

# Improvement of Multilateration (MLAT) Accuracy and Convergence for Airport Surveillance

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**Abstract**— In this paper, we study, evaluate and develop the use of regularization methods to solve the location problem in multilateration systems using Mode-S signals. The Tikhonov method has been implemented as a first application to solve the classical system of hyperbolic equations in multilateration systems. Some simulations are obtained and the results are compared with those obtained by the well established Taylor linearization and with the Cramér-Rao Lower Bound analysis. Significant improvements are found for the application of Tikhonov method.

**Keywords**—multilateration; regularization methods; localization; air traffic control.

## I. INTRODUCTION

Nowadays, Mode-S Multilateration systems are a feasible option to be used in the Air Traffic Control (ATC) technological infrastructures, so much so that the European Organization for the Safety or Air Navigation (EUROCONTROL) published in its report “*The ATM Surveillance Strategy for ECAC*” [1] that these systems will be one of three pillars of the ground based surveillance infrastructure for 2020. These systems exploit the SSR Mode-S (and Mode A/C) signals in order to calculate the position of aircrafts and vehicles in the coverage area. They perform the localization by solving a system of hyperbolic equations based on TDOA technique; the pertaining algorithms run at real time in a CPS (Central Processor System) [2].

In some scenarios, it is common to find a typical problem for the system of hyperbolic equations to be solved; i.e., the coefficient matrix has a very large condition number [3]. This problem is defined in the literature as an ill-conditioned problem and the consequence of this is that, when the system of equation is solved, the solution is not correct or it has a big error regarding to the exact solution. The mathematical interpretation of this problem goes back to the three conditions of Jacques Hadamard [4], namely, the solution exists, the solution is unique and the solution depends continuously on the data. If at least one of these conditions is not satisfied the problem becomes ill-conditioned. On the other hand, the effects of this problem in the multilateration systems accuracy have been highlighted in [5-6].

Some ill-conditioned problems can be also found in other fields as image processing [7], electromagnetic scattering [8] or geophysics [9]. In these fields, this problem has been solved by applying a group of methods called regularization methods. These methods basically convert the ill-conditioned problem in a well-conditioned problem where the three Hadamard's conditions are satisfied. In this paper, the authors study and apply one of these methods to solve the ill-conditioned problem in multilateration systems.

It is important to emphasize that no specific reference in the literature has been found on this topic, with the remarkable exception of that published in [10], which is an application for passive location system with angle of arrival measurements.

## II. LOCATION PROBLEM IN MODE-S MULTILATERATION

In Mode-S multilateration (MLAT) systems, a number of ground stations (at least three for 2D or four for 3D) are placed in some strategic locations around the airport or the area to be covered. The system uses the Mode-S transmission and asynchronous transponder (Mode-S) replies as well as the responses to interrogations elicited by the MLAT system. Then, the signal is sent to a CPS (Central Processing Station) where the transponder position is calculated. This calculation is based on the Time Difference of Arrival (TDOA) principle, where the intersections of multiples hyperbolas (or hyperboloids), which have been created with the relative time differences, are computed. Each of these hyperbolas follows the expression shown in (1).

$$TDOA_{i,1} = \frac{1}{c} \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} - \frac{1}{c} \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \quad (1)$$

where  $c$  is the velocity of light,  $(x,y,z)$  the unknown target position (aircraft position) and  $(x_i,y_i,z_i)$  is the known position of the  $i$ th station ( $i=1$  denotes the reference station). Linearizing (1) by Taylor series expansion [11-12] is the most accepted strategy to solve an inverse problem with the hyperbolic equations, in order to estimate the target position. In the current literature, the solution of this inverse problem has been presented as an iterative procedure in the sense of the Least-Squares (LS) [11-12]. Denoting the unknown target position as  $\theta = [x, y, z]^T$  and comprising the system measurements (for a

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number of  $N_s$  ground stations) in a  $N_s - 1$  vector  $\hat{\mathbf{m}} = [TDOA_{i,1}, \dots, TDOA_{N_s,1}]^T$ , the final formulation of that method can be summarized as follows,

$$\hat{\boldsymbol{\theta}}^k = \left( \mathbf{G}(\hat{\boldsymbol{\theta}}^{k-1})^T \mathbf{G}(\hat{\boldsymbol{\theta}}^{k-1}) \right)^{-1} \mathbf{G}(\hat{\boldsymbol{\theta}}^{k-1})^T \hat{\mathbf{m}}_{\Delta}(\hat{\boldsymbol{\theta}}^{k-1}) + \hat{\boldsymbol{\theta}}^{k-1} \quad (2)$$

