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“Four-Stroke Internal Combustion Engine Crankshaft Fundamentals”

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ABSTRACT

This thesis covers general topics concerning the forces, oscillations, manufacturing processes, and design trends of a four stroke internal combustion engine crankshaft and the prior knowledge required to obtain a precise understanding of the crankshaft working conditions and the parameters that affect said conditions.

Prior to defining the different forces acting on a crankshaft, basic piston dynamics are explained to establish relations between piston movement and crankshaft rotation since piston and connecting rod masses are a large source of force on the crankshaft. Geometric relations are used to derive kinematic equations that describe piston motion. By considering an ideal Otto cycle and the piston motion equations, a relation between cylinder gas pressure and crankshaft rotation is obtained. These equations are then used to derive the inertial and gas forces acting on the crankshaft.

These equations are then used to create a software tool via MATLAB that allows users to input certain engine characteristics and obtain several graphs illustrating torques, piston motion, and gas pressure. The user can change independent parameters to observe how these affect different graphs.

Manufacturing processes and their advantages and disadvantages are covered, alongside the different materials, their characteristics, and applications. Finally, the iterative crankshaft design process and key design focus points and their corresponding limitations are covered.

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1. INTRODUCTION

The purpose of this thesis is to achieve a better and more concise understanding of the principals that make up a crankshaft. The main topics that will be covered are the different configurations and architectures found in modern day four stroke engine crankshafts, the different manufacturing processes commonly used, a review of the basic piston motions that are essentially the source of loads present on the crankshaft, a further look into the forces and vibrations generated by the piston motions, concepts and techniques utilized to balance crankshafts, various crankshaft configurations, and the development of a software tool using MATLAB to calculate the different kinematics and dynamics involved in a cranktrain assembly. The mathematical expressions covered in this thesis will be based on a single cylinder engine; however, the information and basis covered by the rest of the topics is applicable to engines with multiple cylinders. In addition, many studies performed on multi-cylinder engines are done by simplifying the engine in question into a small portion containing a single cylinder, and, consequently, a single portion of the crankshaft.

1.1 Objectives

The overall objective of this thesis is to derive and analyze the kinematics and dynamics involved in the crankshaft and its components to achieve an understanding of how they are influenced by each other to later apply this knowledge during the design phase. To do this, the various kinematics of the piston-connecting rod assembly will be explained in detailed by studying existing graphs that illustrate piston position, velocity, and acceleration and will afterwards be supported by deriving the kinematic equations. Once the piston-connecting rod kinematics have been covered, the dynamics of the crankshaft will be studied together with the derived piston kinematics; first by defining different forces acting on the crankshaft and then by obtaining equations formulating these forces and their consequent torques. The main objective of this part is to obtain a relation between the kinematics of piston-connecting rod assembly and the dynamics suffered by a crankshaft to be able to create a software tool via MATLAB that will enable the kinematic and dynamic analysis of any engine given its characteristics.

After covering design considerations, crankshaft configurations, and manufacturing processes one should be able to determine the adequate procedures needed to design a

successful crankshaft by selecting initial dimensions, a crankshaft configuration in compliance with the engine, appropriate material, and its manufacturing process.

1.2 The Crankshaft

The core of any reciprocating engine is the cranktrain, made up of the crankshaft, connecting rods, pistons, and the flywheel. Even though this thesis will be focused primarily on the crankshaft, connecting rods and their configurations will also be addressed. “A crankshaft is a mechanical part able to perform a conversion between reciprocating motion and rotational motion” [1]. The main purpose of this conversion is to create useful work. In an internal combustion engine, the reciprocating motion of the pistons connected to the crankshaft via connecting rods is transformed into rotational motion which is eventually transferred to the wheels of the vehicle. In the case of a piston compressor, the rotational motion (provided by either a viscous fluid, electric motor, etc.) is converted into reciprocating motion consequently compressing a fluid.

A crankshaft is made up of at least two centrally located coaxial axes, known as “main” journals, and one or more offset axes, known as “rod” or “crankpin” journals or crank throws. The main journals get their name since they house the supporting or “main” bearings which attach to the crankcase. As the main journals rotate around its own axis, the crankpin journals describe a circle whose diameter is twice the size of its offset with the main journals. This diameter described by the crankpin journals is also the engine stroke, which is the distance the piston travels in its cylinder.

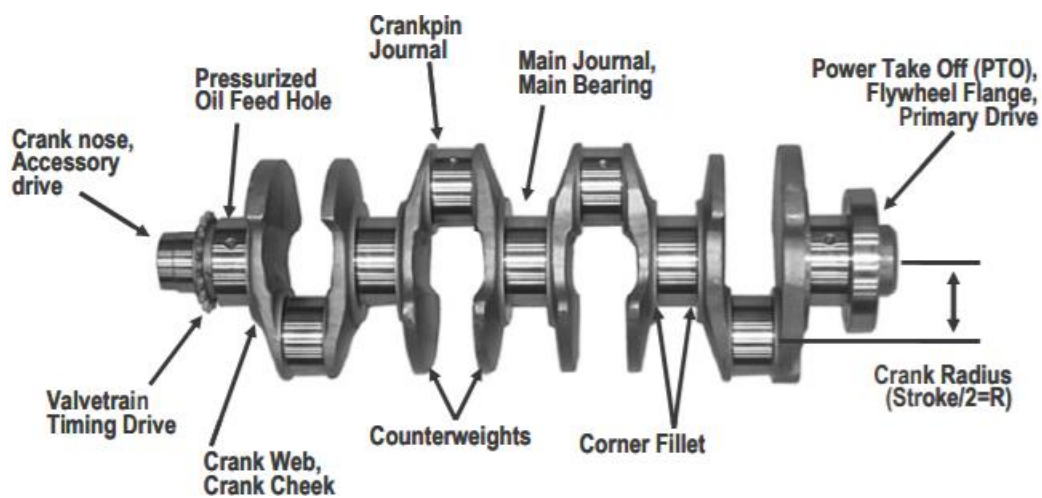


Fig. 1.1: Representation of a crankshaft and its important components [2, p. 16-2]

As seen in the figure above, the primary drive typically has a flywheel attached to reduce characteristic four-stroke engine vibrations and to conserve inertia maintaining smoother power delivery to the rest of the vehicle components.

The connecting rod, also known as conrods, is an important component of the crankshaft since it exerts forces on the crankshaft of important characteristics that will later be studied during the analyses portions of the thesis. The simplest representation of a connecting rod is a beam with two pins on both of its ends as shown in figure 1.2. The end of the connecting rod attached to the crankshaft suffers high rotational speeds while the opposite end, attached to the piston, suffers from lower rotational speeds. It is one of the components that suffers the most stress in the engine. [2, p. 16-2]

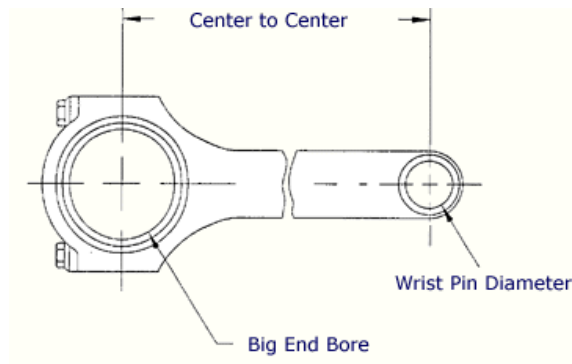


Fig. 1.2: Representation of a simple connecting rod [3]

2. INTERNAL COMBUSTION ENGINE BASICS

An internal combustion (IC) engine utilizes a piston to compress a fuel mixture and turn it into mechanical energy through its combustion. This combustion is induced by either a spark or by high compression, otherwise known as a spark ignition engine and compression ignition engine, respectively. To achieve fuel mixture combustion, generating power, an IC engine goes through a cycle composed of several strokes. The two most common are two stroke and four stroke IC engines. As mentioned earlier in the introduction, only four stroke IC engines will be considered. The next figure demonstrates a simple IC engine architecture and components:

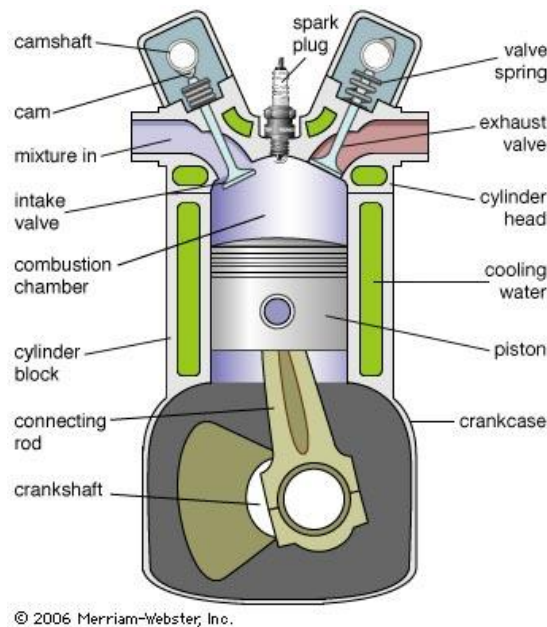


Fig. 2.1: Spark ignition IC engine [4]

The four strokes of an internal combustion engine are: intake, compression, ignition (power), and exhaust. In a four-stroke engine, to achieve one power cycle four strokes are required. One stroke is equivalent to a 180° rotation of the crankshaft; therefore, to complete four strokes (or one power cycle) the crankshaft must rotate twice or 720° . [5] When the piston is at its highest position, it is known as top dead center or TDC, and when it's at its lowest position, it is known as bottom dead center or BDC.

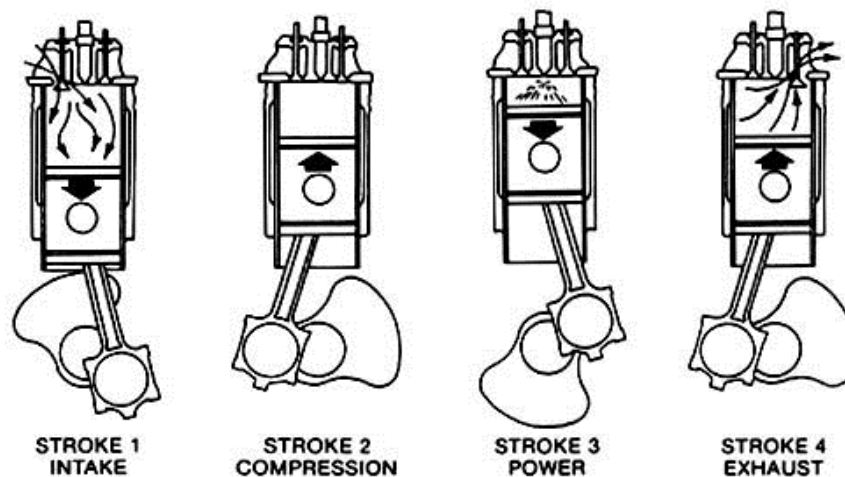


Fig. 2.2: IC engine strokes. Note: valve timing is also considered in the figure. [6]

During the first stroke, intake, the piston starts at TDC and begins to descend towards BDC. As the piston descends, the intake valve opens and an air-fuel mixture (for a spark ignition IC engine) is absorbed into the cylinder. Once the piston reaches BDC, the intake valve closes and the piston changes direction towards TDC. As of now, the crankshaft has rotated 180° . As the piston ascends, the air-fuel mixture is compressed causing the temperature inside the cylinder to increase. When the piston reaches TDC the compression stroke is complete, and a spark is produced provoking an explosion in the combustion chamber (cylinder volume at TDC). This explosion releases a high amount of energy which accelerates the piston back down towards BDC completing the power stroke. Finally, the piston starts to ascend again and at the same time the exhaust valve is open to be able to release the combustion products. The piston has traveled a total of four stroke or 720° of crankshaft rotation to produce one power cycle.

This previously described cycle is summarized for an ideal IC engine. In reality, the valve timings do not occur instantly, nor do they occur right when TDC or BDC is reached. The spark is not ignited at TDC either; generally, it occurs before TDC to help obtain the maximum piston acceleration since the combustion of all the air-fuel mixture doesn't happen instantaneously. The firing of the spark before TDC is known as advanced ignition timing and is not a constant value throughout the RPM range of an engine. The advanced ignition timing must occur even earlier as the RPMs increase to compensate the time it takes to complete the combustion.

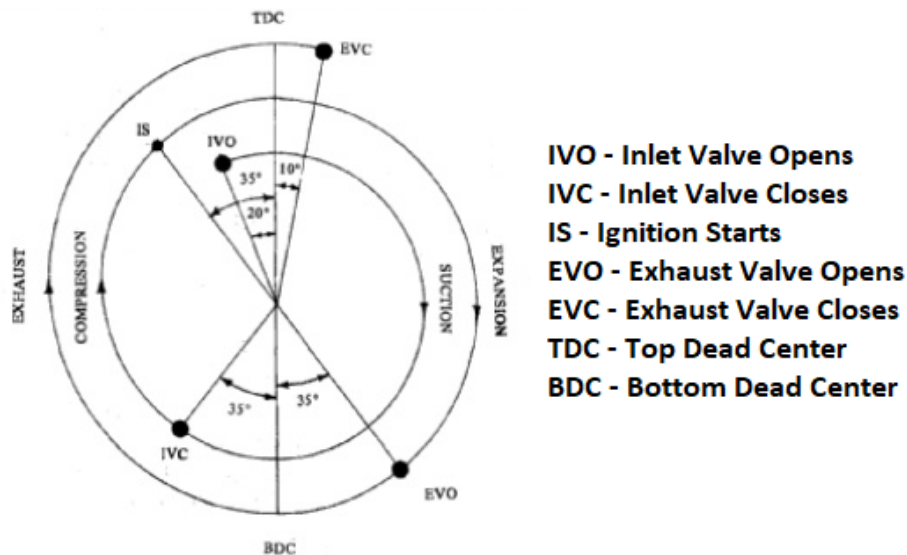


Fig. 2.3: Valve and ignition timing for a four stroke spark ignition IC engine [7]

Figure 2.3 demonstrates the timing distribution of a spark ignition IC engine. Note how the valve openings and closings occur earlier and later than described previously. The reason behind this is mainly due to the inertia of the gases inside the cylinder which need time to accelerate and flow. As a result, at the end of the exhaust stroke and beginning of the induction stroke both intake and exhaust valves are open at the same time. This is known as valve overlap. Also, the ignition occurs 35° before TDC. Advanced ignition timing is crucial for engine performance and for determining piston acceleration which translates into forces that act upon the crankshaft.

To evenly distribute the power cycles, these 720° are divided by the number of cylinders. It is natural to distribute the power cycles to not only have a constant delivery of power but to also reduce vibrations produced during the operation.

3. PISTON DYNAMICS BASICS

The study of piston motion and dynamics is crucial to understand the forces acting on the engine, specially the crankshaft. The crankshaft, connecting rods, and pistons make up the cranktrain which is the mechanism that captures the energy liberated upon combustion of the fuel mixture in the combustion chamber. The fuel mixture is injected, compressed, and ignited within the combustion chamber producing a large amount of thermal energy. This thermal energy is absorbed and transformed into work by the linearly travelling piston which in return transfers this work into the crankshaft through the connecting rods. This thermal energy which makes its way to the crankshaft is main reason behind the stresses produced in the cranktrain components. More importantly, the way the piston moves due to the thermal energy released upon combustion causing said stresses is going to be the main topic of this section.

3.1 Piston-connecting Rod Assembly

The connecting rod is made up of two bores or pins as seen in figure 1.2. The smaller of the two bores is attached to the piston via a cylindrical pin, known as wristpin. As a result, the bore and wristpin are constrained in the cylinder's axis and follows the piston's linear movement; consequently, maintaining the linear reciprocating motion since the connecting rod's big bore is rotating around the rod journal bearing (bearing on the crank throw).

3.2 Piston Motion Characteristics

The characteristics of the piston motion is directly related to the crankshaft-connecting rod mechanism geometry. Firstly, it is important to clarify two key positions the piston has within the cylinder: top dead center and bottom dead center. Top dead center, TDC, is the position known when the piston is at its highest point within the cylinder (when the volume in the cylinder is smallest), and bottom dead center, BDC, is the position known when the piston is at its lowest point within the cylinder (when the volume in the cylinder is largest).

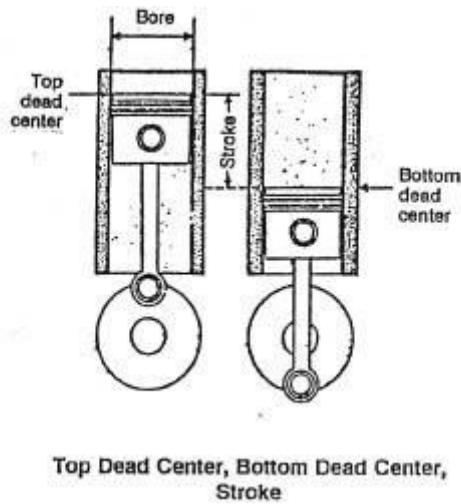


Fig. 3.1: TDC vs BDC and how the stroke of the engine is related to these positions. [8]

An important geometrical characteristic in the crankshaft-connecting rod-piston mechanism is the ratio between connecting rod length and the engine stroke, known as rod to stroke ratio. The rod to stroke ratio is decisive when it comes to vibrations, balance characteristics, performance characteristics, and piston motion asymmetry. [9] In order to generalize further explanations and clarifications, the following descriptions will be based on a zero wristpin offset cranktrain configuration. A zero wristpin offset is when the axis of symmetry of the cylinder intersects with the center of the crankshaft. However, general descriptions are also applicable to non-zero offset wristpin configurations as well.

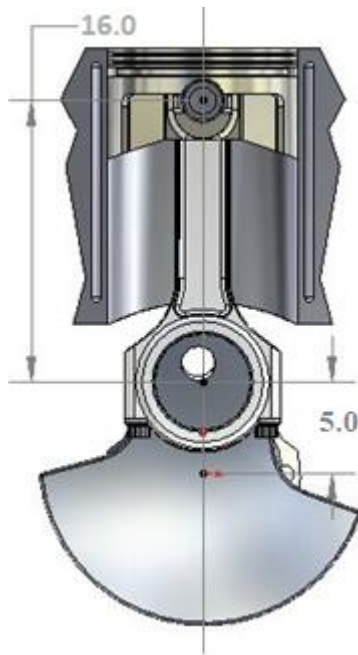


Fig. 3.2: Example cranktrain mechanism with zero wristpin offset. Note: measurements are in centimeters [10]

The cranktrain mechanism in figure 3.2 will serve as an example for the upcoming concepts. In this case, the rod length is 16 cm and the stroke is the crankshaft diameter, 10 cm; therefore, the rod to stroke ratio is $16/10 = 1.6$.

Piston motion asymmetry refers to the unequal length the piston travels from TDC to BDC. In other words, the motion of the piston 90° before and after TDC is **not** the same as the motion of the piston 90° before and after BDC. When the crankshaft is within 90° from TDC, the rotation of the crankshaft displaces the piston more than when the crankshaft is within 90° from BDC. This motion asymmetry is an important factor when it comes to properties such as engine operation, performance, and engine durability.

