THE INVISIBLE HEARTBEAT

The beauty and soul of mathematics

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We will look at mathematical aesthetic experience from a point of view close to the one of analytical psychology. We will analyse the myth that the universe is a harmonic entity that can be described through mathematics, giving it its beauty, and study the unconscious remnant of this archetypal idea in contemporary science. Mathematical beauty appears, finally, as a link between the Archetype of the Cosmos and the wholeness of the psyche.

1. Introduction

Mathematics is often seen as a strictly rational activity where emotions and feelings do not intervene, but among mathematicians it is usual to refer to their activity and motivation in other words: it is common to hear of the joy and pleasure that stems from persevering in a task, and the results gained after long days of uncertain work. This mathematical enjoyment is far from being a mere intellectual experience and, conversely, involves complex feelings. A vivid example of the expression of this emotional sensibility can be found in the micro-history of statistician Thomas Royen's proof of Gaussian correlation inequality, who explains that the demonstration came to his head while brushing his teeth on July 17 2014:

> The "feeling of *deep joy and gratitude*" that comes from finding an important proof has been reward enough. "It is like a *kind of grace*," he said. "We can work for a long time on a problem and suddenly *an angel*—[which] stands here poetically for the mysteries of our neurons—brings a good idea" (Wolchover 2017. Emphasis added).

Many times this pleasure is linked to an aesthetic experience. Thus, traditionally in mathematical texts there may be talk of elegant demonstrations, subtle arguments or particularly beautiful objects and constructions (Hardy 1992), (Rota 1997), (Russell 1919). The great mathematician Jacques Hadamard, who along with Charles-Jean de la Vallée-Poussin demonstrated the theorem of prime numbers, emphasised the role of mathematical beauty in explaining the emotional attachment of mathematicians with their science. In his book on the psychology of the processes of mathematical invention, he quotes Paul Valéry, writing:

> It may be surprising to see emotional sensibility invoked apropos of mathematical demonstrations which, it would seem, can interest only the intellect. This would be to forget the feeling of mathematical beauty, of the harmony of numbers and forms, of geometric elegance. This is a true aesthetic feeling that all real mathematicians know, and surely it belongs to emotional sensibility (Hadamard 1954, 31).

The relationship between mathematics and beauty is rightfully one of the important chapters of the philosophy of mathematics, but it is also an elusive and recurring issue. Traditionally, some "objective" elements are listed that would give a result, a proof, or the construction of a mathematical object its beauty. Some of these elements would be, for example, minimalism and the elementality of resources employed, or the degree of surprise in the face of particularly counterintuitive results or those that relate apparently unconnected concepts to each other. But it is also worth highlighting the enumeration of other—hardly objectifiable—elements that convey a great deal of information about the psychic trace of mathematical work such as the "inevitable" quality (inevitability in Hardy's words) or the feeling of "depth" in a result (Hardy 1992), (Russell 1919) [1].

In this work I will evade the supposed objective elements and I will concentrate on various psychological aspects of the mathematical task, placing the mathematical aesthetic experience in its rightful place: the soul. We will also see that in the past, both collectively and consciously, the emotional sensitivity linked to the mathematical aesthetic experience was deployed in a way that resembles a religious attitude, as uncomfortable and connoted that word is, especially among the contemporary mathematicians. We will also examine the unconscious traces of this spiritual connection to the task of the contemporary mathematicians (Royen's testimony is eloquent in this sense). I believe that becoming aware of this spiritual filiation can be used to find a good frame of reference to explain the phenomena of aesthetic experience and the "unreasonable" effectiveness of mathematics

[1] For a critique of the role of the element of surprise in the mathematical aesthetic experience, see (Rota 1997). A revealing list of adjectives used by the mathematical community to refer to results and proofs can be found in (Johnson, Steinerberger 2019).

to describe the world, phenomena that today are regarded with perplexity (Wigner 1960). It would also serve to restore meaning to the unconscious motivation of the work of many contemporary mathematicians, and would allow this work to be incarnated at the same time in a centuries-old tradition and in a soul-making way close to the process of individuation.

2. Psychological aspects of the mathematical aesthetic experience

The relationship between beauty and transcendence is perfectly reflected in the first verses of Keats's Endymion. These could serve as a substitute for an academic definition of beauty which we do not have, verses that also link the experience of beauty to a form of emotion:

A thing of beauty is a joy for ever:
Its loveliness increases; it will never
Pass into nothingness; but still will keep
A bower quiet for us (...)

Joy, charm, quiet and appeals to eternity that clearly evoke pleasure, order and transcendence. Perhaps it is not too far to say that, according to the poet, beautiful things help us feel the influence of an invisible and benevolent presence, and that his verses suggest the experience of beauty as a form of religious experience. For this reason it is interesting to contrast what Keats's verses convey with the experience of the mathematical beauty referred to by Bertrand Russell in a profusely cited fragment that has become paradigmatic when it comes to explaining the mathematical aesthetic experience.

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty –a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry (Russell 1919).

The choice of words and the difference between that conveyed in this passage with that of Keats's verses is eloquent: severe perfection (*stern*), coldness, austerity, purity, and an excellence that seems antagonistic to joy, which pales in the presence of what is greater than the human being. In the last instance, Keats and Russell describe two forms of transcendental experience, but the benevolent atmosphere of Endymion's verses is suppressed in Russell's text due to the overwhelming direct experience of numen, by a majestic, tremendous and fascinating aura (Otto 1970).