where  $k = 1, \dots, K$ ;  $\mathbf{G}$  is the  $(N_s - 1) \times 3$  Jacobian matrix of the  $N_s - 1$  hyperbolic equations (1),  $\hat{\boldsymbol{\theta}}^0$  is the starting point required for this method,  $\hat{\mathbf{m}}_{\Delta} = \hat{\mathbf{m}} - \mathbf{m}(\hat{\boldsymbol{\theta}}^{k-1})$  and  $\mathbf{m}(\hat{\boldsymbol{\theta}}^{k-1})$  is a  $(N_s - 1) \times 1$  vector comprising the *TDOA* (see (1)) quantities evaluated at the partial solution  $\hat{\boldsymbol{\theta}}^{k-1}$ . Finally, because this method is based on an iterative procedure,  $K$  is the number of refinement iterations.

The solution provided by (2) is the minimum residual norm solution and the matrices product  $(\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T$  is known as the pseudoinverse matrix  $\mathbf{G}^\dagger$  [3]. For some scenarios, due to the system geometry, to the measurements noise and to the starting point quality, this inverse problem is ill-conditioned and therefore, the solution obtained by (2) is not correct or diverges with very large errors. The numerical reason is because the pseudoinverse matrix does not satisfy the three Hadamard's conditions [3-4].

On the other hand, one feasible option to avoid the above problem is to use a horizontal projected version of the Taylor-series expansion method and solve it with the pseudoinverse matrix. This option, although the corresponding coefficient matrix is initially well-conditioned, has the disadvantage that it adds a spatial bias due to the projection from 3D to 2D in the coefficient matrix but not in the measurements. As it will be shown in the results, this option normally is more useful for surface movements surveillance.

In this paper, we use the Tikhonov regularization [13] method to solve the iterative procedure of Taylor-series expansion and to avoid those errors due to the ill-conditioned problem.

### III. SOLUTION OF LOCATION PROBLEM IN MODE-S MULTILATERATION BY TIKHONOV REGULARIZATION

This method was originally and independently derived by Phillips [14] and Tikhonov [13] and it has been used to solve the ill-conditioned problems in an important number of applications in engineering and science. The main idea of this method is to incorporate a priori information about the size and smoothness of the final solution. This a priori information is in the form of semi-norm. Generally, Tikhonov regularization leads to minimize a function that takes the following form,

$$\arg \min \{ \|\mathbf{A}\boldsymbol{\theta} - \hat{\mathbf{m}}\|_2^2 + \lambda^2 \|\mathbf{L}\boldsymbol{\theta}\|_2^2 \} \quad (3)$$

where  $\mathbf{A}$  is the exact coefficient matrix for the inverse problem,  $\lambda$  is called regularization parameter and  $\mathbf{L}$  is called regularization matrix. The regularization parameter  $\lambda$  controls the importance given to the regularization term  $\|\mathbf{L}\boldsymbol{\theta}\|_2$ .

Using the Tikhonov regularization concept, the likelihood function [12] for the Mode-S location problem can be expressed as follows,

$$\Lambda(\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{N_s-1}{2}} \det(\mathbf{N}(\boldsymbol{\theta}))^{\frac{1}{2}}} e^{-\frac{1}{2}((\hat{\mathbf{m}} - \mathbf{m}(\boldsymbol{\theta}))^T \mathbf{N}(\boldsymbol{\theta})^{-1} (\hat{\mathbf{m}} - \mathbf{m}(\boldsymbol{\theta})) + \lambda^2 (\mathbf{L}\boldsymbol{\theta})^T (\mathbf{L}\boldsymbol{\theta}))} \quad (4)$$

where  $\mathbf{N}(\boldsymbol{\theta})$  is the covariance matrix of the TDOA measurements noise and *det* denotes the determinant operator. The maximum likelihood solution of (4) is that  $\hat{\boldsymbol{\theta}}$  which minimizes the following function,

$$Q(\boldsymbol{\theta}) = \left\{ (\hat{\mathbf{m}} - \mathbf{m}(\boldsymbol{\theta}))^T \mathbf{N}(\boldsymbol{\theta})^{-1} (\hat{\mathbf{m}} - \mathbf{m}(\boldsymbol{\theta})) + \lambda^2 (\mathbf{L}\boldsymbol{\theta})^T (\mathbf{L}\boldsymbol{\theta}) \right\} \quad (5)$$

