uc3m | Universidad Carlos III de Madrid

Degree in Aerospace Engineering

2018-2019

Degree's Thesis

TRAJECTORY OPTIMIZATION IN THE CIRCULAR RESTRICTED THREE BODY PROBLEM

Claudio Fernández de Heredia Pérez de Zabalza

Supervisors

Manuel Sanjurjo Rivo

David Morante González

July 8^{th}, 2019

SUMMARY

The main goal of this thesis is to prove the validity of an optimizer based on the MATLAB program GPOPS, for solving complex problems on orbital transfer dynamics.

Nowadays, optimization is one of the most important fields in space exploration. The high cost of space missions makes that every project developed must be perfectly studied, calculated, optimized and well executed. Hence, optimization programs appear as a very suitable solution for trying not only to prove the validity of a solution already designed, but also to provide a more optimal solution for any problem proposed .

In this study, a program has been developed based on an open source program called GPOPS, a MATLAB software for solving optimal control problems. This tool is able to return an optimal solution for problems approached as a dynamical system whenever it has a consistent initial guess. This initial guess generally is the mission wanted to be analysed and optimized. It is very important for the optimizer, since it will have a strong dependence on the performance of the tool, for example, computation time and accuracy.

The problem studied has been a Jovian transfer previously proposed in a different study. This journey implies moving from the Jovian moon Callisto, to an other Jovian moon, Ganymede, using an interesting element from Space mechanics, invariant manifolds, a really useful element for Low Thrust manoeuvres. This mission in particular, has as main objective the exploration of these celestial bodies, being the transfer time not a key factor. This fact ends with a multi-revolution Low Thrust manoeuvre.

During this thesis, different solutions were analysed such as: three and two body problem for the dynamics point of view, inertial and non-inertial reference systems and different mathematical methods.

After the results obtained, it can be concluded that the tool was able to prove the validity of the solutions or initial guesses given. However, after all the time spent working on the program, it has been demonstrated that this tool has a considerable room of improvement and a considerable number of upgrades have been proposed.

Keywords: Optimization; Space; Three Body problem; Jupiter; Invariant Manifolds

DEDICATION

Many thanks to my supervisors, Dr. Manuel Sanjurjo and David Morante, for showing me the beautiful world of Orbital Mechanics and its secrets.

Many thanks to my friends and close people, for not giving up and keep gifting their support every day.

And last but not least, my deepest appreciation to my family, for all the love and dedication received from them.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

During its History, the human being has set limits to itself for later, surpassing them. Firstly with ground and sea transport, humans were able to cover longer distances or heavier cargo, in less time. Later, they had the dream of flying and in the last century, they did it. The world was connected thanks to the airplane. Finally, the last big advance in transport was achieved at April 12^{th}, 1961 humans reached the space.

Space transport probably is one of the key factors of the technological development of the last part of the previous century and the first part of the current one i.e, the creation of a global communication network with satellites. This milestone has provided the development of the web, computers, cellphones, smartphones, which at the ends came up with real time communications through all the globe, breaking the term of distance. As a consequence, nowadays everyone can talk about globalization, one of the pillars of 21^{st} century.

Concurrently, it has helped to share knowledge worldwide, producing one of the periods in human history with highest technological development with the 3^{rd} *Industrial Revolution* in the last part of the previous century and nowadays, with the 4^{th} *Industrial Revolution*.

In the beginning of space exploration, efficiency was not the main goal. During the Cold War, for the United States and the Soviet Union, their main objective was to break the limits faster and surpass its contrary, no matter how much it will cost. Humans reaching the Moon together with the end of the Soviet Union years after, caused that the interest in space was lost significantly, mainly in terms of social popularity, due to its extremely high cost. This caused an extremely reduction in space investment by governments which were the main agents in this sector. Space industry needed to change its business model.

In 21^{st} Century, the appearance of the private sector in space transport has changed the industry. As in air transport, lot of companies are moving forward a common objective, **efficiency**. This fact reduces cost, making space more affordable and profitable and therefore more attractive to investment.

For the next decades, reducing costs should be the next step for space exploration. Companies such us *Space X* or *Blue Origin* in United States or *PLD Space*, *Deimos*, *Zero to Infinity* in Europe, are introducing this model with very positive results.

Nowadays, Low Earth Orbits and Geostationary Orbits are almost conquered, lot of business stills being in these distances such us the ones developed by the companies named later, but now, the humankind is looking for new opportunities outside the Earth. These new challenges seem to be located in the Solar System specially, in the nearest neighbour, Mars

1.1. Motivation

This study was born with an initial purpose, finding different options to reach Mars, specifically carrying to MARS a satellite range. This fact is crucial for the development of Mars as a suitable stop for human kind, however, developing a satellite network in Mars is not as "easier" as in Earth. The main reasons are:

1. Firstly, one main reason is distance. The mean distance between Earth and Mars is approximately 225 millions of kilometres. An extremely large value if it is compared to LEO orbit which has a range of 200 to 2000 kilometres or to GEO which is reached at an altitude of 36000 kilometres.

   In addition, satellites maintenance if required, is extremely easier near Earth where a wide amount of resources can be obtained unlike Mars. Moreover, if maintenance is not possible, replacing the satellite is easier for Earth Orbits than Mars orbits, unless Mars has a satellite factory, which at short term seems not possible.

2. Secondly, is launch mass. The last generation of GPS satellites, the III generation, has a mean launch mass of 3800 kg for single satellite [1]. For completing a GPS constellation on Earth, 24 satellites are required. Hence, if a symply constellation of satellites such as a terrestrial one is desired, at least 8 launches of a Falcon Heavy rocket, the most powerful rocket on the market nowadays [2], would be needed[1].

3. And last but not least is the project's cost,. This element is extremely related with the launch mass and actually is the most relevant. The higher is the launch mass, the higher is the cost of the mission.

After this short and pessimistic analysis of the possibility to implement a satellite network on Mars, a possible solution came up, **smallsats**. This technology is able to solve the main threats exposed previously:

- Firstly, the launch mass is reduced, since this technology has a branch of masses significantly lower in comparison with usual Earth satellites, such us the ones used at GPS constellations. These values generally oscillates between 1 kg to 200 kg.

- Secondly, a new alternative appears. A wide amount of smallsats can be packed together and then launched with an equivalent mass as the payload of a nominal satellite. Reducing the amount of launches needed to perform a constellation.

- Thirdly, and related with the first fact, the cost of the mission is reduced significantly since the payload for the rocket is shortened or less launches are needed to perform the mission.

[1] This conclusion has been done taken into account literally its specifications [2], probably making a deep analysis, this value should increase.

- Through all this analysis, the assumption of using chemical propulsion for reaching Mars was the one used, since Electrical propulsion can not put up with the transfer. Now, with smallsats the opportunity appears. This last fact is the key and later the main base for this study.

The possibility of using Electric Propulsion opens the possibility for a Low Thrust manoeuvre. This type of orbits increases complexity in the design phase and as stated at the beginning, efficiency is a key factor for the current business model in space transportation. Hence, every designed mission must be, from the beginning, fully optimized.

The idea of an optimizer

Achieving the initial purpose of carrying to Mars a suitable satellite constellation, is a long term project and therefore too extent for a single thesis. Its requires lots of different disciplines such us, telecommunications, propulsion design, systems design among others. But the purpose still there.

So following the trends of the industry, an optimizer for Low Thurst Manoeuvres was decided to be done. This tool, although should be designed for Earth-Mars transfers, finally is designed for every mission desired , as long as the user knows the dynamics of the system and has already a mission designed for being optimized.

This program, at its concept, would help mission designers to optimize every mission needed if and only if previously it was designed and viable. But this tool, after its design should pass a validity process. This is the main motivation of the project, developing an optimizing tool and later prove its validity.

1.2. Objective

The main target of this thesis is to design and prove an optimizing program based on an open source MATLAB optimizer software called *GPOPS*. For proving the validity of the tool, a Circular Restricted 3 Body Problem (CR3BP) using invariant manifold dynamics will be studied based in a Jovian transfer from Callisto to Ganymede already solved such us the one in [3] will be studied.

1.3. Outline

The report's structure has been designed in order to follow the program structure:

In Chapter 2, the nature of the optimizer is explained as well as its structure and mathematical methodology.

In Chapter 3, all the concepts regarding the Initial guess used are explained. The method used, the basis for its development and how it was designed. Finally, in this chapter the auxiliary initial guess used is also explained.

In Chapter 4 all the background regarding physics is explained. Firstly, the main problem used for the study, the Circular Restricted 3 Body Problem. Secondly, the general assumptions followed during the project. To conclude, the secondary dynamics of the project divided mainly in two, according to the reference system used: the inertial approach and the non-inertial approach.

In Chapter 5, the results obtained after all the simulations are plotted. In order to follow the same structure as the one in Chapter 4, firstly, the solutions obtained from the initial guess are plotted. Secondly, the other mission proposal calculated is shown. Lastly, the difference between both solutions is defined.

To conclude, Chapter 6 evaluates all the solutions given, the validity of the tool and states some possibilities for future work.

1.4. Regulatory framework

The whole project was developed in Spain, mainly in la Comunidad de Madrid. In addition, no dangerous materials or equipment has been managed during the process. Neither dangerous places such us factories or laboratories were needed for doing the project. Hence, all the elements needed were an office and a computer.

In consequence, the legal framework that comprehend this project is the one related with offices and devices with screen.

In conclusion, the legislation considered is the one which includes aeronautical projects, since space legislation was not found for Spain, however, it should be very similar. Hence, the following legal framework is considered:

(i) The relationship company-worker in the Comunidad de Madrid for an aeronautical project is recolected in the collective bargaining agreement 28006522011990 of the Official Bulletin of the Comunidad de Madrid, number 60 from March 10^{th}, 2018.

(ii) The project legislation regarding labour risks is defined at *Ley 31/1995* from November 8^{th}.

(iii) *Real Decreto 486/1997* from April 14^{th}, establishes the minimum regulation of Health and Security in work places.

(iv) And finally, *Real Decreto 488/1997* from April 14^{th}, standardizes the jobs which have equipment with screens.

In addition, this tool as a Software project for academic purposes has not specific regulations regarding type of project. Moreover, since the tool is an open software, it has free access to operation and changes. However, if it is desired to include the software in a project from a company, there exist some standards that could be followed:

(i) IEEE 1074: Developing Software Life Cycle Processes, where all the processes for the life cycle of a Software are specified.

(ii) IEEE 1219: Software Maintenance where all the process regarding software maintenance and execution are standardized.

1.5. Socio-Economic Environment

This section encompass all the elements which the projects affects socially and economically. It is divided in two main parts: the Project's Budget and the socio-economic impact of the project.

1.5.1. Project's Budget

In this part, all the costs involved in the project's generation are taken into account except the indirect costs such as work place, light and gas. These expenses have been assumed as 0 since they were all provided by the university, and they have not been included in the budget.

Next, the project's budget together with the explanation of each element has been written:

Description	Price
COMPUTING DEVICE The computing device has been a portable computer. Specifically an *HP Probook 430 G2*.	430 €
MATLAB LICENSE The tool used for all the project. Since the Licence is provided by the university, it has null cost	0 €
ENGINEERING HOURS The nominal cost of a engineer has been set in 25 €/hour. This price is for an engineer fully dedicated to an optimization task and has been assumed according to market values	8750 €
Total	9180 €

TABLE 1.1. PROJECT'S BUDGET

For the part of engineering hours, 350 hours of project were introduced. Although a nominal project like this should last 300 hours, more time was needed for developing the project.

Concurrently with the project's specifications a Gantt diagram was performed for showing how the engineering hours were distributed during the project.

