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A third hypothesis on the origin of the redshift: Application to the Pioneer 6 data

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Abstract

We propose a third general mechanism (beyond the Doppler and Einstein effects) to account for the redshift. We model the interstellar medium by a low density Fermi gas and its interaction with light by standard QED. Applying the techniques of the stochastic limit of quantum field theory, we prove a general *redshift theorem*. This result fits satisfactorily both Pioneer 6 and solar limb redshift data.

1. Introduction

The ordered shift towards lower frequencies - redshift - of (both emission and absorption) spectral lines in the range 10^9–10^{15} Hz of far away astrophysical sources (galaxies, quasars, etc.) is usually explained in terms of Doppler effect (and interpreted, in the frame of Friedman cosmological models, as a consequence of the space expansion). The only known alternative explanation, in the context of standard cosmology, is the gravitational redshift (or Einstein effect), caused by the difference of the gravitational potential between the source and the observer. This effect can be very important to explain local effects on the surface of collapsed stars, but cannot be invoked for explaining global (in particular cosmological) effects, because the tidal effects near the surface of a collapsed star cause a large broadening of the spectral lines.

However, it is known that there are some astrophysical sources whose redshift can be hardly explained in terms of the known (Doppler or Einstein) effects: this is the so-called *anomalous redshift* problem (see, e.g., Ref. [1]). Most of the scientific community explains these anomalies (e.g. clusters of galaxies and quasars with discordant redshifts, but whose morphology strongly suggests a physical association) as optical effects (e.g. casual alienations of extragalactic objects located at very different distances). However, this explanation seems rather in contrast with the increasing number of observational data providing statistical evidence for a physical correlation of high-redshift sources (quasars or galaxies) with low-redshift galaxies, see, e.g., Refs. [2,3] (see also Refs. [4,5], where the only possible "orthodox" explanation, based on the hypothesis of far quasars brightened by gravitational microlensing of individual stars of the near galaxy, is discussed and ruled out on statistical bases); moreover it does not apply to the Pioneer 6 experiment (see

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below). This has stimulated the search for a cause of these anomalies but, up to now, no explanation survived a detailed analysis and comparison with observations. In particular, the hypothesis that the light could be redshifted because of dissipative processes (e.g. scattering-like interaction with interstellar media) occurring to the travelling photon during its long journey has been proposed various times by various authors ("tired-light hypothesis"). This hypothesis has been discarded on the basis of the fact that a sequence of scattering-type interactions with any medium, independently of the mediating force, would imply: (i) an angular dispersion, which should destroy the observed point-like image of the source; (ii) a broadening of the spectral lines, much wider than those effectively observed (see, e.g., Ref. [9]).

Quite recently, James and Wolf [6] proposed a scattering-like mechanism with a fluctuating medium that should mimic the Doppler shift, for a suitable choice of the correlation properties of the medium. However, this model implies a diffusion of light in all directions with defocusing of the source. Moreover, both the shift and the intensity of light turn out to be strongly direction dependent; in particular the shift is blue for small scattering angles, where the intensity takes its maximum value (see Refs. [7,8]), in contrast with observations (particularly evident in the Pioneer 6 experiment, where the scattering angle is zero).

However, some ideas and techniques, recently introduced in the so-called *stochastic limit of quantum field theory* [10], suggest a new approach to the tired-light hypothesis, which seems to elude the Zel'dovich objection. The basic new intuition given by this theory is that any quantum system interacting with an environment presents shift phenomena if its dynamics is studied in a suitable time scale. In particular, in this paper we use the techniques of the so-called *low density limit* (LDL), that applies to any system in which the environment is a very rarefied gas (cf. Ref. [11]).

Our main result is the proof of a *redshift theorem* (cf. formula (19) below) that shows that the standard QED interaction can produce a negative, i.e. red, shift. In the range of applicability of the theory (low density, in our case) this is a general phenomenon, independent of the specific parameters [2]. We point out that our results, here obtained in a rigorous way by means of the LDL, can also be obtained, in the second order approximation, by the traditional perturbation theory (cf. Ref. [12]).