It is very interesting to see how the mathematical aesthetic experience referred to in Russell's passage has become commonplace among those who do not cultivate mathematics, who often believe that the relationship of mathematicians with their science is "cold". But it is even more curious and surprising to see how often this passage is cited—and tacitly accepted—among the mathematical community. I say curiously, because the Russellian experience does not seem to leave space for the emotional element, indispensable in the creative processes, as noted by Hadamard (Hadamard 1954)[2]. Nor does it seem that we can find the high psychic significance and transformational potential that is expected from contact with the *sublime* (for an example, see Royen's testimony at the beginning or also the considerations about the experience of the sublime in (Schmidt 2019)). Reading Russell's passage one can feel that the transcendental experience reflected seems sterile: by placing mathematical beauty on a plane far superior to human experience, by placing it outside the soul, coldness breaks in. In the words of C.G. Jung, "if we accept the fact that a God is absolute and beyond any human experience, it leaves me cold. I can not affect him, nor does he affect me" (Jung, 1967).

In short, Russell's passage conveys an impression of sterility that is opposed to the fact that mathematical beauty is widely regarded as a fruitful criterion in science. In recent work, Nobel Prize winner Frank Wilczek (Wilczek 2015) agrees with this fact and mentions several examples in which relying on the beauty of a theory led to astonishing discoveries: Urbain Le Verrier and John Couch Adams relying on the beauty of the Newtonian theory of gravitation—which at that time was challenged by the anomalous behaviour of

[2] Hadamard tells us how the affective element constitutes an essential part of any discovery or invention. He exemplifies it in the analysis of the life and work of Henry Poincaré, in which we see that the sense of beauty was an indispensable means in his research. Also on the affective element, Arthur Koestler dedicates passages inspired and documented in his work "Act of creation" (Koestler 1964).

Uranus—predicted the existence of a new planet that until then had not been observed, Neptune. Heinrich Hertz could produce and observe radio waves while relying on the beauty of Maxwell's equations, and Paul Dirac predicted the existence of antiparticles that would be detected shortly thereafter using a "strange and beautiful" equation. It is also well known that faith in beauty has led to errors. An example that already belongs to the epics of science was the attempt by Johannes Kepler to create a planetary system in which the distances of the orbits (that were considered to be circular) from the sun to the six planets that were then known were given in terms of spheres inscribed and circumscribed in platonic solids. Kepler had to correct his model for a small observational discrepancy and had to consider that the orbits should not be circular but elliptical. The huge disappointment that Kepler felt when he verified that the elliptical orbits adjusted better than the circles to the observed data is well known. Etymologically, *élleipsis* means *insufficiency* and, therefore, the ellipse was considered a defective figure, without the aesthetic and metaphysical value attributed to the circle (McAllister 1996, 178). Today, Kepler's three laws for planetary motion and their derivation based on the laws of Newton are considered a beautiful piece of mathematical goldsmith work (Fitzpatrick 2012). There are also critical positions regarding the fertility of beauty as a guiding criterion: a critical and solidly documented approach can be found in (Hossenfelder 2018).

At this point I would like to raise the possibility that the experience of beauty in mathematics may be a phenomenon of projection, that is to say an unconscious and autonomous process by which we place qualities in mathematical objects that belong to our personal and collective psychic reality. We will later see how the beauty of these objects is often referred to as qualities attributed by analytical psychology to certain representations of the Selbst, the archetype of the wholeness of the psyche. Some of these qualities have religious characteristics and show how a burden has been placed on mathematics of a very old kind of yearning: the desire for the universe to be a Cosmos, an orderly reality. In fact, we will see how this myth was explicit over many centuries and how consequently, dedication to mathematics was lived as soul-making task. We will also see, as with the advent of the mechanistic paradigm and the process of secularisation in scientific motivations, this belief remained unconscious. Therefore, its manifestation in contemporary science would also be a phenomenon of projection. Mathematics, then, would be the symbolic recipient of this yearning, the place where the Archetype Cosmos is projected.

3. Mathematical beauty, religious sentiment and the Archetype Cosmos

It is usual to present science as a discipline that approaches reality in an objective way. This story is part of its own myth. In reality however, one only need break the surface of the history of scientific thought to discover that this is impregnated—when not driven—by a large number of non-explicit cultural assumptions, worldviews and aspirations that are very often unconscious, in such a way that the image of the world presented to us by science is a reflection of the beliefs and longings of each time[3].

The main assumption on which science is built is based on reality having an order that is accessible to our minds. Insofar as this assumption is not usually explicit, we can consider that it is an approach that exerts its influence in an unconscious way. This assumption is the expression of a desire beating through the heart of Western scientific thought, even through its secularised scientific thinking of the 21st century. A second assumption, valid today with renewed vigour[4], is that this order cannot be anything other than a mathematical order. Mathematics, then, has become the cultural depositary of the projection of the Archetype Cosmos and its implicit beauty.