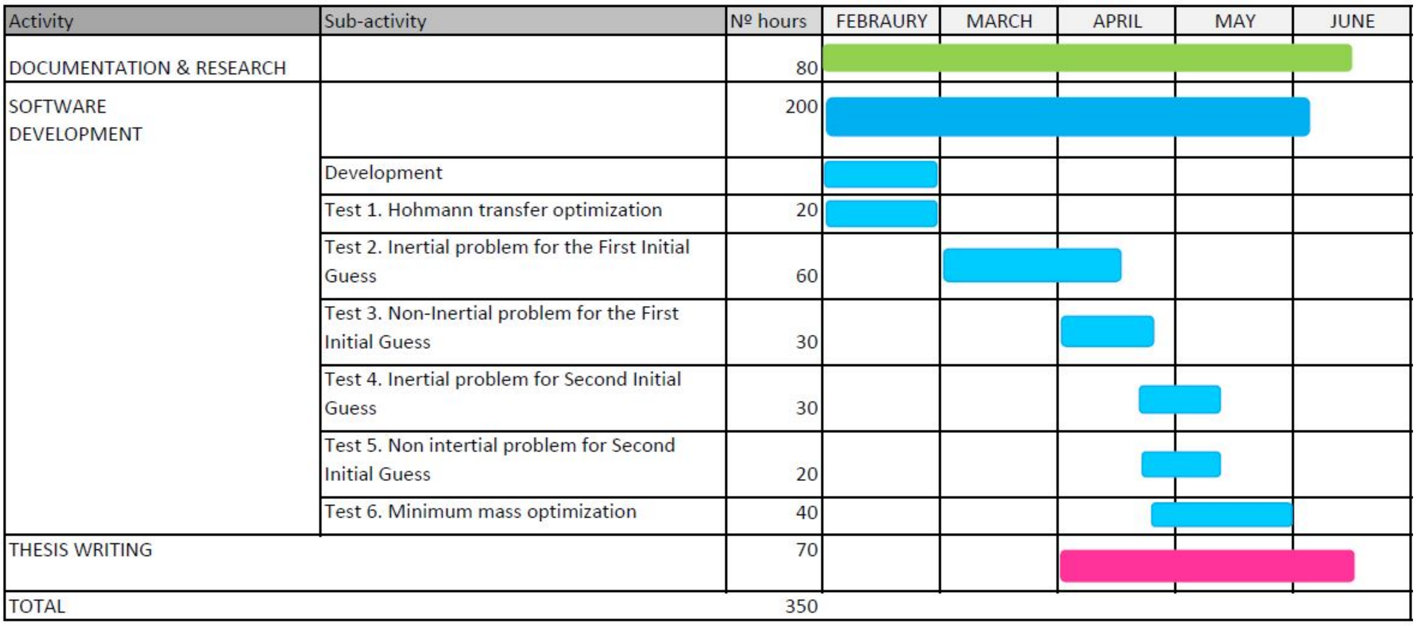


Fig. 1.1. Gantt diagram of the project

In this diagram is reflected all the process done in the project. However, in this report not all the process done have been introduced since they are not relevant for the project itself. Test 1 was a simple training for getting used to the tool and Test 6 did not reach any useful conclusion. Nevertheless, they have been taken into account for the process pricing.

1.5.2. Socio-economic impact of the tool

Space industry is continuously changing the economy and society in the 21st century. Its effect can be observed in all the industries specifically in telecommunications. However, it is just the beginning.

Enabling Space transportation for different places in the Solar Systems can produce a higher impact for the socio-economic environment than the one produced by events such as the Silk road utilise by Marco Polo or the Discovery of America.

Those elements produced a significant change in human history since it allowed to develop new and a wide quantity of jobs, new places to live, new countries or new economic agents. This effect will be higher for Space Exploration, since the number of people affected by such a type of change is extremely larger than the living population from those days.

This project and its future work could be a catalyst for this development since they could save an important quantity of money to the industry.

One of the main advantages of this sector is that for the last decades, most of the projects have been completely studied, its safety factor has increased and the industry has reduced its price significantly.

The tool developed could help in the future to do this work faster and therefore, save money in two ways: with the mission cost itself due to mass reduction or time reduction and with the cost of mission designed, since the optimal solution could be reached faster and easier.

Putting specific numbers for this improvements is very difficult due to the variety of mission types and prices in the space industry, however, its main effect has been explained wide enough.

2. THE OPTIMIZER

The tool used during all the project is a MATLAB based program called DMG tool [4]. Likewise, this tool is based in an open software called GPOPS which corresponds to Gauss Pseudospectral Optimization Software.

This program was developed by the union of MIT, Draper laboratory and the University of Florida and published under a Simple Public License. This tool works mainly with a Gauss Pseudospectral Collocation Method, which shortly is defined as an orthogonal method where the collocation methods are the *Legendre-Gauss* points[2].

On the other side, the DMG tool was a modified software which implements the Hermite Simpson Collocation Method for obtaining a Non linear Optimization Problem from the continuous optimal control problem.

2.1. Structure of the program

The structure of this report is based on the structure of the program itself. This software as sketched in Figure 2.1 is composed mainly by three pillars: Initial Guess, dynamics of the problem and Mathematical method used.

The two first elements will be explained in detail at Chapters 3 and 4 respectively. The mathematical method used, as said before, is the Hermite Simpson Collocation method.

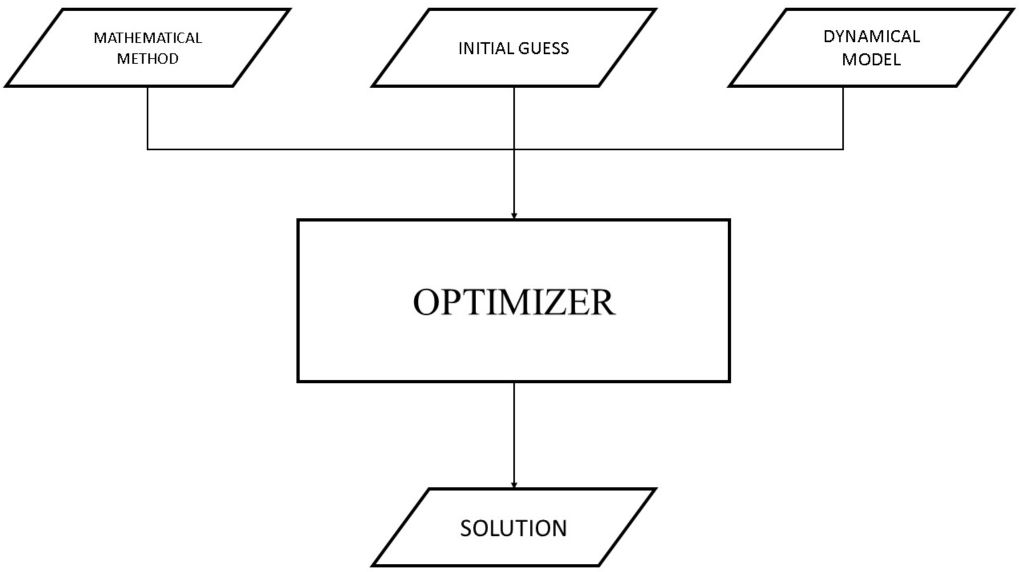


Fig. 2.1. Inputs for the DMG tool

[2]The basis for this method is not needed to know for the understanding of the work done since other method will be used, however, a brief description was done. For more information please go to [5].

2.2. The Hermite Simpson Collocation method

This method requires a set of equations to state the dynamics of the problem and an initial guess. This initial guess is divided in intervals where its limits are called nodes, therefore, the number of intervals will depend on the number of nodes chosen. These intervals are produced in the state and control variables of the problem, leading to the intervals shown in Figure 2.2.

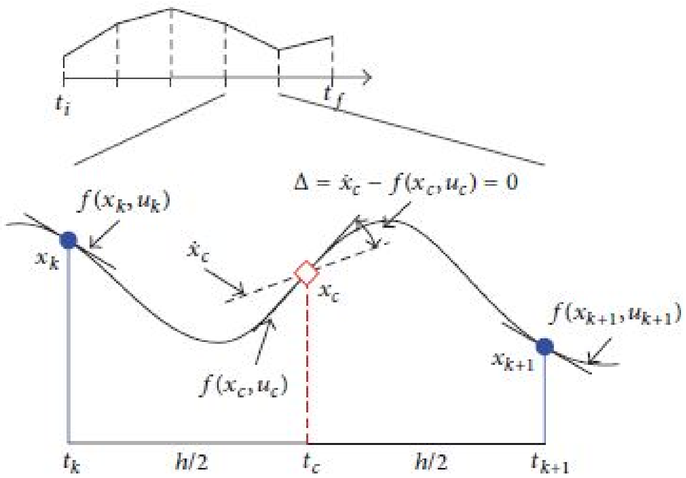


Fig. 2.2. Hermite Simpson Collocation method sketch from [6]

The blue dots of Figure 2.2 represent those nodes having a set of points $[x_k,\ u_k,\ x_{k+1},\ u_{k+1}]$. These represent the state variable as x, and the control variable as u, for the interval of time $[t_k,\ t_{k+1}]$. The dynamics provided are used for obtaining the time derivatives of the nodes ending with four pieces of information that gives the possibility of coming up with a third order Hermite polynomial. However, this polynomial is not able to satisfy the dynamics provided for the whole interval $[t_k,\ t_{k+1}]$, since it is only able to solve the ones at the nodes.

Let $[x_c,\ u_c]$ at t_c the middle point between the nodes, called the *collocation point*, represented at Figure 2.2 as the red diamond. Making $\Delta = \dot{x}_c - f(x_c, u_c) = 0$, a polynomial can be obtained that satisfies all the points from the whole interval $[t_k,\ t_{k+1}]$.

The detailed procedure of the method is derived as follows [6]. The function 2.1 represents the state variable in the interval chosen $[t_k,\ t_{k+1}]$, in a third order form:

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{2.1}$$

yielding with its time derivative,

$$\dot{x}_t = a_1 + 2a_2 t + 3a_3 t^2 \tag{2.2}$$

where $[a_0,\ a_1,\ a_2,\ a_3]$ are the coefficients of the polynomial.

In order to simplify the notation, the time interval will be $[0, h]$, giving that the values of the nodes for the state variable are $x(0)$ and $x(h)$ and its time derivatives are $\dot{x}(0)$ and $\dot{x}(h)$. Using equations 2.1 and 2.2 yields with:

$$\begin{bmatrix} x(0) \\ \dot{x}(0) \\ x(h) \\ \dot{x}(h) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & h & h^2 & h^3 \\ 0 & 1 & 2h & 3h^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{2.3}$$

This system helps to obtain the coefficients for the interpolated polynomial:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{h^2} & -\frac{2}{h} & -\frac{3}{h^2} & -\frac{1}{h} \\ \frac{2}{h^3} & \frac{1}{h^2} & -\frac{2}{h^3} & -\frac{1}{h^2} \end{bmatrix} \begin{bmatrix} x(0) \\ \dot{x}(0) \\ x(h) \\ \dot{x}(h) \end{bmatrix} \tag{2.4}$$

Then, the coefficients can be obtained since the values for the nodes are known and the parameter h also. With them, the collocation point and its time derivative can be obtained as the next formulas:

$$x_c = x\left(\frac{h}{2}\right) = \frac{1}{2}(x_k + x_{k+1}) + \frac{h}{8}[f(x_k, u_k) - f(x_{k+1}, u_{k+1})] \tag{2.5}$$

$$\dot{x}_c = \dot{x}\left(\frac{h}{2}\right) = -\frac{3}{2h}(x_k - x_{k+1}) - \frac{1}{4}[f(x_k, u_k) + f(x_{k+1}, u_{k+1})] \tag{2.6}$$

At the same time, the control variable at the collocation point is calculated with linear interpolation,

$$u_c = \frac{u_k + u_{k+1}}{2} \tag{2.7}$$

To conclude, as set at the beginning of the section, the solver tries to reduce to 0 the integration defect, $\Delta = \dot{x}_c - f(x_c, u_c)$, which using the information given in equation 2.6 can be expressed as:

$$\Delta = \dot{x}_c - f(x_c, u_c) = x_k - x_{k+1} + \frac{h}{6}[f(x_k, u_k) + f(x_{k+1}, u_{k+1}) + 4f(x_c, u_c)] \tag{2.8}$$

The NPL solver will use the values $[x_k,\ \ u_k,\ \ x_{k+1},\ \ u_{k+1}]$ for achieving its objective of minimizing the integration defect and therefore, the polynomial obtained will be approximated to the true dynamics of the problem within the accuracy of numerical integration.

To sum up, the optimizer requires several elements in order to work such us the dynamics of the problem, the initial conditions and final conditions of the problem and the mathematical method. But what is more critical for the optimizer is the initial guess.

