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Aggregation Across Agents in Demand Systems

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AGGREGATION ACROSS AGENTS IN DEMAND SYSTEMS

INTRODUCTION

This is a brief survey of the topic of aggregation across agents in demand systems. The purpose of this paper is not to give a complete account of the existing literature in the field, but to illustrate in a simple way three major steps in the development of economic theory on this subject. The first step is represented by the contributions of Gorman and Nataf, both in 1953, partially anticipated by Antonelli (1886); the second concerns the articles published by Muellbauer in 1975 and 1976; the third refers to Lau (1977a, 1977b, 1982). Further contributions by Jorgenson, Lau and Stoker (1982), Gorman (1981) and Heineke and Shefrin (1986,1987) are also reviewed.

To simplify matters, the problem which these authors try to answer might be summarised in the following way. Assume that the consumption behaviour of each agent is well described by means of individual demand functions which link the consumption of each good to prices and total individual expenditure. Do functions exist which relate aggregate consumption to prices and aggregate expenditure or income? Or do functions exist which relate aggregate consumption to prices and any index (or set of indices) which summarizes the effects of the distribution of individual expenditures? Let us take for granted that these *aggregate relations* do exist; do they share the same properties of the individual demand functions?

The theoretical relevance of these questions is evident. The relationships between aggregate variables must be connected in some way to individual behaviour. Relationships between aggregates are frequently posited in macroeconomic theory (and in econometric studies) without being accompanied by a rigorous analysis of the link. Even theories which claim to be microfounded and provide a well specified microtheory do not consider adequately the link between macro and micro. It is common practice to suppose that the macrorelation has the same form and properties as those of the microfunctions. This hypothesis is frequently justified by assuming the existence of a *representative agent*; however the implications deriving from the latter assumption are hardly ever made explicit.

Although the problems mentioned above mainly concern the consistency between micro and macro-theory, an aggregation problem might also arise within the horizon of the microeconomic discipline. Standard consumer theory regards the economic unit to be indifferently either an agent or a household. But as Samuelson (1956) points out, the existence of household demand functions is not a less troublesome problem than the existence of demand functions for the entire economy.

The implications for econometric work are also evident. If the relationships between aggregates do not exist, there is no point in estimating demand systems with aggregate data. On the other hand, if aggregate relations do exist but do not share the same properties of microrelations it is not correct to impose on aggregate systems restrictions derived from standard consumer theory such as homogeneity and the symmetry of the Slutsky matrix.

Formally, in its simplest version, the aggregation problem can be formulated in the following way. There are n consumers and m goods. The ith agent demands for the m goods, $\boldsymbol{y}_i = (y_{i1}, \ldots, y_{im})$, are given by the (vector) *microrelation*

$$\boldsymbol{y}_i = \boldsymbol{f}_i(x_i, \boldsymbol{p}), \tag{1}$$

where x_i is the total expenditure of consumer i and $\boldsymbol{p} = (p_1, \ldots, p_m)$ is the price vector[1]. The aggregate demand of the m goods is denoted by

$$\boldsymbol{y} = \sum_i \boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{G}(x_1, \ldots, x_n, \boldsymbol{p}),$$

and aggregate expenditure is $x = \sum_i x_i$. The crucial question is to find under what conditions does a *macrorelation* $\boldsymbol{F}(x, \boldsymbol{p})$ exist such that

$$\boldsymbol{y} = \boldsymbol{G}(x_1, \ldots, x_n, \boldsymbol{p}) = \boldsymbol{F}(x, \boldsymbol{p}). \tag{2}$$

[1] From now on, we shall use the terms expenditure and income as synonymous; indeed the difference between the two concepts is not relevant in this context. Moreover, we suppose that the price vector is the same for all agents.

It is easily seen that $\boldsymbol{F}(x, \boldsymbol{p})$ exists if individual incomes are always the same proportion of aggregate income; i.e. $x_i = r_i x$, $i = 1, \ldots, n$, $\sum r_i = 1$. However, in this paper we shall assume that the micro-incomes are not linked by any particular relationship[2].

If the latter hypothesis holds we can easily prove (Theorem 1) that the macrorelation (2) exists as long as the microrelations take the Gorman form; i.e. they are linear in income and have the same slope. This result has a simple intuitive explanation: equation (2) requires the aggregate consumption to depend solely on aggregate income, regardless of how this income is distributed among the consumers; this means that an increase in aggregate income must be spent in the same way, irrespective of which consumer is the recipient and whatever is his initial income. This condition is extremely restrictive and is widely contradicted by empirical evidence.

Muellbauer and Lau attempt to derive, in different ways, less severe aggregation conditions. Both reach the goal by modifying the formulation of the problem with particular reference to the definition of the macrorelation. According to the new definitions, the meaning and the properties of the aggregate relation are different from those of equation (2). In Muellbauer (1975, 1976), for instance, the value of the macrofunction is allowed to vary when a transfer of income between consumers takes place. In other words, the aggregate consumption is not independent, unlike equation (2), of income distribution. However, Muellbauer's macrofunction retains an important feature of (2): it is similar to the microfunctions as far as the properties and the functional form are concerned. It might then be regarded as an individual demand function: the demand function of the representative agent. In Lau (1977b, 1982) and Jorgenson, Lau, Stoker (1982), even this property is lost; the macrorelation has no longer any formal similarity to the

[2] Pierce (1963) deals with the properties of aggregate relations when individual incomes are proportional. In the same setting Chipman (1974) proves, following Eisenberg (1961), that if the agents have homothetic preferences (but not necessarily identical) the aggregate relations (2) are integrable. For further details and references see Shafer and Sonnenschein (1982). More recent work which takes into account restrictions on the distribution of income is Hildenbrand (1983) and Chiappori (1985). Wold and Jureen (1953), Malinvaud (1956, 1981) and Stoker (1982,1984,1986) assume that individual incomes move in such a way that the income distribution depends upon either a unique parameter or a small number of parameters.

individual demand functions and therefore it may not be regarded as the function of a representative agent. Nevertheless, it has a simple economic interpretation and useful properties for empirical studies. Lau's macrorelation exists (Theorem 6) under much weaker conditions than Gorman's and Muellbauer's, as we shall see below.

To sum up, we notice that in order to obtain sensible aggregation conditions we had to drop two unreasonable requirements: the former is that the income distribution does not affect aggregate consumption and the latter is that the aggregate relation has the same characteristics as those of the microrelations.

The outline of the paper is as follows. In the first section we shall consider the results of Gorman (1953, 1961), Antonelli (1886) and Nataf (1953, 1964), and clarify the link between Gorman's problem and that of the existence of a macrorelation such as (2). The equivalence between the concepts of representative agent and *community preference field* is shown in Appendix 1. The second section deals with the contributions of Muellbauer, with particular reference to Muellbauer (1976). In the third section the Lau's concept of *exact aggregation* (1977a, 1977b, 1982) is defined and discussed. Some results of Jorgenson, Lau and Stoker (1982), Gorman (1981) and Heineke and Shefrin (1986, 1987) are presented in section 4. The last section provides some concluding remarks. The proofs of Theorems 3, 4, and 6 are given in Appendix 2. All the proofs (except that of Theorem 5) assume the differentiability of the microrelations.

1. GORMAN

The solution to the problem of existence of an aggregate relation such as (2) is given by the following theorem.

Theorem 1. *Macrorelation (2) exists if and only if microfunctions (1) take the form:*

$$y_i = a_i(p) + b(p)x_i. \tag{3}$$

The functional form (3) is known as the *Gorman polar form*: the microrelations are linear in income and have the same slopes for all agents.

The equality of slopes is the Theil's (1954) condition of *perfect aggregation*; the difference with respect to the result of Theorem 1 is that in Theil the microrelations are linear by hypothesis[3].

Proof[4]. Necessity. Differentiating both members of (2) with respect to x_i we obtain:

$$\frac{\partial \boldsymbol{f}_i(x_i, \boldsymbol{p})}{\partial x_i} = \frac{\partial \boldsymbol{F}(x, \boldsymbol{p})}{\partial x}.$$

The term on the left-hand side does not depend on $x_j, j \neq i$. Therefore the right-hand side does not depend on $x_j, j \neq i$ and does not depend on x either. Consequently $\partial \boldsymbol{f}_i / \partial x_i$ may only depend on $\boldsymbol{p}$ and for an appropriate function $\boldsymbol{b}(\boldsymbol{p})$ the following relation holds:

$$\frac{\partial \boldsymbol{f}_i(x_i, \boldsymbol{p})}{\partial x_i} = \boldsymbol{b}(\boldsymbol{p}).$$

Integrating with respect to x_i yelds (3).
Sufficiency. If the microrelations have the form (3) it follows immediately that macrofunction (2) exists and is:

$$\boldsymbol{F}(x, \boldsymbol{p}) = \sum_i \boldsymbol{a}_i(\boldsymbol{p}) + \boldsymbol{b}(\boldsymbol{p})x.$$

■

Imposing the *adding up* condition, i.e. $\boldsymbol{f}_i \cdot \boldsymbol{p} = x_i$, we obtain the further restrictions $\boldsymbol{a}_i(\boldsymbol{p}) \cdot \boldsymbol{p} = 0$ and $\boldsymbol{b}(\boldsymbol{p}) \cdot \boldsymbol{p} = 1$.

Gorman (1953, 1961) problem is strictly related to that of the existence of an aggregate relation such as (2). Gorman assumes that microrelations (1) are *integrable* (i.e. derive from the maximization of a utility function) and looks for the existence conditions of a *community preference field.* Postulating the existence of a community preference

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[3] According to Theil's definition, perfect aggregation is still possible when microrelations have different slopes and the aggregate explanatory variable is a linear combination of the microincomes. But if the explanatory variable is assumed to be the aggregate income, then there is perfect aggregation only when the slopes are equal.

[4] A proof which does not require the differentiability of the microfunctions is in Jorgenson, Lau, Stoker (1982).

field amounts to requiring the aggregate relation to exist and to be integrable. In other words, the macrofunction may be thought of as the demand function of an optimizing agent – the representative agent – whose income is equal to the aggregate income. We refer to Appendix 1 for a more detailed account on the notions of community preference field and representative agent and for the proof of the equivalence between the two concepts.

The conditions of existence and integrability of (2) are also studied by Nataf (1953, 1964) and by Antonelli (1886), who tackles this problem in terms of indirect utility functions (see Appendix 1). The solution is given by the following theorem.

Theorem 2. *(Antonelli-Gorman-Nataf). Assuming that microrelations (1) are integrable, macrorelation (2) exists and is integrable if and only if the individual Engel curves for the same good (i.e. the demand curves in the income-commodity space*[5]*) are linear and parallel, that is, take the form (3).*

Proof[6]. From Theorem 1, we know that microfunctions of the form (3) are necessary and sufficient for macrofunction (2) to exist. In order to prove the theorem, it only remains to show that if the microfunctions are integrable, then macrorelation (2), if it exists, is integrable.

