# Building initial models of rotating white dwarfs with SPH

José Miguel Blanco-Iglesias
Departament de Física,
Universitat Politècnica de Catalunya (UPC)
Barcelona, Spain
jose.miguel.blanco@upc.edu

Domingo García-Senz
Departament de Física,
Universitat Politècnica de Catalunya (UPC) and
Institut d'Estudis Espacials de Catalunya (IEEC)
Barcelona, Spain
domingo.garcia@upc.edu

Rubén M. Cabezón
sciCORE,
Universität Basel
Basel, Switzerland
ruben.cabezon@unibas.ch

***Abstract*—A general procedure to build self-gravitational, rotating equilibrium structures with the Smoothed Particle Hydrodynamics (SPH) technique does not exist. In particular, obtaining stable rotating configurations for white dwarf (WD) stars is currently a major drawback of many astrophysical simulations. Rotating WDs with low internal temperatures are connected with both, explosive and implosive scenarios such as type Ia supernova explosions or neutron stars formation. Simulations of these events with SPH codes demand stable enough particle configurations as initial models. In this work we have developed and tested a relaxation method to obtain equilibrium configurations of rotating WDs. This method is straightforward and takes advantage of the excellent mass and angular momentum conservation properties of the SPH technique. Although we focus on rigid rotation and its potential applications to several Type Ia supernova scenarios, we also show that our proposal is also able to provide good initial models in differential rotation, which has the potential to benefit many other types of simulations where rotation plays a capital role, like disk evolution and stellar formation.**

## I. Introduction

The smoothed particle hydrodynamics (SPH) technique has been widely used in astrophysics to study highly dynamical, geometrically distorted, and often catastrophic, events such as star formation [1], the merging of compact objects [2], supernova explosions [3], [4] or the modelling of large-scale structure in the Universe [5].

In the particular case of the merging of two white dwarf (WD) stars, the final outcome could be a type Ia supernova explosion (SNe Ia), being this double-degenerate scenario (DD) one of the favoured production channels for these explosions [6]. Another SNe Ia explosion channel involves a single WD accreting mass from a companion, non-degenerate star, through the Roche-Lobe overflow. This second possibility is called the single-degenerate scenario (SD) channel for SNe Ia. Both scenarios involve large amounts of angular momentum, so the question arises on how to adequately model self-gravitating fast-spinning rotators with SPH.

Unfortunately, there is no general procedure to build such self-gravitational, rotating equilibrium structures with SPH. A numerical scheme to handle thick disks in presence of pressure gradients was developed by [7] and there are several recipes to approach the initial conditions prior the dynamic merging of two WDs [8]. In this communication we develop and test an easy, albeit practical, scheme to build degenerate, zero-temperature, rotating structures in equilibrium. These structures can be used as suitable initial models to study the explosion of a rotating WD in the SD scenario as well as the outcome of the WDs merging in the DD scenario.

In Section II we describe the physical foundations of our proposal. Section III presents the application of the method to build three-dimensional, zero-temperature, white dwarfs in rigid rotation. The scheme developed in Section III is also applied to the initial setting of two interacting WDs in a compact binary system. The extension of the scheme to handle differential rotation is explained in Section IV. Finally, we present a summary of our findings and the prospects for future work in the conclusions.

## II. Relaxing rotating white dwarfs with SPH

Non-rotating equilibrium configurations of WDs can be obtained by relaxing a sample of particles with initial spherical coordinates $(r, \phi, \theta)$. These SPH particles are radially distributed according to the density profile, but randomly in angles $\phi, \theta$. Usually a damping force is added to the momentum equation so that, after a few sound-crossing times, the sample of particles relaxes to a stable configuration [9]. It is worth noting, however, that as the mass of the WD approaches the Chandrasekhar-mass limit such equilibrium is not perfect and the degenerate star undergoes small radial oscillations. Fortunately, the thermonuclear explosion of a massive WD is so fast that it is enough to keep the equilibrium only during a few sound-crossing times, $t_{sc}$ (typically $t_{sc} \sim 0.4$ s at $\rho \simeq 10^9$ g.cm$^{-3}$)[1].

