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The deterministic nature of the fracture location in the dynamic tensile testing of steel sheets

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Abstract

This paper investigates the key mechanisms which determine the fracture location in the dynamic tensile testing of steel sheets. For that purpose we have conducted experiments and finite element simulations. Experiments have been performed using samples with six different gauge lengths, ranging from 20 mm to 140 mm, that have been tested within a wide spectrum of loading velocities, ranging from 1 m/s to 7.5 m/s. Three are the key outcomes derived from the tests: (1) for a given gauge length and applied velocity, the repeatability in the failure location is extremely high, (2) there is a strong interplay between applied velocity, gauge length and fracture location and (3) multiple, and largely regular, localization patterns have been observed in a significant number of the experiments performed using the samples with the shorter gauge lengths. Our experimental findings are explained using the finite element simulations. On the one hand, we have shown that variations in the applied velocity and the gauge length alter the processes of reflection and interaction of waves taking place in the sample during the test, which leads to the systematic motion of the plastic localization along the gauge (as experimentally observed). On the other hand, we have detected that the emergence of multiple localization patterns requires of short and equilibrated specimens with uniform stress and strain distributions along the gauge. We conclude that the experimental and numerical results presented in this paper show that, in absence of significant material and/or geometrical defects, the location of plastic strain localization in the dynamic tensile test is deterministic.

Keywords:

Dynamic tensile experiments, Finite element simulations, Stress waves, Strain localization, Dynamic fracture

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28 1. Introduction

29 In the decade of the 40's, the pioneering publications of Nadai and Manjoine [1], De Forest
30 et al. [2], Clark [3], Parker and Ferguson [4] and Manjoine [5] represented a significant progress in
31 the research of the dynamic tensile test. These works, motivated by the celebrated papers of Mann
32 [6, 7], definitely showed that high velocity tests are essential to reveal the true dynamic properties
33 of materials. It was recognized that the performance of some materials under dynamic loading is
34 different from that observed under static conditions. For the first time, the effect of velocity on
35 the capacity of metallic materials to absorb energy was demonstrated. Within this context, special
36 mention requires the thorough experimental investigation conducted in the Guggenheim Aeronau-
37 tical Laboratory of the California Institute of Technology (directed at that time by Theodore Von
38 Kármán) with the aim of evaluating the impact endurance limit of different metals used in aircraft
39 construction [8, 9, 10, 11]. Note that this extensive experimental research was directly driven by
40 industrial concerns. In Beardsley and Coates [9] words *"with the current improvements in aircraft*
41 *structural design methods, resulting in more efficient structures in which the material is worked at*
42 *higher stresses, it is becoming increasingly more necessary to consider the effects of dynamic loading*
43 *on the structure"*.

44 During the following years, with the continuous support of the aeronautical sector, the efforts
45 were focused on developing a theoretical framework to explain the experimental findings. Thus,
46 Clark and co-workers published a series of papers [12, 13, 14, 15] in which the theory of the
47 elastic and plastic strain propagation developed by Von Kármán and others [16, 17, 18, 19, 20] was
48 used to interpret in a rational manner the experimental data. A key outcome of these theoretical
49 investigations was to show that the strain rate in impact tests varies from point to point along
50 the specimen, and for a given point it is also dependent upon time [14]. This behaviour, which
51 is accentuated as the impact velocity increases, was identified as the main problem of the tension
52 impact test to study the influence of the rate of strain on the properties of metals.

53 The following decades, especially after the development of the tension version of the Hopkinson-
54 bar technique in the early 60's [21], were very much focused on overcoming this drawback. The
55 belief that the use of very short specimens minimizes the importance of the inertia loads and allows
56 to neglect the intervention of strain propagation phenomena within the specimen became widely
57 accepted [22, 23] and the dynamic stress-strain characteristics of different metallic materials were

58 published, see for instance the works of Nicholas [24, 25, 26]. On the other hand, the works of
59 Lubliner [27] and Botte et al. [28, 29] strengthened the idea that the essential character of the
60 tensile impact test is the non-uniformity in time and space of the state variables of the material.
61 If long specimens are used the parameters which define the state of the material (stress, strain
62 and particle velocity) assume different values in the different sections of the specimen, and they
63 change with time. Botte et al. [28] explicitly stated that numerical analysis becomes indispensable
64 to investigate the spatial-temporal variation of the field variables in detail.

65 Thus, the advent of computational mechanics gave new impetus to the analysis and understand-
66 ing of the impact tensile test [30, 31, 32]. The finite element method has been widely used over
67 the last years in the design of tensile specimens suitable to extract the true dynamic properties of
68 metallic materials [33, 34, 35]. Within this context, it has to be highlighted the work of Rusinek
69 et al. [36] who reviewed the performance of six different specimen geometries loaded in impact ten-
70 sion. Driven by the earlier work of Nemes and Eftis [31], Rusinek et al. [36] paid special attention
71 to the interplay between necking inception, impact velocity and specimen geometry. They showed
72 that, as soon as the impact velocity is such that the strain propagation effects become relevant,
73 the necking moves away from the central point of the sample (where it locates under quasi-static
74 conditions). This observation, which agrees with previous experimental results published by Wood
75 [37], suggests that the necking inception in the dynamic tensile test is a deterministic process.
76 Nevertheless, whether the nature of the necking location is deterministic or random is still a con-
77 troversial issue, as can be seen from the number of recent publications dealing with this precise
78 topic [38, 39, 40].

79 With the aim of clarifying this controversial issue, in this investigation we have performed an
80 extensive experimental and numerical campaign that reveals the deterministic character of the
81 necking (and fracture) location in the dynamic tensile test. We have carried out dynamic tensile
82 experiments using steel sheet specimens with six different gauge lengths (20 *mm*, 40 *mm*, 60 *mm*,
83 80 *mm*, 100 *mm* and 140 *mm*) for seven impact velocities (1 *m/s*, 1.75 *m/s*, 2.5 *m/s*, 3.75 *m/s*,
84 5 *m/s*, 6.25 *m/s* and 7.5 *m/s*). Similarly to the experiments reported by Wood [37], we have
85 observed that the fracture location moves systematically from side to side of the sample with the
86 variations in impact velocity and gauge length. Further, for each combination of gauge length and
87 applied velocity several repeats are performed which show an extremely high repeatability in the

necking (and failure) location. A key, and very unusual, experimental finding of this work is the multiple, and largely regular, localization patterns that have been observed in a significant number of the shortest samples tested. We have explained all these experimental findings with finite element simulations performed in ABAQUS/Explicit [41]. Thus, in agreement with the experiments, the computations have shown that variations in the applied velocity and gauge length lead to the systematic motion of the plastic localization along the gauge. Further, our numerical calculations serve to prove that the emergence of multiple localization patterns is associated to equilibrated specimens with low slenderness ratios and *hardly* subjected to the influence of stress waves.

2. Experimental setup and mechanical characterization

2.1. Material and specimens

The material of this study is annealed AISI 430 stainless steel. Its chemical composition is given in Table 1.

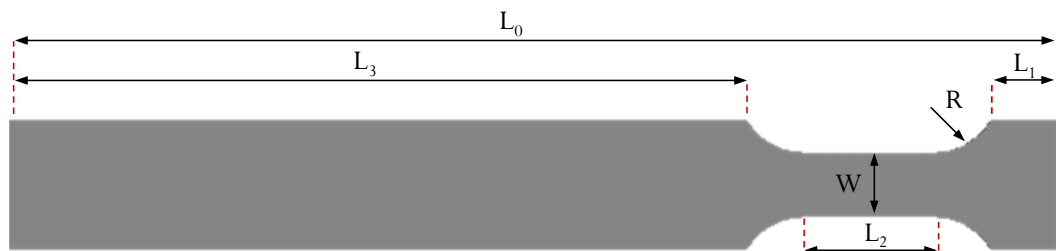
Fe	C	Mn	P	S	Si	C	Ni
Balance	0.12 max.	1.00 max.	0.04 max.	0.03 max.	1.00 max.	16.00 - 18.00	0.5 max.

Table 1: Chemical composition of the AISI 430 stainless steel (wt %) as taken from [42].

The AISI 430 is one of the most widely used ferritic stainless steels. It shows excellent stress corrosion cracking resistance and good resistance to pitting and crevice corrosion in chlorine environments. Typical consumer product applications include automotive trim and molding and furnace combustion chambers. Industrial and commercial applications range from interior architectural applications to nitric acid plant equipment and oil refinery equipment [42].

The material is supplied in plates of thickness $h = 1 \text{ mm}$ from which tensile specimens are machined. The specimens' geometry and dimensions are shown in Fig. 1. The impacted side is the left side of the specimen in the figure (and therefore the clamped side is the right side). L_0 , L_1 , L_2 , L_3 , W and R denote respectively the overall length of the sample, the length of the grip section of the clamped side, the length of the gauge, the length of the grip section of the impacted side, the width of the gauge and the radius of the fillets. The specimens are machined by laser cutting with accuracy of $\pm 0.1 \text{ mm}$. We distinguish between samples used in the quasi-static tests and samples

112 used in the dynamic tests. The quasi-static specimens, identical to those used in [43], have a gauge
 113 length of 20 mm. Note that the quasi-static tests are a requisite to characterize the mechanical
 114 response of the material rather than a specific goal of this investigation. The dynamic samples are
 115 machined with six different gauge lengths: type 1 with 20 mm, type 2 with 40 mm, type 3 with 60
 116 mm, type 4 with 80 mm, type 5 with 100 mm and type 6 with 140 mm. The dynamic tests are
 117 performed in order to uncover the interplay between specimen gauge length, the impact velocity
 118 and the fracture location, as further discussed in section 3. Whether it is a quasi-static or dynamic
 119 experiment, at least three repeats are conducted.



Static specimens

L_0	L_1	L_2 (gauge length)	L_3	W	R
120	41.34	20	41.34	10	10

Dynamic specimens

	L_0	L_1	L_2 (gauge length)	L_3	W	R
Type 1			20	421.34		
Type 2			40	401.34		
Type 3	500	41.34	60	381.34	10	10
Type 4			80	361.34		
Type 5			100	341.34		
Type 6			140	301.34		

Figure 1: Geometry and dimensions of the specimens used in the static and dynamic experiments.

120 2.2. Quasi-static testing

121 The quasi-static experiments at room temperature were conducted using a servo-hydraulic test-
 122 ing machine INSTRON 8516 100kN under displacement control. We tested specimens whose loading
 123 direction formed angles of 0° (parallel), 45° and 90° (perpendicular) with the rolling direction of the
 124 plate. The goal was to investigate whether the material displays anisotropy caused by the rolling of
 125 the plate. Experiments were conducted for three (initial) strain rates: $\dot{\epsilon}_0 = 10^{-3} s^{-1}$, $\dot{\epsilon}_0 = 10^{-2} s^{-1}$
 126 and $\dot{\epsilon}_0 = 10^{-1} s^{-1}$. In all the experiments the axial strain in the specimen is calculated relying on

127 the cross-head displacement of the machine which has been corrected with knowledge of the elastic
 128 modulus of the material as described, for instance, in [44].

129 Fig. 2 shows stress-strain curves obtained from specimens tested at 10^{-3} s^{-1} , that have been cut
 130 following the three different orientations (0° , 45° , 90°) investigated. It is shown that the orientation
 131 plays a minor role in the material behaviour since the three curves (practically) overlap. The yield
 132 stress and the strain hardening of the material are mild, and the onset of flow localization occurs
 133 for ~ 0.2 .

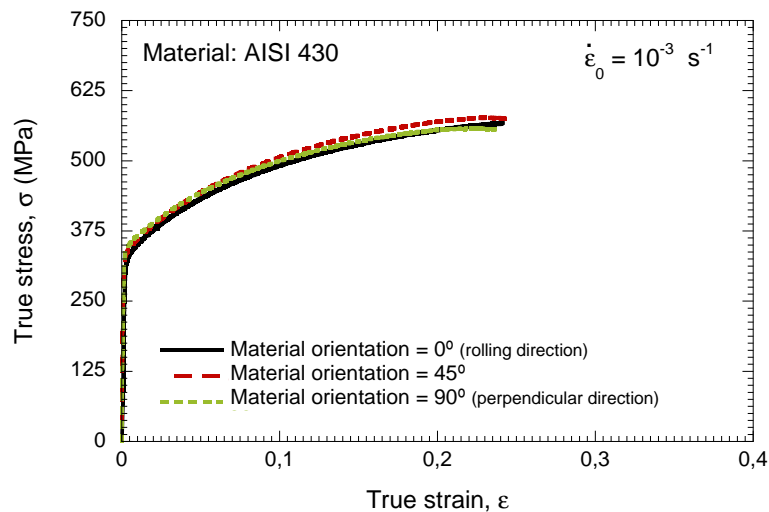


Figure 2: Experimental stress-strain curves for AISI 430 at $T_0 = 300 \text{ K}$ and 10^{-3} s^{-1} .