Differentiating (2) with respect to p_k we obtain $\partial F/\partial p_k = \sum_i \partial f_i/\partial p_k$. Differentiating the same relations with respect to x_i we have $\partial F/\partial x = \partial f_i/\partial x_i$ for all i. Hence it holds that

$$\frac{\partial F}{\partial p_k} + \frac{\partial F}{\partial x} y_k = \sum_i \left(\frac{\partial f_i}{\partial p_k} + \frac{\partial f_i}{\partial x_i} y_{ik} \right).$$

The term on the left-hand side is nothing other than the kth column of the Slutsky matrix of the macrorelations; hence, the equality establishes that this matrix is simply the sum of the Slutsky matrices of

[5] According to Gorman's terminology, the Engel curves are not the demand curves in the income-commodity space, but the income-expansion paths in the m-commodity space. Nevertheless, it is easily seen that the former are linear and parallel if and only if the latter are (see also Gorman 1953, p. 78).

[6] In Shafer and Sonnenschein (1982) there is a proof based on the approach of the theory of revealed preferences which does not require the differentiability of the microrelations.

the individual demand functions. Since the latter are symmetric and negative semidefinite, the former must also share the same properties. Furthermore, it can be easily seen that the macrorelation, if it exists, is homogeneous of zero degree in prices and aggregate income, due to the same property as the microfunctions. ■

Microrelations (3) imply that the income-expansion paths of different optimizing agents at each given price vector are parallel stright lines, i.e., agents have *quasi-homothetic* preferences. The intercepts of these lines are not necessarily zero, because of the term $\boldsymbol{a}_i(\boldsymbol{p})$. However, as we have already seen, the adding up condition implies $\boldsymbol{a}_i(\boldsymbol{p}) \cdot \boldsymbol{p} = 0$. Hence, if there is a good, say h, such that $a_{ih} > 0$, then there is at least one good, say k, such that $a_{ik} < 0$. Therefore, when incomes are close to zero, the demand for some goods is negative. If the microfunctions are defined for every non-negative income and both adding up and non-negativity are imposed, then $\boldsymbol{a}_i(\boldsymbol{p}) = \boldsymbol{0}$ for all i, so that income-expansion paths must pass through the origin and agents must have homothetic preferences. Gorman (1953, 1961) solves the problem by assuming that the income-expansion paths are only defined above a given indifference surface, i.e. the demand curves are defined for incomes over a certain level.

The aggregation conditions found by Gorman are extremely stringent. Agents are required to possess a substantially similar consumption behaviour with possible differences in tastes being confined to the term $\boldsymbol{a}_i(\boldsymbol{p})$. Moreover, an increase in income must always be spent in the same proportions regardless of the initial income level of the agent.

We can attempt to obtain less stringent conditions as to the form of the microrelations by slightly modifying the formulation of the problem. For instance, we may require the explanatory variable which appears in the macrorelation to be a generic function of the micro-incomes, instead of the aggregate income. In this case the macrorelation becomes:

$$\sum_i \boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{F}(g, \boldsymbol{p}), \tag{4}$$

where $g = g(x_1, \ldots, x_n)$. The conditions of existence of (4) are the *consistent aggregation* conditions of Green (1964, ch. 5).

In this definition the aggregate relation looses a straightforward economic meaning, in that the interpretation of the function g is not obvious; however, if (4) is integrable, g may still be regarded as the income of a representative agent. In fact, as Muellbauer (1975, 1976) emphasized, there is no need to require the income of the representative agent to be equal to the aggregate income; the only condition really needed is that there exists an integrable macrorelation which transform the function g of micro-incomes into aggregate consumption. Furthermore, under certain conditions that are provided further on, g can be interpreted as an index describing the distribution of income.

It should be noted that the definition of the aggregate explanatory variable given in (4) does not preclude the macrorelation accounting for "composition" effects: i.e. it is not excluded that a transfer of income from one consumer to another modifies the aggregate consumption. Indeed, if g is different from x, it does not necessarily remain unchanged when a mean preserving redistribution of income takes place. Consequently, if g varies (and (4) is not costant in g) aggregate consumption changes as well.

The following theorem establishes the existence conditions for (4).

Theorem 3. *Macrorelation (4) exists if and only if the microrelations take the form:*

$$\boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{a}_i(\boldsymbol{p}) + \boldsymbol{b}(\boldsymbol{p}) g_i(x_i). \tag{5}$$

We refer to appendix 2 for the proof of this theorem.

Theorem 3 requires the microrelations to be linear not with respect to income but instead with respect to some function of income (identical across different goods but not necessarily identical across agents). The functional form (5) is therefore more general than (3). However, it is worth emphasizing that the greater generality is only apparent. If the microrelations are required to obey the adding up condition $\boldsymbol{f}_i \cdot \boldsymbol{p} = x_i$, we are brought back to the Gorman form. Indeed, if (5) holds, adding up implies $g_i(x_i)\boldsymbol{b}(\boldsymbol{p}) \cdot \boldsymbol{p} + \boldsymbol{a}_i(\boldsymbol{p}) \cdot \boldsymbol{p} = x_i$; therefore $g_i(x_i)$ must be linear in income. The microrelations must then be linear in income and result in form (3).

2. MUELLBAUER

Muellbauer's approach differs from Gorman's in two main respects. In the first place, the income of the representative agent is not bound to be the aggregate income but instead it may be any function of microincomes (in the more general version of microincomes and prices). As we have already seen, this broader definition of the macrorelation is not sufficient for less stringent aggregation conditions to be achieved. Secondly, and this is the essential point, Muellbauer does not postulate the existence of aggregate relations for the *quantities* demanded but instead for the *budget shares* of each good. The two things do not coincide because the income of the representative agent is, in general, different from aggregate income. Indeed, suppose that the aggregate budget share W_h of good h is equal to the representative agent budget share of the same good. Then the aggregate consumption is $W_h x/p_h$, whereas the consumption of the representative agent is $W_h g/p_h$. If $g \neq x$, that is the income of the representative agent is different from the aggregate income, then the two quantities demanded differ.

Adopting this approach Muellbauer obtains more general conditions of existence and integrability than Gorman does: the microrelations may be non-linear in income but must obey the GL (*generalized linearity*) form or the PIGL (*price independent generalized linearity*) form[7]. On this basis Muellbauer (1975) exhorts econometricians to work with demand systems which are consistent with the PIGL form on which can be imposed, in a theoretically meaningful sense, the symmetry of the Slutsky matrix.

In this paper we focus on the existence conditions of Muellbauer's macrorelation; if the agents are optimisers, the integrability of the macrorelation does not result in any further restriction in the form of the microrelations (see Muellbauer (1976) pp. 983-4). Furthermore, we only consider the restrictions that aggregation places on the microrelations and not on individual expenditure functions; for more details on the latter we refer to Muellbauer (1975, 1976).

Formally Muellbauer's argument can be set out as follows. The aggregate budget share of good h is $W_h = \sum_i f_{ih} p_h / x$. The macrorelation

[7] See also Deaton and Muellbauer (1980).

is defined by $\boldsymbol{W} = \boldsymbol{W}(g, \boldsymbol{p})$, where $g = g(x_1, \ldots, x_n, \boldsymbol{p})$ (notice that g also depends on prices). Hence the macrofunction exist *if and only if* the following relation holds:

$$\sum_i \boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{Q}(g, \boldsymbol{p})x, \tag{6}$$

where $Q_h(g, \boldsymbol{p}) = W_h/p_h$.

Theorem 4. *(Muellbauer). A necessary and sufficient condition for macrorelation $\boldsymbol{W}(g, \boldsymbol{p})$ to exist is that the microrelations take the form*

$$\boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{a}_i(\boldsymbol{p}) + \boldsymbol{b}(\boldsymbol{p})x_i + \boldsymbol{c}(\boldsymbol{p})g_i(x_i, \boldsymbol{p}), \tag{7}$$

where at least one of these conditions is met: (i) $\sum_i \boldsymbol{a}_i(\boldsymbol{p}) = \boldsymbol{0}$; (ii) $\boldsymbol{c}(\boldsymbol{p}) = \boldsymbol{0}$.

For the proof[8] see Appendix 2. The macrorelation $\boldsymbol{W} = \boldsymbol{W}(g, \boldsymbol{p})$ corresponding to (7) is given by $\boldsymbol{W}(g, \boldsymbol{p}) = \hat{\boldsymbol{b}}(\boldsymbol{p}) + \hat{\boldsymbol{c}}(\boldsymbol{p}) \sum_i g_i(x_i, \boldsymbol{p})/x$, with an obvious change in notation. The income of the representative agent can be chosen as

$$g(x_1, \ldots, x_n, \boldsymbol{p}) = \frac{\sum_i g_i(x_i, \boldsymbol{p})}{x}. \tag{8}$$

Requiring microrelations to satisfy the adding up condition, we obtain that (7) can be written in such a way so that the following relations hold[9]: $\boldsymbol{c}(\boldsymbol{p}) \cdot \boldsymbol{p} = 0, \boldsymbol{a}_i(\boldsymbol{p}) \cdot \boldsymbol{p} = 0$ and $\boldsymbol{b}(\boldsymbol{p}) \cdot \boldsymbol{p} = 1$. Then, when the problem is defined in terms of budget shares, the summability condition does not bring us back to the Gorman form. The form (7) where adding up and homogeneity are imposed is the GL form.

[8] Theorem 4 summarises results 2A and 2B of Muellbauer 1976. Theorem 2B of Muellbauer contains two imprecisions: the term $\boldsymbol{a}_i$ does not depend on $\boldsymbol{p}$ and condition (ii) is missing. Both of them must be eliminated for the Gorman form to be a particular case of Muellbauer's.

[9] Adding-up implies $x_i - \boldsymbol{a}_i(\boldsymbol{p}) \cdot \boldsymbol{p} = \boldsymbol{b}(\boldsymbol{p}) \cdot \boldsymbol{p}\, x_i + \boldsymbol{c}(\boldsymbol{p}) \cdot \boldsymbol{p}\, g_i(x_i, \boldsymbol{p})$. If g_i is not linear in x_i we must have $\boldsymbol{c}(\boldsymbol{p}) \cdot \boldsymbol{p} = 0$, which in turn implies $\boldsymbol{a}_i(\boldsymbol{p}) \cdot \boldsymbol{p} = 0$ and $\boldsymbol{b}(\boldsymbol{p}) \cdot \boldsymbol{p} = 1$. If g_i is linear in x_i we are back to the Gorman case, which involves the same restrictions.

It is worth emphasizing that the GL form, apart from homogeneity and adding up, does not place any restriction on the microfunction of good h in itself[10]. The restriction concerns the relation between microfunctions of different goods which are required to be linear in the same two variables x_i and $g_i(x_i, \boldsymbol{p})$. This is where the term generalized linearity comes from. Moreover, the (vector) coefficients $\boldsymbol{b}(\boldsymbol{p})$ and $\boldsymbol{c}(\boldsymbol{p})$ must be identical across agents. This means that the individual demanded bundles, for any given $\boldsymbol{p}$, are linear combinations of the same two vectors (apart from the term $\boldsymbol{a}_i(\boldsymbol{p})$). In other words, the Engel curves of different agents must lie on parallel planes.

