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An analytical approach to the design of liquid crystal microlenses

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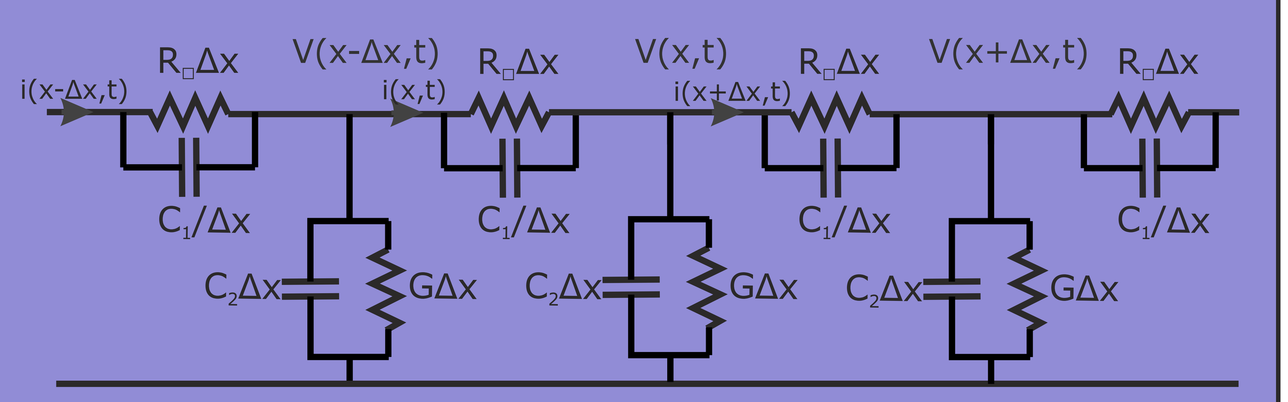
ABSTRACT

New analytical expressions for designing liquid crystal microlenses have been proposed. Equations involved, based on a novel equivalent electric circuit (EEC), lead to optimum designs of liquid crystal lenses in which lens diameter ranges from few micrometers to one millimeter. Manufacturing parameters of the lenses have been designed for fulfilling focal length and diameter requirements. Some of these requirements are the thickness, the electrode configuration, and the driving scheme (modal or hole patterned). Some liquid crystal microlenses have been manufactured and an electrooptic characterization has been carried out in order to compare the measurements with the analytical approach.

ELECTRICAL MODEL AND GOVERNING EQUATIONS

ELECTRICAL MODEL

- The design of LC microlenses has been determined from a novel proposed EEC.
- This scheme is a transmission line that includes distributed capacitances that describe the effect of the typical parasitic capacitances between teeth of the comb electrode for LC microlenses.



$$R_{sq} = \frac{\rho}{t}\left(\frac{\Omega}{sq}\right); \quad C_1 = \varepsilon_0 \varepsilon'_1 \frac{d}{k}(F) \quad C_2 = \frac{\varepsilon_0 \varepsilon'_2}{d}\left(\frac{F}{m^2}\right); \quad G = \frac{\sigma}{d}\left(\frac{S}{m^2}\right)$$

$$\frac{\partial V^2(x)}{\partial x^2} = \frac{\omega i C_2 + G}{\frac{1}{R_\square} + \omega i C_1} \cdot V(x)$$

GOVERNING EQUATIONS

- Resolving the differential equation, some constructives parameters can be predicted. The equation below determines the maximum lens radius before there is a need for a high resistivity layer:

$$r_{CRIT} > d \cdot acosh\left(\left|\frac{V_{side}}{V_{center}}\right|\right)\sqrt{\frac{2\varepsilon'_1}{\varepsilon'_2 \cdot \kappa}} \quad (1)$$

k is a parameter that depends on fringe field effects and is related to the electrode and diameter dimensions [1].

- Modal lens ($r > r_{CRIT}$): If Z_{C1} is bigger than high resistivity layer.
- Hole patterned ($r < r_{CRIT}$): If Z_{C1} is smaller than the high resistivity layer.

MODAL LENS

$$R_{sq} \approx \frac{d \cdot 2acosh^2\left(\left|\frac{V_{side}}{V_{center}}\right|\right)}{r^2(\varepsilon_0 \varepsilon'_2 \omega)} \quad (2)$$

- The value of the sheet resistance is estimated as proportional to the thickness and inversely proportional to the square of lens radius and frequency.
- The smaller the diameter is, the more resistivity is necessary.