The initial guess is so important due to the direct effect into the capacity of the optimizer for obtaining a solution. If the initial guess introduced does not fulfil the dynamics used at the beginning, the first iteration of the program, which is this input guess, will produce that the optimizer gets away from the solution in the beginning, requiring more time for the convergence of the solution. Generally, for a bad initial guess,[3] the optimizer does not reach a solution.

Next all the basis needed for the development of the initial guess will be shown. This was one of key point for the development of the project.

[3]i.e. a transfer designed for two body problem and the dynamics introduced are for three body problem, or a transfer where the initial conditions given to the optimizer are not the ones at the guess.

3. INITIAL GUESS

As sketched before, new trends in space industry are moving forward optimization, low cost, economically friendly projects and multifunction missions. They can perform a wide variety of objectives for a single mission, implying that the mission performed must be optimized at maximum. Generally, after analysing all the variables involved in the mission, it ends up with two main variables for optimizing: minimizing the fuel mass consumed during the mission, or minimizing the time that the execution lasts. Both have its strengths and weaknesses.

The aim of this chapter is to define properly the initial base of the project, since later, it will prove the validity of the optimization tool.

3.1. Choosing the main objective

Minimizing the fuel mass implies that the spacecraft needed is lighter and therefore its launch cost is lower, but it has a main drawback. Usually, the execution time for this type of mission increases significantly, which implies mainly in an increase of two important costs of the project: the cost in spacecraft systems since they need to last longer and be more reliable, and the cost of the workforce since they will need to get involved in the mission more time.

On the other side, minimizing time has other type of consequences on the cost. Since fuel mass is less taken into account, the spacecraft will be heavier raising the price of the launch but, probably, systems will last shorter being more cheaper.

In this particular mission and following the objectives used by Fantino & Castelli in [3], time spent in the transfer will be the one minimized. Nevertheless, the optimization problem has been designed for taking into account both objectives, minimizing time and mass in the case that both cases want to be analysed. However, in this study, only time reduction will be studied.

Before introducing the initial guess given and the method used to obtain it, it seems convenient to introduce the system and the elements used for designing the input data used.

3.2. Lagrange Points

One of the singularities when studying the orbital dynamics of a system are the Lagrange Points. This particular points are obtained when the third body in a Three Body problem (explained in depth at Chapter 4, Section 4.1.1) moves with the same frequency as the primaries, the bodies of higher mass. In those points there exist an equilibria between all the elements of the problem, where from the point of view of the third body the system is motionless.

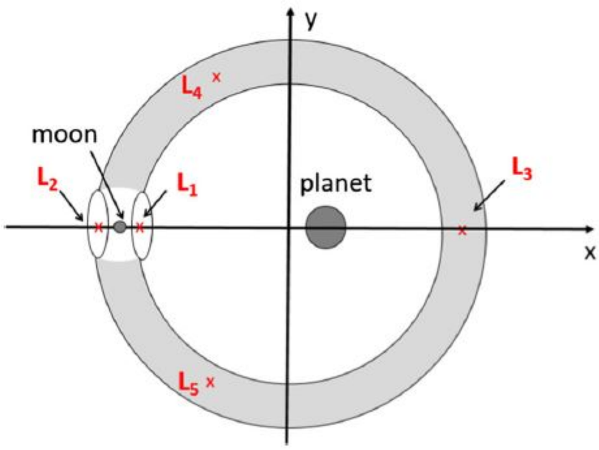


Fig. 3.1. Example of a general Planet-Moon system with its Lagrange Points from [7]

In Figure 3.1, the 5 possible Lagrange Points that a system as the one used in the study can have, are plotted. However, for the project its self only the two first points L_1 and L_2 are going to be used since the other three of them, are not accessible due to potential energy issues. This not accessible region is represented as the grey zone at Figure3.1.

The concept of Lagrange points is significant for space exploration, mainly for investigation. Missions such us LISA among others [8], use the stability of these singularities to observed different objects and phenomena in Universe. However, this positions are very difficult to be maintained among time and therefore expensive. Extremely accurate systems are needed to achieve this goal, such us Micro-Propulsion or very accurate Real Time positioning.

3.3. Lyapunov orbits

Although using Lagrange Points, LP, seems very suitable for observing and mapping the Jovian moons following the main objective of the mission, it adds a significant amount of complexity to the problem.

At this point, a concept in line with the Lagrange Point appears. As specified before an special interest comes for Points L_1 and L_2, which in addition as specified in Figure 3.1, are the closest coordinates to the moon, and advantage for the mission.

These points after a linearised analysis of the potential equation, appears as saddle points. This ends in the *Lyapunov theorem* which states that: "*there is a family of periodic orbits surrounding each of these points; one can think of this as meaning that one can 'go into orbit about these points'*"[7].

Those orbits can provide the same advantages as the ones given by the LPs but adding one more advantage, reaching and maintaining those orbits are easier than LP. Maintaining the orbit is similar to a conventional orbit around a celestial body hence, is easier since there exist a wide amount of knowledge and practice in this area.

Reaching the trajectory is easier since there exist some ballistic movements that can end in those Lyapunov Orbits, the invariant manifolds.

3.4. Invariant manifolds

L_1 and L_2 points as saddle points of the system, experiment two different behaviours: a tendency of getting closer to the point and a tendency of getting away from it. Those tendencies are:

- Heteroclinic behaviour which is defined as the movement that makes a mass getting away from the main LP to the other. This can be defined as the non stable part of the saddle point. This trajectory is useful for getting away from one LP.
- Homoclinic trajectory. This element, on the contrary of hetereoclinic trajectories, tends to move the mass to the LP and in particular to the Lyapunov's orbit, LO. This is the stable behaviour of the system and is useful for trying to reach a LO of the desire Lagrange Point, since is an stable manifold.

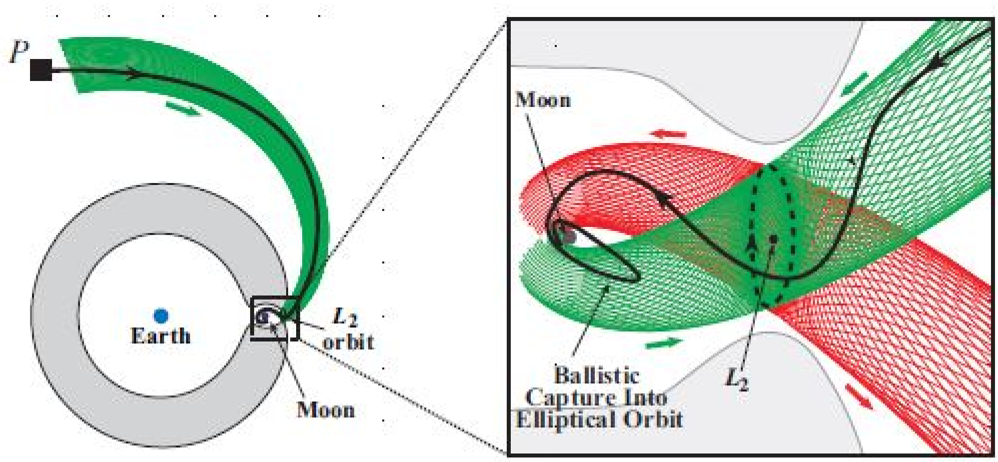


Fig. 3.2. Example of an invariant Manifold for the Earth-Moon system

Figure 3.2, shows an stable manifold, homoclinic trajectory as the green coloured and an unstable manifold as the red coloured. Both types of trajectories are really important for the method used on the initial guess since a combination of both trajectories are the one used.

3.5. The basis for the method

Now, that the concept of invariant manifold has been given, let's introduce the methodology used at [3], in order to design its proposal for a transfer mission.

3.5.1. Transfer design

Knowing in depth the basis of invariant manifolds dynamics, it can be concluded that joining an heteroclinic and homoclinic manifolds from different systems could derivative in an interesting solution for a transfer.

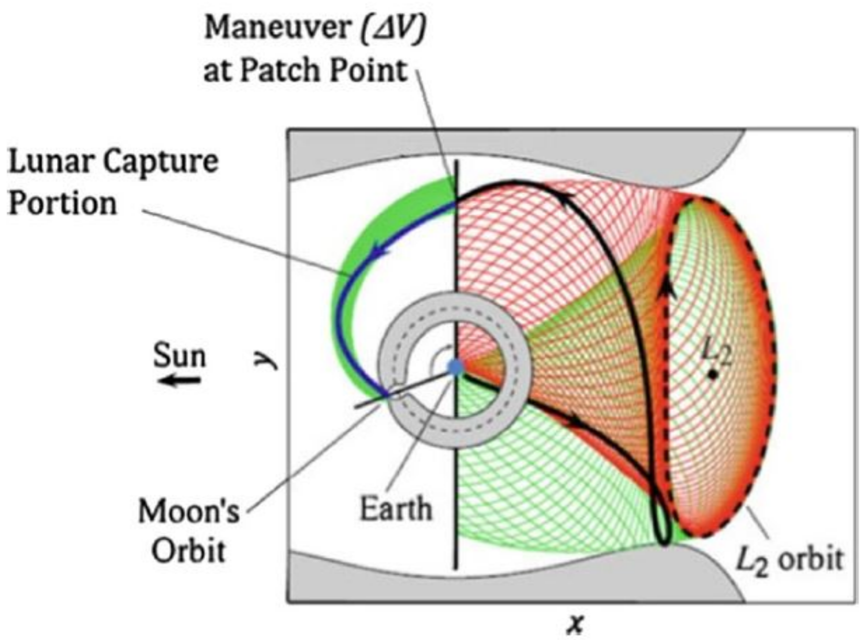


Fig. 3.3. Example ofthe method used in a different system, in this case, a transfer from Earth-Sun L_2 to Moon-Earth L_1.[3]

For example, let be the red mapping of Figure 3.3, the unstable manifold of L_2 of an Earth-Sun system and the green path, the stable manifold of the L_1 for a Earth-Moon system. By a manoeuvre, ΔV, at the matching point between both manifold, both paths can be connected and by a ballistic trajectory, the moon can capture the spacecraft, ending in a Lyapunov orbit at Earth-Moon L_1, since is the stable path.

However, this matching paths are not always as easier to match as sketched in Figure 3.3. Since both systems have its own rotatory systems, the possible manifolds trajectories will not always coincide, or if they coincide, the manoeuvre at the Patch Point required will not always be as easier as shown. Specially for a Jovian transfer, where the moons rotates significantly faster than in an interplanetary system as Earth-Sun system. Figure 3.4 shows perfectly that depending on the value of that α_0 angle which is a function of time, the system can be aligned or not.

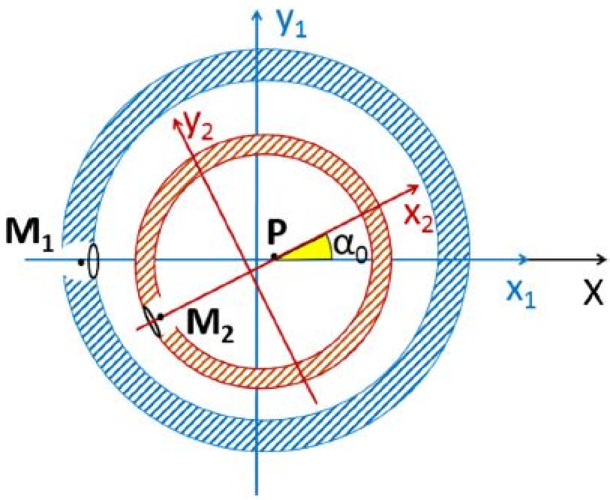


Fig. 3.4. An example for the Jovian system with its Lyapunov orbits drawed [3].

