TITLE: Criteria for rapid sliding II. Thermo-hydro-mechanical and scale effects in Vaiont case.

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CRITERIA FOR RAPID SLIDING. II. THERMO-HYDRO-MECHANICAL AND SCALE EFFECTS

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ABSTRACT:

Thermally induced excess pore pressures have been included into a two wedge evolutive model of Vaiont landslide. The problem requires the solution of a system of four coupled balance equations for the shear bands and the surrounding rock as well as the joint equation of motion of the entire slide. The model predicts the high velocities observed and is consistent with other data (slide geometry, residual strength, conditions on the sliding surface). The interpretation of a sensitivity analysis suggests that there exists a threshold permeability band, in the range10^{-8} to 10^{-9} m/s, which separates potentially fast motions from slow motions. This conclusion is maintained if the scale of the landslide is reduced.

1 INTRODUCTION

In the companion paper (Alonso and Pinyol , 2009), an attempt was made to determine the run-out of Vaiont landslide taking, as a reference model, an evolutive two-wedge representation of cross Section 5 (Figure 7 of Alonso and Pinyol, 2009). Starting at a condition of near equilibrium at t = 0, it was assumed that the strength of the plane separating the two wedges could degrade as shearing displacements developed along this plane during the motion. It was found that, even in the extreme case of a fast and complete loss of cohesion acting on this plane (an unlikely event), the slide maximum velocity did not exceed 4.5 m/s. In order to explain the estimated high velocities of the slide (30 m/s), a consistent mechanism or physical process, leading to a total loss of basal shear strength, has to be found.

The favorite explanation of a number of published contributions on the subject is associated with the development of frictional heat at the sliding surface. In some papers (Uriel and Molina, 1977; Nonveiller, 1987) the frictional heat is assumed to take the pore water to the equilibrium state between liquid and vapour phases. In Uriel and Molina (1977) the phase diagram of water provides a criterion to find the water/vapour pressure. Nonveiller (1987) assumes a linear decrease of rock strength with temperature in the shear zone. In other approaches (Hendron and Patton, 1985; Voigt and Faust, 1982; Vardoulakis, 2002) the increase in pore pressure is related to the dilation of pore water as temperature increases and to temperature-induced plastic collapse of the shearing band (in the case of Vardoulakis).

In all cases, the fluid pressure developed at the sliding surface reduces the effective normal stress and hence, the available strength.

2 HEATING EXPERIMENTS ON LOW PERMEABILITY CLAYEY ROCKS

Consider the simple experiment of heating a sample of saturated clay in a microwave oven. In the experiment performed a thermocouple temperature sensor was inserted into a specimen of Opalinus clay, which had been maintained in a humid chamber to ensure saturation. Opalinus clay is a low permeability soft clayey rock of marine origin. Clay minerals (illite, illite-smectite mixed layers, chlorites, kaolinites) dominate its mineralogical composition (40% to 80%). Quartz, calcite, siderite, pyrite, feldspar and organic carbon are also present. Natural porosity varies between 4% and 12% (Bossart *et al., 2002)*. Pore water has a concentration of 20 g/l of sodium chloride. Permeability coefficients (Darcy) varying between $0.8 \cdot 10^{-13}$ m/s and $7.3 \cdot 10^{-13}$ m/s, Young's modulus ranging between 1000 and 7000 MPa, and uniaxial compressive strength varying between 9 and 18 MPa have been reported for this clay shale by several authors (Thury and Bossart, 1999; Bock, 2001; Muñoz, 2007) on the basis of "in situ" and laboratory tests. This soft clayey shale is in some respects (low permeability, clay minerals of low shearing resistance, low porosity) similar to the Mälm clay and marl layers found in the position of Vaiont basal sliding surface

A thermal pulse having a nominal power of 1400 watts was applied during 40 s. The recorded temperature is shown in Figure 1. The specimen broke, accompanied by a clearly audible cracking noise, shortly before the end of the application of the heating pulse. At that time the temperature reached values in excess of 160ºC (Figure 1). The shale specimen cracked in an explosive manner and was reduced to small fragments.

The following explanation can be given for this phenomenon. When the temperature of a saturated porous material increases, the solid matter, as well as the water in pores, dilates. Probably, local equilibrium of temperature is achieved soon, and therefore the temperatures of water and solid skeleton will be essentially equal.

The volume of pore water and solid skeleton will increase in a direct proportion to their thermal dilation coefficients, β_w and β_s, respectively. The associated volumetric strains, for a common change in temperature, $d\theta$ can be written:

$$\begin{aligned} \mathrm{d}\varepsilon_{vw} &= \frac{\mathrm{d}V^w}{V^w} = -\beta_w \mathrm{d}\theta \\ \mathrm{d}\varepsilon_{vs} &= \frac{\mathrm{d}V^s}{V^s} = -\beta_s \mathrm{d}\theta \end{aligned} \qquad (1\ \mathrm{a,b})$$

where V^w and V^s are the volumes occupied by water and solid particles respectively. Negative symbol indicates that compressions are positive. β_w is substantially higher than β_s. Typical values for β_w and β_s are $3.4 \cdot 10^{-4}$ ºC^{-1} and $3.0 \cdot 10^{-5}$ ºC^{-1}. Water dilates almost one order of magnitude more than solid particles. Thermal dilation of water and solid will result in an internal volumetric expansion. The soil expansion is explained by a decrease in effective stress. Therefore, in a saturated porous media, if the total stress does not change, pore water pressure has to increase in order to reduce the effective stress. The increase in pore pressure will be proportional to the soil or rock stiffness. In the absence of external stresses, tensile effective stresses will develop. They may be able to overcome the tensile strength of the soil/rock and lead to a failure in tension, as observed in the experiment.

The volumetric strains associated with temperature changes are far from being negligible. For instance, for an increase of temperature from 4 ºC to 50 ºC, an average volumetric strain of 1.5% is derived for the water. It is concluded that the heat induced expansive strain may cause a substantial increase in water pressure in an impervious stiff rock. For a given rate of increase of temperature the attained pore water pressure will be the result of two competing mechanisms: the rate of increase of water volume, directly related to the rate of increase of temperature, and the rate of dissipation, governed by the permeability of the porous material (and also by the rock stiffness, in a process similar to the more familiar consolidation phenomenon). For a given rate of temperature increase, the lower the soil or rock permeability and the stiffer the soil or rock, the higher the pore water pressure developed. Stiff clays and, particularly, clayey rocks are therefore prone to develop significant temperature-induced pore water pressures. The pore water pressure was not measured in the simple experiment described but, interestingly, a small amount of liquid water – presumably escaped from the specimen- was also observed on the floor of the oven after the broken rock fragments were removed.

A second microwave experiment, with a totally different material, a discarded highly pervious porous stone was also run. The measured temperature is shown in Figure 1. No cracking noise was heard during heating and the specimen remained intact. Some water was also seen to escape from the stone. Unlike the previous experiment, when the temperature measured by the thermocouple sensor reached 100ºC, it remained constant at this temperature during the application of the power pulse. The water in this case behaved as it is to be expected in a free volume of water at atmospheric pressure: when the vaporization (boiling) temperature is reached, water evaporates in the pores and the boiling temperature remains constant, at 100ºC, because the heat input is “spent” in vaporizing the remaining liquid water.

Another interesting information of these experiments is that the pore water in the shale specimen increased its temperature well beyond 100ºC (it reached a peak value of 170ºC with no symptoms of decreasing during the power input phase). Pore water in the claystone is, in a significant proportion, adsorbed by clay minerals and this prevents its vaporization.

The phase diagram of water provides additional information on the conditions leading to the vaporization of water. At increasing pressure, the temperature for vaporization also increases. For instance, at a pressure of 120 m of water (1.2MPa), which is the pore pressure likely acting at the lower horizontal sliding surface of a representative section of Vaiont (Hendron and Patton, 1985) at the beginning of the failure-see the companion paper-, the boiling water temperature raises to 200ºC approximately.