HOLE PATTERNED

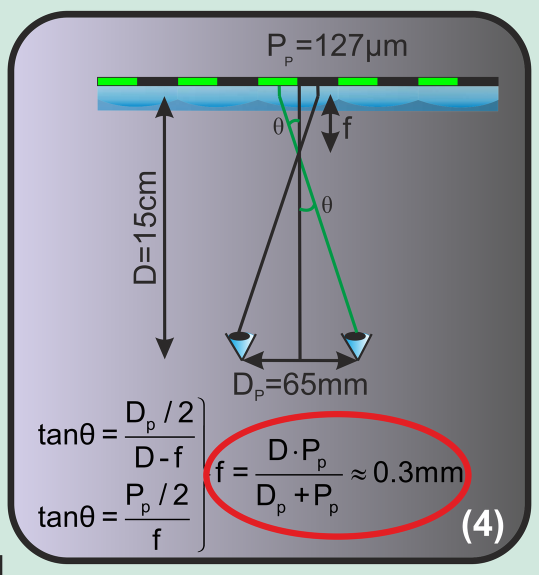
$$\frac{r}{d} = acosh(|\alpha|)\sqrt{\frac{\varepsilon'_1}{\varepsilon'_2 \cdot k}} \quad (3)$$

- This equation is essential for LC hole patterned microlens manufacture.
- It gives the key of the dimensions design for generating phase gradients similar to those in conventional lenses.

MICROLENS DESIGN

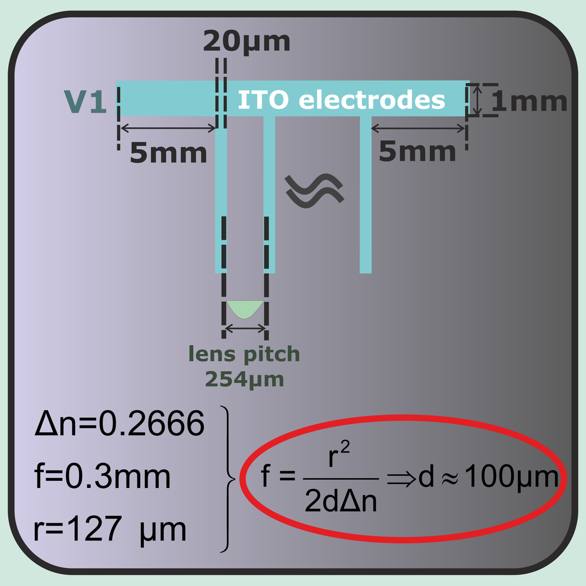
- A specific application has been chosen to validate the proposed equations: a LC based autostereoscopic display for a mobile.
- Optimum autostereoscopic viewing distance determines the focal length.

1 Viewing Distance



- Display dimensions and focal length (for an optimum 3D viewing) determine the LC lens thickness.
- A 200ppp display is employed.

2 Mask Design



3 µLens Design

- Considering:
  - Voltage at the lens center = V_{th}
  - and voltage at the lens side = V_{sat}

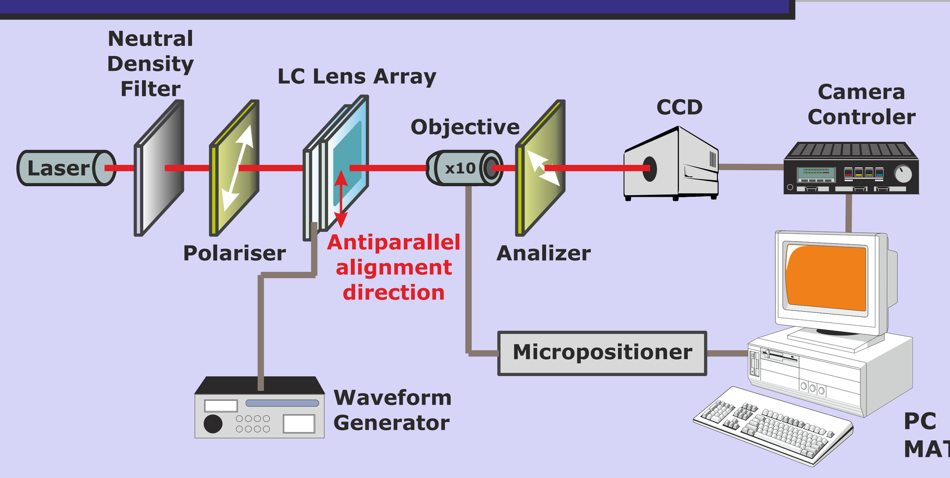
Following Eq. (1): r_{CRIT} = 132µm → Required radius = 127µm, $r_{req} < r_{CRIT}$ → HOLE PATTERNED