134 Similarly, we have observed that for 10^{-2} s^{-1} and 10^{-1} s^{-1} the orientation barely affects the
 135 stress-strain characteristics of the material. Relying on these observations we assume that the
 136 in-plane mechanical behaviour can be considered isotropic. From now on, all other experimental
 137 results we show are obtained from specimens cut parallel to the rolling direction.

138 Additionally to quasi-static room temperature tests, we conducted experiments at elevated
 139 temperatures $T_0 = 375 \text{ K}$, $T_0 = 425 \text{ K}$ and $T_0 = 475 \text{ K}$. A heating furnace SERVOSIS Split
 140 was installed on a servo-hydraulic testing machine INSTRON 8516 100kN. The experiments were
 141 conducted under displacement control. For all these tests, the (initial) strain rate was 10^{-2} s^{-1} .
 142 Fig. 3 shows that the stress-strain characteristic is slightly shifted downwards as the testing tem-
 143 perature increases, revealing the temperature sensitivity of the material within the range of testing
 144 temperatures considered.

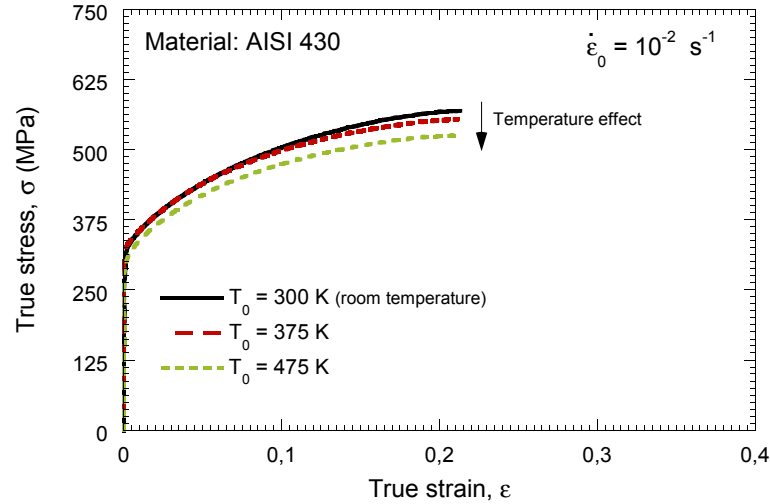


Figure 3: Experimental stress-strain curves for AISI 430 at 10^{-2} s^{-1} and three different testing temperatures $T_0 = 300 \text{ K}$, $T_0 = 375 \text{ K}$ and $T_0 = 475 \text{ K}$.

145 2.3. Dynamic testing

146 Dynamic tensile tests at room temperature are conducted using a high-speed testing machine
 147 Instron VHS within the range of impact velocities $1 \text{ m/s} \leq V_0 \leq 7.5 \text{ m/s}$. For the dynamic
 148 samples shown in Fig. 1, this set of impact velocities leads to a wide range of (initial) strain rates
 149 $7.15 \text{ s}^{-1} \leq \dot{\epsilon}_0 \leq 375 \text{ s}^{-1}$.

150 The gripping system incorporated in the Instron VHS is the so-called Fast Jaw system. This
 151 system relies on two gripping faces being initially held apart by a pair of angled wedges. The
 152 actuator initially accelerates downwards with the specimen passing freely between the grips. At
 153 the desired location the wedges are knocked out by a set of adjustable rods. This action releases
 154 the force of four pretensioned bolts, so causing a set of grips to clamp onto the specimen surface,
 155 applying the high velocity loading. This explanation, and further details on the operation mode of
 156 the Instron VHS machine, can be found in the work of Battams [45].

157 Note that the ringing period of the raw data registered from the machine is $\sim 157 \mu\text{s}$. This
 158 value corresponds to an eigenfrequency of the piezoelectric load cell of $\sim 6.4 \text{ kHz}$, as further verified
 159 using the Welch's Power Spectral Density estimation preimplemented in MATLAB. A band-pass
 160 Butterworth IIR Filter with a zero-phase forward and reverse procedure (to correct the associated
 161 delay of the signal) has been designed in MATLAB to filter the raw stress-strain curves. As
 162 further discussed by Rusinek et al. [33], this type of filtering process is usually applied to analyse
 163 the stress-strain characteristics obtained from dynamic tensile experiments performed using fast

164 servo-hydraulic machines.

165 Fig. 4 shows stress-strain curves obtained for different loading rates using specimens with gauge
 166 length $L_2 = 20 \text{ mm}$. Dynamic (filtered) experimental curves for $\dot{\epsilon}_0 = 87.5 \text{ s}^{-1}$ and $\dot{\epsilon}_0 = 250 \text{ s}^{-1}$
 167 are compared with the stress-strain characteristic obtained for $\dot{\epsilon}_0 = 10^{-3} \text{ s}^{-1}$. The material shows
 168 significant strain rate sensitivity within the range of strain rates tested.

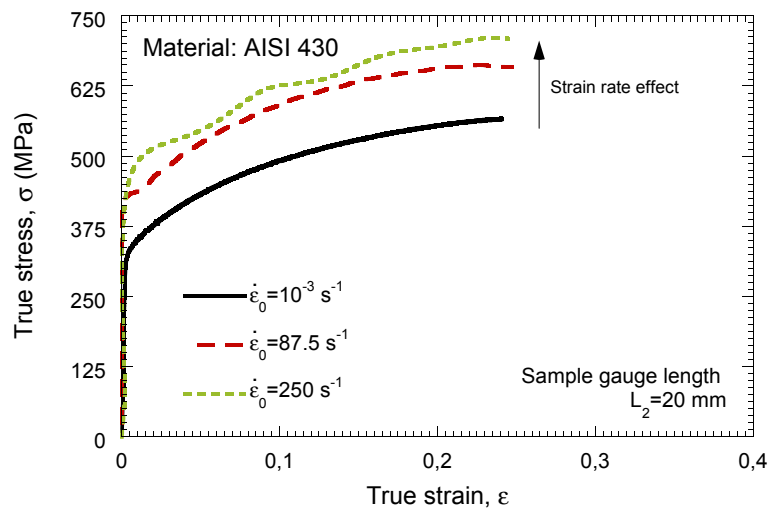


Figure 4: Experimental stress-strain curves for AISI 430 at $T_0 = 300 \text{ K}$ and three different initial strain rates: $\dot{\epsilon}_0 = 10^{-3} \text{ s}^{-1}$, $\dot{\epsilon}_0 = 87.5 \text{ s}^{-1}$ and $\dot{\epsilon}_0 = 250 \text{ s}^{-1}$.

169 3. Analysis and results: experiments

170 In this section we show selected dynamic experiments for different gauge lengths and impact
 171 velocities. The goal is to show an experimental verification of the deterministic character of the
 172 flow localization in the dynamic tensile test. The complete set of dynamic experiments that we
 173 have carried out is shown in Appendix A.

174 Fig. 5 shows three post-mortem samples with gauge length $L_2 = 100 \text{ mm}$ tested at $V_0 = 5 \text{ m/s}$.
 175 It has to be highlighted that, in the three repeats conducted of this test, we have obtained the same
 176 failure location. The specimen fails close to the clamped (opposite) side. According to Rodríguez-
 177 Martínez et al. [46], the fact that the failure is located away from the middle of the gauge clearly
 178 indicates that the specimen is not in (complete) equilibrium during loading. As discussed in the
 179 introductory section, the lack of equilibrium in dynamic testing of long tensile samples was reported,
 180 for instance, by Lubliner [27] and Botte et al. [28, 29]. Moreover, note that plastic localization
 181 develops by the intersection of a pair of necking bands that, in agreement with the theoretical

182 and numerical predictions reported by Storen and Rice [47] and Zhang and Ravi-Chandar [48], are
 183 aligned with the directions of zero stretch rate. One of these two bands, the one which develops
 184 faster, leads to the final fracture of the specimen. Note that there is (relatively) little reduction
 185 of the samples-width within the area surrounding the failure location. The width-reduction of the
 186 samples is largely uniform along the gauge.

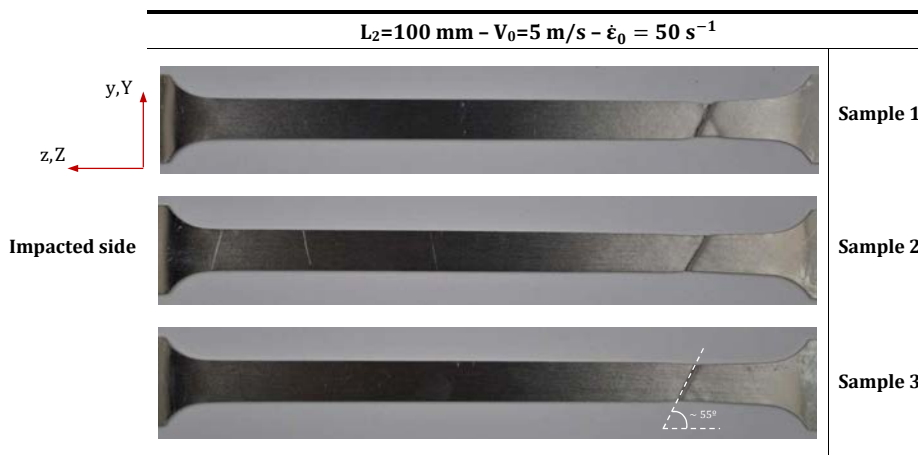


Figure 5: Three post-mortem samples with gauge length $L_2 = 100 \text{ mm}$ tested at $V_0 = 5 \text{ m/s}$.

187 The repeatability in the failure location of the dynamic samples is further illustrated in Fig. 6
 188 where we show three post-mortem samples with gauge length $L_2 = 140 \text{ mm}$ tested at $V_0 = 1.75 \text{ m/s}$.
 189 The failure of the sample always occurs close to the middle of the gauge. This does not necessarily
 190 imply that the sample is in equilibrium, but it simply exposes that the failure location depends on
 191 the applied velocity and the gauge length, as further discussed in sections 3.1 and 3.2. In other
 192 words: (1) if the failure locus is located away from the middle of the gauge we know that the sample
 193 is not in equilibrium but (2) the fact that the failure locus is located in the middle of the gauge
 194 does not ensure that the sample is in equilibrium, see Rodríguez-Martínez et al. [46] for details.
 195 Moreover, it has to be noted that, in comparison with the results shown in Fig. 5, now there is
 196 larger width-reduction of the gauge in the vicinity of the fracture point. The pair of localization
 197 bands are located inside a necked region in the $\{Y, Z\}$ plane. The width-reduction is not uniform
 198 along the gauge. The aspect ratio of the specimen gauge seems to play a strong role in the failure
 199 location and in the failure pattern, as further discussed in forthcoming sections of this paper.

200 To be noted that, as detailed in Appendix A, we have obtained very high repeatability in the
 201 failure location for all the gauge lengths explored and within the whole range of impact velocities
 202 tested. This indicates that, rather than being random, the position where the flow localization

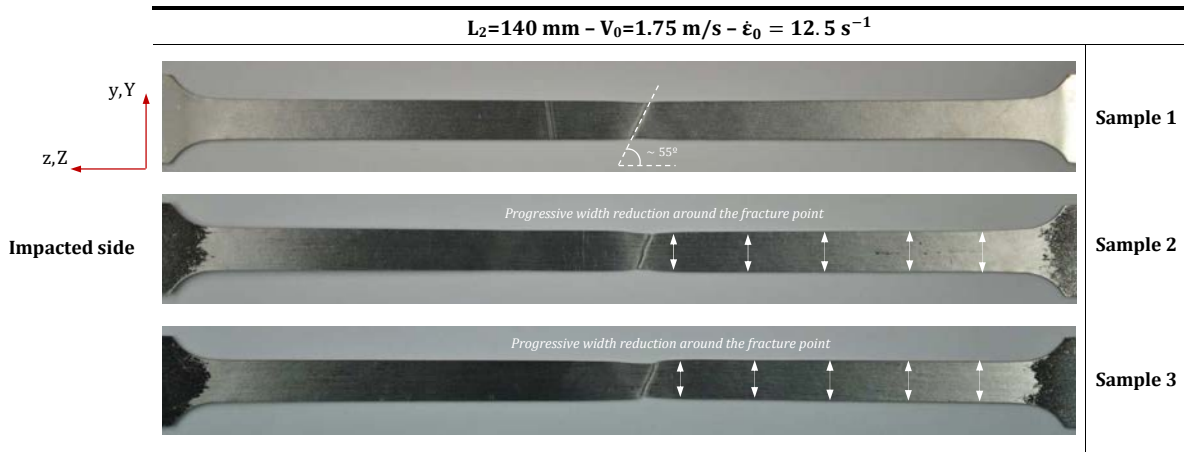


Figure 6: Three post-mortem samples with gauge length $L_2 = 140 \text{ mm}$ tested at $V_0 = 1.75 \text{ m/s}$.

occurs is deterministic. Exceptions occurred in few cases for which one of the three repeats programmed showed different failure location than the other two. In these selected cases we decided to perform an additional test after which we always had three (of four) samples with the same failure location. This failure location was assumed to be the representative of such sample geometry and loading conditions. The fact that one of the tests is not providing the same fracture location than the other three is simply attributed to the inherent uncertainties surrounding experimentation. Our belief is that slight variations in (1) the pressure applied by the jaws to fix the samples during testing and/or (2) the actual velocity applied by the machine are responsible for the small scatter that we have registered in the fracture location.