Additional information on the effect of temperature on low permeability clayey rocks is the large scale Heating Experiment (HE), performed in the Monterri underground research laboratory (Switzerland), within the context of nuclear waste disposal research. The HE experiment is described in detail in EUR (2006) and in Muñoz (2007). The scheme (inset) given in Figure 2 summarizes the concept of the experiment. A cylindrical heater −which simulates the waste−, is located in a centered position in a vertical borehole excavated in Opalinus clay from the floor of a tunnel. A ring of compacted bentonite blocks was placed around the heater. Piezometers and temperature sensors were located at different radial distances. The response of two pairs of sensors (piezometer, thermocouple) located at a radial distance of 0.65 m from the axis of the borehole, at two different elevations (z = -5 m and z = -6.5 m; z = 0 m corresponds to the floor of the niche where the experiment was located), is shown in Figure 2. Maximum temperature at the bentonite-borehole wall contact was limited to 100 ºC (Figure 2a).

As temperature increased (at a rate of 0.25 ºC/day) until it reached a value of 35 ºC, pore water pressures also increased at measured rates of 0.012 MPa/day and 0.007 MPa/day in the two sensors, until they reached maximum values of 0.9 MPa and 0.55 MPa respectively. Note that a substantial pressure peak developed before the pore pressure began to decrease when the rate of temperature increase slowed down. The low permeability of Opalinus clay explains the continuous accumulation of pore pressure due to the relatively slow rate of increase of temperature. When the (permeability controlled) dissipation rate of excess water pressure dominated the process, the pore water pressure began to drop, at an essentially constant temperature.

The maximum excess water pressure recorded in this experiment (0.9 MPa) is relatively large, in absolute terms. Such water pressure is equivalent to the weight per unit area of a column of rock of a height of 40 m (if the rock had a bulk specific weight of 22.5 kN/m^3). The base of such a column (of Opalinus clay), if heated is in the location of the piezometer QB19/3 in Figure 2, will reach a zero vertical effective stress and, therefore, it will not be able to develop any frictional shear strength.

3 THE PROBLEM

A common observation in translational and rotational slides is that deformations are confined to sliding surfaces of very small thickness. Direct observations of sliding surfaces in clayey materials indicate that their thickness is very small, typically in the range of a few millimetres. One example is given in Figure 3, which shows a portion of the sliding surface of Cortes landslide (Alonso *et al.*, 1992). The sliding surface was easily identified, when it was exposed after a large excavation, because of its greenish-gray colour, in contrast with the brown tonalities of the marl layer, two meters thick, where it was embedded. Massive limestone strata, which essentially slid as a rigid body, covered the marl layer. The thickness of the striated layer ranged between 3 and 5 mm.

The thickness of shear bands has been reported by several authors (Morgenstern and Tchalenko, 1967; Roscoe, 1970; Vardoulakis, 1980; Scarpelli and Wood, 1982; Desrues, 1984). An important conclusion of basic research is that shear band thickness is related to a characteristic grain size. For instance, Vardoulakis (2002) proposes a value $e \approx 200 d_{50\%}$. Grain size analysis of specimens recovered from Vaiont sliding surface (Hendron & Patton, 1985; Tika & Hutchinson, 1999) indicate that $d_{50\%} \approx$ 0.01 mm. The reported direct observation at Cortes slide is not far from the thickness suggested by the preceding relationship. Vaiont landslide was significantly bigger than Cortes slide, but they had some similarities. In both cases rigid and massive limestone and marl banks slid on a fairly continuous layer of clay. It was reported by Hendron and Patton (1985) that, in Vaiont, this clay strata had a thickness in the order of one meter. It is expected, however, that the sliding surface itself had a reduced thickness, probably a few millimetres, as in Cortes.

Consider now in the sketch of Figure 4a a representative cross section of Vaiont and in Figure 4b the 1m thick clay layer where the sliding surface was located. The shear band proper will be located within the clay layer (Figure 4c). Its thickness is many orders of magnitude smaller than the horizontal and vertical dimensions of the slide. In fact, the thickness of the clay layer is no particularly relevant.

If the slide moves as a rigid body with a velocity v_{max}, shear straining, which will be concentrated on the shear band, will induce an average shearing strain rate of

$$\dot{\gamma} = \frac{v_{\max}}{2e} \tag{2}$$

where 2e is the thickness of the shear band. Therefore, during the sliding motion all the straining work will be concentrated inside the band. The volumetric deformation of the clay material which constitutes the band will be very small compared with the extremely large shear deformations induced by sliding on a thin clay band. Therefore, the rate of work input per unit volume of band material will be essentially given by

$$\dot{W} = \tau_f \dot{\gamma} = \frac{\tau_f v_{\max}}{2e} \tag{3}$$

where τ_f is the shear strength offered by the shear band. This work input will be transformed entirely in heat, following the first principle of thermodynamics. Therefore, the band will increase its temperature, and in view of the tests discussed before a pore water pressure, in excess of the initially existing, will develop. Note that the work of volumetric deformations are neglected compared with the shear work.

The excess pore pressure is essentially caused by the thermal dilation of the water. Therefore, despite its potential large effect in modifying effective stresses, the absolute amount of the increment of water volume in the band will be very small. Its dissipation will take place in the immediate vicinity of the band. In other words, the band and its “zone of influence” will have a small thickness (Figure 4d), similar to the thickness of the band itself.

It then becomes reasonable to assume that, for the purposes of investigating the behaviour of pore pressures in the band and its vicinity, the band is essentially a planar feature located within an infinite domain. The lateral extent of this band is very large compared with its thickness and, in addition, points within the band are similar to each other. Water and energy transfer out of the band will take place in a direction normal to the band. The problem of the interaction of the band and its surroundings becomes a one dimensional problem in which the spatial coordinate (z) is directed normal to the band plane (Figure 4d).

The equation of motion of the slide was derived in the companion paper for the two-wedge model (Equation 18 together with Equations 19 and 20). The slide velocity depends on the water pressures existing on the sliding surface (terms *U1* and *U2* in equations 18-20). But now pore water pressures will not depend only on the hydrostatic water conditions but on the additional pore pressures developed in the band as a result of its heating. These pore pressures depend on the work input into the shear band and therefore on the slide velocity which is the unknown variable of the problem. The problem of finding the pore water pressures in the band when it is sheared by means of the application of a boundary velocity will be approached by formulating the three conservation equations for solid, water and energy. They will be written for a general (three dimensional) case but the solution will be found for the one-dimensional case described before. The solution of this problem will enable also the calculation of temperatures in the shear band. The final step will be the solution of the equations of motion of the slide.

4 BALANCE EQUATIONS INSIDE AND OUTSIDE THE SHEAR BAND

Consider a point in a shear band (in general of argillaceous material) of thickness $2e$ surrounded by the rock substratum. The z axis follows the direction of the gradient of water pressure and temperature generated within the band (Figure 4d). In Pinyol and Alonso (2009) the balance equations for the thermo-hydro-mechanical processes taking place in the band and its immediate vicinity are given. They are summarized here together with the main assumptions introduced.

Heat balance

In a fast slide advective terms are negligible as well as heat conduction. These assumptions, which were checked numerically in the case of a fast planar slide leads to the following simplified heat balance equations:

$$\frac{\partial}{\partial t}\left(\rho c_m \theta\right) = \left(\rho c_m\right)\frac{\partial \theta}{\partial t} = H\left(t\right) \tag{4}$$

where the average storage capacity $\rho c_m = (1-n)\rho_s c_s + n\rho_w c_w$ is assumed to be constant. ρ is the density and *c* the specific heat (subscript *s* indicates solid particles and *w*, water). *n* is the band porosity, θ the band temperature and $H\left(t\right) = \dot{W}$ is the heat input due to the dissipation of the frictional work.

Mass balance equation

Mass balance of solid and water combine into the following relationship:

$$\frac{n}{\rho_w}\frac{D\rho_w}{Dt} + \frac{\left(1-n\right)}{\rho_s}\frac{D\rho_s}{Dt} + \mathbf{div}\cdot\left(\mathbf{v}\right) + \frac{1}{\rho_w} div\cdot\left(\rho_w \mathbf{q}\right) = 0 \tag{4}$$

where **v** is the velocity vectors of the particles and **q** the velocity vector of the water with respect to particles.

