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Supporting Novice Mathematics Teacher Educators Teaching Elementary Mathematics Content Courses for the First Time

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Abstract: *In order to be effectively prepared by a teacher education program, prospective elementary teachers (PTs) need to experience high quality mathematics instruction in their mathematics content courses. The instructors of these courses typically consist of individuals (mathematicians and mathematics educators) with ranging experiences, from tenured faculty members to first-year assistant professors or graduate students. This paper explores how to support novice mathematics teacher educators (MTEs) who are teaching elementary content coursework for PTs for the first time. We detail and describe how to implement three systems for supporting novice MTEs: working with a mentor, being provided with educative curriculum materials, and working in a collaborative teaching environment. We close by discussing specific challenges associated with these supports, and call for more institutions to share how they have successfully implemented systems to support novice MTEs.*

Keywords: elementary mathematics teacher educators, mentoring, educative curriculum, collaborative teaching

Introduction

In order to improve the quality of mathematics education, we must improve the quality of mathematics preparation of prospective elementary teachers (PTs). However, because the knowledge, backgrounds, and expertise of mathematics teacher educators (MTEs) varies

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substantially even within a single elementary teacher education program, this is not an easy feat. Indeed, the population of MTEs includes tenured/tenure track faculty members, instructors, and graduate students, many of whom do not have experience teaching young children themselves (Masingila, Olanoff, & Kwaka, 2012).

Teaching mathematics to PTs is unlike teaching mathematics to non-PTs (e.g., students enrolled in mathematics courses who do not plan on becoming classroom teachers) for a number of reasons. We will revoice two primary differences that have been noted by other researchers (Welder, Appova, Olanoff, Taylor, & Kulow, 2016; Zopf, 2000). First, the content differs in important yet subtle ways. Non-PTs and PTs may need to learn mathematical content, but PTs need to learn a specialized form of mathematical knowledge, namely *mathematical knowledge for teaching* (MKT) (Ball, Thames, & Phelps, 2008) in order to support diverse learners once they enter the field.

Second, while non-PTs do not possess substantial knowledge of the mathematical content which they are learning (Zopf, 2000), PTs typically have already had extensive exposure to the mathematical content they are learning and often think they already know all they need to know to teach. For example, they have mastered standard algorithms but are unable to explain why they work (Ball, 1988; Ma, 1999; Thanheiser, 2009). Consequently, MTEs face the unique challenge of preparing PTs to teach content that the PTs themselves believe they already know (Thanheiser, 2018). Therefore, effectively preparing PTs includes addressing their beliefs about what it means to know mathematics, to learn mathematics, and to teach mathematics for *understanding*.

Because PTs are a unique population of students who are (re-)learning elementary mathematics content (see Castro Superfine, Prasad, Welder, Olanoff, & Eubanks-Turner, 2020,

this issue; Johnson & Olanoff, 2020, this issue) that they believe they already know, it can be challenging to support MTEs, especially novice MTEs teaching a mathematics content course for the first or second time. Indeed, a majority of novice MTEs feel unprepared and report a lack of training, resources, and support at their institutions (Goodwin et al., 2014; Masingila et al., 2012). Contrary to these findings, the first and second authors of this paper felt very well prepared when teaching a content course for the first time, in spite of having no prior experience teaching young children or working with PTs. Moreover, the third and fourth authors of this paper (as experienced MTEs) have had great success supporting novice MTEs at their own institutions. From both perspectives, we identified three overarching support factors that contribute to this success: 1) productive mentorships, 2) educative curriculum materials, and 3) collaborative teaching environments. In this paper, we elaborate on each of these three support factors, particularly for supporting the work of novice MTEs, and offer a discussion of additional ways in which we (the authors of this chapter) implemented these support factors in our own professional practice as MTEs.

We organized this paper by first reviewing the literature on the three support factors and how they contribute to the developing expertise of novice MTEs. Second, we drew upon our own experiences (as MTEs) at various institutions to describe practical ways to implement these support factors in the profession. Our hope is that other MTEs in the field will find this helpful and be able to use our detailed success stories to support their own work and the work of novice MTEs at their own institutions.

Supporting Novice Mathematics Teacher Educators: Background

Much has been written in the mathematics education literature about how to support novice classroom teachers. However, we know much less about how to effectively support and

prepare MTEs (Even, 2008; Goos, 2009; Masingila et al., 2012; Zaslavsky & Leikin, 2004). Here we describe three primary systems embedded in some teacher education programs to support novice MTEs. Specifically, we review research on mentorships, designing educative curricula, and teaching in collaborative environments.

Productive Mentorships

While there are many benefits to working with a mentor (e.g., Hobson, Ashby, Malderez, & Tomlinson, 2009; McIntyre & Hagger, 1996), many novice MTEs report having unsuccessful relationships with their mentor or not having a mentor at all (e.g., Masingila et al., 2012; Yow, Eli, Beisiegel, McCloskey, & Welder, 2016). For example, of the 69 novice MTEs that Yow et al. (2016) surveyed, 55% of participants were not assigned a mentor and 20% were assigned a mentor who was not helpful. Furthermore, according to Masingila and colleagues (2012), very few institutions have regularly scheduled meetings for novice MTEs and even fewer institutions allow novice MTEs to co-teach or observe another MTE teaching content courses, even though the benefits of observing and debriefing have been shown to be highly valuable for novices (Hobson et al., 2009). Other studies show similar trends that display the variability in the effectiveness of mentoring (e.g. Hardy, 1999; Oberski, Ford, Higgins, & Fisher, 1999; Smith & Maclay, 2007). In some cases, a mentorship can even have a negative effect on novice MTEs as a result of the power imbalance between the novice and experienced MTE (e.g., Beck & Kosnick, 2000). Therefore, it is vital to establish productive mentorships to effectively support novice MTEs.

To facilitate productive mentorships, the interactions between novice MTEs and the experienced mentors should be focused around developing *mathematical content knowledge for teaching teachers* (MKTT) (Castro Superfine et al., this issue; Masingila, Olanoff, & Kimani,

2018; Superfine & Li, 2014; Zopf, 2000). MKTT includes the specialized form of content knowledge that MTEs need in order to effectively prepare PTs (Superfine & Li, 2014), analogous to MKT (Ball et al., 2008). One way to help novice MTEs develop MKTT is to allow them to observe an *experienced* MTE teaching a content course—one who has substantial MKTT, scholarly activity in the field of mathematics education, and either has experience teaching young children or has worked closely with young children in some way (similar to the way Appova and Taylor (2017) have defined an "expert MTE"). Allowing novice MTEs to observe an experienced MTE provides opportunities to develop their own understanding of how to best support PTs, engage PTs in mathematical activities appropriate for targeting mathematical knowledge for teaching (Ball et al., 2008), and motivate PTs to learn mathematics for understanding (Masingila et al., 2018). Accompanying these observations with deliberate reflections about what the novice MTEs notice while observing an experienced MTE teach provides a space for novice MTEs to consider how they will teach their own class, what instructional moves the experienced MTE uses to engage PTs, and how PTs engage in learning (Masingila et al., 2018).

Masingila, Olanoff, and Kimani (2018) discuss how they were able to form an effective mentoring group as an experienced MTE (Masingila) worked with two graduate students (Olanoff and Kimani). By allowing Olanoff and Kimani to observe Masingila teach every lesson before they taught the same lesson themselves, Olanoff and Kimani were able to watch how an experienced MTE engages PTs in developing MKT. In addition, the three MTEs met weekly to facilitate discussions around what they were noticing in their observations of Masingila and were given opportunities to reflect upon their observations and think about how they would implement similar pedagogical moves in their own classes.