A special case of the GL form is obtained when g, the income of the representative agent, does not depend on $\boldsymbol{p}$. From (8) it is easy to see that if g is constant in $\boldsymbol{p}$ then also g_i must be constant in $\boldsymbol{p}$ for all i. Hence, if the representative income cannot depend on $\boldsymbol{p}$ the microrelations must obey the form

$$\boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{a}_i(\boldsymbol{p}) + \boldsymbol{b}(\boldsymbol{p})x_i + \boldsymbol{c}(\boldsymbol{p})g_i(x_i). \tag{9}$$

Imposing homogeneity we obtain either, (i) $g_i = d_i x_i^{\alpha}$ or (ii) $g_i = x_i \log(x_i/d_i)$, where α and d_i are constants (see Muellbauer 1976). The form (9), where adding up and homogeneity are imposed is the Muellbauer's PIGL form. Form (ii) is also known as PIGLOG. Note that, unlike (7), equation (9) places a restriction on the form of f_{ih} regardless of the form of f_{ik}'s, $k \neq h$: prices must combine with income in a proper way as in (5).

Both GL and PIGL are more general than Gorman's microfunctions. First, they are not necessarily linear in income. Second, differences in tastes are not confined to the term $\boldsymbol{a}_i(\boldsymbol{p})$, because of the presence of the function g_i. In the PIGL case, for instance, differences across agents may be captured by the parameters d_i, which can be interpreted as individual characteristics.

An example may be useful to illustrate a possible empirical application of Muellbauer's analysis[11].

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[10] Indeed, we can always reduce f_{ih} to the form (7) by setting $a_{ih}(\boldsymbol{p}) = b_h(\boldsymbol{p}) = 0$, $c_h(\boldsymbol{p}) = 1$ and $g_i(x_i, \boldsymbol{p}) = f_{ih}(x_i, \boldsymbol{p})$.

[11] See Muellbauer (1975) and Deaton and Muellbauer (1980, pp. 156–57).

Example 1. Let us consider the PIGL case (i) and assume that agents have identical tastes, so that, without loss of generality, we can write $g_i = x_i^\alpha$. In this case the income of the representative agent, given by (8), becomes $g = \sum_i x_i^\alpha / x$. A methodology that we could follow in order to calculate g is to combine expenditure distribution data with an estimate of the unknown parameter α, obtained from household budget data. For instance, if α turns out to be equal to 2, that is, the microfunctions are quadratic, then $g = (\sigma^2/\bar{x}^2 + 1)\bar{x}$, where $\bar{x}$ and σ^2 are respectively the mean and the variance of the income distribution. If σ^2 is known we can easily compute the income of the representative agent and use it to fit the macroequation.

In general microrelations such as (9) do not allow for a macrofunction like $\boldsymbol{F}(g, \boldsymbol{p})$ for the quantities. However, if we permit the aggregate relation to have *two* functions of micro-incomes as its arguments (in addition to prices) instead of having just one, then aggregation is also possible for quantities. It follows immediately that, for instance, (9) admits a macrorelation of the type $\boldsymbol{y} = \boldsymbol{F}(x, g, \boldsymbol{p})$, with $g(x_1, \ldots, x_n) = \sum_i g_i(x_i)$. In addition it is not hard to envisage that in further increasing the number of the explanatory macro-variables more general microrelations than (9) might be aggregated.

3. LAU

There are two main differences between Muellbauer's and Lau's approaches. The first difference concerns the definition of the *microrelations*; Lau's microfunctions include *individual attributes* as well as income as arguments. Individual attributes can be any demographic variable, such as the age of the household head or the size of the family, which helps to explain differences in household consumption patterns of families with the same income. The introduction of these types of variables is not new in consumer theory; in addition to the references given in Jorgenson, Lau and Stoker (1982) we can also quote the section devoted to aggregation in Friedman (1957).

The second and most important difference with respect to Muellbauer's approach concerns the definition of the *macrorelation*. According to Muellbauer's definition, the macrorelation $\boldsymbol{W} = \boldsymbol{W}(g, \boldsymbol{p})$ has only one explanatory variable in addition to prices, namely g. On the contrary, Lau's macrorelation may include several explanatory variables. These variables must be regarded as "indices" of the joint distribution of microincomes and individual attributes. It may be helpful to give an example to make this point clearer.

Example 2. Let us suppose for simplicity that individual attributes do not enter the microrelations, so that individual income is the only explanatory variable other than prices. Let these microrelations be $\boldsymbol{y}_i = \boldsymbol{a}(\boldsymbol{p})x_i + \boldsymbol{b}(\boldsymbol{p})x_i^2 + \boldsymbol{c}(\boldsymbol{p})x_i^3$. Summing over individuals yelds:

$$\boldsymbol{y} = \boldsymbol{a}(\boldsymbol{p})x + \boldsymbol{b}(\boldsymbol{p})\mu_2 + \boldsymbol{c}(\boldsymbol{p})\mu_3, \tag{10}$$

where $\mu_k = \sum x_i^k$ are moments of the income distribution. According to Lau's definition, equation (10) is a valid macrorelation. Aggregate consumption is not bound to depend on either the aggregate income alone, or a single function of microincomes. The "moments" μ_2 and μ_3 are variables which capture the further effects on aggregate consumption arising from the features of the income distribution.

It is obvious that in this definition the macrorelation loses its formal similarity with the microfunctions, since in general it possesses a different number of explanatory variables. Consequently, it can no longer be regarded as the demand function of a single agent. If the aggregate relation has only two explanatory variables, then a representative agent interpretation is still possible, under suitable assumptions, for the budget shares. But if the number of the explanatory variables is greater than two, the representative agent does not exist, either for the quantities or for the budget shares.

Example 3. To clarify this point, take Example 2 and set $\boldsymbol{c}(\boldsymbol{p}) = \boldsymbol{0}$. Multiplying every line h of equation (10) by p_h/x we obtain $\boldsymbol{W} = \boldsymbol{W}(g, \boldsymbol{p}) = \boldsymbol{a}^*(\boldsymbol{p}) + \boldsymbol{b}^*(\boldsymbol{p})\, g$, where $a_h^*(\boldsymbol{p}) = a_h(\boldsymbol{p})p_h$, $b_h^*(\boldsymbol{p}) = b_h(\boldsymbol{p})p_h$ and $g = \mu_2/x$. Therefore, the representative consumer exists for the budget shares and the microfunctions are aggregable in Muellbauer's sense. On the other hand, if $\boldsymbol{c}(\boldsymbol{p}) \neq \boldsymbol{0}$, so that three explanatory variables enter equation (10), then no macrofunction of the form $\boldsymbol{W}(g, \boldsymbol{p})$ exists.

Formally Lau's microfunctions are defined by

$$\boldsymbol{y}_i = \boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}), \tag{11}$$

where $\boldsymbol{A}_i$ is the vector of individual attributes. The macrofunction is defined by

$$\boldsymbol{y} = \boldsymbol{F}\big(g_1(x_1, \dots, x_n, \boldsymbol{A}_1, \dots, \boldsymbol{A}_n), \dots, g_q(x_1, \dots, x_n, \boldsymbol{A}_1, \dots, \boldsymbol{A}_n), \boldsymbol{p}\big), \tag{12}$$

with $q < n$. There is *exact aggregation* when the macrofunction (12) exists and satisfies the following conditions:

(*i*) the functions g_ℓ, $\ell = 1, \dots, q$, are symmetric, i.e they are invariant with respect to the substitution of the pair $x_i, \boldsymbol{A}_i$ for the pair $x_j, \boldsymbol{A}_j$, for all i, j;

(*ii*) the functions $g_1, \dots, g_q$ are functionally independent, i.e. there is no non-constant function $L(\cdot)$ such that $L(g_1, \dots, g_q) = 0$;

(*iii*) there exists a good, say k, and q vectors of prices $\boldsymbol{p}^{(1)}, \dots, \boldsymbol{p}^{(q)}$ such that the functions $F_k(g_1 \dots, g_q, \boldsymbol{p}^{(1)})$, ..., $F_k(g_1, \dots, g_q, \boldsymbol{p}^{(q)})$ are invertible in the g_ℓ's.

The symmetry condition guarantees that the *indices* g_ℓ do not change when the joint distribution of income and attributes does not change. Non-symmetric functions cannot be regarded as indices of a distribution in that a distribution does not change when the characteristics of two agents are swapped. Condition (*ii*) implies that none of the functions $g_1, \dots, g_q$ can be expressed in terms of the others. Condition (*iii*) ensures that (12) cannot be written in terms of a smaller number of arguments g_ℓ. Consider for instance the macrorelation $\boldsymbol{F}(g_1, \dots, g_q, \boldsymbol{p}) = \boldsymbol{F}^*(g(g_1, \dots, g_q), \boldsymbol{p})$; this function depends on a single index, namely g, so that there is no way to determine uniquely the indices $g_1, \dots, g_q$, even though they are functionally independent.

Note that the number of indices g_ℓ is not equal to the number of explanatory micro-variables, as is the case in Green (1964). Furthermore, the g_ℓ's are functions of incomes as well as individual attributes, whereas in Green (1964) and Nataf (1948) each index can only depend on the corresponding micro-variables[12]. The following is an important preliminary result obtained by Lau (1982).

Theorem 5. *(Lau) A necessary condition for exact aggregation is that*

[12] In our opinion, the claim of Van Daal and Merkies (1984, p. 122) that Lau's theorem is a special case of Nataf's (1948) theorem is unfounded.

the microrelations (11) are of the type:

$$\boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \boldsymbol{f}(x_i, \boldsymbol{A}_i, \boldsymbol{p}) + \bar{\boldsymbol{a}}_i(\boldsymbol{p}). \tag{13}$$

Proof. This result solely depends on the symmetry of the functions g_ℓ. The symmetry property means that exchanging the pairs $x_s, \boldsymbol{A}_s$ for $x_r, \boldsymbol{A}_r$ in $\boldsymbol{F}(g_1, \ldots, g_q, \boldsymbol{p})$ will not affect the values of the function. In other words, by giving an agent the income and the attributes of another agent and vice versa, the aggregate consumption of each good must remain the same. Hence, $\sum \boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \sum_{i \neq s, i \neq r} \boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}) + \boldsymbol{f}_s(x_r, \boldsymbol{A}_r, \boldsymbol{p}) + \boldsymbol{f}_r(x_s, \boldsymbol{A}_s, \boldsymbol{p})$. After eliminating the identical terms and reordering we obtain

$$\boldsymbol{f}_s(x_r, \boldsymbol{A}_r, \boldsymbol{p}) - \boldsymbol{f}_r(x_r, \boldsymbol{A}_r, \boldsymbol{p}) = \boldsymbol{f}_r(x_s, \boldsymbol{A}_s, \boldsymbol{p}) - \boldsymbol{f}_s(x_s, \boldsymbol{A}_s, \boldsymbol{p}).$$

The terms on the right-hand side do not change with x_s and $\boldsymbol{A}_s$, whereas those on the left hand side do not vary with x_r and $\boldsymbol{A}_r$; therefore, both of them may only depend on prices. Since this is true for all s and r, all the microfunctions differ by a term which only depends on prices. ■

The requirement that the g_ℓ functions are symmetric means that all the microrelations must be the same for all agents up to an additive term which only depends on prices. This is of course a strong restriction. However, the introduction of individual attributes among the explanatory micro-variables reduces its stringency to the extent to which differences in preferences will result in differences in the values of the attributes. The following example may be useful to clarify this point.