The following set of constitutive relationships are introduced:

- Water density:

$$\rho_w = \rho_w^0 \exp\left[\alpha_w\left(p_w - p_w^0\right) - \beta_w\left(\theta - \theta^0\right)\right] \tag{5}$$

Superscript (º) indentifies a reference state. The compressibility coefficient, α_w, against pore water pressure, p_w, changes and the thermal dilation coefficient, β_w, are assumed to be constant-.

- Solid density:

$$\rho_s = \rho_s^0 \exp\left[-\beta_s\left(\theta - \theta^0\right)\right] \tag{6}$$

where β_s is the thermal coefficient of solid particles. In Equation (6) solid grains are assumed to be incompressible.

- The volumetric strain rate ($\dot{\varepsilon}_{vol} = \mathbf{div}\cdot\left(\mathbf{v}\right)$) will be expressed in terms of changes in effective stress in the band by means of a compressibility coefficient m_v:

$$\mathbf{div}\cdot\left(\mathbf{v}\right) = -m_v\left(\frac{\partial \sigma_n}{\partial t} - \frac{\partial u_w}{\partial t}\right) \tag{7}$$

where u_w is the excess pore pressure over a constant (hydrostatic, for instance) water pressure.

In Equation (7), σ_n is the total normal stress against the band. Note that this stress, in average terms, will decrease on the upper wedge of Vaiont slide and it will be approximately constant on the lower wedge.

- Darcy's law provides **q**:

$$q = -\frac{k}{\rho_w g}\left[\mathrm{grad}\left(p_w\right) + \rho_w g\,\mathrm{grad}\left(z_g\right)\right] \simeq -\frac{k}{\gamma_w}\frac{\partial u_w}{\partial t} \tag{8}$$

where the hydraulic conductivity of the band, *k*, is assumed to be constant and z_g is the vertical coordinate. Specific weight of water $\gamma_w = \rho_w g$ has been introduced. Only the gradients of excess pore pressure are significant in this problem.

If Equations (5)-(8) are introduced in (4), material derivatives are approximated by the Eulerian ones and the heat balance equation (3) is taking into account, the following differential equation may be written for the shear band:

$$-\left[n\beta_w + (1-n)\beta_s\right]\frac{\partial\theta}{\partial t} + (n\alpha_w + m_v)\frac{\partial u_w}{\partial t} - \frac{k}{\gamma_w}\frac{\partial^2 u_w}{\partial z^2} = 0 \tag{9}$$

Outside the shear band no heat is generated and the balance equations of solid mass, water mass and energy lead to:

$$(n_r\alpha_w + m_{vr})\frac{\partial u_w}{\partial t} - \frac{k_r}{\gamma_w}\frac{\partial^2 u_w}{\partial z^2} = 0 \tag{10}$$

where the subscript r refers to the rock outside the band.

5 DYNAMICS OF VAIONT SLIDING GEOMETRY

The analysis of the infinite slide presented in Pinyol and Alonso (2009) is useful to understand the thermo-hydraulic process that takes place in a shear band and its effect of the overall slide motion. However, the geometry of the Vaiont slide introduces significant changes which will be presented here. The slide is now divided in two wedges (1 and 2), following the discussion presented in the companion paper.

The analysis follows the calculation procedure developed for the infinite slide: mass and energy balance have to be written for the shear bands limiting the two wedges and the overall dynamic equilibrium of the two wedges has to be satisfied.

Considerer in Figure 5 the geometry of Vaiont. The lower wedge (wedge 2), resting on a horizontal plane, supports (passively) the unstable upper wedge (wedge 1) which slides on a inclined plane. This geometry was used in the first paper for the analysis of static and dynamic equilibrium of the slope without considering the effect of water dilation due to the heat generated on the basal shear band.

Changes in geometry have to be considered in a dynamic analysis. Figure 5b indicates the evolving geometry of the slide when a displacement *s* is considered. The initial basal length of wedge 1 (L_1^0) is reduced to:

$$L_1 = L_1^0 - s \tag{11}$$

It is accepted that this displacement directly contributes to increasing the initial basal length of Wedge 2 (L_2^0) to:

$$L_2 = L_2^0 + s \tag{12}$$

Length *h* (see Figure 5) can be obtained, for a given displacement *s*, knowing that:

$$\frac{h}{L_1} = \frac{h^0}{L_1^0} \tag{13}$$

because triangles AVB and A'VB' are similar.

The volume of the wedge 1 for a given displacement can be obtained as

$$V_1 = \frac{1}{2} L_1 h \ \cos\left(\frac{\alpha}{2}\right) \tag{14}$$

Volume reduction of wedge 1 contributes to increase the volume of the wedge 2, by the same amount, and therefore its current volume becomes,

$$V_2 = V_2^0 + \left(V_1^0 - V_1\right) \tag{15}$$

Wedge weights (W_1 and W_2) and masses ($M_1 = W_1/g$ and $M_2 = W_2/g$; *g* is the gravity acceleration) can be computed from these volumes. A specific weight of the rock (γ_r = 23.5 kN/m^3) was used in calculations.

5.1.1 Balance equations

Mass and energy balance of the lower shear band and equilibrium conditions for the entire moving mass will be written separately for each wedge. Forcing the slide to move as a single unit, the governing equations of the movement of the landslide will be obtained.

The effective interaction forces across the common plane (VB'; see Figures 5 and 6) between the two wedges have two components, *N'* and *S*, normal and tangential to the plane. Forces due to hydrostatic pore water pressures acting P_{w_1}, P_{w_2}, $P_{w\mathrm{int}}$ and P_{wf} will be controlled by the reservoir water level which will be considered constant during the landslide.

Since the shear resistant forces of each wedge (T_1 and T_2) are different (although a unique frictional angle is considered, normal resultant forces on the basal planes, N'_1 and N'_2, need not to be equal), the work input into the bounding shear bands of the two wedges will be different. Therefore, two different values for the shear band temperature (θ_1 and θ_2) and for the excess pore water pressures (u_{w1} and u_{w2}) will be developed in the two wedges. Specific balance equations should be written for each one of the two wedges. To avoid confusions, each part of the shear band will be denoted by shear band 1 or 2 according to the wedge involved. The same thickness and material properties will be assumed for the two bands.

Consider first the one-dimensional balance equations already developed for a planar band and its vicinity. They will now be directly applied to wedge 1. The z_1-direction corresponds to the normal direction of the shear band 1. The generated heat (H_1) in the shear band 1 is expressed as

$$H_1(t) = \tau_{f1}(t)\frac{v_{\max}(t)}{2e} \text{ for } z_1 \in [-e, e] \tag{16}$$

The frictional strength (τ_{f1}) will be derived from equilibrium conditions as done previously for the infinite slide.

Neglecting conduction and diffusion of heat, heat balance in the shear band 1 reads

$$H_1(t) = \rho c_m \frac{\partial \theta_1(t)}{\partial t} \text{ for } z_1 \in [-e, e] \tag{17}$$

Mass balances inside and outside of the shear band 1 are, (see equations 9 and 10)

$$-\left[(1-n)\beta_s + \beta_w n\right]\frac{\partial \theta_1(t)}{\partial t} + \left[m_v + n\alpha_w\right]\frac{\partial u_{w1}(z_1,t)}{\partial t} - m_v\frac{\partial \sigma_{v1}(t)}{\partial t} = \frac{k}{\gamma_w}\frac{\partial^2 u_{w1}(z_1,t)}{\partial z_1^2} \text{ for } z_1 \in [-e, e] \tag{18a}$$

$$\left[m_{vr} + n_r\alpha_w\right]\frac{\partial u_{w1}(z_1,t)}{\partial t} - m_v\frac{\partial \sigma_{v1}(t)}{\partial t} = \frac{k_r}{\gamma_w}\frac{\partial^2 u_{w1}(z_1,t)}{\partial z_1^2} \text{ for } z_1 \in (-\infty, -e] \cup [e, \infty) \tag{18b}$$

Regarding Wedge 2, the generated heat can be expressed as

$$H_2(t) = \tau_{f2}(t)\frac{v_{\max}(t)}{2e} \text{ for } z_2 \in [-e, e] \tag{19}$$

valid in the normal direction (z_2) to the shear band 2. The heat balance will be given by

$$H_2(t) = \rho c_m \frac{\partial \theta_2(t)}{\partial t} \text{ for } z_2 \in [-e, e] \tag{20}$$

Likewise, mass balance inside and outside of the shear band 2 is written as

$$-\left[(1-n)\beta_s + \beta_w n\right]\frac{\partial \theta_2(t)}{\partial t} + \left[m_v + n\alpha_w\right]\frac{\partial u_{w2}(z_2,t)}{\partial t} - m_v\frac{\partial \sigma_{v2}(t)}{\partial t} = \frac{k}{\gamma_w}\frac{\partial^2 u_{w2}(z_2,t)}{\partial z_2^2} \text{ for } z \in [-e, e] \tag{21a}$$

$$\left[m_{vr} + n_r\alpha_w\right]\frac{\partial u_{w2}(z_2,t)}{\partial t} - m_v\frac{\partial \sigma_{v2}(t)}{\partial t} = \frac{k_r}{\gamma_w}\frac{\partial^2 u_{w2}(z_2,t)}{\partial z_2^2} \quad \text{for } z_2 \in (-\infty, -e] \cup [e, \infty) \tag{21b}$$

These expressions complete the balance equations for the two shear bands.