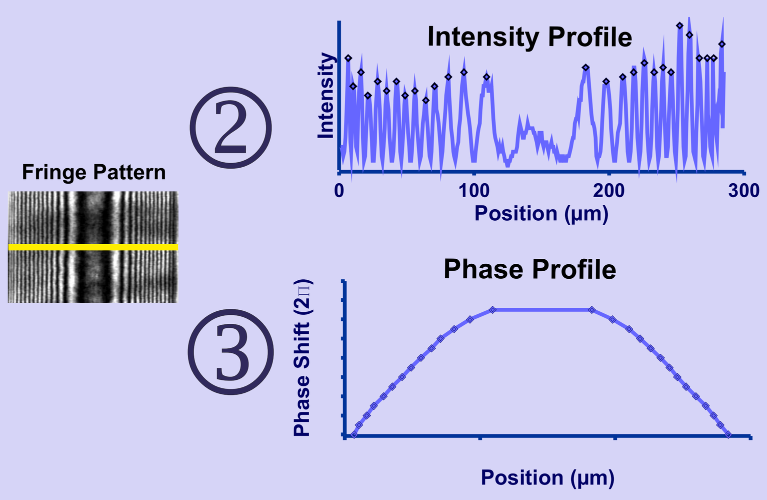
Following Eq. (3): r/d = 1.15

- Equation (3) indicates that our design is going to generate a proper phase profile.

HOLE PATTERNED LENS DESIGN:
Diameter = 254µm
Thickness = 100µm
NLC: MDA-98-1602
Birefringence: 0.2666
Expected focus: 0.3mm

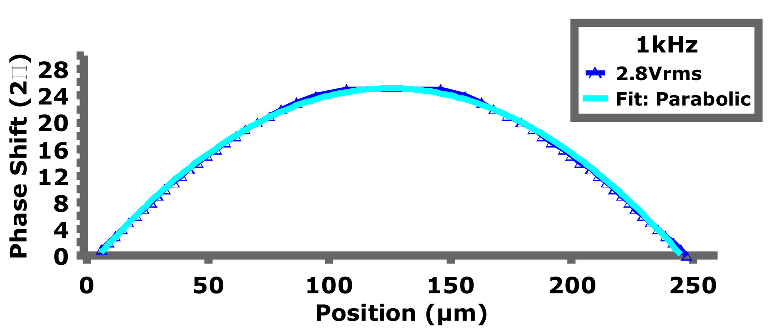
EXPERIMENTAL RESULTS





- Fringe pattern of the interference experiment reveals dark and clear zones corresponding to constructive and destructive interferences.
- Two neighbouring fringes have a phase difference of 2π.
- Microlenses have been placed between crossed polarizers with the alignment direction at π/4 from the input linear polarization.

Hole Patterned
Cell gap≈100-120µm
Surface area: 3.18cm²
Lens Pitch: 254µm



If the ratio diameter/thickness is correct, then LC lens produces appropriate voltage gradient.

FRESNEL APROX. + GRIN LENS FOCAL LENGHT
Focal length based on Phase Shift

$$\Delta\Phi = \frac{2\pi}{\lambda}\int_{-r}^{r} n_{eff}(\theta)dz = \frac{2\pi}{\lambda}\Delta n d \qquad f = \frac{r^2}{\Delta n 2d} \quad \Bigg\} \quad f = \frac{\pi r^2}{\lambda \Delta\Phi} \quad (5)$$

- A quasi-parabolic profile is achieved.
- Total phase shift of the profile is 25 2π.
- By equation (5), focal length is 0.5mm, that is similar to the expected focus.

CONCLUSIONS

- A novel EEC for LC microlenses has been proposed. Required equations for LC microlens design have been extracted through this electric model.
- These equations give the key for the design of the lens dimensions and for generating a phase gradient similar to those in conventional lenses.
- LC microlenses, for a specific application, has been designed and manufactured with the previous design equations.
- Experimental characterization validates the values of the lens design extracted from the analytical approach. Deviations in the focal length obtained may be due to diferences between theoretical and experimental birefringences.
- Equations involved are intended as a tool to be used by researchers in lenticular design.