The role of mathematics in scientific activity dates back to the traditions of Pythagorean and Platonic inspiration[5], but its entry into the nucleus of scientific thought takes place in the Renaissance when the natural philosophers of the time and even later, steeped in a

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[3] For a presentation of the evolution of scientific thought in the context of the development of Western culture see (Tarnas 1991). A vision centered on the field of mathematics can be found in (Kline 1980). Three classic texts, complementary in background and form, that have served to question the story of science as a linear process resulting from the systematic application of an objective methodology can be found in (Kuhn 1957), (Kuhn 1962) and (Feyerabend 1975). In a recent work, biologist Rupert Sheldrake (Sheldrake 2012) examines ten dogmas that, according to the author, limit the creativity of science by being unconsciously assumed. Also noteworthy are the archetypal figures that Arthur Koestler identified in the motivational impulses of scientific activity (Koestler 1964, 255-258).

[4] I am referring to the progressive mathematisation of biology and social sciences.

[5] Contrary to what is usually presented in the texts of history of mathematics, the figure of Pythagoras represents the survival of the different Mystery traditions of the Mediterranean and, at the same time, a philosophical evolution of these that incorporates elements of the pre-scientific tradition that have reached us through the enormous influence of Plato. In this sense, the school of Plato mediates between the brief but decisive, Greek "illustration" —which incorporated rational discourse into philosophy, determining what is acceptable in Western philosophy and what is not—and the ancient form of knowledge based on the pre-eminence of the supernatural and ritual as a way of accessing knowledge (Dodds 1973). It is probable that the origin of the doctrines of "scientific" Pythagoreanism can be found in the works of Philolaus from Croton and Archytas from Tarent in the second half of the 5th century BC and the first half of the 4th century BC. These works trace a border between ancient Pythagoreanism, more interested in the doctrines of the soul and the formation of a philosophical way of life, and a new Pythagoreanism, of mathematical and cosmological character that would be imposed following the death of Pythagoras (Burkert 1972) and (Hernández de la Fuente 2014).

recently rediscovered Platonism, began to develop a physics in which the universe was described mathematically. Evidently, in ancient times there were mathematical models of nature (astronomical mechanisms, for example), the novelty being that mathematical descriptions were considered the literal expression of the *laws* according to which the world had been ordered: the authentic geometric order of the world (Barfield 1957)[6]. The history of science, and in particular that of mathematics, shows us how this inquiry into the geometry of nature had a spiritual character. In short, the explicit (and therefore conscious) assumption of the works of Copernicus, Kepler, Galileo, Pascal—or Newton's very eloquent writings—was that the universe was designed according to mathematical principles. In the words of Leibniz, "Cum Deus calculat et cogitationem exercet, fit mundus" (with his calculations and thoughts God made the world). Let's look at two explicit examples:

> To make this systeme therefore with all its motions, required a Cause which understood & compared together the quantities of matter in the several bodies of the Sun & Planets & the gravitating powers resulting from thence, the several distances of the primary Planets from the Sun & secondary ones from Saturn Iupiter & the earth, & the velocities with which these Planets could revolve at those distances about those quantities of matter in the central bodies. And to compare & adjust all these things together in so great a variety of bodies argues that cause to be not blind & fortuitous, but very well skilled in Mechanicks & Geometry (Newton 1692).

This "Cause" that plans the universe, is a "Being" that does it beautifully:

> This most beautiful System of the Sun, Planets, and Comets, could only proceed from the counsel and dominion of an intelligent and powerful being (...) This Being governs all things, *not as the soul of*

[6] We can observe that the anthropocentric concept of the *law* of nature can be opposed to the concept of nature's *habit*, perhaps also anthropocentric, posed by some thinkers influenced by the discovery of the evolution of the species, which opened up the possibility that the universe was also an evolving being, not governed by immutable principles (laws) but by habits that would evolve in time in a similar way to organisms. See (Sheldrake 2012) citing F. Nietzche, C.S. Pierce, W. James, H. Bergson and A.N. Whitehead.

> *the world*, but as Lord over all (Newton 1729, 387-393. Emphasis added).

In the words of Arthur Koestler (Koestler 1964), the first science emerging from the Renaissance was inspired by an "oceanic feeling of religious mysticism". This approach can be followed until well into the eighteenth century. For example, in the defence of theological terms of the principle of minimal action by Maperius and Euler (Kline 1980)[7]:

> For since the fabric of the universe is most perfect, and is the work of a most wise Creator, nothing whatsoever takes place in the universe in which some relation of maximum and minimum does not appear (Euler 1744).

These testimonies, among many others, show how from a psychological point of view scientific activity was lived as a way of enriching the soul (a soul-making task): In mathematics, the natural philosopher contemplated the elements with which the creator had built the world. Geometry was pre-existing to creation, was co-eternal to the mind of God, as Kepler said, it was God himself (Koestler 1964). The harmonic relations of mathematical objects, the presence of symmetry or the simplicity of arguments in their definitive form comprised a very elevated form of beauty and therefore, from a Platonic perspective, assimilated to good. The contact with this beauty unfailingly had to transform the soul of the natural philosopher. The beauty of the universe, expressed in the beauty of mathematics, was a reflection of the beauty and kindness of the creator's mind. It is impossible not to wonder about the effect the cultivation of mathematics had on the psyche of someone who consciously believed —as Newton did—that when one geometrized the thought of God was re-traced.