5.1.2 *Dynamic equilibrium of the two wedges*

Dynamic equilibrium equations were derived in the companion paper. They will be written for each wedge. For directions parallel and normal to the basal sliding plane are:

$$W_1(t)\sin(\alpha) - T_1(t) - N'_{\text{int}}(t)\cos\left(\tfrac{\alpha}{2}\right) - Q_{\text{int}}(t)\sin\left(\tfrac{\alpha}{2}\right) - P_{w\text{int}}c\,\text{os}\left(\tfrac{\alpha}{2}\right) = M_1(t)\frac{\mathrm{d}v_{\max}}{\mathrm{d}t} \tag{22a}$$

$$W_1(t)\cos(\alpha)-N_1'(t)+N'_{\text{int}}(t)\sin\left(\tfrac{\alpha}{2}\right)-Q_{\text{int}}(t)\cos\left(\tfrac{\alpha}{2}\right)+$$
$$+P_{w\text{int}}\sin\left(\tfrac{\alpha}{2}\right)-P_{w1}-u_{w1}(t)L_1(t)=0 \tag{22b}$$

The shear resistance force on the base of the wedge 1 (*T_1*) is expressed, following Mohr – Coulomb strength criterion, as:

$$T_1(t)=N_1'(t)\tan\left(\varphi_b'\right) \tag{23}$$

The mobilized shear force on the common plane between wedges is given by:

$$Q_{\text{int}}(t)=c_r'h(t)+N_{\text{int}}'(t)\tan\left(\varphi_r'\right) \tag{24}$$

where φ'_b is the effective residual friction angle of the sliding surface, c'_r is the effective cohesion of the rock, and φ'_r, the effective friction angle of the rock. The values of these strength parameters are indicated in Table 1. These values have been justified in the companion paper.

Table 1 Strength parameters of the sliding rock mass.

Sliding mass material			
Parameter	**Symbol**	**Value**	**Unit**
Cohesion	c_r'	762.24	MPa
Friction angle	φ_r'	38	º

The water pressure force due to the presence of a water table of height h_w acting against wedge 1 is:

$$P_{w1}=\frac{h_w^2\gamma_w}{2\sin(\alpha)} \tag{25}$$

The water pressure force acting against the right boundary of wedge 1 (Figure 6b) is calculated as:

$$P_{w\text{int}}=\frac{h_w^2\gamma_w}{2\cos(\alpha)} \tag{26}$$

and the value of P_{w2} is given by:

$$P_{w2}(t)=L_2(t)h_w\gamma_w \tag{37}$$

Dynamic equilibrium expressions for wedge 2 (parallel and normal to the slide direction, respectively) are

$$N'_{\mathrm{int}}(t)\cos\left(\tfrac{\alpha}{2}\right)-Q_{\mathrm{int}}(t)\sin\left(\tfrac{\alpha}{2}\right)-T_2(t)=M_2(t)\frac{\mathrm{d}v_{\max}}{\mathrm{d}t} \tag{28}$$

$$W_2(t)-N'_2(t)+N'_{\mathrm{int}}(t)\sin\left(\tfrac{\alpha}{2}\right)+Q_{\mathrm{int}}(t)\cos\left(\tfrac{\alpha}{2}\right)+P_{w\mathrm{int}}\sin\left(\tfrac{\alpha}{2}\right)+P_{wf}\sin(\beta)-P_{w2}(t)-u_{w2}(t)L_2(t)=0 \tag{29}$$

The shear resistance on the base of Wedge 2 (T_2) is given by

$$T_2(t)=N'_2(t)\tan(\varphi'_b) \tag{30}$$

Note that these equations depend on the displacement, *s*, travelled by the wedges.

If Equations (22) to (30) are properly combined, a single motion equation for the total slide mass is obtained as follows:

$$\begin{aligned}&t_{W_1}W_1(t)+t_{W_2}W_2(t)+t_{P_{w\mathrm{int}}}P_{w\mathrm{int}}+t_{P_{wf}}P_{wf}+t_{P_{w1}}P_{w1}+\\&+t_{P_{w2}}P_{w2}(t)+t_{u_{w1}}u_{w1}(t)L_1(t)+t_{u_{w2}}u_{w2}(t)L_2(t)+\\&+t_{c_r}c'_rh(t)=\left(t_{M_1}+t_{M_2}\right)\frac{\mathrm{d}v_{\max}(t)}{\mathrm{d}t}\end{aligned} \tag{31}$$

where the t_i coefficients depend on the section geometry and on the cohesive and frictional parameters of the materials involved as indicated in Appendix 1.

The strength acting on the basal sliding surface of the two wedges is found as the ratio of the total resistance forces T_1 or T_2 and the current base lengths L_1 or L_2. They are given by:

$$\begin{aligned}\tau_1(t)=&[r_{W_1}W_1(t)+r_{W_2}W_2(t)+r_{P_{w\mathrm{int}}}P_{w\mathrm{int}}+r_{P_{wf}}P_{wf}+r_{P_{w1}}P_{w1}+r_{P_{w2}}P_{w2}(t)+\\&+r_{u_{w1}}u_{w1}(t)L_1(t)+r_{u_{w2}}u_{w2}(t)L_2(t)+r_{c'_r}c'_rh(t)]/[r_{M_1}M_1(t)+r_{M_2}M_2(t)]\end{aligned} \tag{32}$$

$$\begin{aligned}\tau_2(t)=&[s_{W_1}W_1(t)+s_{W_2}W_2(t)+s_{P_{w\mathrm{int}}}P_{w\mathrm{int}}+s_{P_{wf}}P_{wf}+s_{P_{w1}}P_{w1}+s_{P_{w2}}P_{w2}(t)+\\&+s_{u_{w1}}u_{w1}(t)L_1(t)+s_{u_{w2}}u_{w2}(t)L_2(t)+s_{c_r}c'_rh(t)]/[s_{M_1}M_1(t)+s_{M_2}M_2(t)]\end{aligned} \tag{33}$$

where coefficients "*r*" and "s" are function of geometry and of wedge masses. They are collected in Appendix 1.