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An analytical approach to the design of liquid crystal microlenses

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New analytical expressions for designing liquid crystal microlenses have been proposed. Equations involved, based on a novel equivalent electric circuit, lead to optimum designs of liquid crystal lenses in which lens diameter ranges from few micrometers to one millimeter. Manufacturing parameters of the lenses have been designed for fulfilling focal length and diameter requirements. Some of these requirements are the thickness, the electrode configuration, and the driving scheme (modal or hole patterned). Some liquid crystal microlenses have been manufactured and an electrooptic characterization has been carried out in order to compare the measurements with the analytical approach.

Introduction

Liquid crystal (LC) lenses have been an important research field for many years. Lenses based on LC technology were first reported more than 30 years ago. Recent research applications have further extended the interest in mechanisms and new structures that exploit the typical anisotropy of LC mixtures for developing new functionalities. Some examples are tunable-focusing optical zoom systems (cell phones, cameras, picoprojectors, night vision of hand-carried weapons) [1] or spectacles and autostereoscopic devices [2]. Many of the reported topologies have focused clear efforts on reduction of the number of the control electrodes and the amplitude of the control voltage. But only experimental observations have demonstrated practical restrictions on those devices so far. For example, at the micrometric scale, modal control technique [3] has shown some constructive restrictions (very high resistivity layers are necessary). On the other hand, hole patterned LC lenses [4] only work for very small diameters.
In this work, we propose an analytical approach that describes the designing equations for manufacturing LC microlenses. A useful protocol has been derived for this purpose. Transmission line theory models the electrical response of the lenses. The equivalent electric circuit (EEC) proposed is a transmission line that includes coplanar capacitances. These capacitances describe the physical effect of the fringe field between electrodes placed at the same surface of a LC microlens. The aim of optimizing the manufacturing protocol is to reduce the spherical aberration caused by shrinking and flatness effects, and to achieve tunable lenses.

Resolving manufacturing restrictions

The design of LC microlenses has been determined from a novel proposed EEC. This scheme is a transmission line that includes distributed capacitances that describe the effect of the typical parasitic capacitances between teeth of the comb electrode for LC microlenses. For modal lenses, the most important equations derived from the ECC is the modal limit given by Eq. 1,

$$r_{CRIT} > d \cdot a\,cosh(|\alpha|)\sqrt{\frac{2 \cdot \varepsilon'_1}{\varepsilon'_2 \cdot \kappa}} \quad (1),$$

Microlens radius must meet the previous inequation for getting a working modal lens. d is the LC thickness, ε'_1 and ε'_2 are the LC real permittivities considered in the parallel and perpendicular directions to the glasses, respectively, α is the ratio between the voltage at the control electrode and at the lens center, and κ is a parameter that depends on fringe field effects and is related to the electrode and diameter dimensions. Considering this restriction, modal lens concept is based on the inclusion of a high resistivity layer for achieving the proper voltage gradient (α).If the lens radius is minor than r_{CRIT}, LC impedance itself governs the electrical gradient and high resistivity layer inclusion is not a constraint for manufacturing the devices. Resistivity of the high resistivity layer can be obtained from the ECC as Eq. 2,

$$R_{\square} \approx \frac{d \cdot 2a\,cosh^2(|\alpha|)}{r^2\left(\varepsilon_0 \varepsilon'_2 \omega\right)} \quad (2),$$

Different materials and techniques have been proposed and reported to manufacture the high resistivity layers for modal devices. This design at micrometric scale is not a simple task and sometimes it can require the application of unusual physical properties of the materials [5]. If no high resistivity layer is required, ratio between radius and thickness of the lens is described by Eq. 3,

$$\frac{r}{d} = a\,cosh\left(|\alpha|\right)\sqrt{\frac{\varepsilon'_1}{\varepsilon'_2 \cdot \kappa}} \quad (3),$$

This equation is essential for LC microlens manufacture, since it gives the key of the dimensions design for generating phase gradients similar to those in conventional lenses.