The roots of this way of enriching a soul by immersing it in the eternal through contact with its symbols (numbers, geometrical figures) could be traced back to the ancient Pythagoreans, which we certainly know attributed symbolic nature to numbers. Some authors have pointed out that the contemplation of mathematical beauty was assimilated

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[7] It is interesting to note the paradox that, over the years, (spiritual) research in the design of the great plan that promoted the discovery of new laws (today we would say the creation of new physical models) became conforming to the idea of a creator who launches a mechanism and lets it evolve, without taking any special care. *Deism* will be a key step towards the paradigm of the universe-mechanism.

into a form of worship. According to (Gorman 1979), for Pythagoreans the soul was purified when contemplating mathematical truths and leaving behind the physical senses. In this framework, numbers are comparable to the Gods insofar as they are pure and immutable. When the mind meditates on numbers it is communicating with the Gods. Contemplation is, therefore, a selfless form of prayer that does not ask for the favour of the Gods. Arthur Koestler adds more details, including the etymological link between *contemplation* and *theory*:

> (…) (the Pythagoreans) transformed the Orphic mystery cult into a religion which considered mathematical and astronomical studies as the main forms of divine worship and prayer The physical intoxication which had accompanied the Bacchic rites was superseded by the mental intoxication derived from philo-sophia, the love of knowledge. It was one of the many key concepts they coined and which are still basic units in our verbal currency. The cliché about the 'mysteries of nature' originates in the revolutionary innovation of applying the word referring to the secret rites of the worshippers of Orpheus, to the devotions of stargazing. (…) 'Pure science' is another of their coinages; it signified not merely a contrast to the 'applied' sciences, but also that the contemplation of the new mysteria was regarded as a means of purifying the soul by its immersion in the eternal. Contemplation of the 'divine dance of numbers' which held both the secrets of music and of the celestial motions became the link in the mystic union between human thought and the *anima mundi* (Koestler 1964, 259-260).

At this point it is difficult to avoid an analogy between mathematical work after the scientific revolution and the alchemical work studied by Jung and his school.

> But there were always a few for whom laboratory work was primarily a matter of symbols and their psychic effect. (…) Although their labours over the retort were a serious effort to elicit the secrets of chemical transformation, it was at the same time —

> and often in overwhelming degree—the reflection of a parallel psychic process which could be projected all the more easily into the unknown chemistry of matter since that process is an unconscious phenomenon of nature, just like the mysterious alteration of substances. What the symbolism of alchemy expresses is the whole problem of the evolution of personality described above, the so-called individuation process (Jung 1968, par. 40).

It is important to point out that while these were conscious ideas, natural philosophers could *participate* in the cosmos. In fact, this point is the basis on which the Platonic theory of knowledge rests, that the psyche can access the ultimate structure of reality (mathematical-ideal) because it *participates* in this reality. For Plato and his school, the psyche has had access to this reality at a stage prior to our life; it is only a matter of recovering what is already known (*Anamnesis*) (Kline 1980, p. 21). Under these explicit (conscious) assumptions, the non-sensory relationship between the natural philosopher and the world—and therefore his experience of science and the cultivation of mathematics—could be fruitful and transformative: a beautiful cosmos is discovered, fruit of beautiful elements—mathematics—which are both *in the soul and the world* at the same time. In fact, the distinction between the mind and the world has not yet been outlined in a determined way, the Cartesian distinction between *res cogitans* and *res extensa* is not still in force.

Ironically, the physics and mathematics developed during the aforementioned *period of participation* consolidated the image of the world as a mechanism. This idea, which is already in the treatise of astronomy *De sphaera mundi* (c. 1230) by Johannes de Sacrobosco was adopted by the heroes of the scientific revolution, such as Kepler and later Leibniz, and was advocated by the deists of the Enlightenment as a result of the publication of Newton's *Principia Mathematica* in 1687. It is interesting to note here that the appearance of the metaphor of the clock mechanism is linked to the Archetype Cosmos and how this archetype is also linked to an aesthetic and transcendental experience which is part of the individuation, that persists in the contemporary world: see, for example, the final dream of the four hundred dreams sequence of Wolfgang Pauli analysed by Jung, where the Nobel Prize in Physics refers to the vision of the "world clock" as the cause of an impression of "the most sublime harmony" (Jung 1968b, 203-214) and (Jung 1969, 64-105): we note the parallelism with the testimonies of Royen, Valéry and Russell referred to above. The experience of the sublime in analysis and the individuation is treated in depth in (Schmidt 2019).

4. Mathematical crises and changes in the aesthetic perception

It is still a paradox that mathematical physics, based on an oceanic feeling of mysticism, caused the emergence of the mechanical conception of the world that has finally been imposed not only in the physical sciences—despite being in recession today[8]—but also in areas such as medicine, biology and even psychology (Riskins 2016), (Shamdasani 2003) and (Sheldrake 2012). The mechanical paradigm ended up formally cutting out the transcendent motivation from scientific activity. Some consequences of the establishment of this paradigm have been the progressive de-soulment and disenchantment of the world[9] and also the progressive withdrawal of one's soul from the men and women of modernity and the contemporary world (Tarnas 1991). Regarding the object of our study, it is notable that despite being stripped of its role as a *sacred language* establishing a link between the world and the soul of the scientist (the participation we have referred to), mathematics will continue to be the language of science in the secularised world from the nineteenth to the twenty-first century, revealing that the Platonic belief of an underlying order of nature (the Archetype Cosmos) often remains unconscious.