Example 4. Let us assume that the demand function of agent i is $\boldsymbol{h}_i(x_i) = \boldsymbol{h}(x_i; \boldsymbol{t}_i)$, where $\boldsymbol{t}_i$ is a vector of parameters characterizing the preferences of the agent. Suppose then that $\boldsymbol{t}_i$ depends on *observable* individual attributes $\boldsymbol{A}_i$ by way of the relation $\boldsymbol{t}_i = \boldsymbol{t}(\boldsymbol{A}_i; \boldsymbol{v})$, where $\boldsymbol{v}$ is a vector of parameters which is common to all agents. The demand can then be expressed by means of the function

$$\boldsymbol{h}^*(x_i, \boldsymbol{A}_i; \boldsymbol{v}) = \boldsymbol{h}(x_i; \boldsymbol{t}(\boldsymbol{A}_i; \boldsymbol{v}));$$

in other words, we passed from a microfunction specified in terms of heterogeneous parameters to a microfunction of the observable attributes $\boldsymbol{A}_i$ and the parameters $\boldsymbol{v}$ which are identical for all agents.

It must be noticed that, as Hildenbrand (1985) pointed out, the individual preferences are not reducible to observable variables, that is the relation $\boldsymbol{t}_i = \boldsymbol{t}(\boldsymbol{A}_i; \boldsymbol{v})$ in the example above does not necessarily hold. If this were the case it would not be possible, even theoretically, to entirely model the differences in parameters by means of individual attributes. Although the "attributes" model is not completely general, it may still be the more fruitful approach for certain purposes. The estimation of demand systems with aggregate time series data by means of the "parameters" model requires the $\boldsymbol{t}_i$'s to be constant over time. This is a strong assumption which is not needed in the "attributes" model. Indeed, changes of preferences over time may be approximately modelled by means of the temporal pattern of the observable variables.

The following theorem is called *the fundamental theorem of exact aggregation.*

Theorem 6. *(Lau) Macrofunction (12) exists, is continuously differentiable and satisfies conditions (i), (ii) and (iii), if and only if the microrelations can be written in the following form:*

$$\boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \boldsymbol{b}_1(\boldsymbol{p}) g_1^*(x_i, \boldsymbol{A}_i) + \cdots + \boldsymbol{b}_q(\boldsymbol{p}) g_q^*(x_i, \boldsymbol{A}_i) + \boldsymbol{a}_i(\boldsymbol{p}), \qquad (14)$$

where the functions $b_{\ell k}, \ell = 1, \ldots, q$ and the functions $1, g_1^*, \ldots, g_q^*$ are linearly independent.

For the proof[13] we refer to Appendix 2.

Theorem 6 establishes that the microrelations must be linear in certain functions of incomes and attributes which are the same for all agents and goods. Moreover, the coefficients $\boldsymbol{b}_\ell$ are identical across agents; hence, for any given $\boldsymbol{p}$, individual Engel Curves lie on parallel hyperplanes. The functions g_ℓ^* are not required to be linear in income, as in Gorman, and need not be at most two in number as is the case in Muellbauer. The generalisation of the definition of macrofunction

[13] As is shown in Russel (1982), Lau's theorem can also be demonstrated on the basis of Richmond's (1976) results.

accomplished by Lau involves then a wide extension of the class of aggregable microfunctions, not only with respect to the Gorman form but also with reference to the PIGL form of Muellbauer[14].

If exact aggregation holds (i.e. the microrelations take the form (14)), we can choose the indices g_ℓ of the macrorelation (12) so as to be $g_\ell = \sum_i g^*_\ell(x_i, \boldsymbol{A}_i)$, $\ell = 1, \ldots, q$. Therefore, a corollary to the theorem is that the explanatory macrovariables must be expressible as sums of functions, each of them depending solely on x_i and $\boldsymbol{A}_i$. Indexes of the distribution which do not satisfy this property, such as the Gini index, are not allowed to enter the macrofunction as arguments.

Finally, note that in (14) the coefficients $\boldsymbol{b}_\ell$'s cannot depend on individual attributes. However, the presence of many functions g^*_ℓ allows us to admit microfunctions which, at first glance, would not seem to conform to (14).

Example 5. Let us consider, for instance, the function $f_{ih} = \hat{b}_h(\boldsymbol{p})\phi_h(\boldsymbol{A}_i)\hat{g}(x_i)$. Indeed, we cannot define either $b_h(\boldsymbol{p}) = \hat{b}_h(\boldsymbol{p})$ and $g^*(x_i, \boldsymbol{A}_i) = \phi_h(\boldsymbol{A}_i)\hat{g}(x_i)$ because g^* must be identical over goods, or $b_h(\boldsymbol{p}) = \hat{b}_h(\boldsymbol{p})\phi_h(\boldsymbol{A}_i)$, because b_h cannot depend on individual attributes. Nevertheless, such microrelations can be reduced to form (14) introducing as many g^*_ℓ's as goods, and defining $g^*_\ell(x_i, \boldsymbol{A}_i) = \phi_\ell(\boldsymbol{A}_i)\hat{g}(x_i)$, $b_{\ell h}(\boldsymbol{p}) = 0$ for $\ell \neq h$ and $b_{\ell h}(\boldsymbol{p}) = \hat{b}_h(\boldsymbol{p})$ for $\ell = h$. This would not be possible in Muellbauer's approach, where only two functions g^*_ℓ are admissible in the microrelations.

4. FURTHER CONTRIBUTIONS

4.1. Lau's theorem is used in Jorgenson, Lau and Stoker (1982), where a demand system for the United States based on the theory of exact aggregation — the *trascendental logarithmic model* — is proposed and estimated. In the paper the authors specify various particular forms of (14), by analysing the constraints placed on the microrelations by

[14] As a matter of fact, it is not possible to reduce exactly the PIGL form to Lau's form. The last term on the right-hand side of (9) admits a certain degree of heterogeneity in the microfunctions of the different agents, because Muellbauer does not require the index function $g(x_1, \ldots, x_n)$ to be symmetric.

the hypotheses of non-negativity of the demand functions, adding up, homogeneity and symmetry of the Slutsky matrix.

We do not intend to go through the details of Jorgenson, Lau and Stoker's work but we mention the following results. First, requiring that (i) aggregate consumption of each good is zero when all individual incomes are zero and (ii) individual consumption is non-negative, it is always possible to chose the g_ℓ^* in such a way that $g_\ell^*(0, \boldsymbol{A}_i) = 0$ for all ℓ and $\boldsymbol{a}_i(\boldsymbol{p}) = \boldsymbol{0}$ for all[15] i. Hence, assumptions (i) and (ii) completely remove the chance of modelling differences in preferences by means of differences in the parameters of the microfunctions; nevertheless, it remains possible to express them through the values of the attributes $\boldsymbol{A}_i$.

Secondly, imposing adding up it turns out that aggregate income can always be chosen as an explanatory macrovariable. Indeed, adding up implies

$$\boldsymbol{b}_1(\boldsymbol{p}) \cdot \boldsymbol{p}\, g_1^*(x_i, \boldsymbol{A}_i) + \cdots + \boldsymbol{b}_q(\boldsymbol{p}) \cdot \boldsymbol{p}\, g_q^*(x_i, \boldsymbol{A}_i) = x_i.$$

At least one of the vector products on the left-hand side is different from zero. Let us say that it is $\boldsymbol{b}_1(\boldsymbol{p}) \cdot \boldsymbol{p}$. Then $g_1^* = \beta_1(\boldsymbol{p})x_i + \beta_2(\boldsymbol{p})g_2^* \cdots + \beta_q(\boldsymbol{p})g_q^*$, where $\beta_1(\boldsymbol{p}) = 1/\boldsymbol{b}_1(\boldsymbol{p}) \cdot \boldsymbol{p}$ and $\beta_\ell(\boldsymbol{p}) = -\boldsymbol{b}_\ell(\boldsymbol{p}) \cdot \boldsymbol{p}/\boldsymbol{b}_1(\boldsymbol{p}) \cdot \boldsymbol{p}$. Hence, equation (14) can be written as

$$\boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \boldsymbol{c}_1(\boldsymbol{p})x_i + \boldsymbol{c}_2(\boldsymbol{p})g_2^*(x_i, \boldsymbol{A}_i) + \cdots + \boldsymbol{c}_q(\boldsymbol{p})g_q^*(x_i, \boldsymbol{A}_i), \quad (15)$$

with $\boldsymbol{c}_1(\boldsymbol{p}) = \boldsymbol{b}_1(\boldsymbol{p})\beta_1(\boldsymbol{p})$ and $\boldsymbol{c}_\ell(\boldsymbol{p}) = \boldsymbol{b}_\ell(\boldsymbol{p}) + \boldsymbol{b}_1(\boldsymbol{p})\beta_\ell(\boldsymbol{p})$. Therefore, we may choose $g_1(x_1, \ldots, x_n, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_n) = \sum x_i = x$. Moreover, imposing on (15) adding up yelds $\boldsymbol{c}_1(\boldsymbol{p}) \cdot \boldsymbol{p} = 1$ and $\boldsymbol{c}_\ell(\boldsymbol{p}) \cdot \boldsymbol{p} = 0$, $\ell = 2, \ldots, q$.[16]

Finally, it is worth mentioning the special case where the indices g_ℓ^* appearing in the microfunctions are two in number (i.e. $q = 2$) and depend solely on income. Imposing homogeneity of zero degree of the microfunctions as well as summability, Jorgenson, Lau and Stoker find

[15] On this point see Jorgenson, Lau, Stoker (1982, p. 105). Notice that assumptions (i) and (ii) are implied by adding up and non-negativity of individual consumptions.

[16] Adding up implies $0 = (\boldsymbol{c}_1(\boldsymbol{p}) \cdot \boldsymbol{p} - 1)x_i + \boldsymbol{c}_2(\boldsymbol{p}) \cdot \boldsymbol{p}\, g_2^*(x_i, \boldsymbol{A}_i) + \cdots + \boldsymbol{c}_q(\boldsymbol{p}) \cdot \boldsymbol{p}\, g_q^*(x_i, \boldsymbol{A}_i)$. The result follows immediately from the linear independence of the g_ℓ^*'s.

that the former of the two explanatory microvariables is bound to be income, and the latter, income multiplied either by the logarithm of income or by a power of income — namely, these conditions bring us back to the PIGL form of Muellbauer.

4.2. Let us consider a special case of Lau's theorem. If the individual attributes A_i are constant over time, we do not need to include them explicitly in microrelations (10) which can be rewritten as $\boldsymbol{y}_i = \boldsymbol{f}_i(x_i, \boldsymbol{p})$. Accordingly, macrorelation (11) becomes

$$\boldsymbol{y} = \boldsymbol{F}(g_1, \ldots, g_q, \boldsymbol{p}) \tag{16}$$

where $g_\ell = g_\ell(x_1, \ldots, x_n)$ with $\ell = 1, \ldots, q$.