Solving (5) by Taylor-series expansion, the estimation for the unknown target position in the Tikhonov sense takes the following form,

$$\hat{\boldsymbol{\theta}}_{\lambda}^k = \mathbf{A}_{\lambda}^{-1}(\hat{\boldsymbol{\theta}}_{\lambda}^{k-1}) \hat{\mathbf{m}}_{\Delta}(\hat{\boldsymbol{\theta}}_{\lambda}^{k-1}) + \hat{\boldsymbol{\theta}}_{\lambda}^{k-1}, \quad k = 1, \dots, K \quad (6)$$

where  $\mathbf{A}_{\lambda}^{-1}$  is known in the literature as the regularized inverse matrix of Tikhonov [13] and it is defined as follows,

$$\mathbf{A}_{\lambda}^{-1} = (\mathbf{G}^T \mathbf{N}(\boldsymbol{\theta})^{-1} \mathbf{G} + \lambda^2 \mathbf{L}^T \mathbf{L})^{-1} \mathbf{G}^T \mathbf{N}(\boldsymbol{\theta})^{-1} \quad (7)$$

It is worth to say that, due to the fact that the covariance matrix  $\mathbf{N}(\boldsymbol{\theta})$ , for real applications, is often not known because it depends on the true target position, in practice it is common remove it from (7), assuming an identity matrix.

The choice of regularization parameter  $\lambda$  and regularization matrix  $\mathbf{L}$  is the most critical aspect to make a correct use of the procedure described above. Firstly, the choice of the regularization matrix is directly connected with the statistics of the target position vector  $\boldsymbol{\theta}$ . If the components of  $\boldsymbol{\theta}$  are assumed to be non-random and uncorrelated, a standard choice of the regularization matrix is  $\mathbf{L} = \mathbf{I}_3$ , where  $\mathbf{I}_3$  is a  $3 \times 3$  identity matrix.

On the other hand, the choice of the regularization parameter value is not as straightforward as the choice of regularization matrix. In the literature there exist a considerable number of methods and procedures to calculate/estimate an approximated regularization parameter value. These methods provide good results for a variety of applications (e.g. image processing, biologic computer, remote sensing, electromagnetic scattering, etc.) and they are basically based on the solution of an optimization problem, i.e., find a parameter that satisfies some equalities [15] or find a parameter that minimizes some special functions [16-18]. However, it is worth to say that, due to nature of these methods, they introduce a significant computational load and therefore the computation time can be not acceptable for real time location in Mode-S Multilateration.

In this work, we evaluate the problem for several regularization parameters values (no more than three) and then we choose as true solution the one which corresponds with the minimum residual error. This option is feasible for this application because the typical size of the coefficients matrices (Jacobian matrix) is normally smaller than  $10 \times 3$ .

In general, the residual error for an inverse problem is given by,

$$error_j = \frac{\|G(\hat{\theta}_{\lambda_j})\hat{\theta}_{\lambda_j} - \hat{m}\|_2}{\|\hat{m}\|_2}, \quad j = 1, \dots, \text{total of } \lambda \quad (8)$$

Remembering that for Taylor-series expansion method, the matrix  $G$  is an approximation of an exact coefficient matrix, then (8) could not be a correct value for the residual error regarding to the true target position  $\theta$ . Therefore, in this work, we propose to calculate the residual error by replacing the regularized solution  $\hat{\theta}_{\lambda_j}$  in the non-linear TDOA function (1), instead in the matrix  $G$ , as follows,

$$error_j = \frac{\|h_{\lambda_j} - \hat{m}\|_2}{\|\hat{m}\|_2}, \quad j = 1, \dots, \text{total of } \lambda \quad (9)$$

where the vector  $h_{\lambda_j}$  is given by,

$$h_{\lambda_j} = \begin{bmatrix} TDOA_{i,1}(\hat{\theta}_{\lambda_j}) \\ \vdots \\ TDOA_{N_s,1}(\hat{\theta}_{\lambda_j}) \end{bmatrix}_{(N_s-1) \times 1}, \quad i = 1, \dots, N_s \quad (10)$$

#### IV. RESULTS

Preliminary results are shown to validate the improvement of the system accuracy and its convergence by applying the Tikhonov method in the iterative procedure of Taylor-series expansion. Two scenarios have been simulated; the first one is the operating system of Linate Airport (Milan, Italy) and the second one is a multilateration system which is well described and studied by Cramér-Rao Lower Bound -CRLB- analysis in [6].