In a context where religious sentiment is detached from scientific research, mathematics is subjected to both periods of great creativity and subsequent crises on the nature of its foundations (Kline 1980). Periods of crisis parallel the periods of convulsion and social and cultural effervescence of its time. Neither mathematics of the late nineteenth or twentieth century are excluded from the process of extreme sophistication of knowledge and the confinement of the psyche in academic labyrinths (remember the sarcasm with which the writer and physicist Ernesto Sabato reflects on his "One and the Universe" on the extreme artificiality of the definition of number 1 proposed by Burali-Forti, binding it to the future

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[8] The survival of the mechanistic paradigm and its advancement in the life sciences occurs while gradually retreating the interpretation of physics: in 1931, James Jeans wrote the following for the general public:

> To-day there is a wide measure of agreement which on the physical side of science approaches almost to unanimity that the stream of knowledge is heading towards a non-mechanical reality; the Universe begins to look more like a great thought than like a great machine. Mind no longer appears to be an accidental intruder into the realm of matter... we ought rather to hail it as the creator and governor of the realm of matter (Jeans 1931).

[9] See the emphasis added in the second Newton's quoted text.

of ignorance[10]). The effect of the complexity of these labyrinths on the soul is again projected onto nature thanks to the advent of non-Euclidean geometries and topology, or the appearance of objects such as Cantor's sets, the curves of Peano or others linked with the paradoxes of measure theory. Objects that favour the emergence, in the twentieth century, of new geometric shapes that we *believe* present in nature (Mandelbrot 1982): the obsessive recurrence of fractals, the voluptuousness of turbulence and the complexity of chaos and other phenomena provided to us by the theory of dynamic systems are all examples. All these structures are the result of a new form of order, an evolution of the Archetype Cosmos, and all of them hold a numinous fascination for many of those who contemplate them.

The progressive appearance of these new objects thus leads to an evolution in the aesthetic experience of mathematics, which gains in sophistication and evolves in parallel with what happens in the visual arts with the birth of the avant-garde, sometimes mutually influencing one another (Pont 1982). This is also in keeping with the rest of the cultural phenomena of the moment and what the essayist Joan Fuster brilliantly described as "the discrediting of reality". In witnessing this parallelism and assuming that the evolution of the aesthetic sense is influenced by changes to both the collective consciousness and unconscious, we can legitimately ask ourselves if the evolution of discovery is linked to the evolution of our aesthetic sense, a question analysed by (McAllister 1996).

In the next section we will see that despite these changes in aesthetic sensibility and despite the secularity in the motivations of science, the Archetype Cosmos continues to beat at the heart of contemporary physics and mathematics. In the geometry of the so-called *standard model* as in complexity theory survives the idea of *invariance* and *symmetry*, which are the symbols par excellence of this archetype and also of the archetype of the wholeness of the psyche.

5. Fascination, mandalas, symmetry: Platonism's unconscious legacy

We have seen how some authors have referred to the *contemplation* of mathematical beauty as a form of prayer. Mathematical objects would be seen as recipients of psychic content and symbols. For millennia we have known that the contemplation of symbols is a way of

[10] This definition can be consulted, for example, in (Ewald 1996, 1029).

accessing another form of consciousness. We can follow this knowledge from the Orphics and the Pythagoreans through to a 20th century philosopher like Maria Zambrano —who tells us that in contact with the symbols the word cannot but distil its purest language (Colinas 2019). It is difficult not to relate this form of contemplation of mathematical objects with the ancient Hindu tradition of the contemplation of *yantras*, diagrams used to conduct meditation. Yantras are instruments that connect with unconscious psychic structures. This form of contemplation, stripped of its mystical element, persists in the common fascination that certain mathematical objects exert on us: we are once again made aware that fascination is one of the key elements of the Numinous experience.

Symmetry is one of these properties of mathematical objects that is simultaneously depository (by projection) of our yearning for a transcendental order, the expression of the wholeness of the psyche, and a vehicle of contemplation. Jung's well-known passage on the harmony of mandala is even cited, in this last sense, in an excellent mathematical popularisation text (Du Satoy 2008):

> I sketched every morning in a notebook a small circular drawing, (...) which seemed to correspond to my inner situation at the time.... Only gradually did I discover what the mandala really is: (...) the Self, the wholeness of the personality, which if all goes well is harmonious (Jung 1965).

It is interesting to highlight the equivalence between the harmony that stems from symmetry and wellbeing ("if all goes well").

In a current textbook of mathematical methods for physics we are told that it is worthwhile studying the symmetry not only for its beauty but also for its fundamental role in physics:

> For mathematicians, symmetry is worth studying simply for the sake of its beauty, but symmetry is also very important in physics, because it allows us to at least partially understand situations that would be otherwise be too complicated. Gauge theories are among the most beautiful, symmetrical laws of physics we know, and our

> current theories of electromagnetism, the strong and weak forces, and gravity are all gauge theories (Baez, Muniain 1994).