In Section 5 we apply the *redshift theorem* to the data produced in the Pioneer 6 experiment (see Refs. [13,14]). This experiment consists of a set of careful measurements performed by the Pioneer 6 spacecraft, launched into an orbital trajectory around the sun, in order to pass behind it. The information was transmitted through amplitude modulations of a spectrally pure carrier wave (with frequency $\nu = 2292$ MHz). A set of observations by Goldstein [13] of some parameters of the carrier wave (signal power, center frequency and bandwidth), showed an unexpected redshift of the carrier wave, while the spacecraft was passing behind the solar corona. In fact, once all Doppler effects were cancelled, the redshift turned out to be symmetric and increasing from 10^{-8} to 5×10^{-8} when the distance of the light ray (between the spacecraft and the earth) from the center of sun was decreasing from 10 to 3 solar radii. Such a behavior, in the absence of any observable bandwidth-redshift correlation, cannot be related to any known physical effect.

All attempts suggested up to now to explain this phenomenon have met the same difficulty: the need of appealing to some ad hoc hypothesis, on which experimental checks are impossible. In particular, we recall some models by Merat, Pecker, Roberts and Vigier (cf. Refs. [20-23]), based on the hypothesis of the existence of a bath of (light) scalar neutrals, interacting with the incident photon (with a non-zero rest mass) by a new kind of strong interaction or a new type of inelastic photon-photon interaction.

Our goal is to give an explanation to the Pioneer 6 data *based only on standard QED, without any ad hoc hypothesis*: the photon travelling from the Pioneer 6 to the earth is supposed to interact with the electrons and ions of the solar corona *by the usual e.m. interaction*. Our result is that, in the stochastic limit, the frequency of the wave is shifted towards the red by a quantity which is directly proportional to $\int_{-\infty}^{\infty} T_d(x) n_d(x)\, \mathrm{d}x$, where $T_d(x)$ and $n_d(x)$ are the temperature and the density of the coronal plasma along the trajectory of the photon (labeled by d).

[2] Our shift can be related to the vacuum polarization diagram: from this point of view it can be interpreted as a kind of Lamb shift, induced by a gas of charged particles.

This effect is observable since the corona plasma is so rarefied that the Compton scatterings give a negligible contribution to the observable effects, so that the point-like image of the source is not destroyed and the spectral lines are not broadened. More exactly, the photons which undergo a Compton scattering are less than 10^{-9} of the total incoming photons; moreover, they are completely randomized in direction and frequency so that they are scattered away from the light ray, escaping observation.

In Section 5 we insert in our redshift formula the actual density and temperature of the solar corona (along the photon trajectory), and we show that our result fits the Pioneer 6 data.

Finally, we show that our theory can also take into account the center-to-limb variation of the redshift of the solar lines. This variation, observed since the beginning of the century and confirmed by recent observations (see, e.g., Refs. [15,16]), is not taken into account by any standard theory (the gravitational redshift is 2.16×10^{-6}, independently of the position on the solar disk). As far as we know, the only attempt to explain this effect, avoiding any ad hoc hypothesis, is due to Marmet [17,18], and is founded on the theoretical existence of inelastic collisions of the photons with neutral H_2 in the chromosphere. According to this author, these collisions would imply a small energy loss without angular dispersion. However, while the Marmet mechanism does not apply to the Pioneer 6 experiment (since the solar corona gas is completely ionized), our theory takes into account both the Pioneer 6 and the solar limb data.

A still unsolved problem is the presence, in the redshift formula, of a divergent integral (here treated as a phenomenological parameter), whose renormalization will be the subject of a future work.

2. The physical model

The free e.m. field is modeled by the standard second quantized Hamiltonian,

$$H_{\text{e.m.}} \doteq \sum_{\omega\alpha} \hbar\omega b^{\dagger}_{\omega,\alpha} b_{\omega,\alpha}, \qquad (1)$$

where $b^{\dagger}_{\omega,\alpha}$ and $b_{\omega,\alpha}$ are, respectively, the (bosonic) creation and annihilation operators associated to the e.m. field mode with wave number ω and polarization vector $\boldsymbol{\epsilon}^{\alpha}_{\omega}$ $(\alpha = 1, 2)$.