Therefore in order to be able to design a solution for a Jovian transfer using manifolds it seems convenient to analyse the different possible positions for the path point and choosing one feasible solution. Figure 3.5 shows those possibilities:

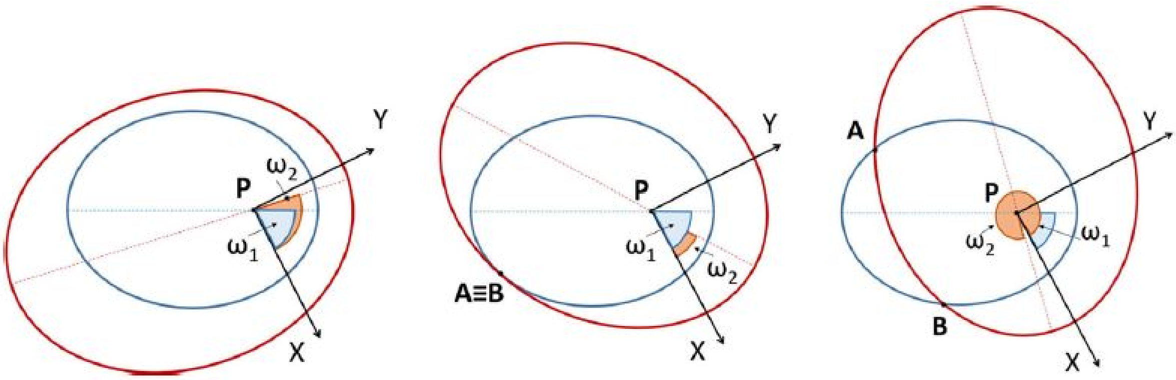


Fig. 3.5. Different positions of the manifolds trajectories [3].

- First option disables the opportunity of having a patch point between both transfers, since there not exist a conection between both trajectories. So it will not be a possible solution.
- Second option, a tangent intersection between both trajectories, reflects a common point between both transfers and insinuates that this solution could be the most easier for the patch point manoeuvre, since a low ΔV would be needed for switching trajectories.
- Finally, the last option, an intersection between both trajectories, comes up with the possibility of having two different manoeuvres for reaching the destiny but with more abrupt changes in the trajectory.

In conclusion, the tangent path seems to be the most suitable solution for the transfer shortly presented, since requires less complexity and a lover value of ΔV and therefore lower fuel mass required. This fact ends with a cheaper manoeuvre. Nevertheless, this implies that the trajectory designed will be limited by the positions of both primaries and therefore time. In consequence, the design should look for the exact moment of action.

3.5.2. The trajectory design

After knowing all the possibilities of joining two different manifolds trajectories from different systems, the designing of the mission can be finally defined as a transfer which goes from L_1 point of Callisto-Jupiter system through an unstable invariant manifold, to L_2 of Ganymede-Jupiter system through an stable manifold, ending in a Lyapunov orbit at L_2.

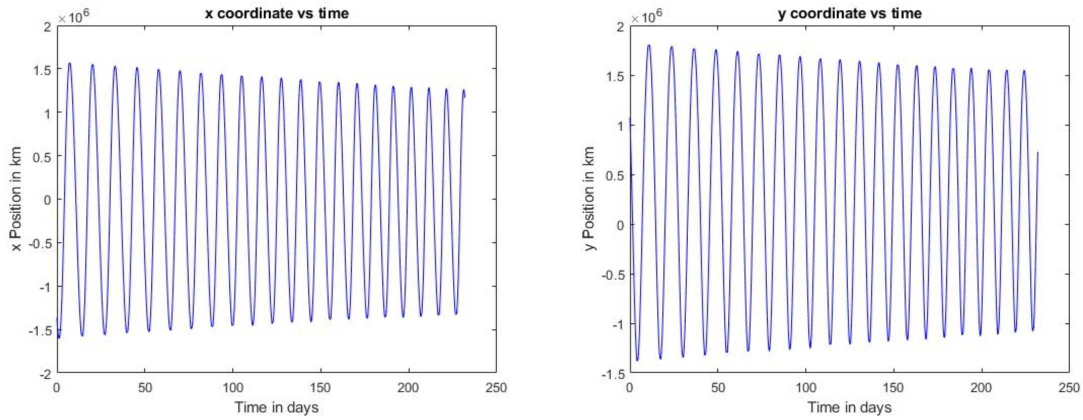
In order to proceed, 4 different phases were proposed since different dynamics should appear during the transfer:

1. The dynamics under the sphere of influence, CI, of the first Moon and the Lyapunov orbit.
2. A CR3BP where the system is Spacecraft-Callisto-Jupiter. This region goes from the CI of the first moon until the Patch manoeuvre.
3. A CR3BP where the system is Spacecraft-Ganymede-Jupiter. This region goes from the Patch manoeuvre to the CI of the second moon.
4. Finally, in this last region, the dynamics under the sphere of influence, CI, of the second Moon and the Lyapunov orbit of L_2 will appear.

To conclude, for proving the validity of the optimizer, the second and third phase will be the ones studied, and therefore its designed trajectory will be the initial guess used for the project.

3.6. The initial guess and the trajectory designed

Before introducing all the mathematical background employed in this study, it seems convenient to present the solution of the method previously named and also, the initial guess used:



(a) X coordinate vs time in days (b) Y coordinate vs time in days

Fig. 3.6. Initial position vectors respect to the inertial frame

The frame chosen for plotting this is the inertial frame exposed at 4.1.1. Furthermore, the propulsion of the spacecraft in this problem is also significant, later explained at 4.2. Bellow, the graph for the initial guess of the thrust orientation is plotted. It is significant to sign that these angles represent the thrust orientation with respect to the inertial reference frame previously stated.

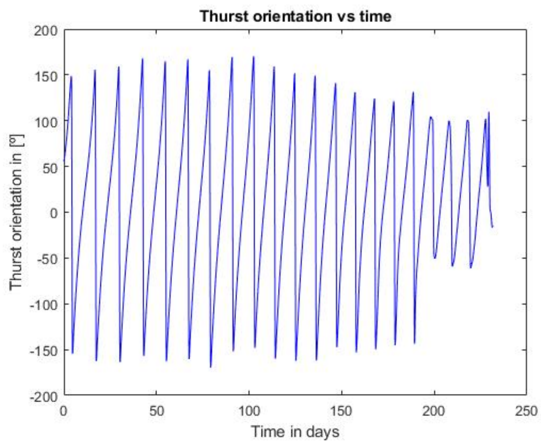


Fig. 3.7. Thrust orientation vs time in days

3.7. Q-LAW optimisation as an alternative input

Working with the initial guess is one of the tests that the designed tool must pass. However, for completing this analysis and encouraging its validity, other guess was analysed.

This guess was an optimal solution obtained by the Q-LAW optimization tool. Since this solution is already optimized taking into account time, it will be difficult for the optimizer to return a better solution. But if the optimizer is able to give the same solution or almost it, it will be an extra argument that could prove the validity of the program.

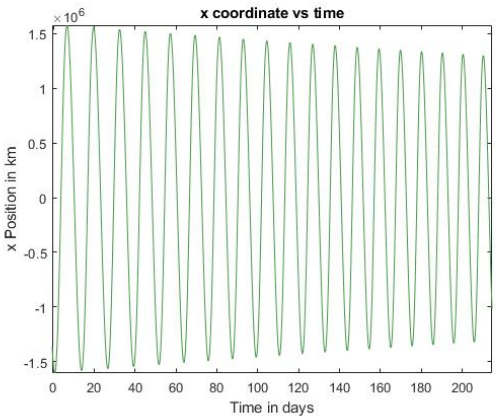
This optimization method provides the optimal thrusting angles and the coast phases for a transfer between two orbits around a central body. This fits perfectly with the problem description. Although the functionality of the algorithm is not needed[4], it is defined as *"efficient and robust to generate an optimal, well-spread Pareto-front of the transfer time versus the required propellant mass for an orbit transfer between two arbitrary orbits"* at [9], concluding the suitability of the method.

3.7.1. Q-LAW solution

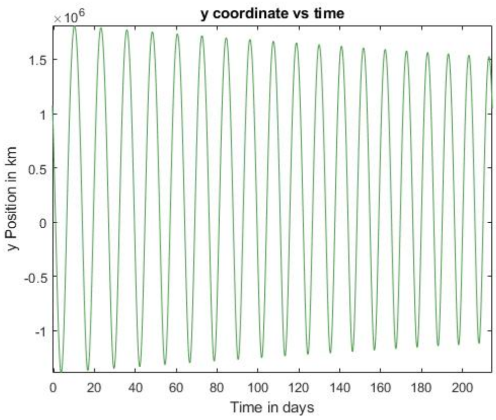
Before introducing the solution given, it is important to highlight that there exits some differences between the solutions provided. As it can be seen at first sight, if a comparison is done between the previous and the following graphs, these differences later explained in Chapter 5, can be mainly summarized in a difference of mission endurance of approximately 20 days.

However, these solutions have common initial and final conditions. This configuration was done in order to obtain feasible conclusions after the optimization process.

As it was done for the initial guess, the plots of the solution used for the optimizer as input should be known.



(a) X coordinate vs time in days



(b) Y coordinate vs time in days

Fig. 3.8. Initial position vectors respect to the inertial frame for the Q-LAW

[4]For mote information of this method, the reader can find it at [9]

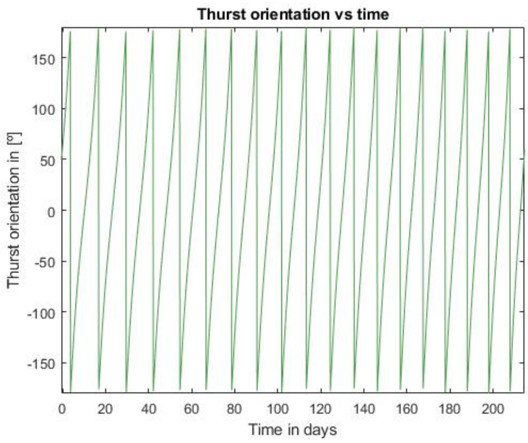


Fig. 3.9. Thrust orientation vs time in days for the Q-LAW method

To conclude this chapter, it seems that the Q-LAW solution is simpler than the initial solution since it needs less revolutions for reaching its target destination. Hence, probably solving this with the optimizer could end with a faster solution.

4. DYNAMICAL MODEL

The objective of this chapter is to explain all the physical background used to develop this study. Firstly the main theoretical dynamics of the problem. Secondly, the assumptions taken during the project will be shown. Then, the main approaches of the reference systems used for the problem will be explained, and lastly the optimizer and its methods will be explained.

4.1. Dynamics of the problem

One of the key factors for developing the optimizer is the dynamics of the system. This element will be strongly dependent on the tool's ability of reaching a solution.

4.1.1. Three Body problem

The Three Body Problem (3BP) is one of the most used methods for calculating a transfer orbit, which involves mainly two massive bodies (primaries) such as planets, stars or moons, and finally, the spacecraft. This approximation of mechanical orbits is established under the following assumptions:

- All the bodies involved in the problem are assumed to be point masses.
- The mass of the spacecraft can be simplified as a massless object, since the primaries of the problem are extremely more massive than it.
- All the elements are contained in the same plane for this case, therefore all the transfer is done at the XY plane.
- The two primaries rotate with constant angular velocity through the center of the system, the barycentre.
- An inertial reference frame can be placed at the barycentre of the system.
- An non-inertial rotating reference frame called the *synodic reference frame* can be used. This rotating frame has its origin at the barycentre of the system and it is rotating with a speed which produces that the X axis is always collinear to the vector that connects the two primaries.

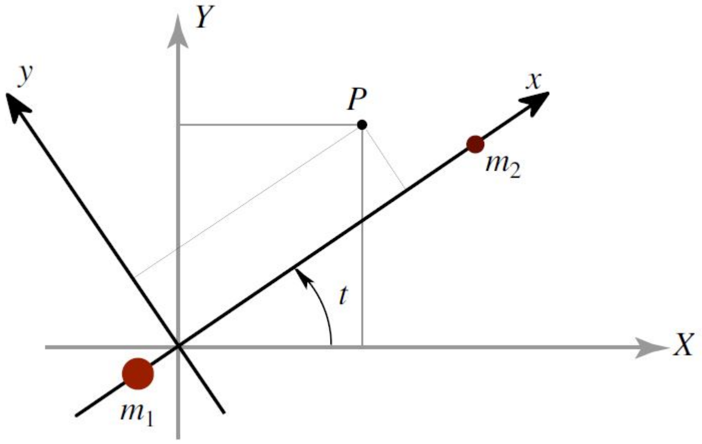


Fig. 4.1. This figure obtained from [7] shows the 3BP and its transformation to a *synodic reference frame* which has a counter clockwise rotation. The z-axis points out the plane, not represented in this image since the problem is simplified to a planar XY system.