3.1. Influence of loading velocity on the location of flow localization

In this section we analyse the influence of loading velocity on the fracture location. Fig. 7 shows seven samples with gauge length $L_2 = 60 \text{ mm}$ tested at different velocities. For the smallest impact velocity that we have explored $V_0 = 1 \text{ m/s}$ the failure location occurs close to the impacted side. Increasing the impact velocity changes the place where the failure occurs. Thus, for $V_0 = 1.75 \text{ m/s}$, $V_0 = 2.5 \text{ m/s}$, $V_0 = 3.75 \text{ m/s}$, $V_0 = 5 \text{ m/s}$ and $V_0 = 6.25 \text{ m/s}$, we observe that the sample breaks near the clamped side. Finally, for the highest velocity tested $V_0 = 7.5 \text{ m/s}$ the fracture location moves again to the impacted side. Note that such a strong interplay between impact velocity and failure location has been found for the largest sample gauge lengths investigated. These experimental results bear a definite resemblance to those recently reported by Osovski et al. [39], Rittel et al. [40] and Rotbaum et al. [49] using cylindrical samples, and confirm the numerical predictions

223 reported by Rusinek et al. [36] and Rodríguez-Martínez et al. [50] using flat samples who claimed
 224 that the failure location in the dynamic tensile test is very much controlled by the impact velocity.
 225 Since the sample is initially at rest, the fact that the fracture location is controlled by the impact
 226 velocity means that the dynamic effects (stress waves and inertia) dictate the fracture location. We
 227 will further deepen into these experimental findings using the finite element calculations in section
 228 6.1.

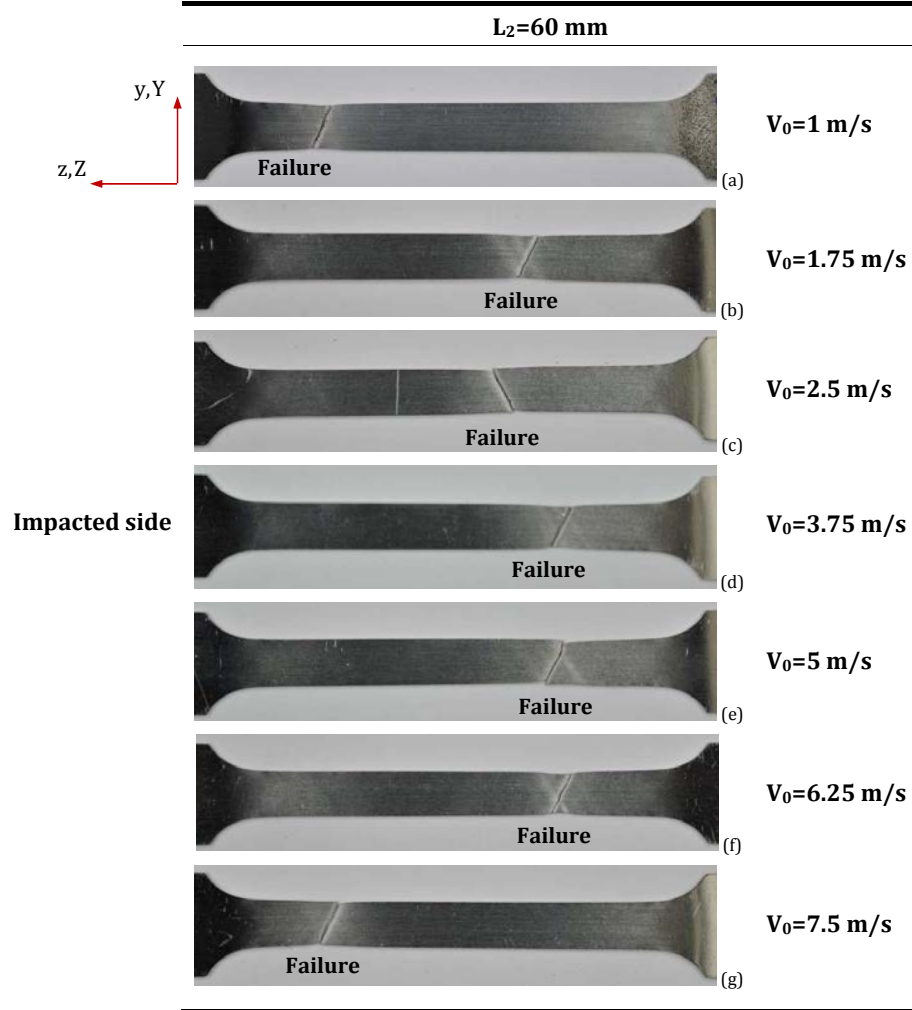


Figure 7: Seven post-mortem samples with gauge length $L_2 = 60 \text{ mm}$ tested at: (a) $V_0 = 1 \text{ m/s}$, (b) $V_0 = 1.75 \text{ m/s}$, (c) $V_0 = 2.5 \text{ m/s}$, (d) $V_0 = 3.75 \text{ m/s}$, (e) $V_0 = 5 \text{ m/s}$, (f) $V_0 = 6.25 \text{ m/s}$, (g) $V_0 = 7.5 \text{ m/s}$.

229 3.2. Influence of specimen gauge length on the location of flow localization

230 Relying on the experimental results shown above, we expect that the gauge length will play a
 231 role in the fracture location. For different gauge lengths the stress waves need different times to go
 232 over the entire gauge, which alters the processes of reflection and interaction of waves taking place

233 in the sample during the test. Further, we expect that the gauge length will affect the fracture
 234 pattern. Note that the gauge length determines the aspect ratio (slenderness) of the gauge which,
 235 on the basis of the results shown in Figs. 5 and 6, plays a role in the failure mode.

236 Fig. 8 shows six specimens with different gauge lengths tested at $V_0 = 5 \text{ m/s}$. In the case
 237 of $L_2 = 20 \text{ mm}$ the failure occurs in the middle of the gauge with negligible (localized) width-
 238 reduction near the fracture location. To be noted that, instead of having a single localization point
 239 which leads to fracture as in the specimens shown in Figs. 5, 6 and 7, there are traces of multiple
 240 localization bands all along the gauge. This key (and very uncommon) finding will be discussed in
 241 detail in the next section. By now, we just focus on the role played by gauge length in the fracture
 242 location. It is observed that for $L_2 = 40 \text{ mm}$ the failure is no longer in the middle of the gauge but
 243 close to the impacted side, whereas for $L_2 = 60 \text{ mm}$, $L_2 = 80 \text{ mm}$ and $L_2 = 100 \text{ mm}$ the fracture is
 244 located near the clamped side. Surrounding the failure point, the thinning of the sample along the
 245 Y direction increases with the gauge length. Finally, for the greatest gauge length $L_2 = 140 \text{ mm}$
 246 the fracture location is located in the middle of the gauge. There is a significant reduction of the
 247 width of the gauge around the fracture point. The sample straining is not uniform along the gauge.

248 A close relation between gauge length, failure location and failure pattern has been found for
 249 all the impact velocities tested, which confirms the control that dynamic effects (stress waves and
 250 inertia) have over the failure location and the failure mode of the sample. Further, we claim that the
 251 extensive experimental campaign that we have conducted in this investigation strengthens the idea
 252 that the failure location in the dynamic tensile test is deterministic. Instead of being controlled by
 253 random-type effects as intrinsic material defects, the failure location is governed to a large extent
 254 by dynamic phenomena.

255 *3.3. Multiple localization pattern*

256 Multiple, and largely regular, localization patterns have been observed in a significant number of
 257 the experiments performed using the samples with the shorter gauge lengths. Four of these samples
 258 are shown in Fig. 9. For $L_2 = 20 \text{ mm}$ we have found multiple necking bands in $\sim 45\%$ of the
 259 samples tested at velocities larger than $V_0 = 3.75 \text{ m/s}$. For $L_2 = 40 \text{ mm}$ the multiple localization
 260 pattern is observed in $\sim 35\%$ of the experiments. For $L_2 = 60 \text{ mm}$ we only have observed multiple
 261 necking bands in two samples tested at $V_0 = 1.75 \text{ m/s}$ and $V_0 = 5 \text{ m/s}$. For all the samples with
 262 $L_2 = 80 \text{ mm}$, $L_2 = 100 \text{ mm}$ and $L_2 = 140 \text{ mm}$ only a pair of necking bands are formed, these being

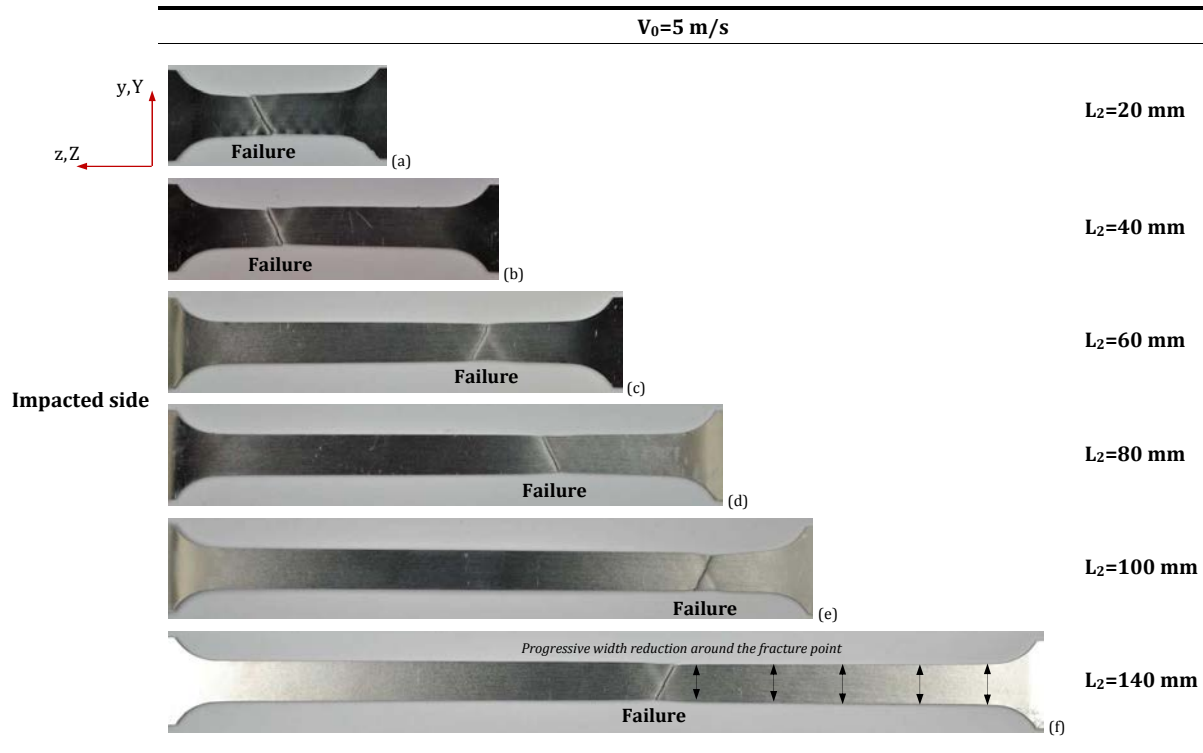


Figure 8: Six post-mortem samples with different gauge lengths tested at $V_0 = 5 \text{ m/s}$: (a) $L_2 = 20 \text{ mm}$, (b) $L_2 = 40 \text{ mm}$, (c) $L_2 = 60 \text{ mm}$, (d) $L_2 = 80 \text{ mm}$, (e) $L_2 = 100 \text{ mm}$, (f) $L_2 = 140 \text{ mm}$.

263 responsible for the specimen fracture. It follows from previous results that short samples tested
 264 at high impact velocities are more prone to develop multiple localization bands. This behaviour
 265 may be explained based on the following premises: (1) the shortest samples (shortest aspect ratios
 266 L_2/W in Fig. 1) are the most equilibrated during testing [22, 29], develop the most uniform strain
 267 distribution along the gauge and do not show (localized) width-reduction near the fracture point;
 268 (2) increasing impact velocity boosts the role played by inertia in the material response [51, 52].
 269 These two ideas are developed below:

- 270 1. A tensile sample with constant cross section tested under *perfect* mechanical equilibrium shall
 271 develop uniform strain distribution along the gauge (i.e. constant width-reduction along the
 272 gauge) leading to regular and symmetric localization and failure patterns (in the absence of
 273 significant material defects). In the absence of *perfect* equilibrium, the specimen is susceptible
 274 to show variability in the strain field along the gauge (i.e. variable width-reduction along the
 275 gauge) leading to irregular and unsymmetrical localization and failure patterns. On these
 276 basis, it is reasonable to assume that a specimen tested under conditions close to equilibrium
 277 is more likely to develop regular and symmetric localization and failure patterns than a

sample tested under loading conditions which are far from mechanical equilibrium, as further discussed in section 6.3.