Summarizing the preceding results, the system of equations to be solved includes the balance equations for the two shear bands (2+2 equations) and the equation for the dynamic equilibrium of the entire landslide (one equation):

$$\left[(1-n)\beta_s+\beta_w n\right]\frac{H_1(t)}{(\rho c)_m}+\left[m_v+n\alpha_w\right]\frac{\partial u_{w1}(z_1,t)}{\partial t}-m_v\frac{\partial \sigma_{v1}(t)}{\partial t}=\frac{k}{\gamma_w}\frac{\partial^2 u_{w1}(z_1,t)}{\partial z_1^2}\quad \text{for } z_1\in[-e,e]$$

$$\left[m_{vr}+n_r\alpha_w\right]\frac{\partial u_{w1}(z_1,t)}{\partial t}-m_{vr}\frac{\partial \sigma_{v1}(t)}{\partial t}=\frac{k_r}{\gamma_w}\frac{\partial^2 u_{w1}(z_1,t)}{\partial z_1^2}\quad \text{for } z_1\in(-\infty,-e]\cup[e,\infty)$$

$$\left[(1-n)\beta\partial_s+\beta_w n\right]\frac{H_2(t)}{\rho c_m}+\left[m_v+n\alpha_w\right]\frac{\partial u_{w2}(z_2,t)}{\partial t}-m_v\frac{\partial \sigma_{v2}(t)}{\partial t}=\frac{k}{\gamma_w}\frac{\partial^2 u_{w2}(z_2,t)}{\partial z_2^2}\quad \text{for } z\in[-e,e] \quad (33\text{ a to e})$$

$$\left[m_{vr}+n_r\alpha_w\right]\frac{\partial u_{w2}(z_2,t)}{\partial t}-m_{vr}\frac{\partial \sigma_{v2}(t)}{\partial t}=\frac{k_r}{\gamma_w}\frac{\partial^2 u_{w2}(z_2,t)}{\partial z_2^2}\quad \text{for } z_2\in(-\infty,-e]\cup[e,\infty)$$

$$t_{w1}W_1(t)+t_{w2}W_2(t)+t_{P_{w\mathrm{int}}}P_{w\mathrm{int}}+t_{P_{wf}}P_{wf}+t_{P_{w1}}P_{w1}+t_{P_{w2}}P_{w2}(t)+$$

$$t_{u_{w1}}u_{w1}(t)L_1(t)+t_{u_{w2}}u_{w2}(t)L_2(t)+t_{c_r}c_r h(t)=\left(t_{M_1}+t_{M_1}\right)\frac{\mathrm{d}v_{\max}(t)}{\mathrm{d}t}$$

where the heat generation rates H_1 and H_2 are given by equations (16) and (19) and shear stresses τ_{1f} and τ_{2f}, by Equations (32) and (33) respectively.

6 COMPUTED RESULTS

The system of Equations (33) was solved and integrated by finite differences. Once in a situation of strict equilibrium the slide was made unstable by rising 10 cm the water level in the reservoir. A forward Euler method of integration was programmed. Figures 7 to 10 presents some results. Calculation ended when the slide reached a displacement of 400 m. The physical explanation of phenomena taking place in the shear band and the response of the slide can be summarized as follows. As soon as wedge is unstable the dynamic equilibrium allows the calculation of some sliding velocity. This velocity and the current shear stress at the shear ban provides some heat input which is introduced into the combined thermo-hydro-mechanical balance equation of the shear band and its vicinity. The set of partial differential equation and a new pore pressure in the shear band is calculated. In the calculations made, the pore pressure on the band axis (which is maximum) is used to modify again the equilibrium conditions and to calculate a new sliding velocity. Velocity and mobilized strength in the band provide a new work input (heat) and the calculation resumes. Results for a “base case” are presented first. The constitutive parameters used in this case are given in Table 2. This set of parameters approximates the case of Vaiont. Note that most of the parameters are physical constants.

No precise laboratory information on the permeability of the clay sliding surface seems to be available. Hendron and Patton (1985) use the value k = 1.6·10^{-10} m/s in their analysis. Vardoulakis (2002) uses k = 1.1·10^{-11} m/s. The high plasticity values consistently measured and the presence of montmorillonite probably favors a low clay permeability. A value k = 1.0·10^{-11} m/s was selected here as a base case. Shear band permeability is one of the key parameters of the model. It is subjected to high uncertainty. A sensitivity analysis, discussed later, was performed to analyze the effect of changing clay permeability. An estimated oedometric coefficient of compressibility equal to $5 \cdot 10^{-10}$ 1/Pa has been used. As it will show later, changes in this parameter between an acceptable range has no a relevant effect on the slide development. Band thickness, an important parameter, was unknown. However, this point will be discussed at length later.

Table 2. Dynamic analysis of Vaiont Parameters for the "Base case".

Parameter	**Symbol**	**Value**	**Unit**
Water			
Density	ρ_w	1000	kg/m^3
Coefficient of compressibility	α_w	$5 \cdot 10^{-10}$	1/Pa
Thermal expansion coefficient	β_w	$3.42 \cdot 10^{-4}$	1/ºC
Specific heat	c_w	$4.186 \cdot 10^{3}$ 1.0	J/kg·ºC cal/ kg·ºC
Solid particles			
Density	ρ_s	2700	kg/m^3
Thermal expansion coefficient	β_s	$3 \cdot 10^{-5}$	1/ºC
Specific heat	c_s	$8.372 \cdot 10^{2}$ 0.20	J/kg·ºC cal/ kg·ºC
Shear band material			
Porosity	n	0.2	-
Permeability	k	$1 \cdot 10^{-11}$	m/s
Compressibility coefficient	m_v	$1.5 \cdot 10^{-9}$	1/Pa
Friction angle (residual)	φ'	12	º
Sliding mass material			

Density	ρ_r	2350	kg/m^3

Calculated isochrones of excess pore water pressure in the shear band below Wedges 1 and 2 are given in Figure 7 for the first 12 seconds of motion, when the slide velocity was 7 m/s. The average excess pore water pressure reached, at t = 10 s, maximum values of 1.7 MPa and 4.5 MPa under wedges 1 and 2 respectively. Note that the unloading associated to the loss of weight of Wedge 1 results in a small pore pressure reduction outside the shear band. This has no effect on the motion, which is controlled by the maximum pore pressures on the center of the band. At t = 10 s the available shear strength in the center of the shear band was already very small and the heat generated (and the associated pore pressure build-up) decreased sharply. As a result, the pore pressure dissipation towards the surrounding soil dominated the following time steps. This explanation can be followed in more detail in Figure 8 where global performance variables for the entire slope have been plotted against time for Wedge 1. The slide reaches a displacement of 230 m 30 s after the initiation of the motion (Figure 8f). At this time the velocity is 27 m/s (close to 100 km/h). These are values consistent with field observations.

Further insight is provided by the evolution of temperature, the drop in strength and the work (or heat) input into the shear band. The maximum temperature calculated in this case is slightly higher than 100ºC. The drop of shear strength is rapid from t = 7 s to t = 14 s. The work performed increases first during this period due to the rapid increase in velocity but it later decays because of the very low value of shear strength. The entire behaviour of the band and, hence, of the landslide, depends in a fully coupled manner on the mass and heat transfer phenomena in the thin shear band and its immediate vicinity.

Changing the permeability of the shear band leads to significant changes in behaviour. It can be checked that a more impervious band leads to minor changes, when compared with the base case. When it is made more pervious pore water pressure dissipation becomes more significant and the effective normal stress (and the shear strength) maintains higher values. The slide also accelerates fast and the high velocities coupled with relatively higher shear strengths lead to larger heat inputs into the band and to higher temperatures. These effects can be followed in Figure 9, where the case for $k = 1 \cdot 10^{-10}$ m/s has been represented.

The effect of the permeability of the shear band can be analyzed in more detail in the comparison plots given in Figure 10. When the band has permeability values of 10^{-8} m/s and below, flow-induced dissipation in the band reduces significantly excess pore pressures (Figure 10b) and the shear strength does not decrease to values which could explain the high velocity reached by the landslide (Figure 10a). Values of band permeability lower than 10^{-9} m/s could all explain the high velocities reached by Vaiont if the remaining parameters are accepted as truly representative of field conditions.

It is concluded that clay permeability in the vicinity of 10^{-8} m/s to 10^{-9} m/s is a critical threshold to determine if the slide can or cannot reach very high velocities. Permeability values below the threshold lead to a catastrophic rapid failure; higher values lead to moderate slide velocities. Interestingly, Figure 10 also points out that for some intermediate permeability values, within the "fast" range, temperature in the band may reach very high values, which may not only vaporize the water but even melting the clay to some degree. If the permeability is lower, calculated temperatures become moderate, lower than 100ºC. This was apparently the case for the estimated permeability of the montmorillonite-rich clay of Vaiont sliding surface.

The discussion is necessarily more complex because shear band permeability is only one of the parameters controlling the development of pore pressures. Relevant parameters are also the band thickness and its stiffness. To some extent permeability and band thickness provide the same information: both are related to the grain size distribution. Narrow or, alternatively, thick shear bands are expected in impervious or pervious materials, respectively. Stiffness is a different type of property and rock-like or soil-like materials may be found for the same mineralogy and grain size distribution. An analysis of the combined effect of permeability and stiffness has been made and will be discussed immediately. But, before, we will examine a very important practical issue, namely, the effect of changing the size of the slide on its dynamic behaviour.

