that satisfies the equilibrium requirements and compatibility between the train and the track assuming that track unevenness is the source of excitation. The train/track interaction is solved using

$$\mathbf{F}_{w/r}(\tilde{\omega}) = [\mathbf{H}_v^{w/r} + \mathbf{H}_r^{w/r} + 1/k_H \mathbf{I}]^{-1} \mathbf{E}_r(\tilde{\omega}), \quad (1)$$

where $\tilde{\omega}$ is the excitation frequency seen in the point of view of the moving frame of reference attached to the train motion due to the unevenness, $\mathbf{F}_{w/r}$ is the vector of the train/track dynamic interaction loads, $\mathbf{H}_v^{w/r}$ is the vehicle receptance matrix at the contact points with the track, \mathbf{I} is an identity matrix, k_H is linearised Hertzian spring considered to be the same in all the contacts, $\mathbf{H}_r^{w/r}$ is the receptance matrix of the track at the interaction points with the vehicle seen in the point of view of the moving frame of reference and \mathbf{E}_r is the vector of frequency spectra of the track unevenness at all the wheel/rail interaction points, $\mathbf{H}_r^{w/r}$ can be obtained assuming a vertical unitary load is applied at both railheads using the 2.5D FEM-BEM model of the track/tunnel/soil system. The model for train/track interaction can be found in [18, 19].

2.2.2. Track/tunnel/soil modelling

The 2.5D FEM-BEM methodology for modelling the track/tunnel/soil system is based on the formulation presented in [7], which uses a regularization scheme of Green's function. In this method, the response is obtained by constructing a global stiffness matrix in a FEM framework. The equation for solving the track/tunnel/soil system modelled with the coupled 2.5D FEM-BEM is given by

$$[\mathbf{K}_0 - ik_x \mathbf{K}_1 + k_x^2 \mathbf{K}_2 + \bar{\mathbf{K}}_s - \omega^2 \mathbf{M}] \bar{\mathbf{U}} = \bar{\mathbf{F}}, \quad (2)$$

\mathbf{K}_0 , \mathbf{K}_1 and \mathbf{K}_2 are the stiffness matrices related to the domain modelled with 2.5D FEM, \mathbf{M} is the mass matrix of the structure and $\bar{\mathbf{K}}_s$ is the dynamic stiffness matrix of the soil obtained from the 2.5D BEM model. The stiffness and mass matrices of the structure are independent of the wavenumber and the frequency, while the stiffness matrix of the soil is a function of them. Moreover, $\bar{\mathbf{F}}$ represents the vector of external forces and $\bar{\mathbf{U}}$ is the vector of displacements, both defined in the 2.5D domain.

2.2.3. Acoustic modelling

As a decoupled approach is followed, normal velocity \bar{V}_n is used as a boundary condition on the collocation points of acoustic BEM. It is computed from the normal displacement \bar{U}_n obtained from the dynamic response of the structure as:

$$\bar{V}_n = i\omega\bar{U}_n, \quad (3)$$

The boundary integral equation is obtained by regularizing the integral equation 5. The regularization scheme applied to obtain this boundary integral equation is similar to the one followed by François the details of regularizing bounded domain integrals is found in Bonnet. The regularised boundary integral equation in matrix form is given by:

$$\bar{H}_b\bar{P}_{nb} = i\rho\omega\bar{G}_b\bar{V}_{nb}, \quad (4)$$

where, \bar{H}_b , \bar{G}_b are matrices related to Green's velocity and pressure computed on the boundary, \bar{P}_{nb} and \bar{V}_{nb} are nodal pressures and velocities on the boundary, respectively. After obtaining the boundary unknowns the pressure in the acoustic space is obtained as:

$$\bar{P}_{nf} = -(\bar{H}_f\bar{P}_{nb} + i\rho\omega\bar{G}_f\bar{V}_{nb}), \quad (5)$$

where, \bar{P}_{nf} is the vector of pressure in the acoustic space, \bar{H}_f , \bar{G}_f are matrices related to Green's velocities and pressures computed in on evaluation points in acoustic space. The derivation of the above equation can be found in [17, 20].