Educative Curriculum Materials

In some teacher education programs, departments have attempted to support novice MTEs by choosing to use or design *educative* curricula (Ball & Cohen, 1996; Davis & Krajcik, 2005; Suppa, 2018). An educative curriculum regards the MTE as a learner when engaging with the materials. Ball and Cohen (1996) argued that curriculum materials "could be designed to place [classroom] teachers in the center of curriculum construction and make teachers' learning central to efforts to improve education" (p. 7). Analogously, we argue that curriculum materials for MTEs could be designed to place MTEs at the center of curriculum design and construction so that MTEs are provided with opportunities to learn about mathematical content for these courses and about PTs.

As such, educative curricula should be transparent in their rationales and help MTEs understand the intentions of curriculum authors (Stein & Kim, 2009) to then make informed curricular decisions to meet the unique needs of PTs. In addition, "curriculum materials could help [MTEs] learn how to anticipate and interpret what [PTs] may think about or do" (Davis & Krajcik, 2005, p. 5). Thus, the purpose of creating and/or using educative curricula is to help MTEs learn more about the mathematical content of the course, the ways PTs are expected to engage with the content, the purpose of this engagement, and pedagogical ways to structure the classroom environment. Consequently, we hypothesize that educative curricula would support novice MTEs to acquire MKTT because of the inclusion of rationales, anticipated PT comments, questions, solution strategies, and suggested ways to respond to PTs (Davis & Krajcik, 2005). These kinds of features are hypothesized to teach the MTE about the ways in which PTs are expected to engage in learning the content and how the MTE can support PTs during this engagement.

Building on these hypotheses, several studies have begun investigating educative curricula. Suppa (2018) found that novice MTEs used an educative curriculum to help them learn about the purpose of each lesson, deeply understand the mathematical content, and learn how PTs might respond to the lesson. This particular curriculum included detailed rationales, precise learning goals, specific anticipations for what PTs might think and do, and suggested responses for MTEs during those moments. The two MTEs identified all of these curriculum features as helpful in supporting their learning of the purpose of each lesson, the intentions of the curriculum authors, and more information about PTs and the mathematical content of each lesson. Suppa (2018) found that the MTEs used specific PT anticipations during enactment as evidenced by their understandings of PT work during enactment and the ways they responded to PTs during class. This study suggests that educative curricula have the potential to effectively support MTEs to develop MKTT.

Collaborative Teaching Environments

A collaborative teaching environment can also support novice MTEs to feel prepared when teaching PTs for the first time (Gallimore, Ermeling, Saunders, & Goldenberg, 2009; Hiebert, Morris, & Glass, 2003; Lampert & Grazianic, 2009; Shaughnessy, Garcia, Selling, & Ball, 2016). Some examples include setting up frequent meetings with colleagues (Gallimore et al., 2009); sharing curriculum and/or assessment materials (Hiebert et al., 2003; Lampert & Graziani, 2009; Morris & Hiebert, 2011); holding common assessments for all PTs; creating and agreeing upon course objectives or learning goals (Morris & Hiebert, 2011); developing syllabi collaboratively; and developing activities collaboratively to address certain learning goals (Beam & Kuennen, this issue; Bryk, Gomez, Grunow, & LeMahieu, 2015; Lewis, Perry, & Hurd, 2009; Morris & Hiebert, 2011).

Collaborating with colleagues can help to alleviate feelings of isolation (e.g., Bryk et al., 2015; Heider, 2005) and ease the transition into a new role as an educator (Golde, 2006). Teaching in a collaborative environment provides novice MTEs a space to freely ask questions and seek help in navigating their new roles. Unfortunately, Masingila et al. (2012) report that very few schools (less than 17% of the nearly 2000 institutions of higher education surveyed) have regularly scheduled meetings or ongoing discussions with MTEs about teaching content courses.

Collaboration that will likely be most successful in supporting novice MTEs should include very focused, deliberate, rich discussions around the mathematical content and pedagogical skills that support PTs in a mathematical content course. For example, Applegate, Dick, Soto, and Gupta (this issue) found that through collaboration and rich discussions of teaching PTs specific lessons on multiplication, the MTEs in their lesson study group gained a deeper understanding of multiplication and increased their own MKTT. Indeed, Yow et al. (2016) surveyed 69 recently graduated doctoral students as new faculty members teaching mathematics content courses for PTs and found that many of these novice MTEs "wished for more conversations about the intricacies of teaching a course, such as syllabus development and discussion facilitation" (p. 63). As Masingila et al. (2012) explain, "unless instructors have been put in situations where they have taught elementary mathematics, they have likely not thought about these mathematical ideas since they themselves were students in elementary school; and when these instructors were elementary school students, they were likely not thinking about these ideas in a deep way" (p. 350). Thus, the participants in such discussions should include at least one knowledgeable MTE who has deep knowledge of the ways in which young children learn mathematics, the ways elementary classroom teachers are expected to engage young

children in learning according to recent reform efforts, and the mathematical and pedagogical content knowledge that aligns with recent reform ideologies and visions. This kind of knowledge will help to promote MKTT during their discussions with novice MTEs.

The results from Yow et al. (2016) and Masingila et al. (2012) indicate that novice MTEs would feel better prepared if they had productive mentorships and had been given opportunities to collaborate with experienced MTEs regarding the specifics of teaching the content course. In addition, Suppa's (2018) results indicate that creating and improving educative curriculum materials for MTEs can help novice MTEs develop MKTT, making them feel more prepared by understanding the intentions of the lessons in a content course and anticipating PTs' questions, solution strategies, misconceptions, and comments.

The purpose of this paper is to describe how to implement these three supports effectively according to our own experiences as both novice and experienced MTEs at various institutions. The literature provides explanations for *why* these three systems can, and in some cases do, support novice MTEs but does not unpack *how* to implement these three systems. It is our hope that providing concrete examples from our own experiences involving effective mentorships, educative curriculum materials, and collaborative teaching environments will help other MTEs and institutions effectively implement these supports as well. We describe how these supporting factors can be successfully implemented according to our own experiences next.

Supporting Novice Mathematics Teacher Educators: Our Experiences

Productive Mentorship: The Power of Classroom Observations

A key support for novice MTEs is establishing a productive mentor-mentee relationship that focuses on developing MKTT (Masingila et al., 2018; Superfine & Li, 2014; Zopf, 2000). The purpose of this relationship is for the mentee (the novice MTE) to learn first-hand from the

mentor (the experienced MTE) about the mathematics content and pedagogy, as well as the student population in teacher preparation courses. Implementing a successful mentor-mentee relationship goes far beyond simply establishing one (Yow et al., 2016). Carefully planned activities must define this relationship such that the mentee can learn from the mentor to be an effective MTE. These activities include ongoing classroom observations of the mentor by the mentee and scheduled opportunities to debrief and discuss what was observed (for example, see the lesson study process of Applegate et al., this issue).

Structuring the mentoring relationship around classroom observations is an effective way for novice MTEs to discover aspects of teaching for which they need mentoring (Hobson et al., 2009; Masingila et al., 2018). We contend that these observations should be guided by observational protocols that focus attention on the specific content and pedagogy of the course, as well as on the unique motivational needs of PTs in attendance. Without any experience working with PTs, novice MTEs might not yet understand the nuances of teaching elementary mathematics content for understanding (Masingila et al., 2012), nor the ways to motivate PTs to meaningfully engage with it. We discuss how our observation protocol helps to focus novice MTEs' attention on content, pedagogy, and PT motivation next.

As mentees, novice MTEs should pay careful attention to the mathematics content being taught by their mentor during classroom observations (Masingila et al., 2018). As part of our observation protocol, novice MTEs note the specific learning goals of the lesson, as well as the mathematical ideas that are introduced and drawn upon to help PTs achieve those goals (see Table 1). It is important to prompt novice MTEs to notice how concepts and procedures are incorporated in a lesson. PTs are expected to understand why procedures make sense, rather than just execute a rehearsed procedure or algorithm. These classroom observations give mentees an

opportunity to think about how concepts, procedures, and prior knowledge can work together to help PTs create meaning around elementary mathematical content.