In summary, gauge theories are mathematical descriptions of nature in which the equations, and therefore their solutions, are *invariant* under certain transformations. This is precisely the definition of symmetry—invariance—and here once again the language is telling.

The set of transformations that leave an object invariant form a mathematical structure called a *group*. In fact, based on the work of Felix Klein and the famous Erlangen Program (1872), geometry is defined as the study of the properties of a set that remain invariant for a certain transformation group (Kline 1972). In other words, *in the contemporary world symmetry is almost synonymous with geometry*.

The objects studied by physics are mostly *fields*. In essence the fields represent how certain properties are distributed in a space. These fields can be represented by differential equations. The equations that model the interactions of the particles in the standard model —but also the classical equations of electromagnetism and special and general relativity— are invariant for certain transformation groups.

In summary, one can say that today "discovering the laws of physics" means discovering which equations have solutions that are invariant for a concrete transformation group or, in other words, that have certain prescribed symmetries (for one more formal definition of symmetry and a simple symmetrical field see the Appendix).

This search for symmetry in contemporary mathematical physics is significant and reveals an unconscious Platonism. At present, there are several forms of mathematical Platonism, but all share in varying degrees two principles: (a) That nature is structured in accordance with mathematical principles that are underlying relations which unify and reveal appearances' ordination; (b) That this structure is accessible to our psyche.

The openly platonic positions of some scientists are known, such as Werner Heisenberg. The most radical form of mathematical Platonism affirms that the world *is* a mathematical construction. This is the position, for example, of the cosmologist Max Tegmark (Tegmark 2008), but the most interesting Platonism in the context of our study is that which is not declared, for revealing the deep motivations of the mathematical task. By this unconscious Platonism *symmetry is a symbol, the depositary of an archetypal content*, and at the same time its repeated search proves that to be manifested, this archetype needs an instrument for its projection[11]. *Symmetry therefore plays the role of mandala in contemporary scientific thought.* Its searching is the expression of the remnant of an unconscious spirituality.

Against these affirmations and in the face of the fact that many mathematical objects are beautiful and elegant even in an "unreasonable" way, there is a response in the form of an anthropic principle that would say: "If these objects were not pleasant and regular, we would not understand them and we would not know how to describe them" (Hazewinkel 2009). I propose integrating this principle and developing it further by adding that *we are capable of developing mathematics and physics insofar as what we discover involves a sense of harmony and beauty that is also of us*. This is a harmony that our psyche also endeavours to restore through contact with mathematical objects and in expression of their beauty. That is why the great cosmic archetype of the harmony of the universe uses the same element (the symmetry) as the archetype of the wholeness of the psyche, the Selbst. I propose therefore, that *beauty is the bond - the continuity element - between the Selbst and the Cosmos.*

6. Conclusion

The thesis we have tried to explain in this work is that the attempt to describe the universe in an orderly way—the attempt to describe a Cosmos—is correlated with the attempt to restore harmony to the whole psyche. This is a process that can be linked to the process of individuation. Historically, this procedure has been explicit, and therefore conscious, through the link between scientific activity and religious stance. In the science of our fully secularised time, this relationship remains through unconscious assumptions and manifests itself in the quest for beauty in physical theories, which then act as authentic mandalas. The etymological relation between theory and contemplation then, remains in force.

Additionally, the "unreasonable effectiveness of mathematics" to describe the world (Wigner 1960) is best explained if we are aware that the image of the world given to us by

[11] Recently, and in the context of a critical essay on the role of the search for beauty in theoretical physics, Sabine Hossenfelder presents a series of testimonies that demonstrate to what extent beauty and symmetry are the vessels of the projection of theories of the community of theoretical physics. The testimonials offer a lot of psychological information and constitute a revealing document (Hossenfelder 2018).

science says perhaps more of the unconscious assumptions and content of our culture, than of a supposed neutral description of the facts. Moreover, the very idea of a neutral description of the facts is itself a myth. In other words, everything is clearer if we are able to see the circular argument: because of the unconscious remnants of Pythagoreanism and Platonism, we project its mathematisable quality onto the universe (which must be read as synonymous with "ordered and harmonious") and we collect, with success and a point of perplexity, the image of a mathematisable universe.

Where does this yearning to build an orderly image come from? Our interior, naturally. The idea of the Cosmos may be nothing other than a projection of the Selbst that needs to be recognised and assimilated. In this sense, and on a strictly personal level, I will add that I also believe mathematics to be beautiful as it participates in the soul of the universe that is neither cold nor austere nor as severe as it is derived from the mechanical paradigm, but lively, diverse and resplendent. We seek beauty, in mathematics among others, because it is a mirror of the most intimate, sublime and is finally constitutive of each and every one of us. And because it is a bond with this part of the universe, beauty is the invisible heartbeat that drives us to become what we really are.

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Appendix. Symmetry

Formally, an object is symmetrical if some of its properties remain *invariant* under certain transformations. For example, the position of an equilateral triangle is invariant by any reflection with respect to its medians and also by rotations of 0º, 120º and 240º.