To further elaborate on piston motion asymmetry, the next two figures demonstrate the distances travelled when the piston moves from TDC to 90° clockwise and then to BDC. [10]

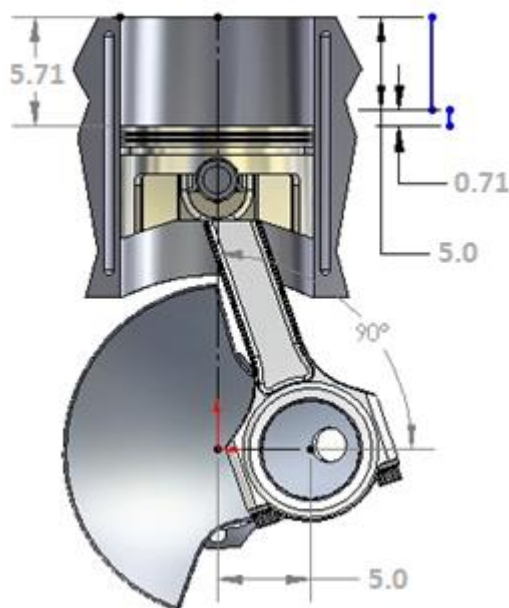


Fig. 3.3a: Piston travel dimensions from TDC to 90° . Note: Measurements in centimeters [10]

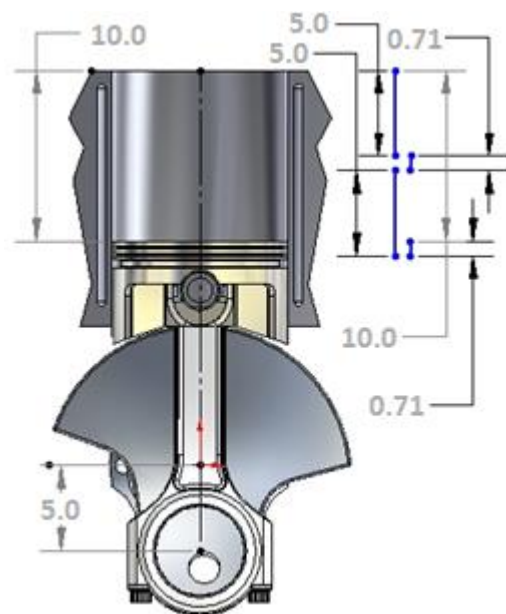


Fig. 3.3b: Piston travel dimension from TDC to 180° . Note: Measurements in centimeters [10]

Figure 3.3a represents the piston travel when the crankshaft rotates from TDC to 90° . On the right, the distance traveled is shown to be 5.71 cm which is 0.71 cm more half of the engine stroke; in other words, 57% of the engine stroke. The reason behind this travel distance is because as crankpin moves downwards 5 cm, it also moves horizontally 5 cm which puts the connecting rod at an angle with the vertical plane. This is known as the

cosine effect. If the length of the connecting rod was to be projected onto the vertical plane, the length of the connecting rod's vertical projection would be 15.3 cm, 0.7 cm shorter than 16 cm. This dynamic "shortening" of the connecting rod effectively adds 0.7 cm to the 5 cm downward motion produced by the rotation of the crank throw, as seen on the right side of figure 3.3a.

Since during the first 90° of the piston traveled 57% of the stroke, it is evident that after the next 90° the piston will only have to travel 43% of the stroke until it is at BDC. The reason behind this is the same as mentioned before: as the crank throw rotates and moves downwards 5 cm, it also travels horizontally. However, this time it travels horizontally back towards the center of the crankshaft, and, thus, "regaining" the actual length of the connecting rod. Again, the cosine effect is the culprit, but this time it acts as if the connecting rod lengthens, regaining back the 0.7 cm it has lost previously, since the horizontal motion is now inwards. As a result, since the connecting rod "gains" 0.7 cm, the half-stroke only has to cover 4.29 cm. This description can be seen in figure 3.3b where to the left the dimensions of the entire stroke is found, and to the right blue lines indicate the half-stroke distance travelled.

The crankshaft rotation at which the half-stroke occurs depends on the rod to stroke ratio. In the example used, with a rod to stroke ratio of 1.6, the half stroke occurs at 82° of rotation after TDC, as seen in figure 3.4. As the rod to stroke ratio increases, the crankshaft position for a half-stroke becomes closer to 90°.

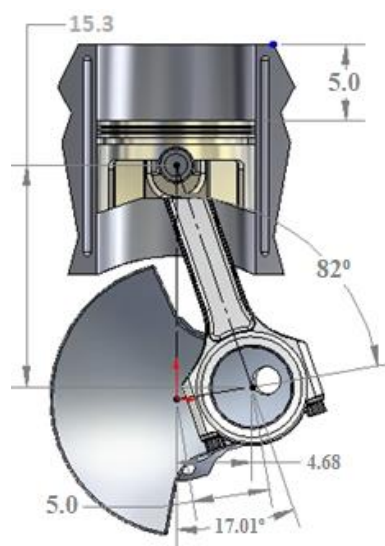


Fig. 3.4: Half stroke position and dimensions. Note: dimensions are in centimeters [10]

In conclusion, the connecting rod is not parallel to the cylinder's axis of symmetry other than at TDC and BDC. As a result, the cosine effect is present whenever the piston is not at TDC or BDC. This causes the effective length of the connecting rod at any position other than BDC and TDC to be the connecting rod's center-to-center length multiplied by the cosine of angle between the connecting rod and the vertical plane. For this example, the effective length of connecting rod at half stroke would be the following:

$$16 \text{ cm} * \cos(17.01^\circ) = 15.3 \text{ cm} \quad (3.1)$$

These 15.3 cm are represented to the left of figure 3.4.

3.3 Piston Velocity

Velocity is the instantaneous rate of change of position. In many applications, velocity is measured with respect to time as the reference variable. However, when it comes to piston velocity, it is more practical and intuitive to use crankshaft rotation as the reference variable since this way it is easy to see how the crankshaft degree of rotation affects the piston velocity. By definition, a linearly reciprocating motion suffers from constant velocity change. In the case of piston travel, the velocity of the piston is 0 at TDC and BDC and changes in between these two positions. The maximum velocity of the piston is strongly dependent on the rod to stroke ratio. [10]

Figure 3.5 represent the piston position where maximum velocity occurs for the example configuration used.

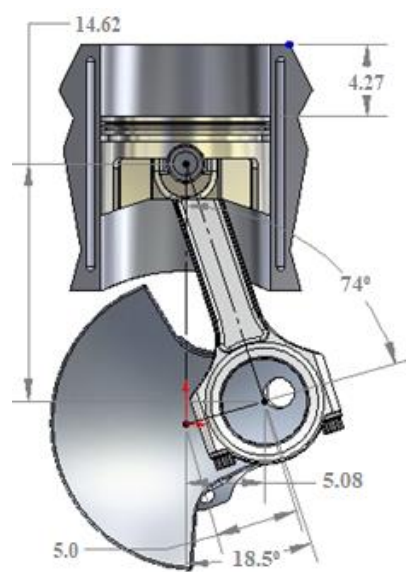
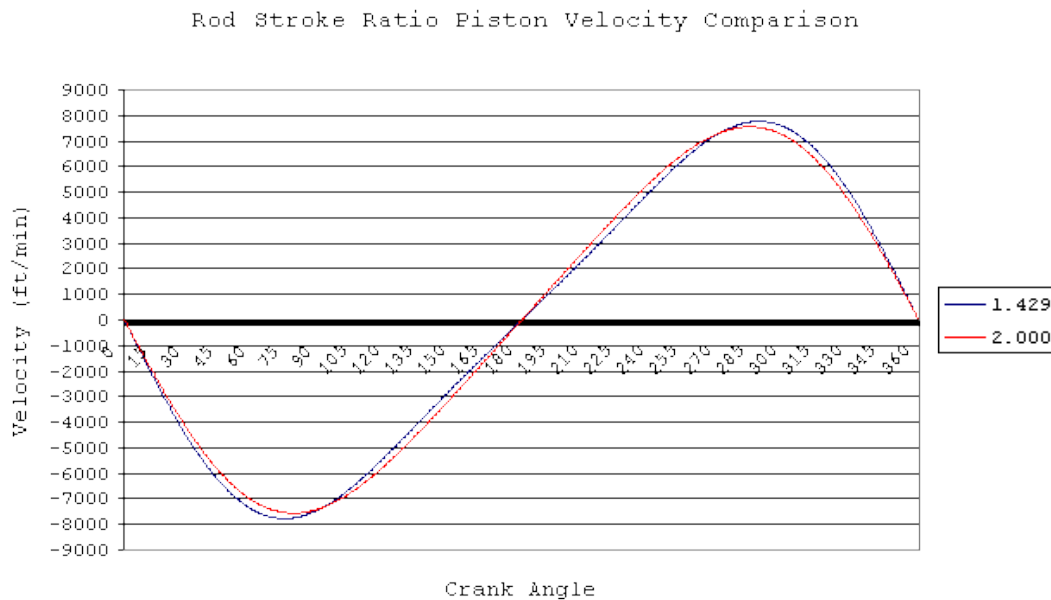


Fig. 3.5: Maximum velocity position. Note: measurements in centimeters [10]

The example configuration reaches maximum velocity 74° of crankshaft rotation before and after TDC. Given a certain engine speed in RPM, the peak piston velocity can be calculated (this will be done in a later section).

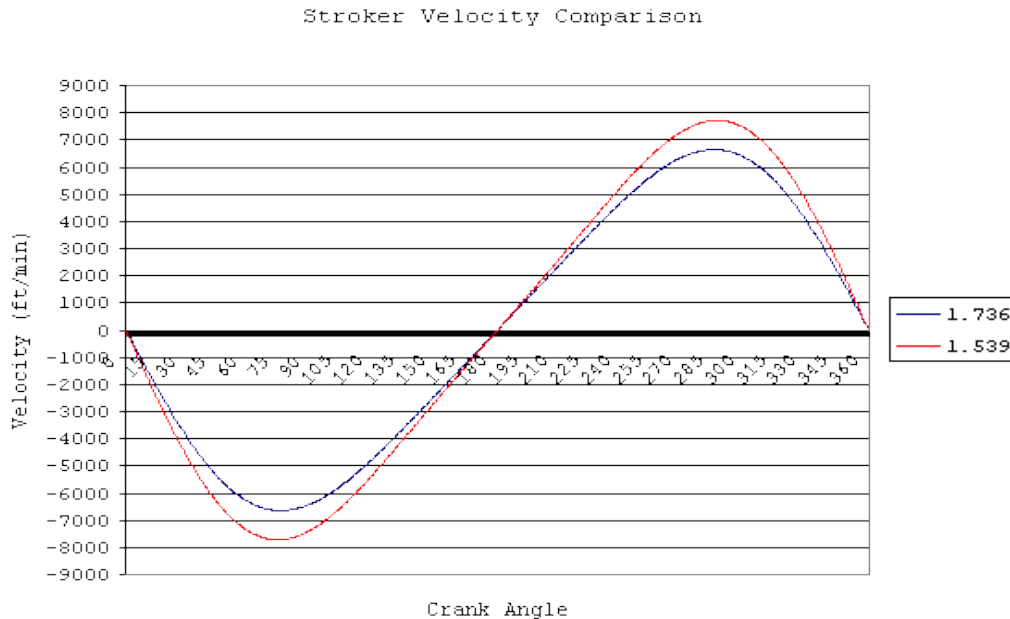
As mentioned earlier, the rod to stroke ratio influences the maximum velocity and it does so in the following manner: a smaller rod to stroke ratio achieves a higher maximum velocity, and the maximum velocity occurs closer to TDC than a larger rod to stroke ratio. This can be observed in graph 3.1. How does changing the connecting rod length and the stroke, individually, affect velocity? Shortening the connecting rod length results in increased piston velocity, but also increases tensile stresses on the connecting rod. Increasing the engine stroke as a similar effect; it results in an increase of piston velocity, but also an increase of stress on the connecting rods. [11]



Graph 3.1: Rod to stroke ratio piston velocity comparison at 8000 RPM. Red line corresponds to a rod to stroke ratio of 2 while the blue line corresponds to a rod to stroke ratio of 1.429. The horizontal axis represents the crankshaft rotation angle in degrees. [11]

The previous graph compares two engines with the same stroke of 9 cm, but with different connecting rod lengths. The engine with the rod to stroke ratio of 1.429 has a rod length of 13 cm while the ratio of 2.000 has a rod length of 18 cm. In conclusion, reducing the connecting rod length, not only increases maximum velocity, but it also causes the maximum velocity to happen sooner.

A common practice in engine tuning is called “stroking” an engine. The practice of stroking an engine is increasing the engine stroke to gain more power by increasing the engine capacity. However, this generally requires shortening of the connecting rod due to geometrical constraints within the engine. Therefore, how would the piston velocity be affected if the connecting rods were to remain unchanged, but the stroke of the engine was increased? To answer this question, the use of the next graph is required:

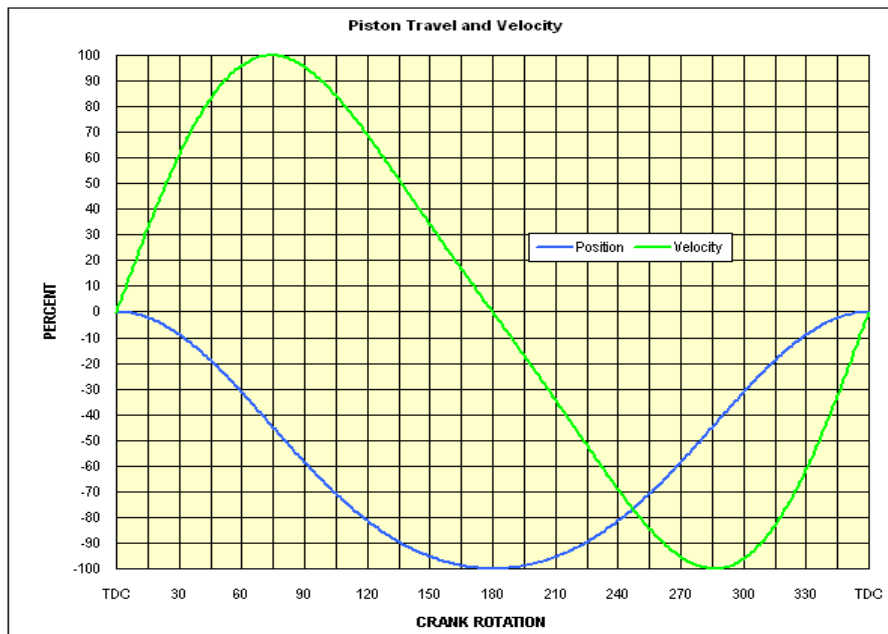


Graph 3.2: Velocity comparison between two engines with the same connecting rod length but different strokes and their corresponding rod to stroke ratio on the right. Both engines at 8000 RPM. The horizontal axis represents the rotation angle of the crankshaft in degrees. [11]

Graph 3.2 compares to engines with the same connecting rod length and different strokes with their respective rod to stroke ratios. From the graph one can observe that the red line, which corresponds to the engine with a larger stroke (lower ratio), has a higher maximum velocity than the engine with the blue line. These results coincide with the description previously stated that increasing the stroke would cause an increase in piston velocity. Even though it may seem as if the point of maximum velocity has remained unchanged, it has slightly shifted. The position of the maximum velocity point depends on the rod to stroke ratio and not just the connecting rod length. In the case of graph 3.2, both engines' rod to stroke ratios are very similar resulting in a small difference in the maximum velocity point. Therefore, to achieve higher maximum velocity and maintain the maximum velocity point, the stroke must be increased while keeping the same rod to stroke ratio (and the same RPM). [12]

The maximum velocity and its position during the cycle has great importance on engine performance. As the rod to stroke ratio is reduced, the point of maximum piston velocity moves closer to TDC, and the maximum piston velocity increases, this results in a stronger induction pulse which in turn is a determining factor for the optimization of the camshaft lob profiles used in the engine at a certain RPM range.

To achieve a visual comparison between piston displacement and velocity, the two can be graphed together with the crankshaft rotation as the reference. However, the graph can become more intuitive for the view if the vertical axes is in terms of piston displacement and velocity percentage. Therefore, 100% would represent maximum velocity while 0% signifies no velocity, and for the displacement the principle applies: 0% indicates TDC while 100% represents BDC.



Graph 1.3: Piston travel and velocity in percentage in terms of maximum displacement and velocity, respectively. Blue line represents piston position while green line represents piston velocity. Note: the negative values for the displacement percentage have been used for a more intuitive approach (the negative sign should be ignored). [10]

In graph 3.3, the positive velocity percentage represents piston travel towards the crankshaft while negative velocity percentage represents travel away from the crankshaft. First, it is important to observe that when the piston is in TDC and BDC ($0^{\circ}/360^{\circ}$ and 180° , respectively) the piston velocity is zero. This is trivial since the piston must come to a complete stop to change direction. Graph 3 also coincides with the half-stroke and

crankshaft rotation relation made in figure 12. The half-stroke corresponds to 50% of piston travel; at 50% the crank rotation is in about 80° after and before TDC.

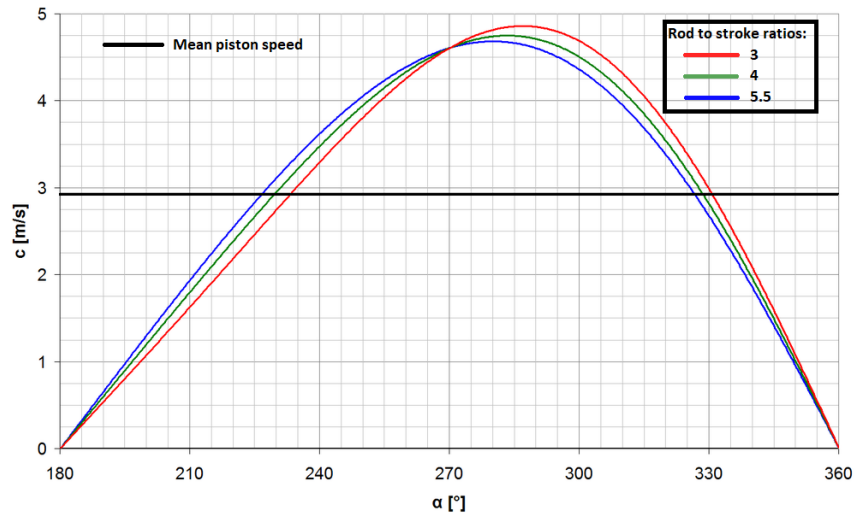
The crank rotation point at which maximum velocity occurs can also be seen with the graph, but most importantly, it allows an understanding of how the piston velocity and distribution is, and that it is not symmetrical like piston travel. More specifically, any point between TDC and maximum velocity (and vice versa) has greater velocity than any point between BDC and maximum velocity (and vice versa). For example, comparing the piston velocity at 30° after TDC with the piston velocity at 30° before BDC, one can easily see which point has more velocity. [10]

3.4 Mean Piston Speed

An alternative way of measuring piston velocity is the mean piston speed. Unlike the previously shown graphs in which the piston velocity has a specific value for every position of the crankshaft rotation, the mean piston speed is the average speed the piston has during a full rotation of the crankshaft, 360°. “Speed” is the key word in the previous sentence since speed is the absolute value of velocity. If mean piston speed measure the average velocity, the mean piston speed would be equal to 0 since velocity considers direction. The mean piston speed depends solely on the stroke and the RPM of the engine and serves as a rule of thumb to evaluate engine design. The formula to calculate the mean piston speed is the following:

$$MPS = 2 * stroke * \frac{RPM}{60} \quad (3.2)$$

The formula above has a factor of 2 multiplying the stroke since the piston travels a total of two strokes during the 360° rotation of the crankshaft. It is important to note that the rod to stroke ratio does not play a role in the mean piston speed. The following graph compares mean piston speed with instantaneous piston velocity of different rod to stroke ratios. [13]



Graph 3.2: Mean piston speed vs various rod to stroke ratio instantaneous piston velocity. Note: the horizontal axis represents the crankshaft rotation and the vertical axis is the piston velocity [14]

Some common mean piston speeds for different types of engines can be seen in the table below:

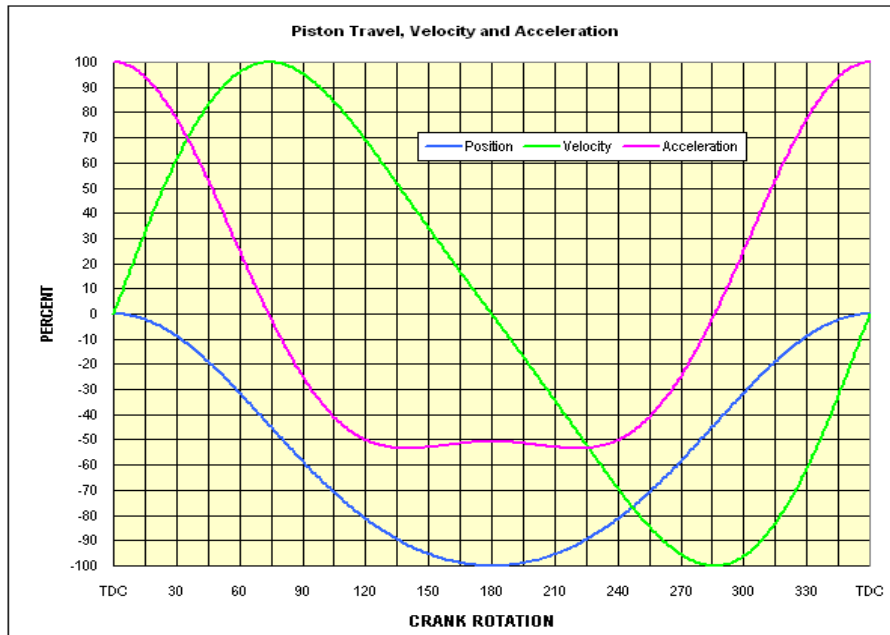
TABLE 1: COMMON MEAN PISTON SPEEDS FOR VARIOUS TYPES OF ENGINES

Engine Type	Low Speed Diesel	Medium Speed Diesel	High Speed Diesel	Medium Speed Petrol	High Speed Petrol	Competition Vehicles
Mean Piston Speed [m/s]	8.5	11	14	16	20-25	25-30

