

## Stability analysis of solid oxide fuel cell systems

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### 1. Introduction

Solid oxide fuel cells (SOFC), with entirely solid structure and high operating temperatures, have attracted research interest in recent years. Unlike other types of fuel cells, low electrode corrosion and low electrolyte losses are assumed due to its solid structure. Furthermore, the high operating temperatures enable SOFC to reach up to 50% to 65% efficiency with excellent impurity tolerance. However, there are several degradation mechanisms in SOFC, such as electrode delamination, electrolyte cracking, electrode poisoning, etc [1]. Most of these degradations are related with the operation conditions, which can be optimized by appropriate control. Since most control algorithms are developed based on the mathematical models, it is important to obtain SOFC control-oriented models. Therefore, this paper aims to develop a SOFC control-oriented model, including the dynamics of inlet manifold, SOFC stack and outlet manifold. Moreover, equilibrium points are characterized and a stability around these equilibrium points analysis is performed. This information can provide guidelines for control strategies design.

### 2. Model of solid oxide fuel cell system

The modelled system is a planar SOFC system, where the dry air (contained 79% N<sub>2</sub> and 21% O<sub>2</sub>) is fed into cathode channel through a compressor. The inlet manifold is the pipeline connected between the pump and SOFC stack. Due to the compressed process, the air temperature between two sides of the inlet manifold is different. For the SOFC stack, oxygen in the air goes through the cathode to the anode reacted with hydrogen. The unreacted gas in the cathode channel will be exhausted by the outlet manifold. In the anode side, the fuel is combined of 97% H<sub>2</sub>, 2% H<sub>2</sub>O, 0.5% CO, and 0.5% CO<sub>2</sub>, which is compressed and stored in a tank. A schematic of the SOFC system is shown in Figure. 1. The proposed control-oriented model is based on the system mass, electrochemical and thermal power balance in the inlet manifold, the outlet manifold and the stack.

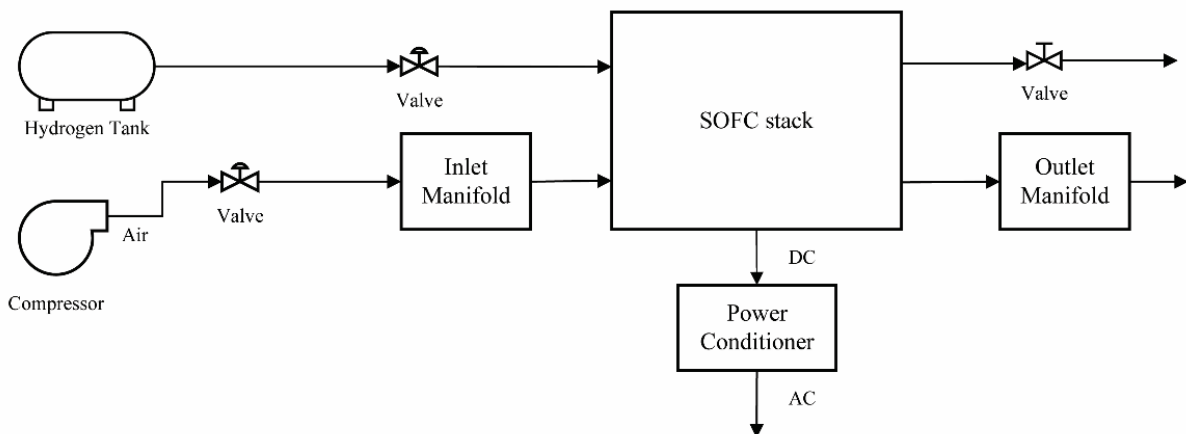


Figure. 1. Block diagram of the considered SOFC system

### 3. Model analysis

#### 3.1 Equilibrium points

Equilibrium points define where the dynamic system can be in steady-state operation. These points play a significant role in dynamic analysis and the controller design. The SOFC system model can be represented by a set of nonlinear differential equations:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u})\end{aligned}\quad (1)$$

where  $\mathbf{x} \in \mathbb{R}^n$  is the state vector;  $\mathbf{u} \in \mathbb{R}^m$  is the input vector; and  $\mathbf{y} \in \mathbb{R}^r$  denotes the output;  $\mathbf{f}(\mathbf{x}, \mathbf{u})$  is the vector field and  $\mathbf{h}(\mathbf{x}, \mathbf{u})$  allows to compute the output from the state variables.