4.1.2. Assumptions taken into account

For this study, further approximations of the 3BP were done in order to simplify the problem and earning time from the simulations[5]. This are the assumptions considered:

1. Since the mass of Jupiter is significantly larger than the Jovians moons, Jupiter is placed at the origin as the barycentre:

$$m_{Jupiter} \sim 10^{27}\ kg \gg m_{Callisto} \approx m_{ganymede} \sim 10^{23}\ kg$$

2. The orbits of the two moons are considered to be circular and contained in the same plane, therefore the problem ends to be a Planar Circular Restricted 3 Body Problem, PCR3BP.

3. The Transfer proposed in the study has its origin out of the sphere of influence of Callisto, and ends in a Lyapunov orbit at L_2 of the Ganymede-Jupiter system. Therefore, the primaries of the problem are Jupiter and Ganymede, since Callisto will not have influence on the dynamics of the problem.

4. The possible *synodic reference frame* of the system rotates at the same speed as Ganymede, hence, it will have a period of approximately 7 days and 3 hours.

In conclusion, with all these previous assumptions and taking into account the Newtonian physics, the dynamics used through the study could be divided in two different groups:

[5] As it can be seen in Table 5.2 time has been one of the main threads of the study. In order to develop suitable results, simplifications were needed.

4.2. Dynamics for the inertial reference frame

In this part, since all the dynamics are done in the inertial frame no accelerations terms due to the rotation of the moon should be taken into account, hence the dynamics could be simplified using Newtonian Dynamics as follows:

$$\sum F = m_{S/C}\ddot{r}_{S/C} = -G\frac{m_{S/C}m_J}{\rho_1^3}\vec{\rho_1} - G\frac{m_{S/C}m_G}{\rho_2^3}\vec{\rho_2} \tag{4.1}$$

Therefore the dynamics can be simplified as:

$$\ddot{r}_{S/C} = -G\frac{m_J}{\rho_1^3}\vec{\rho_1} - G\frac{m_G}{\rho_2^3}\vec{\rho_2} \tag{4.2}$$

Since the optimizer used, needs a dynamical system well defined for proceeding, the dynamics should be decomposed in the two axis of the problem, X and Y. This decomposition can be seen in Equations 4.3, 4.4, 4.5, 4.6.

In addition, for improving the accuracy of the model proposed, the mass variation with time was taken into account, introduced in the dynamical system by equation 4.7. Although it is a low thrust manoeuvre, the propulsion is taken into account and therefore its contribution on the mass variation. This effect is reflected in equations 4.5 and 4.6. Finally the dynamical system remains like:

$$\dot{r}_x = v_x \tag{4.3}$$

$$\dot{r}_y = v_y \tag{4.4}$$

$$\dot{v}_x = -G\frac{m_J}{\rho_1^3}\,\rho_{x1} - G\frac{m_G}{\rho_2^3}\,\rho_{x2} + \frac{T\ cos(\alpha)\ \pi}{m_{S/C}(t)} \tag{4.5}$$

$$\dot{v}_y = -G\frac{m_J}{\rho_1^3}\,\rho_{y1} - G\frac{m_G}{\rho_2^3}\,\rho_{y2} + \frac{T\ sin(\alpha)\ \pi}{m_{S/C}(t)} \tag{4.6}$$

$$\dot{m}_{S/C} = \frac{T\pi}{g_o I_{sp}} \tag{4.7}$$

Where:

- $\vec{r}$ is the position vector in the Cartesian coordinates of the spacecraft.

$$\vec{r} = r_{x1}\vec{i} \ + \ r_{y1}\vec{j}$$

- $\vec{\rho_1}$ stands for the vector respect to the first primary, Jupiter, $\vec{r}_1$.

$$\vec{\rho}_1 = \vec{r} - \vec{r}_1 = \rho_{x1}\vec{i} \ + \ \rho_{y1}\vec{j} = (r_x + (1-\zeta L))\vec{i} + r_y^2\vec{j}$$

- $\vec{\rho_2}$ stands for the vector respect to the second primary, Ganymede, $\vec{r}_2$.

$$\vec{\rho}_2 = \vec{r} - \vec{r}_2 = \rho_{x2}\vec{i} \ + \ \rho_{y2}\vec{j} = (r_x + \zeta L)\vec{i} + r_y^2\vec{j}$$

Being:

- $L = ||\vec{r}_1|| + ||\vec{r}_2||$ and it is the distance between the two primaries.
- $\zeta = \frac{m_G}{m_G + m_J}$ is a non dimensional parameter that relates both primaries

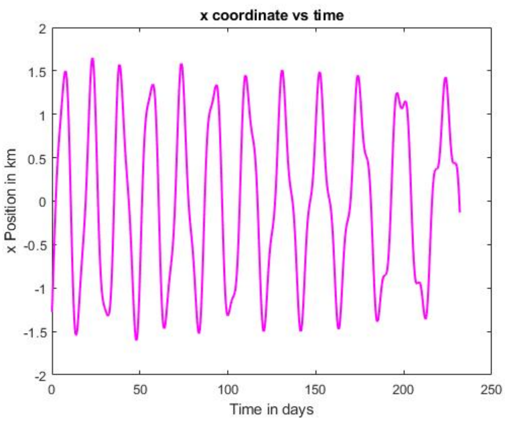
This dynamics are used during all the study for the cases where the inertial reference frame was decided to be used. In the other cases, an synodic reference frame was used.

4.3. *Synodic* reference frame, an alternative solution

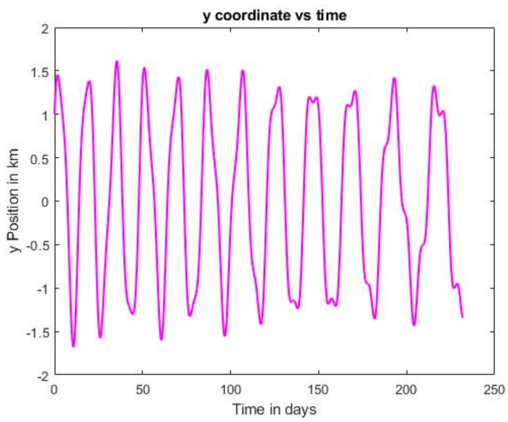
The manoeuvre proposed and the ones obtained, are a multi-revolution transfer. As showed at Figures 3.6a, 3.6b, 3.7 the duration of the transfer is significantly long, approximately 230 days, producing that the complexity of the position graphs is high, due to the numerous turns around the main primary, Jupiter. The number of revolutions in the initial guess is 20 and in the Q-LAW solution 19.

In consequence, a synodic reference frame apparently seems to simplify the initial guess by reducing the number of turns, thanks to its rotation, and therefore, it should be easier to obtain a solution, since the complexity is lower, the optimizer used should arrive to a solution faster. In addition, since the computation is faster, another opportunity appears, the accuracy of the model could be increased.

Before analysing the dynamics of the *synodic* frame, this premise should be proved. The following graphs will show the same data exposed in Figures 3.6a, 3.6b, 3.7 respect to this non-inertial reference frame, which has the same period as Ganymede:



(a) X coordinate vs time in days



(b) Y coordinate vs time in days

Fig. 4.2. Initial position vectors respect to the synodic frame

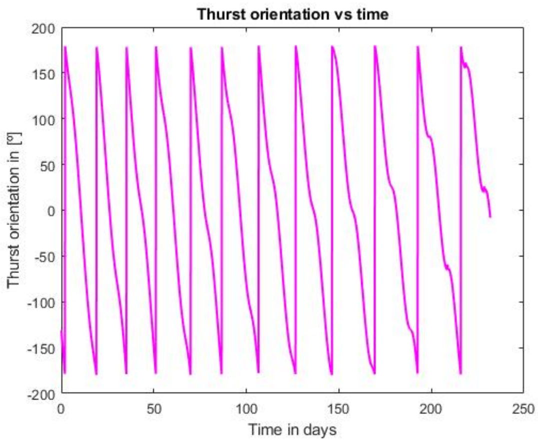


Fig. 4.3. Thrust orientation vs time in days

These plots agree with the premise presented before. Although it seems to be some instabilities on Figures 4.2a and 4.2b, the number of revolutions are reduced. Hence, it comes the opportunity of comparing both types of solution with the two initial guesses. An advantage that later could be used for obtaining conclusions and setting future work for the tool.

4.3.1. Dynamics for the *synodic* reference frame

After proving the validity of the synodic reference frame, its dynamics should be established. Like is a non-inertial reference frame and following the procedure done at Section 4.2, Non-inertial contributions should be taken into account in Equation 4.2:

$$\ddot{r}_{S/C} = -G\frac{m_J}{\rho_1^3}\vec{\rho_1} - G\frac{m_G}{\rho_2^3}\vec{\rho_2} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{\omega} \times \vec{\dot{r}} \tag{4.8}$$

With this information as main premise, the dynamical system for the *synodic* reference frame can be summarized in these equations:

$$\dot{r}_x = v_x \tag{4.9}$$

$$\dot{r}_y = v_y \tag{4.10}$$

$$\dot{v}_x = -G\frac{m_J}{\rho_1^3}\,\rho_{x1} - G\frac{m_G}{\rho_2^3}\,\rho_{x2} + \frac{T\ cos(\alpha)\ \pi}{m_{S/C}(t)} + \omega^2 r_x + 2\omega\dot{r}_y \tag{4.11}$$

$$\dot{v}_y = -G\frac{m_J}{\rho_1^3}\,\rho_{y1} - G\frac{m_G}{\rho_2^3}\,\rho_{y2} + \frac{T\ sin(\alpha)\ \pi}{m_{S/C}(t)}\omega^2 r_y - 2\omega\dot{r}_x \tag{4.12}$$

$$\dot{m}_{S/C} = \frac{T\pi}{g_o I_{sp}} \tag{4.13}$$

4.4. Non dimensional approach for the solutions

As the study was advancing with time, other main issue appeared while computing the solution, it was the computation time. This item although was tried to reduced with the change of reference frame to the synodic, at one point it was not enough.

For minimizing the computation time, the optimizer requires that all the variables of the dynamic system should be at the same magnitude. In space dynamics, the variety of values is significant: masses, distances, gravitational fields are some of the variables involved in the system and all of them have different order of magnitude.

In consequence, reducing all the variables to the same order of magnitude was crucial for the progress of this project. The best procedure to achieve this objective was to make all the variables non dimensional.

In order to complete this objective, one main variable was tended to stay as 1, the gravitational constant of Jupiter, $\mu_{Jupiter}$. Taking this statement as true for all the process the following definitions where done:

$$u = \frac{r_x}{l_c} \qquad v = \frac{r_y}{l_c} \qquad \tau = \frac{t}{t_c} \qquad \bar{\mu} = \frac{\mu}{\mu_{Jupiter}} \tag{4.14}$$

Where the c sub index reflects the characteristic variables of the problem. The characteristic distance is the orbit radius of Ganymede for the circular motion and the time is the one required for being $\mu_{Jupiter} = 1$:

Characteristic Variable	Value
Distance, l_c	1.0704 10^6 $[km]$
Time, t_c	9.8391 10^4 $[s]$ $\approx$ 1 day and 8 hours
Mass(S/C mass) m_C	1000 $[kg]$

TABLE 4.1. CHARACTERISTIC VARIABLES FOR NON-DIMENSIONALISATION

With these definitions the main variables are sketched: length, time, mass. Hence, it is possible reduce all the variables of the problem to dimensionless parameters following the Burckingham Π Theorem of dimensional analysis.