[13, Sec. 2]

3.5 Piston Acceleration

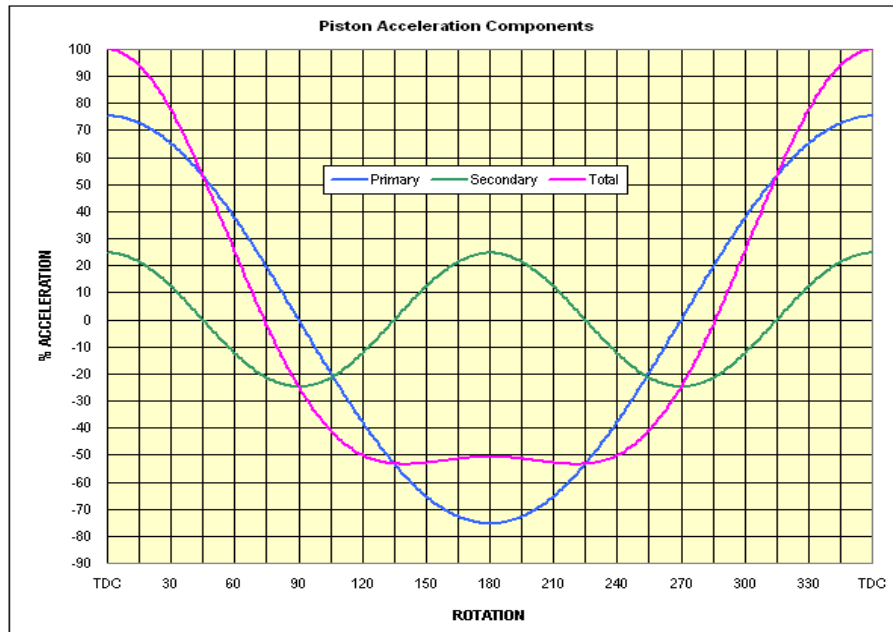
Piston acceleration is the most crucial part when it comes to forces and vibrations exerted on the crankshaft and other engine components. Acceleration is the rate at which the velocity is changing with respect to a reference variable which is usually time. In other words, piston acceleration corresponds to how quickly the piston velocity is changing. In graph 3.3 one can observe that since the maximum velocity is reached 74° after and before TDC the piston must suffer large acceleration (varying with crank rotation). Graph 3.5 demonstrates the acceleration curve for the example being used up until now:



Graph 3.3: Piston travel, velocity, and acceleration in percentage in terms of maximum displacement, velocity, and acceleration, respectively. Blue line represents piston position, green line represents piston velocity, and pink line represents acceleration. Note: the negative values for the displacement percentage have been used for a more intuitive approach (the negative sign should be ignored). [10]

Since the slope of the velocity is greatest just after and before TDC, this means that the acceleration must have the maximum positive value after and before TDC. Following the crankshaft rotation from TDC, the velocity is still increasing, but at a slower rate; therefore, the acceleration values are still positive, but declining. Once the piston velocity reaches maximum, it changes directions; as a result, the acceleration passes through zero (since the velocity's slope at maximum is zero) and changes to negative acceleration. Afterwards, the maximum negative acceleration occurs at about 40° before and after BDC. Note that the maximum negative acceleration is about half of the maximum positive acceleration. After reaching maximum velocity, the velocity's slope becomes more constant, specially 50° before and after BDC. Consequently, the acceleration between these two points is fairly constant as well. [10]

The piston acceleration shown in graph 3.5 is the total piston acceleration. In reality, acceleration is made up of various orders of acceleration. The most important ones for regarding crankshaft balancing are the primary and secondary acceleration orders since they produce substantial vibrations. For this reason, the total acceleration curve in graph 3.5 has an indent at the bottom.



Graph 3.4: Piston acceleration orders and total in terms of percentage with respect to maximum values [10]

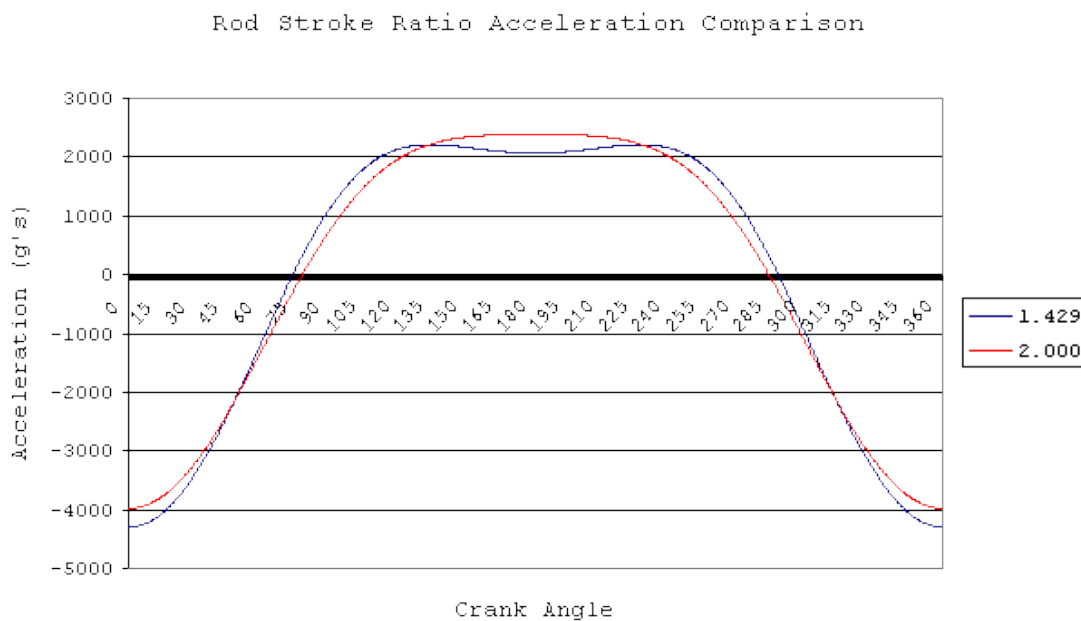
The primary and secondary accelerations add up to make the total acceleration. So, where do these acceleration orders come from? As mentioned before in the section “Piston Motion Characteristics”, as the crankshaft rotates 90° the crank throw travels vertically and horizontally which in turn causes the cosine effect that artificially shortens and lengthens the connecting rod’s length projected on the vertical plane. The primary acceleration is produced by piston motion caused by the vertical projection of the crank throw rotation. As seen in graph 3.6, the primary acceleration (blue line) is a sinusoidal curve that makes up most of the total accelerating and repeats itself once every crankshaft revolution. The primary accelerating is 0% 90° after and before TDC because at these points for only an instant the piston has peak vertical velocity. The maximum values of the primary acceleration take place at TDC and BDC since this is when the piston changes direction. [15]

The secondary acceleration is a direct consequence of the cosine effect. The dynamic connecting rod length-change caused by the cosine effect influences piston motion between TDC and the maximum velocity points by adding and subtracting travel. Therefore, secondary acceleration is produced by the piston travelling faster 90° before and after TDC than 90° before and after BDC. The secondary acceleration curve, in green seen in graph 3.6, is also a sinusoidal curve, but instead has two repetitions per crankshaft

revolution and crosses 0% at 45°, 135°, 225°, and 315°. At these points the dynamic lengthening and shortening are at their peaks. [15]

3.6 Rod to Stroke Ratio Effects on Piston Acceleration

The rod to stroke ratio also has a direct effect on piston acceleration. As the rod to stroke ratio increases, the primary acceleration becomes even more dominate than the secondary acceleration. This is due to the simple fact that since the connecting rod length as increase so does the projected vertical motion. Graphically, this results in a smoother total acceleration curve with less of an indent in the middle of the sinusoidal curve. However, the maximum negative acceleration is smaller the larger the rod to stroke ratio. This is a direct consequence of a smaller maximum piston velocity produced by larger rod to stroke ratios. These effects can be seen in the following graph.

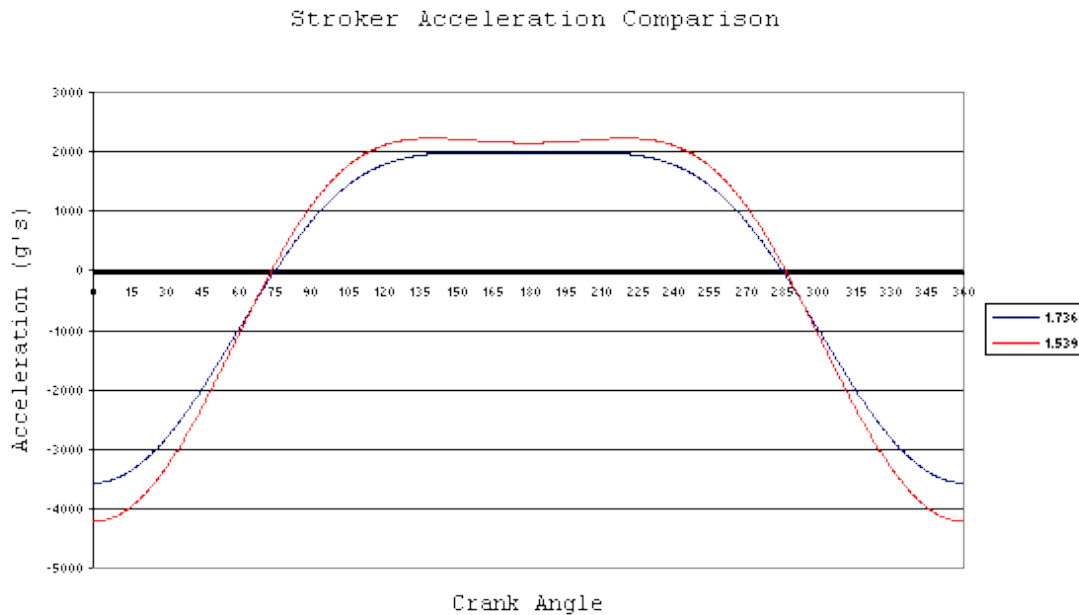


Graph 3.5: Rod to stroke ratio vs piston acceleration at 8000 RPM. Note: vertical axis measures piston acceleration in terms of g's and the horizontal axis is crankshaft rotation. [11]

Graph 3.7 uses actual acceleration values instead of using percentage of maximum acceleration like in graph 3.6. Also, the positive and negative values have been taken in the opposite direction. From the graph above one can see that the rod to stroke ratio has a smaller maximum negative acceleration, but a larger positive acceleration due to larger primary acceleration. Connecting rod failure most commonly happens at the point of

maximum negative acceleration; therefore, correct rod to stroke ratio is important not only for performance but also for engine longevity.

Stroking an engine also has effects on piston acceleration. Again, when an engine is stroked, generally, the connecting rod length is shortened due to geometrical clearance constraints, but for the following example the connecting rod lengths will stay unchanged.



Graph 3.6: Engine stroking effects on piston acceleration. Note: red line has a larger stroke than the blue line. [11]

Graph 3.8 demonstrates the relation between engine stroke and acceleration (with a connecting rod length in common). The red line has a larger stroke than the blue line causing its maximum negative and positive accelerations to be larger. One could argue that the reason behind this is a smaller rod to stroke ratio; however, the difference between the two is not significant enough to produce the shown differences since graph 3.7 shows two configurations with similar maximum piston accelerations but with a larger difference in rod to stroke ratios. The curvature in the red line presents a dent around BDC while the blue line is relatively constant. The explanation for this is the same as seen for graph 3.7: a larger rod to stroke ratio has a more dominant primary acceleration than a smaller stroke.

4. MATHEMATICAL EXPRESSIONS FOR PISTON MOTION

As of now, the piston's motion, velocity, and acceleration have all been defined analytically. The next step is to derive the general equations that describe piston motion, velocity, and acceleration with respect to crankshaft angular position and, finally, with respect to time.

4.1 Piston Motion with Respect to Crankshaft Angular Position

The following diagram illustrates the geometry of a piston-connecting rod-crankshaft mechanism and summarizes its components to help with the derivation of the general mathematical expressions.

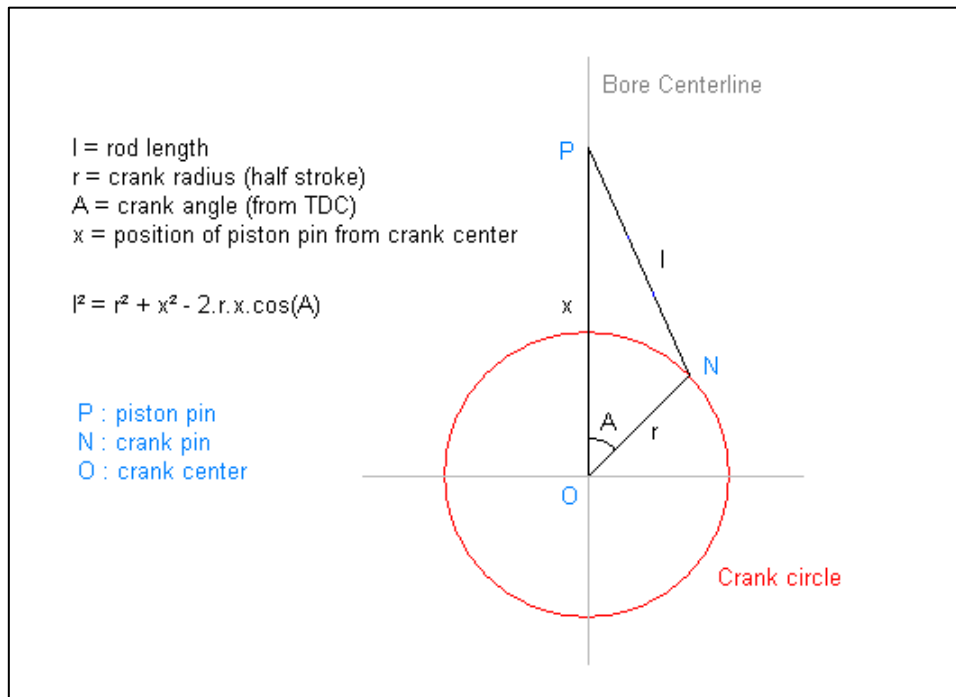


Fig. 4.1: Geometry of a piston-connecting rod-crankshaft mechanism with its corresponding component names. [16]

The figure above represents a geometrical relation amongst the center of the crankshaft (O), the crankpin (N), and the Piston (P) which forms a triangle. This triangle has its corresponding side lengths and the angle for the equations' domain, A. Using the cosine law, the relation amongst the side lengths and the angle is as follows:

$$l^2 = r^2 + x^2 - 2 \cdot r \cdot x \cdot \cos A \quad [17] (4.1)$$

Parting for the cosine law about the equations that describe the reciprocating motion of the piston with respect to the crankshaft rotation angle are obtained. First, the cosine law is arranged:

$$l^2 - r^2 = x^2 - 2 \cdot r \cdot x \cdot \cos A \quad [17] (4.2)$$

Second, a relation between the radius, r , and the angle, A , is inserted into the equation:

$$l^2 - r^2 = x^2 - 2 \cdot r \cdot x \cdot \cos A + r^2[(\cos^2 A + \sin^2 A) - 1] \quad [17] (4.3)$$

Third, the equation is rearranged again to isolate the cosine and sine terms:

$$l^2 - r^2 + r^2 - r^2 \cdot \sin^2 A = x^2 - 2 \cdot r \cdot x \cdot \cos A + r^2 \cdot \cos^2 A \quad [17] (4.4)$$

This way, reducing the quadratic expression to the right of the equals sign results in:

$$l^2 - r^2 \cdot \sin^2 A = (x - r \cdot \cos A)^2 \quad [17] (4.5)$$

Taking the square root of both sides and isolating x , results in an equation for the piston displacement, x , in terms of crank radius, connecting rod length, and crankshaft rotation angle:

$$\boxed{x = r \cdot \cos A + \sqrt{l^2 - r^2 \cdot \sin^2 A}} \quad [17] (4.6)$$

4.2 Velocity

Velocity, by definition, is the rate of change of position. In other words, the first derivative of position:

$$v = x' = \frac{dx}{dA} \quad (4.7)$$

To take the derivative of the expression for x above, the chain rule must be applied for the second term:

$$\boxed{v = x' = -r \cdot \sin A - \frac{r^2 \cdot \sin A \cdot \cos A}{\sqrt{l^2 - r^2 \cdot \sin^2 A}}} \quad [17] (4.8)$$

4.3 Acceleration

The acceleration is the rate of change of velocity, or it is the second derivative of position:

$$a = v' = x'' = \frac{d^2x}{dA^2} \quad (4.9)$$

Again, the chain rule must be applied to derive the velocity with respect to the angle, A. However, since equation for velocity includes a quotient of two functions of A, the quotient rule also has to be applied:

$$a = x'' = -r \cdot \cos A - \frac{r^2 \cdot \cos^2 A}{\sqrt{l^2 - r^2 \cdot \sin^2 A}} - \frac{r^2 \cdot \sin^2 A}{\sqrt{l^2 - r^2 \cdot \sin^2 A}} - \frac{r^2 \cdot \sin A \cdot \cos A \cdot \left(-\frac{1}{2}\right) \cdot (-2) \cdot r^2 \cdot \sin A \cdot \cos A}{(\sqrt{l^2 - r^2 \cdot \sin^2 A})^3} \quad [17] (4.10)$$

After simplifying:

$$a = x'' = -r \cdot \cos A - \frac{r^2 \cdot (\cos^2 A - \sin^2 A)}{\sqrt{l^2 - r^2 \cdot \sin^2 A}} - \frac{r^4 \cdot \sin^2 A \cdot \cos^2 A}{(\sqrt{l^2 - r^2 \cdot \sin^2 A})^3} \quad [17] (4.11)$$

These expressions can be used to calculate the piston motion, velocity, and acceleration for a given stroke and connecting rod length at any given crankshaft rotation angle. Also, the maximum velocities, accelerations, and the points at which they occur can also be found with the equations.

4.4 Piston Motion Equations with Respect to Time

Modifying the previous equations to calculate piston motion as a function of time is relatively easy. However, a relation between crankshaft rotation angle, A, and time must exist. The following equation demonstrates said relation:

$$A = \omega \cdot t \quad (4.12)$$

where $\omega = \frac{2 \cdot \pi \cdot \text{RPM}}{60}$ is angular speed and t is time

Considering the angular velocity to be constant, meaning the engine is spinning at a constant RPM, angular velocity and acceleration are $\frac{dA}{dt} = \omega$ and $\frac{d^2A}{dt^2} = 0$, respectively.

Therefore, to obtain the piston motion equations with respect to time, substituting equation 4.12 into the angle dependent equation and applying the chain rule will yield the equations with respect to time.

The equation for piston travel with respect to time would simply be:

$$x = r \cdot \cos(\omega \cdot t) + \sqrt{l^2 - r^2 \cdot \sin^2(\omega \cdot t)} \quad (4.13)$$

However, the velocity and acceleration equations can be simplified drastically by applying the chain rule. The velocity equation as a function of time would be as follows:

$$v = \frac{dx}{dt} \quad (4.14)$$

After applying the chain rule:

$$\boxed{v = \frac{dx}{dA} \cdot \frac{dA}{dt} = \frac{dx}{dA} \cdot \omega = x' \cdot \omega} \quad [17] (4.15)$$

As for acceleration, the procedure is longer:

$$a = \frac{d^2x}{dt^2} = \frac{d}{dt} \frac{dx}{dt} \quad [17] (4.16)$$

At this point the chain rule is applied:

$$a = \frac{d}{dt} \left[\frac{dx}{dA} \cdot \frac{dA}{dt} \right] \quad [17] (4.17)$$

Afterwards the product rule and the chain rule are applied:

$$a = \frac{d}{dt} \left[\frac{dx}{dA} \right] \cdot \frac{dA}{dt} + \frac{dx}{dA} \cdot \frac{d}{dt} \left[\frac{dA}{dt} \right] \quad [17] (4.18)$$

$$a = \frac{d}{dA} \cdot \left[\frac{dx}{dA} \right] \cdot \left[\frac{dA}{dt} \right]^2 + \frac{dx}{dA} \cdot \frac{d^2A}{dt^2} \quad [17] (4.19)$$

$$a = \frac{d^2x}{dA^2} \cdot \omega^2 + \frac{dx}{dA} \cdot 0 \quad [17] (4.20)$$

The substitution made in equation 4.20 is due to the relations stated after equation 4.12.