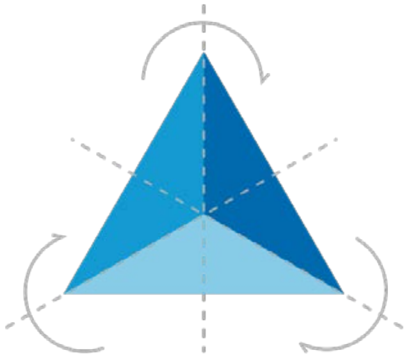

Figure 1. Symmetries of an equilateral triangle

Here is a simple example to illustrate the concept of symmetry in the context of field theory. As we have mentioned, the objects studied by physics are mostly fields. In essence, the fields explain how a certain property is distributed in space. Let's consider the vector field in the plane:

$$\mathcal{X}(x,y) = \left(-x + y + x^3 + xy^2\right)\frac{\partial}{\partial x} + \left(-x - y + x^2 y + y^3\right)\frac{\partial}{\partial y}.$$

Intuitively, we can imagine that at each coordinate point *(x,y)* this field is the velocity vector that would have an object interacting with it. The movement of these objects would be given by the solutions of the differential equation associated with this field:

$$\begin{cases} \dot{x} = -x + y + x^3 + xy^2, \\ \dot{y} = -x - y + x^2 y + y^3. \end{cases}$$

This field is invariant by any planar rotation, that is to say, by a transformation of the form:

$$\begin{aligned} u &= \cos(a)\,x - \sin(a)\,y, \\ v &= \sin(a)\,x + \cos(a)\,y. \end{aligned}$$

This means that the equations that describe the dynamics in both coordinates *(x,y)* and *(u,v)* are the same, which implies that any rotation of the plane transforms solutions of the differential equation into other solutions. Figure 2 represents some of the infinite solutions of the differential equation. It can be observed that the form of these solutions is invariant by rotations.

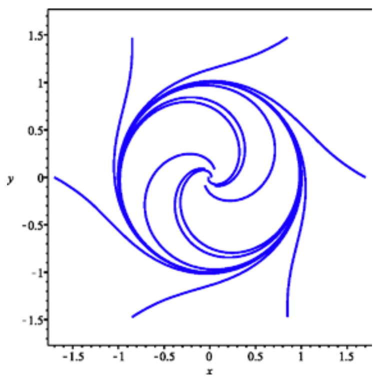

Figure 2.

The set of transformations that leave an object invariant form a mathematical structure called a *group*. The set of all rotations of the plane is known as the SO(2) group, also represented by U(1). For this reason, we say that the field of the example is *invariant* by the action of SO(2). The standard model equations, for example, are invariant to a much more sophisticated transformation group that is denoted by SU(3)×SU(2)×U(1) (Baez, Muniain 1994).

References

Barfield, O. (1957). *Saving the Appearances: a Study in Idolatry*. London: Faber & Faber.

Baez, J., Muniain, J.P. (1994). *Gauge Fields, Knots and Gravity*. Singapore: World Scientific.

Burkert, W. (1972). *Lore and Science in Ancient Pythagoreanism* (Engl. Trans. E.L. Minar Jr). Boston: Harvard University Press.

Colinas, A. (2019). *Sobre María Zambrano.* [About María Zambrano]. Madrid: Siruela.

Dodds, E.R. (1973). *The Greeks and the Irrational.* Berkeley: University of California Press.

Du Sautoy, M. (2008). *Symmetry: A Journey into the Patterns of Nature.* New York: HarperCollins.

Euler, L. (1744). *De Curvis Elasticis, Additamentum I to his Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes*; In W. A. Oldfather, C. A. Ellis & D. M. Brown (1933)

Leonhard Euler's Elastic Curves. *Isis* 20 (1), 72-160. Retrieved July 27, 2019, from http://www.jstor.org/stable/224885

Ewald, W.B. (1996). *From Kant to Hilbert: A Source Book in the Foundations of Mathematics, Vol II*. Oxford: Oxford University Press.

Feyerabend, P. (1975). *Against Method: Outline of an Anarchistic Theory of Knowledge.* New Jersey: Atlantic Highlands, and London: Humanities Press.

Fitzpatrick, R. (2012). *An Introduction to Celestial Mechanics*, Cambridge: Cambridge University Press.

Gorman, P. (1979). *Pythagoras: A Life*. London: Routledge 1979.

Hadamard, J. (1954). *The Psychology of Invention in the Mathematical Field.* New York: Dover Publications. Unaltered reprint of the 1945 first edition by Princeton University Press.

Hardy, G.H. (1992). *A Mathematician's Apology*, Cambridge: Cambridge University Press. Reprinted from the 1940 first edition.

Hazewinkel, M. (2009). Niceness Theorems. In B. Kruglikov, V. Lychagin & E. Straume (Eds.) *Differential Equations: Geometry, Symmetries and Integrability: the Abel symposium 2008*. New York: Springer.

Hossenfelder, S. (2018). *Lost in Math: How Beauty Leads Physics Astray*. New York: Basic Books.

Hernández de la Fuente, D. (2014). *Vidas de Pitágoras* [Pythagoras' lives]. Vilaür: Atalanta.