We will suppose the plasma to be composed of identical non-interacting fermions (it is straightforward to extend our calculations to many different species of particles). Let $T_d(x)$ and $z_d(x)$ denote the temperature and the fugacity of the coronal plasma along the trajectory of the photon (the trajectory of the photon is labeled by d , the impact parameter of the photon with respect to the solar center). In this paper we calculate the energy shift of a one-dimensional e.m. wave (i.e. a stationary state of the e.m. field in a one-dimensional box) caused by a Fermi perfect gas in a Gibbs state, whose parameters $T_d(x)$ and $z_d(x)$ depend on the position. Since the fugacity and the temperature in the solar corona vary on a length scale which is much greater than the typical length (i.e. the mean free path) on which the plasma particles can be considered as bounded, we will calculate the plasma matrix elements considering $T_d(x)$ and $z_d(x)$ as parameters and not as operators with respect to the plasma states (see also Remark 3 below).

If H_{P} is the Hamiltonian of the plasma, the total free Hamiltonian of the e.m. field and the plasma is, in self-explaining notation,

$$H_0 = H_{\text{e.m.}} \otimes 1_{\text{P}} + 1_{\text{e.m.}} \otimes H_{\text{P}}. \qquad (2)$$

The interaction Hamiltonian is the standard second quantized Hamiltonian describing the interaction of the quantum e.m. field with a Fermi gas of particles of mass m (cf. Ref. [24]). Denoting with $a^{\dagger}_{k}$ and a_k the (fermionic) creation and annihilation operators associated to the one particle state of wave number $\boldsymbol{k}$ (we omit the unessential spin index), we have

$$H_{\text{I}} = \sum_{\omega,\alpha} \frac{e}{m}\sqrt{\frac{1}{V}\frac{\hbar^3}{2\omega}} \times \left(b_{\omega,\alpha} \otimes \sum_{\boldsymbol{k},\boldsymbol{k}'} \boldsymbol{k}' \cdot \boldsymbol{\epsilon}^{\alpha}_{\omega} \delta_{\omega/c+\boldsymbol{k},\boldsymbol{k}'} a^{\dagger}_{\boldsymbol{k}} a_{\boldsymbol{k}'} + \text{h.c.} \right), \qquad (3)$$

where V is a normalization volume.

3. Deduction of the shift operator by the LDL

Let us consider the wave operator U_t in the interaction representation associated to H_{I}. It is the solution of the equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}U_t = H_\mathrm{I}(t)U_t,$$
$$H_\mathrm{I}(t) \doteq \exp(\mathrm{i}tH_0)H_\mathrm{I}\exp(-\mathrm{i}tH_0). \tag{4}$$

In the *low density limit* in the stochastic quantum field theory, one considers a spatially confined quantum system (the "system", in our case the e.m. field) coupled to another quantum system (the "reservoir", in our case the plasma of charged particles) through an interaction H_I of form (3). The idea of LDL is to find out an equation that is equivalent to (3) *if the fugacity z of the reservoir is small* (i.e. if the density of the reservoir is small) and if the evolution of the system is considered on times of the order of t/z (i.e., since the fugacity is small, *for very long times*). The fundamental result is that, in a convergence specified in Refs. [11,25,26],

$$\lim_{z\to 0} U_{t/z} = U(t), \tag{5}$$

where $U(t)$ is the solution of a quantum stochastic differential equation (in the sense of Ref. [10]) containing, in the most general case, both purely Hamiltonian terms and noise terms. This equation is exact, in the sense that it has contributions from QED processes with any number of vertices (cf. Ref. [26]).

If we consider only the two vertices contribution (*second order approximation*) we obtain that $U(t)$ satisfies a much simpler differential equation, that never has noise terms (cf. Refs. [27,28]),

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}U(t) = KU(t), \tag{6}$$

where K is called the *drift operator* and has the following form,

$$K = \sum_{\omega\alpha}(\zeta_\omega^+ + \mathrm{i}\xi_\omega^+)b_{\omega,\alpha}^\dagger b_{\omega,\alpha} - (\zeta_\omega^- + \mathrm{i}\xi_\omega^-)b_{\omega,\alpha}b_{\omega,\alpha}^\dagger, \tag{7}$$

where

$$\xi_\omega^+ \doteq \lim_{t\to\infty}\frac{1}{\hbar}\sum_{kk'}|g_{kk'}^{(\omega,\alpha)}|^2 z\exp(-\beta\epsilon_k) \times \delta\left(\frac{\epsilon_k}{\hbar} - \frac{\epsilon_{k'}}{\hbar} + \omega\right), \tag{8}$$

$$\xi_\omega^- \doteq \lim_{t\to\infty}\frac{1}{\hbar}\sum_{kk'}|g_{k'k}^{(\omega,\alpha)}|^2 z\exp(-\beta\epsilon_k) \times \delta\left(\frac{\epsilon_k}{\hbar} - \frac{\epsilon_{k'}}{\hbar} - \omega\right), \tag{9}$$