Finally, the expression for piston acceleration as a function of time is:

$$\boxed{a = x'' \cdot \omega^2} \quad [17] (4.21)$$

5. FORCES ACTING ON A CRANKTRAIN

There are two categories of forces that act on the cranktrain assembly: gas-forces and inertial-forces. The gas forces include the energy released during the combustion. Spark ignition IC engines can produce more than 100 bars of pressure within the cylinder. Given a 10 cm in diameter piston, the force produced on the piston by the 100 bars pressure is about 31.4 kN. This amount of force transferred from the piston to the crank throw results in important bending and torsional moments which in turn cause normal and shear stresses. The gas forces acting on the crankshaft are static forces since the combustion that produces them is considered to take place instantaneously.

The other source of important forces comes from piston acceleration and is a dynamic force. The piston-connecting rod-wrist pin assembly is mass that is constantly being accelerated with an acceleration order of magnitude in the 1000s of g's producing significant forces on the crankshaft. Combustion and acceleration forces are the principal cause for external engine vibrations. It's important to note that when the engine speed is slow, the gas forces dominate over the inertial forces. However, as the engine speed increases the inertial forces become more important and the gas forces can be ignored.

All the following equations derived will be inserted into a MATLAB program to calculate graphs for a given engine configuration to further strength the understand of the equations.

5.1 Decomposition of Force Vectors

The inertial forces can be broken down into several components. The forces acting on the cranktrain mechanism are as follows:

- F_G – Gas forces acting on the piston surface
- F_N – Normal force acting perpendicular to the wristpin's axis
- F_T – Tangential force
- F_R – Radial force
- F_{CR} – Connecting rod force

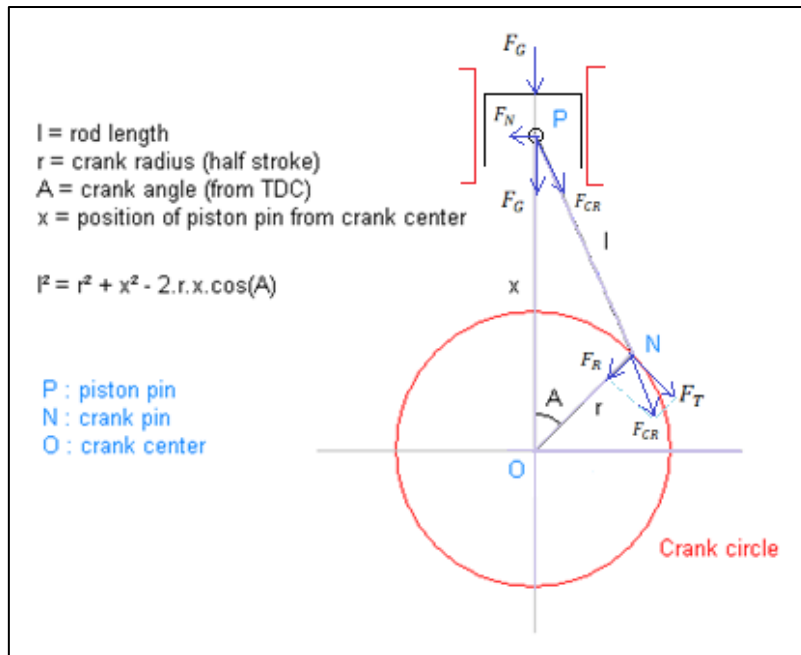


Fig. 5.1: Forces acting on the crankshaft-connecting rod-piston mechanism or cranktrain. [16]

5.2 Gas Force

The gas force, F_G , occurs when the air-fuel mixture combusts and exerts pressure on the piston head during the power stroke. Therefore, the gas force is dependent on crankshaft rotation angle. The gas force multiplied by the half-stroke or crank radius produces torque which varies periodically with crankshaft rotation. In multi-cylinder engines, the torque curves for each individual cylinder are super-positioned. Each with a phase shift with respect to one another that depends on the number of cylinders, crankshaft configuration, and firing order.

The gas force, F_G , is the product between maximum gas pressure and the surface area of the cylinder:

$$F_G = p_{max} \cdot \frac{\pi \cdot B^2}{4} \quad [18] \quad (5.1)$$

where p_{max} is the maximum pressure exerted on the piston and

B is the piston bore

The gas torque, T_G , is the product between instantaneous gas force and instantaneous perpendicular distance between the gas force application point and the crank throw, point N in figure 5.1.

$$T_G = P_G \cdot \left(\frac{\pi \cdot B^2}{4} \right) \cdot r \cdot \sin(A) \cdot \left(1 + \frac{r}{l} \cdot \cos(A) \right) \quad [18] \quad (5.2)$$

Where P_G is the instantaneous gas pressure, r is the crank radius, l is the connecting rod length, and A is the crankshaft rotation angle with respect to the vertical axis.

In order to completely define gas torque, the instantaneous gas pressure must be formulated with respect to the same variables as rest of the equations. To begin, consider the following equation:

$$P_g = p - p_{atm} \quad (5.3)$$

Where p is the instantaneous cylinder pressure and P_{atm} is the atmospheric pressure and is considered constant. Therefore, the instantaneous gas pressure is a gauge pressure measurement, and the problem lies in calculating this value.

To be able to obtain any relation between pressure and crankshaft rotation, we first start by defining the pressure-volume diagram for an ideal Otto cycle (in other words, spark-ignition IC engine). The reason why an ideal cycle is chosen is to be able to mathematically relate different engine parameters, and because an ideal cycle is approximate enough for this application.

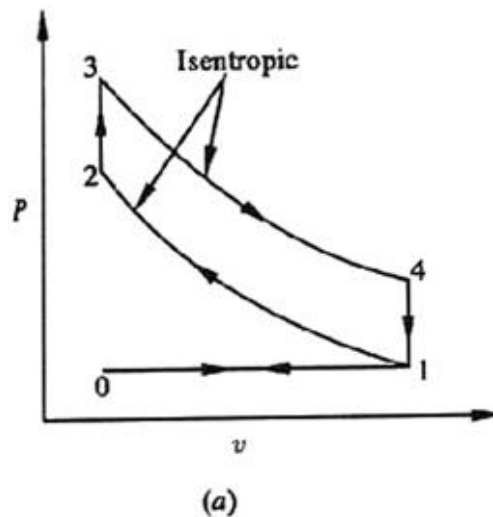


Fig. 3: P-V diagram for ideal Otto cycle. [19]

The vertical axis, P , represents the instantaneous cylinder pressure we are after. It is related to the volume through four processes:

- **0-1: Intake valve opens**

- The opening of the intake valve causes the pressure within the cylinder to remain constant and equal to atmospheric pressure.
- **1-2: Adiabatic compression**
 - The piston moves towards TDC, compressing the volume while no heat is exchanged is modeled by:

$$P \cdot V^\gamma = cte \quad (5.4)$$

or

$$P_1 \cdot V_1^\gamma = P_2 \cdot V_2^\gamma \quad (5.5)$$

Where the subscripts 1 and 2 represent the corresponding pressure and volume values for BDC and TDC, respectively, and γ represents the ratio of specific heats at constant pressure and volume and is considered equal to 1.4. This equation models a polytropic process.

- **2-3: Constant volume pressure increase**
 - The air-fuel mixture is considered to ignite instantaneously producing energy added to the gas at TDC. No volume change occurs during this process which must be considered when formulating the instantaneous cylinder pressure.
- **3-4: Adiabatic expansion**
 - As the piston moves to BDC, the gas expands adiabatically following the same equation as in process 1-2.
- **4-1: Constant volume pressure decrease**
 - The products of the combustion are considered to evacuate from the cylinder immediately causing the pressure to instantly drop to atmospheric.
- **1-0: Exhaust valve opens:**
 - The opening of the exhaust valve causes the cylinder pressure to be constant and equal to atmospheric pressure.

The next figure indicates the different dimensions needed to relate volume and crank angular position.

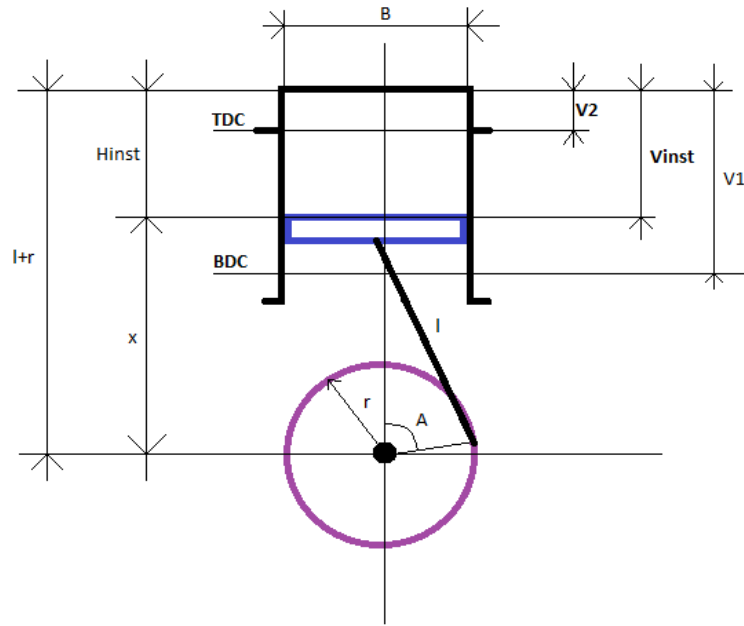


Fig. 4: cranktrain mechanism with volume and position dimensions.

The letters indicate the following:

V_1 is the volume at position 1, V_2 is the volume at position 2, V_{inst} is the instantaneous volume, H_{inst} is the instantaneous height of V_{inst} , and x is the piston's position. These dimensions are defined by the following equations:

$$V_1 = 2 \cdot r + V_{cc} \quad (5.6)$$

$$V_2 = V_{cc} = H_{cc} \cdot \frac{\pi B^2}{4} \quad (5.7)$$

Where V_{cc} is the combustion chamber volume and H_{cc} is the combustion chamber height and is a known value characteristic of the engine.

$$V_{inst} = [(l + r) - x] \cdot \frac{\pi \cdot B^2}{4} + V_{cc} \quad (5.8)$$

Where x is given by the equation 4.6. The equation for the instantaneous volume is essential to relate instantaneous cylinder pressure to crank rotation. The next step is to relate volume and pressure through the polytropic equation. Isolating the pressure in the equation 5.4 yields:

$$P = \frac{cte}{V^\gamma} = cte \cdot V^{-\gamma} \quad (5.9)$$

Where the cte depends on the previous point of the adiabatic process and takes the form of: $P_a \cdot V_b^\gamma$ and is of known value. Therefore, the instantaneous cylinder pressure is as follows:

$$P_{inst} = \frac{cte}{V^\gamma} = cte \cdot V_{inst}^{-\gamma} = P_a \cdot V_a^\gamma \cdot \left\{ [(l+r) - x] \cdot \frac{\pi \cdot B^2}{4} + V_{cc} \right\}^{-\gamma} \quad (5.10)$$

Where x given by is equation 4.6. Thus, the instantaneous cylinder pressure becomes defined solely by one variable, A, the other parameters are known values. Finally, the instantaneous cylinder pressure can be defined for the ideal Otto cycle through a piece-wise function.

- **0-1: $0 \leq A < 180^\circ$**

$$P_{inst} = P_{atm} \quad (5.11)$$

- **1-2: $180^\circ < A < 360^\circ$**

$$P_{inst} = P_{atm} + P_1 \cdot V_1^\gamma \cdot \left\{ [(l+r) - x] \cdot \frac{\pi \cdot B^2}{4} + V_{cc} \right\}^{-\gamma} \quad (5.12)$$

- **2-3: $A = 360^\circ$**

$$P_{inst} = P_3 = P_{max} \quad (5.13)$$

- **3-4: $360^\circ < A < 540^\circ$**

$$P_{inst} = P_{atm} + P_{max} \cdot V_{cc}^\gamma \cdot \left\{ [(l+r) - x] \cdot \frac{\pi \cdot B^2}{4} + V_{cc} \right\}^{-\gamma} \quad (5.14)$$

- **4-1: $A = 540^\circ$**

$$P_{inst} = P_{atm} \quad (5.15)$$

- **1-0: $540^\circ < A \leq 720^\circ$**

$$P_{inst} = P_{atm} \quad (5.16)$$

Other than the dimensions of the engine in question, the only other values necessary to calculate the instantaneous cylinder pressure is the maximum pressure, $P_{max} = P_3$, which is a known approximate value that varies slightly from engine to engine.

Ultimately, everything necessary to calculate the instantaneous gas torque is obtained. When inserting the piece-wise instantaneous cylinder pressure function back into

equation 5.3 the atmospheric pressure terms are canceled resulting in zero instantaneous cylinder pressure for some crank rotation angle intervals. Consequently, during these intervals, the gas torque is also 0.

To summarize, the gas torque is given by the following equation:

$$T_G = P_G \cdot \left(\frac{\pi \cdot B^2}{4} \right) \cdot r \cdot \sin(A) \cdot \left(1 + \frac{r}{l} \cdot \cos(A) \right) \quad (5.17)$$

Where P_G is given by the piece-wise function derived above.

Given the physical dimensions of the single-cylinder engine as well as its maximum pressure, the gas torque can be found at any crank angle rotation.

5.3 Inertial Force

The inertial force, F_I , is produced by the reciprocating mass of the piston-connecting rod assembly. The piston center of mass is taken to be at the wristpin (where the connecting rod attaches); therefore, the entire piston mass is taken into account, but the center of mass of connecting rod depends on the model used. Generally, the simplest model is two mass lumps concentrated at either end of the connecting rod: one reciprocating with the piston and the other rotating with the crank throw. Even though in truth both masses reciprocate and rotate. Nevertheless, since the inertial force only considers the reciprocating mass, the mass lump at the crank throw is not taken into account resulting in about two-thirds of the connecting rod weight considered in the reciprocating mass.

The inertial force can be obtained by multiplying the piston acceleration (acceleration the reciprocating body has) and the reciprocating mass. This force acts parallel to the piston travel.

$$F_I = M_R \cdot \left[r \cdot \cos A - \frac{r^2 \cdot (\cos^2 A - \sin^2 A)}{\sqrt{l^2 - r^2 \cdot \sin^2 A}} - \frac{r^4 \cdot \sin A \cdot \cos A}{(\sqrt{l^2 - r^2 \cdot \sin^2 A})^3} \right] \cdot \omega^2 \quad (5.18)$$

Where M_R is the reciprocating mass and in this case is assumed to be equal to:

$$M_R = M_{piston} + \frac{1}{2} \cdot M_{conrod} \quad (5.19)$$

Where M_{piston} is the mass of the piston and M_{conrod} is the mass of the connecting rod.

The inertial torque is the product between the instantaneous inertial force and the instantaneous perpendicular distance between the inertial force application point and the crank throw, point P and point N, respectively, in figure 5.1.

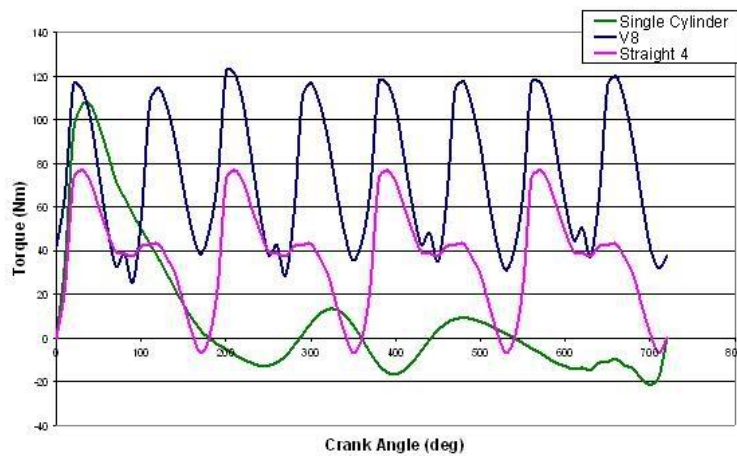
$$T_I = M_R \cdot \left[r \cdot \cos A - \frac{r^2 \cdot (\cos^2 A - \sin^2 A)}{\sqrt{l^2 - r^2 \cdot \sin^2 A}} - \frac{r^4 \cdot \sin A \cdot \cos A}{(\sqrt{l^2 - r^2 \cdot \sin^2 A})^3} \right] \cdot \omega^2 \quad (5.20)$$

$$\cdot r \cdot \sin A \cdot \left(1 + \frac{r}{l} \cdot \cos A \right)$$

$$\boxed{T_I = M_R \cdot r^2 \cdot \sin A \cdot \left(1 + \frac{r}{l} \cdot \cos A \right) \cdot \left[\cos A - \frac{r \cdot (\cos^2 A - \sin^2 A)}{\sqrt{l^2 - r^2 \cdot \sin^2 A}} - \frac{r^3 \cdot \sin A \cdot \cos A}{(\sqrt{l^2 - r^2 \cdot \sin^2 A})^3} \right] \cdot \omega^2} \quad (5.21)$$

Note: the previous equations are formulated for a single-cylinder configuration.

Therefore, the total torque suffered by the crankshaft is the addition of the inertial torque and the gas torque. [18] However, as mentioned earlier, the RPM of the engine influences these torque producing forces in such a way that the higher the RPM the more dominant the inertial forces. This characteristic is easily seen through the gas torque and inertial torque equations. The gas torque is not a function of angular velocity while the inertial torque is a function of the angular velocity squared. Thus, the inertial forces increase exponentially as the angular velocity of the engine increases. The following image depicts torque versus crank angle for various engines at low RPM; thus, gas forces should be dominant.



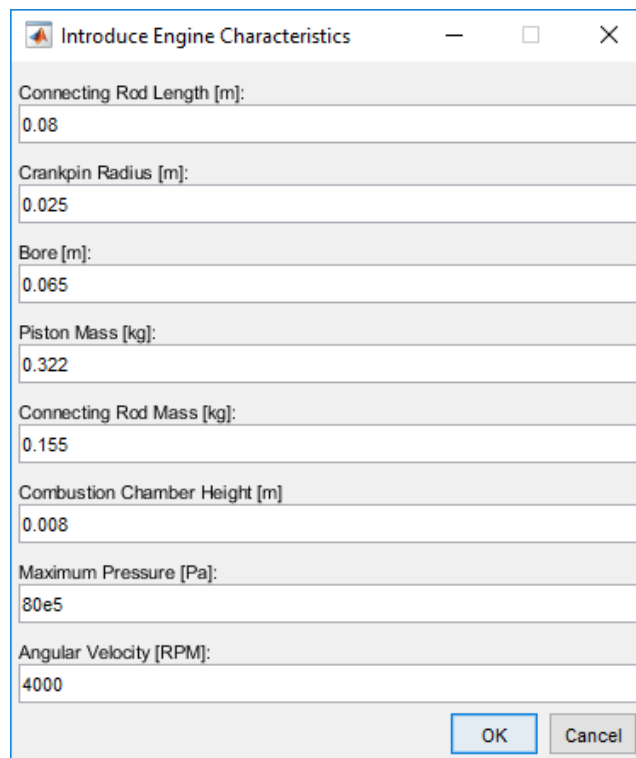
Graph 5.1: Torque versus crank angle for three types of engines at low RPM. Note: TDC occurs at 90°. [20]

Graph 5.1 has three engine configurations: single cylinder, V-type eight-cylinder, and an inline four-cylinder engine. These engines have a power cycle every 720° , 90° , and 180° , respectively (this will be thoroughly covered during the next section). The gas forces acting on the v-type eight-cylinder engine occur with a period of 90° corresponding to the firing intervals. The single cylinder engine fires just once during 720° before 90° causing the torque peak of 110 Nm. However, the oscillations that occur during the rest of the cycle are due solely to inertial forces and have a magnitude much smaller than the gas forces, reinforcing the statement made earlier. A very interesting observation is the discontinuity of the inertial oscillations after 500° of crank angle. At approximately 500° the intake stroke is finishing, and the compression stroke is taking place. This does not occur for the previous two oscillations because the intake and exhaust valves are open. Since the gas is being compressed and energy is being transferred to it, the gas is producing negative torque causing the irregular oscillations. The inline four-cylinder engine has four peak torque values with a period of 180° coinciding with the firing intervals. The oscillations are not as smooth and constant as the eight-cylinder engine since the inertial forces have time to act before the next cylinder is fired. This phenomenon can be seen on the slight “plateau” during the 3 last power strokes. This flat flatter area coincides with the peak inertial forces (demonstrated by the peak inertial torque on the single-cylinder engine) causing the total torque to level out. [21]

6. MATLAB IMPLEMENTATION FOR KINEMATIC AND DYNAMIC CALCULATIONS

One of the main objectives of this thesis was to create a software tool that would allow approximate calculations of the piston kinematics and their consequent dynamic effects on any crankshaft given certain characteristics. The procedure used to achieve this was defining the derived equations in MATLAB with a common variable, the crankshaft angular rotation, which serve to plot the kinematic and dynamic curves to illustrate their development throughout the engine's cycle. This software tool allows users to input any single-cylinder engine characteristics and change said characteristics to visualize how each parameter effects the kinematic and dynamic curves. The full MATLAB script is found in annex 1.