In absence of rotation, the structure of the WD after the relaxation process is spherically symmetric and follows the well-known solution of the Lane-Emden (LE) equation. Therefore, for a zero-temperature electron gas the only parameters determining the density and pressure profiles are the mass of the WD and the electron molecular weight $\{M_{WD},\ \mu_e\}$.

[1]Unless explicitly stated, the c.g.s. system of units is used in this work, as it is customary in astrophysics literature

Unfortunately, there is not a simple description, equivalent to the Lane-Emden equation, but for rotating stars. Assuming that the rotating body has axisymmetric geometry its structure is set by the triad $\{M_{WD},\ J_{WD},\ \mu_e\}$ where $J_{WD}$ is the total angular momentum. For differential rotators, it is necessary to specify the rotational law followed by the angular velocity $\Omega(s)$, where $s$ is the distance to the rotation axis.

Once these parameters are defined, the maximum density ($\rho_{max}$) and radius ($R_{WD}$) of the configuration will be uniquely determined. It is important to note, that those final values ($\rho_{max}$ and $R_{WD}$) are different when rotation is present, even for the same combination of $\{M_{WD},\ \mu_e\}$. Indeed, once the mass and its composition are fixed, a rotating WD will have a lower $\rho_{max}$ and larger $R_{WD}$ in the rotating plane, than a not rotating one.

A general semi-analytical method to obtain self-gravitating structures in rotation is the self-consistent field (SCF) method developed by [10]. Later, Hachisu [11] successfully applied the SCF method to build zero-temperature spinning white dwarfs. In particular, the Hachisu work includes many tables with valuable information such as, for example the value of $\rho_{max}$ and the polar and equatorial radius, as a function of the adopted set of values $\{M_{WD}, J_{WD}, \mu_e\}$. We therefore use the results by Hachisu to test the viability of our relaxation method. When studying the efficiency on nuclear burning in Type Ia Supernova explosions, the dominant magnitude is the density. Therefore, we fix $\rho_{max}$ between rotating and no rotating models which, as a consequence, implies a change in the total mass of the WD.

In this work we have considered the following rotation-law:

$$\Omega(s) = \frac{\Omega_c}{\left(1 + \frac{s^2}{R_c^2}\right)^m}, \tag{1}$$

where $R_c$ is a parameter which sets the size of the central core with nearly rigid rotation $\Omega_c$, and $m$ is a parameter linked to the type of rotation (see Section III). Current choices of $m$ are: $m = 0$ (rigid rotation), $m = 1/2$ and $m = 1$ (shellular). Rigid rotation is also attained for $R_c >> R_{WD}$, independently of $m$.

Our relaxation method works as follows. Firstly, we choose the values of $\rho_{max}$, $M_{WD}$ and $J_{WD}$ from the Hachisu [11] data-tables, and we take $\mu_e = 2$ in the equation of state (EOS). We then build an initial model with spherical symmetry (i.e. without rotation) and maximum density, $\rho_{max}$. Such model is obtained after integrating the Lane-Emden equation with inner boundary condition $\rho(r = 0) = \rho_{max}$. We note that the total mass of the WD differs depending on the rotation profile of the star, so that the maximum density is the same across models. The density profile is then mapped to a 3D distribution of mass particles and their mass re-scaled so that the total mass becomes $M_{WD}$. Next, a velocity is given to each SPH particle so that the total angular momentum is $J_{WD}$,

$$J_{WD} = \sum_b m_b s_b v_b = \left[\sum_b m_b s_b^2 \left(1 + \frac{s_b^2}{R_c^2}\right)^{-m}\right] \Omega_c(t), \tag{2}$$

from which the instantaneous value of $\Omega_c(t)$ is obtained.

$$\Omega_c(t) = \frac{J_{WD}}{\left[\sum_b m_b s_b^2 \left(1 + \frac{s_b^2}{R_c^2}\right)^{-m}\right]}. \tag{3}$$

Note that when $R_c \to \infty$ the angular velocity becomes $\Omega(t) = J_{WD}/I_{WD}(t)$, where $I_{WD}(t)$ is the time dependent momentum of inertia of particles around the rotation axis, which is computed at each integration step during the relaxation.