$$\zeta_\omega^+ \doteq \sum_{kk'}|g_{kk'}^{(\omega,\alpha)}|^2\frac{z\exp(-\beta\epsilon_k)}{\hbar\omega + \epsilon_k - \epsilon_{k'}}, \tag{10}$$

$$\zeta_\omega^- \doteq \sum_{kk'}|g_{k'k}^{(\omega,\alpha)}|^2\frac{z\exp(-\beta\epsilon_k)}{\hbar\omega + \epsilon_{k'} - \epsilon_k}. \tag{11}$$

Here we used the notation

$$g_{k,k'}^{(\omega,\alpha)} \doteq \sqrt{\frac{1}{V}\frac{e^2}{m^2}\frac{\hbar^3}{2\omega}}\,\boldsymbol{k}'\cdot\boldsymbol{\epsilon}_\omega^\alpha\delta_{\omega/c+k,k'}.$$

Comparing (6) with (4) it is easy to see that K plays the role of a *perturbation Hamiltonian* in the free dynamics of the e.m. field induced by the presence of the gas of charged particles. We can say that the dynamics of the e.m. field is induced by a *total effective Hamiltonian* $H = H_{\mathrm{e.m.}} + K$. The Hermitian part of K will be called the *shift operator* and denoted by S. We have

$$S = \tfrac{1}{2}(K + K^\dagger).$$

Remark 1. The approximation procedure that we followed to obtain (6) consists of:

(i) neglecting all terms in z^n, $n \geqslant 2$ with respect to the z terms (LDL);

(ii) neglecting all terms in α^n, $n \geqslant 2$ with respect to the terms in α (second order approximation).

Remark 2. The reason why the drift operator is not Hermitian (and then it induces a non-unitary evolution) is because we are describing a system, the e.m. field, that is not physically isolated.

Before proceeding, we give an explicit form for the shift operator. By using the canonical commutation relations and denoting $\varsigma_\omega = \zeta_\omega^+ - \zeta_\omega^-$, S can take the form

$$S = \sum_{\omega\alpha}\varsigma_\omega b_{\omega,\alpha}^\dagger b_{\omega,\alpha} - \sum_{\omega\alpha}\zeta_\omega^-. \tag{12}$$

We now estimate ζ_ω^-. First of all, we sum with respect to $\boldsymbol{k}'$. Denoting $\boldsymbol{k}\cdot\boldsymbol{\epsilon}_\omega^\alpha =: k_\epsilon$ one obtains

$$\zeta_\omega^- = \frac{1}{V}\frac{e^2}{m^2}\frac{\hbar^3}{2\omega}z\sum_k\exp(-\beta\epsilon_k)k_\epsilon^2 \times (\hbar\omega + \epsilon_{|k-\omega/c|} - \epsilon_k)^{-1}.$$

and, in the continuous limit,

$$\zeta_\omega^- = \frac{e^2}{m^2}\frac{\hbar^3}{2\omega} z \frac{1}{(2\pi)^{3/2}} \times \int \mathrm{d}\boldsymbol{k} \exp(-\beta\epsilon_{\boldsymbol{k}}) k_\epsilon^2 (\hbar\omega + \epsilon_{|\boldsymbol{k}-\omega/c|} - \epsilon_{\boldsymbol{k}})^{-1}. \tag{13}$$

In the following we shall use polar coordinates in which the propagation direction of the wave coincides with the polar axis and the polarization vector (normal to the polar axis) lies on the plane $\varphi = 0$. Taking into account that $\epsilon_k = (\hbar^2/2m)k^2$ and that $k_\epsilon = k\sin\theta\cos\varphi$, we have

$$\frac{2m}{\hbar^2}(\hbar\omega + \epsilon_{|\boldsymbol{k}-\omega/c|} - \epsilon_{\boldsymbol{k}}) = \frac{2m\omega}{\hbar} + (|\boldsymbol{k}-\omega/c|^2 - k^2) = \frac{2m\omega}{\hbar} + \frac{\omega^2}{c^2} - 2\frac{\omega}{c}k\cos\theta$$