7 SCALE EFFECTS

Vaiont was a very large landslide (a mobilized volume close to 300 million m^3 was estimated). A slide 100 times smaller is still a very large landslide. For instance, the 5 million m^3 Cortes landslide, described in Alonso *et al*. (1995), posed a significant threat to the 100 m high Cortes concrete arch dam. Its overall dimensions (length, height) were roughly 1/10 of Vaiont dimensions. Moreover, many dangerous rock and soil slides described in the literature are one order of magnitude smaller than Cortes slide. Vaiont was an extreme case, of very rare occurrence, on a world basis. Therefore, a relevant question is: Is the velocity reached by Vaiont also a common occurrence or, at least, a real possibility in smaller and much more frequent landslides?

A comprehensive answer to this question would require a lengthy analysis of the dynamic behaviour of different types of landslides. But a simple answer can be given if the main characteristics of Vaiont (a displacement type of motion involving a mass of rigid rock, sliding on a clay layer) are maintained and the geometrical dimensions are reduced without any further change in material properties or geometrical arrangement. In fact, if all the dimensions of Vaiont are reduced by a factor of 10, a landslide very similar to Cortes slide is obtained. If this slide becomes (slightly) unstable, how would it evolve if heat-induced water pressure develops at the sliding surface?

A new case has been run, modifying the scale of the Vaiont landslide. The new geometry is defined by reducing the dimensions (lengths and heights) of wedges 1 and 2 (Figure 5a) by a factor 10. The water level was located at the same relative position and the cohesion of the central shearing plane was reduced to bring the slope to the state of strict equilibrium. The remaining properties are also given in Table 2. The motion was triggered by a slight increase (1 cm) of the water level in the lower wedge.

The calculated response of this slide is shown in Figure 11 for a base case (k = $1 \cdot 10^{-11}$ m/s). Calculations were run in time until the slide reached a displacement of 50 m. The calculated heat input into the shear band and the maximum excess pore pressures are now one order of magnitude smaller than in the previous case. As a result, the temperature increase of the band is very moderate (3.5ºC). The shear strength, however, is lost after a few seconds and the slide is able to reach a significant velocity. A maximum value of 9 m/s is obtained at the end of the calculation period. The implication is that this reduced slide may be also dangerous, if the circumstances of the analysis are fulfilled in practice.

As before, band permeability is a key parameter to control the response of the slide. This is shown in detail in Figure 12, which is similar to Figure 10. Band values of permeability of 10^{-9} m/s and larger do not trigger any heat-induced effect. This threshold is obviously associated with the band thickness used in calculations (5 mm), but a more consistent analysis is given below. Since the two-wedge mechanism analyzed has a self-equilibrating mechanism, the small initial triggering effect (increasing water pressure in the shear band by 10 cm) is, in "absorbed" by the changing geometry and the slide comes to rest after some small displacement. If the permeability decreases below this threshold the coupled thermo-hydro-mechanical processes taking place in the band result in a progressive accumulation of pore pressures and in an accelerated slide motion. The critical permeability is the same, irrespective of the size of the slide. Temperature increase in the band, when the slide accelerates ($k<10^{-9}$ m/s) is now quite moderate in most cases. However, for the reasons already explained, there exists some specific k values (in the vicinity of $k = 10^{-10}$ m/s) which result in a strong dissipation of energy at the band and, accordingly, in a significant temperature increase (37ºC are obtained -Figure 12c- at the end of the calculation interval).

Summarizing, smaller slides, similar in shape to the Vaiont case, may also reach significant velocities. It appears that band permeability is a key parameter controlling slide acceleration. Below a certain threshold value (around $k = 10^{-9}$ m/s for the geometry and parameters selected for the case analyzed) the slide may reach a high velocity. It appears that this threshold k value of the sliding band is independent of the size of the slide. However, when the size of the slide decreases, the generated band excess pore pressures and temperatures reduce. In fact, it appears that for slides having the size of a "reduced Vaiont" by a factor of 10 in the scale of dimensions, maximum temperature increments in the shear band will be no more than a few degrees. It turns out that the generated temperature depends also strongly on the thickness of the shear band. Before general conclusions are reached in this regard, it is convenient to perform a sensitivity analysis of the calculated solution when the thickness, permeability and stiffness of the band are varied between acceptable limits.

8 DISCUSSION

A better insight into the physics of the problem is gained if a sensitivity analysis of the main controlling factors is performed. Consider first the issue of the combined effect of band permeability and band thickness. In view of previous results, permeability values in the range 10^{-13} to 10^{-9} m/s and band thickness varying between 0.5 mm and 50 mm have been selected. Then, for each combination (*k,* 2*e*) the movement of the slide with scaled geometry of Vaiont has been calculated. The calculated maximum velocities and band temperatures, for a maximum run out of 50 m, are given in Tables 3 and 4.

Table 3 Calculated maximum velocities of the scaled Vaiont slide for the set of band permeability and band thickness indicated.

Thickness (mm) \ Permeability (m/s)	10^{-13}	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
0.5	9.7	9.3	5.9	0.0	0.0	0.0	0.0
1	10.7	9.4	8.2	0.0	0.0	0.0	0.0
5	11.0	11.0	9.3	5.9	0.0	0.0	0.0
10	10.7	10.7	10.2	7.7	0.0	0.0	0.0
20	10.1	10.1	10.1	8.7	0.0	0.0	0.0
50	8.8	8.8	8.8	8.7	4.4	0.0	0.0

Table 4 Calculated maximum temperatures in the shear band of the scaled Vaiont slide for the combination of band permeability and band thickness indicated.

Thickness (mm) \ Permeability (m/s)	10^{-13}	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
0.1	94.0	787.0	10.4	10.1	10.0	10.0	10.0
0.5	13.6	42.5	305.0	10.0	10.1	10.0	10.0
1	13.1	17.4	71.4	10.0	10.0	10.0	10.0
5	12.8	12.8	13.5	38.7	10.0	10.0	10.0
10	12.6	10.7	12.6	17.1	10.0	10.0	10.0
20	12.2	12.2	12.2	12.6	10.0	10.0	10.0
50	11.6	11.6	11.6	11.6	12.9	10.0	10.0

For values of permeability of 10^{-8} m/s and larger calculated maximum velocities are negligible. In those cases, band temperatures remain at the initial selected value (10ºC) (Table 4).

Since particle size controls band thickness and, to some extent, band permeability, it is accepted that the two parameters are related. In other words, they convey similar information. This is approximately reflected in the shaded band shown in Tables 3 and 4. Combinations outside the central diagonal (a very impervious material developing a thick shear band, for instance) seem unlikely situations. Values in Tables 3 and 4 have been plotted in Figures 13 and 14. They provide the slide response for a wide range of situations in terms of band permeability and band thickness. The trend shown by results in Figure 13 is represented by means of a continuum line. It provides a relationship between slide velocity and band permeability and it includes already the information on the expected band thickness. The figure reinforces the idea of a threshold permeability separating conditions leading to a fast acceleration of the slide and conditions which lead to a halt in the motion after the initial instability.

Calculated temperatures are shown in Figure 14. Temperatures reach a maximum for some intermediate value of permeability, which is related to the band thickness. The maximum value calculated, for the range of band thicknesses examined (0.5 – 50 mm) is 305ºC, for k = 10^{-11} m/s and $2e$ = 0.5 mm, a combination barely outside the band of likely values in Table 4. It is concluded that slides of the size of a tenth of the scale of Vaiont (say slides of a few million cubic meters) develop, in most circumstances, a very moderate increase in band temperature.

A similar analysis, performed now for the actual dimensions of Vaiont is given in Tables 5 and 6 as well as in Figures 15 and 16. Results are qualitatively similar. Maximum velocities now increase to values in the order of 30 m/s (for a specified displacement of 400 m, against 50 m in the scaled case) provided the band permeability is below a threshold value, which is again in the range 10^{-9} m/s to 10^{-8} m/s. The "zero" velocities shown in Table 5 for the high values of permeability are, in fact, very low velocities. In fact, in all the calculated cases, when the slide is made initially unstable by means of a small increment (10 cm) of the level of the phreatic surface, the slide accelerates. However, if no significant excess pore pressures are developed, the change in geometry as the slide displaces forward is able to compensate for the initial unbalance of driving and resisting forces and the velocity reduces again to zero. This behaviour is illustrated in Figure 17. These cases, which imply a drained shear band, correspond to the zero values annotated in Table 5

Table 5 Calculated maximum velocities of Vaiont slide for the set of band permeability and band thickness indicated.