Applying Theorem 5 we immediately see that the microrelations must be identical up to a term depending solely on prices, i.e. $\boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{f}(x_i, \boldsymbol{p}) + \boldsymbol{a}_i(\boldsymbol{p})$. Moreover, the aggregation conditions for (16) derived from Theorem 6 imply microrelations taking the form:

$$\boldsymbol{f}_i(x_i, \boldsymbol{p}) = \sum_{\ell=1}^{q} \boldsymbol{b}_\ell(\boldsymbol{p}) g_\ell^*(x_i) + \boldsymbol{a}_i(\boldsymbol{p}). \tag{17}$$

Microfunctions of this type are studied by Gorman (1981), who derives further restrictions on (17) from the assumption of individual optimizing behaviour. Gorman finds that when adding up, homogeneity and symmetry are imposed, microfunctions (17) must take one of the following forms:

$$\boldsymbol{y}_i = \boldsymbol{b}_1(\boldsymbol{p}) x_i + \boldsymbol{b}_2(\boldsymbol{p}) x_i \log x_i + \boldsymbol{b}_3(\boldsymbol{p}) \sum_{s=2}^{q-1} c_s(\boldsymbol{p}) x_i (\log x_i)^s \tag{18}$$

$$\boldsymbol{y}_i = \boldsymbol{b}_1(\boldsymbol{p}) x_i + \boldsymbol{b}_2(\boldsymbol{p}) \sum_{\sigma \in S}^{\sigma < 0} d_\sigma(\boldsymbol{p}) x_i^{\sigma+1} + \boldsymbol{b}_3(\boldsymbol{p}) \sum_{\sigma \in S}^{\sigma > 0} c_\sigma(\boldsymbol{p}) x_i^{\sigma+1} \tag{19}$$

$$\begin{aligned} \boldsymbol{y}_i = \boldsymbol{b}_1(\boldsymbol{p}) x_i &+ \boldsymbol{b}_2(\boldsymbol{p}) \sum_{\tau \in T}^{\tau > 0} d_\tau(\boldsymbol{p}) x_i \cos(\tau \log x_i) + \\ &+ \boldsymbol{b}_3(\boldsymbol{p}) \sum_{\tau \in T}^{\tau > 0} c_\tau(\boldsymbol{p}) x_i \sin(\tau \log x_i) \end{aligned} \tag{20}$$

where S and T are properly defined finite sets of real numbers. Polynomial microfunctions are a particular case of equation (19). Indeed, when $S = \{-1, 0, 1, \ldots, q-1\}$, equation (19) becomes

$$\boldsymbol{y}_i = \boldsymbol{a}_1(\boldsymbol{p}) + \boldsymbol{a}_2(\boldsymbol{p})x_i + \boldsymbol{a}_3(\boldsymbol{p}) \sum_{\ell=2}^{q} c_{\ell-1}(\boldsymbol{p})x_i^{\ell}, \tag{21}$$

where $\ell = \sigma + 1$, $\boldsymbol{a}_1(\boldsymbol{p}) = \boldsymbol{b}_2(\boldsymbol{p})d_{-1}(\boldsymbol{p})$, $\boldsymbol{a}_2(\boldsymbol{p}) = \boldsymbol{b}_1(\boldsymbol{p})$ and $\boldsymbol{a}_3(\boldsymbol{p}) = \boldsymbol{b}_3(\boldsymbol{p})$.

Optimizing behaviour places restrictions on the functions g_ℓ^*'s as well as on the "coefficients" $\boldsymbol{b}_\ell$'s. First, note that in all the three equations (18), (19) and (20), individual income x_i occurs, so that the aggregate expenditure can always be chosen as an index function for the macrorelation[17] (16). Furthermore, the functional form of the g_ℓ^*'s is restricted[18] to be either $x_i(\log x_i)^s$ as in equation (18), or some power of income $x_i^{\sigma+1}$ as in equation (19) or trigonometric functions such as $x_i \cos(\tau \log x_i)$ and $x_i \sin(\tau \log x_i)$ as in equation (20). Finally, the coefficients $\boldsymbol{b}_\ell$'s of (17) are linked by particular relations: for any given $\boldsymbol{p}$ no more than three of them can be linearly independent. For instance, in the simplest case of equation (18) the coefficients $\boldsymbol{b}_\ell$'s, for $\ell \geq 3$, are all multiples of $\boldsymbol{b}_3$, that is $\boldsymbol{b}_\ell(\boldsymbol{p}) = c_{\ell-1}(\boldsymbol{p})\boldsymbol{b}_3(\boldsymbol{p})$. This means that individual Engel Curves lie on a three-dimensional hyperplane.

The last restriction is important for empirical analysis. In cross section studies, where microrelations such as (17) are specified with more than three functions g_ℓ^*, the rank of the coefficient matrix of the demand system must not exceed three. This restriction provides a simple way to test the hypothesis of individual optimizing behaviour. On the other hand, if optimizing behaviour is assumed, no more than three functions of income can enter the microrelations without imposing some additional restrictions on the coefficients.

In a series of papers, Heineke and Shefrin make some remarks on Gorman's work. Heineke and Shefrin (1986) points out that Gorman's microrelations (18), (19) and (20) cannot approximate arbitrarily closely any integrable demand function. Therefore, imposing integrability, an

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[17] This result depend solely on adding up as we have already seen.

[18] As an intermediate result, note that adding up and symmetry imply that the g_ℓ^*'s functions take the form $x^{\sigma+1}e^{i\tau \log x}(\log x)^s$.

important feature of microequation (17), i.e. the property of approximating every function, is lost.

Moreover, Heineke and Shefrin (1987) study the case where the additional assumption of non-negativity of demanded quantities is imposed. Adding up and non-negativity result in the so called *bounded budget shares* (BBS) condition, i.e. $0 \leq w_{ih} \leq 1$, for all goods h, where $w_{ih} = y_{ih}p_h/x_i$. This condition is required to hold globally, that is for all the levels of individual expenditure. The authors state that when the BBS condition is imposed along with either symmetry or homogeneity, microequations (17) can only take form (20).

Indeed, microequations (18) and (19) do not satisfy BBS. To make this point clearer let us consider the polynomial case and rewrite equation (21) in terms of budget shares. In an obvious notation we obtain:

$$w_i = \hat{a}_1(p)\frac{1}{x_i} + \hat{a}_2(p) + \hat{a}_3(p)\sum_{\ell=2}^{q} c_{\ell-1}(p)x_i^{\ell-1}.$$

This function is not bounded, so that the condition $0 \leq w_{ih} \leq 1$ does not hold for all the levels of income, unless $w_i = \hat{a}_2(p)$. A similar argument applies to equations (18) and (19). On the contrary, BBS may well be satisfied by (20), which is (in terms of budget shares) a periodic and bounded function.

The point of Heineke and Shefrin (1987) is that this result may be obtained without imposing symmetry on equation (17). As a matter of fact, BBS is an extremely stringent requirement which suffices by itself to rule out every polynomial specification of (17). We may question whether such a global condition should be required. From an economic and empirical point of view it is not relevant to know what happens at unrealistically low or high levels of income. In our opinion, it seems more sensible to define the microfunctions over a relevant interval of the expenditure where we may still employ the useful approximating property of polynomial specifications.

5. CONCLUDING REMARKS

The work considered in this paper has attempted to establish what form the microrelations must take for a macrorelation to exist. The differences in the result depend on the differences in the definition of the macrofunction and in particular on the different definitions of the explanatory macrovariables.

A fruitful way to look at the relationship between the achieved results and the assumptions made is the following. Let us assume that the macrofuntions are defined by the relation

$$y_h = F_h(g_{1h}, \ldots, g_{qh}, \boldsymbol{p}),$$

where

$$g_{\ell h} = g_{\ell h}(x_1, \ldots, x_n, \boldsymbol{p}), \qquad \ell = 1, \ldots, q. \tag{22}$$

The demographic variables are omitted for simplicity. If no restriction is placed on the $g_{\ell h}$ functions, apart from satisfying the general form (22), the macrofunction always exists, whatever form the microrelations take. There are two trivial choices for the indices $g_{\ell h}$ which permit aggregation. The first consists in setting $q = 1$ and $g_{1h} = \sum_i f_{ih}(x_i, \boldsymbol{p})$, namely in taking, for every good, as explanatory variable, the aggregate consumption of that good. The latter amounts to setting for every good, $q = n$ and $g_{ih} = x_i$, that is, to take as many explanatory variables as there are agents, and set them equal to the income of the corresponding agent. In the former case, the aggregate relation is a tautology: aggregate consumption is equal to aggregate consumption. In the latter case, it simply expresses aggregate consumption as the sum of individual consumption.

A condition which is imposed by all the authors considered is that $g_{\ell h} = g_\ell$ for every h, or in other words, that the aggregate consumption of each good depends on the same set of explanatory variables. This condition does not place any restriction on the microrelation of a given good in itself, but it binds the microrelations of the different goods to be *similar* to each other; e.g. the microrelations (7) put forward by Muellbauer. In Lau's approach, however, this constraint is not particularly relevant: since the g_ℓ^*'s which appear in (14) may be more than one, nothing prevents some of them from having non-zero coefficients for a good and zero for the others.

A further restriction on the indices g is the symmetry condition imposed by Lau. If the g's are invariant with respect to the exchange of income of one agent with another, the microrelations of different agents must be identical up to a term which solely depends on prices (see Th. 5). The symmetry condition is needed to permit the explanatory macrovariables to be regarded as indices of the income distribution (or indices of the joint distribution of the explanatory microvariables).

A third restriction concerns the number of variables which are allowed to enter the aggregate relation. If there is no limit to the number of explanatory variables, it is always possible to construct a set of indices that uniquely identify the independent microvariables. If, for instance, there are only two agents and income is the only explanatory variable, the mean and the variance permit the identification of the two micro-incomes (even though they are not able to identify to whom of the two agents each income goes). Therefore (when the microrelations are identical) it is always possible to express aggregate consumption as a function of aggregate income and variance.

However, for the macrofunction to be a meaningful concept it seems reasonable to require that the dimension of the space of the explanatory macrovariables be *small* as compared to the dimension of the space of the explanatory microvariables; in other words, the information needed to forecast aggregate consumption by means of the macrofunction must be less than that needed to carry out the forecast by means of the microrelations (see on this point Heineke and Shefrin 1988). With regard to this, it is worth noticing that the limit $q < n$ imposed by Lau does not match the case where the micro-consumptions depend on both income and demographic variables[19]. Indeed, in this case, the explanatory microvariables are more than n and it is still possible to have a reduction in information even with $q > n$.

A further constraint imposed on the explanatory macrovariables by the authors considered in this paper is their independence of prices. This condition results in microrelations where prices combine with incomes (and demographic variables) in a particular way: microrelations must be linear in functions which do not depend on prices (even though the coefficients do depend on prices). This result is part of Lau's fundamental theorem.

Removing prices from the arguments of g's has an obvious meaning in Gorman, but not in Lau or in Jorgenson, Lau and Stoker. If the

[19] See Heineke and Shefrin (1988).

g's have to be regarded as indices of the joint distribution of the independent microvariables it is not clear why an index such as $\sum x_i^{\psi(A_i)}$ should be admissible whereas an index such as $\sum x_i^{\phi(p)}$ should be excluded. In this respect the GL form of Muellbauer is more general than Lau's form. As a matter of fact, the exclusion of prices by Lau is more an assumption needed to simplify the empirical applications than a theoretical requirement. Allowing for prices as arguments in the functions g_ℓ, microfunctions such as

$$\boldsymbol{f}_i(x_i, \boldsymbol{p}) = \boldsymbol{b}_1(\boldsymbol{p}) g_1^*(x_i, \boldsymbol{p}) + \cdots + \boldsymbol{b}_q(\boldsymbol{p}) g_q^*(x_i, \boldsymbol{p}) \tag{23}$$

are aggregable.