Table 1

Novice MTE's Observations of Mathematical Content

Observation Protocol: Guiding Questions
What are the learning goals for the lesson? What mathematical concepts are important for PTs to understand to help achieve the learning goals? What mathematical procedures are important for PTs to understand to help achieve the learning goals? What prior mathematical knowledge is important for PTs to engage in during the lesson? What mathematical ideas do you think were missing from the lesson? What questions do you have about the mathematical content from this lesson?

In a debriefing session after a classroom observation, mentees can use their specific content observations to start conversations with their mentor about the mathematics from the lesson (Hobson et al., 2009). In some cases, mentors might ask mentees if they had any questions about understanding the content themselves, especially since many novice MTEs' past exposure to elementary mathematics topics may have been largely procedural, so the idea of entire courses being dedicated to unpacking the concepts of elementary mathematics may seem foreign to them. These debriefing sessions should occur as soon as possible after the classroom observation occurs in order to help the MTEs remember the intricacies of what transpired in the classroom.

Novice MTEs should also observe the specific pedagogy that is used in these courses (Masingila et al., 2018). Our observation protocol focuses a mentee's attention on how the mentor's pedagogy helps PTs engage with and learn the mathematics (see Table 2).

Table 2

Novice MTE's Observations of Mathematical Pedagogy

Observation Protocol: Guiding Questions
How was the content of the lesson taught?
What specific teacher moves were used by the instructor?
What activities were incorporated in this lesson?
In what ways were the PTs engaging with the mathematics in this lesson?
How do you think the teaching of this lesson helped the PTs learn?
What teaching practices do you think were missing from the lesson?
What questions do you have about the mathematical pedagogy from this lesson?

In order to develop MKT, PTs are expected to unpack and explain their reasoning. Thus, mentees are required to provide opportunities for PTs to develop conceptual understanding in their coursework. Much of developing conceptual understanding in mathematics boils down to having opportunities to productively struggle with challenging ideas and having those ideas made explicit afterwards (Hiebert & Grouws, 2007). Supporting PTs in this way can be difficult for novice MTEs because they have had little or no prior experience teaching in this context. Furthermore, the teaching experiences they may have had (e.g., high school mathematics or undergraduate mathematics for non-PTs) were likely different from what and how they are expected to teach in a PT content course.

During a debriefing session after a classroom observation, experienced MTEs should be ready to engage in a discussion with the novice MTE about the importance of providing PTs opportunities to productively struggle with the content (Hobson et al., 2009). Furthermore, mentors can share specific strategies for how to support and facilitate productive struggle in their classrooms. Such teaching practices can include mindfully structuring mathematical talk around a classroom activity (Kazemi & Hintz, 2014). This can start with private think time, such that all PTs can engage with a task before it is discussed. Then, each PT can share in small groups for a

designated amount of time (about 1 minute per PT). Once all PTs have shared their thinking, the group can create a joint solution and discuss it with the class. The design and implementation of the activity itself can also be structured in ways to encourage productive struggle (Thanheiser et al., 2016; Tobias et al., 2014). Activities should encourage familiar ways to initially engage with the mathematics, only to grow in complexity as the task unfolds (DiNapoli & Marzocchi, 2017). MTEs can scaffold PTs' engagement as the task becomes more demanding by asking timely, non-leading questions about how they are thinking about what they have accomplished thus far and their plans for moving forward (Arbaugh & Freeburn, 2017).

Finally, novice MTEs should recognize the unique motivational needs of the student population taking these courses (Thanheiser, 2018). Our observation protocol incorporates the ways in which PTs are motivated (or not) to learn (see Table 3). In their content courses, PTs often believe they already know all the mathematics needed to teach elementary school (Thanheiser, 2009, 2010, 2018). In reality, most PTs have little conceptual understanding of this content and even less perspective on how young children learn mathematics for understanding. Thus, these beliefs pose a motivational hurdle for MTEs. With their lack of experience in elementary content courses, novice MTEs especially may struggle to understand ways to motivate PTs in this setting.

Table 3

Novice MTE's Observations of PTs' Unique Motivational Needs for Learning

Observation Protocol: Guiding Questions
What aspects of the lesson seemed to motivate PTs to learn? What aspects of the lesson did not seem to motivate PTs to learn? What motivational practices do you think were missing from the lesson? What questions do you have about how to best motivate PTs to learn from this lesson?

After the classroom observation, the debriefing session is a vital space for the mentor and mentee to discuss the motivational strategies that were used (Hobson et al., 2009). One effective strategy that we have used as experienced mentors is to briefly talk one-on-one with PTs about the mathematics content of the course, highlighting to them that they can execute the algorithms but not explain them (Thanheiser, Philipp, Fasteen, Strand, & Mills, 2013). This is essential because children today are expected to learn how to explain algorithms, for example, using a visual model to explain why the common denominator algorithm for adding or subtracting fractions works (see Standard CC.5.NF.2, Common Core State Standards—Mathematics, 2010). Helping PTs realize that they may only know how to execute the algorithm, but not the details of why it works, can be a practical motivator because they can envision using such conceptual knowledge in their future careers.

Related, another effective motivational strategy for MTEs is to connect PTs to young children in the context of learning mathematics. Activities that encourage PTs to apply what they have learned in the course to working with children, such as organizing and attending a Family Math Night (see Appendix A for sample information flyer), can give PTs first-hand experience into how children learn mathematics. Engaging with children to learn mathematics for elementary classroom teaching can be an authentic learning experience for PTs and serve to reconceptualize mathematics learning as enjoyable (see Thanheiser, Philipp, & Fasteen, 2014).

Another way of connecting PTs and children is integrating articles published for classroom teachers in journals such as *Teaching Children Mathematics*, *Mathematics Teaching in the Middle School*, and *Mathematics Teacher: Learning and Teaching PK-12* (to appear in 2020) into the content course (Strand & Thanheiser, 2017). Assigning and discussing readings like “Techniques for Small Group Discourse” (Kilic et al., 2010), “Multicultural Mathematics

and Alternative Algorithms" (Philipp, 1996), and "Tuheen's Thinking about Place Value" (Wickett, 2009) can help motivate PTs to better understand the child's point of view around learning mathematics for understanding. Specifically, these three pieces have been used to increase PTs' understanding of the rationale behind the teaching methods commonly used in their classes, to challenge PTs' beliefs about mathematics, and to address mathematics content via children's thinking, respectively (Strand & Thanheiser, 2017).

Educative Curriculum Materials: Identifying Key Features of a Lesson

Providing educative curriculum materials is another key support for novice MTEs (Suppa, 2018). The purpose of such materials is for the novice MTE to develop MKTT through preparing and engaging with the materials (Ball & Cohen, 1996). To develop MKTT, curriculum materials must support novice MTEs' learning around the nuances of the mathematical content, as well as the pedagogical practices that support PTs' learning. To do so, the curriculum should contain key features, such as precise learning goals of the lesson, detailed rationales for activities, and anticipated PT questions, comments, and misconceptions alongside helpful ways to respond (Davis & Krajcik, 2005; Suppa, 2018). For example, as novice MTEs, the first and second authors used annotated lesson plans designed and continuously improved by expert MTEs at the University of Delaware that contained these features. We share ideas from one of these lesson plans below (Lesson X, an introduction to decimal multiplication; see Appendix B for complete lesson plan[1]).

Precise learning goals for Lesson X, as well as evidence for ways PTs achieved these learning goals, are important features of this educative lesson plan because they gave us a clear

[1] This lesson is intended for PTs who have already demonstrated understanding about place value, addition/subtraction with whole numbers and decimals, and multiplication of whole numbers. Lesson X is one example of a lesson plan coming from a first course of three on elementary mathematics for PTs. These lesson plans have undergone years of lesson study and continuous improvement, thus offering research-based content and pedagogy foci for effective learning. Note that throughout Lesson Plan X, "PST" refers to prospective teacher (PT).

picture of what a successful lesson should accomplish (see Table 4). By specifying the goals and evidence for learning, the lesson plans helped us understand nuances of multiplication in this context. We realized that an important aspect of multiplication is understanding that the meaning of multiplication changes slightly when the multiplier changes from a whole number to a rational number less than one. Also, the inclusion of evidence of PT learning helped us know exactly what to look for as the lesson unfolded.