2.2.4. Fast Computation of BEM

A focus of this work has been the improvement of the computational speed of the entire numerical method. A significant amount of computational time of this method is spent in the computation of the boundary element matrices both in coupled 2.5D elastodynamic FEM-BEM and 2.5D acoustic BEM. The computational time of the boundary element matrices are dependent on number of source/receiver combinations. Thus, by reducing these source-receiver combinations the computational speed of the method can be improved. In this work, the computational speed of boundary element method for both acoustics and elastodynamics have been improved by two strategies.

- The first strategy is a general strategy and involves computing the Green's function for a unique set of source-receiver. For a closed geometry in a homogeneous full-space, the Green's functions are a function of the relative distance between source and receiver points. Relative distances of all the source/receiver combinations are as such contained in a small set of unique source/receiver relative distance combinations. Thus by computing the Green's function for this unique set of source/receiver combination distances, and then mapping them to get the Green's function for the complete set of source receiver combinations, fast computation of Green's functions is accomplished.
- The next strategy involves exploiting the axisymmetric nature of circular tunnels. When the boundary elements of the BEM mesh have the same length the elements of the BEM matrices associated to all combinations of the source/receiver locations can be obtained from elements associated to only one source. Thus, if the elements associated to a reference source are computed, the response for all the other

combinations of source/receiver distances can be easily obtained by multiplying these reference elements with a transformation matrix that takes into account the rotation between this reference source and the other ones followed by appropriately mapping the resulting elements.

These two strategies are employed in the computation of boundary element matrices in 2.5D FEM-BEM and 2.5D acoustic BEM resulting in improving the computational time of numerical methodology presented. The verification and details of the methodology can be found in [19].

3. APPLICATION

The model of the underground tunnel is shown in the Fig. 1. The structure, (the tunnel, the fasteners and the rails), is modelled with finite elements, while the soil is modelled with boundary elements using the 2.5D elastodynamic BEM, the nodes of which are shown in red. BEM matrices are obtained by following nodal collocation scheme. Assuming a weak coupling between elastodynamic and acoustics systems, the structural response is obtained at the points marked in purple. They are used as the input to the acoustic model in terms of structural velocities. The tunnel has an inner radius of 3 m and a thickness of 0.25 m. The mechanical parameters of the rails, tunnel and soil as isotropic elastic media are presented in Table 1.

Table 1: Mechanical properties of the rails, tunnel and soil.

	E [GPa]	ρ [kg/m ³]	ν	D_p	D_s
Rails	207	7850	0.15	0.01	0.01
Tunnel	35	2500	0.15	0.01	0.01
Soil	0.18	2191	0.3	0.025	0.015

The rail fasteners that attach the rails to the tunnel invert consist of a top elastomer just below the rails and a bottom elastomer sandwiched between two metallic plates under the top elastomer. Thus, the fasteners are modelled as a sandwiched system consisting of four layers meshed with 2.5D finite elements. The equivalent properties to be used in the 2.5D FEM model of the fasteners for the two cases are presented in Tables 2 and 3. These equivalent properties are obtained by considering vertical stiffness of the rails fastening system should not be modified when a continuous distributed fasteners are assumed. The thicknesses of the top and bottom elastomers are considered to be 0.007 m and 0.012 m, respectively, and those of the metallic plates are 0.016 m and 0.012 m, respectively for the top and the bottom. Uniformly distributed samples of 2^{12} points in range of $(-10$ to $10)$ have been used as a sampling vector of the wavenumber, as most of the spectral information of the system is contained in this range. The properties and model of the train used for this simulation can be found in Costa et al. [18].