For each scenario, the horizontal (2D) R.M.S error (obtained via Monte-Carlo simulation with 100 trials), the theoretical accuracy provided by the CRLB [6], the bias of the estimator and the spatial convergence are calculated.

##### A. Linate Airport System

The Linate airport system is composed of eight ground stations. For this scenario we have simulated a path of surface movement around the airport. The system layout and simulated

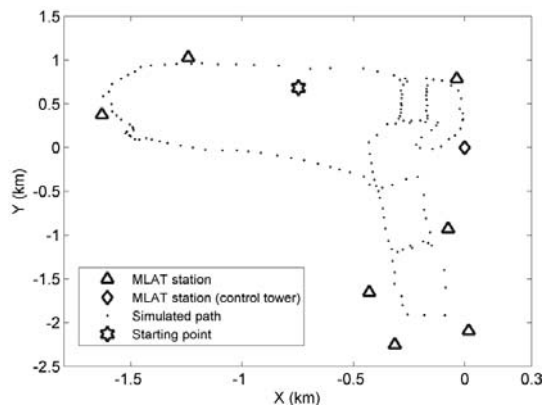


Figure 1. Linate airport system layout.

path are shown in Fig. 1.

For this scenario, the starting point for the Taylor-series expansion method has been assumed to be a fixed point over the airport and it is shown as the star in Fig. 1. For this scenario it has been found that using only one regularization parameter value ( $\lambda = 0.1$ ) is enough to obtain satisfactory results.

Fig. 2 shows the horizontal R.M.S error for the horizontal projection of Taylor-series expansion method and the non-projected (3D) version solved by the pseudoinverse matrix. It also shows the non-projected (full version) Taylor-series expansion method solved by Tikhonov regularization and the corresponding CRLB analysis.

Initially, the CRLB analysis predicts a good accuracy over the entire path, presenting only a few peaks around the points 40 and 50, where the horizontal accuracy is slightly larger than 7 meters. However, for the non-projected Taylor (circles), it can be seen how the ill-conditioned problem avoids the convergence of the method solved by the pseudoinverse matrix, i.e., the R.M.S error tends to infinity in the most of the points. On the other hand, the horizontal projected version obtains acceptable accuracy levels but the effect of the spatial bias is present, for this scenario, in most of the points (more for those points within the 30 and 120). Finally, it is evident how the solution obtained by applying Tikhonov regularization improves both the ill-conditioned problem, which is directly related with the system accuracy and convergence and the spatial bias added for the projected version.

Fig. 3 shows the bias of the estimator for the projected version of Taylor as solved by the pseudoinverse and that one corresponding to the full version of Taylor as solved by the Tikhonov regularization. In this figure it can be noted the improvement, regarding to the spatial bias of the horizontal projection of Taylor-series method, added by using the Tikhonov regularization. This aspect is very important when using tracking algorithms (which are present in all the Air Traffic Control -ATC- systems) because they can improve the R.M.S error of the location algorithm but not the bias. In this way, it is clear to see how Tikhonov method also helps to the tracking algorithms to reach more accurate tracks.

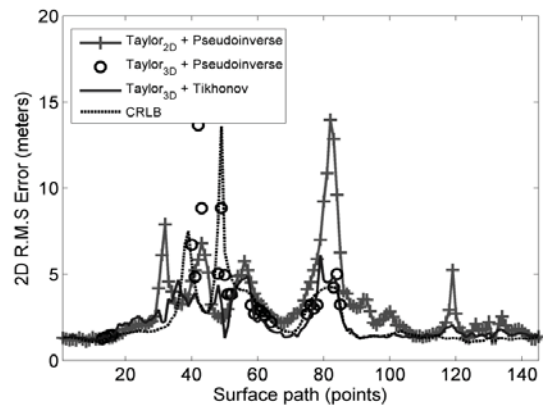


Figure 2. Horizontal accuracy for Linate airport. Each point in the abscissa corresponds to a point in the simulated path.

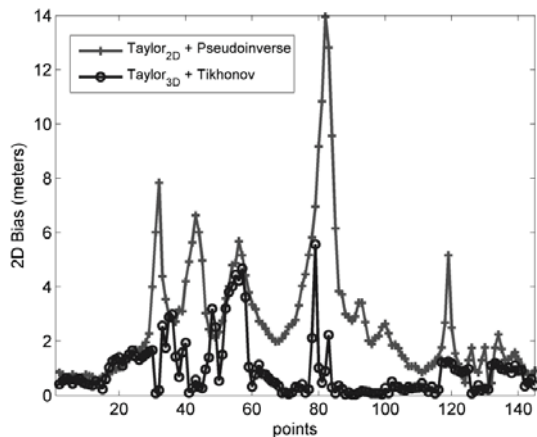


Figure 3. Horizontal bias of the estimator for Linate airport. Each point in the abscissa corresponds to a point in the simulated path.