In the equilibrium points  $\mathbf{x}^* = [x_1^*, \dots, x_n^*]^T$  and the input vector  $\mathbf{u}^* = [u_1^*, \dots, u_n^*]^T$ , the derivatives of the state variables in the system (1) have to be zero, that is  $0 = \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*)$ .

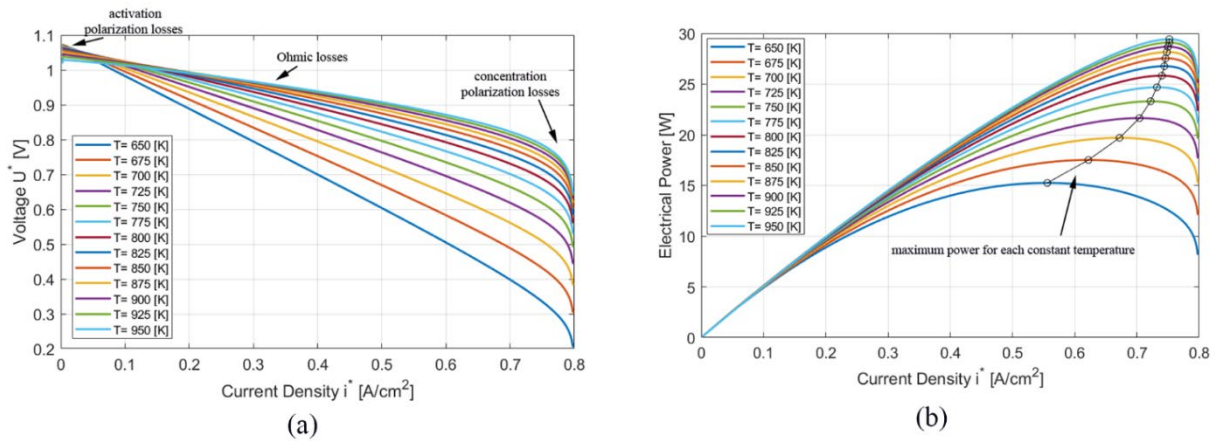


Figure. 2. The results of the equilibrium point analysis

Figure. 2(a). shows equilibrium points of voltage obtained from different current densities and temperatures. Usually for SOFC system, this curve can be splitted in three main parts: activation polarization losses, ohmic losses and concentration polarization losses. Moreover, the evolution of electrical power versus current density for different temperature is depicted in Figure. 2(b). Among those equilibrium points of electrical power, a black curve is corresponding to the maximum power for each constant temperature, which provides a guideline for optimal control in order to obtain the maximum power.

#### 3.2 Stability analysis

In fact, the obtained equilibrium points in section 3.1 might be stable or unstable. The system dynamics can drift away under unstable equilibrium points, where the control technology is unable to apply. In order to determine the stability of equilibrium points, Lyapunov's approach can be performed [2]. First, the model of SOFC system around the equilibrium point is transformed to the following linear system:

$$\begin{aligned}\Delta \dot{\mathbf{x}} &= \mathbf{A} \times \Delta \mathbf{x} + \mathbf{B} \times \Delta \mathbf{u} \\ \Delta \mathbf{y} &= \mathbf{C} \times \Delta \mathbf{x} + \mathbf{D} \times \Delta \mathbf{u}\end{aligned}\quad (2)$$

where  $\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}^*$  is the state variable; the input vector is  $\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}^*$ ; the output vector is  $\Delta \mathbf{y} = \mathbf{y} - \mathbf{y}^*$ ; the matrices are  $\mathbf{A} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}^*, \mathbf{u}=\mathbf{u}^*} \in \mathbb{R}^{n \times n}$ ,  $\mathbf{B} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\mathbf{x}=\mathbf{x}^*, \mathbf{u}=\mathbf{u}^*} \in \mathbb{R}^{n \times m}$ ,  $\mathbf{C} = \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}^*, \mathbf{u}=\mathbf{u}^*} \in \mathbb{R}^{r \times n}$ ,  $\mathbf{D} = \left. \frac{\partial \mathbf{h}}{\partial \mathbf{u}} \right|_{\mathbf{x}=\mathbf{x}^*, \mathbf{u}=\mathbf{u}^*} \in \mathbb{R}^{r \times m}$

The eigenvalue of  $\mathbf{A}$  in the left half of the complex plane implies that the equilibrium points are stable. Figure. 3. shows the maximum eigenvalue and minimum eigenvalue for the studied SOFC system at different equilibrium points. From Figure. 3, all eigenvalues are negative which implies that the linearized system is stable.