When the script is run, an interface appears prompting the characteristics needed to plot the piston position, velocity, acceleration, torque, and gas pressure as a function of the crankshaft's angular position.



The screenshot shows a MATLAB dialog box titled "Introduce Engine Characteristics". It contains several input fields with the following labels and values:

Parameter	Value
Connecting Rod Length [m]:	0.08
Crankpin Radius [m]:	0.025
Bore [m]:	0.065
Piston Mass [kg]:	0.322
Connecting Rod Mass [kg]:	0.155
Combustion Chamber Height [m]	0.008
Maximum Pressure [Pa]:	80e5
Angular Velocity [RPM]:	4000

At the bottom right of the dialog box, there are two buttons: "OK" and "Cancel".

Fig. 6.1: MATLAB software tool prompt interface.

The prompt interface comes with default values that can be changed before proceeding by pressing OK. The parameters are described with their corresponding units. Once the

parameters have been introduced, the software displays several graphs indicating points of interest.

The first graph that shows up displays the inertial torque, gas torque, and total torque, sum of inertial and gas torque.

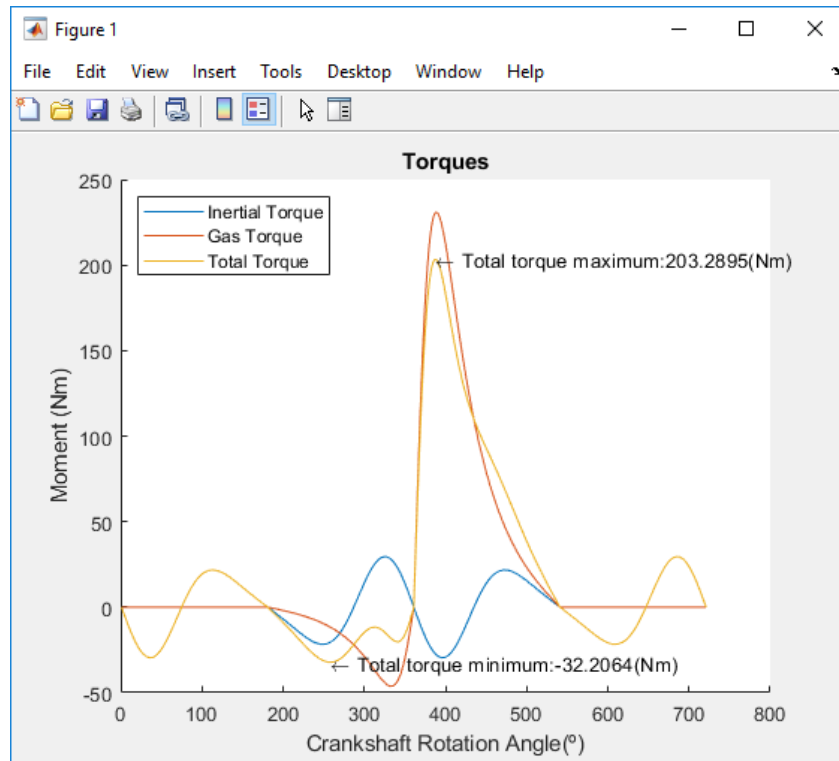


Fig. 6.2: Torque curves generated by MATLAB tool at 4000 RPM.

Figure 6.2 demonstrates the first graph. This graph identifies the different torques and the maximum and minimum values of the total torque. The main purpose of this graph is not only to identify the magnitude of the torques the crankshaft is suffering, but also to vary the engine characteristics that directly affect the shape of these curves. In the chapter “Forces Acting on a Cranktrain”, it was stated that inertial forces became more dominant as the angular velocity of the engine is increased. The effect of maximum pressure on the gas torque was also explained. Therefore, by changing the engine maximum pressure and angular velocity one can see how these parameters effect the curves. It is important to note that the inertial torque curve is hidden by the total torque curve at several points since they completely coincide.

By relaunching the tool and introducing 100 bars and 10000 RPM as the maximum pressure and engine angular velocity, respectively, the following graph appears:

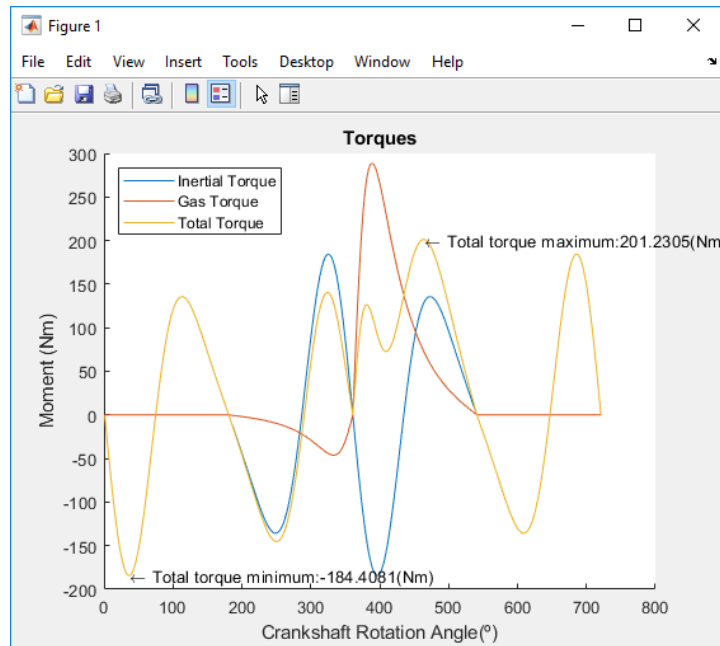


Fig. 6.3: Torque curves at 100 bars and 10000 RPM.

The torque curves have drastically changed. The previous observation stating that the higher the RPM the more significant the inertial torque becomes is very evident in this graph. The maximum and minimum torque values are very similar. The maximum gas torque value is only 100 Nm more than the inertial torque values while in figure 16 the difference is of 200 Nm.

Reverting to the original parameters of figure 6.1, the piston position, velocity, and acceleration curves are as follows:

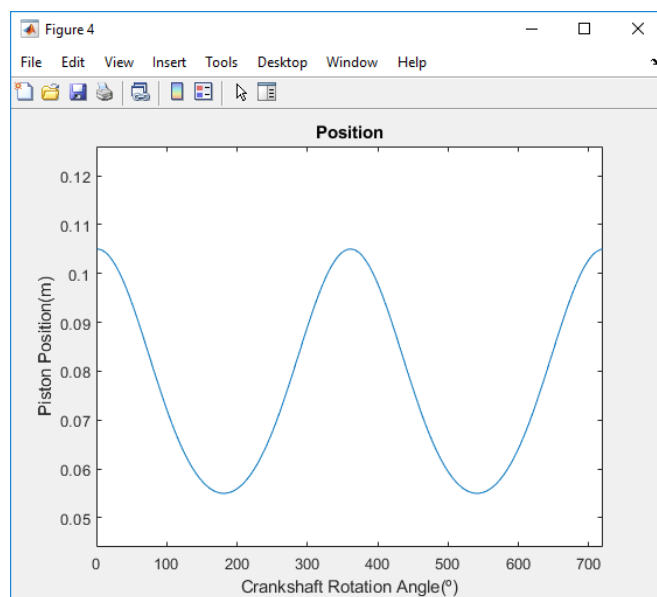


Fig. 5: Piston position. Note: 0 is taken at the crankshaft's axis.

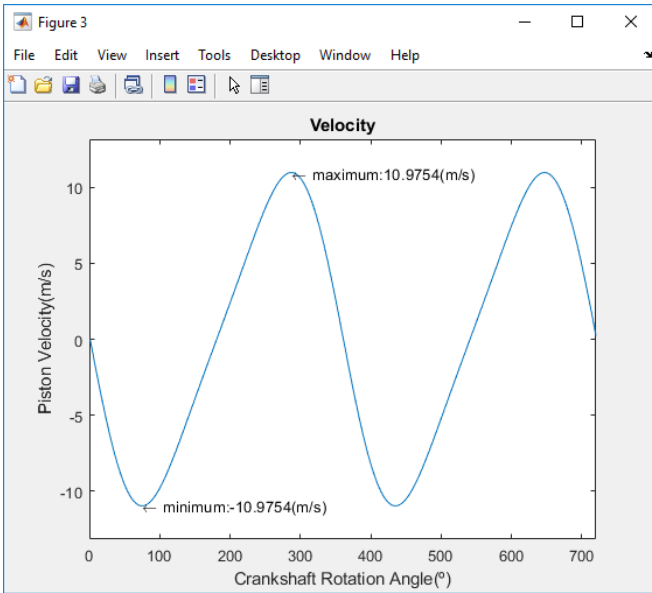


Fig. 65: Piston velocity with its respective maximum and minimum values.

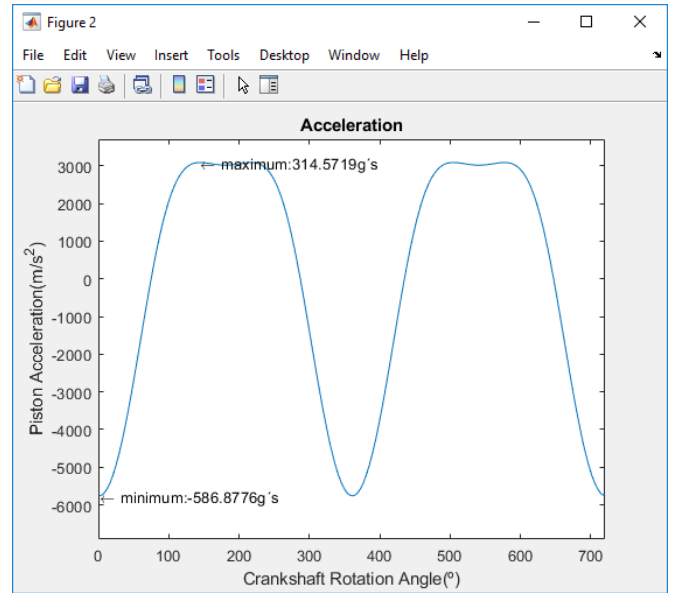


Fig. 6.6: Piston acceleration with its respective maximum and minimum values.

The previous 3 figures permit the user to see exactly what values the piston's position, velocity, and acceleration have at any given crankshaft rotation angle by hovering over a point on the curve with the mouse pointer.

In the chapter "Piston Dynamics Basics", many times refers to the effect the rod to stroke ratio has on the velocity and acceleration curves. The effect of the rod to stroke ratio can be studied by modifying the stroke and connecting rod length in the prompt interface. For example, if the connecting rod's length was to be increase while maintaining the stroke constant, yielding a larger rod to stroke ratio, the acceleration curve would be:

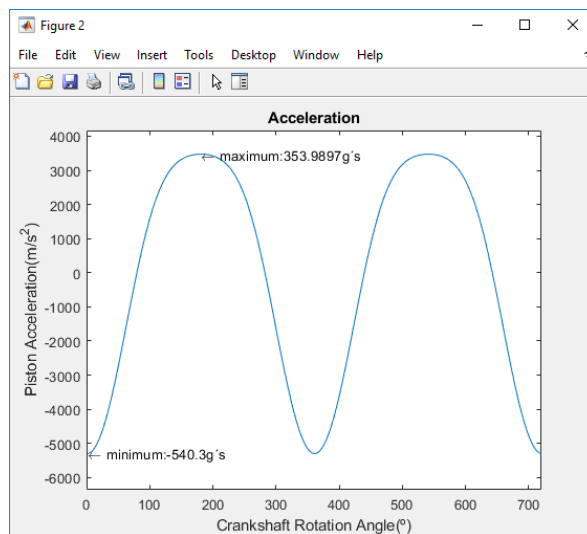


Fig. 6.7: Acceleration curve for a connecting rod length of 0.12 m.

The acceleration curve becomes a more constant and smoother curve at the top which is the same observation that was made in the previous chapters since the secondary acceleration becomes less significant.

Finally, the last graph that is plotted is the gas pressure. This gas pressure is used to calculate the gas torque.

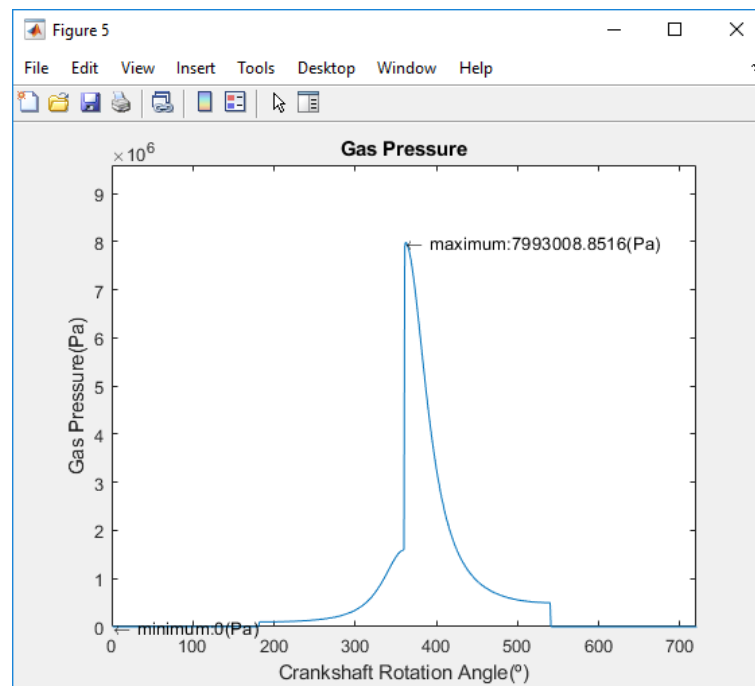


Fig. 6.8: Gas pressure as a function of crankshaft rotation angle

The parameters that dictate the shape of the graph are the stroke, the bore, and the maximum pressure. The non-continuous curve is caused by the ideal Otto cycle from which the piece-wise function was derived. The vertical path at 360° is caused by the assumption that the air-fuel mixture combusts instantaneously.

7. CRANKSHAFT BALANCING

Properly balancing the forces taking place in an engine is essential to design and building a reliable engine that will perform correctly during its service life. In the chapter “Forces Acting on the Crankshaft”, inertial and gas forces are described. Inertial forces are broken down into two components: rotating and reciprocating forces. The gas forces are caused by the cylinder firing. These three components need to be balanced to achieve a properly designed engine crankshaft. The inertial forces can also be broken down into primary and secondary accelerations directly associated with primary and secondary accelerations, respectively. Therefore, to achieve the most balanced crankshaft possible the following parameters must be studied:

- Primary Force Balance
- Secondary Force Balance
- Firing Interval Balance
- Component Balance

7.1 Primary Force Balance

The primary forces are associated with the reciprocating mass of the engine, i.e. the piston's and part of the connecting rod's mass. These masses produce this primary force due to the primary acceleration they suffer when travelling from TDC to BDC and vice versa producing oscillations in the vertical plane. Thus, when a single piston is travelling towards TDC the mass times the primary acceleration produces a positive, upwards, primary force, and when the piston is travelling towards BDC, a negative, downwards, primary force is produced. This upwards and downwards force oscillation is produced once every engine cycle; hence, the named primary forces since it occurs with a frequency of one per cycle.

To overcome these vertical plane oscillations, masses are added to the opposite end of the crankpin, known as counterweights. These counterweights have the main function of balance the crankpin's rotational mass, but also help counteract the piston's reciprocating motion. Figure 15 demonstrates how counterweights balancing works.

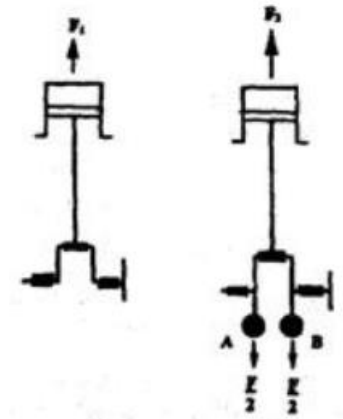


Fig. 7: To the left: single piston crankshaft without counterweights. To the right: counterweights are added. [22]

These counterweights, however, are not ideal when counteracting the entire reciprocating mass since they would produce an imbalance in the horizontal plane. Both crankpin rotational mass and piston reciprocating mass cannot be balanced with the counterweights; thus, if the reciprocating mass is balanced the crankpin mass would not be, producing oscillations horizontally when the piston is in any other position other than TDC and BDC. Therefore, vibrations are sometimes inevitable, especially in single cylinder configurations. [22, Sec. 1]

One method to overcome primary forces in odd cylinder engines is by adding a balancing shaft. A balancing shaft is made up of masses, gears, and a shaft. The rotating gears have more mass on one side than the other and spin in synchrony with the crankshaft via its shaft and another gear placed on the crankshaft. It is important to note that the masses and gears can work independently; there are several balancing shaft configurations. Balancing shafts come in pairs and are depicted below.

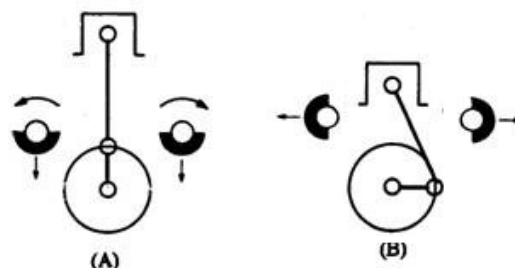


Fig. 7.2: Balancing shafts acting on a single cylinder engine. [22]

The semicircular shapes in the figures represent the weighted gears. The dark area in these gears represent the heavier part. In position A of figure 7.2, as the piston travels

towards TDC, the balancing shaft's gears are rotating downwards, counteracting the primary vertical force. In position B, the piston is travelling from TDC to BDC, and the gears are rotating in opposite directions to cancel out their horizontal forces eliminating their own forces. Hence the reason why balancing shafts come in pairs. Once the piston reaches BDC, the masses will be upwards. [22, Sec. 1]

7.2 Four-cylinder Engine

The reciprocating forces, or primary forces, on a four-cylinder engine are balanced due to its configuration. When two pistons are travelling towards TDC, the other two pistons are travelling towards BDC; thus, canceling each other out. Another important aspect that is consequent of the primary forces is the bending moment. Some engine configurations can suffer from a bending moment produced by the reciprocating mass's distance to a main journal bearing. The distance to a main journal bearing is taken as the reference point since this is where all the forces and moments are supported. Therefore, multiplying the primary force produced by the reciprocating piston mass by the perpendicular distance to the main journal bearing produces a bending moment, also known as rocking couple. Consider the four-cylinder engine in the figure below.

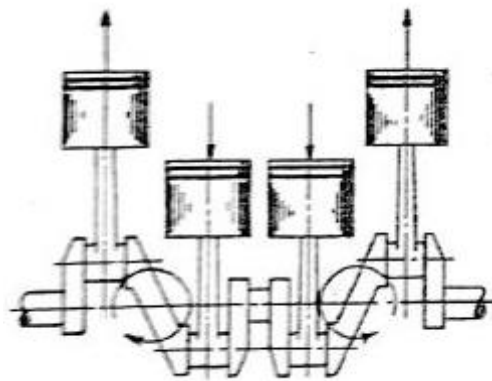


Fig. 7.3: Primary forces and rocking couples acting on an inline four-cylinder engine. [23]

The central main journal bearing is taken as the point of reference. Every piston and its primary forces produces a bending moment with respect to the point of reference. However, in the case of an inline four-cylinder engine, all these bending moments have an equal and opposite bending moment causing the total bending moment to be null. In configurations where this null bending moment does not occur naturally, the previously mentioned balancing shafts can be positioned in the desired distance to minimize the bending moment. [22, Sec. 1]

7.3 Secondary Forces Balance

The secondary forces present in a crankshaft are a direct consequence of the secondary acceleration explained in the chapter “Piston Dynamics Basics” which states the following about secondary acceleration:

“The dynamic connecting rod length-change caused by the cosine effect influences piston motion between TDC and the maximum velocity points by adding and subtracting travel. Therefore, secondary acceleration is produced by the piston travelling faster 90° before and after TDC than 90° before and after BDC.”