Once $\Omega_c(t)$ is known, (1) gives $\Omega(s,t)$, so that the centripetal acceleration of each particle $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$ is obtained. The particle distribution is henceforth relaxed in a co-moving reference frame. We then let the system freely evolve with the SPH code until equilibrium. To accelerate the convergence, the velocities are periodically set to zero. After several sound crossing times the final equilibrium configuration is attained.

## III. Calculated models

Here we mainly focus on the evolution towards equilibrium of models calculated assuming rigid rotation for both, single WDs and double degenerate stars in compact binaries. Some insights of how to handle differential rotation, obeying the rotation law given by (1) with $m = 1/2$ are provided in section IV. We performed all simulations presented here with the hydrodynamics code SPHYNX [12], using the electron zero-temperature EOS,

$$P_e = a\ [x(2x^2 - 3)(x^2 + 1)^{\frac{1}{2}} + 3\ sinh^{-1}x], \tag{4}$$

$$\rho = bx^3, \tag{5}$$

where $a = 6.00{\times}10^{22}$ dynes/cm$^2$ and $b = 9.82{\times}10^5\mu_e$ g/cm$^3$. The mean molecular weight was taken $\mu_2 = 2$ in all calculations. A cautionary remark concerning fast rotation, close to keplerian values, is necessary here. For these models we observed the runaway of some surface particles located on the equatorial plane, which is a numerical artifact. To avoid this and keep the relaxation stable we included a reduction of the pressure in the low-density regions of the WDs. To do that we simply multiply the electron pressure (Eq. 4) by a density cut-off,

$$P = \begin{cases} P = P_e & \rho > \rho_{crit} \\ P = P_e \times \frac{\rho}{\rho_{crit}} & \rho \le \rho_{crit} \end{cases}, \tag{6}$$

where $P$ is the pressure used in the calculations and $\rho_{crit} \simeq 5{\times}10^{-4}\rho_{max}$ being $\rho_{max}$ the maximum density, which usually is attained at the center of the configuration (but not always).

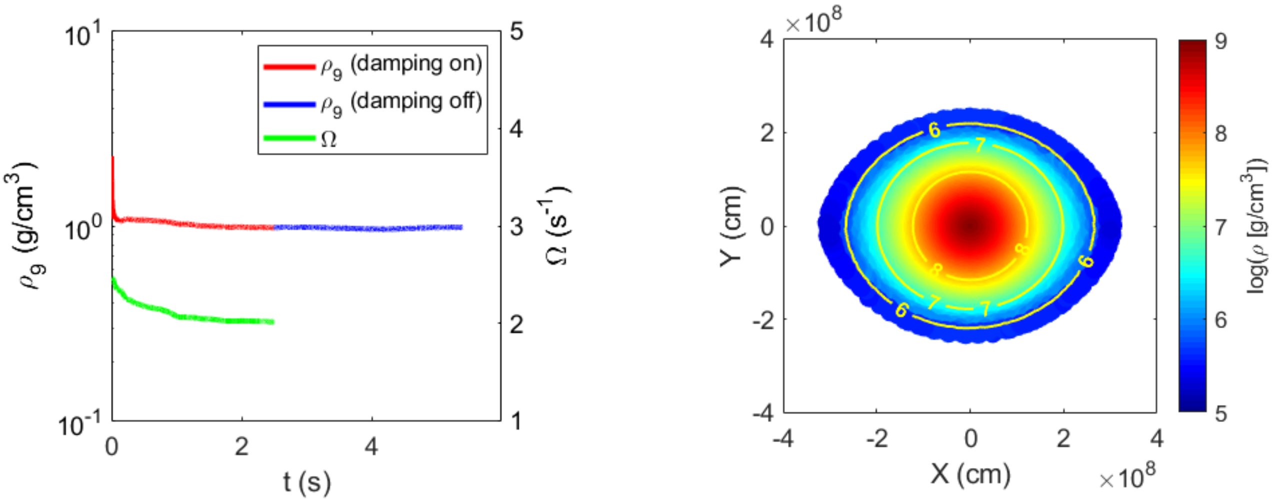


Fig. 1. Simulation results of model $A_2$ in Table I. Left: Evolution of central density (in $10^9$ g/cm$^3$) and angular velocity $\Omega$. After removing the damping at $t = 2.5$ s, the central density remains stable during several sound-crossing, $t_{sc}$, times ($t_{sc} \simeq 0.4$ s). Right: density colormap and iso-density contours of a 2D meridional slice of the rotating WD at $t = 2.5$ s.

