A stochastic dynamic programming approach for integrating supply chain network design and new product diffusion

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Abstract
We focus on the problem of designing a supply chain for a new product. We propose a stochastic dynamic program based on a Stochastic Bass Model of the product demand dynamics. We heuristically solve our model using Monte Carlo simulation and math programming techniques. We apply our approach to the case of a new distribution service for Made-in-Italy wine and food products, benchmarking the performance of our heuristic policy against the performance of an easier to compute heuristic policy and a lower bound.

Keywords*:* Supply Chain Network Design, Facility Location, New Product Diffusion, Stochastic Dynamic Programming

Introduction
A firm's competitive advantage is strongly related to how the firm manages its material and information flows, balances its logistic and manufacturing costs, and matches its supply and demand. Supply Chain Management (SCM) is thus a crucial function for manufacturing and service enterprises. Supply Chain Network Design (SCND) represents a strategic SCM decision that focuses on determining the physical configuration and infrastructure of the supply chain (SC). Facility Location (FL) is one of the most critical SCND steps, as these decisions are difficult to modify even in the middle term. In contrast, transportation, inventory management, and information sharing choices can be readily re-evaluated in response to environmental changes. Moreover, SCM and the design of marketing policies are interrelated: designing a SC without taking into account the evolution of the customers' distribution in a given geographical area, or, vice versa, making a marketing decision without considering logistic constraints can negatively impact a firm's

competitive advantage. Hence, a crucial SCND objective is to capture the interactions between operations and marketing concerns during a product life cycle.

In the product launch phase, when the number and location of potential customers are unknown, the evolution of demand is uncertain. Integrating SCND and New Product Diffusion (NPD) concerns may thus be highly beneficial. This problem involves two key strategic choices: (i) determining the number, location, and size of the facilities (e.g., warehouses); and (ii) allocating customers to each facility. This problem is both dynamic – the decisions about facilities must be made at the beginning of each time period and their impact must be evaluated for the remaining part of the planning horizon – and stochastic – the number and location of customers are uncertain.

The Operations Management and Operations Research literatures on SCND are extensive. However, to the best of our knowledge, the extant literature has paid little attention to FL models for designing the SC of a newly launched product, whose demand evolves in a random fashion. We thus develop and analyze a stochastic and dynamic SCND model for an innovative product/service, integrating both operations and marketing concerns. The Bass Model (BM) and its revised forms have long been used for analyzing and forecasting the market penetration of new products in different sectors (Mahajan et al., 1990). We propose a Stochastic Dynamic Programming (SDP) model based on a Stochastic Bass Model (SBM) of the product demand evolution over a given planning horizon, given a set of potential customers and warehouse locations. SBM allows us to assign to each potential customer a probability of requiring service during each time period in the planning horizon, hence describing the product diffusion process at a disaggregate level.

Due to the difficulty of solving our model to optimality, we propose a heuristic solution procedure integrating Monte Carlo simulation and optimization. We assess the effectiveness of this heuristic policy by estimating an alternative, easier to compute, upper bound (UB) and a lower bound (LB), applied to the industrial case of a new distribution service for Made-in-Italy wine and food products.

Literature Review

Dynamic and Stochastic SCND

Rosenthal et al. (1978) were the first to analyse a Stochastic Dynamic Location problem where servers have to be located in the face of stochastic customer relocations driven by a stationary Markov chain. Berman and Odoni (1982) approached the same issue to minimize travel times and relocation cost. Berman and LeBlanc (1984) proposed an efficient heuristic for the multi-facility and multi-state version of the problem. Carson and Batta (1990) presented a case study on ambulance relocation on the SUNY Buffalo's Amherst campus based on solving a 1-median problem for each considered time period. Jornsten and Bjorndal (1994) studied a dynamic and stochastic FL problem through an aggregation and augmented Lagrangian approach to minimize expected discounted total cost. Current et al. (1997) investigated the *p*-median problem when the number of facilities to be located is uncertain. Vairaktarakis and Kouvelis (1999) considered 1-median problems on a tree with dynamic and stochastic edge lengths and node weights. Focusing on mixed-integer programming models, Romauch and Hartl (2005) analysed a *Stochastic Dynamic Facility Location Problem* to minimize the expected total cost to serve customers during a given planning horizon. They proposed an exact solution method based on SDP, as well as Monte Carlo based heuristics for large size problems. Aghezzaf (2005) considered multi-period strategic capacity planning and warehouse location decisions under uncertain customer demand, using a robust optimization and Lagrangian relaxation method. Klibi et al. (2010) investigated the *Stochastic Multi-Period Location-Transportation Problem*, which integrates FL, customer allocation, and transport decisions, by applying stochastic programming and hierarchical heuristics. To the best of our knowledge, the extant FL literature does not seem to have yet addressed the SCND problem under stochastic expanding demand.