On the one hand, these arguments explain that almost all the specimens that we have tested under (quasi)static loading, and therefore under loading conditions very close to mechanical equilibrium, failed in the middle of the sample, i.e. they have shown a symmetric failure pattern. On the other hand, these arguments also explain that most of the shortest samples (shortest aspect ratio L_2/W) tested under dynamic loading show symmetric localization and failure patterns. Note that in these samples (1) the localization pattern is repetitive and largely symmetric with the respect to the longitudinal and transversal axes of the specimens and (2) the samples fail in (approximately) the middle of the gauge.

2. An equilibrated tensile specimen tested under dynamic loading is prone to develop multiple localization points. This behaviour is frequently observed in the radial expansion of axially symmetric structures like rings [53, 54], tubes [55, 56] and hemispheres [57]. The symmetry of these structures nearly eliminates the effects of wave propagation before the onset of plastic localization, the specimen being tested under loading conditions close to equilibrium. All these experimental works reported that the number of localization points increases with the loading velocity. This experimental finding has been explained by several authors [58, 59] who claimed that inertia, via strain rate, is the main responsible for the development of multiple localization patterns in samples tested under dynamic loading. These arguments explain that we have observed multiple necking bands mostly in those samples that we have tested at the higher strain rates.

4. Constitutive model

The main hypothesis of the constitutive model used to describe the thermoviscoplastic behaviour of the AISI 430 steel centers on the standard principles of Huber-Mises plasticity: additive decomposition of the rate of deformation tensor, isotropic hardening, associated flow rule and plastic power equivalence

$$\boldsymbol{\sigma}^\nabla = \mathbf{C} : \mathbf{d}^e = \mathbf{C} : (\mathbf{d} - \mathbf{d}^p - \mathbf{d}^\theta) \quad (1)$$

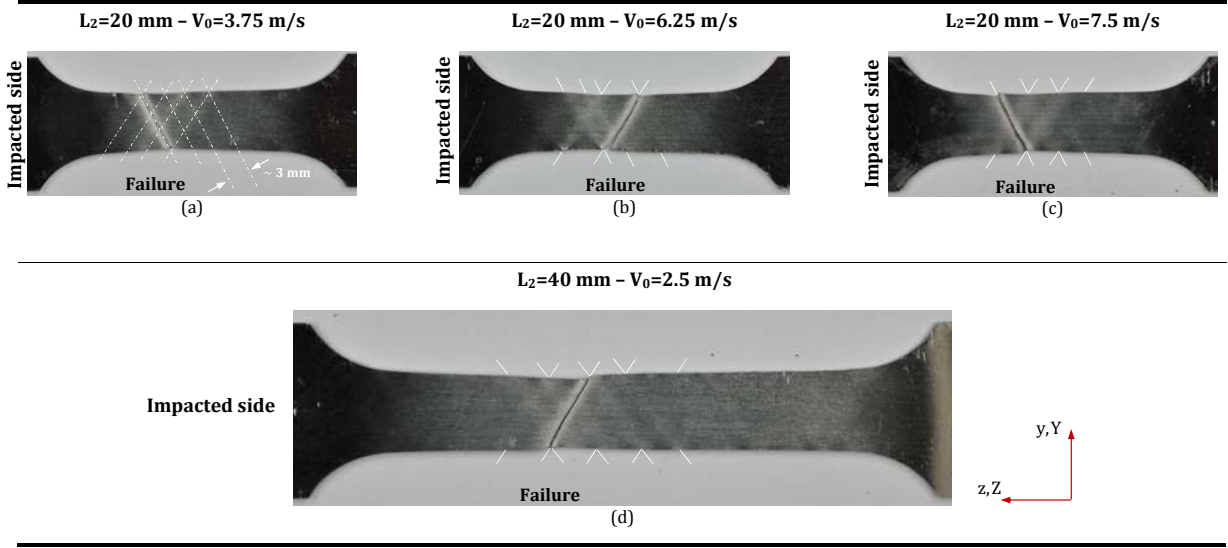


Figure 9: Four post-mortem samples with different gauge lengths tested at different velocities. Multiple localization bands are observed in all of them. (a) $L_2 = 20\text{ mm}$ and $V_0 = 3.75\text{ m/s}$, (b) $L_2 = 20\text{ mm}$ and $V_0 = 6.25\text{ m/s}$, (c) $L_2 = 20\text{ mm}$ and $V_0 = 7.5\text{ m/s}$ and (d) $L_2 = 40\text{ mm}$ $V_0 = 2.5\text{ m/s}$.

$$\Psi = \bar{\sigma} - \sigma_Y = 0 \quad (2)$$

$$\mathbf{d}^p = \frac{\partial \Psi}{\partial \boldsymbol{\sigma}} \dot{\bar{\epsilon}}^p = \frac{3\mathbf{s}}{2\bar{\sigma}} \dot{\bar{\epsilon}}^p \quad (3)$$

304 where $\boldsymbol{\sigma}^\nabla$ is an objective derivative of the Cauchy stress tensor, \mathbf{d} , \mathbf{d}^e , \mathbf{d}^p and \mathbf{d}^θ are the total,
 305 elastic, plastic and thermal rate of deformation tensors respectively, \mathbf{C} is the Hooke tensor for
 306 isotropic elasticity (defined by Young modulus E and Poisson ratio ν), Ψ the yield function, $\bar{\sigma}$ the
 307 equivalent stress, σ_Y is the yield stress, \mathbf{s} the deviatoric stress tensor and $\dot{\bar{\epsilon}}^p$ is the equivalent plastic
 308 strain rate.

309 The yield stress is given as a function of the equivalent plastic strain $\bar{\epsilon}^p$, the equivalent plastic
 310 strain rate $\dot{\bar{\epsilon}}^p$ and the temperature T through the following power-type relation

$$\sigma_Y = A + B (\bar{\epsilon}^p)^n \left(\frac{\dot{\bar{\epsilon}}^p}{\dot{\bar{\epsilon}}_{ref}^p} \right)^m \left(\frac{T}{T_{ref}} \right)^{-\mu} \quad (4)$$

311 The identification of the yield stress parameters is conducted by a numerical regression based on
 312 experimental data obtained (only) with the samples of gauge length 20 mm at different strain rates
 313 and temperatures. Relying on finite element calculations, we have checked that these specimens
 314 reach equilibrium during the experiments. This result agrees with previous observations reported
 315 by Rusinek et al. [36] and Klepaczko [60]. Conventional material constants, elastic parameters and
 316 parameters related to the yield stress for AISI 430 steel are given in Table 2.

Symbol	Property and units	Value
ρ_o	Initial density (kg/m^3)	7740
C_p	Specific heat (J/kgK), Eq. (5)	460
k	Thermal conductivity (W/mK), Eq. (5)	26.1
α	Thermal expansion coefficient (K^{-1}), Eq. (5)	0.00001
E	Young modulus (GPa)	200
ν	Poisson ratio	0.33
A	Initial yield stress (MPa), Eq. (4)	175.67
B	Work hardening modulus (MPa), Eq. (4)	530.13
n	Work hardening exponent, Eq. (4)	0.167
$\dot{\epsilon}_{ref}$	Reference strain rate (s^{-1}), Eq. (4)	0.01
m	Strain rate sensitivity exponent, Eq. (4)	0.0118
T_{ref}	Reference temperature (K), Eq. (4)	300
μ	Temperature sensitivity exponent, Eq. (4)	0.51
β	Taylor-Quinney coefficient, Eq. (5)	0.9

Table 2: Conventional material constants, elastic parameters and parameters related to the yield stress for AISI 430 steel.

317 No doubt, more sophisticated constitutive descriptions could be used to model the material
 318 behaviour (see e.g. [61, 62]). Nevertheless we claim that the simple modelling presented here is
 319 sufficient to develop reliable numerical computations to uncover the key issues which control the
 320 deterministic character of plastic flow localization in the dynamic tensile test.

321 5. Finite element model

322 This section describes the features of the 3D finite element models developed to simulate plastic
 323 strain localization in AISI 430 steel sheets subjected to dynamic tension. The numerical analyses
 324 are carried out using the finite element code ABAQUS/Explicit [41]. To be noted that the goal
 325 of the numerical calculations is not to mimic the experimental tests but to provide new insights
 326 into the role played by dynamic effects (inertia and wave disturbances) and boundary conditions

327 in the deterministic character of the plastic flow localization. For that purpose is enough to use
 328 simple geometrical models which solely consider the gauge of the sample, as further demonstrated
 329 in section 6. This greatly simplifies the interpretation of the finite element results and reduces
 330 the computational cost. Thus, our problem setting is a strip with thickness $h = 1 \text{ mm}$, width
 331 $W = 10 \text{ mm}$ (unless otherwise stated, see section 6.3) and six different lengths L_2 , according to the
 332 six gauge lengths used in the dynamic samples described in Fig. 1. On these geometrical basis, two
 333 different types of finite element models are developed. The idea is that the comparison between
 334 the results obtained with these two models which are described below will allow to explore the
 335 respective influence of dynamic effects and boundary conditions on flow localization. Note that
 336 $\{x, y, z\}$ denotes the Eulerian coordinate system while $\{X, Y, Z\}$ refers to the Lagrangian.

337 • **Model A: No-field configuration.** The solid is initially at rest. The loading conditions
 338 are $V_Z(X, Y, L_2, t) = V_0 = \dot{\varepsilon}_0 L_2$ and $V_Z(X, Y, 0, t) = 0$ (see the Lagrangian coordinate system
 339 defined in the figure). Application of these loading conditions leads to the propagation of
 340 stress waves along the sample [63, 64], precluding –full/complete– mechanical equilibrium.
 341 Within model A we distinguish 2 configurations:

- 342 – **Model A-1.** No additional constraints are imposed to the displacements of the nodes
 343 of the model. This configuration is representative of a typical experimental test.
- 344 – **Model A-2.** The nodes of the workpiece located at the surfaces $\{X, \pm \frac{W}{2}, Z\}$ have
 345 identical displacement along the Y axis during the calculation. Using Hencky strain
 346 as our strain measure, and relying on the incompressibility of the plastic flow, we set
 347 $u_Y(X, \pm W/2, Z, t) = \mp \frac{W}{2} \left(\frac{1}{\sqrt{\varepsilon_0 t + 1}} - 1 \right)$. This configuration tries to emulate an infinitely
 348 long sample along the Y axis.

349 Note that, due to the symmetry of the model, only the $\{X > 0, Y > 0\}$ quarter of the
 350 specimen has been analysed (see Fig. 10).

351 • **Model B: Field configuration.** The initial condition corresponds to an equilibrium config-
 352 uration which *virtually* prevents the generation of stress waves during the loading process. We
 353 say *virtually* because, due to the discretization of the workpiece and the explicit integration
 354 scheme used by the FE code, slight disturbances in the field variables are generated during
 355 the simulations. These little perturbations are required to trigger plastic flow localization as

356 shown by Rusinek and Zaera [65]. Nevertheless, we claim that in comparison with the *no-field*
 357 condition, now the role played by the stress waves in the sample's response is significantly
 358 reduced [46, 59]. The loading conditions are $V_Z(X, Y, \pm \frac{L_2}{2}, t) = \pm \frac{V_0}{2} = \pm \dot{\epsilon}_0 \frac{L_2}{2}$ (see the La-
 359 grangian coordinate system defined in the figure). The initial equilibrium state is obtained
 360 by initializing the velocity, stress, strain and displacement fields in the sample. The initial
 361 conditions in velocity, formulated based on Zaera et al. [59], are $V_X(X, Y, Z, 0) = -\nu \dot{\epsilon}_0 X$,
 362 $V_Y(X, Y, Z, 0) = -\nu \dot{\epsilon}_0 Y$ and $V_Z(X, Y, Z, 0) = \dot{\epsilon}_0 Z$. The initial conditions in stress are
 363 $\sigma_X(X, Y, Z, 0) = 0$, $\sigma_Y(X, Y, Z, 0) = 0$ and $\sigma_Z(X, Y, Z, 0) = \rho_0 C \dot{\epsilon}_0 \frac{L_2}{2}$, where $C = \sqrt{E/\rho_0}$
 364 is the longitudinal elastic wave speed. Note that this procedure for initializing the stress
 365 field has to be limited to the cases for which $\rho_0 C \dot{\epsilon}_0 \frac{L_2}{2} < A$, where it has to be recalled
 366 that A in Eq. (4) defines the initial yield stress of the material. Previous expression im-
 367 plies that the maximum loading velocity V_0 that can be investigated using this procedure
 368 is 8.92 m/s . With the knowledge of the initial stress field, and relying on the Hooke's law,
 369 we calculate the initial strains as $\epsilon_X(X, Y, Z, 0) = -\frac{\nu \rho_0 C \dot{\epsilon}_0 L_2}{2E}$, $\epsilon_Y(X, Y, Z, 0) = -\frac{\nu \rho_0 C \dot{\epsilon}_0 L_2}{2E}$
 370 and $\epsilon_Z(X, Y, Z, 0) = \frac{\rho_0 C \dot{\epsilon}_0 L_2}{2E}$. Using Hencky strain we calculate the initial displacements
 371 as $u_X(X, Y, Z, 0) = -\frac{X}{2} \left(\exp^{-\frac{\nu \rho_0 C \dot{\epsilon}_0 L_2}{2E}} - 1 \right)$, $u_Y(X, Y, Z, 0) = -\frac{Y}{2} \left(\exp^{-\frac{\nu \rho_0 C \dot{\epsilon}_0 L_2}{2E}} - 1 \right)$ and
 372 $u_Z(X, Y, Z, 0) = Z \left(\exp^{\frac{\rho_0 C \dot{\epsilon}_0 L_2}{2E}} - 1 \right)$. It is worth mentioning that this *initialization method-*
 373 *ology* is an original contribution of this paper since it significantly improves the procedure
 374 proposed by Rodríguez-Martínez et al. [46], where only the velocity along the loading direc-
 375 tion was initialized in the so-called field configuration. As for model A, we also distinguish 2
 376 configurations for model B:

- 377 – **Model B-1.** No additional constraints are imposed to the displacements of the nodes
 378 of the model.
- 379 – **Model B-2.** The displacement of the nodes located at the surfaces $\{X, \pm W/2, Z\}$
 380 is prescribed as $u_Y(X, \pm W/2, Z, t) = \mp \frac{W}{2} \left(\exp^{-\frac{\nu \rho_0 C \dot{\epsilon}_0 L_2}{2E}} + \frac{1}{\sqrt{\dot{\epsilon}_0 t + 1}} - 2 \right)$. The first term
 381 inside the parenthesis refers to the displacement due to the initialization of the field
 382 variables while the second term corresponds to the time dependent displacement calculated
 383 based on the incompressibility of the plastic flow, as previously described for model A-2.