TABLE I
MODELS WITH RIGID ROTATION: MODELS A ARE THE SPH CALCULATIONS PERFORMED WITH SPHYNX. MODELS H ARE THE SCF SEMI-ANALYTICAL CALCULATIONS BY HACHISU. LAST COLUMN SHOWS THE POLAR TO EQUATOR RADIUS RATIO F. ENERGIES AND DENSITIES ARE IN C.G.S. UNITS.

| | $M_{wd}$ ($M_\odot$) | $J_{wd}$ ($\times 10^{50}$) | $E_k$ ($\times 10^{50}$) | $E_I$ ($\times 10^{50}$) | $-E_G$ ($\times 10^{50}$) | $\rho_{max}$ ($\times 10^6$) | $R_{eq}$ (km) | F |
|---|---|---|---|---|---|---|---|---|
| $A_1$ | 1.35 | 0 | 0 | 18.1 | 22.6 | 1004 | 2360 | 1 |
| $A_2$ | 1.44 | 0.522 | 0.525 | 18.7 | 24.5 | 980 | 3070 | 0.752 |
| $A_3$ | 1.28 | 0.745 | 0.313 | 6.32 | 9.78 | 102 | 5820 | 0.687 |
| $A_4$ | 0.908 | 0.707 | 0.118 | 1.43 | 2.67 | 9.91 | 9470 | 0.678 |
| $A_5$ | 0.674 | 0.543 | 0.056 | 0.564 | 1.134 | 3.13 | 12000 | 0.663 |
| $H_1$ | 1.35 | 0 | 0 | 18.1 | 22.6 | 1000 | 2460 | 1 |
| $H_2$ | 1.44 | 0.522 | 0.537 | 18.9 | 24.6 | 1000 | 3500 | 0.667 |
| $H_3$ | 1.28 | 0.745 | 0.313 | 6.25 | 9.69 | 100 | 6040 | 0.667 |
| $H_4$ | 0.908 | 0.707 | 0.119 | 1.43 | 2.69 | 10 | 9720 | 0.667 |
| $H_5$ | 0.674 | 0.543 | 0.056 | 0.553 | 1.14 | 3.16 | 12100 | 0.667 |

### *A. Rigid rotation: Single spinning white dwarfs*

In the SD scenario, the accretion of mass from the companion star transfers angular momentum to the compact star. Therefore, it is expected that the WD is spinning at the moment of the explosion. The fingerprint of the rotation in the thermonuclear explosion of a WD has been studied by [13], [14] and [15]. In the latter, the code SPHYNX was used to simulate the explosion of a WD with mass $\simeq 1M_\odot$ in fast rigid rotation. Although it is thought that the accretion of matter from the companion star would lead to differential rotation [16], the assumption of rigid rotation is easy to implement and very useful to gain insight on many physical problems. Moreover, the rigid rotation hypothesis is adequate in those cases where the transport of angular momentum from the surface to the center of the star is very efficient [17] as it could be the case of degenerate objects like WDs.

Here we carry out the relaxation of several WDs with different $\{M_{WD}, J_{WD}\}$, using the hydrocode SPHYNX. An estimation of the accuracy of the resulting equilibrium configurations is done by comparing our results to those by Hachisu [11], being the latter obtained using the SFC method.