In other words, the acceleration before and after TDC is greater than the acceleration before and after BDC. Consequently, the inertial forces before and after TDC are greater than the inertial forces before and after BDC. These differences in accelerations and inertial forces give rise to secondary forces that need to be balanced. In addition, recalling from graph 6, these up and down accelerations occur twice every cycle, giving them the name secondary accelerations. [22, Sec. 4]

The secondary accelerations are always positive during TDC and BDC, unlike primary accelerations, giving rise to secondary forces that are also always positive during TDC and BDC. The following figure depicts primary and secondary forces acting on an inline four-cylinder engine:

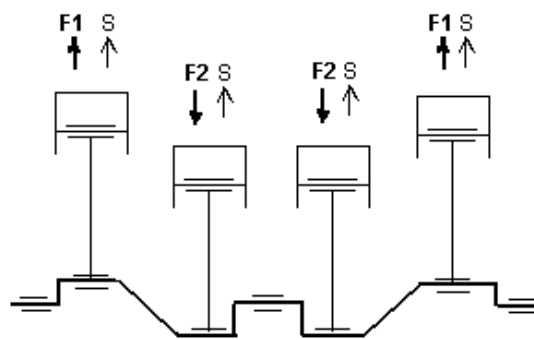


Fig. 7.4: Primary and secondary forces acting on an inline four-cylinder engine. [24]

To balance these secondary forces a balancing shaft-type system is used, known as secondary harmonic balancer. Secondary harmonic balancers work the same way a balancing shaft with gears does, but instead of having a gearing ratio of 1:1, secondary harmonic balancers must have a gearing ratio of 1:2. Meaning for every single rotation of the crankshaft, the harmonic balancers rotate twice. Like balancing shafts, harmonic

balancers come in pairs as well, also to counteract horizontal oscillations produced by the rotating masses by rotating in opposite directions.

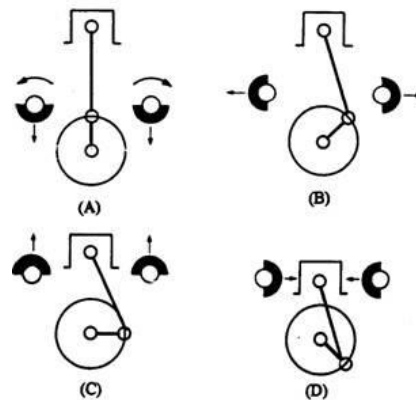


Fig. 7.5: Secondary harmonic balancers and their relative positions to the crankshaft. [22]

The maximum positive secondary forces occur at 0° and 180° while the maximum negative secondary forces occur at 90° and 270° ; therefore, that secondary harmonic balancers must counteract these maximums. Figure 7.5 demonstrates how the balancers rotate with the piston position. In figure 7.5A, the piston is at TDC and as a maximum positive secondary force. As a result, the harmonic balancer has rotated towards the bottom to counteract it. The same occurs in figure 7.5C where at 90° the piston has a maximum negative secondary acceleration and, thus, the counterweight of the balancer is producing an upwards force. It is important to note that the masses on a secondary harmonic balancer are not as large as the masses on a balancing shaft since the magnitudes of the secondary accelerations are less significant than primary accelerations. [22, Sec. 4]

7.4 Firing Order Balance

The firing order of an engine dictates when each individual cylinder is going to be fire, or in which order the power strokes are going to occur. Unfortunately, there are very few ways to balance the forces produced by the firing of cylinders. This firing order imbalance is generally taken account for in the design phase of the crankshaft and is built to withstand the vibrations produced by cylinder firing.

The only way to reduce firing vibrations is by increasing the number of cylinders. However, this engine parameter is one of the first decisive factor in the design and is not generally modified to alter firing order balance. Therefore, generally, the more cylinders an engine has the smoother it will run. [22, Sec. 3]

It is even more desirable that an engine has firing overlap. Whenever an engine has a firing order of less than 180°, firing overlap occurs. Firing overlap takes place when two pistons are in their power stroke during the same time (one starting and another ending its power stroke, for example). Therefore, an overlap of 90° (which is seen in the following section) means that during 90° of crank rotation, two pistons are undergoing a power stroke. Firing overlap is important since it reduces the variation in the output torque by providing a smoother power band. As a result, the more firing overlap, the smoother the engine.

Calculating the firing order of a specific configuration can be done with a straight forward formula:

$$\frac{720^\circ}{\text{Number of cylinders}} = \theta^\circ \text{ per cylinder} \quad (7.1)$$

Where $\theta^\circ \text{ per cylinder}$ expressed the firing order interval, i.e. for a four-cylinder engine the firing interval would be 180° of crankshaft rotation to fire one cylinder.

7.5 Component Balance

Component balance specifically refers to engine components that have a high angular rotation. Generally, these components effect the engine with external vibrations (unlike internal vibrations produced by the previous forces). Therefore, a component that requires careful balancing is the flywheel. The flywheel is attached to one end of the crankshaft to store rotational energy. [25] This mechanical element is balanced individual before assembly, but, ideally, the crankshaft and the flywheel should be balanced as one unit since imperfect assembly at the mating surfaces could cause unwanted external vibrations. [22, Sec. 2] Thus, component balance is a manufacturing and assembly issue that could occur in any engine configuration.

8. COMMON CRANKSHAFT CONFIGURATIONS AND ARCHITECTURE

Crankshafts are generally characterized by their throw configuration, whether they have single or multiple crank throws, and by their connecting rod journals, these can be either single or shared. Single crank throw crankshafts are typically seen in single, twin, and (to some extent) radial cylinder applications. These single crank throw crankshafts usually have a single rod per journal, in the case of a single cylinder engine, or two connecting rods per journal, in the case of a twin cylinder engine. Radial engines can have more than two connecting rods per journal but will not be studied in the remainder of this thesis since this technology is outdated.

A multiple crank throw crankshaft is generally used with a three or more-cylinder engine. Specifically, single connecting rod per crank throw is used for inline engine configurations, while two connecting rods per throw are used for engines with two cylinder banks (such as V, L, or boxer configurations). [2, p. 16-4]



Fig. 8.1: On the left: Single crank throw with a single connecting rod. On the right: four crank throws with two connecting rods per throw for a total of eight cylinders. [26] [27]

The arrangement of the crankshaft plane depends predominately on three factors:

- Cylinder arrangement
- Number of cylinders
- Strokes of the engine

For this study, all the engines considered in further sections will be considered to have four strokes unless otherwise stated.

Another important aspect of the power cycles that is crucial when determining stresses on a multi-crankpin crankshaft is the cylinder firing order. The cylinder firing order is the sequence in which the power stroke takes place in each cylinder. The primary reason to utilize one firing over sequence over another is to achieve a smoother engine. A smoother running engine has a more linear acceleration curve, less harmonic effect and less crank deflection which translates into less stress on the crankshaft and main bearings. A correct firing order can also reduce “hot-spots” in the engine by avoiding or reducing the firing of adjacent cylinders which causes the cylinder wall in between those cylinders to increase in temperature. [28]

8.1 Cylinder Arrangement

Cylinder arrangement refers to the orientation and disposition of the pistons and respective cylinders in an engine. Many times, engines are characterized by adjectives such as V-type, inline, boxer, etc. These adjectives describe the orientation the cylinders have with respect to each other on the plane perpendicular to the crankshaft. The figure below illustrates common cylinder arrangements with their respective names.

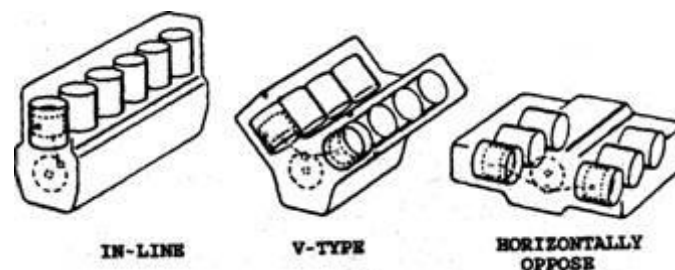


Fig. 8: Common cylinder arrangements [29]

The number of cylinders in an engine is accompanied by the cylinder arrangement to describe the engine to the greatest extent possible. For example, the engine to the right of figure 8.2 is called an in-line six-cylinder engine. The engine in the middle of figure 8.2 depending on its crankshaft configuration could have two different names: crossplane v-type eight-cylinder engine or single plane v-type eight-cylinder engine.

8.2 Crankshaft Planes

Crankshafts with multiple crank throws can be arranged in a single plane or multiple plan arrangements. A single-plane crankshaft has all its crank throws on the same plane, either at 0° or at 180° . For example, a single-plane inline four-cylinder engine shown below has

cylinders (or connecting rods) number one and four and 0° while cylinders two and three are at 180° as seen on figure 8.3 to the left.

It is important to note the following pattern: since the cylinders have a certain order (1, 2, 3, and 4), the stroke that each cylinder is undergoing must also have the same order. For example, when both cylinders 1 and 4 are at top dead center (meaning they're at their highest position, as seen in figure 8.3 on the left) only one of them can be starting a power stroke. As a result, the other must be ending the exhaust stroke. The other two cylinders 2 and 3, which are at bottom dead center, are undergoing the end of the power and the start of the compression strokes, respectively. 1-3-4-2 is the most common firing order for an inline four-cylinder engine.

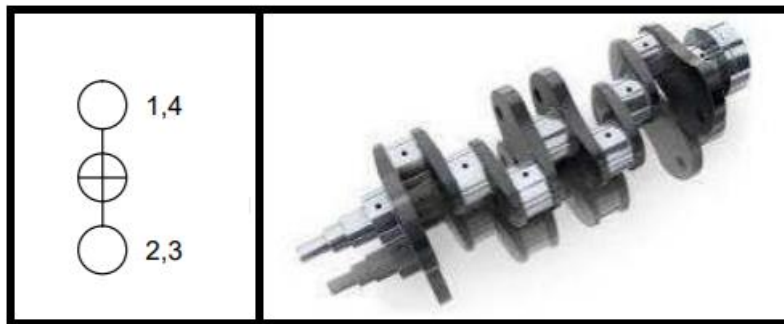


Fig. 8.3: Inline four-cylinder crankshaft distribution. [2, p. 16-5]

Next, two more single-plane crankshaft configurations and their stroke cycle will be discussed.

8.3 Horizontally opposed four-cylinder engine

A horizontally opposed four-cylinder engine has two banks of cylinder pairs at 180° with respect to each other. However, since the number of cylinders is still the same as the previous example, each crank will be rotated or spaced at 180° intervals. The objective of a horizontally opposed four-cylinder engine is to make it more compact (shorter) than an inline four. As a result, cylinders 1 and 3 are on side of the engine while cylinders 2 and 4 are on the other side. This makes the engine more compact since the four cylinders do not have to be side by side anymore and can “overlap” as seen in figure 10. In the inline configuration, both cylinders 1 and 2 were at top dead center at the same time; however, this is not possible given the arrangement of a horizontally opposed four-cylinder engine. Instead, the diametrical opposition of the cylinder is conserved; meaning cylinders 1 and

2 are on opposite ends of the crankshaft's diameter as well as cylinders 3 and 4 are with each other.

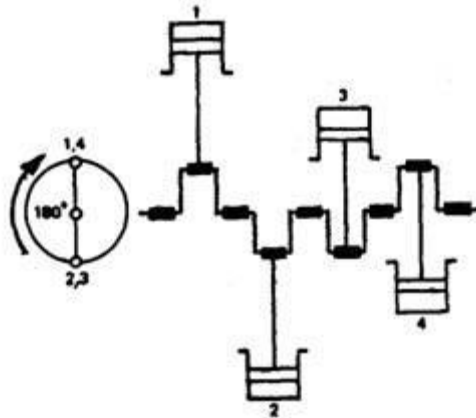


Fig. 8.4: horizontally opposed four-cylinder engine crankshaft. **Note:** circle on the left indicates crankshaft throw position and not piston position. [30]

The firing order for this configuration is as follows: 1-4-2-3. Therefore, assume that cylinder one is starting its power stroke (descending), cylinder 2 is starting its intake stroke, and both cylinder 3 and 4 are ascending to top dead center starting their exhaust and compression strokes, respectively. Once cylinders 1 and 2 are at bottom dead center and cylinders 3 and 4 at top dead center, cylinder 4 fires and cylinder 1 starts its exhaust stroke, cylinder 2 starts its compression stroke, and cylinder 3 starts its intake stroke. This sequence repeats until cylinder 3 is fired, completing the firing order. [31]

8.4 Eight Cylinder V-type Engine

An eight-cylinder V-type engine has two cylinder banks with four cylinders in each bank. These banks form an angle with respect to one another; hence, getting the name V-type. To find out the crankpin spacing, the procedure would be to divide 720° by 8 and the spacing, or firing interval, would turn out to be 90° , resulting in a multi-plane crankshaft (more specifically in a two-plane or crossplane arrangement). However, a single-plane or flat-plane can be achieved by placing two connecting rods per crank throw which effectively multiplies the 90° spacing by 2 equaling a 180° firing interval. It is important to note that a crossplane arrangement can still be achieved by placing two connecting rods per crank throw; however, this will be discussed in the following section. One way to look at a flat-plane V8 arrangement is by imaging two inline four-cylinder engines put together at an angle. Figure 8.5 shows a flat-plane V8 arrangement.

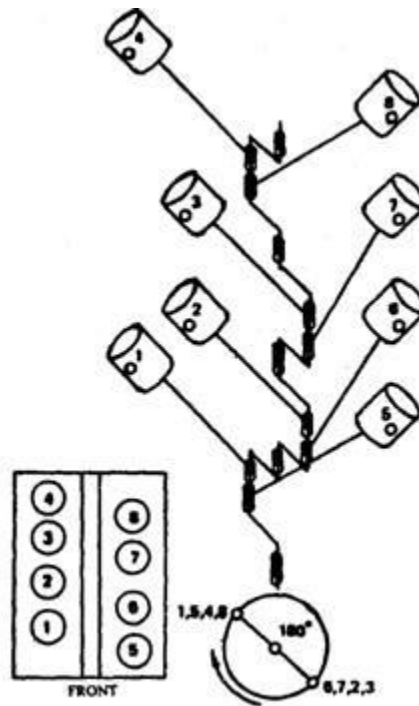


Fig. 8.5: Flat-plane V-type eight-cylinder crankshaft arrangement. Image in the bottom left represents the engine block view with the respective cylinder numbering. [32]

The firing order for a flat-plane V-type eight-cylinder engine varies from one manufacturer to another. A common firing order that Ferrari uses is 1-5-3-7-4-8-2-6.

One of the reasons for the use of this firing order is straight forward. As mentioned before, this flat-plane eight-cylinder arrangement is equivalent to combining two inline four-cylinder. This can be easily seen in figure 8.5 if one considers the bottom left image and imagines either bank to be two single inline four-cylinder arrangements. One can recall the firing order for an inline four-cylinder as 1-3-4-2. Consequently, the other bank with cylinder numbers 5, 6, 7, 8 would have a firing order of 5-7-8-6. Therefore, if both cylinder banks were to be combined and fired in a certain order, it's logical that Ferrari would choose a firing order overlapping the two previous inline four-cylinder firing orders: 1-5-3-7-4-8-2-6. [33]

8.5 Multiple Plan Crankshafts

A multi-plan crankshaft arrangement is where the crank throws are on two or more planes. This arrangement is generally seen in engines with more than four cylinders but can be applied to four-cylinder engines as seen later. A common crankshaft configuration seen in American eight-cylinder V-type engines is the dual-plane or crossplane crankshaft. Like the single-plane V8 engine described in the previous section, this crankshaft

arrangement also utilizes two connecting rods per crank throw. The reason behind using two connecting rods per crank throw is the same as using a V-type engine: compactness. The same V-type engine configuration can be achieved by using one connecting rod per crank throw, but that would duplicate the number of crank throws and completely defeat the purpose of using a V-type engine, since the result would be very similar to that of an inline eight-cylinder engine. However, six-cylinder V-type engines are more common in both said configurations.

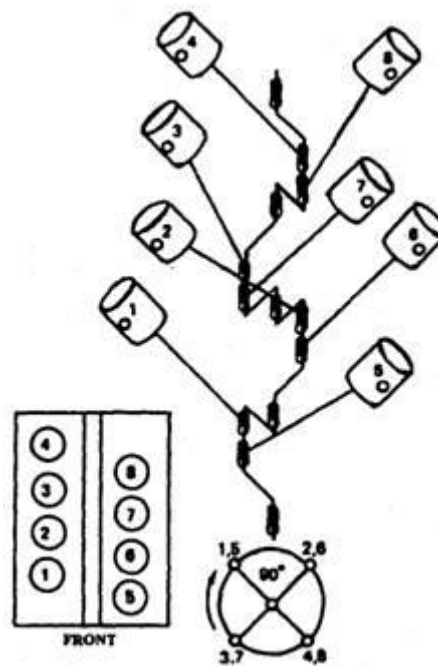


Fig. 8.6: Crossplane V-type eight-cylinder crankshaft arrangement. Image in the bottom left represents the engine block view with the respective cylinder numbering. [32]

In a crossplane V-type eight-cylinder engine as seen in figure 8.6, the firing of cylinders is equally spaced out on intervals of 90° . There is one main journal bearing in between each crank throw for a total of five main journal bearings. These crankshaft configurations tend to be more popular than the previously described flat-plane V8 engines because the crossplane crankshafts have a superior dynamic balance. [34, Sec. B]

To understand the firing order of this configuration, let's study the above figure as if the engine was frozen in time:

1. Piston 1 is assumed to be at top dead center after completing its compression stroke and about to begin its power stroke and piston 5 is halfway into its compression stroke.
2. Pistons 3 and 7 are halfway into its exhaust stroke and beginning of its exhaust stroke, respectively. P
3. Pistons 4 and 8 are starting their compression stroke and are midway into an intake stroke, respectively.
4. Pistons 2 and 6 are mid power stroke and at the beginning of the intake stroke, respectively. [34, Sec. B]

Consequently, after seven 90° rotations of the crankshaft the firing ordering is: 1, 5, 4, 8, 6, 3, 7, 2. This translates into eight power strokes in less than 720° , while a four-cylinder flat-plane engine would require $2 * 4 \text{ power strokes} * 180^\circ \text{ per power stroke} = 1440^\circ$ for 8 power strokes, which is twice the rotation require. However, this last observation is trivial since all four stroke internal combustion engines require 720° per power stroke independently of their crankshaft configuration or number of cylinders. The only determining factor for the number of power strokes within 720° of crankshaft rotation is the number of cylinders. It is also important to note, as seen in the bulleted numbers above, piston 1 and piston 2 are both in their power stroke. This is due to a 90° firing overlap which in turn provides a smoother engine.

In the case of a twelve-cylinder engine, $\frac{720^\circ}{12} = 60^\circ$, one power stroke takes place every 60° . This results in a very smooth distribution of power strokes providing a very responsive engine with greater torque and perfect dynamic balance due to a 120° overlap. [28] This crankshaft configuration requires one connecting rod per crank throw which can result to be expensive and heavy. Applying the same principle as in the eight-cylinder V-type engine, by connecting two connecting rods per crank throw, the engine can be slightly shortened. However, this has the same effect as it does on the eight-cylinder V-type engine (as well as all types of engine configurations): the power stroke timing is reduced to one power stroke every 120° as seen in the figure below.

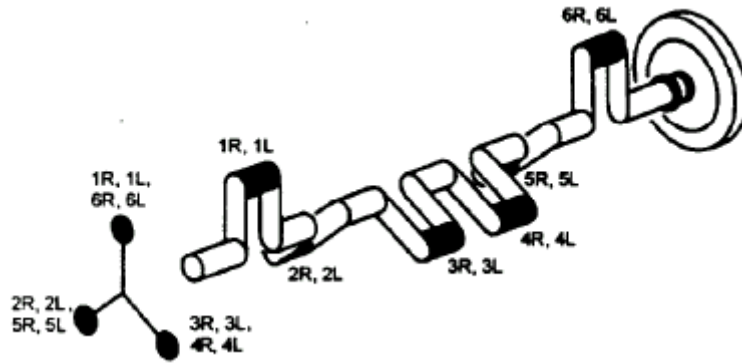


Fig. 8.7: Twelve-cylinder V-type engine with two connecting rods per crankpin resulting in a 120° power stroke spread. [32]

The following two multiplane crankshaft configurations are rarer in production vehicles, but still essential to understand these configurations in a generic manner.

8.6 Inline Five-cylinder Engine

This arrangement is the first one to have an odd number of pistons, and, consequently, an odd number of crank throws. However, this doesn't add any characteristic to the configuration. As in most cases, the choice of the crankshaft configuration used comes down to requirements, space available, and budget.