384 Note that, due to the symmetry of the model, only the $\{X > 0, Y > 0, Z > 0\}$ eight of the

385 specimen has been analysed (see Fig. 10).

386

387 Models A-2 and B-2 will serve to explain the role played by boundary conditions in the post-
 388 uniform elongation of the sample and, specifically, in the failure pattern. Further, the fact that the
 389 boundary condition $u_Y(X, \pm W/2, Z, t)$ imposed to the models A-2 and B-2 emulates an infinitely
 390 long sample in the Y axis will serve to highlight the influence of the sample slenderness on the
 391 formation of multiple localization patterns.

392 Moreover, we have considered a *fully coupled* thermo-mechanical framework in which, assuming
 393 no heat flow at the workpiece boundaries, the relationship between the spatial-temporal variation
 394 of the temperature T and the dissipative and thermoelastic heat generation rates is as follows

$$k\nabla^2 T - \rho C_p \dot{T} = -\beta \boldsymbol{\sigma} : \mathbf{d}^p + \alpha (3\lambda + 2\mu) T_0 \mathbf{d}^e : \mathbf{1} \quad (5)$$

395 where k is the thermal conductivity, ρ is the current material density, C_p is the specific heat,
 396 β is the Taylor-Quinney coefficient and α is the thermal expansion coefficient. Moreover λ and μ
 397 are the Lamé constants, and T_0 is the initial temperature that has been set to 300 K in all cases.
 398 Note that $\mathbf{d}^e : \mathbf{1}$ is the trace of the elastic rate of deformation tensor.

399

400 The finite element models are meshed using eight node coupled displacement-temperature solid
 401 elements, with reduced integration and hourglass control (*C3D8RT*). The elements have an initial
 402 aspect ratio 1 : 2 : 1 with dimensions $0.166 \times 0.333 \times 0.166 \text{ mm}^3$ for all the models that we have
 403 built. We have checked that, with the increase of plastic deformation in the workpiece, the shape of
 404 the elements evolves, approaching an aspect ratio closer to 1 : 1 : 1 at the time of flow localization.
 405 According to Zukas and Scheffer [66], such an element shape is optimal for describing dynamic events
 406 like high rate flow localization. Further, a mesh convergence study has been performed, and the
 407 time evolution of different critical output variables, namely stress, strain and necking inception,
 408 were compared against a measure of mesh density until the results converged satisfactorily (see
 409 Appendix B for details). Note that, in our modelling, viscosity, inertia and thermal conductivity
 410 act as potent regularization factors that help to the well-posedness of the problem at hand

411 [67, 68]. We hold that this minimizes the spurious influence of the mesh in the solution of the
 412 boundary value problem.

413

414 The set of constitutive equations describing the material behaviour presented in section 4 are
 415 implemented in the finite element code through a user subroutine following the procedure developed
 416 by Zaera and Fernández-Sáez [69]. For integration of the set of constitutive equations in a finite
 417 deformation framework, incremental objectivity is achieved by rewriting them in a corotational
 418 configuration [70, 71], defined in ABAQUS/Explicit by the polar rotation tensor. The stress is
 419 updated with the radial return algorithm

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{trial} - 3G\Delta\bar{\varepsilon}^p \frac{\mathbf{s}_{n+1}}{\bar{\sigma}_{n+1}} \quad (6)$$

420 where G is the elastic shear modulus and $\boldsymbol{\sigma}_{n+1}^{trial}$ is the trial stress is defined by

$$\boldsymbol{\sigma}_{n+1}^{trial} = \boldsymbol{\sigma}_n + \mathbf{C} : \Delta\boldsymbol{\varepsilon} \quad (7)$$

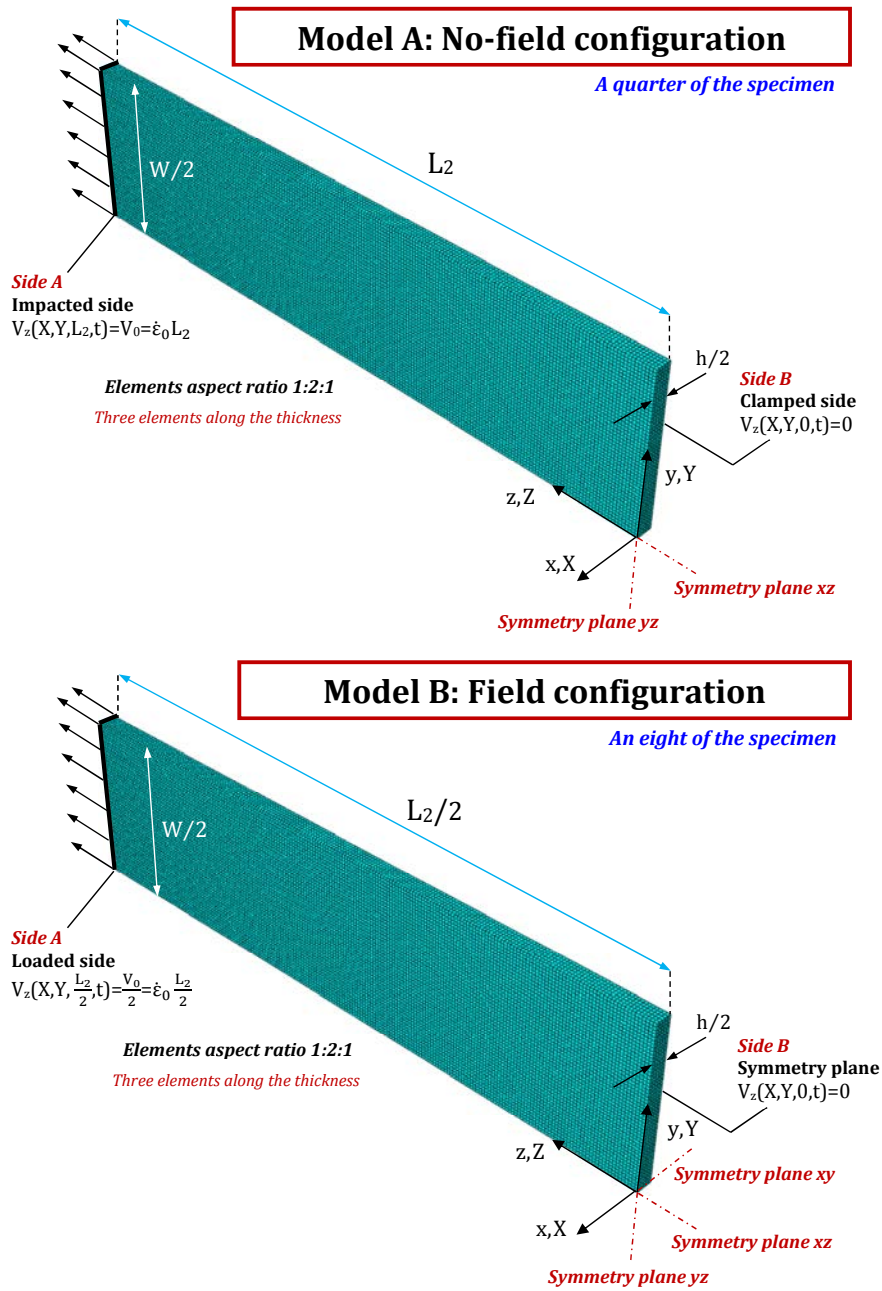
421 According to the properties of radial return, the equivalent stress may be updated with the following
 422 equation

$$\bar{\sigma}_{n+1} = \bar{\sigma}_{n+1}^{trial} - 3G\Delta\bar{\varepsilon}^p \quad (8)$$

423 and the yield condition Eq. (2) which, coupled to Eq. (4), permits to obtain the equivalent plastic
 424 strain increment $\Delta\bar{\varepsilon}^p$.

425 6. Analysis and results: finite element simulations

426 Next, the experimental findings reported in section 3 are further explained relying on the results
 427 obtained from the finite element simulations.



Finite element models

L_2 (mm)					
Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
20	40	60	80	100	140

Figure 10: 3D finite element models. Mesh, dimensions, boundary conditions and loading conditions of models A and B.

428 *6.1. Influence of loading velocity on the location of flow localization*

429 In order to obtain further insights into the interplay between the impact velocity and the
 430 location of flow localization, we rely on finite element simulations conducted using the model A-1.
 431 As described in section 5, within the models built in this paper, the A-1 is the most similar to a
 432 typical experimental arrangement in terms of initial, loading and boundary conditions. Fig. 11
 433 shows contours of equivalent plastic strain $\bar{\epsilon}^P$ in the Lagrangian configuration (undeformed shape)
 434 for $L_2 = 60 \text{ mm}$ and various loading velocities. The range of loading velocities analysed in the
 435 calculations is wider than the range covered by the experiments in order to reveal, to the full extent,
 436 how the point of localization varies sequentially from side to side of the sample with the increase
 437 of the loading velocity. Note that, irrespective of the impact velocity, the plastic strain localization
 438 takes the form of a pair of necking bands that follow the directions of zero stretch rate, as shown
 439 in the experimental results reported in section 3.

440 In the case of $V_0 = 0.125 \text{ m/s}$, the smallest velocity explored, the localization of plastic defor-
 441 mation is located at the clamped end. The increase in applied velocity moves the localization point
 442 towards the impacted side, where it remains until reaching $V_0 = 7.5 \text{ m/s}$. Then, plastic localization
 443 occurs near the clamped end. For $V_0 = 10 \text{ m/s}$ the localization point is back to the impacted side
 444 while for $V_0 = 15 \text{ m/s}$ it takes place, again, near the clamped end. Such a systematic motion of the
 445 localization point along the sample continues taking place if we keep increasing the applied speed,
 446 until the critical impact velocity (CIV) is attained for $V_0 \approx 80 \text{ m/s}$. When the CIV is reached the
 447 applied velocity is such that it generates a plastic wave which induces (*instantaneous*) flow local-
 448 ization [63]. Thus, for velocities above the CIV the localization of plastic deformation inevitably
 449 occurs (*instantaneously*) at the impacted side, as shown by Klepaczko [72] and Rusinek et al. [36].
 450 Note that such a strong influence of the impact velocity on the location of flow localization has
 451 been found for all the gauge lengths investigated, the so-called types 1-6 in Fig. 10.

452 It is important to realize that the specific locations of flow localization predicted by the nu-
 453 merical calculations do not agree with their experimental counterparts shown in Fig. 7. While we
 454 highlight the qualitative agreement between numerical calculations and experiments, we acknowl-
 455 edge the lack of quantitative agreement. Besides the simplified geometry that we have analysed,
 456 we think that there are some other factors, that can hardly be overcome, responsible for this dis-
 457 agreement (quantitative, but not qualitative, disagreement). For instance, there are *uncertainties*

458 intrinsic to the experimental setup related to the loading condition (the actual applied velocity is
 459 surely not a *perfect* step-function) and the boundary conditions (the system used to attach the
 460 sample does not ensures a *perfect* embedding). We hold that these *uncertainties* make virtually
 461 impossible to build a finite element model to mimic the experiments with the accuracy required
 462 to predict the specific location of flow localization. Moreover, while in the experiments the stress
 463 waves may be transmitted to the machine through the jaws, we do not consider this scenario in
 464 our modelling. Nevertheless, we hold that our (simple) calculations are in qualitative agreement
 465 with the experiments and show the interplay between the fracture location and the loading velocity.
 466 Further, these calculations provide an additional proof of the deterministic character of location of
 467 plastic strain localization in the dynamic tensile test.