In Fig. 1, evaluators A and B are considered for showing vibration response of the structure, while evaluators a and b are used for computing the noise response in following sections. Figure 2 shows the time histories of the vertical rail velocities for the two cases of fastener stiffness. Sub-figure (a) is the time history for the stiffest fastener in this study (case 1: Table 2) followed by the one which is a softer fastener (case 2: Table 3) in subfigure (b). Figure 3 shows the frequency content for the vertical component of the vibration velocity of rail and tunnel evaluators in one-third octave bands for the two cases

Table 2: Fasteners properties for case 1.

	E [MPa]	ρ [kg/m ³]	ν	$D_p = D_s$
Top elastomer	1.15	1329	0.45	0.05
Top plate	$207 \cdot 10^3$	7850	0.30	0.01
Bot. elastomer	2.7	1329	0.35	0.05
Bottom plate	$207 \cdot 10^3$	7850	0.30	0.01

Table 3: Fasteners properties for case 2.

	E [MPa]	ρ [kg/m ³]	ν	$D_p = D_s$
Top elastomer	0.3	1200	0.45	0.05
Top plate	$207 \cdot 10^3$	7850	0.30	0.01
Bot. elastomer	0.7	1200	0.35	0.05
Bottom plate	$207 \cdot 10^3$	7850	0.30	0.01

of the fasteners stiffness. From the figures, it can be seen that the application of rail pads in the rails fastening system implies a reduction of the vibration in the tunnel invert as well as an increasing of the rails vibration. This behaviour is not clearly seen in the tunnel evaluator A, because of the specific combination of vibration modes of the tunnel. It is seen that the track with softer fasteners (case 2) is a better solution than the track with stiffer one (case 1) in order to reduce the vibration in the tunnel. In general, the track in case 2 is shifting the vibration spectra in the tunnel and the rails to lower frequencies.

For the noise response of the system, the evaluator locations in the acoustics space, where the pressure field is computed, are a and b in Fig. 1. The BEM nodes for the 2.5D acoustic BEM are denoted in purple in Fig. 1 and the normal velocities on all these nodes obtained from the structural response computed previously by 2.5D elastodynamic FEM-BEM are used as boundary condition for 2.5D acoustic BEM. For the case when rails are not considered, the nodes in the tunnel invert just below the rails are used as nodes in 2.5D acoustic BEM. Zero velocity condition is ascribed to these nodes. A small damping ratio of 0.025 is considered for modelling the acoustic domain.

Figure 4 shows the pressure levels obtained inside the tunnel for the acoustics evaluator a and b for each of the two cases of rail fastening systems. The most important observation that one can obtain from this figure is that the noise field inside the tunnel is completely influenced at all frequencies by the rail noise radiation. The noise field in the tunnel radiated only because of the tunnel structure is affected by the fasteners stiffness in a similar way as the vibration response. In contrast, the noise field is affected in a completely different way when the rail contribution is considered. From the point of view of the different rail fastening systems studied, the frequency content of the noise behaves in similar way as the vibration response previously described.

4. CONCLUSION

In this article, a methodology to predict re-radiated noise in underground tunnels is presented. The elastodynamics problem is solved using coupled 2.5D elastodynamic FEM-BEM. The acoustic analysis is accomplished by using a 2.5D acoustic BEM considering a weak coupling with the elastodynamic problem. The 2.5D acoustic BEM and the 2.5D elastodynamic FEM-BEM methodologies developed in this work are