Table 4

Learning Goals and Evidence of Learning for Lesson X: Introduction to Decimal Multiplication

Learning Goal	Evidence of Learning
PTs will understand the meaning of the number sentence a x $b = c$.	PTs will explain that a is the number of groups, b is the size of each group, and c is the whole.
PTs will be able to use the "of" interpretation of the multiplication symbol.	PTs will understand, for instance, that 0.7 x 8 means "Find seven tenths of 8" OR "Find seven groups of one-tenth of 8" OR "Find seven one-tenths of 8."
PTs will use the meaning of multiplication to model multiplication number sentences with decimal multipliers less than one.	PTs will solve multiplication number sentences with graph paper models. In these models, they will give explanations that explain in detail each of the relevant steps and concepts (see Appendix B for sample explanation)

While precise learning goals help MTEs to understand the overall purpose and aims of a lesson, anticipated PT reactions and questions help MTEs to prepare responses to PTs during enactment while maintaining a focus on the learning goals for the lesson. Indeed, due to inexperience, interpreting and productively responding to PTs' questions and concerns in-the-moment can be particularly challenging for novice MTEs. By including common reactions from

past PTs' engagement with the activity in the curriculum, novice MTEs can prepare for these conversations and use the provided, evidenced-based MTE responses to help them during those challenging moments. For example, in Lesson X, one particular area of concern is helping PTs develop a flexible concept of one whole as they work on the Meaning of Decimal Multiplication Activity. For the problem 0.1 x 2 = ?, or "find one-tenth of two," PTs have commonly represented the multiplicand with two separate drawings of regions with areas of 1 unit^2 and operated on them separately (see Figure 1, left). This operating-on-each-measuring-unit-separately approach is described in the lesson plan and, although mathematically valid, is discouraged (see Table 5). This strategy is problematic because PTs often memorize the mechanics of operating on separate measuring units and skip the conceptual steps of operating upon one general whole, which is essential in conceptual development of operations with fractions. Instead, instructors are encouraged to help PTs represent the multiplicand as one whole region with an area of 2 unit^2 (see Figure 1, right) and the lesson plan includes possible ways to respond to PTs who use the former strategy (see Table 5).

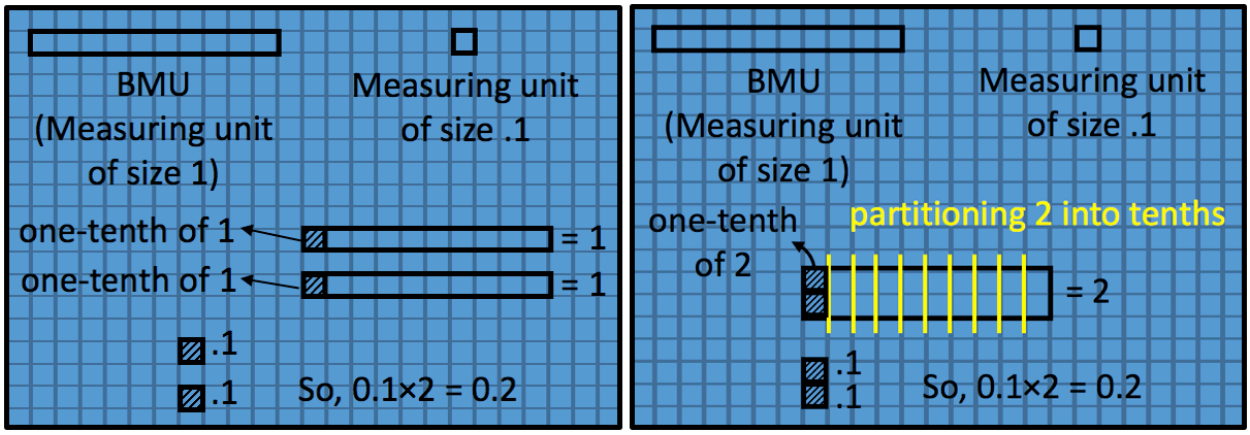

Figure 1: PT-generated area models for the number sentence $0.1 \times 2 = ?$.

Table 5

Anticipated PT Action and Possible MTE Response for Finding One-Tenth of 2

PT Action	MTE Response
PTs draw 2 wholes to represent 2. To find one tenth of 2, they shade one tenth of one whole and then shade one tenth of the other whole. The answer is then readily apparent. It is 0.1 (the shaded one tenth of the whole is a measuring unit of size 0.1) + 0.1 = 0.2.	At this point, discourage this approach (the "operating-on-each-measuring-unit -separately" approach). The PTs should be able to consider 2 as the whole [the 20 squares as a whole] and be able to find one tenth of the whole [be able to break the entire 20 squares into 10 equal parts of 2 squares each]. They should be able to directly model the meaning of the number sentence ("find one tenth of 2" as opposed to "find one tenth of 1 and find one tenth of 1").

These kinds of anticipations and rationales for why this strategy should be discouraged are vital for novice MTEs because we did not have the foresight to see *why* modeling decimal multiplication in this way could be problematic for developing meaning around fractions. This example of an anticipated PT response, strategy, and instructor response in Lesson X helped us, as novice MTEs, prepare for such an outcome and carry out the activity in ways to help PTs achieve the current (and future) learning goals.

We would like to reiterate that the lesson plan described here was created by and for MTEs at the University of Delaware. The kinds of educative features that now exist in this curriculum were developed gradually by refining and improving the curriculum materials for over a decade. This specific curriculum was so effective in preparing the first and second authors to teach PTs because, in part, it richly considered the local learning environment; the educative

features were so specific to the context at the University of Delaware, the population of PTs at this institution, and the population of MTEs teaching these content courses.

At this time, we are unaware of commercial curricula that possess these types of features at a rich enough level to be considered "educative" for MTEs. However, we refer readers to Kuennen and Beam (2020, this issue) for ideas on how to deeply unpack a specific task and Applegate et al. (2020, this issue) for ways to systematically study and gradually improve tasks. If rationales and anticipations for how PTs will engage with specific tasks are included in a *written artifact* that can be given to novice MTEs (and thus serve as a written curriculum—even if only partial), this curriculum would then be educative for the novice MTE. Therefore, we encourage MTEs to initiate and implement a system of continuous improvement or lesson study (Applegate et al., 2020, this issue; Bryk, 2015; Bryk et al., 2015) at their own institutions using their current curriculum materials to gradually refine and improve the materials to include these kinds of curricular features (e.g., precise learning goals, detailed rationales, anticipated PT questions and comments, and suggested MTE responses). Furthermore, we believe that improving curriculum materials is done more effectively working in a collaborative teaching environment, which brings us to our third support factor.

Collaborative Teaching Environment: The Importance of Regular (Weekly) Feedback

A third effective factor for supporting novice MTEs involves establishing a collaborative teaching environment, a commonly absent practice at many institutions (Masingila et al., 2012). In addition to sharing course materials (e.g., lesson plans, assignments, assessments) amongst all MTEs, we contend that structuring a collaborative environment around weekly MTE meetings is paramount for the development of novice MTEs. The purpose of these meetings is for MTEs to look back and look ahead – to share about the effectiveness of lessons just enacted and to discuss

expected challenges of upcoming class sessions. When MTEs share the same course materials and goals, these weekly meetings make it easier for novice MTEs to reflect on past experiences, understand the goals and challenges of upcoming lessons, and plan ahead for future instruction. Thus, it is important for these instructor meetings to be guided by a shared agenda that focuses on collaboratively looking back and ahead to help all MTEs participate in developing MKTT (Masingila et al., 2018).