Jeans, J.H. (1931) *The Mysterious Universe.* Cambridge: Cambridge University Press. Retrieved May 5, 2019, from https://archive.org/details/TheMysteriousUniverseSirJamesJeans

Johnson, S.G.B., Steinerberger S. (2019) The Universal Aesthetics of Mathematics. *The Mathematical Intelligencer* 41 (1), 67-70. DOI 10.1007/s00283-018-09857-5

Jung, C.G. (1967). *Commentary on the Secret of the Golden Flower (1929).* In *Collected Works of C.G. Jung, Vol. 13.* Princeton NJ: Princeton University Press.

Jung, C.G. (1968). *Psychology and Alchemy (1944).* In *Collected Works of C.G. Jung, Vol. 12.* Princeton NJ: Princeton University Press.

Jung, C.G. (1968b). *Individual dream symbolism in relation to the alchemy: a study of the unconscious processes at work in dreams. 3. The symbolism of the mandala. III. The vision of the world clock* In *Collected Works of C.G. Jung, Vol. 12*. Princeton NJ: Princeton University Press.

Jung, C.G. (1969). *Psychology and Religion. 3. The history and psychology of a natural symbol.* In *Collected Works of C.G. Jung, Vol. 11*. Princeton NJ: Princeton University Press.

Jung, C.G. & Jaffé, A. (1965). *Memories, Dreams, Reflections.* New York: Random House.

Kline, M. (1972). *Mathematics through from ancient to modern times.* Oxford: Oxford University Press, 1972. Retrieved July 27, 2019, from https://archive.org/details/MathematicalThoughtFromAncientToModernTimes

Kline, M. (1980). *Mathematics: The Loss of Certainty.* Oxford: Oxford University Press.

Koestler, A. (1964). *The Act of Creation.* New York: The Macmillan Company.

Kuhn, T.S. (1957). *The Copernican Revolution.* Boston: Harvard University Press.

Kuhn, T. S (1962). *The Structure of Scientific Revolutions.* Chicago: University of Chicago Press.

Mandelbrot, B. (1982). Des monstres de Cantor et de Peano à la géométrie fractale de la nature [From Cantor and Peano's monsters to the fractal geometry of nature]. In F. Guénard & G. Lelièvre (Eds.), *Penser les mathemátiques* [Thinking mathematics]. Paris: Éditions du Seuil.

McAllister, J.W. (1996). *Beauty and Revolution in Science.* Ithaca NY: Cornell University Press. Retrieved September 10, 2019, from https://openaccess.leidenuniv.nl/handle/1887/10158

Newton, I. (1729). *The Mathematical Principles of Natural Philosophy.* (Engl. Trans. A. Motte). London. Retrieved March 3, 2019, from http://inters.org/Newton-Scholium-Principia-Mathematica

Newton, I. (1692). Original letter from Isaac Newton to Richard Bentley, dated 10 December 1692 Source: 189.R.4.47, ff. 4A-5, Trinity College Library, Cambridge, UK. Retrieved March 3, 2019, from http://www.newtonproject.ox.ac.uk/view/texts/normalized/THEM00254

Otto, R. (1970). *The Idea of the Holy.* (Engl. Trans. J.W. Harvey). New York: OUP. Reprint of the 1923 edition. Originally published 1917. Retrieved May 23, 2019, from https://archive.org/details/theideaoftheholy00ottouoft

Riskins, J. (2016). *The Restless Clock: A History of the Centuries-Long Argument over What Makes Living Things Tick.* Chicago: University of Chicago Press.

Rota, G.-C. (1997). The Phenomenology of Mathematical Beauty. *Synthese* 111 (2), 171-182. DOI 10.1023/A:1004930722234

Russell, B. (1919). *The Study of Mathematics. Mysticism and Logic: And Other Essays.* London: Longman. Retrieved July 27, 2019, from https://archive.org/stream/mysticism00russuoft/mysticism00russuoft_djvu.txt

Schmidt, M. (2019). Beauty, ugliness and the sublime. *Journal of Analytical Psychology*, 64, I, 73-93.

Shamdasani, S. (2003). *Jung and the Making of Modern Psychology: The Dream of a Science.* Cambridge: Cambridge University Press.

Sheldrake, R. (2012). *The Science Delusion: Freeing the spirit of enquiry.* London: Coronet.

Tarnas, R. (1991). *The Passion of the Western Mind.* New York: Ballantine books.

Tegmark, M. (2008). The Mathematical Universe. *Foundations of Physics* 38 (2), 101-150. arXiv:0704.0646v2 [gr-qc]

Pont, J.-C. (1982). Peinture et géométrie au XXe siècle [Painting and geometry in the twentieth century]. In F. Guénard & G. Lelièvre (Eds.), *Penser les mathemátiques* [Thinking mathematics]. Paris: Éditions du Seuil.

Wigner, E. (1960). The Unreasonable Effectiveness of Mathematics in the Natural Sciences. *Communications in Pure and Applied Mathematics*, Vol. 13 (I), 1-14.
DOI 10.1002/cpa.3160130102

Wilczek, F. (2015). *A Beautiful Question: Finding Nature's Deep Design.* London: Allen Lane.

Wolchover, N. (2017). A Long-Sought Proof, Found and Almost Lost. Quanta Magazine March 28, 2017. Retrieved April 21, 2019, from https://www.quantamagazine.org/statistician-proves-gaussian-correlation-inequality-20170328/

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