We provide a representative example of the evolution towards equilibrium of one of our rotating models in Fig. 1 (left), which depicts the evolution of the maximum density $\rho_9$ and angular velocity $\Omega$. As we can see, both magnitudes evolve in a similar manner. They start from relatively high values, decrease fast during a couple of tenths of second, and stabilize at $t \simeq 1$ s. To check that the star was stable, we removed the damping at $t = 2.5$ s, so that the system evolves freely. As it can be seen, the central density remained stable during at least $\Delta t \simeq 2$ s ($\simeq 5$ times the sound-crossing time). We show a color-map slice of the density at $t = 2.5$ s in Fig. 2 (right). The white dwarf is neatly oblated with a polar

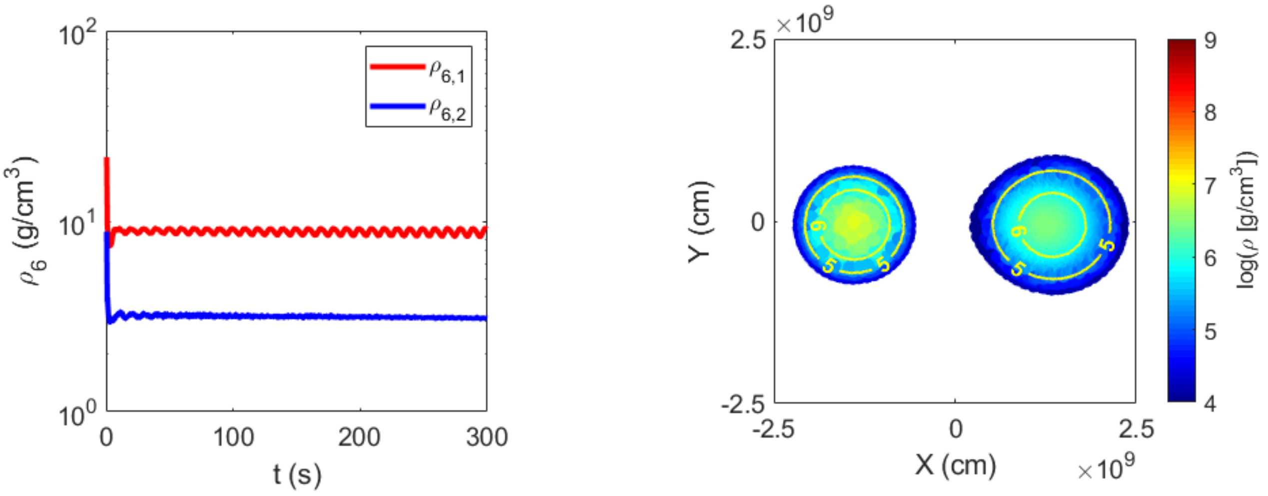


Fig. 2. Initial setting of the DD scenario. left: Evolution of the central desity of both WDs during the relaxation. Right: Slice in the orbital plane depicting the density colormap of each white dwarf at $t \simeq 300$ s

to equator radius $\frac{R_p}{R_{eq}} = 0.75$ which is $\simeq 12\%$ larger than that given by the SCF method. This is, however, the largest discrepancy found across all the calculated models in Table I. The differences with respect the results by Hachisu remain, for the most part, below $4\%$.

Table I summarizes the relevant information of the calculated models. Models A refer to the SPHYNX calculations and H refer to the SFC models by Hachisu. Models $A_1$ and $H_1$ are non rotating, with a mass close to the Chandrasekhar-mass limit. As we can see, the fit is excellent. The larger discrepancy, $\simeq 4\%$, is in the radius of the configuration. Actually this is something expected, because a white dwarf nearing the Chandrasekhar-mass limit has a not well-defined scale-length. We thus expect that the larger differences with respect the SFC method affect the equatorial radius and the oblateness of the WDs at $\rho_9 \geq 1$. Therefore, our relaxation method is able to match the Hachisu results in a wide range of stellar masses, $0.67 M_\odot \leq M_{WD} \leq 1.44 M_\odot$. The lower mass is close to that of a standard WD and the higher mass is actually at the Chandrasekhar-mass limit of a non-rotating white dwarf.

### *B. Rigid rotation: Initial models of compact double degenerate binaries.*

A straightforward and timely extension of the relaxation procedure described above can be used to generate suitable initial models to study the DD production channel for SNe Ia. In the DD channel, a couple of WDs settled in a compact orbit get closer because of gravitational-wave radiative losses. At some point the gravitational pulling from the more massive WD breaks the lighter compact star and an accretion disk around the surviving WD forms. The further accretion of the debris would provoke the explosion of the initially more massive white dwarf [6].