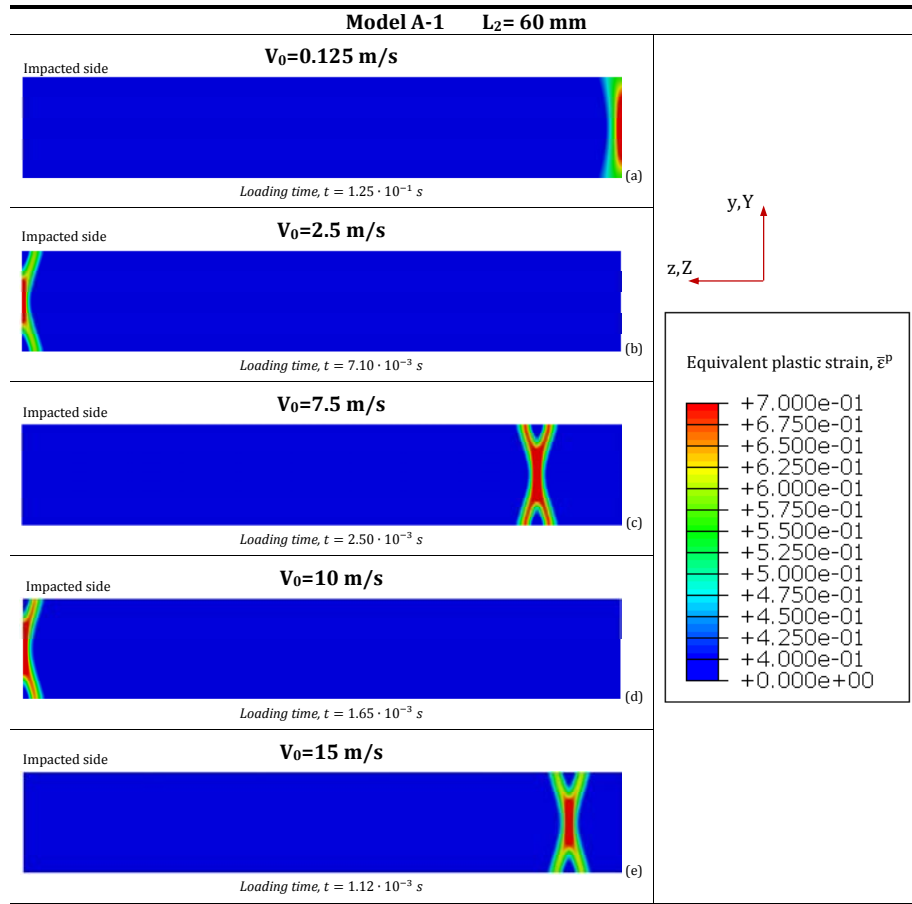


Figure 11: Finite element results. Model A-1. Contours of equivalent plastic strain $\bar{\epsilon}^P$ in the Lagrangian configuration (undeformed shape) for $L_2 = 60 \text{ mm}$ and various impact velocities. (a) $V_0 = 0.125 \text{ m/s}$, (b) $V_0 = 2.5 \text{ m/s}$, (c) $V_0 = 7.5 \text{ m/s}$, (d) $V_0 = 10 \text{ m/s}$ and (e) $V_0 = 15 \text{ m/s}$.

468 *6.2. Influence of specimen gauge length on the location of flow localization*

469 This section aims at further deepen into the relationship between the sample gauge length and
 470 the location of flow localization that was revealed in section 3.2. For that purpose we rely on finite
 471 element simulations conducted using the model A-1. Fig. 12 illustrates contours of equivalent
 472 plastic strain $\bar{\varepsilon}^p$ in the Lagrangian configuration (undeformed shape) for $V_0 = 5 \text{ m/s}$ and various
 473 gauge lengths. Note that, irrespective of the sample length, the plastic strain localization takes the
 474 form of a pair of necking bands.

475 In the case of $L_2 = 20 \text{ mm}$, the shortest gauge length explored, the localization of plastic
 476 deformation is located roughly at the center of the sample. The increase of the gauge length affects
 477 the location of flow localization which occurs at the impacted end for $L_2 = 40 \text{ mm}$, $L_2 = 60 \text{ mm}$
 478 and $L_2 = 80 \text{ mm}$. For $L_2 = 100 \text{ mm}$ two localization points are detected. The main one (the
 479 most developed) takes place at the impacted end, while the secondary one appears at the clamped
 480 site. For $L_2 = 140 \text{ mm}$ a single localization point appears at the clamped site. Such a systematic
 481 motion of the localization point along the sample continues taking place if we keep increasing
 482 the sample gauge length. Note that such a strong influence of the gauge length on the location
 483 of flow localization has been found for all the applied velocities investigated within the range
 484 $0.125 \text{ m/s} \lesssim V_0 \lesssim 80 \text{ m/s}$ (below the CIV).

485 Moreover, it has to be highlighted that the case $L_2 = 100 \text{ mm}$ shown in Fig. 12 is a transient
 486 state, halfway between the localization patterns of $L_2 = 80 \text{ mm}$ and $L_2 = 140 \text{ mm}$. As such, it
 487 reveals the nature of the role played by the sample length in the location of flow localization. We
 488 recall here that the gauge length determines the time required by the elastic strains to travel over
 489 the whole gauge and, as such, it controls the processes of reflection and interaction of stress waves
 490 which dictates the locations where the build up of plastic deformation occurs. These results shall
 491 be understood as an additional proof of the deterministic character of the flow localization in the
 492 dynamic tensile test.

493 It is a fact that, because of a number of reasons already discussed in previous section, our
 494 calculations do not predict the specific location of flow localization observed in the experiments
 495 (qualitative agreement, quantitative disagreement), see Fig. 8. Nevertheless, we hold that they
 496 help to provide a proper interpretation of our experimental findings and contribute to reveal the key
 497 mechanisms which reside behind the interplay between the gauge length and the fracture location.

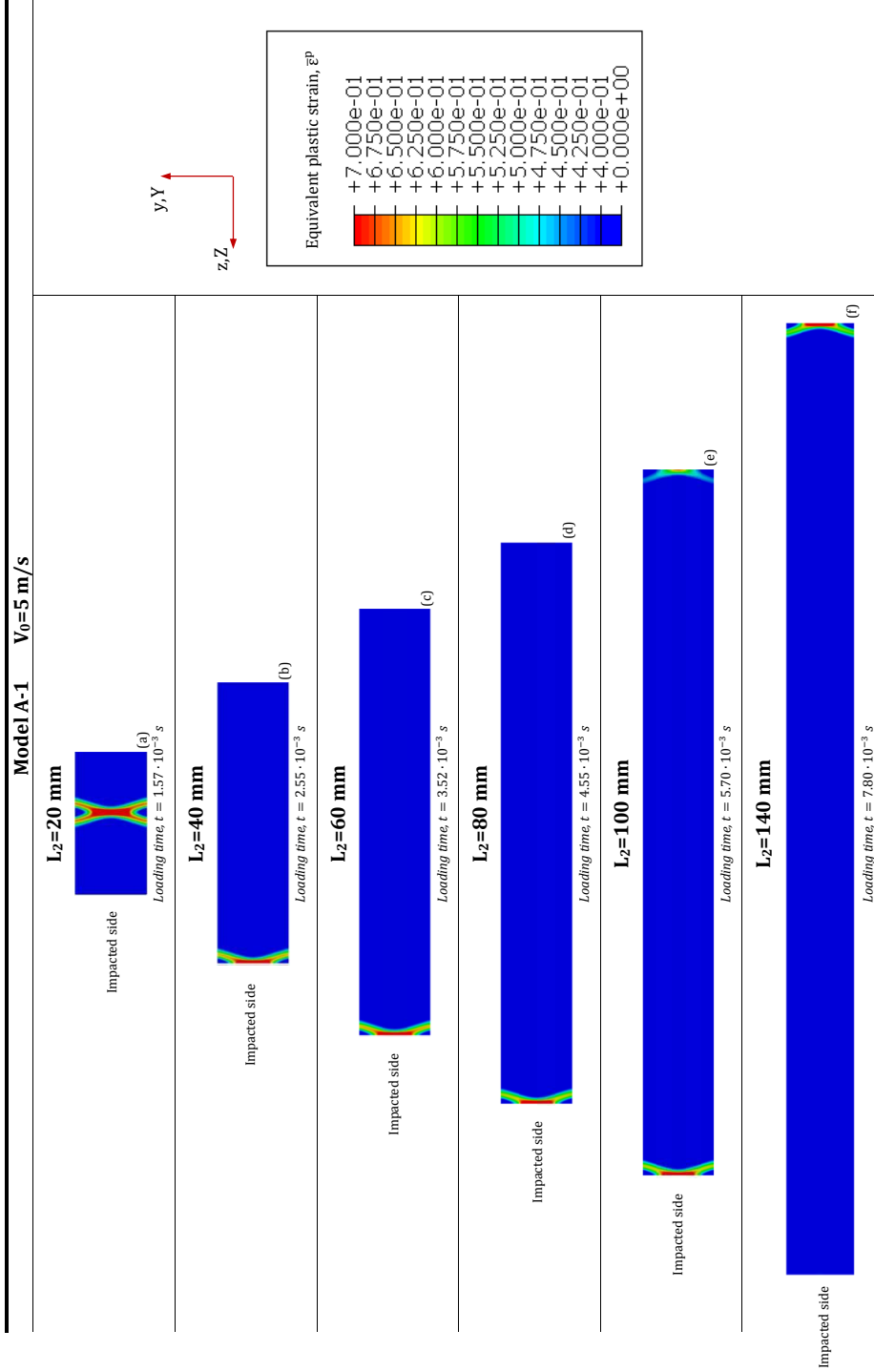


Figure 12: Finite element results. Model A-1. Contours of equivalent plastic strain ϵ^p in the Lagrangian configuration (undeformed shape) for $V_0 = 5 \text{ m/s}$ and various gauge lengths. (a) $L_2 = 20 \text{ mm}$, (b) $L_2 = 40 \text{ mm}$, (c) $L_2 = 60 \text{ mm}$, (d) $L_2 = 80 \text{ mm}$, (e) $L_2 = 100 \text{ mm}$ and (f) $L_2 = 140 \text{ mm}$.

498 *6.3. Multiple localization pattern*

499 In this section we aim at uncovering the role played by the initial conditions, the boundary
500 conditions and the sample slenderness on the formation of multiple localization patterns. The way
501 in which these factors either favour or preclude the emergence of multiple necking bands has been
502 hardly investigated in the literature [73], thus we intend to give some indications about it here.

503 Fig. 13 shows contours of equivalent strain rate in Eulerian (deformed shape) configuration
504 for $V_0 = 5 \text{ m/s}$ and $L_2 = 20 \text{ mm}$. The results for model A-1 are depicted in Fig. 13(a) while
505 the results of model B-1 are illustrated in Fig. 13(b). We have determined the localization strain
506 $\bar{\varepsilon}_l^p$ in the calculations following the procedure reported elsewhere [63, 74]. The localization strain
507 is assumed as given by the condition $\frac{d\bar{\varepsilon}^p}{dt} = 0$, where $\bar{\varepsilon}^p$ is measured within the unloading zone
508 which surrounds the localized region. The localization strain obtained for model A-1 is $\bar{\varepsilon}_l^p \approx 0.25$
509 while for model B-1 is $\bar{\varepsilon}_l^p \approx 0.34$. The retardation of flow localization registered for model B-
510 1 is caused by the initialization of the field variables (see section 5) which minimizes the stress
511 propagation phenomena, boosting mechanical equilibrium and delaying plastic localization [59].
512 This observation agrees with the theoretical and numerical results presented by different authors
513 [75, 73] who showed that the stress waves disturbances represent a limiting factor for the material
514 ductility.

515 Note that in Fig. 13 we show the deformed shape in order to have a clear perception of the
516 straining of the samples during the process of plastic localization. Thus, we point out that the
517 development of the pair of localization bands is accompanied by a substantial reduction of the
518 width of the sample near the localization area. As shown in Fig. 8, such kind of localization
519 pattern with a single pair of bands inside a necked region (local width reduction) is representative
520 of the largest samples tested. However, it does not find correlation with the experimental failure
521 pattern observed for $V_0 = 5 \text{ m/s}$ and $L_2 = 20 \text{ mm}$, for which multiple localization bands and little
522 width reduction near the fracture location were observed (see Fig. 8). This mismatch between the
523 numerical calculation and the experimental counterpart is mostly attributed to the simplicity of our
524 finite element model which only takes into account the gauge of the sample. In the experimental
525 sample, the fillets and the gripping sections increase the momentum of inertia of the cross section
526 (along the Y direction). We assume that this opposes to the local width reduction near the failure
527 point, enhancing the formation of multiple necking bands. This statement is confirmed with Fig. 14,

528 where we show contours of equivalent strain rate for model A-2 in Fig. 14(a) and model B-2 in Fig.
 529 14(b). As for Fig. 13, the loading velocity is $V_0 = 5 \text{ m/s}$ and the sample length is $L_2 = 20 \text{ mm}$.
 530 The Eulerian (deformed shape) configuration is depicted. The localization strain corresponding
 531 to model A-2 is $\bar{\epsilon}_l^p \approx 0.85$ while for model B-2 the specimen never reaches the condition of full
 532 localization. Thus, we have:

- 533 • Because of the difference in the initial conditions, model A-2 shows lower ductility than model
 534 B-2.
- 535 • Because of the difference in the boundary conditions, model A-2 shows larger ductility than
 536 model A-1 and model B-2 shows larger ductility than model B-1.