Well-designed meetings give novice MTEs an opportunity to reflect on a previously taught lesson and seek feedback from other MTEs who recently enacted the same lesson (see Applegate et al., this issue). While looking back, novice MTEs should evaluate the effectiveness of key aspects of the enacted lesson plan (see Table 6). This includes reflecting on how well PTs were ready to engage with the learning goals of the enacted lesson, the ways in which PTs demonstrated their understanding of those learning goals, and the pedagogy used to help PTs achieve those learning goals. Also, the discussion of these reflections may result in annotating a lesson plan(s) to potentially make the materials more educative for future novice MTEs.

Table 6

Looking Back: Reflecting on an Enacted Lesson

Focus of Reflection	Guiding Questions
Readiness of PTs	Were the PTs ready to engage with the lesson's content? How do you know?
	What homework questions did PTs have?
	About what past content are PTs still confused?
Demonstrated Understanding of Learning Goals by PTs	In what ways did the PTs demonstrate their understanding of each learning goal?
	In what ways did the PTs demonstrate their misunderstanding of each learning goal?
Pedagogy of MTE	What pedagogy was effective in helping PTs achieve each learning goal?
	What pedagogy was ineffective in helping PTs achieve each learning goal?
Annotations of Lesson Plan	What would you add, remove, or edit about the lesson plan based on your experience enacting it? Why?

While reflecting on an enacted lesson, novice MTEs must consider how ready the PTs were to engage with the lesson's content. Novice MTEs can take note of the questions PTs ask and/or the ways in which PTs draw upon prerequisite knowledge during the lesson. For example, for an introductory lesson on multiplication of decimals, PTs must have an understanding of the "groups of" meaning of multiplication as well as the multiplicative relationship between digits in adjacent places of a decimal number. If PTs are still struggling to make sense of what multiplication means or the relationship between one-whole and one-tenth, for example, then perhaps they are not quite ready to explore the ideas of decimal multiplication. Weekly instructor meetings provide a perfect setting for consistent discussion of PT readiness. New MTEs may

struggle to see the intricate connections between mathematical content and how PTs' understanding of one idea influences their readiness to engage with another (Masingila et al., 2012). Novice MTEs can share what they noticed from their enactment, and more experienced MTEs can help steer the discussion toward key concepts that PTs must know to indicate their readiness. Plus, any consensus around ways in which PTs were not ready to engage with a lesson's content informs potential annotations to that lesson plan or an earlier one.

After teaching a lesson, one of the most important things to consider is if PTs demonstrated understanding of the mathematics content. Thus, as novice MTEs reflect on a lesson's enactment, they must determine the extent to which PTs achieved the learning goals, and the effectiveness of the pedagogy used to help PTs achieve those goals. As mentioned in the previous section (see Table 4), educative lesson plans should describe what specific evidence of PTs' understanding looks like (Davis & Krajcik, 2005). However, it may be difficult for novice MTEs to recognize such evidence in practice due to their limited experience working with PTs.

For instance, in Lesson X, an important learning goal for PTs is to use the "of" interpretation of the multiplication symbol. A common misconception among PTs is that multiplication always results in a bigger product, which interferes with their understanding when the multiplier is less than one (Olanoff, Lo, & Tobias, 2014). The lesson plan states that PTs can demonstrate evidence of their understanding by interpreting the number sentence 0.7 x 8 as "find seven tenths of 8." Novice MTEs may struggle, though, to distinguish a conceptual understanding from a perfunctory understanding of the "of" interpretation of the multiplication symbol. It is possible that PTs will write the correct interpretation of the number sentence, but fail to realize that the product of 0.7 and 8 will be smaller than the multiplicand, 8. Thus, this is an important topic of discussion for a weekly instructor meeting. Novice MTEs can describe

exactly how PTs were showcasing their understanding of this learning goal, and more experienced MTEs can ask for more convincing evidence, if necessary. Further, novice MTEs can share the teacher moves they chose to support PTs' achievement of this learning goal, and experienced MTEs can contribute their own pedagogical practices applied in the same activity to offer their expertise. As always, any consensus as a result of these discussions can result in annotating the curricular materials to potentially improve the effectiveness of the lesson.

It is just as important to collaboratively anticipate challenges for an upcoming lesson as it is to collaboratively reflect on an enacted lesson. While looking ahead, novice MTEs should apply what they know about their PTs to predict particular teaching and learning obstacles that may arise in an upcoming lesson (see Table 7).

Table 7

Looking Ahead: Anticipating Challenges for Enacting a Lesson

Anticipated Challenge	Guiding Questions
Learning Goals and Connected Mathematics	Which learning goals may be particularly challenging for PTs to achieve? Why?
	What previously-encountered mathematical ideas are connected to these learning goals? How?
Anticipating What PTs Will Do and How to Respond	In what ways do you predict PTs will engage with the mathematics in the lesson?
	What questions do you predict PTs will ask as they engage with the mathematics in the lesson?
	In what ways could you respond to PTs' questions as they engage with the mathematics in the lesson?
Annotations of Lesson Plan	What would you add, remove, or edit about the lesson plan as you prepare to enact it? Why?

This includes taking the time to understand how the learning goals of the lesson build off of previously encountered mathematical content and anticipating what PTs will do and how to respond.

As part of our weekly instructor meeting agenda, novice MTEs should try to predict some specific challenges associated with PTs achieving the learning goals in the upcoming lesson. It may also help for novice MTEs to think about the ways in which previously-encountered mathematics content exists within these learning goals. All MTEs should draw on their experiences teaching PTs to make educated guesses about the learning goals that will be most challenging. But, by definition, novice MTEs have little experience working with PTs and may not be able to foresee potential obstacles. It can be helpful to think about learning goals as a collection of mathematical ideas, with past concepts applying to and informing current concepts.

For example, consider the third learning goal listed for Lesson X: PTs will use the meaning of multiplication to model multiplication number sentences with decimal multipliers less than one (see Table 4). This learning goal contains several mathematical ideas, including the meaning of multiplication, the meaning of decimal numbers, the meaning of numbers between zero and one, and sketching models of multiplication. Undoubtedly, previous lessons have contained learning goals associated with each one of these mathematical ideas, and novice MTEs have had experiences facilitating activities in previous lessons with these PTs that explore these mathematical ideas. Therefore, novice MTEs can draw on their experiences working with these PTs around previously encountered mathematics to inform their predictions about the ways in which PTs may struggle with achieving the current learning goals.

Unpacking how current mathematics content is informed by past mathematics content is not easy, and thus discussing these mathematical relationships in a collaborative instructor

meeting is encouraged. More experienced MTEs can help novice MTEs understand these mathematical relationships and nuances and how they apply to making predictions about PTs and their achievement of learning goals. In some cases, the discussions around predicting these specific challenges can lead to changing the curriculum materials somehow, perhaps to include more transparent details about how different mathematical ideas in the course are connected.

Anticipating what PTs will do and how to respond is one of the most important aspects of planning to teach a lesson (Suppa, 2018). Due to inexperience, novice MTEs can find it difficult to predict the specific ways PTs will try to solve a problem. Related, novice MTEs may be caught off-guard when PTs ask unanticipated questions. Therefore, it's important to discuss all the potential ways PTs may attempt to solve a problem so more experienced MTEs can share their insight. As mentioned in the previous section, educative lesson plans should include anticipated PT responses to all activities, as well as corresponding MTE responses. However, due to lack of experience, novice MTEs may have difficulty interpreting these anticipations, requiring help from experienced MTEs. After discussion, adding specific anticipated PT actions/questions with corresponding MTE moves to the lesson plan is encouraged.

