# Art and Math Using an Equally Linked Tetrahedron

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## Abstract

The tetrahedron is a well-known polyhedron. The tetrahedron can be shown with a set of lines (bars) made of 3 points (joints) from the different elements; for instance an edge is a line of 2 vertices and the same edge. The equally linked property means that every element is connected with the same amount of links. The paper explains this property with tetrahedrons and shows different prototypes that can become nice objects.

## Introduction

"Les anciens ont fort bien remarqué qu'entre les problèmes de géométrie, les uns sont plans, les autres solides et les autres linéaires"[1], René Descartes wrote it at *La Geometrie*. It goes without saying that he referred to classical geometry of dimension 2 (2D) with planes and dimension 3 (3D) with solids (volumes) as well as dimension 1 (1D) with lines. Later on, mathematicians used higher dimensions and manage to physically represent them as images or sculptures.

In classical geometry, the reference objects in 2D are the polygons and in 3D they are the polyhedra; there are also the the polytopes in multidimensional spaces (nD). From our side, geometrical figures have linear bars made of 2 end joints and a named "middle joint". Circular bars are formed by 3 middle joints. Then, as a polygon, the triangle has 3 edges and 3 vertices; in our way, a triangle has 3 connected linear bars in the end joints and a circular bar; briefly, a triangle has 4 bars and 6 joints. In a similar way, as a polyhedron, there is the tetrahedron as 4 triangles with shared edges; as a 3D object, it shapes a volume that it is also called a cell. In our way, the tetrahedron has also "2D bars" called planar bars.

From our side, within the Mobile Numbers Technology (MNT) initiative as a work in progress [2, 3], we will refer to the several tetrahedrons with an equally linked property: a geometrical figure linked by bars of 3 joints and, overall, each joint is linked with the same number of bars. Then, with this property, we refer to equally linked 3L, 6L or 7L tetrahedrons.

## The equally linked 3L tetrahedron

The equally linked 3L tetrahedron (Figure 1) has 10 joints (4 end joints and 6 middle joints) and 10 bars (6 linear and 4 circular). Each joint is linked with 3 bars (Figure 1). When we think the bars as beams, there is another interesting feature regarding "stability" whenever we nail several joints. As the bar has 3 joints, we can also think the beam with 3 joints; then if you nail 2 out of 3, the beam becomes stable or, in other words, the bar becomes uniquely determined. It also applies with the tetrahedron that becomes "stable" with 4 fixed joints, depending on the choice of the four fixed joints.

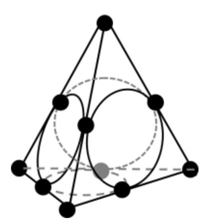


**Figure 1:** *The equally linked 3L tetrahedron with 10 joints and 10 bars*

## The equally linked 7L tetrahedron

The equally linked 7L tetrahedron has 15 joints (Figure 2) and 45 bars (Figure 3). The joints are 4 vertex joints, 6 edge joints, 4 centroid joints and 1center joint, each joint is linked with 7 bars.