537 Since the effect of the initial conditions in the material ductility was already discussed above,
 538 we analyse here the role played by the boundary conditions. It has to be recalled that, as described
 539 in section 5, the boundary conditions applied to models A-2 and B-2 are such that all the nodes
 540 located at the surfaces $\{X, \pm \frac{W}{2}, Y\}$ have identical displacement along the Y axis during the calcu-
 541 lation (thus impeding the local width reduction of the sample). The application of such boundary
 542 conditions, which try to emulate an infinite plate along the Y direction (see section 5), delays flow
 543 localization and promotes the emergence of multiple localization bands. These results suggest that:

- 544 • If the metallic sheet has a large slenderness L_2/W such that it mostly behaves like a rod
 545 then: (1) flow localization is promoted and (2) a single pair of necking bands contained in
 546 the $\{X, Z\}$ plane are formed inside a necked region contained in the $\{Y, Z\}$ plane.
- 547 • If the metallic sheet shows a short slenderness L_2/W such that it mostly behaves like a plate
 548 then: (1) flow localization is delayed and (2) multiple necking bands contained in the $\{X, Z\}$
 549 plane are formed.

550 In order to deepen into the previous two observations, we carry out additional numerical calcu-
 551 lations for models A-1 and A-2 in which different values of W have been explored: 2 mm , 10 mm
 552 (reference width as shown in Fig. 10), 30 mm , 40 mm , 80 mm , 140 mm , 280 mm , 560 mm
 553 and 600 mm . In order to maintain the longitudinal inertial resistance to motion of the specimen
 554 we have used for all the computations the same applied velocity $V_0 = 5 \text{ m/s}$ and sample length
 555 $L_2 = 20 \text{ mm}$. Recall that for model A-1 the surfaces $\{X, \pm \frac{W}{2}, Z\}$ are free of constrains (in such

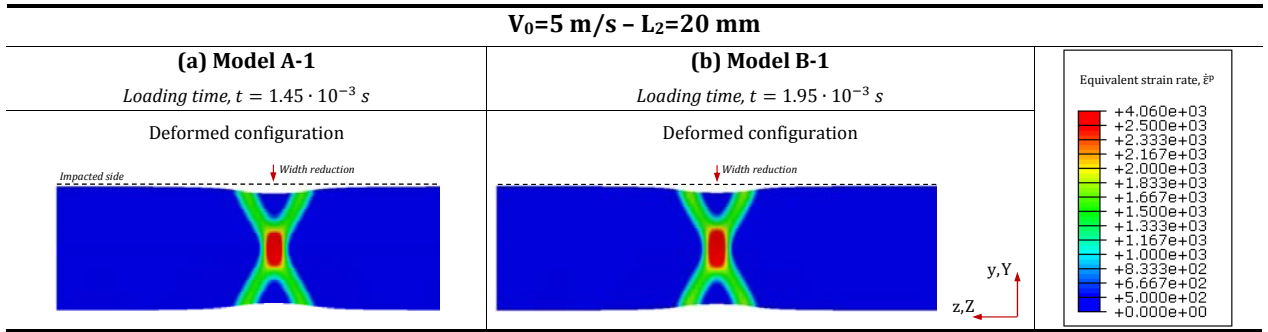


Figure 13: Finite element results. Contours of equivalent strain rate $\dot{\epsilon}^p$ in Eulerian (deformed shape) configuration for $V_0 = 5 \text{ m/s}$ and $L_2 = 20 \text{ mm}$. (a) Model A-1, loading time $t = 1.45 \cdot 10^{-3} \text{ s}$. (b) Model B-1, loading time $t = 1.95 \cdot 10^{-3} \text{ s}$.

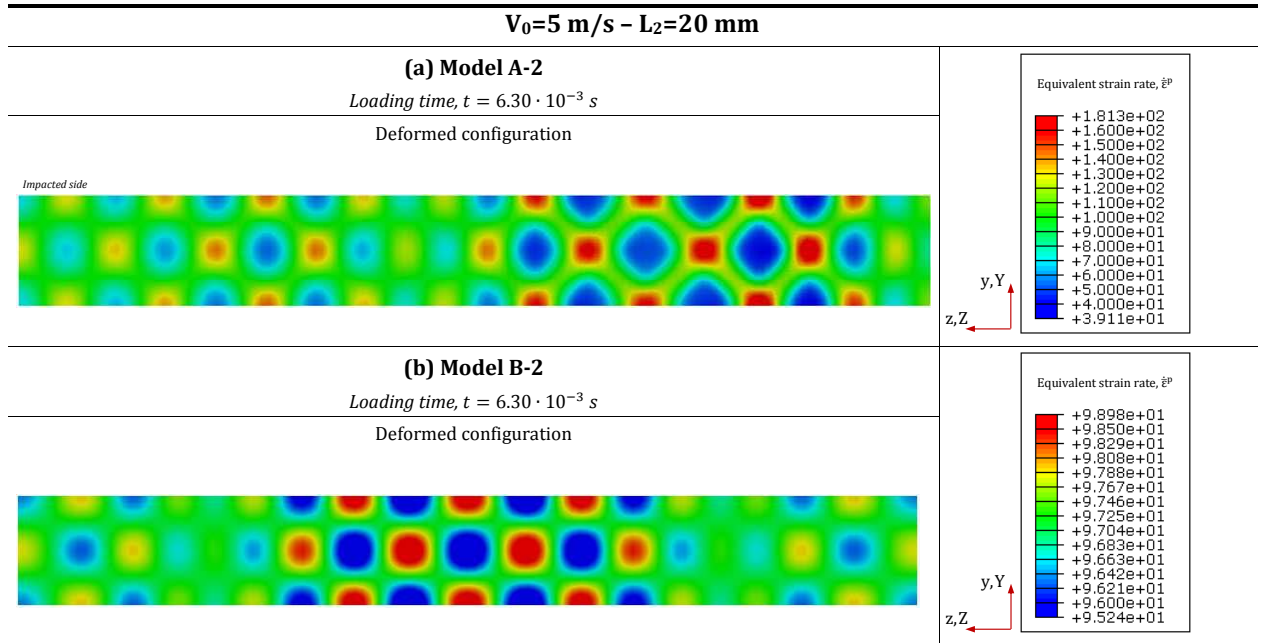


Figure 14: Finite element results. Contours of equivalent strain rate $\dot{\epsilon}^p$ in Eulerian (deformed shape) configuration for $V_0 = 5 \text{ m/s}$ and $L_2 = 20 \text{ mm}$. (a) Model A-2, loading time $t = 6.30 \cdot 10^{-3} \text{ s}$. (b) Model B-2, loading time $t = 6.30 \cdot 10^{-3} \text{ s}$.

556 a sense this configuration is representative of an experimental test) whereas for model A-2 all the
 557 nodes of the surfaces $\{X, \pm \frac{W}{2}, Z\}$ undergo the same displacement along the Y direction. Fig. 15
 558 shows the localization strain $\bar{\epsilon}_l^p$ versus the sample slenderness L_2/W .

- 559 • Model A-1: there is a significant increase of the localization strain with the decrease of
 560 sample slenderness within the greatest values of L_2/W considered. Nevertheless, the rise of
 561 $\bar{\epsilon}_l^p$ becomes gradually reduced as L_2/W decreases, such that within the range $L_2/W < 0.1$
 562 the localization strain tends asymptotically to ~ 0.39 . We have observed that the localization
 563 pattern evolves from a single pair of bands inside a necked region for large values of L_2/W
 564 to multiple necking bands for short values of L_2/W . This interplay between the specimen
 565 slenderness and the failure pattern finds good correlation (qualitative agreement) with the
 566 experimental trends shown in Fig. 8.

567 Note that irrespective of the ratio L_2/W the sample is subjected to uniaxial tension during
 568 the process of homogeneous deformation. It is only after the perturbation of the fundamental
 569 solution, within the post-uniform deformation regime (after the diffuse localization and prior
 570 to the full localization [52, 76, 77]), when samples with different aspect ratios L_2/W may
 571 behave in a different manner due to the development of stress gradients along the Y direction.

- 572 • Model A-2: the localization strain tends to infinity for the greatest values of L_2/W studied.
 573 The imposed boundary condition in the sample-surfaces $\{X, \pm \frac{W}{2}, Z\}$ does not allow to develop
 574 a necked region contained in the $\{Y, Z\}$ plane (the natural localization pattern of the samples
 575 that mostly behave like a rod, see Fig. 13) and the specimen ductility *virtually* tends to
 576 infinity. Finite values of the localization strain are found for $L_2/W < 2$. For this range of
 577 the ratio L_2/W the localization strain decreases non-linearly with the decrease of the sample
 578 slenderness. This drop becomes gradually mitigated as L_2/W decreases, such that within the
 579 range $L_2/W < 0.1$ the localization strain tends asymptotically to ~ 0.39 .

580 Within the range $0.1 < L_2/W < 2$ flow localization is reached but, in comparison with the
 581 model A-1, the process requires the investment of a greater amount of external work. The
 582 sample undergoes localization but, due to the imposed boundary conditions, without following
 583 the natural pattern of the specimen. For $L_2/W < 0.1$ the imposed boundary conditions do
 584 not affect the localization process, thus models A-1 and A-2 provide very similar localization

585 strain and failure pattern. Then, the samples with aspect ratio $L_2/W < 0.1$ can be considered,
 586 for all purposes, as infinite plates. This is further illustrated in Fig. 16 where, for models
 587 A-1 and A-2, we show contours of equivalent plastic strain $\bar{\varepsilon}^p$ in the Eulerian configuration
 588 (deformed shape) for $L_2 = 20 \text{ mm}$ and $W = 280 \text{ mm}$ ($L_2/W = 0.0714$). We observe that the
 589 failure pattern is now characterized, irrespective of the model selected (either A-1 or A-2),
 590 by the emergence of multiple necking bands contained in the $\{X, Z\}$ plane.

591 The finite element calculations presented in this section explain the experimental observations
 592 previously reported in section 3.3, and illustrate the effect that the specimen slenderness and the
 593 boundary conditions have on the emergence of multiple localization patterns.

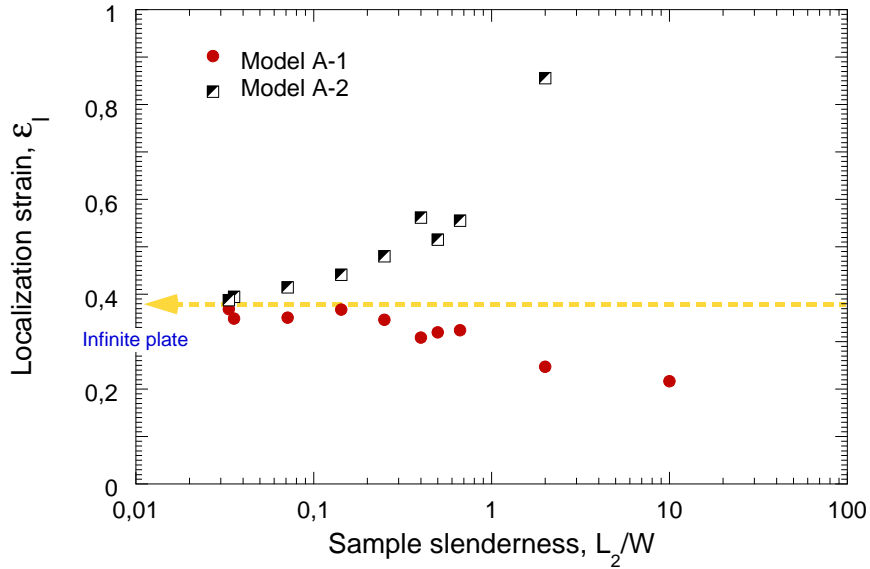


Figure 15: Finite element results. Models A-1 and A-2. Localization strain $\bar{\varepsilon}_l^p$ versus sample slenderness L_2/W .

594 Note that, while our simple geometrical models neglect the influences of the shoulders of the
 595 specimen as well as possible wave transmissions and reflections from/to the machine in the location
 596 of flow localization, they capture the essential features of the interplay between fracture location,
 597 loading velocity and sample size observed in the experiments.