However, it must be pointed out that the greater formal generality of (23) is not very relevant. In fact, it is always possible to use polynomial approximations of (23) which obey the form (14). To make this point more clearly let us suppose that there are only two goods and p is the ratio between the prices. Microfunctions such as (23) may then be adequately approximated (in the neighbourhood of a point common to all the agents) by means of functions of the type $\boldsymbol{f}^*(x_i, p) = \sum_{r,s} \boldsymbol{c}_{rs} p^s x_i^r$.

Finally, it is worth noticing that microfunctions such as $\boldsymbol{f}_i(x_i, A_i) = \boldsymbol{f}(x_i, A_i)$ (we omit prices for simplicity and suppose that A_i is a scalar) are approximated by functions of the form:

$$\sum_{r,s} \boldsymbol{d}_{rs} x_i^r A_i^s. \tag{24}$$

If we denote by $\mu_{rs} = \sum x_i^r A_i^s$ the raw moments of the joint distribution of A_i and x_i, then polynomial (24) is aggregable with the moments μ_{rs} as arguments of the macrofunction. Therefore, provided that all the moments needed for (24) to closely approximate the microfunctions are allowed to enter the aggregate relation, any microfunction is aggregable with the sole condition that the agents have the same parameters[20].

[20] A similar point of view is expressed by Fortin (1989).

APPENDIX 1

In this appendix we shall show the equivalence between the notion of *community preference field* introduced by Gorman and the concept of *representative agent.* The problems of the existence of a community preference field (CPF) and that of the existence of a representative agent (RA) appear as particular formulations of the *aggregation problem.* Indeed, their solution not only involves the existence of a macrorelation but also requires that certain properties are preserved when we pass from micro to macro.

1. The definition of a RA is intuitively bound to the idea that market behaviour can be thought of as the behaviour of a single agent. In the case of optimizing agents with an expenditure function[21] such as $x_i = e_i(\boldsymbol{p}, u_i)$, the definition of RA for demand functions is the following: *Definition A1:* A representative agent exists if and only if a function $E(\boldsymbol{p}, u_0)$ exists, which possesses the same properties as those of the individual expenditure functions $e_i(\boldsymbol{p}, u_i)$ and such that

$$\frac{\partial E(\boldsymbol{p}, u_0)}{\partial p_h} = \sum_i \frac{\partial e_i(\boldsymbol{p}, u_i)}{\partial p_h}, \qquad h = 1, \ldots, m \tag{25}$$

where $u_0 = u_0(u_1, \ldots, u_n)$. The income of the representative agent is $x_0 = E(\boldsymbol{p}, u_0)$.

Multiplying (25) by the corresponding prices and summing over goods we obtain, by the Euler theorem,

$$E(\boldsymbol{p}, u_0) = \sum_i e_i(\boldsymbol{p}, u_i) \tag{26}$$

from which $x_0 = \sum_i x_i = x$, namely, the income of the RA is equal to the aggregate income.[22] Equivalent definitions of A1 are possible by

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[21] Given the standard assumptions on preferences, the $e_i(\boldsymbol{p}, u_i)$'s are linearly homogeneous and concave in $\boldsymbol{p}$ and increasing in u_i.

[22] Muellbauer (1976, condition R) gives a definition of the RA for the budget shares; this definition is less restrictive than A1 because it does not imply that $x_0 = x$.

substituting (26) for (25) or also in terms of indirect utility by imposing the condition

$$V(\boldsymbol{p}, x) = u(v_1(\boldsymbol{p}, x_1), \ldots, v_n(\boldsymbol{p}, x_n)), \tag{27}$$

where $V = E^{-1}$ and $v_i = e_i^{-1}$. Slightly different versions of condition (27) are analysed in Green (1964, ch. VII) and Antonelli (1886).

The existence of a RA is a sufficient condition for the existence of demand macrorelations of the type $y_h = F_h(\boldsymbol{p}, x)$. Substituting the indirect utility functions $u_i = e_i^{-1}(\boldsymbol{p}, x_i)$ and the aggregate utility index $u_0 = E^{-1}(\boldsymbol{p}, x_0)$ in (25), we obtain equation (2): $y_h = G_h(\boldsymbol{p}, x_1, \ldots, x_n) = F_h(\boldsymbol{p}, x)$. On the other hand, if the macrorelations $F_h(\cdot)$ are integrable then there exists an "aggregate" expenditure function $E(\boldsymbol{p}, u_0)$ to which macrorelations (2) can be traced back. It is then possible to redefine the notion of RA in terms of macrorelations:
Definition A1bis: There exists a representative agent if and only if (i) there exist macrorelations of the type $F_h(\boldsymbol{p}, x) = G_h(\boldsymbol{p}, x_1, \ldots, x_n)$ with $x = \sum x_i$; and (ii) the $F_h(\boldsymbol{p}, x)$ are integrable.

2. The notion of CPF, introduced by Gorman (1953), concerns the possibility for the entire community of establishing a preference relation, over the aggregate endowments of the economy, which enjoys the same properties as the individual preferences. We shall illustrate this problem starting from the original formulation of Social Indifference Curve (SIC) given by Scitovsky (1942) (see also Samuelson (1956)).

A SIC is given by the set of vectors of aggregate endowments of the economy which enable us to obtain the same distribution of individual utilities $\bar{\boldsymbol{u}} = (\bar{u}_1, \ldots, \bar{u}_n)$ and ensure for each agent the same marginal rate of substitution between goods. Figure 1 and 2 illustrates how these curves are obtained in a simple case with two goods 1 and 2, and two agents a and b. Given a vector of aggregate endowments $\boldsymbol{y} = (y_1, y_2)$ we draw the Edgeworth box and the corresponding contract curve; the pareto optimum corresponding to the point $\boldsymbol{q}$ singles out a combination of the utilities of the two agents $\bar{\boldsymbol{u}} = (\bar{u}_a, \bar{u}_b)$. The corresponding SIC, $Y(\bar{\boldsymbol{u}})$ is the curve drawn in fig. 1. It is easily obtained by looking at fig. 2 where the two individual indifference curves corresponding to $\bar{u}_a$ and $\bar{u}_b$ are drawn. Indeed, the curve is derived by taking the sum of the bundles which lie on the same indifference curves and minimize the expenditure of each agent for every given price level.

Fig. 1

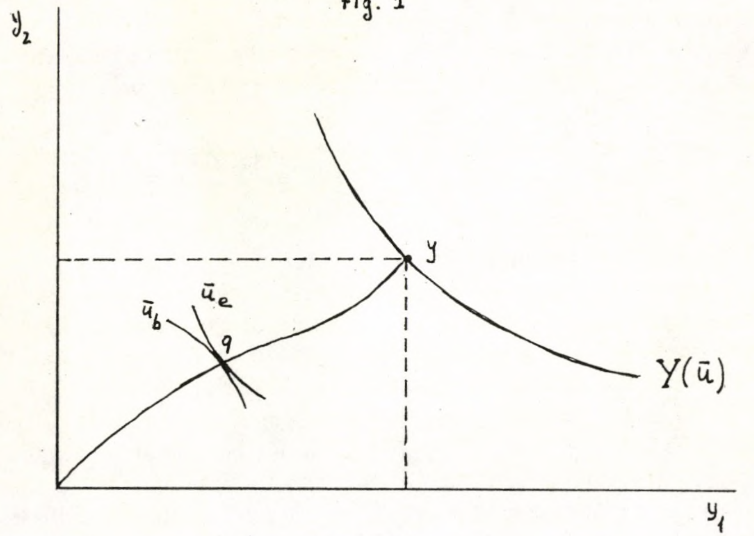
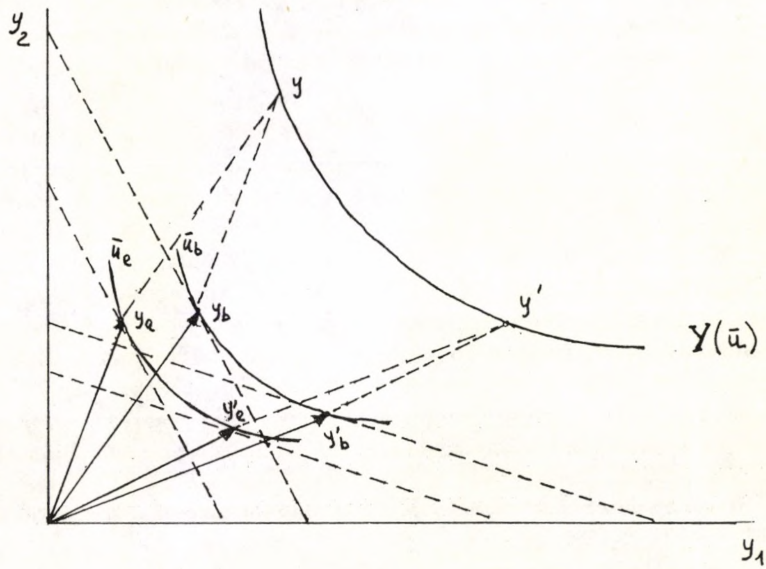

y_2
y
$\bar{u}_e$
$\bar{u}_b$
q
$Y(\bar{u})$
y_1

Fig. 2


y_2
y
$\bar{u}_e$
$\bar{u}_b$
y_e
y_b
y'
$Y(\bar{u})$
y'_e
y'_b
y_1

From this simple graphical analysis it can easily be seen that each point on a SIC of Scitovsky $Y(\boldsymbol{u})$ is given by the sum of the hicksian individual demand for a given vector of utilities.

Proposition A1: Each vector $\bar{\boldsymbol{u}}$ has a corresponding curve $Y(\bar{\boldsymbol{u}})$ and each point $\boldsymbol{y}$ on this curve has a corresponding vector of prices[23] $\boldsymbol{p}(\boldsymbol{y}, \bar{\boldsymbol{u}})$; moreover

$$\text{if} \quad \boldsymbol{y} \in Y(\bar{\boldsymbol{u}}) \quad \text{then} \quad \boldsymbol{y} = \sum_{i \in A} \boldsymbol{h}_i(\boldsymbol{p}, \bar{u}_i), \quad A = \{a, b\}$$

where $\boldsymbol{h}_i(\boldsymbol{p}, \bar{u}_i)$ is the hicksian demand vector.

Starting from a point of aggregate endowments $\boldsymbol{y}$ we associate the set of distribution of utilities corresponding to the pareto optima contained in the relative Edgeworth box. To every $\boldsymbol{y}$ corresponds a utility frontier $U(\boldsymbol{y})$, as drawn in fig. 3, namely, the set of the distributions of utility obtainable with the aggregate endowment $\boldsymbol{y}$. Similarly, with any point fixed on the frontier $U(\boldsymbol{y})$ we associate a SIC passing through the point $\boldsymbol{y}$; i.e. fixing $\bar{\boldsymbol{u}}$ we obtain $Y(\bar{\boldsymbol{u}})$; fixing a different distribution of utilities beetwen the agents $\boldsymbol{u}' = (u'_a, u'_b)$ we find a different indifference curve for the community, $Y(\boldsymbol{u}')$, which will pass through the same point of aggregate endowments $\boldsymbol{y}$, since $\boldsymbol{u}' \in U(\boldsymbol{y})$. In general, however, it will have a different slope; in other words, the SIC are intersecting.