Finally, Fig. 4 shows the spatial convergence for a specific Monte-Carlo trial. In this figure it can be observed how the solution by Tikhonov regularization allows the Taylor-series expansion to ensure the convergence to the true point.

#### B. MLAT System for a Takeoff Line

This system is composed for four stations and it is well analyzed in [6]. The layout of the simulated scenario is shown in Fig. 5.

For this scenario, the starting point for the Taylor-series expansion method has been obtained by means of the closed form algorithm described in [19]. This algorithm is based on spherical intersections and it does not need a starting point but, as it is shown in the results, it is also affected for the ill-conditioning of the problem due to the system geometry. The horizontal coordinates of the starting point ( $x,y$ ) are taken from the closed form algorithm and the vertical coordinate ( $z$ ) is simulated as the barometric altitude, i.e., with a bias of 40m regarding to the real target height. Also for this scenario it has been found that only using one regularization parameter value ( $\lambda = 0.1$ ) is enough to obtain satisfactory results.

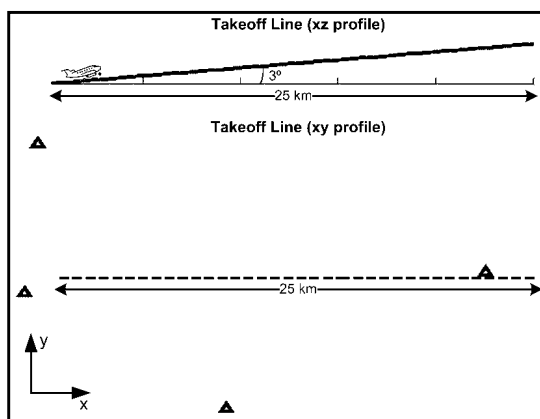


Figure 5. Layout of the MLAT system for a takeoff line.

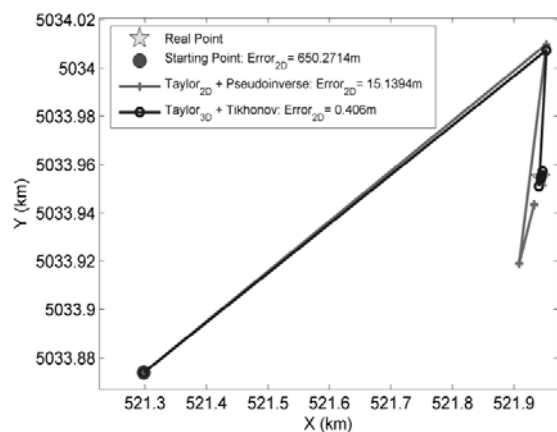


Figure 4. Spatial convergence for one trial.

The amount of ill-conditioning of this scenario is significantly greater than that of the first scenario. It is because the number of stations here (four) is much smaller than the first one (eight). This effect can be noted in the CRLB analysis shown in Fig. 6 since the theoretical accuracy diverges for points within 0 and 5 km and for those around 20 km. On the other hand, due to the fact that for this scenario, the target height is increasing with the distance, the vertical separation of this with the plane of the ground stations considerably affect the accuracy provided by the horizontal projection of Taylor-series method (crosses) and the spatial bias added by this is considerably large for points beyond 15 km.

Due to the ill-conditioning, it can be observed that, for this scenario, the accuracy levels provided by the full version of Taylor, using the pseudoinverse matrix, diverges far from the theoretical accuracy values (CRLB) for points within 0-5 km and 15-20 km. In contrast, the closed form algorithm presents a more stable accuracy but it is also affected by system geometry (Dilution of Precision -DOP-). Finally, it is evident the significant improvement, of the system accuracy, obtained by applying Tikhonov regularization. The accuracy for this option is stable for the whole of takeoff line and it is not larger than 25

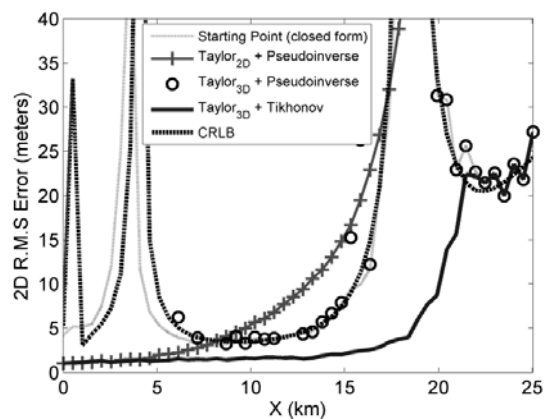


Figure 6. Horizontal accuracy over the takeoff line.