SCM and Innovation Diffusion Theory

The relevant Marketing literature is the one that incorporates SC dynamics into a diffusion model (DM). Jain et al. (1991) investigated the influence of supply restrictions on innovations by developing a DM to capture the impact of procurement dynamics on the growth of the demand for a new product. Ho et al. (2002) added lost sales to this model. Kumar and Swaminathan (2003) used BM in a setting with fixed production capacity to study the interactions between manufacturing and marketing/sales decisions (e.g., backlog) during the product lifecycle. Laínez et al. (2010) developed a mixed integer nonlinear programming model that optimizes SC and strategic marketing decisions to maximize corporate value. Graves and Willems (2005) examined optimal SC configuration for a new product emphasizing safety stock placement and sourcing decisions. Amini and Li (2011) proposed an integrated hybrid optimization model for safety stock placement in a SC, considering the dynamics of the demand diffusion process. Amini et al. (2012) compared the performance of different production–sales policies for new product diffusion. These papers demonstrate the benefits of integrating NPD and SCM models. However, it seems that the existing literature has not fully investigated the integration of a DM into a SCND optimization model.

The Stochastic Dynamic Programming Model for Supply Chain Network Design

Problem Description and Model Formulation

We are concerned with the problem of integrating the strategic location of warehouses within a considered area and the tactical allocation of customers to warehouses to minimize overall logistics costs subject to the constraint of satisfying customer demand. We divide a given planning horizon into T time periods, each with the same length (e.g., one year). At the start of each time period, we need to select a subset of a given set I of eligible facilities (warehouses) and assign them to serve the demand of a subset of customers that are foreseen to require the product during a given time period. These decisions are contingent on the current set of operating warehouses and state of the customer adoption process.

More specifically, the *location problem* in each time period involves deciding which warehouses to open/close before demand is known for the period. We denote by $w^{(t-1)}$ the array that describes the status of each warehouse at the *start* of period t, with 0 and 1 respectively indicating a closed and open warehouse, before deciding which facilities to open/close or maintain in operations in this period. As in the *Dynamic Uncapacitated Facility Location Problem* (Van Roy & Erlenkotter, 1982; Klose & Drexl, 2005), a closing/opening option is available in each time period for each open/closed facility (the costs of opening and closing facility i in period t are $O_i^{(t)}$ and $C_i^{(t)}$ and these decisions are modelled using the binary variables $o_i^{(t)}$ and $c_i^{(t)}$, respectively). We assume facilities have infinite capacity compared to delivered quantities. This assumption is particularly reasonable for third party warehouses.

The diffusion of the new product is described by SBM (Niu, 2002). After facilities are located in period t, conditional on the set $D^{(t-1)}$ of all the customers that have already joined the system, a subset $N^{(t)}$ of customers from the set of all remaining potential customers joins the market in period t according to SBM. This constitutes a *scenario* (this terminology is as in Romauch and Hartl (2005)). Given $D^{(t-1)}$, we denote by $K_t(D^{(t-1)})$ the set of possible scenarios in period t, and by $P_{tk}(D^{(t-1)})$ the probability that scenario k in set $K_t(D^{(t-1)})$ occurs. The demands of the new customers that enter the system in period t and of those already in the system have to be fully met. We include all these customers in set $D^{(t)}$ (that is, $D^{(t)} = D^{(t-1)} \cup N^{(t)}$), and allocate them to the available warehouses. The binary variable $x_{ikj}^{(t)}$ is set equal to 1 if customer j is allocated to facility i under scenario k in period t and 0 otherwise.

Analogously to Gaur and Fisher (2004), we assume that there is a single homogeneous product with deterministic and possibly time-varying demand for each customer. We use the integer valued parameter $q_j^{(t)}$ to indicate the demand in period t of a customer j that has joined the system (in the current period or any of the previous periods).