TABLE II
MODELS OF DD.

| | $M_{wd1}$ ($M_\odot$) | $M_{wd2}$ ($M_\odot$) | $J_{sys}$ ($\times 10^{50}$) | $\frac{1}{\beta}$ | $\rho_{max1}$ ($\times 10^6$) | $\rho_{max2}$ ($\times 10^6$) | Period (s) |
|---|---|---|---|---|---|---|---|
| $DD_1$ | 0.705 | 0.705 | 5.203 | 5 | 5.70 | 5.70 | 58 |
| $DD_2$ | 0.796 | 0.606 | 5.209 | 4.2 | 9.40 | 3.20 | 67 |

A key technical point of the simulations of the DD scenario is how to set the immediate initial conditions prior the merging. A common procedure is to build spherically symmetric single white dwarfs and put them at some initial distance. Afterwards the two WDs are artificially displaced a small fraction of the initial distance so that they begin to feel the tidal forces. After a sequence of approaching-relaxing steps the two WDs settle in the final configuration at the verge of the catastrophic merging [18].

A somehow more elegant and fast procedure is to relax both stars once they are already settled in a close orbit just prior the merging. To do that we first set the orbit parameters so that the gravitational pulling onto the surface of the less massive WD becomes a sizable fraction of its own gravity ($\beta$). That constraint leads to the following expression for the distance,

$$D_{1,2} = R_2 \times \left(1 + \sqrt{\frac{M_{WD1}}{\beta\ M_{WD2}}}\right). \qquad (7)$$

Once $D_{1,2}$ is known, and assuming a circular orbit, we calculate the velocity of the center of mass of each star, $M_{WD1}$ (the more massive) and $M_{WD2}$. The total angular momentum of the binary system $J_{sys}$ is calculated so that we can benefit from the scheme developed above, in Sections II and III-A.

As an example we have calculated the stable initial con-

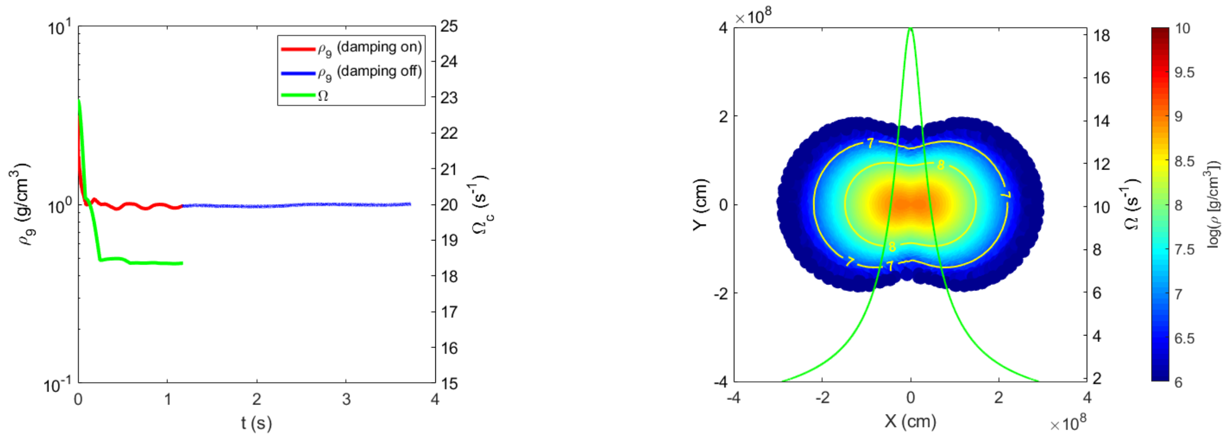


Fig. 3. Differential Rotation. Model A7 in Table III. Superposed to the density color map is the 1D profile of angular velocity (green) along a one-dimensional cut on the equator plane.

figurations in two cases. We have first considered a system of twin stars with masses $M_{WD1} = M_{WD2} = 0.705$ $M_\odot$ and parameter $\beta = 1/5$. The second case is for $M_{WD1} = 0.796$ $M_\odot$, $M_{WD2} = 0.606$ $M_\odot$, and $\beta \simeq 1/4$. In this work we focus on pairs of white dwarfs where the orbital movement was the only contribution to the total angular momentum $J_{sys}$. Nevertheless, including the contribution of the WDs self-rotation in compact binaries is not difficult because tidal coupling enforce synchronous spin and orbital periods. A summary of the parameters used in these simulations is given in Table II.