598 7. Summary and conclusions

599 In this paper we have investigated whether the nature of the fracture location in the dynamic
 600 tensile testing of metallic sheets is deterministic or random. For that purpose we have carried
 601 out experiments and finite element simulations. The results have revealed some key mechanisms

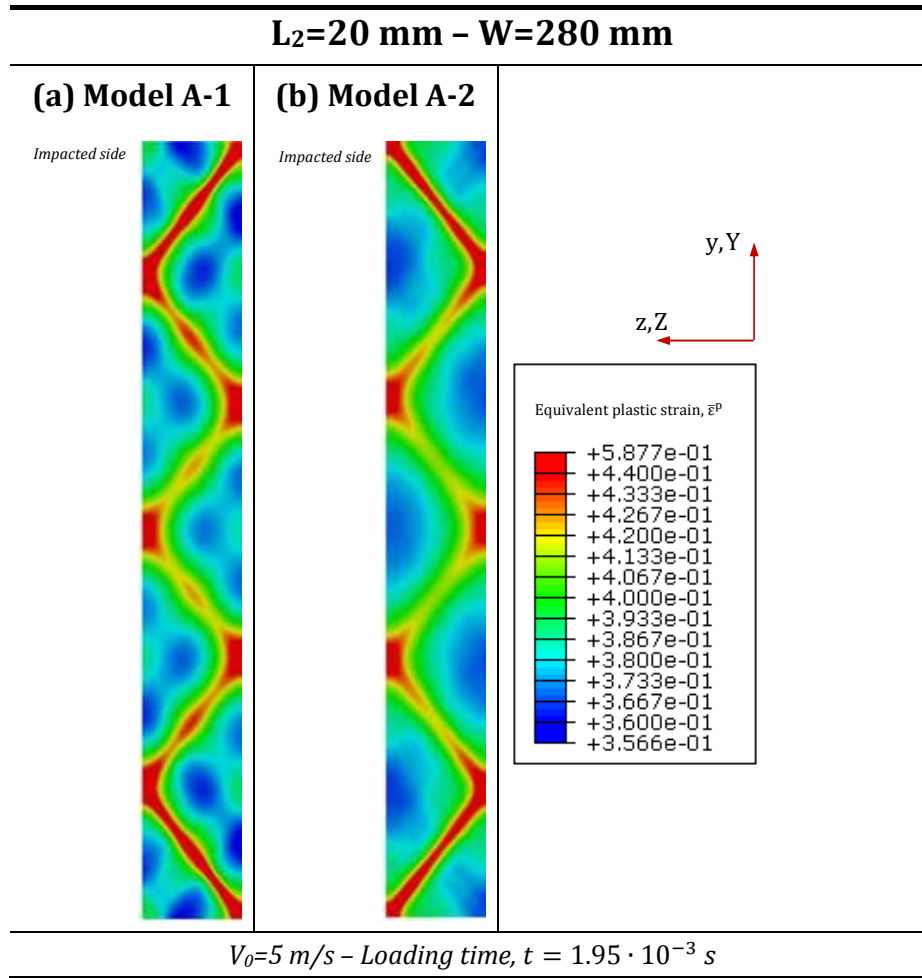


Figure 16: Finite element results. Contours of equivalent plastic strain $\bar{\epsilon}^P$ in the Eulerian configuration (deformed shape) for $L_2 = 20 \text{ mm}$ and $W = 280 \text{ mm}$, i.e. $L_2/W = 0.0714$. Applied velocity $V_0 = 5 \text{ m/s}$, loading time $t = 1.95 \cdot 10^{-3} \text{ s}$. (a) Model A-1. (b) Model A-2.

602 which play a dominant role in the development of flow localization and subsequent fracture of the
603 specimen.

604 • Experiments: we have conducted a comprehensive experimental campaign in which a large
605 number of specimens with different gauge lengths ranging from 20 *mm* to 140 *mm* have
606 been tested at velocities varying from 1 *m/s* to 7.5 *m/s*. For each combination of gauge-
607 length/applied-velocity we have carried out several repeats which have revealed an extremely
608 high repeatability in the fracture location. This is a key experimental finding of this paper
609 which shows that the fracture location is not random but deterministic.

610 Moreover, we claim that the deterministic character of the fracture location is directly con-
611 nected with the intervention of dynamic effects (stress waves and inertia) during the test. We
612 further investigate this statement paying specific attention to the role played by the applied
613 velocity and the gauge length, since these factors control to a large extent the processes of
614 reflection and interaction of waves taking place in the sample during the test. For different
615 impact velocities we have different magnitudes of the stress waves induced in the specimen,
616 while for different gauge lengths the stress waves need different times to go over the gauge.
617 Thus, we claim that the systematic motion from side to side of the sample that shows the
618 fracture location with the variations in impact velocity and gauge length is an additional
619 proof of the deterministic character of the strain localization process.

620 Nevertheless, it is not only the failure location which depends on the applied velocity and the
621 gauge length, but the failure pattern also does. While short samples tested at high velocities
622 are prone to develop multiple and highly regular localization bands, large samples tested at
623 low velocities use to develop a single pair of bands inside a necked region. We conclude that
624 the emergence of multiple localization bands is favoured in those samples with low slenderness
625 for which the strain field along the gauge is kept highly uniform during the loading process.

626 • Finite element simulations: previous experimental findings have been further explained using
627 numerical calculations. For that purpose, to consider a simple geometrical model which
628 solely accounts for the gauge of the sample has proven to be sufficient. Different initial
629 and boundary conditions have been used, leading to four distinctive numerical configurations
630 named in section 5 as models A-1, A-2, B-1 and B-2.

631 Model A-1, for which the initial and boundary conditions are representative of a typical
632 experimental test, has been used to check the interplay between the location of plastic strain
633 localization, the applied velocity and the gauge length. In qualitative agreement with the
634 experiments, the computations predict that the location of plastic localization changes with
635 variations in the impact velocity and the slenderness of the sample. This reinforces the idea
636 that stress waves and inertia are main factors which control flow localization.

637 Moreover, the confrontation of the results obtained from models A-1, A-2, B-1 and B-2 allowed
638 to point out two key issues. The first one refers to the increased ductility registered in the
639 calculations for which the field variables (velocity, stress, strain and displacement) have been
640 initialized. In agreement with different works available in the literature, we have shown that
641 the stress waves, under specific loading conditions, may represent a limiting factor for the
642 sample ductility. The second key issue refers to the role played by the boundary conditions
643 in the specimen ductility and localization pattern. We have shown that the application of
644 boundary conditions representative of an infinite plate (infinite width) to a sheet with finite
645 width may lead to a substantial increase of the sample ductility and a strong modification
646 of the localization pattern which (always) takes the form of multiple necking bands. From
647 previous statement we have derived two relevant conclusions: (1) if the metallic sheet has a
648 large slenderness such that it mostly behaves like a rod then flow localization is promoted and
649 a single pair of necking bands contained inside a necked region are formed, (2) if the metallic
650 sheet shows a short slenderness such that it mostly behaves like a plate then flow localization
651 is delayed and multiple necking bands are formed. Note that previous conclusions (1) and
652 (2) agree with our experimental findings.

653 All in all, in this paper we have emphasized the deterministic character of the fracture location
654 in the dynamic tensile test. Moreover, the combination of an extensive experimental work with
655 detailed numerical calculations has brought some insights into the key factors which control flow
656 localization and fracture in dynamically loaded metallic sheets. Special attention has to be paid
657 to the fact that the specimen ductility, far from being a material property, is highly dependent on
658 the sample size, the initial conditions and the boundary conditions.

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664 **Appendix A. Complete set of dynamic experiments**

665 In Table A.3 we show the complete set of dynamic experiments, providing the fracture location
666 in each case.

Velocity (<i>m/s</i>)	Specimen	Fracture location					
		$L_2 = 20$ mm	$L_2 = 40$ mm	$L_2 = 60$ mm	$L_2 = 80$ mm	$L_2 = 100$ mm	$L_2 = 140$ mm
1	1	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>
	3	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>
	4	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>Clamped</i>	<i>N/A</i>	<i>N/A</i>
1,75	1	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Impact</i>	<i>Impact</i>	<i>Centre</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Impact</i>	<i>Impact</i>	<i>Centre</i>
	3	<i>Centre</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Impact</i>	<i>Impact</i>	<i>Centre</i>
	4	<i>N/A</i>	<i>Impact</i>	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>
2,5	1	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Centre</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Centre</i>
	3	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Centre</i>
	4	<i>Centre</i>	<i>N/A</i>	<i>Clamped</i>	<i>Clamped</i>	<i>N/A</i>	<i>N/A</i>
3,75	1	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Impact</i>	<i>Clamped</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>
	3	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Impact</i>	<i>Clamped</i>
	4	<i>N/A</i>	<i>N/A</i>	<i>Clamped</i>	<i>Clamped</i>	<i>N/A</i>	<i>Clamped</i>
5	1	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Centre</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>
	3	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Centre</i>
	4	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>Centre</i>
6,25	1	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Impact</i>	<i>Clamped</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>
	3	<i>Centre</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Impact</i>	<i>Clamped</i>
	4	<i>Centre</i>	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>Impact</i>	<i>N/A</i>
7,5	1	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Impact</i>	<i>Clamped</i>
	2	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>
	3	<i>Centre</i>	<i>Impact</i>	<i>Impact</i>	<i>Clamped</i>	<i>Clamped</i>	<i>Clamped</i>
	4	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>N/A</i>	<i>Clamped</i>	<i>N/A</i>

Table A.3: Complete set of dynamic experiments. For each test we indicate the fracture location.

667 **Appendix B. Mesh sensitivity analysis**

668 In order to check the mesh independence of our numerical calculations we have carried out
669 computations using three different mesh densities:

670 • Mesh 1: the elements dimensions are $0.166 \times 0.333 \times 0.166 \text{ mm}^3$ (reference configuration).

671 • Mesh 2: the elements dimensions are $0.125 \times 0.250 \times 0.125 \text{ mm}^3$.

672 • Mesh 3: the elements dimensions are $0.100 \times 0.200 \times 0.100 \text{ mm}^3$.

673 Fig. B.17. shows finite elements results obtained using these three mesh densities for the
 674 model A-1, the loading velocity $V_0 = 5 \text{ m/s}$ and the gauge length $L_2 = 20 \text{ mm}$. We illustrate the
 675 equivalent plastic strain $\bar{\varepsilon}^p$ versus the normalized specimen coordinate $\bar{Z} = \frac{z}{L_2}$ for the loading time
 676 $t = 1.45 \cdot 10^{-3} \text{ s}$. The excursions of strain represent the necking bands. The results corresponding
 677 to the three different mesh densities practically overlap to each other, which confirms that our
 678 computations are largely insensitive to the mesh size. Therefore, in order to have the smallest
 679 computational time, the coarser mesh (Mesh 1) was used in all the numerical simulations shown in
 680 this paper.

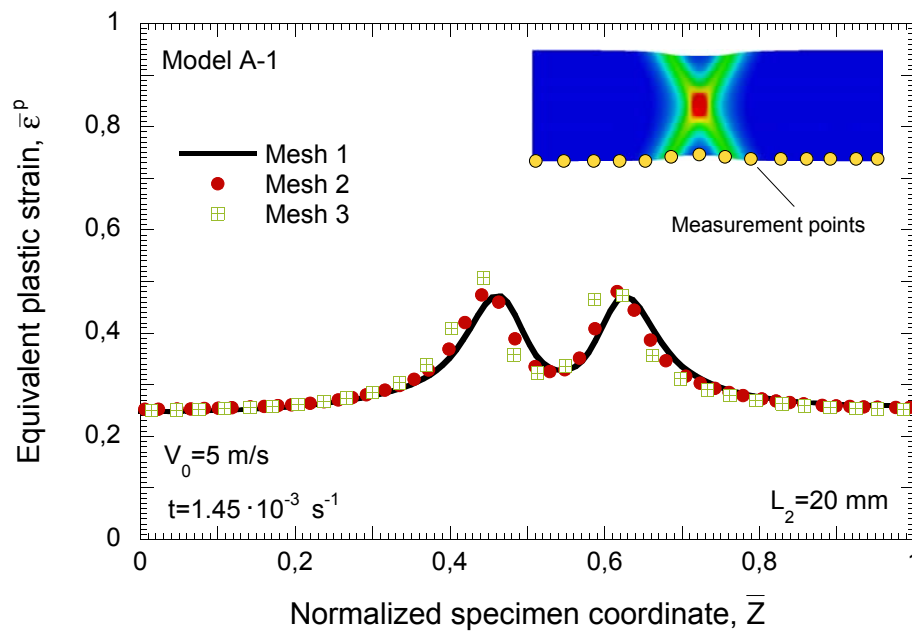


Figure B.17: Finite element results. Model A-1. Equivalent plastic strain $\bar{\varepsilon}^p$ versus the normalized specimen coordinate $\bar{Z} = \frac{z}{L_2}$. Gauge length $L_2 = 20 \text{ mm}$. Impact velocity $V_0 = 5 \text{ m/s}$. Loading time $t = 1.45 \cdot 10^{-3} \text{ s}$. Results are shown for mesh 1, mesh 2 and mesh 3.

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