We model inventory and transportation costs as proposed by Shen and Qi (2007). In particular, we use an Economic Order Quantity model (Harris, 1913; Wilson, 1934) for facility inventory management. If n and h are, respectively, the number of shipments per year from the supplier and the inventory holding cost per period per unit, we express the total inventory cost incurred at all facilities under scenario k in period t as

$$\sum_{i\in I} \sum_{j\in D_k^{(t)}} x_{ikj}^{(t)} q_j^{(t)} \frac{h}{2n}. \qquad (1.)$$

If customer j is allocated to facility i in period t we incur the transportation cost $l_{ij}^{(t)}$. This cost is the product of the total number of shipments to each customer per year, n, the distance d_{ij} from facility i to customer j, and the cost $\bar{l}$ for each kilometre covered. When the customer allocation is decided, the space used at facility i in period t is captured by the variable $u_i^{(t)}$. The unit cost of space at facility i in period t is $m_i^{(t)}$.

We denote by $V_t\left(D^{(t-1)}, w^{(t-1)}\right)$ the optimal value function of our SDP model in period t and state $\left(D^{(t-1)}, w^{(t-1)}\right)$, that is, before observing the customers that enter the market in period t. This function satisfies the following Bellman equation:

$$\begin{aligned} &V_t\left(D^{(t-1)}, w^{(t-1)}\right) \\ &= \min \sum_{i\in I}\left[O_i^{(t)} o^{(t)} + C_i^{(t)} c^{(t)}\right] \\ &+ \sum_{k\in K_t\left(D^{(t-1)}\right)} P_{tk}^{(t)}\left(D^{(t-1)}\right)\left\{\sum_{i\in I} m_i^{(t)} u_{ik}^{(t)} + \sum_{i\in I}\sum_{j\in D_k^{(t)}} x_{ikj}^{(t)}\left[\frac{q_j^{(t)} h}{2n} + l_{ij}^{(t)}\right]\right\} \\ &+ \sum_{k\in K_t\left(D^{(t-1)}\right)} P_{tk}^{(t)}\left(D^{(t-1)}\right) V_{t+1}\left(D_k^{(t)}, w^{(t)}\right) \qquad (2.) \end{aligned}$$

s.t.

$$D_k^{(t)} = D_k^{(t-1)} \cup N_k^{(t)}, \forall k \in K_t\big(D^{(t-1)}\big), \tag{3.}$$

$$o_i^{(t)} \leq 1 - w_i^{(t-1)},\ \forall i \in I, \tag{4.}$$

$$c_i^{(t)} \leq w_i^{(t-1)},\ \forall i \in I, \tag{5.}$$

$$w_i^{(t)} = w_i^{(t-1)} + o_i^{(t)} - c_i^{(t)}, \forall i \in I\ , \tag{6.}$$

$$x_{ikj}^{(t)} \leq w_i^{(t)}, \forall i \in I, k \in K_t\big(D^{(t-1)}\big),\ j \in D_k^{(t)}, \tag{7.}$$

$$\sum_{i \in I} x_{ikj}^{(t)} = 1, \forall k \in K_t\big(D^{(t-1)}\big), j \in D_k^{(t)}, \tag{8.}$$

$$\sum_{j \in D_k^{(t)}} x_{ikj}^{(t)} q_j^{(t)} \leq u_{ik}^{(t)}, \forall i \in I, k \in K_t\big(D^{(t-1)}\big), \tag{9.}$$

$$u_{ik}^{(t)} \geq 0, \forall i \in I, k \in K_t\big(D^{(t-1)}\big), \tag{10.}$$

$$x_{ikj}^{(t)} \in \{0,\ 1\}, \forall i \in I, k \in K_t\big(D^{(t-1)}\big), j \in D_k^{(t)}, \tag{11.}$$

$$w_i^{(t)}, o_i^{(t)}, c_i^{(t)} \in \{0,\ 1\}, \forall i \in I. \tag{12.}$$

The objective function (2) includes three parts. The first part accounts for the total location costs, that is, the costs of opening/closing facilities in period t. The second part accounts for the expected total customer allocation cost in period t, that is, the sum of warehousing, inventory holding, and transportation costs resulting from serving customers in this period. The third part includes the expected optimal total cost in the remaining part of the planning horizon. Constraints (3) update the set of customers that have joined the market in period t under scenario k. Constraints (4) and (5), respectively, make sure that a warehouse is open only if it was previously closed and that a warehouse is closed only if it was previously open. Constraints (6) update the operating status of the warehouses. Constraints (7) and (8) enforce the condition that a customer is assigned only to an operating warehouse and that all customers are assigned to exactly one warehouse, respectively. Constraints (9) require that sufficient space is available at each warehouse to serve all the demands of customers assigned to this warehouse. Constraints (10) impose non-negativity conditions on the used warehousing space. Constraints (11) and (22), respectively, enforce binary conditions on the customer allocation variables and the warehouse related variable.