The evolution of the central densities of both WDs, as well as that of the orbital angular velocity is shown in Fig. 2 (left). We also provide the equilibrium configuration of the WDs in the same figure (right), obtained in the comoving non-inertial frame of reference located at the center of mass. After the relaxation, we checked that the final structure remains stable during at least one complete orbit in the inertial frame. Although the final binary configuration is stable, it is at the verge of the Roche-Lobe overflow. To trigger the catastrophic merging of the WDs it is enough to slightly shrink the distance $D_{1,2}$.

## IV. Differential rotation

It is feasible to apply the proposed relaxation method to handle differential rotation. In particular, we were able to build stable initial models obeying the rotation-law given by (1), for $m = 1/2$ and $m = 1$. These cases are usually referred as $v$-constant and $j$-constant profiles in the literature. As it can be seen in Table III, we find a good agreement among our models and those reported by Hachisu for the case $m = 1/2$. For these models $R_c = 0.1R_{eq}$, where the value of $R_{eq}$ was taken from the work by Hachisu. For the $m = 1$ cases we were able to produce stable initial models only if $R_c \geq 0.3R_{eq}$, otherwise the rotation in the central region of the WD was too high for our method to converge to the correct values. Obtaining stable models with strong differential rotation would probably demand higher resolution and more elaborated initial trial models than the naïve Lane-Emden configurations considered here. This is left for a future work.

TABLE III
Models with differential rotation: Models A are the SPH calculations (Eq. 1, with $m = 1/2$). H are the SCF semi-analytical models by Hachisu. Last column shows the polar to equator radius ratio F. Energies are in CGS units

| | $M_{wd}$ $M_\odot$ | $J_{wd}$ $10^{50}$ | $E_k$ $10^{50}$ | $E_I$ $10^{50}$ | $-E_G$ $10^{50}$ | $\rho_{max}$ $10^6$ | $R_{eq}$ km | F |
|---|---|---|---|---|---|---|---|---|
| $A_6$ | 1.65 | 0.993 | 1.92 | 22.9 | 32.3 | 970 | 2680 | 0.698 |
| $A_7$ | 1.99 | 1.86 | 4.78 | 29.1 | 45.5 | 980 | 2900 | 0.510 |
| $H_6$ | 1.65 | 0.993 | 1.96 | 23.3 | 32.8 | 1000 | 2720 | 0.667 |
| $H_7$ | 1.99 | 1.86 | 4.87 | 29.6 | 46.1 | 1000 | 2940 | 0.500 |

We show a summary of our results for a two cases with $m = 1/2$ in Table III and Fig 3. The two models differ from the SCF calculations in less than 5%, being stable enough for further hydrodynamic calculations. Figure 3 (left), shows the evolution of the maximum density and central value of the angular velocity for model $A_7$. The profile of the angular velocity $\Omega$ has been superposed to the density color-map (right). It is clear that the maximum density is not located at the center of mass of the configuration, which is a typical signature of models with high angular momentum. The value of the angular velocity $\Omega$ is maximum at the center of the configuration and is very high, $\Omega(s = 0) \simeq 18$ $s^{-1}$.

## V. Conclusion

A common problem in simulating the evolution of compact objects with SPH is that there is not a general procedure to obtain stable initial models when these objects are spinning fast. In this communication we propose and test an easy and versatile relaxation scheme to build stable rotating models of cold WDs. As detailed in section II, our method relies in the exceptional angular momentum conservation properties of the SPH technique.

We apply our method to get stable rotating configurations of WDs with different masses and angular momentum. We were able to build stable models of rotating white dwarfs spanning a wide mass-range, $0.67M_{\odot} \leq M_{WD} \leq 2M_{\odot}$, with both, rigid and differential rotation. The main magnitudes: central densities, total kinetic, internal and gravitational energies, equatorial and polar radius, match the semi-analytical results by Hachisu [11], in general, within a few percent. Given the current uncertainties in the particular rotation-law followed by these compact objects, that precision is enough to explore many issues concerning to their evolution, either for explosion or collapse scenarios. Additionally, we showed that our method is also able to produce stable configurations when it is applied to a pair of white dwarfs orbiting in a compact binary system. This last scenario is of capital importance to understand the double-degenerate route to Type Ia supernova explosions.