Conclusion

In this paper, we have unpacked three support factors that contribute to the development of novice MTEs. We have included specific recommendations from the literature and detailed descriptions of our own success stories on the implementation of these factors when teaching content courses to prospective elementary classroom teachers. Overall, we argue that consistently observing a mentor (expert) MTE in the classroom supports the development of a novice MTE, particularly by focusing on the ways mathematical content, pedagogy, and PTs' motivations are addressed in the lesson enactments. Second, discussing the key features and

teaching practices embedded into the educative curriculum materials supports novice MTEs by making clear the learning goals, rationales, and anticipated PT questions within each lesson. Finally, we suggest incorporating regular (weekly) instructor meetings to offer an environment in which novice MTEs can learn from collaborative reflections on the enacted lessons and be able to brainstorm and anticipate specific challenges for upcoming lessons as well.

Although we have offered specific methods by which we implemented these three support factors in our own practices (within various institutions), we would like to mention that they did not come without challenges. Establishing effective mentorships and norms for observing mentors while teaching is a nontrivial task that requires a knowledgeable mentor and frequent meetings with a deliberate focus on developing MKTT. Developing, refining, and improving an educative curriculum requires cycles of data collection to help inform improvements on how to effectively support the novice MTEs via these curricula. Finally, collaborative teaching environments are typically not effective if the MTEs do not share the same lessons, or even learning objectives, with PTs. Therefore, effectively implementing all three of these supports will require patience and persistence among all participating MTEs. One should expect to gradually improve these support structures over time to meet the unique needs of novice MTEs within the constraints of teacher education programs and institutional contexts.

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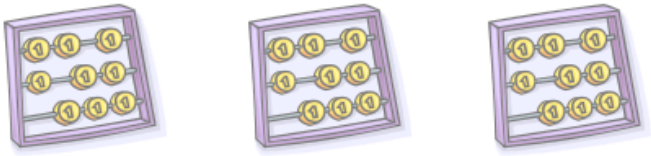
Appendix A: Family Math Night Sample Information Flyer



Please join us for the fifth annual

FAMILY MATH NIGHT!

Wednesday December 3rd

6:00 pm - 7:30 pm

PIZZA & SOFT DRINKS included

Have fun with teachers, parents, and children. A variety of activities **for all grades (K – 6)** will be hosted by Portland State University students planning to become elementary school teachers.

Let the children show you what they are learning! Explore new activities together. Explore mathematics activities designated to address the Common Core State Standards for Mathematics. Leave with ideas to continue mathematics learning at home.

Appendix B: Lesson Plan for Lesson X: Introduction to Decimal Multiplication (included with permission from the University of Delaware)

Topic: Introduction to Decimal Multiplication

Learning Goals:

1. Pre-service teachers will understand the meaning of the number sentence $a \times b = c$. That is, they will understand that a is the number of groups or copies, b is the size of the group or copy, and c is the whole.
2. Pre-service teachers will be able to use the "of" interpretation of the multiplication symbol to model multiplication number sentences with multipliers less than one. For example, they will understand that 0.7×8 means "Find seven tenths of 8" OR "Find seven groups of one-tenth of 8" OR "Find seven one-tenths of 8."
3. Pre-service teachers will use the meaning of multiplication to figure out why $.1 \times 2 = .2$ (and $.01 \times 2 = .02$). They will figure out how they can use this information to determine the value of $.3 \times 2$ ($.03 \times 2$).

Pre-service teachers will develop and show these understandings by (a) carrying out the following mathematical actions and (b) giving explanations that involve these actions:

1. The pre-service teachers will be able to explain the meaning of the number sentence $a \times b = c$; i.e., a is the number of groups, b is the size of the group, and c is the whole. They will be able to use the meaning of the number sentence to model multiplication number sentences on graph paper. (See #2 below.)
2. Pre-service teachers will solve multiplication number sentences with graph paper models. In these models, they will give explanations that explain in detail each of the relevant steps and concepts. An appropriate explanation of a graph paper solution that involves an area model for $0.7 \times 8 = ?$ follows:

> *"First, .7 × 8 means, 'Find seven groups of one-tenth of 8 (or 'Find seven tenths of 8 or 'Find seven one-tenths of 8'). I chose my BMU[i] to be 10 squares on the graph paper. Then a measuring unit of size .1 is 1 square; I drew and labeled these measuring units here. Now I can represent the quantity 8 as 8 BMUs, or 80 squares. Now I need to find one tenth of 8, so I partitioned 8 into 10 equal parts (shows the partitioning by partitioning the 80 squares into 10 equal parts). One of these parts is one tenth of 8. So, one tenth of 8 is 8 squares. I redrew 8 squares here and labeled it 'one tenth of 8.' (I can also see that one tenth of 8 is .8 because 8 mus of size .1 fit into 8 squares.) Now I need seven groups of one tenth of 8, which is 56 squares. So, I drew them here and labeled the 56 squares 'seven tenths of 8.' Now this quantity is my answer. But I need to find the numerical value of this quantity. I assign a numerical value by seeing how many measuring units of each type fit in. Five BMUs fit in, so this area has a numerical value of 5. I showed how 5 BMUs fit in by circling and labeling them each with 'BMU' (or measuring unit of size 1). Six measuring units of size 0.1 fit into the remaining amount. I showed how 6 MUs of size 0.1 fit in by*

circling and labeling them each with '0.1.' So .7 × 8 = 5.6."

Equipment:

- Graph Paper
- Document Camera

Associated Files:

- Handout 1
- Handout 2
- Homework

Associated Text:
Mathematics for Elementary School Teachers, Bassarear, 2nd Edition
Section 3.2, pages 135-139

Time: 0-10 minutes

Activity Flow – Part 1 – Homework Discussion

Ask the pre-service teachers if they have questions from the Lesson 12 homework. Answer only those questions.

Student Responses
Some students use a ratio idea to draw out ¼ of a quantity or 1/100 of a quantity. For example, for ¼ of a quantity, some pre-service teachers shade 1 out of every 4 circles (ratio idea). For 1/100 of 300 squares, some pre-service teachers split the 300 squares into three 100-square areas and then take one square out of each of the three 100-square areas. If the pre-service teachers use a ratio idea like this, they are frequently unable to solve it any other way; they cannot partition the original quantity into 4 or 100 equal parts. instructors can ask the pre-service teachers to look at the picture of the three 100-square areas and ask them if the picture looks like 1/100 of the total amount or if it looks like another fractional quantity was found first. Most will say that it looks like 1/3 of the quantity was found first and then 1/100 of each of the three quantities was found, which does not directly illustrate finding 1/100 of the original quantity.

Time: 10-75 min.

Activity Flow – Part 2 – The Meaning of Decimal Multiplication

Rationale
This activity continues to focus on the "groups of" or "copies of" meaning of the multiplication sign. In this activity, students extend their understanding of multiplication to decimal quantities. The examples, and the order in which they are posed to the students, were chosen to allow the students to create meaning for decimal multiplication, and to build on their ideas about making something ten times as big or as small. Therefore if the instructor changes

the examples and/or the order, he/she should have a rationale for doing so. The pedagogy in this activity involves direct instruction. See the section entitled "History of the Lesson" at the end of this lesson to understand the rationale for this pedagogical choice.

Activity

For homework, we represented 4 × 7 = ? on graph paper. Now let's try to extend our ideas about multiplication to decimals.

Tell students to turn to the Lesson 13 Classwork in their packet.