Solution Approach

Solving our SDP model to optimality on realistic instances is computationally intractable, due the curve of dimensionality (Falsini, 2011). We thus solve this model heuristically by integrating Monte Carlo simulation and math programming. We label as *scenario policy* (SP) the resulting heuristic policy. This policy is evaluated by repeating the following steps for a given number of samples, and averaging the total cost computed on each sample to obtain an estimate of the value function of this policy in the initial state and stage:

Step 1. Initialize t to 1 and set the total cost to 0.

Step 2. At the beginning of period t, sample a scenario of interest that includes a set of customers that adopt the product in this period, as well as a set of analogous additional scenarios. For each of these scenarios, sample a set of scenarios for each remaining time period in the planning horizon. Label as a trajectory a sequence of scenarios from period t through T.

Step 3. Solve the deterministic version of our SDP model formulated on this set of trajectories. This is a math program that does not include a value function in its formulation.

Step 4. Extract from the resulting optimal solution of this math program the array that describes the operating condition of the warehouses in period t and the set of allocated customers in period t for the scenario of interest generated in Step 2. Cumulate the corresponding period t costs of opening/closing warehouses and allocating customers (that is, the space, inventory, and transportation costs). Add these cumulated costs to the total cost.

Step 5. Increment t by 1. If t equals $T + 1$ then stop. Otherwise return to Step 2.

We benchmark the performance of SP against an estimated hindsight information LB and the performance of a *myopic policy* (MP). We estimate this LB by generating a set of trajectories, optimizing a set of deterministic multi-period facility location and customer allocation math programs, each formulated on one of these trajectories (that is, each such math program uses hindsight information about the customers that enter the market in each time period), and averaging the corresponding optimal solutions. MP makes facility location and customer allocation decisions in a given period only considering the information available in that period. We evaluate the MP performance in a manner similar to how we evaluate the SP performance, except that in Step 2 we only consider scenarios pertaining to period t, rather than trajectories from period t through period T.

Model Validation

Case Study Description: A New Made-in-Italy Food and Wine Supply Chain

We validated our modelling approach by applying our heuristic solution method to a project conceived by the Operations Research Group at the Department of Enterprise Engineering, “Tor Vergata” University of Rome, and funded by the Italian Ministry of Economic Development. This project aims at defining a new distribution format for Made-in-Italy food and wine goods both on national and international markets through an e-commerce platform.

We used the following values for our model parameters:

- *Length of the planning horizon*: 4 years.
- *Potential customers*: 1,400 Italian restaurants with fairly uniform distribution over all the Italian territory. We set the SBM innovation and imitation parameters equal to 0.03 and 0.38, respectively, on the basis of the meta-analysis of Sultan (1990). Once a customer adopts the product, the demand of this customer is estimated to be 1,600 product units per year.

- *Potential facilities*: 28 logistics platforms that provide warehousing services for the small quantities of product foreseen by the project business plan.
- *Logistics costs*: The transportation cost is fixed at 1.09 €/km (Falsini et al., 2009). Distances are computed using a Geographic Information System. The estimation of the remaining logistics costs is based on third-party logistics prices for renting and managing a square meter in controlled-temperature warehouses, without quantity discounts.

Results and Sensitivity Analysis

We created a set of 8 test instances by varying the warehouse opening cost O (€/facility) in the set {5,000, 20,000, 50,000, 100,000}; for simplicity, this cost does not depend on the facility or the time period, and hence we suppress the subscript and superscript from the O notation. According to common practice, we assumed either weekly or monthly shipments to customers. Thus, the total number of shipments per year to each customer, n, varies in the set {12, 52}. Given that warehouses are rented, we set the cost of closing a facility to zero. On each instance, we evaluated the LB and the two UBs corresponding to MP and SP, respectively denoted as UB-MP and UB-SP, using 10 samples from the beginning through the end of the time horizon (albeit this is a small number of samples, the sample values of our bounds, that is, their values on each sample, are quite stable). We conducted our computational study on a laptop with an Intel Core i5-460M processor (2,53GHz, 3MB L3 cache) and 4 GB DDR3 of memory. On average, evaluating SP, MP, and LB took 80.8, 36.5, and 5.0 CPU seconds.