and then, with $\cos\theta =: x$,

$$\zeta_\omega^- = \frac{e^2}{m}\frac{\hbar c}{2\omega^2} z \frac{1}{(2\pi)^{3/2}} \pi \int_{-1}^{1} \mathrm{d}x\,(1-x^2) \times \int_0^{+\infty} \mathrm{d}k\, k^4 \exp(-k^2/k_T^2) \left(\frac{mc}{\hbar} + \frac{\omega}{2c} - kx\right)^{-1}, \tag{14}$$

with $k_T \doteq \sqrt{2mk_\mathrm{B}T/\hbar^2}$. For frequencies in the range $\omega \in (10^9, 10^{16})$ Hz and for $T \simeq 10^6$ K, one has $\omega/2c \ll k_T \ll mc/\hbar$. Then, on the interval on which

$$k^4 \exp(-k^2/k_T^2)\left(\frac{mc}{\hbar} + \frac{\omega}{2c} - kx\right)^{-1}$$

is not exponentially zero we can assume

$$\left(\frac{mc}{\hbar} + \frac{\omega}{2c} - kx\right)^{-1} \simeq \frac{\hbar}{mc}\left(1 + \frac{\hbar}{mc}kx\right).$$

The integrals with respect to x and k are then elementary, and

$$\zeta_\omega^- = \frac{1}{2}\frac{e^2}{m}\frac{\hbar c}{2\omega^2} z \frac{1}{(2\pi)^{3/2}} \pi \frac{\hbar}{mc} k_T^5 \sqrt{\pi}. \tag{15}$$

In the low density assumption ($z \ll 1$), the fugacity z is directly proportional to the density,

$$n = 2\pi^2 \int_0^\infty \mathrm{d}k\, k^2 \left(\frac{1}{z}\exp(k^2/k_T^2) + 1\right)^{-1} \simeq 2z\pi^2 \int_0^\infty \mathrm{d}k\, k^2 \exp(-k^2/k_T^2) = z k_T^3 \frac{1}{2\pi^{3/2}}.$$

Replacing this in (15), writing $\alpha = e^2/\hbar c$ and using $k_T = \sqrt{2mk_\mathrm{B}T/\hbar^2}$ one finds

$$\zeta_\omega^- = (\tfrac{1}{2}\pi)^{3/2} \alpha n \frac{(\hbar c)^3}{(\hbar\omega)^2}\frac{k_\mathrm{B}T}{mc^2}. \tag{16}$$

In the same way it is possible to show that

$$\varsigma_\omega = (\tfrac{1}{2}\pi)^{3/2} \alpha n \frac{(\hbar c)^3}{\hbar\omega}\frac{k_\mathrm{B}T}{(mc^2)^2}. \tag{17}$$

We now take into account that the thermodynamic parameters T and n might vary according to the position assuming for the shift operator S the following form (cf. (12), (17) and (16)),

$$S = \sigma T_d(x) n_d(x) \left(\sum_{\omega\alpha} \frac{\hbar}{mc^2}\frac{1}{\omega} b_{\omega\alpha}^\dagger b_{\omega\alpha} - \sum_{\omega\alpha}\frac{1}{\omega^2}\right), \tag{18}$$

where $\sigma \doteq (\frac{1}{2}\pi)^{3/2}\alpha\hbar c(k_\mathrm{B}/m)$.

Remark 3. Form (18) is a consequence of the fact that the dependence of T and n on the position can be neglected if we calculate averages with respect to the plasma states (like we have done till now) but not if we want to calculate averages with respect to the e.m. field states. In fact the plasma particles can be considered as bounded in a volume of typical dimension equal to the mean free path while the probability to observe a monochromatic photon is constant all over the normalization volume.

Remark 4. Since S is by definition Hermitian, we should use a shift operator of the form $\frac{1}{2}(S + \text{h.c.})$. However, it is possible to show that this operator and the operator S defined by Eq. (18) only differ by a quantity that is negligible for $L \to \infty$.