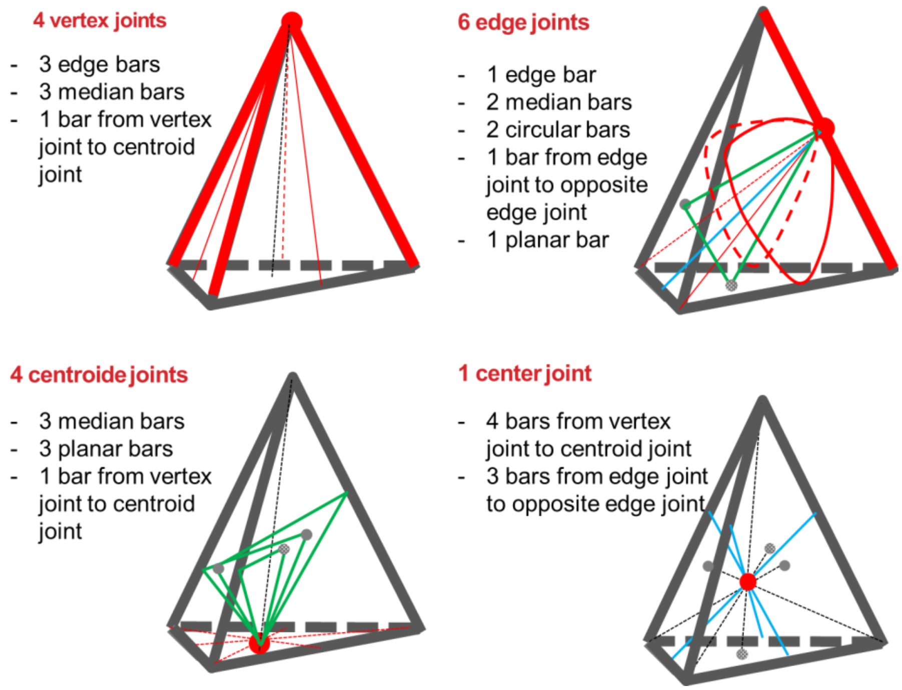



**Figure 2:** *The equally linked 7L tetrahedron with the 7 links of each joint.*

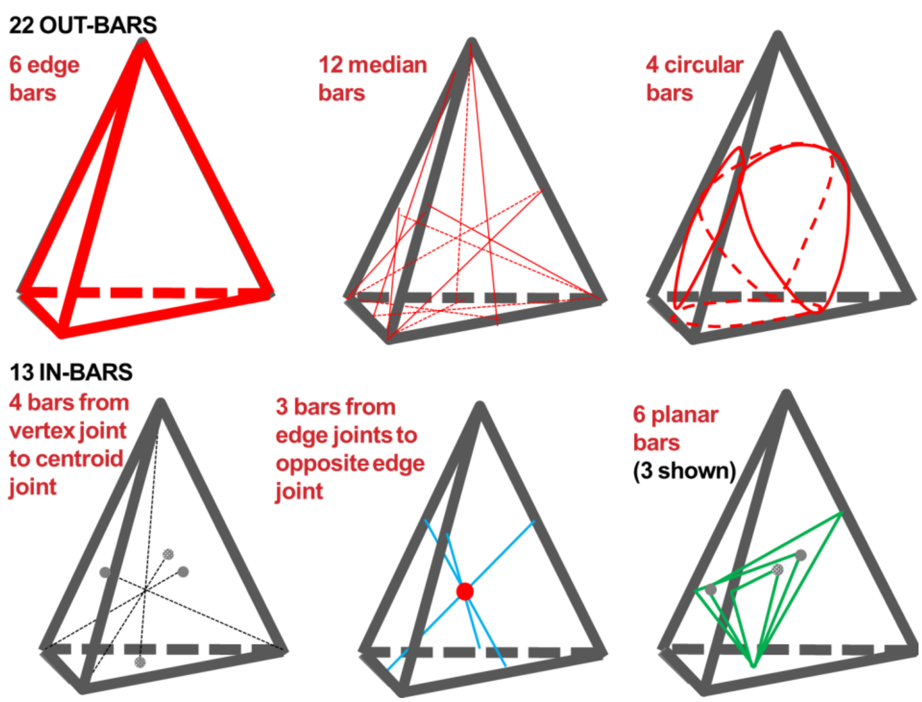



**Figure 3:** *The 45 bars of the equally linked 7L tetrahedron*

## The prototype of equally linked 6L tetrahedron

The equally linked 6L tetrahedron (Figure 4) has 14 joints and 28 bars. Each joint is linked with 6 bars.