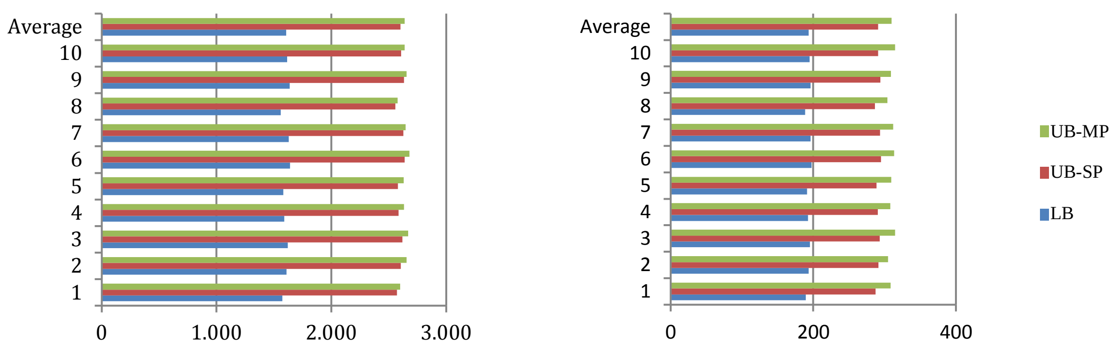

Figure 1 – Estimated bounds (1,000 €) for the parameter pairs (O = 5,000, n = 12), left, and (O = 20,000, n = 12), right.

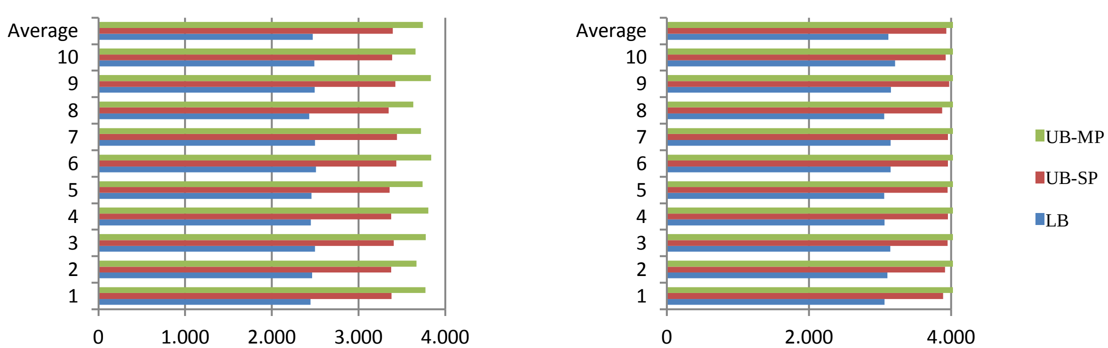

Figure 2 – Estimated bounds (1,000 €) for the parameter pairs (O = 50,000, n = 12), left, and (O = 100,000, n = 12), right.

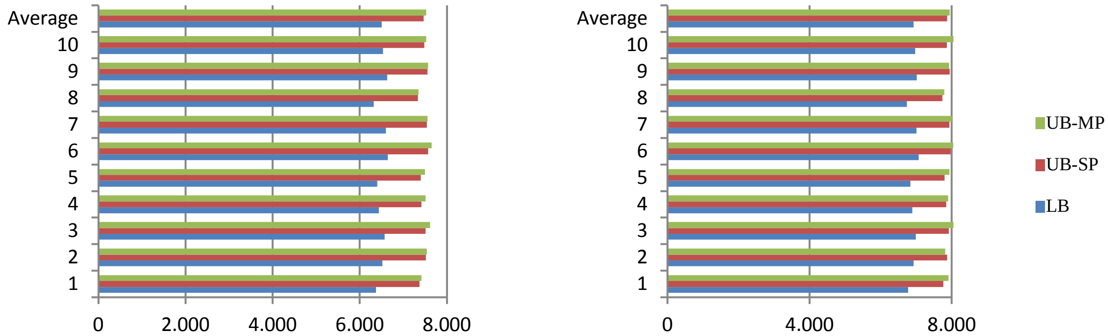

Figure 3 – Estimated bounds (1,000 €) for the parameter pairs (O = 5,000, n = 52), left, and (O = 20,000, n = 52), right.