4. The energy shift of a monochromatic photon

We now calculate the energy shift $\hbar\Delta\omega$ of a monochromatic e.m. wave of frequency ω and polarization α. This is, at the first order, the mean value of the shift operator with respect to the unperturbed (one-dimensional) state $\Phi_{\omega,\alpha} \doteq (1/\sqrt{L})\exp[i(\omega/c)x]\epsilon^{\alpha}_{\omega}$ of frequency ω and polarization $\epsilon^{\alpha}_{\omega}$. Using $b^{\dagger}_{\omega'\alpha}b_{\omega'\alpha}\Phi_{\omega,\alpha} = \delta_{\omega\omega'}\Phi_{\omega,\alpha}$ and $\Phi^{*}_{\omega,\alpha}\Phi_{\omega,\alpha} = 1/L$, we have

$$\hbar\Delta\omega = \int_L \mathrm{d}x\, \Phi^{*}_{\omega,\alpha} S \Phi_{\omega,\alpha} = \frac{\sigma}{L}\int_L \mathrm{d}x\, T_d(x) n_d(x) \left(\frac{\hbar}{mc^2}\frac{1}{\omega} - \sum_{\omega\alpha}\frac{1}{\omega^2}\right).$$

For $L \to \infty$, we can neglect $(\hbar/mc^2)(1/\omega)$ and replace $(1/L)\sum_{\omega'}$ with

$$\frac{1}{(2\pi)^{1/2}}\frac{1}{c}\int \mathrm{d}\omega.$$

With

$$\rho \doteq \left(\frac{2}{\pi}\right)^{1/2}\frac{1}{c}\sigma = \tfrac{1}{2}\pi\alpha\hbar\frac{k_B}{m}$$

we have

$$\hbar\Delta\omega = \rho\int_{-\infty}^{\infty} \mathrm{d}\omega\, \frac{1}{\omega^2}\int_{-\infty}^{\infty} \mathrm{d}x\, T_d(x) n_d(x). \tag{19}$$

Eq. (19) shows that the frequency of a monochromatic photon interacting with a gas of charged particles is shifted by a *negative quantity,* that is directly proportional to the integral of the product of the density and temperature of the gas calculated along the trajectory of the photon. We call this result the *redshift theorem.*

5. Comparison with experimental data

We now show that this dependence on the plasma parameters $n_d(x)$ and $T_d(x)$ fits very well the Pioneer 6 data.

Let r be the distance from the solar center, $R_\odot$ the solar radius, x the coordinate along the trajectory of the photon and d its impact parameter (with respect to the solar center), measured in solar radii. The temperature T can be considered roughly equal to 1.5×10^6 K, in all solar corona.

For $r/R_\odot \geqslant 2.5$ the following phenomenological formula (see, e.g., Ref. [29]) holds,

$$n(r) \simeq \left(\frac{10^2}{(r/R_\odot)^6} + \frac{1}{(r/R_\odot)^2}\right) n_0,$$
$$r/R_\odot = \sqrt{[(x/R_\odot)^2 + d^2]^2}, \tag{20}$$

where $n_0 = 10^6\ \mathrm{cm}^{-3}$. Then we have

$$\int_{-\infty}^{\infty} \mathrm{d}x\, T_d(x) n_d(x) \simeq T\int_{-\infty}^{\infty} \mathrm{d}x\, n_d(x) \simeq \pi R_\odot n_0 T\left(\frac{3}{8}\frac{10^2}{d^5} + \frac{1}{d}\right).$$

As a consequence, the frequency shift is

$$z_\omega \doteq \frac{\Delta\omega}{\omega} = -B_\omega\left(\frac{3}{8}\frac{10^2}{d^5} + \frac{1}{d}\right), \tag{21}$$

where

$$B_\omega = \pi^2\alpha\frac{k_B T}{m} n_0 R_\odot \frac{1}{\omega}\int_0^{\infty} \frac{1}{\omega'^2}\,\mathrm{d}\omega' \tag{22}$$

is a dimensionless constant, depending on a divergent integral [3]. This is not surprising, since we have performed a perturbative energy shift calculation that is affected by the usual QED divergences.

If and how is it possible to remove the divergence will be the subject of a future work. For the moment, we treat B_ω, given by the divergent expression (22), as a phenomenological parameter. Choosing this parameter as $B_\omega = 2 \times 10^{-7}$, we fit the whole set of experimental data of Pioneer 6 (let us stress that this result is obtained by the choice of a single phenomenological parameter).

This can be seen in Fig. 1, in which the expression

$$-z_d = 2 \times 10^{-7}\left(\frac{3}{8}\frac{10^2}{d^5} + \frac{1}{d}\right).$$

of the redshift is plotted versus the non-dimensional impact parameter d.