Microlens design protocol

Focal length and diameter are identified as the two main parameters describing a lens for most applications. Considering that LC lenses are in fact GRIN (GRadient INdex) lenses, an estimation of their focal length, f_{GRIN}, (Eq. 4) becomes simple taking into account focusing of parallel rays,

$$f_{GRIN} = \frac{r^2}{2d\Delta n} \quad (4),$$

Previous parameters leave only two adjustable variables, thickness and birefringence. The smaller the thickness, the lower the response time and the greater the material saving. However, larger thicknesses are required if applications demand large focal tunability ranges with short focal lengths. Later condition can also be achieved by higher birefringence materials, allowing thinner device thicknesses. Once the LC mixture has been selected, birefringence as a function of external voltage can be extracted experimentally and by simulations. By experimental characterization, the working voltage can be measured. Threshold voltage (V_{th}) is related to the voltage at the lens center and saturation voltage (V_{sat}) to the voltage at the edge. If the voltage limits are exceeded, some unwanted effects appear, such as flatness in the lens center or shirking effect at the edges. Next, the parameter $\alpha = V_{th}/V_{sat}$, which quantifies the voltage gradient, is estimated. Finally, by evaluation of Eq. 1, the need to include a high resistivity layer is reveled; it is estimated by Eq. 2. If the lens does not follow a modal design, the voltage gradient is estimated by Eq. 3. In parallel, birefringence is modeled by simulation using Frank-Oseen equations. Mathematically, if an external perturbation, as an electrical field, is introduced in the system, the deformation of molecular director can be estimated minimizing the Gibbs free energy.

Results and conclusion

A specific application has been chosen to validate the proposed protocol: a LC based autostereoscopic display for a mobile. The commercial nematic liquid crystal MDA-98-1602 from Merck has been selected due to its high birefringence, $\Delta n = 0.2666$. Two kinds of devices have been manufactured for the study: a monopixel cell and a LC lenticular array. Birefringence has been measured by spectroscopy on the monopixel cell and compared to simulations (Fig. 1). Both birefringences follow Kerr effect (also called the quadratic electro-optic effect) from about 1.3Vrms to 3Vrms. Then, according to the protocol, $\alpha = 2.3$ in order to avoid undesirable effects.

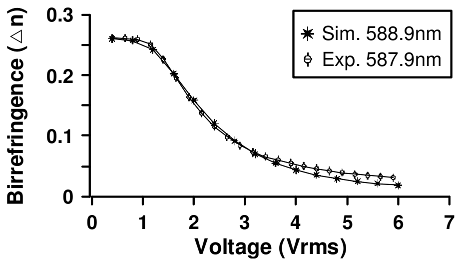

Figure 1: Experimental and simulated birefringences at 588nm.

For the requirements of the autostereoscopic application, a cylindrical lens array has been manufactured with lenses of 254 μm diameter (for a 200 ppp display). A chrome mask consisted on a comb electrode pattern was designed for this purpose. There was no need of including a high resistivity layer due to the specifications. Considering a minimum 3D distance of 15 cm, an interpupilar distance of 65 mm and a pixel pitch of 127 µm, the minimum focal length is 0.3 mm (geometrical optics). The value of the LC birefringence and the focal length determine the LC lens thickness (Eq. 4), that is 100 µm. So, the minimum radius to employ a high resistivity layer would be r_{CRIT} = 132 µm; that is, with lenses of 127 µm, modal control is not required. Also, following Eq. 3, ratio between the hole diameter (D = 2r) and the thickness (d), gives D/d = 2.3 (ε'_1 = 12.74 and ε'_2 = 7.86, extracted from simulations) that is a similar value than final design, D/d = 2.5. Electrooptic characterization of the device was carried out through the analysis of the phase profile generated from the interference pattern. The result, shown in Fig. 2, is the phase profile of a LC lens with 0.3 mm focal length for 2.8 V_{rms}.

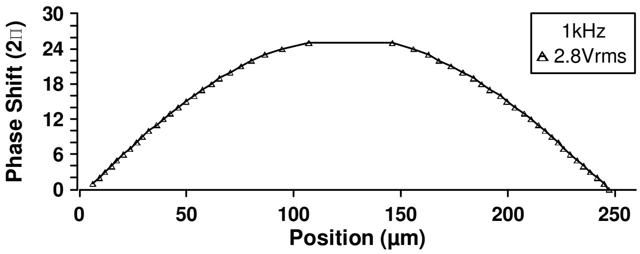

Figure 2: Experimental phase retardation profile for a lens of the array

Experimental results validate the values of the lens design extracted from the analytical approach. Equations involved are intended to be a tool for researchers relating to lenticular optimal designs. It means time saving and maximization of material resources avoiding test cell manufacture.

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