On your graph paper solve 0.1 × 2 = ?. Say to the students:

It is hard to think about one tenth groups of 2. So we think of this as one tenth of 2.
We said that the × sign means "groups of" or *"copies of." People originally developed the four operations for whole numbers, but when they developed ideas about fractional and decimal quantities and numbers, they extended their ideas about the four operations to these new kinds of quantities and numbers. This leads to some difficulties in the interpretation of multiplication. We are able to get around this difficulty by thinking of the × sign as "of" when dealing with multipliers less than one. If you use this interpretation of the meaning of the × sign here, how would you use the graph paper to solve the multiplication, 0.1 × 2 = ?*

Student Responses

Students should first write down the meaning of the multiplication number sentence: ".1 × 2 = ?" **means** "Find one tenth of 2." (In past semesters, instructors have said this means "point one of 2." This statement on the part of instructors can prevent students from making sense of the lessons on multiplication. The phrase "Find point one of 2" does not convey to the students what they need to do to model the problem. In contrast, the phrase "Find one tenth of 2" does suggest what needs to be modeled: Build the quantity that is equal to 2 BMUs and then find one tenth of it. In addition, in past semesters, students have failed to begin problems by writing down the meaning of the multiplication number sentence. This interfered with their ability to model the problem. On the graph paper, students can pick a BMU that has an area of 10 squares. (Other choices are, of course, possible.) They should draw and label the BMU. Then they should draw a measuring unit of size .1 and label it '.1.' Then they should use 20 squares to represent 2. They should draw the quantity and label it '2.' Next they should partition the 20 squares into ten equal parts. They should write, "One of these equal parts [two squares] is equal to one tenth of 2." They should redraw the 2 squares and label the 2 squares, "one tenth of 2" Conceptually, they should be able to consider this quantity as a separate entity, separate from the whole from which it was obtained; this idea is emphasized by redrawing it "outside of and separate from the whole" from which it was obtained. This quantity [2 squares] is the answer, but they still have to find the numerical value of this quantity. Since all quantities are assigned numerical values based on the BMU, they need to figure out how many MUs of each type fit into the two squares. The student should fit the measuring unit of size .1 into the quantity by partitioning the quantity into 2 units of size .1 and labeling each of the measuring units 'measuring unit of size.1.' Since 2 MUs of size .1 fit into the two squares, the numerical value of the 2 squares must be .2. Therefore, .1 × 2 = .2. For additional examples of how we

want students to model these problems, see the homework solutions for Lesson 14 in the Appendix of the packet.	
Student Responses	**Teacher Responses**
Students shade in 2 squares of the quantity that represents 2 and do all subsequent actions within the quantity of 20 squares that represents 2.	This kind of part-of-a-whole representation can be limiting and has been shown to lead to misconceptions in fraction learning. Encourage the students to redraw the 2 squares as a separate quantity. Students should be able to consider fractional quantities as entities in themselves—i.e., separate quantities that are assigned a numerical value based on their relationship to the BMU. If students feel compelled to shade and then "drag the whole 20 squares around with them" throughout the solution, it can interfere with problem-solving as the problems get harder. It also interferes with fraction learning. For example, if a student feels compelled to show 2/3 as 2 of 3 shaded parts and 4/5 as 4 of 5 shaded parts, it becomes difficult to model and create meaning for the number sentence 2/3 + 4/5 = ?.
Students draw 2 BMUs to represent 2. To find one tenth of 2, they shade one tenth of one BMU and then shade one tenth of the other BMU. The answer is then readily apparent. It is .1 (the shaded one tenth of the BMU is a MU of size .1) + .1 = .2.	At this point, discourage this approach (the "operating-on-each-measuring-unit-separately approach"). The pre-service teachers should be able to consider 2 as the whole [the 20 squares as a whole] and be able to find one tenth of the whole [be able to break the entire 20 squares into 10 equal parts of 2 squares each]. They should be able to directly model the meaning of the number sentence ("find one tenth of 2" as opposed to "find one tenth of 1 and find one tenth of 1"). The approach of operating on each measuring unit separately is problematic for 4 reasons. First, the ability to find a fraction of a whole will transfer to fraction multiplication in Math 252, whereas the idea of operating on each measuring unit separately is, in general, easily applied to multiplication problems that involve decimals but not fractions. Second, we have found that pre-service teachers who operate on each measuring unit separately are frequently doing this in a rote manner. For example, for .1 × 2.3 = ?, they represent 2.3 as two BMUs and 3 MUs of size .1. Then they mechanically shade "one row" of each of the 2 BMUs and shade "one row" of each of the 3 MUs of size .1. The answer is readily seen: It is .1 + .1 + .01 + .01 + .01 = .23. They memorize this procedure: Shade a row of each measuring unit, then add up the numerical values of the shaded pieces. Third, even when the approach is not mechanically applied, it allows the pre-service teachers to skip conceptual steps, steps that are essential in understanding (and re-thinking each time one does a problem like this) the meaning of a multiplication number sentence and the meaning of numerical assignment. To understand this, contrast the "operating-on-each-measuring-unit-separately solution" of .1 × 2.3 = ? with the approach that involves operating on the whole. In the latter approach, the BMU is chosen as 100 squares, for example, and 2.3 is represented as 230 squares. To find one tenth of the whole, 2.3, students partition the whole into 10 equal parts of 23

	squares each. This focus on the whole, rather than on the individual measuring units, draws attention to the meaning of the number sentence; it means 'partition 2.3 into 10 equal parts and find the numerical value of one part.' Students redraw the 23 squares as a separate entity, use the BMU to construct a MU of size .1 (10 squares) and a MU of size .01 (1 square), fit 2 MUs of size .1 and 3 MUs of size .01 into the 23 squares by drawing them into the quantity and labeling each of these measuring units, and conclude that the numerical answer must be .23. In the operate-on-each-measuring-unit-separately approach, the conceptual step of dividing the whole quantity (230 squares) into 10 equal parts is unnecessary; students only need to know how to divide measuring units into 10 equal parts. Moreover, in this approach, the final conceptual step of constructing and fitting MUs into the final quantity is not explicit; when using the operate-on-each-measuring-unit-separately approach, the shaded portions in their pictures are already divided into MUs. They can, in effect, read off the answer without this step because the act of dividing measuring units of any size into tenths or hundredths or thousandths, etc. will always result in a quantity that is equal to another measuring unit. The operating-on-the-whole approach requires explicitly fitting in MUs to determine the numerical answer whereas the approach of operating on each MU separately allows one to read off the answer without thinking explicitly about the idea of assigning numerical values to quantities. Fourth, these missing conceptual steps are needed for fraction multiplication. For example, if you want to find $\frac{1}{4} \times \frac{2}{3}$, the approach of operating on each measuring unit separately would involve representing $\frac{1}{4}$ of $\frac{1}{3}$ and $\frac{1}{4}$ of $\frac{1}{3}$ in a picture. The numerical answer cannot be readily read off here as it can with decimal problems: You have to find the numerical answer by using the BMU to identify a single type of measuring unit that will fit into the resulting quantity and to see how many measuring units of the identified type will fit in. Unlike decimal multiplication problems, the answer is not readily apparent without this step. Thus allowing students to skip these conceptual steps in Math 251 does not serve them well in Math 252. We want to encourage approaches that transfer well to fractions. **Whenever the instructor discourages a student strategy or way of thinking, the instructor should tell the pre-service teachers why he/she is discouraging the strategy. Explain the cognitive consequences of particular strategies and why they might be limiting (as the above discussion has done). Make sure that you also point out that while this information applies to them, it also applies to their future students. They should be aware of the potential strengths and limitations of particular conceptions and ways of thinking because they will be teachers. Therefore when**

	you share this information with them, you are accomplishing two things: You are helping them to develop more productive ways of thinking and you are giving them information *about* more productive and less productive ways of thinking that they can apply to their future teaching.
Students choose a BMU of one square and partition two squares into 10 equal parts to find one tenth of 2.	Remind students that it is more convenient to select a BMU that helps them to represent the problem and find the solution. If they select wisely, their representation will do more of the cognitive work for them.

Discuss the problem $.1 \times 2 = ?$ by having a student present his/her solution on an overhead. Ask the standard questions (What is the BMU? What are your other mus? How did you represent 2? How did you model one tenth of 2? How did you find the final answer from your model?).