4.4.1. Non dimensional dynamics for the inertial frame

Applying the dimensionless changes taken into account at Equation 4.14 and the variables set at Table 4.1, the dynamical system showed at equations 4.3, 4.4, 4.5, 4.6 and 4.7 is changed as follows:

$$\dot{u} = \bar{v}_x \tag{4.15}$$

$$\dot{v} = \bar{v}_y \tag{4.16}$$

$$\ddot{u} = \frac{1}{||\bar{\rho}_1||^3}\, u - \frac{\mu_G}{||\bar{\rho}_2||^3}\, v + \frac{\bar{T}\ cos(\alpha)\ \pi}{\bar{m}_{S/C}(\tau)} \tag{4.17}$$

$$\ddot{v} = \frac{1}{||\bar{\rho}_1||^3}\, u - G\frac{m_G}{||\bar{\rho}_2||^3}\, v + \frac{\bar{T}\ sin(\alpha)\ \pi}{\bar{m}_{S/C}(\tau)} \tag{4.18}$$

$$\dot{m}_{S/C} = \frac{\bar{T}\pi}{\bar{g}_o\bar{I}_{sp}} \tag{4.19}$$

Where:

$$\bar{\rho}_1 = \frac{\rho_1}{L}$$

$$\bar{\rho}_2 = \frac{\rho_2}{L}$$

This process simplified significantly the complexity of the dynamic system and the computation time was reduced importantly.

4.4.2. Non dimensional dynamics for the *synodic* frame

Next, the synodic system should be non dimensional, leading with the following formulae:

$$\dot{u} = \bar{v}_x \tag{4.20}$$

$$\dot{v} = \bar{v}_y \tag{4.21}$$

$$\ddot{u} = \frac{1}{||\bar{\rho}_1||^3}\, u - \frac{\mu_G}{||\bar{\rho}_2||^3}\, v + \frac{\bar{T}\ cos(\alpha)\ \pi}{\bar{m}_{S/C}(\tau)} + u + 2\dot{v} \tag{4.22}$$

$$\ddot{v} = \frac{1}{||\bar{\rho}_1||^3}\, u - G\frac{m_G}{||\bar{\rho}_2||^3}\, v + \frac{\bar{T}\ sin(\alpha)\ \pi}{\bar{m}_{S/C}(\tau)} + v - 2\dot{u} \tag{4.23}$$

$$\dot{m}_{S/C} = \frac{\bar{T}\pi}{\bar{g}_o\bar{I}_{sp}} \tag{4.24}$$

Finally, the main physical background for the study has been exposed and the following step will be the presentation of all the results obtained and the validation of the tool.

5. RESULTS

Through all this section, the simulations done during the project will be shown. Two main groups of simulations can be differentiated as set at Chapter 3: the first taking as input the trajectory proposed at [3] and the one obtained with the Q-LAW program.

Both trajectories will be introduced in the optimizer in order to improve this solution or at least to reach it, proving that the optimizer is able to produce a more optimal solution or that the solution designed should be the best and the optimizer can reach the same output than other optimization tools, such us Q-LAW optimization.

In order to follow the same structure as the one used at Chapter 3, first, the solution based on the Solution proposed at [3] will be analysed, later the one designed by the Q-LAW program will be studied.

5.1. First simulations with the initial guess

In the beginning, it is convenient to describe which are the main assumptions taken for performing this very first analysis:

- The main objective of the transfer is to end up in a stable homoclinic invariant manifold for ending in a planar Lyapunov orbit of L2 Jupiter-Ganymede, with a final orbital radius of 1.0704 $10^6 km$ away from Jupiter[6].
- The trajectory proposed ends after 231.84 days which approximately is 231 days and 20 hours.
- The available thrust acceleration is 5 10^{-8} km/s. This fact is controlled thanks to the maximum available thrust force and the mass of the spacecraft lately exposed.
- Since the main objective is minimizing endurance of the mission, this manoeuvre is done at maximum available thrust.
- For this very first attempts, the influence of Ganymede will be assumed to be zero. This simplification will help to not having initial noise perturbations for the optimizer and reaching a solution faster for a one phase optimization.

[6]Note that this value is a 2-D approximation of the Ganimede's orbit.

As matter of fact, these assumptions will be the same for all the simulations done at this study. Keeping these variables constant for all the simulations such us mass of the spacecraft, initial position or final orbital elements, will help lately to perform a more complete analysis of the solutions.

In addition, the main numerical specifications for the mission derived from the assumptions, are exposed in the following table:

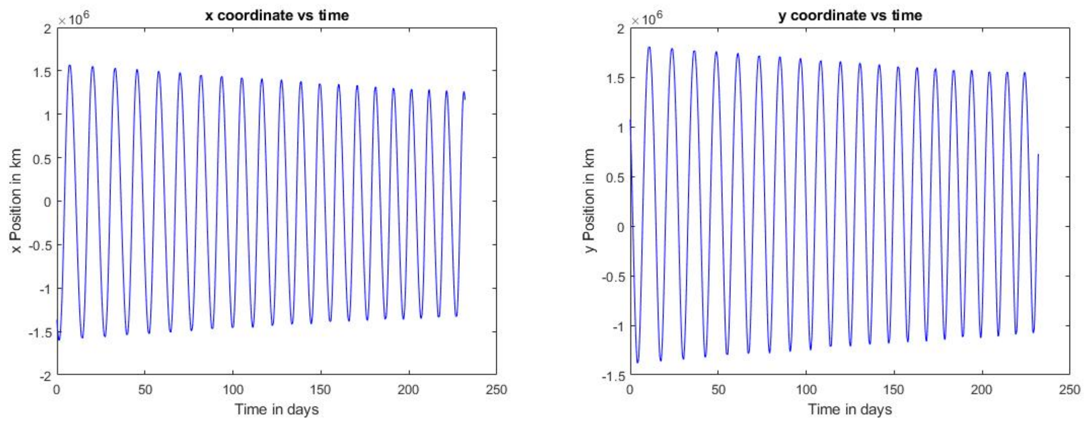
Variable	**Value**
S/C mass	1000 $[kg]$
Specific Impulse	2500 $[s]$
Maximum available thrust	0.5 $[N]$
Gravitational constant of Jupiter	1.2668 $[m^3/s^2]$
Initial radius, Callisto‘s orbital radius	1.8827 10^6 $[km]$
Final radius, Ganymede‘s orbital radius	1.0704 10^6 $[km]$

TABLE 5.1. MAIN PARAMETERS FOR THE MISSION

After setting the main parameters specified in table 5.1 and introducing the initial guess and the dynamics for the system specified in Chapters 3 and 4, the optimizer was run and the following results were obtained. Both solutions were analysed: the one with an inertial reference frame an the one with the *synodic* reference frame.

5.1.1. Initial guess optimization for the inertial reference frame

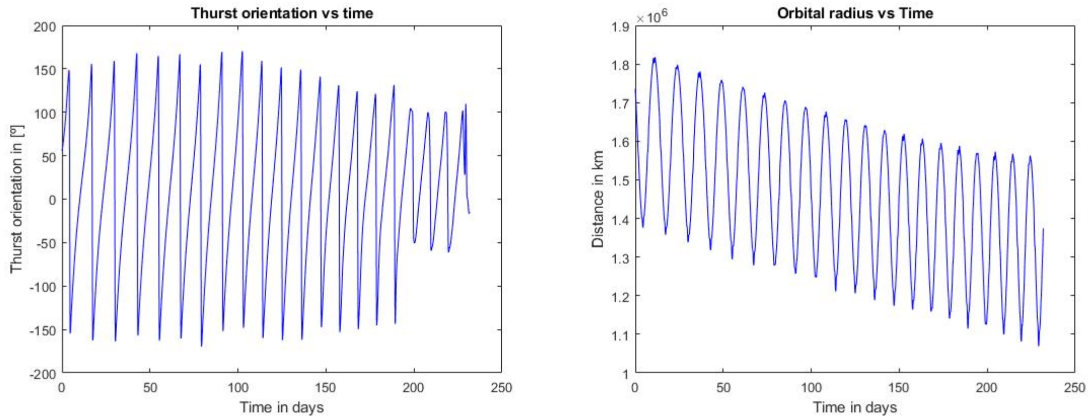
The order to proceed in all the sections where the solutions are plotted will be, first, a double plot of the evolution of the coordinates x and y with time and finally, a double plot of the evolution with time of the orbital radius and the thrust orientation.



(a) X coordinate vs time in days

(b) Y coordinate vs time in days

Fig. 5.1. Initial position values of the position vector



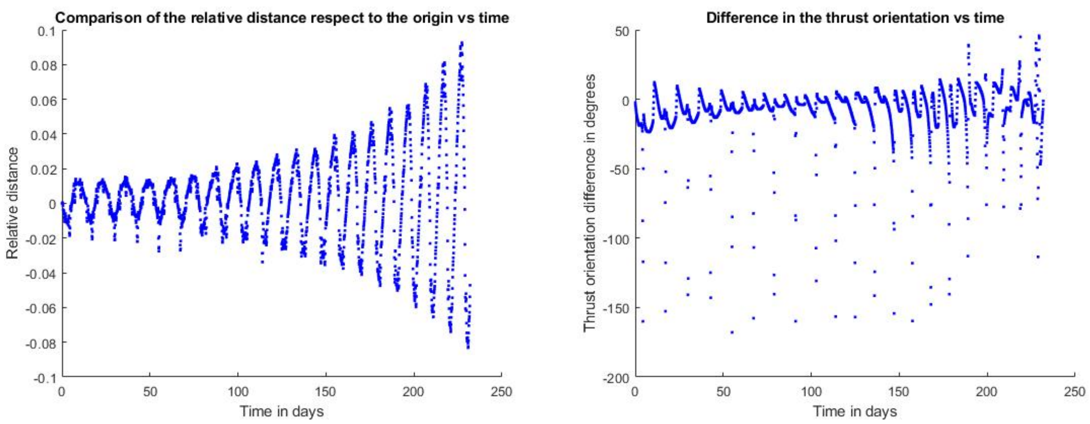
(a) Thrust orientation vs time in days (b) Orbital radius vs time in days

Fig. 5.2. Thrust orientation and orbital radius evolution

The exact value for the endurance of the mission after the optimizer is 232.33 days which approximately is 232 days and 8 hours. Therefore, the optimizer has return a solution that is almost similar to the one proposed.

Although it seems that the solution is nearly similar, this affirmation should demonstrated. One possibility is to show the difference between the values of the thrust orientation and the orbital radius for the initial guess and the solution obtained.

The orbital radius comparison figure will be plotted respect to relative values, in order to see clearly if the difference between solutions is significant. The initial guess value will be the reference value, since is the one to be optimized.



(a) Relative comparison between the orbital radius (b) Comparison between the thrust orientation angles

Fig. 5.3. Thrust orientation and orbital radius comparison

Both figures show that the performances of both missions are very similar until the last part of the mission. This ending phase which corresponds with the manoeuvre for reaching the ending position, apparently, has a different solution for the optimizer.

Analysing both graphs individually some aspects must be highlighted:

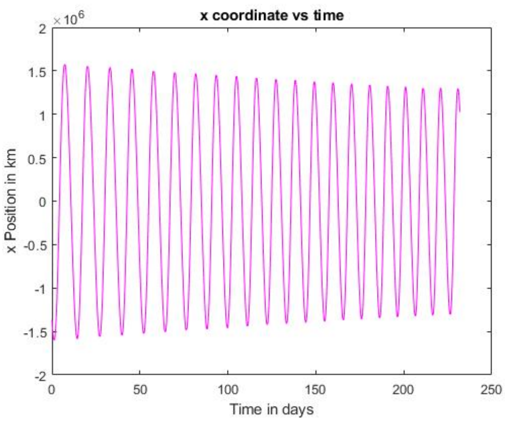
1. Figure 5.3a shows that the difference in the orbital radius do not exceed a 10% in all the mission. Moreover, in the first halve of the mission duration, this difference does not surpass the 2%.

2. Before analysing Figure 5.3b , it should be pointed out that those values that seemed to be away from the main set of points should not be taken into account. This difference is caused by the interpolation that was needed to do in order to being able to compare both solutions. This method makes that between each revolution[7], a higher number of points are located in this interval, therefore, when comparing both solutions some points are cut off from the original solution.