An inline five-cylinder engine has a power stroke every 144° (which is equivalent to $720^\circ/5$ cylinders). There must be a total of five crank throws since the odd number of cylinders cannot be divided equally; thus, having five planes. As a result, there is a total of six main journals and their corresponding bearings. To save space, some designs remove the main journals between the first and last two crankpins; thus, providing a shorter crankshaft. [34, Sec. A]

The firing order for an inline five-cylinder engine is: 1, 2, 4, 5, 3, with respect to the cylinders shown in figure 8.8. Determining the firing order in an inline five-cylinder engine (or any engine with an odd number of cylinders) is immediate since each cylinder has its own interval, unlike the inline four-cylinder engine with has two pistons at 0° and two pistons at 180° .

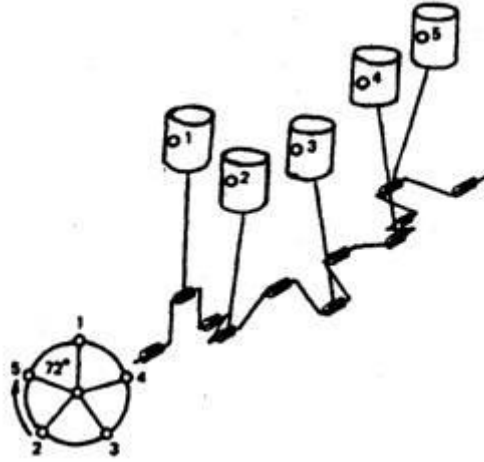


Fig. 8.8: Inline Five-cylinder crankshaft configuration. Bottom left image represents the spacing between the crankpins and their respective numbers. [34]

Since the engine is rotating clockwise and has a power stroke every 144° (2 times 72°), assuming cylinder 1 is the first to fire, the next in line would be cylinder 2, after another 144° of crankshaft rotation cylinder 4 fires, and so on. Considering that cylinder 1 is at top dead center, 0° , and that bottom dead center is at 180° , the strokes taking place at the given instance of figure 14 is the following:

1. Piston 1 is at top dead center at the beginning of its power stroke
2. Piston 4 and 5 are 72° from TDC in the intake stroke and exhaust stroke, respectively.
3. Pistons 2 and 3 are 36° from bottom dead center on their compression and power strokes, respectively.

From points 1 and 3, it is evident that both pistons 1 and 3 are in a power stroke which signifies that, unlike in an inline four-cylinder configuration, a five-cylinder engine has firing overlap. Specifically, the firing overlap of an inline five-cylinder arrangement is $180^\circ - 144^\circ = 36^\circ$. [35]

9. CRANKSHAFT MANUFACTURING

The three main techniques used to manufacture a crankshaft are through forging, casting, and machining. Each of these techniques has its pros and cons and are key to achieve the desired crankshaft characteristics for its application. Before the manufacturing method is chosen, an important design characteristic must be considered: whether the crankshaft in question is a one-piece or an assembled crankshaft.

An assembled crankshaft is made up of several components that are manufactured separately, then assembled together via welds or bolts. This type of crankshaft manufacturing design is used for crankshaft that are either abnormally big or small. When manufacturing huge crankshafts, such as marine engines, the dimensions of the crankshaft not only generally do not fit in standard machinery, but also, transportation of individual pieces facilitates the entire process. For small cases, an assembled crankshaft allows manufacturing costs to decrease since circular individual pieces do not require complex machining and can be manufacturing in a common lathe. However, the main disadvantage assembled crankshafts have are the tolerances. Since the individual pieces have to be aligned and assembled together, to get the proper tolerances the pieces have to be assembled together with large force and, afterwards, go through an alignment process. Misalignment of the components can cause failure.

One-piece crankshafts, as the name suggests, is made up as a single unit by either forging, casting, or machining. This type of design allows for high production manufacturing, better overall tolerances once the final processes are complete, and do not need aligning. However, since these types of crankshafts are produces as a single unit, only plain bearings can be used since ball bearings cannot be fit any longer. Another disadvantage one-piece crankshafts present is an additional drilling operation for lubrication channels which requires an oil pump. Since plain bearings are used, lubricating oil is essential to reduce friction between surfaces. [36]

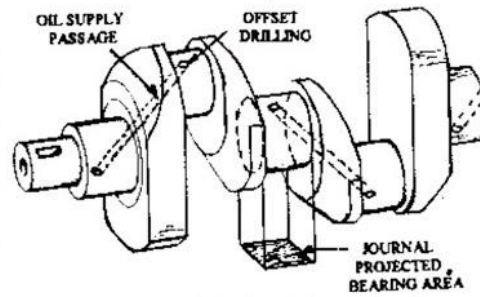


Fig. 9.1: cross drilling used to make oil channels in one-piece crankshafts. [36]

9.1 Forging Manufacture

The art of shaping metal by plastically deforming metal by applying forces to obtain the desired geometry is known as forging. Forging can be subdivided into temperature ranges: cold, warm, and hot forging. As their names imply, cold forging takes place when a metal is shaped when it is at ambient temperature, hot forging occurs when the metal is heated to high temperatures allowing an easier manipulation of the geometry while applying force, and warm forging takes place at a temperature between the two previous processes. [37] Due to the crankshafts' complicated geometries, hot forging is implemented to manufacture them.

There are many types of hot forging processes. The one generally used for crankshaft manufacturing is called closed die forging. "Closed die forging is a forging process in which dies (called tooling) that contain a pre-cut profile of the desired part move towards each other and covers the workpiece in whole or in part. The heated raw material, which is approximately the shape or size of the final forged part, is placed in the bottom die. The shape of the forging is incorporated in the top or bottom die as a negative image. Coming from above, the impact of the top die on the raw material forms it into the required forged form." [38] Therefore, the forged material must have good forgeability (take shape without failing) and must have a low enough strength that allows the easy flow during the process. Steel alloys are used to forge crankshaft and engine components.

Generally, forged pieces are characterized with good strength properties due to the internal grain structures that arise during the forging process. However, this may not be the case in a crankshaft application since the displacements required to form the complex crankshaft geometry result, in some areas, in fractured internal grain structures. Even

though all hot forged pieces undergo heat treatment during the hot forge itself, post-processing heat treatments are required to maximize their service life.



Fig. 9.2: Fiber flow in a forged crankshaft [39]

After being forged, crankshafts have a raw shape that must be machined into its final state. Heat treatments are required before and after the machining. The pre-machining heat treatment is used to produce uniformity within the internal grain structure resulting in easier machining since heterogeneous grains can cause difficulties and irregular cuts.



Fig. 9.3: Forged raw crankshaft [40]

Post-machining heat treatments are applied to fillets and bearing contact areas to harden and improve fatigue since these areas have concentrated stresses and are the most prone to failure.

9.2 Casting Manufacture

Casting is a process in which metal is molten and then poured into a mold with the negative space of the desired object. This molten metal then solidifies and keeps the shape of the mold. Casting allows for complex designs and even hollow areas which is convenient and economical for crankshaft manufacturing. There are different types of mold used in the casting industry; however, sand casting is the most common. Sand casting uses a non-reusable mold where sand is compacted around the mold's pattern to define the walls that make up the negative space. [41] Iron is the material commonly

used to cast crankshafts. The heat treatments used for cast manufacturing are similar to those of forge manufacturing. Post-process hardening of fillets and bearing surfaces are required.

9.3 Casting vs Forging

Given no economical limitations forging would be the choice of bulk crankshaft manufacturing since it provides better mechanical properties than casting. However, since normally limitations are economical, whether forging or casting is used depends ultimately on the application of the crankshaft. Forging is generally more expensive than casting since the investment in machinery for the different processes is higher. Therefore, consideration of the application is crucial for staying within budget. In many cases, however, both processes can be suitable for one given application. The following comparisons should facilitate deciding which of the two processes best fits any given application.

1. The properties that arise from casting are not as predictable as the ones obtained by forging since casting works with a molten metal and can produce inconsistencies in the overall internal structure. Forging naturally creates strengthening effects during working making it a more reliable process. [42, p. 15]
2. Forging exhibits grain flow and anisotropic strength while casting exhibits an isotropic grain formation and strength. Forging takes advantage of grain flow by preemptively orientating the grain flow in direction that require the most resistance. Casting exhibits dendrites produced from cooling that result in a weaker structure. [42, p. 16]
3. Despite the fact that machinery is more expensive for forging processes, casting suffers from strict quality control processes that increase costs. These controls include melting and cooling control that supervise alloy segregation. Alloy segregation is undesired since post-process heat treatments will have an inconsistent reaction, comprising structural integrity. These dangers do not occur in forging since heat treatments are more predictable in absence of alloy segregation. [42, p. 16]

It is evident that forging is superior to casting; however, as mentioned earlier, if the application permits the use of casting, it is a more than viable technique to produce a crankshaft.

The flow chart summarizes the operations both manufacturing techniques use.

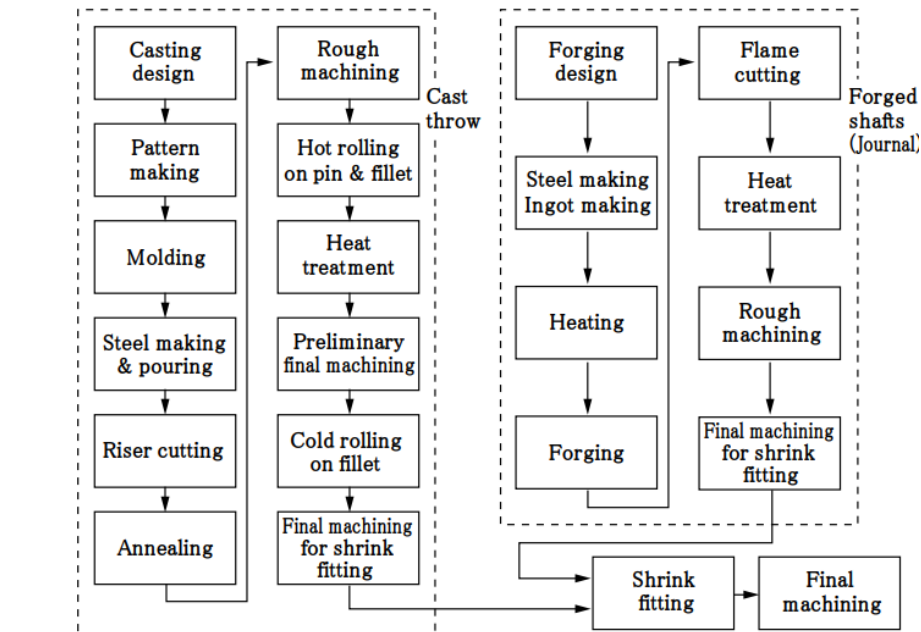


Fig. 9.4: Flow sequence for casted and forged crankshafts. Note: a sequence in the chart can have several operations. [43]

9.4 Machining Manufacture


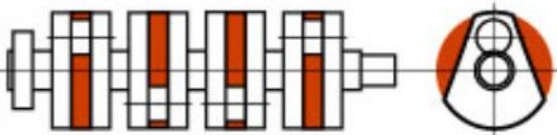


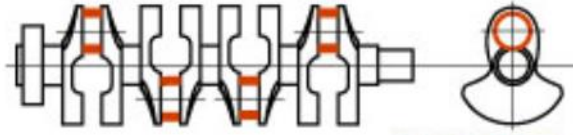
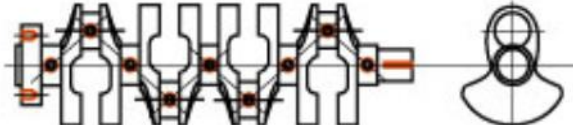
Machining a crankshaft from a billet is the go to for specific and high-end applications. With this method, the crankshaft is machined straight from a round high-alloy steel bar which undergoes several operations, in either one sophisticated machine or several machining equipment. The machining processes used post-forging coincide with most of the processes used in pure machining after initial shaping. However, the initial shaping in machining manufacture is what makes this technique stand out. This technique allows for the designer to change aspects of the crankshaft on the fly and position counterweights to the designer's satisfaction. However, the use of precise multiple axis CNC machinery is not cheap causing this manufacturing method to be the most expensive and reserved for special applications such as racing engines with large angular speeds and power.


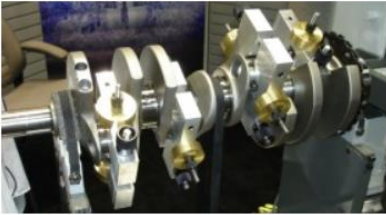
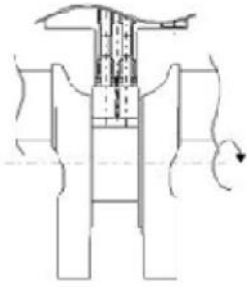


The machining process for a crankshaft varies from one design to another; however, the machining processes that cover an overall order of operations that can be implemented to

manufacturing a crankshaft is as follows (in order): cutting and centering, turning, rotary broaching, internal milling, external milling, oil hole drilling, roller burnishing, and end machining. [44] It is important to note that each of these operations may require several steps to complete.

The following table summarizes the specific order of operations followed by a crankshaft manufacturer, Pure Performance Motorsport.

TABLE 2: MACHINING PROCESS OPERATIONS USED BY PURE PERFORMANCE MOTORSPORT TO MANUFACTURE CRANKSHAFTS OUT OF BILLET. THE PROCESS ORDER IS FROM TOP TO BOTTOM.

Operation	Description	Illustration
Turning	The cylindrical billet has transverse portions cut away to form the main journals. This process leaves a margin for later surface finishing.	
Milling	The crankpins are roughly milled, and weight is shaved; however, margin is left for later finishing. (substitutive operation for rotary broaching)	
Milling	More weight is shaved by initially shaping crankpin ends and removing internal portions of the crankshaft. (substitutive operation for rotary broaching)	
Surface Hardening	Surface hardening is produced by shot peening. Shot peening produces a compressive stress layer by cold working the surface through the impact of metal or ceramic particles. [45]	
Milling	A second milling process is done on the crankpin journals to further reduce weight and reach design specifications. Margin is left for polishing.	
Oil Hole Drilling	During this process, oil holes and bolt holes are drilled; also, key slots are made.	

Polishing	The main and rod journals are polished	
Dynamic Balancing	A dynamic balancing adjustment process is performed to reduce produced oscillations. After measuring oscillations, a balancing process is performed.	
Wrapping (Roller Burnishing)	The crankpins and main journals undergo a wrapping operation that improves their roundness reducing friction. This process also increases surface fatigue resistance.	
Surface Treatment	Surface strength and surface fatigue resistance is increased through heat treatment.	
Final Corrections and Inspection	Finally, after the last heat treatment, the crankshaft is inspected for bends and is corrected accordingly.	

[47]

From table 2, it becomes evident that machining a crankshaft from billet is a lengthy process that requires various specialized tools. More importantly, the crankshaft output is bottle necked to the number of multi-axis CNC machines. Even though casting and forging require machining, the machining operations are far fewer and can be performed on simpler CNC machines; hence, why forging and casting is used for bulk manufacturing and why machining a crankshaft is reserved for special applications.

9.5 Crankshaft Materials

Up until now the materials used for manufacturing crankshafts has been taken for granted, but, like the manufacturing processes, the material used depends heavily on its application. Despite the manufacturing process, the materials used in crankshaft must

have certain qualities and requirements. In terms of performance requirements, a crankshaft must be resistant to fatigue produced by torsional and bending loads. Surface fatigue in the bearing areas must be high, either by good post-process heat treatability or by naturally good surface wear characteristics. Moreover, a crankshaft should exhibit high strength and stiffness. These two terms are often confused, however. The strength of a crankshaft represents its ability to withstand stress while the stiffness is the ability to withstand deflection given stress.

Manufacturing requirements are generally aimed towards forging and casting since machining works directly on a billet with characteristics similar to the finish product. Therefore, when it comes to forging, the material used must have a uniform and consistent response to hardening. Uneven hardening of the crankshaft is completely undesirable. Once the crankshaft has been forged, and consequently hardened, the material must be relatively easy to machine. Surface treats are present during the final stages of crankshaft manufacturing; therefore, the material selected must present predictable responses to various surface treatments such as fillet roller burnishing or nitriding. [48]

The following table demonstrates several steel alloys and their applications.

TABLE 3: COMMON IRON ALLOYS USED FOR CRANKSHAFT MANUFACTURING AND THEIR CHARACTERISTICS.

Steel Alloy	Nominal Percentages of Alloying Elements						Description
	C	Mo	Cr	Si	Ni	Mn	
1%-Chromium-molybdenum	0.4	0.3	1.2	-	-		Crankshafts made from this steel are used for medium to high power output vehicles. The chromium in this steel makes this alloy ideal for corrosion resistance and high temperature applications
2.5%-Nickel-chromium-molybdenum	0.31	0.55	0.65	-	2.5		Used usually for high power output vehicles. This material presents high fatigue and corrosion resistance as well as high strength and toughness at high temperatures.
3%-chromium-molybdenum	0.15	0.5	3	-	-		This alloy presents high fatigue and strength characteristics ideal for high output vehicles

Manganese-molybdenum	0.38	0.3	-	-	-	1.5	This steel is generally used for engines with a medium power output. The main advantage of this material is its high resistance to wear and its ease of surface hardening
Nodular Cast Iron	2.5-4	1-3	-	0.3-5	-	-	Main alloy used for crankshaft casting. Exhibits great surface hardness, strength, and good fluidity.

10. CRANKSHAFT DESIGN TRENDS

The process required to design a crankshaft is not linear. It is a process that requires several iterations and analysis. The iterative process can be outlined and is as follows.

1. **Engine Configuration Data** – the first step in designing a crankshaft is by knowing the engine for which the crankshaft is going to be designed:
 - a. Bore and stroke of the engine
 - b. Number of cylinders and their arrangement
 - c. Component masses such as piston and connecting rod as well as connecting rod length.
2. **Engine Output Data** – the intended use of the engine and the power output desired is a key factor for stress calculations which influence design greatly.
 - a. Maximum Torque at RPM
 - b. Maximum Power at RPM
 - c. Maximum engine angular velocity
 - d. Expected cylinder pressure pattern
3. **Initial Dimensional Values** – Initial dimensional values for crankshaft characteristic parts are chosen either based on previous experiments or other available comparisons. Characteristic parts include: main journal bearing diameter, crank throw diameter, fillet radius, and web thickness.
4. **Calculation of Reciprocating and Rotating Masses** – based on the dimensions obtained previously the masses of the crankshaft assembly can be calculated. It is a known fact that part of the connecting rod reciprocates while the other part rotates. In this step, the portions of the connecting rod that reciprocate and rotate are calculated. Other masses that are calculated include: crank throw and counterweight (rotating) and piston assembly (reciprocating).
5. **Calculation of Pressure and Inertia Forces** – The pressure and inertia forces are calculated throughout the entire RPM range of the engine in question. However, most importantly, the pressure and inertia forces are calculated at maximum power and torque since at those points the stresses will be maximum.
6. **Reaction Forces** – The reaction forces absorbed by the main journal bearings are calculated. This gives the criteria to properly select the main journal bearings.

7. **Bending Moments** – Bending moments are calculated allowing the design to determine whether balancing shafts are necessary and if the stresses produced by the bending moments are significant.
8. **Failure Criteria** – Failure criteria is applied to determine successful and unsuccessful design considerations. The design margin and safety coefficient are calculated with the considered material fatigue.
9. **Iteration** – The process is repeated the required amount until successful preliminary calculations are obtained.

This iterative process is used to calculate forces and to obtain a preliminary design for a crankshaft. The calculations and values obtained through this process must be tested and examined through software. [2, p. 16-12]

10.1 Limiting Crankshaft Parameters

Like in every manufactured element submitted to external stresses, there are several parts that limit the mechanical performance of the element as a whole. The crankshaft has inconsistent cross sections causing inconsistent structure stiffness resulting in stress concentration. The stress concentration is the focus point of crankshaft design and the areas where the stress concentration takes place are common to all crankshafts. These areas are the fillets. A fillet is a manufacturing process where the edge between two surfaces is rounded to reduce stress. [50] The fillets in the crankshaft are located between the crankweb and the journals as seen in the following figure:

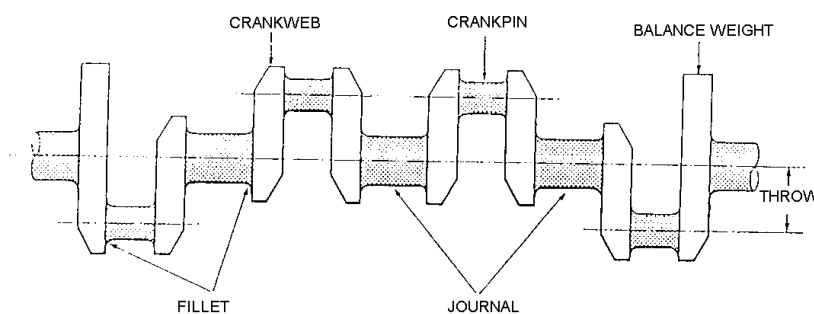


Fig. 10.1: Crankshaft fillets [26]

These areas are constantly exposed to alternating compressive and tractional forces causing fatigue to be a major concern. Even though post-processing heat and surface treatments are made to strengthen these areas, there are key factors in the design phase

that improve fillet strength and fatigue resistance. Crankpin overlap, fillet radius, and the crankweb thickness are all factors that improve resistance. The challenge here is optimizing between crankpin overlap and crankweb thickness and their masses since as both factors increase in dimensions, the crankshaft becomes stiffer, but also, heavier.