Permeability (m/s) / Thickness (mm)	10^{-13}	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
0.5	29.00	29.17	26.50	0.01	0.00	0.00	0.00
1	29.75	29.00	28.00	11.90	0.00	0.00	0.00
5	33.90	31.11	29.15	26.37	0.00	0.00	0.00
10	33.00	33.00	29.58	27.92	10.90	0.00	0.00
20	32.60	33.00	31.94	28.64	21.11	0.00	0.00
50	32.00	33.00	31.97	30.29	26.16	0.00	0.00

Table 6 Calculated maximum temperatures of Vaiont slide for the set of band permeability and band thickness indicated.

Thickness (mm) \ Permeability (m/s)	10^{-13}	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
0.5	108.0	1011.0	9280.0	65.51	13.13	10.00	10.00
1	43.0	257.0	2390.0	20461.0	11.74	10.00	10.00
5	41.6	41.6	106.1	956.1	12.18	10.32	10.00
10	42.0	41.6	42.5	249.0	2150.6	10.17	10.13
20	40.28	42.0	40.2	68.9	565.27	10.40	10.07
50	39.30	39.0	38.2	38.5	101.13	10.23	10.03

Calculated temperatures, shown in Figure 16, are now higher. Again some critical *k* values lead to a very high work input into the shear band and a fast increase of band temperature. In these cases, other phenomena, not included in the formulation (water vaporization, rock melting) may arise. It seems, however, that estimated conditions at Vaiont led to a moderate increase in band temperature, in the order of 100ºC.

Finally, a sensitivity analysis on the effect of band stiffness was performed. Band permeability and thickness were fixed at 10^{-11} m/s and 5 mm respectively. These values (see Figure 15) are located in the "fast region" of the motion. m_v was changed from 10^{-1} MPa^{-1} (a relatively soft band material) to 10^{-3} MPa^{-1} (a stiff material). Calculated velocities were in all cases very close to the average value (30 m/s) found for the base case.

The preceding discussion suggests that the permeability of the band (and its immediate vicinity) is the key parameter controlling the dynamics of the motion.

9 CONCLUSIONS

A seemingly convincing explanation for the accelerated motion of Vaiont relies on the development of excess pore pressures generated by the temperature increase of the sliding surface. This is a consequence of the slide motion itself. A key condition to explain the phenomenon is the existence of a basal sliding plane located in a layer of low permeability high plasticity clay in residual conditions. Then, the self-feeding mechanism of pore pressure generation in the sliding surface may eventually lead to very high sliding velocities (> 30 m/s) which are reached in a few seconds (~15 seconds) even if proper account is given to the self-stabilizing evolving geometry of the slide and even if progressive failure mechanisms, potentially acting on internal shearing surfaces, are not considered.

Slide geometry and strength properties of the sliding surface(s) are not enough to understand the dynamics of Vaiont. Three parameters have been found important to explain the motion: the thickness of the sliding band, its permeability and its (confined) stiffness. Permeability is the major player. This is because it includes, in an approximate manner, the information provided by the shear band thickness. In fact, shear band thickness and permeability are both related to the particle size distribution of the band material. The sensitivity analysis performed has also shown that below a certain permeability threshold (established around 10^{-8} to 10^{-9} m/s), the maximum pore pressure development in the shear band, which is the value controlling the shear strength, is not much affected by the band thickness, within a reasonable range of values. Above this threshold permeability value, pore pressure dissipation is enough to de-activate the process of pore pressure build-up and, therefore, the slide does not accelerate. In other words, the threshold permeability identified marks the transition from a potentially risky slide to a safe one. Of course, this conclusion is valid for the slide geometry analyzed (cross-section 5 of Vaiont) and it should not be extended to other sliding configuration without further analysis. The one-dimensional compressibility coefficient, m_v, of the band material has a very limited effect, unless it reaches very high values, typical of soft clay, an unlikely situation in practice.

In very large landslides (the case of Vaiont), when conditions for accelerated motion exist, there are critical combinations of band permeability and band thicknesses that result in a substantial and rapid increase in shear band temperature. This is a natural outcome of the formulation and it is a consequence of the existence of small –but not negligible- shear strength in the shear band and an increasing shear strain rate as the sliding velocity increases. The permeability of the band in these cases is low enough to maintain a significant pore pressure in the band but high enough to maintain a non negligible effective normal stress. The calculated temperatures (hundreds or even thousands of ºC) are enough to induce water vaporization and rock melting. These phenomena are not covered by the formulation developed, which only explains water pressure increase as a result of thermal dilation effects. The estimated sliding band parameters, in the case of Vaiont, lead to a moderate increase in temperature (< 100 ºC), which is not able to vaporize the interstitial pore water of the clayey band.

When the size of the slide decreases the temperature generated in the band decreases also because the work input into the band decreases. A reduction of Vaiont dimensions by a factor of 1/10 still leads to a very large slide (a few million cubic meters), which has been analyzed. Sliding band temperatures are substantially lower in this case. For an impervious band ($k < 10^{-9}$ m/s) maximum temperature increments are moderate (a few degrees). Even in extreme cases, for critical *k* values of the sliding band, it is unlikely for the temperature to raise more than 100 ºC. Vaporization (and certainly rock melting) is excluded in these cases. Since most slides do not reach, in practice, such a volume (a few million cubic meters), water vaporization and rock melting are extreme phenomena of rare occurrence.

The fact that temperature increases will likely remain moderate or low in most slides does not prevent, however, the development of significant velocities. The reason is that the reduced increase in pore water pressure in those cases is also matched by a reduced normal effective stress on the sliding surface. Therefore, the condition of zero effective stress may also be reached during motion. However, the smaller the slide is, the shorter the sliding path necessary to substantially change its geometry, to evolve to another type of motion, or to be affected by another geometrical restriction to its motion. These considerations, added to the reduced momentum of the slide, tend to limit the danger associated with smaller slides.

Although the two-wedge analysis described provides a reasonable explanation for the final catastrophic motion of the slide, the previous history of landslide creep-like displacements (see the companion paper) cannot possibly be reproduced with the model developed here . Other phenomena such as viscous strength components at the failure surface or the strength degradation of the rock mass could be invoked to approximate the measured velocities prior to failure. Additional limitations can be identified both in the model and in the available information: the geometry has been maintained two-dimensional and as simple as possible; pore water pressures prevailing at the failure surface were never measured; the effect of previous rainfall regime is essentially unknown; the actual conditions (in particular, the continuity of the high plasticity clay layer) of a significant proportion of the sliding surface remain buried by the slide and are essentially unknown, etc. Therefore, complexities and uncertainties around Vaiont are far from being resolved. However, it remains as a fascinating case and a permanent source of inspiration in the field of landslide analysis.