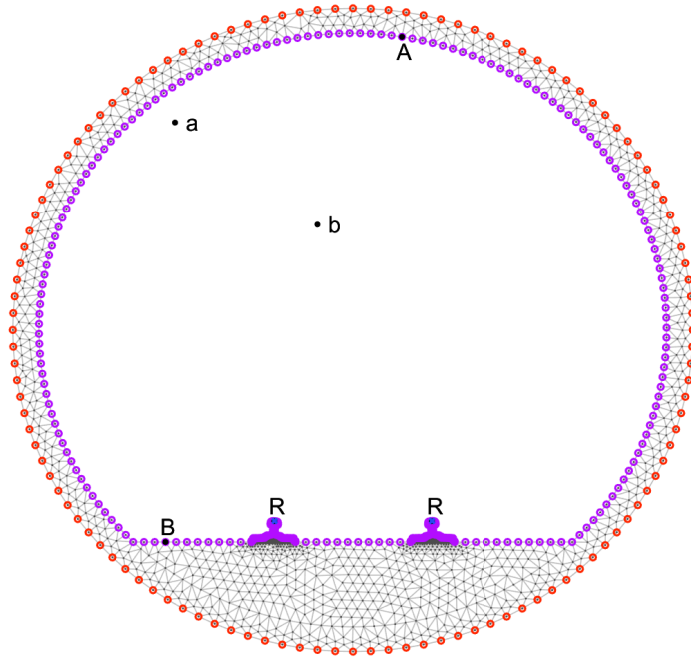


Figure 1: Geometry of the simple tunnel studied in this article. Considering the center of the tunnel inner circumference at $(y = 0, z = 0)$, vibration response evaluators A and B are located at $(y = 0.5, z = -2.96)$ m and $(y = -1.79, z = 2.14)$ m respectively; the rail response is obtained at the evaluators R, placed on top of each of the rails; acoustic response evaluators a, and b are located at $(y = -1.7, z = -2.14)$ m and $(y = -0.34, z = -1.07)$ m respectively.

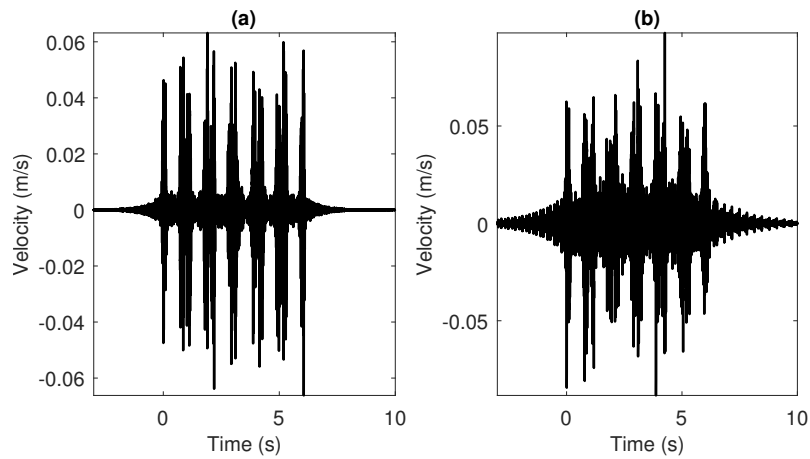


Figure 2: Time histories of the rail velocity for case 1 (a) and case 2 (b).

optimised in terms of computation time. The methodology is used to study the effect of two different kind of rail fastening system on noise and vibration system. The results for noise radiation are obtained with and without rail contribution. The important findings of the study are summarised:

1. Softer rail pads are more efficient solution than stiffer ones in terms of vibration reduction in the tunnel.
2. The noise field inside the tunnel induced by structure without the rails contribution