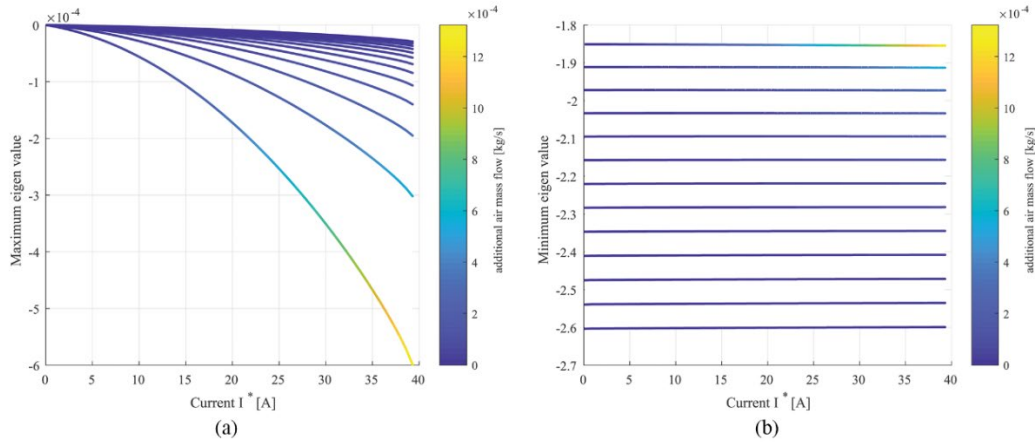


Figure. 3. Eigenvalue of the linearized system

### 3.3 Comparison between the linear model and nonlinear model

In order to verify the linearized model in section 3.2, a small step signal is used as an input to determine the difference between the linear model and nonlinear model. As depicted in Figure. 4, the curve profile of the linear model is almost the same as the nonlinear model. Thus, we can design control strategy based on the linear model.

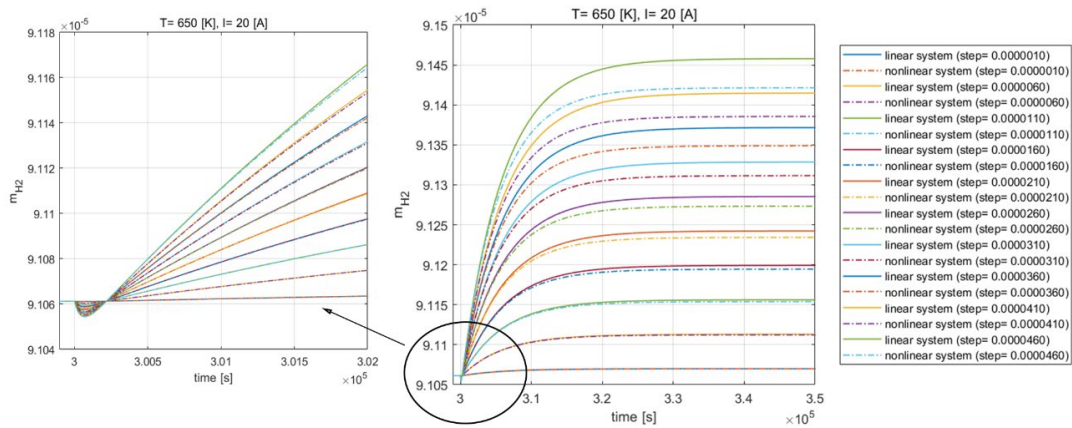


Figure. 4. Comparison between the linear model and nonlinear model

## 4. Conclusions

In this paper, a SOFC control-oriented model is developed and model analysis is performed. Specifically, the equilibrium point analysis shows the maximum power for each constant temperature. Moreover, the stability analysis implies that the linearized model of SOFC system can operate stably around the equilibrium. Besides, the dynamic performance of the linearized is almost the same as the nonlinear model. In the future, the optimal control for SOFC system can be design based on this analysis.

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