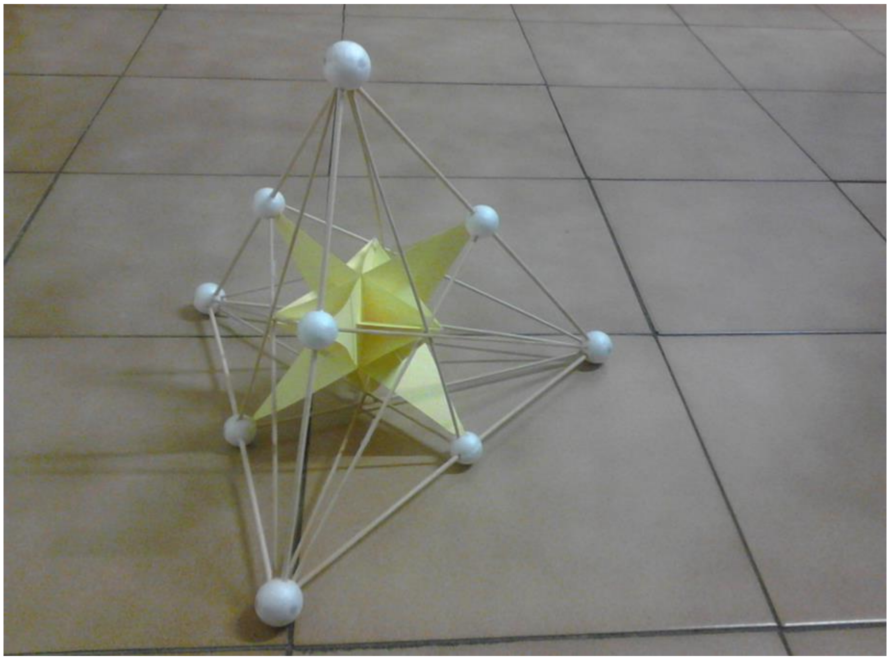


***Figure 4:*** *Mockup of equally linked 6L tetrahedron. Vertex and edge joints as white balls but centroid joints are not. Edge and median out-bars are made of wood and planar bars are made of yellow paper; circular bars are not shown.*

From the mockup above, there is an understanding that equally linked tetrahedrons can be split in interesting pieces: a bar structure (with or without circular bars), a uniform star polyhedron… Figure 5 shows different examples.

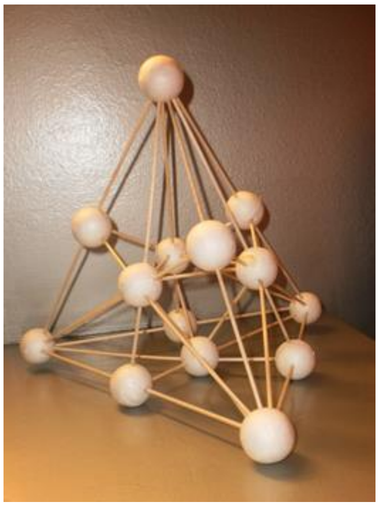
*(a)*  *(b)* 

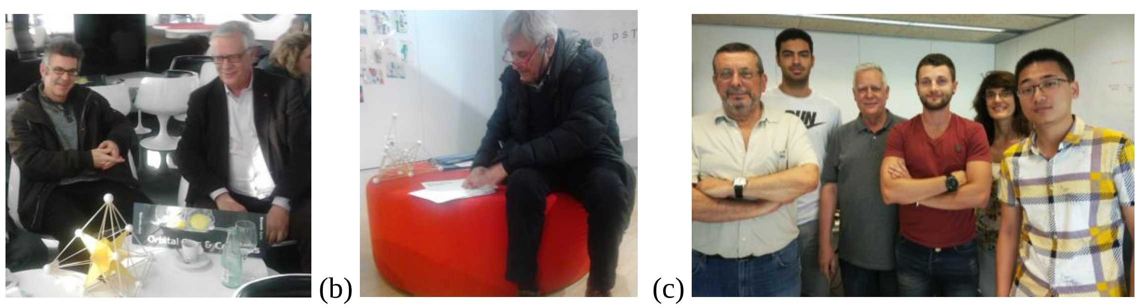
***Figure 5:*** *Tetrahedron as bar structure: (a) with 14 joints but central joint, (b) with straight bars*

## Summary and Conclusions

Regarding equally linked tetrahedrons, whenever you have the final object, they become a beautiful sculpture; in other words, a piece of art. Other market applications are under consideration. As they belong to the MNT initiative, future applications are expected. Briefly, a novelty that brings about new possibilities.

## Acknowledgements

Authors acknowledge the collaboration of different actors in the deployment (Figure 6). Design, graphs and icons, these works are based on research and functional development of J. Tarrés, whereas mockups were done by Luis Sanchez Cuenca and some graphical works designed by T. Vinyes. We also acknowledge the MNT works of 35 students from University of Girona as company internships. At this time, the core deployment is formed by Manuel Moreno, Universitat Politècnica de Catalunya; Joan Josep Sunyol, Universitat de Girona; Timo Tiihonen, Jyväskylä University; Luis Sanchez Cuenca, Toni Vinyes, and Josep Tarrés. Different companies and organizations (MMACA, Museu de Matemàtiques de Catalunya;…) are also related.



(a) (b) (c)

***Figure 6:*** *Photos from deployment of equally linked tetrahedron within Mobile Numbers Technology (MNT) initiative: (a) Manuel Moreno and Josep Tarrés, (b) Luis Sanchez Cuenca, model builder, (c) Toni Vinyes, graphic designer with some members of MNT team*

## References

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