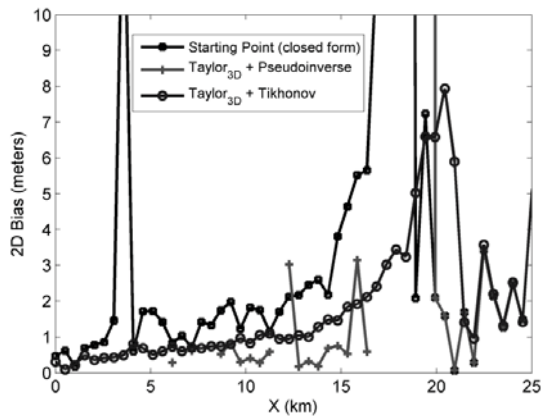


Figure 7. Horizontal bias of the estimator over the takeoff line.

meters. It is worth to say that this solution is below the CRLB values because also the CRLB is affected by the ill-conditioning of the problem, specifically that part due to the system geometry.

Fig. 7 shows the bias estimator for the solutions obtained by the closed form algorithm and by the full version of Taylor-series using both pseudoinverse matrix and Tikhonov regularization. Firstly, it can be noted that for a few points close to 10 km and 15 km, the bias of the solution obtained by pseudoinverse is smaller (nor more than 1 m) than that of Tikhonov method. It can be explained because in the case of well-conditioned problems the pseudoinverse matrix is the solution with minimum norm [3] and in contrast Tikhonov always adds certain amount of bias [13]. The important aspect is that, if the correct regularization parameter value is chosen, this amount of bias can be neglected regarding to the rest of the options to improve the problem (i.e., the horizontal projection of Taylor-series method). Moreover, due to the ill-conditioned problem, for the rest of the points, the bias added by pseudoinverse matrix solution is infinity and for most of the points the bias added by the closed form algorithm has been found greater than that of Tikhonov regularization.

Finally, Fig. 8 shows the spatial convergence for a specific Monte-Carlo trial; in this figure it can be noted how the regularization of the location problem ensures the convergence also for this scenario.

## V. CONCLUSIONS

The implementation of Tikhonov regularization to solve the inverse problem of Taylor-series expansion, for location in Multilateration systems, has been described and evaluated. The theoretical aspects of the method with a practical strategy to calculate the regularization parameter have been described.

For the scenarios simulated here, significant improvements, for the system accuracy and convergence, have been found with the implementation of Tikhonov regularization. For both scenarios, it was found that the regularization of the location problem significantly mitigates the ill-conditioning due to the system geometry, i.e., those points where the CRLB analysis predicts poor accuracy levels; to the measurements noise, i.e.,

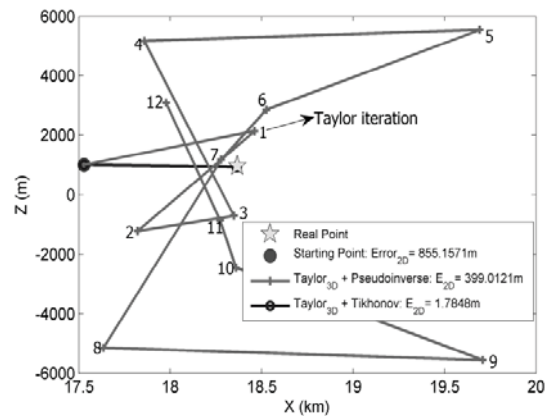


Figure 8. Spatial convergence for one trial over the takeoff line.

those points where the CRLB predicts good accuracy levels but the solution obtained by the pseudoinverse matrix diverges; and also due to the quality of the starting point for Taylor-series expansion method.

For both scenarios it was found that a regularization parameter value of  $\lambda = 0.1$  was enough to obtain satisfactory results but, it is worth to say that in the situations where the problem is better conditioned, it is necessary to use, at least, one or two more values smaller than  $\lambda = 0.1$ , i.e., the smaller the amount of ill-conditioning the smaller should be  $\lambda$ .

The regularization of the location problem is more useful for those situations where the vertical separation between the ground stations and the target is quite small or for those situations with a small number of stations.

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