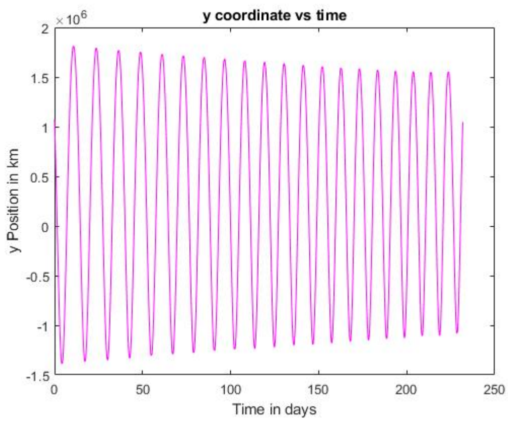
5.1.2. Initial guess optimization for the *synodic* reference frame

At first sight, it seems that the optimizer at least, is able to stablish the initial guess as the more optimal solution. The next step is to see if analysing the system respect to a synodic reference frame gets a solution in less time, since this method according to Chapter 4 should simplify calculations to the optimizer, since the initial guess itself is simpler.

After applying the change of reference system and the non inertial dynamics, the optimizer returned a solution whose mission endurance was 231 days and 16 hours approximately, which is sorter than the one acquired with the problem studied with an inertial reference frame.



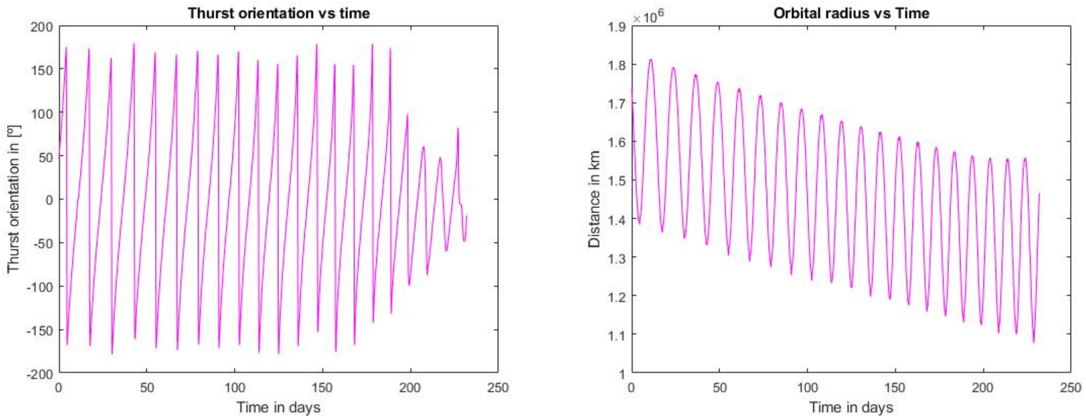
(a) X coordinate vs time in days



(b) Y coordinate vs time in days

Fig. 5.4. Initial position values of the position vector for the synodic method

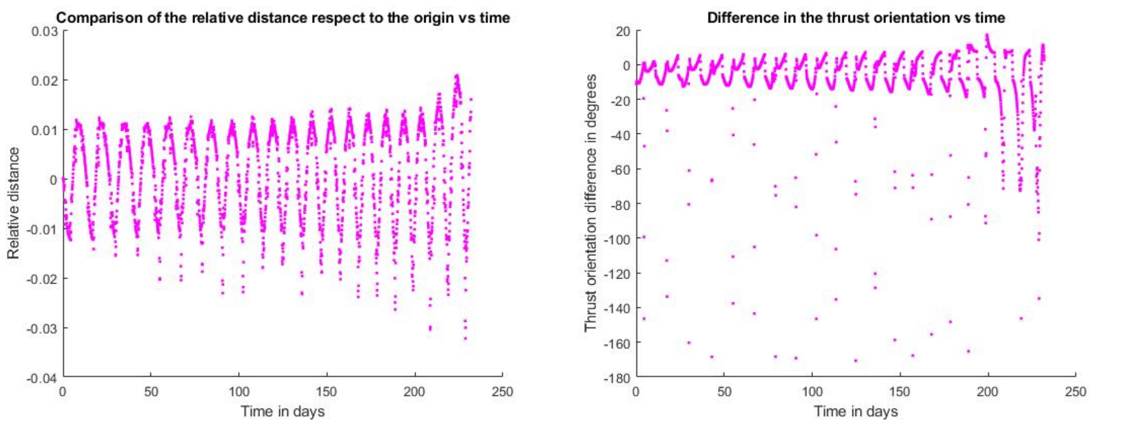
[7]Here represented with the abrupt change in the graph from 180 degrees to -180 degrees



(a) Thrust orientation vs time in days (b) Orbital radius vs time in days

Fig. 5.5. Thrust orientation and orbital radius evolution for the synodic method

Apparently, this method seems to follow the main objective of this study, showing the validity of the designed tool. Nevertheless, as performed previously, it seems convenient to observe the evolution of both solutions along time and its comparison. Hence, as sketched previously in Figure 5.3, both comparing plots were obtained.



(a) Relative comparison between the orbital radius (b) Comparison between the thrust orientation angles

Fig. 5.6. Thrust orientation and orbital radius comparison for the *synodic* method

At first sight, the difference between one solution and the other seems to be even more similar than the previous solution, reaching a maximum relative difference between both solutions at the orbital radius of a 3.5%.

At Figure 5.6b, again these remote points appear. However, if the graph is precisely evaluated it appears that the difference between the initial guess and the synodic solution is significantly small except at the ending part of the mission. Another time, the solution gives a different approach for the approximation to the final position.

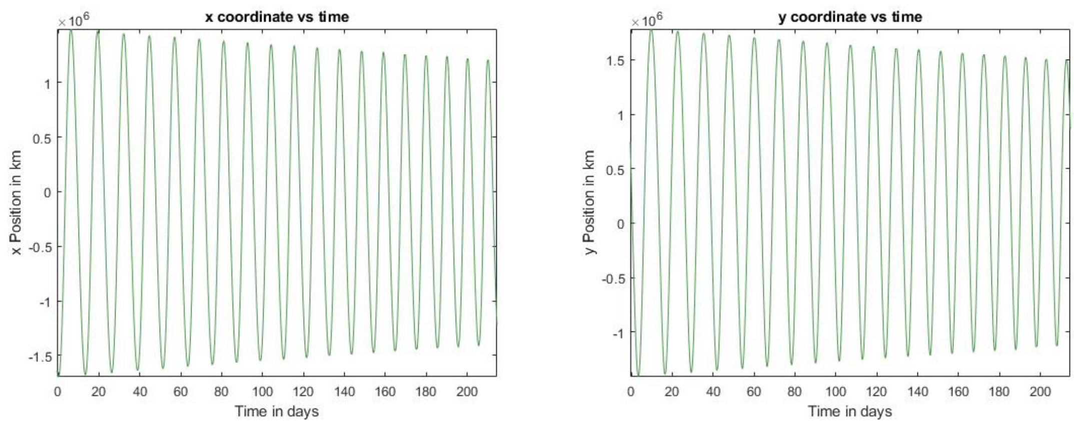
5.2. The Q-LAW optimization

After analysing the initial guess proposed and ending with a solution slightly better, the second possible solution should pass through the optimizer and see its effects.

As stated previously, this solution improves considerably the initial guess having a mission endurance of 214 days and 13 hours. This implies a 8% reduction of mission time respect to the initial guess.

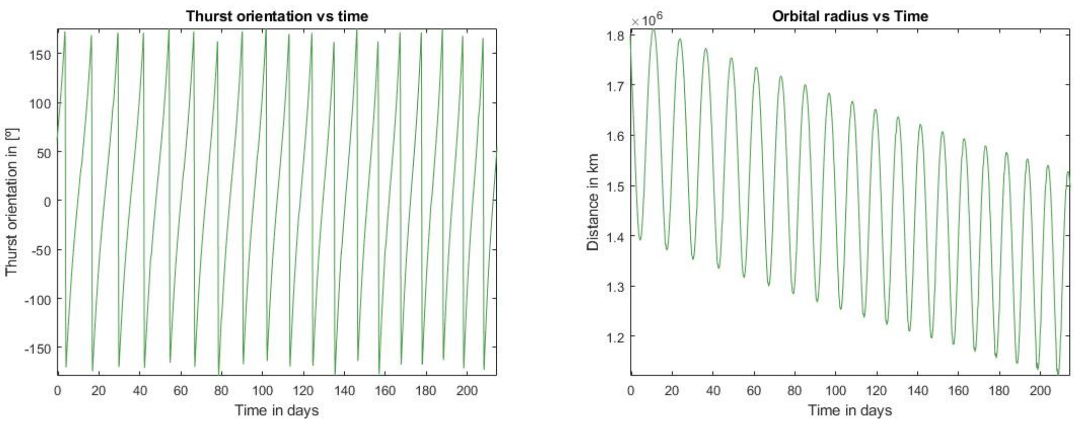
For this process the optimizer was only able to reach a plausible solution for the *synodic* method proving again, that using this reference system, helps the optimizer to work under better conditions.

The optimizer was not able to converge into a solution for the inertial problem, but with the same internal configuration and only changing the dynamics from an inertial problem to the non-inertial, the program finally reach the next solution.



(a) X coordinate vs time in days (b) Y coordinate vs time in days

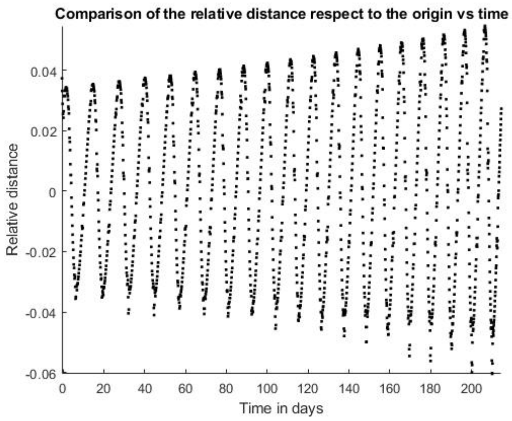
Fig. 5.7. Initial position values of the position vector for the synodic method of the Q-LAW



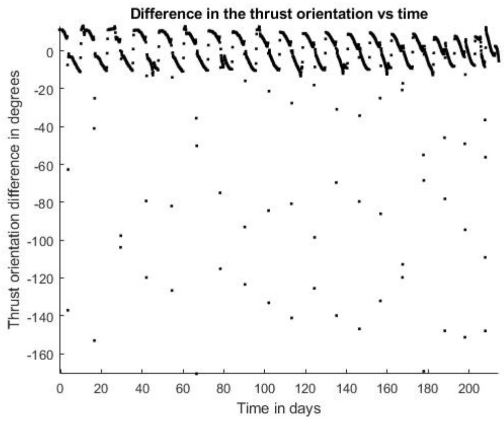
(a) Thrust orientation vs time in days (b) Orbital radius vs time

Fig. 5.8. Thrust orientation and orbital radius evolution for the synodic method of the Q-LAW solution

The solution extracted from the optimizer gave a mission endurance of 215 days, slightly worse than the original extracted from the Q-LAW. However, in order to confirm if both solutions are similar, again, a comparison between input and output should be done. This will show if at least the optimizer is able to give a similar solution to the one given[8]. Next, the same comparison done at previous sections is shown.



(a) Relative comparison between the orbital radius



(b) Comparison between the thrust orientation angles

Fig. 5.9. Thrust orientation and orbital radius comparison for the *synodic* method of the Q-LAW solution

Finally, these Figures show that the solution tends to be very similar to the one proposed by the Q-LAW. In addition, for the final part of the mission, the solution extracted is similar to the one proposed, on the contrary to the first solutions obtained.

5.3. Difference between both solutions

Before taking considerations about the solutions obtained, first, the difference between both solutions and in consequence the difference between initial guesses should be defined.

The initial solution is more an heuristic and analytical solution which means that although for its design was optimized, the trajectory used increments significantly its complexity, appearing different solutions with lower endurance. In addition, these types of solutions fits worse with the optimizer method, complicating the production of a solution. However, as results show, the solution seems to be suitable for the dynamics given.

Moreover, it is important to see that the solution proposed at [3] has an academic purpose, and for the industry point view probably will be less recommended than the Q-LAW solution.

[8]Please note that the Q-LAW is a well known method which already gives an optimized solution

On the other hand, Q-LAW guess is a numerical solution of the transfer program, which does not take into account the invariant manifolds dynamics. In consequence, it fits better with the optimizer tool not only because of its numerical nature, but also because of the fact that this solution is simpler than the previous one given, since it has less revolutions.

This factor makes a faster and more accurate solution for the same number of nodes chosen at the optimizer configuration, which in the end is a great advantage. This enables also to increase the number of nodes for reaching a more accurate solution.