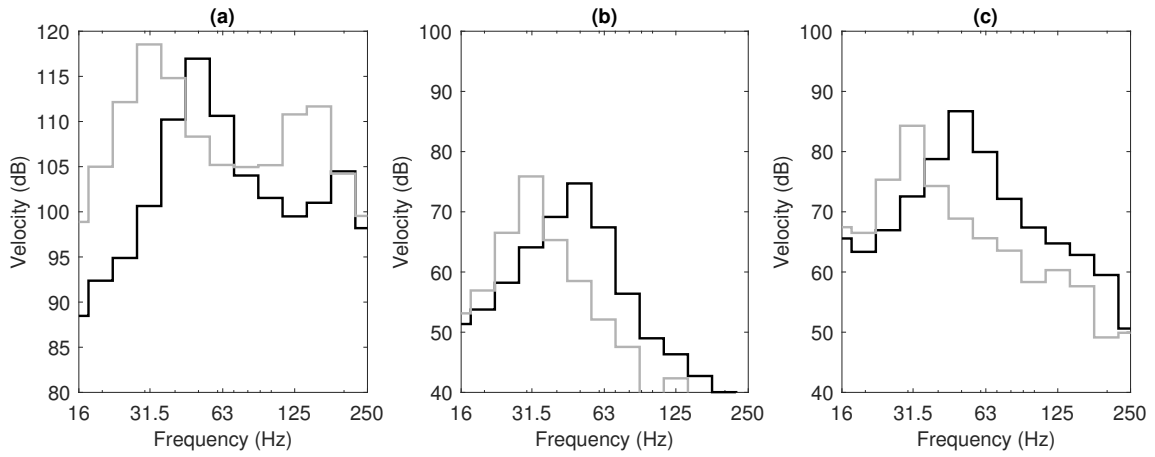


Figure 3: Vertical component of the velocity levels in dB (dB reference 10^{-8} m/s) in one-third octave bands for the rail (a), the tunnel evaluator A (b) and the tunnel evaluator B (c). Black lines represent case 1, grey lines represent case 2.

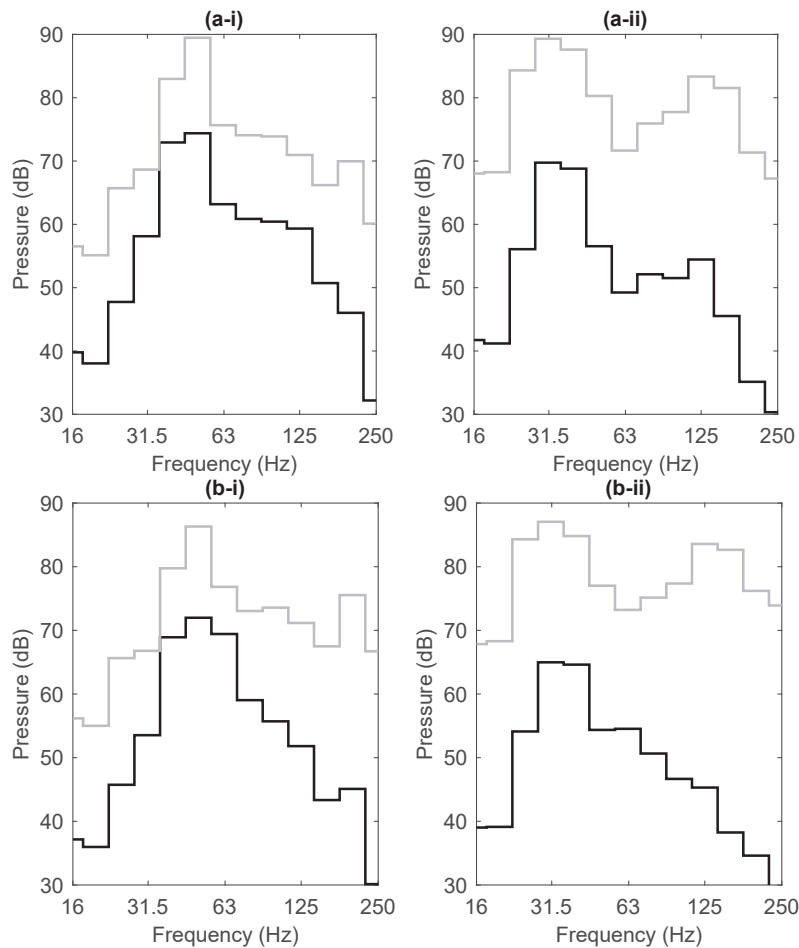


Figure 4: Pressure levels considering the rail contribution (grey line) and without the rail contribution (black line) for the acoustic evaluators a (a) and b (b) for the cases 1 (i) and 2 (ii)

is affected by the fasteners stiffness in a similar way as the vibration response is affected. In contrast, when the rail contribution is considered, the noise level in the tunnel increases as the fasteners stiffness decreases.

3. If train vehicle and wheels are not considered, it is observed that the noise field inside the tunnel is completely control at all frequencies by the rail noise radiation.

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