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Appendix Parameters of the balance equations for the dynamic analysis of two interacting wedges

Parameters which complete the dynamic equilibrium equation of the two wedges (Equation 31) are:

$$t_{w_1} = \left(-\tan\varphi_b'\tan\varphi_r' - \tan^2\varphi_b'\right)\sin\left(\tfrac{\alpha}{2}\right)\cos\alpha + \left(\tan\varphi_r' + \tan\varphi_b'\right)\sin\left(\tfrac{\alpha}{2}\right)\sin\alpha +$$
$$\left(\tan\varphi_b' - \tan^2\varphi_b'\tan\varphi_r'\right)\cos\left(\tfrac{\alpha}{2}\right)\cos\alpha + \tan\varphi_b'\tan\varphi_r'\cos\left(\tfrac{\alpha}{2}\right)\sin\alpha - \cos\alpha\,\sin\alpha$$

$$t_{w_2} = \left(\tan\varphi_b' - \tan^2\varphi_b'\tan\varphi_r'\right)\cos\left(\tfrac{\alpha}{2}\right) + \left(\tan^2\varphi_b' + \tan\varphi_b'\tan\varphi_r'\right)\sin\left(\tfrac{\alpha}{2}\right)$$

$$t_{P_{w\mathrm{int}}} = -2\tan\varphi_r'\sin\left(\tfrac{\alpha}{2}\right)\cos\left(\tfrac{\alpha}{2}\right)\left(1+\tan^2\varphi_b'\right)$$

$$t_{P_{wf}} = \left(\tan^2\varphi_b' + \tan\varphi_b'\tan\varphi_r'\right)\sin\left(\tfrac{\alpha}{2}\right)\cos\beta + \left(1-\tan\varphi_b'\tan\varphi_r'\right)\cos\left(\tfrac{\alpha}{2}\right)\sin\beta +$$
$$\left(\tan\varphi_b' - \tan^2\varphi_b'\tan\varphi_r'\right)\cos\left(\tfrac{\alpha}{2}\right)\cos\beta + \left(\tan\varphi_b' + \tan\varphi_r'\right)\sin\left(\tfrac{\alpha}{2}\right)\sin\beta$$

$$t_{P_{w1}} = \tan^2\varphi_b'\tan\varphi_r'\cos\left(\tfrac{\alpha}{2}\right) + \tan\varphi_b'\tan\varphi_r'\sin\left(\tfrac{\alpha}{2}\right) +$$
$$+\tan^2\varphi_b'\sin\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b'\cos\left(\tfrac{\alpha}{2}\right)$$

$$t_{P_{w2}} = \tan^2\varphi_b'\tan\varphi_r'\cos\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b'\tan\varphi_r'\sin\left(\tfrac{\alpha}{2}\right) -$$
$$-\tan^2\varphi_b'\sin\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b'\cos\left(\tfrac{\alpha}{2}\right)$$

$$t_{u_{w1}} = \left(\tan^2\varphi_b'\tan\varphi_r' - \tan\varphi_b'\right)\cos\left(\tfrac{\alpha}{2}\right) + \left(\tan^2\varphi_b' + \tan\varphi_b'\tan\varphi_r'\right)\sin\left(\tfrac{\alpha}{2}\right)$$

$$t_{u_{w2}} = \left(\tan^2\varphi_b'\tan\varphi_r' - \tan\varphi_b'\right)\cos\left(\tfrac{\alpha}{2}\right) - \left(\tan^2\varphi_b' + \tan\varphi_b'\tan\varphi_r'\right)\sin\left(\tfrac{\alpha}{2}\right)$$

$$t_{c_r} = \cos\left(\tfrac{\alpha}{2}\right)\sin\left(\tfrac{\alpha}{2}\right)\left(1+\tan^2\varphi_b'\tan\varphi_r'\right)$$

$$t_{M_1} = \sin\left(\tfrac{\alpha}{2}\right)\left(\tan\varphi_r' + \tan\varphi_b'\right) + \cos\left(\tfrac{\alpha}{2}\right)\left(\tan\varphi_r'\tan\varphi_b' - 1\right)$$

$$t_{M_2} = -\sin\left(\tfrac{\alpha}{2}\right)\left(\tan\varphi_r' + \tan\varphi_b'\right) + \cos\left(\tfrac{\alpha}{2}\right)\left(\tan\varphi_r'\tan\varphi_b' - 1\right)$$

Coefficients “*r*” and “*s*” in equations (32) and (33) are given by:

$$r_{W_1} = M_1\cos\alpha\left(\cos\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \tan\varphi_b'\sin\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b'\cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right) +$$
$$M_2\left(\cos\alpha\sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \sin\left(\tfrac{\alpha}{2}\right)\sin\alpha - \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\sin\alpha + \cos\alpha\cos\left(\tfrac{\alpha}{2}\right)\right)$$

$$r_{W_2} = M_1\tan\varphi_b'\left(\sin\left(\tfrac{\alpha}{2}\right) - \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right)$$

$$r_{P_{w\mathrm{int}}} = M_1\sin\left(\tfrac{\alpha}{2}\right)\left(\cos\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - 2\cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\tan\varphi_b'\right) + M_2\tan\varphi_r$$

$$r_{P_{wf}} = M_1\cos\beta\tan\varphi_b'\left(\sin\left(\tfrac{\alpha}{2}\right) - \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right)$$

$$r_{P_{w1}} = M_1\left(\cos\left(\tfrac{\alpha}{2}\right) + \tan\varphi_r' \sin\left(\tfrac{\alpha}{2}\right) + \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) + \tan\varphi_b' \tan\varphi_r' \cos\left(\tfrac{\alpha}{2}\right)\right) -$$
$$- M_2\left(\cos\left(\tfrac{\alpha}{2}\right) + \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right)$$

$$r_{P_{w2}} = M_1 \tan\varphi_b' \left(\tan\varphi_r' \cos\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\right)$$

$$r_{u_{w1}} = M_1\left(\sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) - \cos\left(\tfrac{\alpha}{2}\right) + \tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right) -$$
$$- M_2\left(\sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \cos\left(\tfrac{\alpha}{2}\right)\right)$$

$$r_{u_{w2}} = \mathrm{M}_1 \tan\varphi_b' \left(\cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \sin\left(\tfrac{\alpha}{2}\right)\right)$$

$$r_{c_r} = M_1\left(\sin^2\left(\tfrac{\alpha}{2}\right) - \cos^2\left(\tfrac{\alpha}{2}\right) + 2\sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\right) - M_2$$

$$r_{M_1} = \cos\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'$$

$$r_{M_2} = -\tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) + \cos\left(\tfrac{\alpha}{2}\right) + \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'$$

and

$$s_{W_1} = M_2 (\cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_b' \sin\alpha + \sin\left(\tfrac{\alpha}{2}\right)\sin\alpha - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_b' \cos\alpha -$$
$$\cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' \tan\varphi_b' \cos\alpha)$$

$$s_{W_2} = M_1\left(\cos\left(\tfrac{\alpha}{2}\right) + \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right) +$$
$$+ M_2\left(\tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \cos\left(\tfrac{\alpha}{2}\right)\right)$$

$$s_{P_{w\mathrm{int}}} = M_1\left(\sin\left(\tfrac{\alpha}{2}\right)\cos\left(\tfrac{\alpha}{2}\right) - \sin^2\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right) +$$
$$+ M_2 \tan\varphi_r' \left(-\cos^2\left(\tfrac{\alpha}{2}\right) + \sin^2\left(\tfrac{\alpha}{2}\right) - 2\cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right)\right)$$

$$s_{P_{w2}} = M_1 \cos\beta\left(\cos\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right) +$$
$$+ M_2 \cos\beta\left(\cos\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) + \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right)$$

$$s_{P_{w1}} = M_2 \tan\varphi_b' \left(\sin\left(\tfrac{\alpha}{2}\right) + \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right)$$

$$s_{P_{w2}} = M_1\left(\sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \cos\left(\tfrac{\alpha}{2}\right)\right) +$$
$$+ M_2 (\tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \cos\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right))$$

$$s_{u_{w1}} = M_2 \tan\varphi_b' \left(\sin\left(\tfrac{\alpha}{2}\right) + \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'\right)$$

$$s_{u_{w2}} = M_1 \tan\varphi_r' \left(\sin\left(\tfrac{\alpha}{2}\right) - \cos\left(\tfrac{\alpha}{2}\right)\right) +$$
$$+ M_2\left(\tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) - \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' - \cos\left(\tfrac{\alpha}{2}\right)\right)$$

$$s_{c_r} = M_1 + M_2\left(-\sin^2\left(\tfrac{\alpha}{2}\right) + \cos^2\left(\tfrac{\alpha}{2}\right) + 2\sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\right)$$

$$s_{M_1} = \cos\left(\tfrac{\alpha}{2}\right) - \sin\tfrac{\alpha}{2}\tan\varphi_r' - \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) - \tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'$$

$$s_{M_2} = -\tan\varphi_b' \cos\left(\tfrac{\alpha}{2}\right)\tan\varphi_r' + \tan\varphi_b' \sin\left(\tfrac{\alpha}{2}\right) + \cos\left(\tfrac{\alpha}{2}\right) + \sin\left(\tfrac{\alpha}{2}\right)\tan\varphi_r'$$