For supporting this last assertion, a table of the time spent by the tool in order to produce a solution is given for all the solutions done in the study.

Reference frame	Input used	Nº of nodes	Nº of nodes per revolution	Computation time
Inertial	Initial Guess	600	30	50 - 60 minutes
Synodic	Initial Guess	500	25	30 - 40 minutes
Inertial	Q-LAW	650	34	Did not converge
Synodic	Q-LAW	500	26	10 - 20 minutes

TABLE 5.2. PERFORMANCE OF THE OPTIMIZER

This table sets the main limitation of the tool, time of computation. This limitation probably is given by the low capacity of computation of the device used.

Moreover, this table also represents two important facts:

- The synodic solution is easier to be produced as stated in Chapter 4. Lower number of nodes needed and lower computation time
- The Q-LAW initial guess is easier to be computed having a significant lower computation time.

6. CONCLUSIONS

After all the simulations done, it is time to analyse what has been obtained. So first, let's see which were the objectives proposed at the very beginning.

The main objective was to design an optimizing tool to then prove its capability of providing an efficient solution for a transfer problem.

Although during all the simulations a simplification of this problem was done ending with a two body problem, the solutions provided by the tool were near to the ones proposed, hence, it seems that the tool could achieve those solutions if the 3 Body dynamics were implanted.

In addition, it should be taken into account that the initial guess used was designed in order to be a very optimal solution, and the second guess obtained from the Q-LAW program is also an optimal solution, where time was tried to be minimized. Therefore, if a solution, which initially was not designed to be optimal, is introduced at the optimizer as an initial guess, it should end up with its optimal form.

Apart from that, it is important to highlight that the optimizer has been working under high restrictions, founded after the deep analysis done to the obtained data, such us:

- The initial and final positions were continuously fixed according to the initial and final values of the initial guess, being very difficult to reduce the endurance of the mission.

  For reducing significantly this parameter, typically a revolution of the trajectory should be avoided. This fact implies high computing duties for the tool, since it will need to get away from the initial guess in order to converge. In addition, the configuration of the program should be slightly modified for adapting to this change.

  Other possible option is to set as free the final position, but this will end with an incongruence with the method done for the first initial guess. However, for the initial guess from the Q-LAW optimization this alternative could be suitable, since its trajectory is not designed for a specific method.

- Since the computation time for each simulation[9] lasted significant period of time as sketched in Table 5.2 , some simplifications in the optimizer configuration were needed to be done, such us the number of nodes parameter.

  Since the computation time of the tool is directly related with the number of nodes that the optimizer used in order to achieve the optimal solution, the number of them were reduced for all the computations.

[9]The reader should take into account that the converged simulations were not done all at first try, adding a significant amount of time to the computing hours of the project.

This parameter is not only related with time, it is also strongly related with the capability of the optimizer to reach a solution and its quality. The number of nodes used make that the program divides the initial guess introduced in different intervals and then, tries to vary those intervals in order to achieve a solution.

For a multi-revolution problem as the one proposed, the parameters do oscillate several times during the whole mission life, specifically 20 times. Hence, and according to the experience obtained during the simulation process and expressed in Table 5.2, for providing at least a suitable solution, it should have 20 nodes for each revolution, what ends to a minimum number of 400 nodes.

In addition, the optimizer can perform multi-phase problems. This method divides the mission in different parts, where different configurations can be used such us dynamics, number of nodes and mathematical method.

This increases significantly the complexity of the problem but at the same time, it gives the opportunity of using a wide set of combinations for reaching different types of solutions. This element confirms that the tool is able to reach an answer for a widespread set of statements and gives to the program an important development capacity.

Finally, it can be stated that the study has reached its main objective, setting the DMG tool as a useful program for Mechanical dynamics optimization, even with the limitations given to the solutions proposed. These yields with a significant amount of future projects with this tool.

6.1. Future work

Before introducing this section where some of the possibilities that the DMG tool can have for the future is exposed, it is significant to highlight one important fact.

This program should have together with, it a good computation capacity, not only to improve the accuracy of the solution but also for reducing the computation duration. One of the most significant weaknesses of the study has been the computation time. This has handicapped significantly the possibility of improving the tool.

After that, the future works should be divided in order of complexity, ending with at first sight, the best solution that the optimizer can reach.

6.1.1. Adding the 2-D ephemerides of the primaries to the problem

There exists different tools that can provide the characteristics of celestial bodies along different periods of time, this includes variables such as position and velocities. Although generally these data is in three dimensions, a least squares solution should be able to give exact accurate data for the state of the primaries in the 2-D space.

This improvement opens new possible solutions for the system since the initial and final conditions can be replaced with positions depending on time, instead of fixed values. Hence, the optimizer could have more flexibility for reaching solutions than for the previous methods used.

On the other hand, this adds complexity to the calculations, therefore the equipment required for running the tool should improve as well.

6.1.2. Implementing the 3-D model to the model

All the study and the previous improvement has been done under the planar motion. One significant improvement, specially if it is desired to be extremely accurate according to reality, is the implementation of a 3-D system and a 3-D dynamics.

This possible solution, again increases complexity and probably more than the one stated before, but at the same time, opens more possibilities for reaching solutions, specially for the first initial guess. Having the 3-D model for the joins of the invariant manifolds together with the 3-D ephemerides, could give more possibilities for joining the trajectories than the ones specified in Figure 3.5, ending with a different designed mission, easier to be optimized.

6.1.3. Dividing the problem in different phases

This solution probably is the most convenient for this tool and it can works also for 2-D.

As set at the beginning of Chapter 5, the solution was needed to be reduced to a 2 Body problem in order to achieve a solution, since the noise data produce by the Final moon made some perturbations to the dynamics of the system that the optimizer could not lead with the, although it was not under the sphere of influence of the moon.

In addition, as set in Chapter 3 the initial guess had four phases where different dynamics were taken into account. The initial guess was an approximation of two different CR3BP where the two moon contributions were considered.

Hence, by dividing the problem in different phases, where the dynamics and configuration can be modified in each phase, it can open the optimizer to a new procedures.

6.1.4. The interplanetary problem

The main motivation of this study was to design a tool for a Mars-Earth transfer, a fact that finally was not able to be proved in the study.

An interplanetary mission by definition has different parts where different dynamics and different assumptions should be taken into account. Hence, in addition with the previous section, this need fits perfectly with a multiphase approach by the optimizer.

6.1.5. Reaching a complete optimization tool

Finally, the result after implementing these proposals, ends with a tool that could deal with any type of transfer mission.

In consequence, any designed transfer can end up with an optimization, which at the end could save high quantities of money to the trajectories developers, the university and the space industry.

BIBLIOGRAPHY

[1] WIKIPEDIA. (2019). Gps block iiia, [Online]. Available: https://en.wikipedia.org/wiki/GPS_Block_IIIA.

[2] S. X. Space Exploration Technologies Corporation. (2017). Falcon heavy, [Online]. Available: https://www.spacex.com/falcon-heavy.

[3] E. Fantino and R. Castelli, "Efficient design of direct low-energy transfers in multi-moon systems," *Celestial Mechanics and Dynamical Astronomy*, 2016.

[4] D. M. González. (2019). Dmg tool, [Online]. Available: https://github.com/uc3m-aerospace/DMG.

[5] A. V. Rao *et al.*, *User's manual for gpops version 2.3: A matlab software for solving multiple-phase optimal control problems using the gauss pseudospectral method*, 2009.

[6] F. Topputo and C Zhang, "Survey of direct transcription for low-thrust space trajectory optimization with applications," in *Abstract and Applied Analysis*, Hindawi, vol. 2014, 2014.

[7] W. Koon, M. Lo, J. Marsden, and S. Ross, *Dynamical Systems, the Three-Body Problem and Space Mission Design*, ser. Interdisciplinary Applied Mathematics. Springer New York, 2017. [Online]. Available: https://books.google.es/books?id=L0poAAAACAAJ.

[8] A. M. García, "Earth-moon low energy trajectory generation," Master thesis, Universidad Carlos III, 2018.

[9] G. I. Varga and J. M. S. Pérez, "Many-revolution low-thrust orbit transfer computation using equinoctial q-law including j2 and eclipse effects," *AIAA/AAS Paper*, pp. 15–590, 2016.

[10] Y. Lu *et al.*, "Design and optimization of low-energy transfer orbit to mars with multi-body environment," *Science China Technological Sciences*, vol. 58, no. 10, pp. 1660–1671, Oct. 2015.

[11] H. Kim, "Advanced mission design: Interplanetary super highway trajectory method," PhD thesis, 2015.

[12] D. M. González, "A comparative study of optimal control algorithms using ideal frame references," Degree thesis, Universidad Politécnica de Madrid, 2014.

[13] B. A. Conway, *Spacecraft trajectory optimization*. Cambridge University Press, 2010, vol. 29.

[14] J. Nocedal and S. Wright, *Numerical optimization*. Springer Science & Business Media, 2006.

[15] H. D. Curtis, *Orbital mechanics for engineering students*, eng, 4th ed. : Butterworth-Heinemann, 2019.

[16] D. Garg *et al.*, “A unified framework for the numerical solution of optimal control problems using pseudospectral methods,” *Automatica*, vol. 46, no. 11, pp. 1843–1851, 2010.

[17] J. T. Betts, “Survey of numerical methods for trajectory optimization,” *Journal of guidance, control, and dynamics*, vol. 21, no. 2, pp. 193–207, 1998.

[18] C. Conley, “Low energy transit orbits in the restricted three-body problems,” *SIAM Journal on Applied Mathematics*, vol. 16, no. 4, pp. 732–746, 1968.

[19] K. E. Davis, R. L. Anderson, D. J. Scheeres, and G. H. Born, “The use of invariant manifolds for transfers between unstable periodic orbits of different energies,” *Celestial Mechanics and Dynamical Astronomy*, vol. 107, no. 4, pp. 471–485, 2010.

[20] R. Fitzpatrick, “Newtonian dynamics,” 2011.

[21] R. L. Anderson and M. W. Lo, “Role of invariant manifolds in low-thrust trajectory design,” *Journal of guidance, control, and dynamics*, vol. 32, no. 6, pp. 1921–1930, 2009.

[22] G. Mingotti, F. Topputo, and F. Bernelli-Zazzera, “Optimal low-thrust invariant manifold trajectories via attainable sets,” *Journal of guidance, control, and dynamics*, vol. 34, no. 6, pp. 1644–1656, 2011.

[23] L. S. Pontryagin, *Mathematical theory of optimal processes*. Routledge, 2018.

[24] A. V. Rao, “A survey of numerical methods for optimal control,” *Advances in the Astronautical Sciences*, vol. 135, no. 1, pp. 497–528, 2009.

[25] V. Szebehely, “Theory of orbits: The restricted problem of three bodies,” Yale univ New Haven CT, Tech. Rep., 1967.

ANNEX A. GLOSARY

GPOPS	Gauss Pseudospectral Optimization Software
LEO	Low Earth Orbit
GEO	Geosynchronous Equatorial Orbit
L1	First Lagrange point of the system
L2	Second Lagrange point of the system
$3BP$	Three Body Problem
$CR3BP$	Circular Restricted Three Body Problem
NLP	Non Linear Problem
LP	Lagrange Point
LO	Lyapunov Orbit
S/C	Spacecraft
$r_{S/C}$	Position vector o the spacecraft
G	Universal Gravitational constant
m_J	Mass of Jupiter
$m_{s/c}$	Mass of the spacecraft
m_G	Mass of Ganymede
$\vec{\rho}$	Spacecraft-Primary position vector
v	Velocity vector
α	Thrust Angle respect to the reference system
t	Time variable
T	Thrust
π	Throttle
g_o	Gravitational acceleration of the Earth
I_{sp}	Specific impulse of the spacecraft
L	Distance between primaries
ζ	Primaries mass non dimensional parameter
ω	Angular velocity of the reference system
u	Non-dimensional x component of the position vector
l_c	Characteristic length
v	Non-dimensional y component of the position vector
τ	Non-dimensional time variable
$\bar{\mu}$	Non-dimensional gravitational constant
t_c	Characteristic time
$\mu_{Jupiter}$	Gravitational Constant of Jupiter