If we extrapolate Eq. (20) below 2.5 solar radii, we can obtain an estimate of the redshift of the spectral lines coming from the solar limb and from the center of the solar disk. Taking into account that a photon

[3] Notice that the main contribution to the shift comes from "light" particles; i.e., in the solar corona, from the electrons.

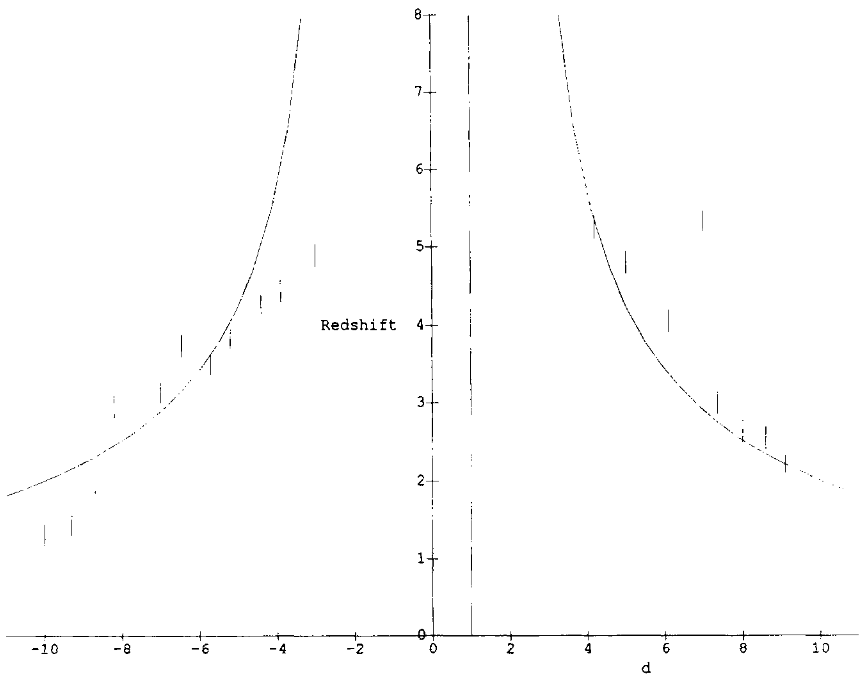

Fig. 1. Redshift (in units of 10^{-8}) versus the impact parameter d (in solar radii). The redshift data are taken from Refs. [13,14]; the error bars are realistic (as given by the quoted authors).

from the solar limb interacts only with half the solar corona, we have

$$z_\omega = -\tfrac{1}{2}B_\omega\left(\frac{3}{8}\frac{10^2}{d^5}+\frac{1}{d}\right)\bigg|_{d=1} = -3.8\times10^{-6}$$

(for a comparison, the gravitational shift is -2.16×10^{-6}, independently of the position on the solar disk). In a similar way, our formula gives, for the radiation coming from the center of the solar disk, the shift $z_\omega = -1.3\times10^{-6}$. Both these data are in good agreement with experiments (see Refs. [15,16]).

6. Some concluding remarks and open questions

(i) Our theory, though very rough and simple, predicts that a monochromatic e.m. wave, interacting with a low density plasma of charged particles, undergoes a frequency shift toward the red.

We point out that *the physical phenomenon that we consider has nothing to do with a Compton scattering or any other collisional process*. Compton scattering with the particles of the solar corona actually occurs, but gives a negligible contribution to the observable effects. In fact, taking into account the very low density of the solar corona (about 10^5 fermions/cm^3 at a distance from the sun of 4 solar radii) and the Compton cross section at low energies (6.5×10^{-25} cm^2), the photons which undergo a Compton scattering are less than 10^{-9} of the total incoming photons; moreover, they are completely randomized in direction and frequency so that they are scattered away from the light ray, escaping from observation.

(ii) In the present work we model the photon as a monochromatic stationary wave in an arbitrarily large box. This assumption completely delocalizes the photon along its trajectory and so it does not allow for a dynamical treatment of the problem. We suspect that the divergence in Eq. (22) is somehow related to this assumption, that we hope to overcome in a future work.

(iii) Eq. (19) shows that our shift is proportional to the integral of the product of the density and temperature of the gas calculated along the trajectory of the photon. This suggests a possible way to explain the discordant redshifts of physically correlated astrophysical sources (e.g. quasar–galaxy and galaxy–galaxy associations). Unfortunately, for giving a numerical estimate, we should have some more knowledge of the temperature and density of the plasma surrounding the quasars.

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