Proposition A2: If the SIC's intersect the utility frontiers will also intersect.

This can be shown by the following simple graphical argument. Given a point $\boldsymbol{y}$ in the commodity space, draw the relative frontier $U(\boldsymbol{y})$, as in figure 3; given two points $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}' \in U(\boldsymbol{y})$, draw the corresponding SIC, $Y(\bar{\boldsymbol{u}})$ and $Y(\boldsymbol{u}')$ as in fig. 4, i.e. intersecting in $\boldsymbol{y}$. Next, take two aggregate endowments $\boldsymbol{z} \in Y(\boldsymbol{u}')$ and $\boldsymbol{y}' \in Y(\bar{\boldsymbol{u}})$, with $\boldsymbol{y}' > \boldsymbol{z}$. Since the endowment $\boldsymbol{z}$ permits us to achieve $\boldsymbol{u}'$ then the endowment $\boldsymbol{y}'$ will enable us to achieve combinations of utilities even greater than $\boldsymbol{u}'$. Let us choose another point $\boldsymbol{u}''$ on the utility frontier $U(\boldsymbol{y})$, such that the corresponding SIC, $Y(\boldsymbol{u}'')$, has a greater slope in the point $\boldsymbol{y}$. Next, choose a point of aggregate endowments $\boldsymbol{w} \in Y(\boldsymbol{u}'')$ greater than $\boldsymbol{y}'$, i.e. $\boldsymbol{w} > \boldsymbol{y}'$; applying a similar argument we come to the conclusion that

[23] It can be shown that the vector $\boldsymbol{p}$ is normal to the tangent of the SIC in that point of aggregate endowments.

Fig. 3

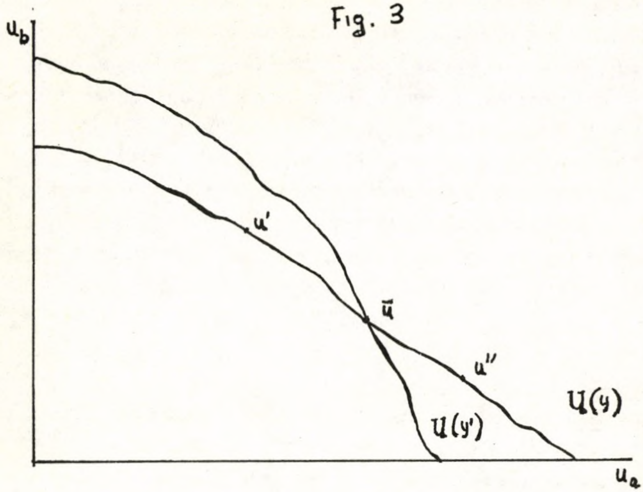
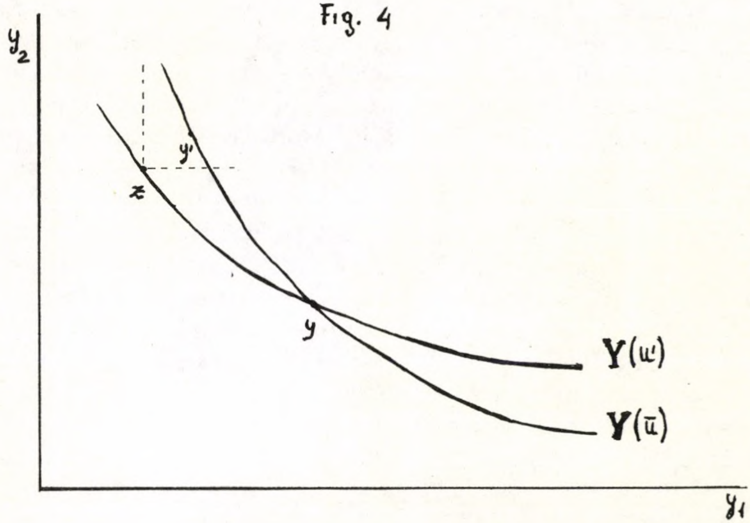


Fig. 4



the endowment $\boldsymbol{y}'$ enable us to obtain combinations of utilities less than $\boldsymbol{u}''$. Putting the two cases together it must necessarily be true that the frontier $U(\boldsymbol{y}')$ crosses $U(\boldsymbol{y})$ at least in one point as it is shown in fig. 3.

If the two frontiers intersect, both points of aggregate endowments $\boldsymbol{y}$ and $\boldsymbol{y}'$ are not tied by any (transitive) preference relation; indeed, in a neighbourhood on the left of the intersecting point, $\boldsymbol{y}'$ is "socially" preferred to $\boldsymbol{y}$ since one of the two agents can still be compensated by the other and viceversa in a neighbourhood on the right. If a unique frontier corresponded to $\boldsymbol{y}$ and $\boldsymbol{y}'$ then the aggregate endowments would have been, *in any case*, indifferent for both agents; this is the idea underlying the notion of community preference field given by Gorman.

Using a similar graphical argument the following proposition can be demonstrated:

Proposition A3: The SIC's do not intersect if and only if the frontiers do not intersect.

It is now easier to understand the definition of CPF:

Definition A2 (CPF): A unique CPF exists when neither the SIC's nor the utility frontiers intersect.

When a CPF exists, we can associate to each point on a SIC the same utility frontier; in other words, the points on a SIC are equivalent for all the members of the community since they provide them with the same opportunities. For every point North-East of a SIC there is only one curve which passes through it. Moreover, this curve corresponds to a superior frontier and is then preferred to the previous one by all agents; i.e. there exists a monotone preference relation over the aggregate quantities of goods which holds true for the entire community.

From the analysis we have developed so far, a more operative definition of CPF is obtainable. Indeed, because of proposition A1 there is only one price $\boldsymbol{p}(\boldsymbol{y}, \boldsymbol{u})$ associated to a point lying on a SIC. If there exists a CPF then, all the SIC passing through a point $\boldsymbol{y}$ must coincide; moreover a unique price and all the utility combinations lying on the same frontiers must correspond to the point $\boldsymbol{y}$, that is:

$$\boldsymbol{y} = \sum_i \boldsymbol{h}_i(\boldsymbol{p}, \bar{u}_i) = \sum_i \boldsymbol{h}_i(\boldsymbol{p}, u'_i) \tag{28}$$

for all $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}'$ on the same frontier. Given a function $u_0(\boldsymbol{u})$, constant for all vectors $\boldsymbol{u}$ on the same frontier and monotone in $\boldsymbol{u}$, it must hold:

$$\boldsymbol{y} = \boldsymbol{H}(\boldsymbol{p}, u_0) = \sum_i \boldsymbol{h}_i(\boldsymbol{p}, u_i) \tag{29}$$

3. It is now easier to go back to the definition of RA given at the beginning. First, note that the hicksian demand functions on the right-hand side of (29) may be rewritten as $H_h(\boldsymbol{p}, u_0) = \sum_i (\partial e_i(\boldsymbol{p}, u_i)/\partial p_h)$ with $h = 1, \ldots, m$. In order to re-obtain the definition A1, it is then sufficient to show that the functions $H_h(\boldsymbol{p}, u_0)$ are integrable. This, in turn, is also obvious, noticing that the jacobian matrix of $\boldsymbol{H}(\cdot)$ is the sum of symmetric and negative semidefinite matrices (i.e. the matrices of the second derivatives of the expenditure functions). Then, there must exist a function $E(\boldsymbol{p}, u_0) = x_0$ such that

$$\frac{\partial E(\boldsymbol{p}, u_0)}{\partial p_h} = H_h(\boldsymbol{p}, u_0), \qquad h = 1, \ldots, m.$$

Finally, it can be easily shown that $E(\boldsymbol{p}, u_0)$ is linearly homogeneous in $\boldsymbol{p}$ and monotone in u_0, and this proves that the existence of a CPF is a sufficient condition for the existence of a RA.

In order to demonstrate the equivalence between the two concepts, it only remains to prove necessity. If a RA exists the inverse image of $\boldsymbol{y}$ corresponds to a unique pair of values $(\boldsymbol{p}, u_0)$, (where the prices have already been normalized). For all pairs of utility distributions $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}'$ such that $u_0(\bar{\boldsymbol{u}}) = u_0(\boldsymbol{u}')$ (and only for those), we obtain, from (25), that (28) must hold. From a geometric point of view, this condition means that the SIC, $Y(\bar{\boldsymbol{u}})$ and $Y(\boldsymbol{u}')$ passing through the same point $\boldsymbol{y}$, must have, at that point, the same slope which is given by the normal to the vector $\boldsymbol{p}$. In other words, we have established that all the SIC passing through a point do not intersect and thus, by proposition A3, frontiers do not intersect either. This is exactly the definition A2 (CPF).

APPENDIX 2

In this appendix we shall provide the proof of Theorems 3, 4 and 6.

Proof of Theorem 3. Necessity. Differentiating equation (4) for good h with respect to x_i we have:

$$\frac{\partial f_{ih}(x_i, \boldsymbol{p})/\partial x_i}{\partial F_h(g, \boldsymbol{p})/\partial g} = \frac{\partial g(x_1, \ldots, x_n)}{\partial x_i}. \tag{30}$$

The function on the righ-hand side does not depend on $\boldsymbol{p}$ and it is the same for all h's. The same must thus hold for the ratio on the left-hand side. Hence, for some functions $\hat{b}_h(x_i, \boldsymbol{p})$, $\bar{b}_h(g, \boldsymbol{p})$, $g_i^*(x_i)$ and $H(g)$, it must hold:

$$\begin{aligned} \frac{\partial f_{hi}}{\partial x_i} &= \hat{b}_h(x_i, \boldsymbol{p}) g_i^*(x_i) \\ \frac{\partial F_h}{\partial g} &= \bar{b}_h(g, \boldsymbol{p}) H(g), \end{aligned} \tag{31}$$

with $\hat{b}_h(x_i, \boldsymbol{p}) = \bar{b}_h(g, \boldsymbol{p})$. In the latter equality, the function on the left-hand side does not depend on x_j, for $j \neq i$; therefore $\bar{b}_h(g, \boldsymbol{p})$ is constant in g and $\hat{b}_h(x_i, \boldsymbol{p})$ is constant in x_i. Thus, $\hat{b}_h(x_i, \boldsymbol{p}) = \bar{b}_h(g, \boldsymbol{p}) = b_h(\boldsymbol{p})$. Substituting in (31) and integrating we obtain equation (5).

Sufficiency. It can be immediately verified that if the microrelations have the form (5), the aggregate relation exists with $g(x_1, \ldots, x_n) = \sum_i g_i(x_i)$.

Proof of Theorem 4. Sufficiency. Taking the sum of (7) and dividing by x it can be checked that $\boldsymbol{Q}(g, \boldsymbol{p})$ exists either with $g = \sum g_i(x_i)/x$ (case (*i*)) or with $g = x$ (case (*ii*)). Note that in case (*ii*) (i.e. $\boldsymbol{c}(\boldsymbol{p}) = 0$), equation (7) reduces to the Gorman form.