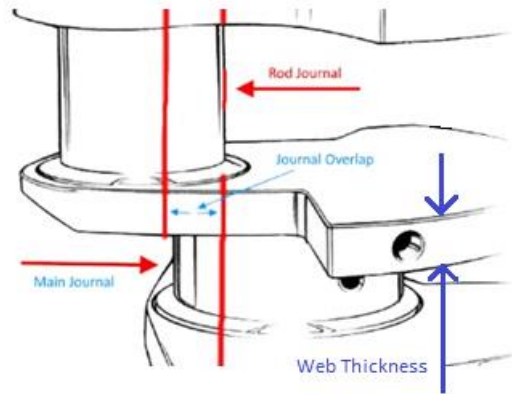


Fig. 10.2: Crankpin overlap and crankweb thickness illustration [51]

Varying the crankweb thickness is straight forward and, usually, the only drawback increasing it presents is additional mass which translates into more inertia. However, increasing the crankpin overlap is more complicated. The following formula shows how crankpin overlap is calculated:

$$e_{overlap} = \frac{(D_c + D_m - r)}{2} \quad [2, \text{eq. (16.1)}] \quad (10.1)$$

Where D_c is the crankpin diameter, D_m is the main journals diameter, and r is the stroke.

The stroke of the engine is a design variable and cannot be changed. Therefore, changing either of the diameters is an option. The drawback of increasing either diameter to increase crankshaft stiffness is that as the diameter increase produces bearing friction (due to more surface area) which in turn reduces fuel economy and power output. [2, p. 16-14]

10.2 Initial Crankshaft Design Dimensions

Parting values for any design is necessary. These parting or initial values usually come from previous experiences and designs, and the iterative process mentioned at the beginning of this chapter is used to obtain a design satisfactory enough to be put through a finite elements analysis. However, an adequate and conservative starting point can be obtained through the relations summarized in the next table which takes the piston bore as the reference value:

TABLE 4: INITIAL CRANKSHAFT DESIGN DIMENSIONS

Characteristic	Initial Design Dimension
Piston bore diameter	D
Distance between cylinders, S	$1.20 \cdot D$
Crankpin diameter, D_c	$>0.6 \cdot D$
Crankpin journal width, W_c	$0.35 \cdot D$ and $W_c / D_c > 0.3$
Main journal diameter, D_m	$0.75 \cdot D$ and $> D_c$
Main journal width, W_m	$0.40 \cdot D$ and $W_m / D_m > 0.3$
Crankweb thickness, e_w	$0.25 \cdot D$
Crankpin fillet radius, r_{cf}	$0.04 \cdot D$ and $> 0.05 \cdot D_c$
Main journal fillet radius, r_{mf}	$0.04 \cdot D$

[2, P. 16-16]

The dimensions stated in table 4 are illustrated in figure 10.3.

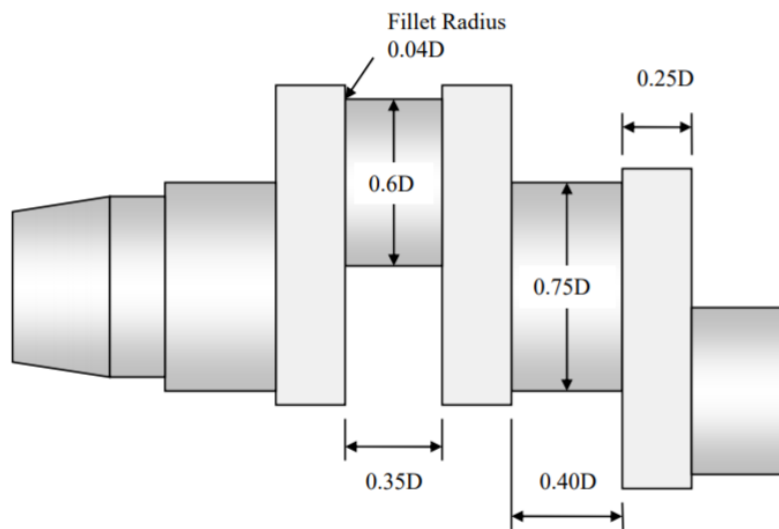


Fig. 10.3: Initial crankshaft design dimensions illustration. [2, Fig. 16.24]

These dimensions are known to be conservative and often need to be reduced and adjusted through iterations.

11. CONCLUSIONS

The objective of this thesis was to obtain a better understanding of how the crankshaft works by including important characteristics to be considered in the designing phase. The forces produced by the piston-connecting rod mechanism are valid for a conservative approximation of initial crankshaft design. However, the methodology used to calculate the gas force and its corresponding torque, while its useful as a representative value, it does not define the real four stroke Otto cycle; thus, is not adequate for final crankshaft design.

A way to improve the potential errors present in gas force and torque is to obtain a least squares regression curve to fit a real Otto cycle and relate it to crankshaft rotation angle; thus, replacing the piece-wise function modelled after an ideal Otto cycle. Another way to improve the instantaneous cylinder gas pressure is by defining a function that models real life behavior of the ratio of specific heats at constant pressure and volume, γ . This parameter is known to be affected by air-fuel ratio and by temperature which is constantly varying within the cylinder. Despite the inaccuracies present in the formulation, the tool developed with MATLAB is still relevant for crankshaft design since it allows the user to create a stepping stone from which to start off.

It is evident that successful crankshaft design requires a lot of experience and powerful software. With standards set so high in today's industries, without prior finite element analysis a crankshaft design would never make it past initial dimensioning. Even though the use of a finite element analysis software was not covered, with the information and criteria given, one should be able to identify if a crankshaft design is successful or not through FEA software results since the areas with the highest amount of stress on the crankshaft will remain the same despite the configuration, that is, the fillets.

Ultimately, the objectives of the thesis were accomplished. The kinematics and dynamics acting on the cranktrain were derived through simple geometric analysis, and then were integrated into a MATLAB software tool to obtain the configurations' kinematic and dynamic characteristic curves. Engine balancing required by the cranktrain dynamics is essential information to understand the considerations needed to be made when designing an engine. Also, understanding the manufacturers' choices of certain crankshaft arrangements further proves that crankshaft design is an experience-based field. The

manufacturing processes covered give knowledge of how different application-based crankshafts differ in fabrication from one to another. In addition, the materials used for crankshaft manufacturing provide information on the different requirements of material properties different types of crankshafts need. Finally, all the previous information combined with the iterative design process provides the reader with enough information to address how different forces affect different crankshaft parameters and what measures can be taken to improve on the design.

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ANNEX 1

MATLAB Crankshaft Program Code

```
%Dialog Box
prompt = {'Connecting Rod Length [m]:','Crankpin Radius (half-stroke)[m]:','Bore
[m]:','Piston Mass [kg]:','Connecting Rod Mass [kg]:','Combustion Chamber Height
[m]','Maximum Pressure [Pa]:','Angular Velocity [RPM]:'};
title = 'Introduce Engine Characteristics';
dims = [1 60];
definput = {'0.08','0.025','0.065','0.322','0.155','0.008','80e5','4000'};
answer = inputdlg(prompt,title,dims,definput);
VectorDatos=str2double(answer)
clear title
%constantes
l=VectorDatos(1,1);
r=VectorDatos(2,1);
B=VectorDatos(3,1);
Mp=VectorDatos(4,1);
Mcon=VectorDatos(5,1);
Hcc=VectorDatos(6,1);
Pmax=VectorDatos(7,1);
w=VectorDatos(8,1)*((2*pi)/60);
A=0;
Patm=100000;
gam=1.4;
%Masa Oscilante
MR=Mp+(0.5*Mcon);
%Volumen de la cámara de combustión
Vcc=Hcc*((pi*B^2)/4);
%Presión instantánea 0<A<180
Pins1=Patm;
%Presión instantánea A=360
Pins3=Pmax;
%Presión instantánea A=540
Pins5=Patm;
while 0<=A && A<=180
    %haciendo las funciones derivables
    Eq1=((r^2)*((cosd(A)^2)-(sind(A)^2))/sqrt((l^2)-(r^2*(sind(A)^2)));
    Eq2=((r^4)*((cosd(A)*sind(A)))/(sqrt((l^2)-(r^2*(sind(A)^2))))^(3);
    % Posición del pistón
    x=r*cosd(A)+sqrt((l^2)-(r^2*(sind(A))^2);
    %Velocidad del pistón
    v=(-(r*sind(A))-((r^2)*((cosd(A)*sind(A)))/(sqrt((l^2)-(r^2*((sind(A))^2))))))*w;
    %Aceleración
    a=((r^2*sind(A)^2)/sqrt(l^2-r^2*sind(A)^2)-(r^2*cosd(A)^2)/sqrt(l^2-r^2*sind(A)^2)-
    (r^4*cosd(A)^2*sind(A)^2)/(l^2-r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
    %Fuerzas Iniciales
    F1=MR*(r*cosd(A)-Eq1-Eq2)*w^2;
    %Momento Torsor
    Tl=(MR*r*sind(A)*(1+(r/l)*cosd(A))*((r^2*sind(A)^2)/sqrt(l^2-r^2*sind(A)^2)-
    (r^2*cosd(A)^2)/sqrt(l^2-r^2*sind(A)^2)-(r^4*cosd(A)^2*sind(A)^2)/(l^2-
    r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
    %Fuerza del Gas
    Fg=(Pmax*pi*B^2)/4;
```



```

%Volumen instantáneo
Vins=((1+r)-x)*(pi*B^2)/4+vcc;
P=Pins1;
%Presión del gas
Pg=P-Patm;
Tg=Pg*((pi*B^2)/4)*r*sind(A)*(1+(r/l)*cosd(A));
A=A+1;
VPg(A,1)=Pg;
VX(A,1)=x;
VA(A,1)=A;
VV(A,1)=v;
VAC(A,1)=a;
VTl(A,1)=Tl;
VTg(A,1)=Tg;
end
while 180<A && A<360
%haciendo las funciones derivables
Eq1=((r^2)*((cosd(A)^2)-(sind(A)^2))/sqrt((1^2)-(r^2*(sind(A)^2)));
Eq2=((r^4)*((cosd(A)*sind(A)))/sqrt((1^2)-(r^2*(sind(A)^2))))^(3);
% Posición del pistón
x=r*cosd(A)+sqrt((1^2)-(r^2*(sind(A))^2));
%Velocidad del pistón
v=(-(r*sind(A))-((r^2)*((cosd(A)*sind(A)))/sqrt((1^2)-(r^2*(sind(A))^2))))*w;
%Aceleración
a=((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
(r^4*cosd(A)^2*sind(A)^2)/(1^2-r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
%Fuerzas Iniciales
F1=MR*(r*cosd(A)-Eq1-Eq2)*w^2;
%Momento Torsor
Tl=(MR*r*sind(A)*(1+(r/l)*cosd(A))*((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^4*cosd(A)^2*sind(A)^2)/(1^2-
r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
%Fuerza del Gas
Fg=(Pmax*pi*B^2)/4;
%Volumen instantáneo
Vins=((1+r)-x)*(pi*B^2)/4+vcc;
%Presión instantánea 180<A<360
Pins2=Patm+((Patm*((2*r*(pi*B^2)/4)+vcc)^gam)*(Vins)^(-gam));
P=Pins2;
%Presión del gas
Pg=P-Patm;
Tg=Pg*((pi*B^2)/4)*r*sind(A)*(1+(r/l)*cosd(A));
A=A+1;
VPg(A,1)=Pg;
VX(A,1)=x;
VA(A,1)=A;
VV(A,1)=v;
VAC(A,1)=a;
VTl(A,1)=Tl;
VTg(A,1)=Tg;
end
if A==360
%haciendo las funciones derivables
Eq1=((r^2)*((cosd(A)^2)-(sind(A)^2))/sqrt((1^2)-(r^2*(sind(A)^2)));
Eq2=((r^4)*((cosd(A)*sind(A)))/sqrt((1^2)-(r^2*(sind(A)^2))))^(3);
% Posición del pistón
x=r*cosd(A)+sqrt((1^2)-(r^2*(sind(A))^2));
%Velocidad del pistón

```

```

v=(-(r*sind(A))-((r^2)*((cosd(A)*sind(A))))/(sqrt((1^2)-(r^2*((sind(A))^2))))))*w;
%Aceleración
a=((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
(r^4*cosd(A)^2*sind(A)^2)/(1^2-r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
%Fuerzas Iniciales
F1=MR*(r*cosd(A)-Eq1-Eq2)*w^2;
%Momento Torsor
T1=(MR*r*sind(A)*(1+(r/l)*cosd(A))*((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^4*cosd(A)^2*sind(A)^2)/(1^2-
r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
%Fuerza del Gas
Fg=(Pmax*pi*B^2)/4;
%Volumen instantáneo
Vins=((1+r)-x)*(pi*B^2)/4+vcc;
P=Pins3;
%Presión del gas
Pg=P-Patm;
Tg=Pg*((pi*B^2)/4)*r*sind(A)*(1+(r/l)*cosd(A));
A=A+1;
VPg(A,1)=Pg;
VX(A,1)=x;
VA(A,1)=A;
VV(A,1)=v;
VAC(A,1)=a;
VT1(A,1)=T1;
VTg(A,1)=Tg;
end
while 360<A && A<540
%haciendo las funciones derivables
Eq1=((r^2)*((cosd(A)^2)-(sind(A)^2)))/sqrt((1^2)-(r^2*(sind(A)^2)));
Eq2=((r^4)*((cosd(A)*sind(A))))/(sqrt((1^2)-(r^2*(sind(A)^2))))^(3);
% Posición del pistón
x=r*cosd(A)+sqrt((1^2)-(r^2)*(sind(A))^2)
%Velocidad del pistón
v=(-(r*sind(A))-((r^2)*((cosd(A)*sind(A))))/(sqrt((1^2)-(r^2*((sind(A))^2))))))*w;
%Aceleración
a=((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
(r^4*cosd(A)^2*sind(A)^2)/(1^2-r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
%Fuerzas Iniciales
F1=MR*(r*cosd(A)-Eq1-Eq2)*w^2;
%Momento Torsor
T1=(MR*r*sind(A)*(1+(r/l)*cosd(A))*((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^4*cosd(A)^2*sind(A)^2)/(1^2-
r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
%Fuerza del Gas
Fg=(Pmax*pi*B^2)/4;
%Volumen instantáneo
Vins=((1+r)-x)*(pi*B^2)/4+vcc;
Pins4=Patm+((Pmax*(Vcc)^gam)*(Vins)^(-gam));
P=Pins4;
%Presión del gas
Pg=P-Patm;
Tg=Pg*((pi*B^2)/4)*r*sind(A)*(1+(r/l)*cosd(A));
A=A+1;
VPg(A,1)=Pg;
VX(A,1)=x;
VA(A,1)=A;
VV(A,1)=v;

```

```

VAC(A,1)=a;
VT1(A,1)=T1;
VTg(A,1)=Tg;
end
while 540<=A && A<=720
    %haciendo las funciones derivables
    Eq1=((r^2)*((cosd(A)^2)-(sind(A)^2))/sqrt((1^2)-(r^2*(sind(A)^2)));
    Eq2=((r^4)*((cosd(A)*sind(A)))/(sqrt((1^2)-(r^2*(sind(A)^2))))^3);
    % Posición del pistón
    x=r*cosd(A)+sqrt((1^2)-(r^2*(sind(A))^2));
    %Velocidad del pistón
    v=(-(r*sind(A))-((r^2)*((cosd(A)*sind(A)))/(sqrt((1^2)-(r^2*((sind(A))^2))))))*w;
    %Aceleración
    a=((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
    (r^4*cosd(A)^2*sind(A)^2)/(1^2-r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
    %Fuerzas Iniciales
    F1=MR*(r*cosd(A)-Eq1-Eq2)*w^2;
    %Momento Torsor
    T1=(MR*r*sind(A)*(1+(r/l)*cosd(A))*((r^2*sind(A)^2)/sqrt(1^2-r^2*sind(A)^2)-
    (r^2*cosd(A)^2)/sqrt(1^2-r^2*sind(A)^2)-(r^4*cosd(A)^2*sind(A)^2)/(1^2-
    r^2*sind(A)^2)^(3/2)-r*cosd(A))*w^2;
    %Fuerza del Gas
    Fg=(Pmax*pi*B^2)/4;
    %Volumen instantáneo
    Vins=((1+r)-x)*(pi*B^2)/4+Vcc;
    P=Pin5;
    %Presión del gas
    Pg=P-Patm;
    Tg=Pg*((pi*B^2)/4)*r*sind(A)*(1+(r/l)*cosd(A));
    A=A+1;
    VPg(A,1)=Pg;
    VX(A,1)=x;
    VA(A,1)=A;
    VV(A,1)=v;
    VAC(A,1)=a;
    VT1(A,1)=T1;
    VTg(A,1)=Tg;
end
VTg2=VTg;
Vsuma=(VT1+VTg2);
figure(5)
plot(VA,VPg)
title('Gas Pressure')
xlabel('Crankshaft Rotation Angle(°)')
ylabel('Gas Pressure(Pa)')
[valPg, idxPg] = max(VPg);
txt = ['\leftarrow maximum:' num2str(valPg) '(Pa)'];
text(idxBg,valBg,txt)
[valPg2, idxPg2] = min(VPg);
txt2 = ['\leftarrow minimum:' num2str(valPg2) '(Pa)'];
text(idxBg2,valBg2,txt2);
axis([0 720 min(VPg)*1.2 max(VPg)*1.2])
figure(4)
plot(VA,VX)
title('Position')
xlabel('Crankshaft Rotation Angle(°)')
ylabel('Piston Position(m)')
axis([0 720 min(VX)*0.8 max(VX)*1.2])

```

```

figure(3)
plot(VA,VV)
title('Velocity')
xlabel('Crankshaft Rotation Angle(°)')
ylabel('Piston Velocity(m/s)')
[valV, idxV] = max(VV);
txt = ['\leftarrow maximum:' num2str(valV) '(m/s)'];
text(idxV,valV,txt)
[valV2, idxV2] = min(VV);
txt2 = ['\leftarrow minimum:' num2str(valV2) '(m/s)'];
text(idxV2,valV2,txt2);
axis([0 720 min(VV)*1.2 max(VV)*1.2])
figure(2)
plot(VA,VAC)
title('Acceleration')
xlabel('Crankshaft Rotation Angle(°)')
ylabel('Piston Acceleration(m/s^2)')
[valAC, idxAC] = max(VAC);
[valAC2, idxAC2] = min(VAC);
txt = ['\leftarrow maximum:' num2str(valAC/9.81) 'g's'];
text(idxAC,valAC,txt)
txt2 = ['\leftarrow minimum:' num2str(valAC2/9.81) 'g's'];
text(idxAC2,valAC2,txt2);
axis([0 720 min(VAC)*1.2 max(VAC)*1.2])
figure(1)
hold on
plot(VA,VT1)
[valVT1, idxVT1] = max(VT1);
[valVT12, idxVT12] = min(VT1);
plot(VA,VTg2)
[valVTg2, idxVTg2] = max(VTg2);
[valVTg22, idxVTg22] = min(VTg2);
plot(VA,VSuma)
[valVSuma, idxVSuma] = max(VSuma);
[valVSuma2, idxVSuma2] = min(VSuma);
txt = ['\leftarrow Total torque maximum:' num2str(valVSuma) '(Nm)'];
text(idxVSuma,valVSuma,txt)
txt2 = ['\leftarrow Total torque minimum:' num2str(valVSuma2) '(Nm)'];
text(idxVSuma2,valVSuma2,txt2);
xlabel('Crankshaft Rotation Angle(°)')
ylabel('Moment (Nm)')
title('Torques')
legend({'Inertial Torque','Gas Torque','Total Torque'},'Location','northwest')
hold off

```