An immediate additional application of our method may consist in building 3D stable models of fast-rotating neutron stars, either isolated or in binary systems. That task should not be difficult because the equation of state of neutron stars can also be described assuming zero temperature. Another potential field of application is to build steady accretion disks, especially in those cases where the dynamical effects of pressure gradients are important.

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## References

[1] Springel, V., Hernquist, L., "Cosmological smoothed particle hydrodynamics simulations: a hybrid multiphase model for star formation" , MNRAS 339, 289, 2003

[2] Lorén-Aguilar, P., Guerrero, J., Isern, J., Lobo, J. A. and García-Berro, E., "Gravitational wave radiation from the coalescence of white dwarfs", MNRAS 356, 627, 2005

[3] García-Senz, D., Bravo, E., "Type Ia Supernova models arising from different distributions of igniting points", A&A, 430, 585, 2005

[4] Cabezón, R. M., Pan, K.-C., Liebendörfer, M., Kuroda, T., Ebinger, K., Heinimann, O., Perego, A. and Thielemann, F.-K.,"Core-collapse supernovae in the hall of mirrors. A three-dimensional code-comparison project" , A&A, 619A, 118C, 2018

[5] Springel, V., "The cosmological simulation code GADGET-2", MNRAS, 364, 1105, 2005

[6] Hillebrandt, W., Kromer, M., Röpke, F. K. and Ruiter, A. J., "Towards an understanding of Type Ia supernovae from a synthesis of theory and observations", Frontiers of Physics, Vol 8, 116, 2013

[7] Raskin, C., Owen, J.M., "Examining the Accuracy of Astrophysical Disk Simulations with a Generalized Hydrodynamical Test Problem", ApJ 831, 26, 2016

[8] Dan, M., Rosswog, S., Guillochon, J. and Ramirez-Ruiz, E., "Prelude to A Double Degenerate Merger: The Onset of Mass Transfer and Its Impact on Gravitational Waves and Surface Detonations", ApJ, 737, 89, 2011

[9] García-Senz, D., Bravo, E. and Serichol, N., "A Particle Code for Deflagrations in White Dwarfs. I. Numerical Techniques", ApJS, 115, 119, 1998

[10] Ostriker, J. P., Mark, J. W.-K., "Rapidly rotating stars. I. The self-consistent-field method", ApJ, 151, 1075, 1968

[11] Hachisu, I., "A versatile method for obtaining structures of rapidly rotating stars", ApJS, 61, 479, 1986

[12] Cabezón, R. M., García-Senz, D. and Figueira, J., "SPHYNX: an accurate density-based SPH method for astrophysical applications", A&A, 606, A78, 2017

[13] Pfannes, J. M. M., Niemeyer, J. C., Schmidt, W. and Klingenberg, C., "Thermonuclear explosions of rapidly rotating white dwarfs". I. Deflagrations", A&A, 509, A74, 2010

[14] Pfannes, J. M. M., Niemeyer, J. C. and Schmidt, W., "Thermonuclear explosions of rapidly rotating white dwarfs. II. Detonations", A&A, 509, A75, 2010

[15] García-Senz, D., Cabezón, R. M. and Domínguez, I., "Surface and Core Detonations in Rotating White Dwarfs", ApJ, 862, 27, 2019

[16] Yoon, S.-C., Langer, N., "On the evolution of rapidly rotating massive white dwarfs towards supernovae or collapses", A&A, 435, 967, 2005

[17] Piro, A. L., "The Internal Shear of Type Ia Supernova Progenitors During Accretion and Simmering", ApJ, 679. 616, 2008

[18] Lorén-Aguilar, P., Isern, J. and García-Berro E., "High-resolution smoothed particle hydrodynamics simulations of the merger of binary white dwarfs", A&A, 500, 1193, 2009