If the student work was confusing, solve the problem yourself on a graph paper transparency and write down the following explanation:
The number sentence "$.1 \times 2 = ?$" means "Find one tenth of 2." I let my BMU be 10 squares. [Draw and label the BMU.] Then I represent 2 as 20 squares. [Draw the quantity that represents 2 and label it '2'.] [Write down:] To find one tenth of 2, I partition this quantity into 10 equal parts. One tenth of 2 means one of ten equal parts of 2. [Partition the quantity into 10 equal parts and label one of the parts "one tenth of 2."] Now I will redraw this quantity. [Redraw the 2 squares and label it "one tenth of 2."] This quantity is the answer but now I have to find the numerical value of this quantity. A BMU does not fit into this quantity, so I need to construct some smaller measuring units. A measuring unit of size .1 is 1 square. [Draw and label the measuring unit of size .1.] Since this measuring unit will fit in 2 times, the answer is .2. [Partition the quantity of 2 squares into 2 groups of size .1 each, and label each part '.1.'] So $.1 \times 2 = .2$. The instructor should emphasize how the resulting quantity in the picture is one-tenth as big as the original amount. Remind them this corresponds to a procedural rule that they should already know — i.e. that $.1 \times 2 = 0.2$ because "we move the decimal point over to the left 1 place."

The instructor should emphasize that in a multiplication number sentence, the numerals play different roles. In $a \times b = ?$, a is an operator and b is a quantity—i.e., an amount of stuff. First we draw b, the quantity or amount of stuff. Then a tells us what to do to b—i.e., it tells us to make a certain number of copies of b and/or to partition b into some number of equal parts and to take some number of those parts. For example, in $4.2 \times 7 = ?$, we draw 7 BMUS because the 7 represents a quantity or a physical amount. The 4.2 is an operator, not an amount of stuff; it tells us to make 4 copies of 7 and it tells us to partition 7 into ten equal parts and to take 2 of those equal parts. The instructor should contrast this with addition and subtraction number sentences: In $a + b = ?$ and $a - b = ?$, a and b are both quantities or amounts of stuff.

After discussing the modeling for 0.1×2, have a similar discussion for 0.01×2. First, discuss the meaning of the number sentence and have the pre-service teachers write the meaning

beneath the number sentence (i.e. 'Find one-hundredth of 2'). Then, ask pre-service teachers to make an area model of 0.01 × 2 on graph paper. Next, have a pre-service teacher present his/her work on an overhead transparency. Ask the standard questions (What is the BMU? What are your other mus? How did you represent 2? How did you model one tenth of 2? How did you find the final answer from your model?).

If the student work was confusing, solve the problem yourself on a graph paper transparency and write down the following explanation:
The number sentence ".01 × 2 = ?" means "Find one hundredth of 2." I chose my BMU to be 100 squares on graph paper. Then 2 is represented as 200 squares. [Draw 200 squares and label this area '2.'] First I will find one hundredth of 2. If I partition 200 squares into 100 equal parts [show the partitioning of the quantity '2' into 100 equal parts on your transparency], each of the parts is 2 blocks and one of the parts is equal to one hundredth of 2 [label one part "one hundredth of 2"]. I will redraw the 2 blocks here; this area is one hundredth of 2 [label the 2 blocks "one hundredth of 2"]. A measuring unit of size 0.1 is 10 squares [draw and label the measuring unit of size .1] and this won't fit into the resulting quantity of 2 blocks. So, I have to find smaller measuring units. A measuring unit of size .01 is 1 square. [Draw and label the measuring unit of size .01.] Since this measuring unit fits in two times, the numerical value of this quantity is .02. [Partition the quantity of 2 squares into 2 groups of size .01 each, and label each part '.01.'] So .01 × 2 = .02. The instructor should emphasize how the resulting quantity is one-hundredth as big as the original amount. Remind them this corresponds to a procedural rule that they should already know — i.e. that .01 × 2 = 0.02 because "we move the decimal point over to the left 2 places."

Always begin each problem that involves the modeling of multiplication number sentences by writing out the meaning of the number sentence and require the pre-service teachers to do so as well. When the pre-service teachers are able to write out the meaning of the number sentence, they are usually able to graph it. Conversely, pre-service teachers who cannot write out the meaning of the number sentence often make pictures that show a lack of understanding of one or more concepts.
A helpful strategy for the instructor is this: When you write out the meaning of the number sentence, represent the operator (the multiplier) with words and represent the quantity (the multiplicand) with a numeral. This helps the students distinguish between the numeral in the equation that is an operator and the numeral in the equation that is an amount of stuff. So, for example, the meaning of .7 × 0.8 = ? can be written by the instructor as "Find seven groups of one tenth of 0.8" or "Find seven tenths of 0.8" or "Find seven one tenths of 0.8." Never say this means "point 7 of 0.8" These phrases do not convey the meaning and they do not help the students distinguish between the roles of .7 and .8; i.e., they do not help them understand that one of these numerals represents an operator, while the other represents an amount of stuff.

Answer any questions students might have about the first two multiplication models.

Tell the pre-service teachers to turn to the next portion of the Lesson 13 Classwork in their packet. Assign each group to a pair of multiplication problems (one has a multiplier with only tenths and the other has a multiplier with only hundredths). Give pre-service teachers about 10

minutes to complete their two problems.

Model each of the following multiplication problems on graph paper using an area model. Choose the basic measuring unit carefully. Make sure you get the answer from your picture and not from a computation. Begin each problem by writing the meaning of the number sentence. For example, .01 × 5 means, "Find one hundredth of 5."

Set 1: $0.9 \times 1.4 = ?$ and $0.04 \times 3 = ?$

Set 2: $0.7 \times 0.8 = ?$ and $0.05 \times 0.4 = ?$

Set 3: $0.6 \times 5 = ?$ and $0.06 \times 5 = ?$

After pre-service teachers have finished their models, have groups share their work on an overhead transparency. Make sure they answer the standard questions if they do not offer them in their presentation (e.g., What is the BMU? What are your other MUs? How did you represent 2? How did you model one tenth of 2? How did you find the final answer from your model?).

After the students solve the problem, tell them they can read the number sentence in these ways: "7 tenths of 0.8" or "7 groups of one tenth of 0.8" or "7 copies of one tenth of 0.8" or "7 one-tenths of 0.8."

If the student work was confusing, solve the problem yourself on a graph paper transparency and write down a model explanation like the ones described above. The instructor should emphasize to the students how they can use their knowledge of how to find one-tenth of a quantity or one-hundredth of a quantity to find as many tenths or hundredths of the quantity as they need.

Time: 75 min.

Activity Flow – Part 3 – Homework

Today, we have been looking at the meaning of multiplication. We have also represented multiplication computational problems on graph paper. For your homework, you will continue to practice representing and solving decimal multiplication problems. Please complete the Lesson 13 Homework in your packet.

History of the lesson:

Many versions of this lesson were written and tested. In previous versions, we began the topic of multiplication with a homework assignment that asked pre-service teachers to independently invent models for number sentences like 0.2×0.7. The pre-service teachers shared their models on the next class day. We then tried to build on their invented models to develop the interpretations of multiplication used in this lesson. This approach was not successful. The invented models were usually incorrect and were not productive in terms of allowing us to move to more sophisticated models and interpretations of multiplication. Many

pre-service teachers held onto their (incorrect) original models and interpretations; i.e., the beginning assignment that asked them to invent their own models interfered with the development of more sophisticated understandings. Largely through direct instruction, this lesson leads them to more productive interpretations of the meaning of multiplication that allow them to extend their understandings to harder problems, to fraction multiplication, and to different contexts involving multiplication (e.g., the by-places interpretation of multiplication that will be developed in Math 251 is used in the standard multiplication algorithm and the all-at-once interpretation that will be developed in Math 251 is used in fraction multiplication).

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[i] The term "BMU" refers to the "basic measuring unit" or the measuring unit of size 1. The term "MU" refers to "measuring unit" and represents any measuring unit other than 1, such as a measuring unit of size 0.1 or 10. These terms are used throughout Lesson Plan X.