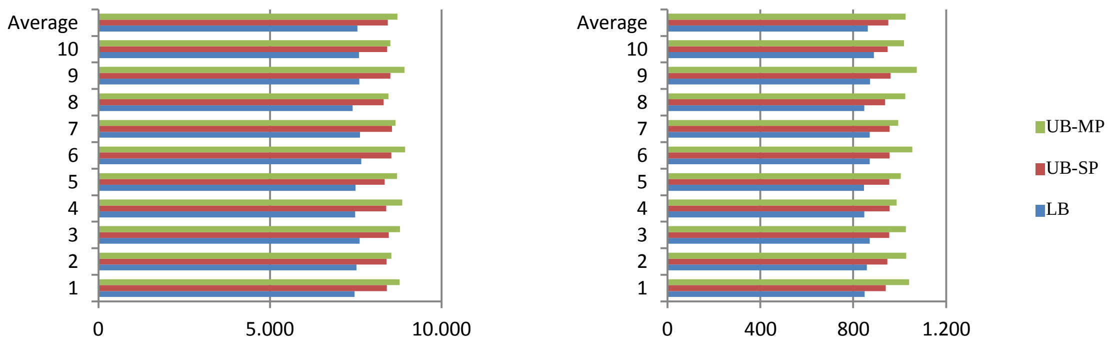

Figure 4 – Estimated bounds (1,000 €) for the parameter pairs (O = 5,000, n = 52), left, and (O = 100,000, f = 52), right.

Figures 1-4 display our computational results for each of the 8 instances. Each chart displays the computed sample bounds on each of the 10 samples used and the estimated bounds, that is, the averages of these sample bounds (the horizontal axis is in 1,000 € and the vertical axis displays the scenario and average labels).

When increasing the warehouse opening cost from 5,000 to 100,000 €, the average percentage difference between UB-SP and UB-MP changes from –1.3% to –8.8% in the case of monthly shipments (n = 12), and from –0.7% to –7.2% in the case of weekly shipments (n = 52). The performance of SP relative to MP thus improves for larger values of the opening cost. This is due to MP opening warehouses more gradually than SP, which results in MP incurring a larger transportation cost (incidentally, SP opens most of the warehouses in the first period, because this policy uses information about the demand during the entire planning horizon). This behaviour is exacerbated when the cost of opening a warehouse increases. In this case, the number of operating warehouses under MP decreases, and this negatively affects the transportation cost incurred by MP.

Upon increasing the cost of opening a warehouse from 5,000 to 100,000 €, the average percentage difference between UB-SP and LB decreases from 62.1% to 26.0% with monthly shipments, and from 15.0% to 10.0% with weekly shipments. The sub-optimality of UB-SP thus improves when opening a warehouse becomes more expensive. Intuitively, when the cost of opening a warehouse increases, both SP and the LB related solutions (one per sample) open fewer warehouses, which tend to be located in a central position in each market area. The difference between UB-SP and LB decreases because of the increased similarity between the SP decisions and the LB related solutions.

Conclusions

We focus on designing a supply chain network when launching a new product. We formulate a stochastic dynamic programming model that uses a stochastic Bass model to describe the demand evolution during a given planning horizon. We propose a heuristic policy that integrates Monte Carlo simulation and math programming. We benchmark the performance of this heuristic against the one of an alternative, simpler to evaluate, heuristic and a lower bound. Application of our methodology to a new Made-in-Italy food and wine product distribution network shows that the proposed heuristic outperforms the simpler heuristic by 0.7 to 8.8%, and the sub-optimality of the proposed heuristic varies in between 10.0% (weekly shipments) and 62.1% (monthly shipments). We conjecture that the larger sub-optimality of the proposed heuristic with monthly shipments is due to our estimated lower bound being looser with monthly shipments rather than weekly shipments. We defer to further research the investigation of this issue.

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