Necessity. Differentiating equation (6) for good h with respect to x_i and reordering, we have:

$$\frac{\partial Q_h}{\partial g} \frac{\partial g}{\partial x_i} x = \partial f_{ih}/\partial x_i - Q_h(g, \boldsymbol{p}). \tag{32}$$

Then the following relation holds

$$S_h(g, \boldsymbol{p}) = \frac{\partial Q_h/\partial g}{\partial Q_1/\partial g} = \frac{\partial f_{ih}/\partial x_i - Q_h(g, \boldsymbol{p})}{\partial f_{i1}/\partial x_i - Q_1(g, \boldsymbol{p})} = R(x_i, g, \boldsymbol{p}). \tag{33}$$

Differentiating the last function with respect to x_j it can be easily seen that $\partial S_h/\partial g = \partial R/\partial g$. On the other hand, taking into account (32) we find that

$$\frac{\partial R}{\partial g} = \frac{\frac{-\partial Q_h}{\partial g}\big(\frac{\partial Q_1}{\partial g}\frac{\partial g}{\partial x_i}x\big) + \frac{\partial Q_1}{\partial g}\big(\frac{\partial Q_h}{\partial g}\frac{\partial g}{\partial x_i}x\big)}{\big(\frac{\partial Q_1}{\partial g}\frac{\partial g}{\partial x_i}x\big)^2} = 0.$$

Therefore $S_h(g, \boldsymbol{p})$ does not depend on g, so that we can write $S_h(g, \boldsymbol{p}) = c_h(\boldsymbol{p})$. Moreover, integrating the first equality in (33) we obtain $Q_h(g, \boldsymbol{p}) = b_h(\boldsymbol{p}) + c_h(\boldsymbol{p})Q_1(g, \boldsymbol{p})$. Substituting S_h and Q_h in (33) we have:

$$c_h(\boldsymbol{p})\frac{\partial f_{i1}}{\partial x_i} = \frac{\partial f_{ih}}{\partial x_i} - b_h(\boldsymbol{p}).$$

Reordering and integrating with respect to x_i:

$$f_{ih}(x_i, \boldsymbol{p}) = a_{ih}(\boldsymbol{p}) + b_h(\boldsymbol{p})x_i + c_h(\boldsymbol{p})f_{i1}(x_i, \boldsymbol{p}).$$

Setting $g_i(x_i, \boldsymbol{p}) = f_{i1}(x_i, \boldsymbol{p})$ we obtain (7). Finally, substituting (7) into (6) we find conditions (i) and (ii).

Proof of Theorem 6. Let us define

$$\begin{aligned} g'_\lambda(x_1, \ldots, x_n, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_n) &= \sum_i f_k(x_i, \boldsymbol{A}_i, \boldsymbol{p}^{(\lambda)}) \\ &= F_k(g_1, \ldots, g_q, \boldsymbol{p}^{(\lambda)}) - \sum_i \bar{a}_{ik}(\boldsymbol{p}^{(\lambda)}) \qquad \lambda = 1, \ldots, q, \end{aligned} \tag{34}$$

where $\bar{a}_{ik}$ and f_k are as in (13). By invertibility of the F_k's (condition (iii)) we can express each of the g_ℓ's as a function of the g'_λ's, so that we can write:

$$\sum_i \boldsymbol{f}_i(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \boldsymbol{F}(g_1, \ldots, g_q, \boldsymbol{p}) = \boldsymbol{F}'(g'_1, \ldots, g'_q, \boldsymbol{p}). \tag{35}$$

Differentiating line h of equation (35) with respect to x_i and the elements of the vector $\boldsymbol{A}_i$, $i = 1\ldots,n$, we obtain:

$$\begin{pmatrix} \partial f_h/\partial x_1 \\ \vdots \\ \partial f_h/\partial x_n \\ \partial f_h/\partial A_{11} \\ \vdots \\ \partial f_h/\partial A_{nt} \end{pmatrix} = \begin{pmatrix} \partial g_1'/\partial x_1 & \ldots & \partial g_q'/\partial x_1 \\ \vdots & \ddots & \vdots \\ \partial g_1'/\partial x_n & \ldots & \partial g_q'/\partial x_n \\ \partial g_1'/\partial A_{11} & \ldots & \partial g_q'/\partial A_{11} \\ \vdots & \ddots & \vdots \\ \partial g_1'/\partial A_{nt} & \ldots & \partial g_q'/\partial A_{nt} \end{pmatrix} \begin{pmatrix} \partial F'/\partial g_1' \\ \vdots \\ \partial F'/\partial g_q' \end{pmatrix}, \quad (36)$$

where $f_h(x_i, \boldsymbol{A}_i, \boldsymbol{p})$ is the h-th element of the vector $\boldsymbol{f}(x_i, \boldsymbol{A}_i, \boldsymbol{p})$ in (13) and t is the number of individual attributes in the vector $\boldsymbol{A}_i$.

By the definition of the g_ℓ''s, the first row of the matrix in (36), say matrix B, depends only on $x_1, \boldsymbol{A}_1$, the second on $x_2, \boldsymbol{A}_2$ and so on. Let us suppose that the columns of B were linearly dependent whatever the values of $x_1, \ldots, x_n, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_n$. Then $\sum_\ell \alpha_\ell \partial g_\ell'/\partial x_i = 0$ and $\sum_\ell \alpha_\ell \partial g_\ell'/\partial A_{is} = 0$, for $s = 1, \ldots, t$ and $i = 1, \ldots, n$, where the α_ℓ cannot depend on $x_i, \boldsymbol{A}_i$, all i, and at least one of them is non-zero. Integrating the above equalities and taking into account equation (34) we obtain $\sum_\ell \alpha_\ell g_\ell' - c = \sum_\ell \alpha_\ell F_k(g_1, \ldots, g_q, \boldsymbol{p}^{(\ell)}) - d = 0$, where c and d are constants. This contradicts the hypothesis of functional independence of the g_ℓ's (condition (ii)). Hence the columns of B are linearly independent and the rank of B is q. Therefore we can find a non-singular square submatrix $(q \times q)$ of B, say B', and solve for the vector $(\partial F'/\partial g_1' \quad \ldots \quad \partial F'/\partial g_q')$. Since $q < n$, there is at least one agent, say agent j, such that B' does not depend on either x_j or A_{js}, $s = 1, \ldots, t$. Hence the vector $(\partial F'/\partial g_1' \quad \ldots \quad \partial F'/\partial g_q')$ is independent of $x_j, \boldsymbol{A}_j$. But this vector is symmetric, since it depends on the symmetric functions g_ℓ'. If it does not depend on $x_j, \boldsymbol{A}_j$, it cannot depend on $x_i, \boldsymbol{A}_i$, $i \neq j$, either. Therefore it must be a function of $\boldsymbol{p}$ and we can write:

$$\frac{\partial f_h}{\partial x_i} = \sum_\ell b_{\ell h}(\boldsymbol{p}) \frac{\partial g_\ell'}{\partial x_i} = \sum_\ell b_{\ell h}(\boldsymbol{p}) \frac{\partial f_k(x_i, \boldsymbol{A}_i, \boldsymbol{p}^{(\ell)})}{\partial x_i}, \qquad i = 1, \ldots, n$$

$$\frac{\partial f_h}{\partial A_{is}} = \sum_\ell b_{\ell h}(\boldsymbol{p}) \frac{\partial f_k(x_i, \boldsymbol{A}_i, \boldsymbol{p}^{(\ell)})}{\partial A_{is}}, \qquad i = 1, \ldots, n; \quad s = 1, \ldots, t.$$

Integrating we have:

$$f_h(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \sum_\ell b_{\ell h}(\boldsymbol{p}) g^*_\ell(x_i, \boldsymbol{A}_i) + a_h(\boldsymbol{p}), \qquad h = 1, \ldots, m, \tag{37}$$

where $g^*_\ell(x_i, \boldsymbol{A}_i) = f_k(x_i, \boldsymbol{A}_i, \boldsymbol{p}^{(\ell)})$. Equation (14) is obtained by (13).

The functions $1, g^*_1, \ldots, g^*_q$ are linearly independent because of assumption (*iii*). If they were linearly dependent, the functions $1, g'_1, \ldots, g'_q$ would be linearly dependent, so that $\sum_\ell \alpha_\ell g'_\ell - c = \sum_\ell \alpha_\ell F_k(g_1, \ldots, g_q, \boldsymbol{p}^{(\ell)}) - c = 0$, where at least one of the α_ℓ's is non-zero, contrary to condition (*ii*). Finally, let us set $h = k$ in equation (37) and sum over individuals to get $F_k(x_i, \boldsymbol{A}_i, \boldsymbol{p}) = \sum_\ell b_{\ell k}(\boldsymbol{p}) g'_\ell(x_i, \boldsymbol{A}_i)$. For each $\boldsymbol{p}^{(\lambda)}$ we have:

$$g'_\lambda - \sum_\ell b_{\ell k}(\boldsymbol{p}^{(\lambda)}) g'_\ell = 0, \qquad \lambda = 1, \ldots, q.$$

Linear independence of the g'_ℓ's implies $b_{\ell k}(\boldsymbol{p}^{(\lambda)}) = \delta_{\ell\lambda}$, where $\delta_{\ell\lambda}$ is the Kroneker's delta. Therefore $\det[b_{\ell k}(\boldsymbol{p}^{(\lambda)})] = 1$ and the functions $b_{\ell k}(\boldsymbol{p})$ are linearly independent.

Sufficiency. Summing equations (14) over individual and defining $g_\ell = \sum_i g^*_\ell$ we obtain an equation like (12) where the g_ℓ's are symmetric and therefore satisfy condition (*i*). Let us suppose that $L(g_1, \ldots, g_q) = 0$. Differentiating with respect to x_i and the elements of the vector $\boldsymbol{A}_i$, $i = 1 \ldots, n$, we obtain:

$$\begin{pmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} \partial g^*_1(x_1, \boldsymbol{A}_1)/\partial x_1 & \ldots & \partial g^*_q(x_q, \boldsymbol{A}_q)/\partial x_1 \\ \vdots & \ddots & \vdots \\ \partial g^*_1(x_1, \boldsymbol{A}_1)/\partial x_n & \ldots & \partial g^*_q(x_q, \boldsymbol{A}_q)/\partial x_n \\ \partial g^*_1(x_1, \boldsymbol{A}_1)/\partial A_{11} & \ldots & \partial g^*_q(x_q, \boldsymbol{A}_q)/\partial A_{11} \\ \vdots & \ddots & \vdots \\ \partial g^*_1(x_1, \boldsymbol{A}_1)/\partial A_{nt} & \ldots & \partial g^*_q(x_q, \boldsymbol{A}_q)/\partial A_{nt} \end{pmatrix} \begin{pmatrix} \partial L/\partial g_1 \\ \vdots \\ \partial L/\partial g_q \end{pmatrix}.$$

The columns of the matrix are linearly independent because the functions $1, g^*_1, \ldots, g^*_q$ are linearly independent. Hence, $\partial L/\partial g_\ell = 0$ for all ℓ, so that $L(\cdot)$ is constant in the g_ℓ's and condition (*ii*) is satisfied. Condition (*iii*) follows from linear independence of the $b_{\ell k}(\boldsymbol{p})$'s.

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