



This is a postprint version of the following published document:

Velasco, J.A., Rigoni, V., Soroudi, A., Keane, A. y Amaris, H. (2019). optimising Load Flexibility for the Day Ahead in Distribution Networks with Photovoltaics. In *2019 IEEE Milan PowerTech*.

DOI: <https://doi.org/10.1109/PTC.2019.8810963>

© 2019 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Optimising Load Flexibility for the Day Ahead in Distribution Networks with Photovoltaics

Jose Angel Velasco*, Valentin Rigoni†, Alireza Soroudi†, Andrew Keane† and Hortensia Amaris*

*Universidad Carlos III de Madrid

Electrical Engineering Department, Madrid, Spain

†University College Dublin

Energy Institute, UCD

Dublin, Ireland

Abstract—In this paper a methodology is proposed to calculate the load demand flexibility that could be activated within the next 24-hours for solving the technical impacts of contingencies that may come up in an unbalanced low voltage distribution networks with high penetration of intermittent DG sources. The methodology is formulated within a Demand Response program environment via load shifting as flexibility enabler mechanism. To achieve that, a non-linear optimisation problem is formulated based on an unbalanced optimal power flow, which allows the determination of the load flexibility that each Demand Response customer could provide at the request of the Distribution System Operator. The demand as well as weather conditions are forecasted for the day ahead. The optimisation problem is solved in a sequence fashion, within a daily framework, splitting the whole problem in optimisation blocks. In each block, the flexible load demand is obtained and the load demand forecasting its updated for the upcoming blocks based on the changes in the scheduled load demand. The methodology is applied to a real distribution network with the load data received from the smart metering infrastructure. The results obtained show the strength of the methodology in solving the technical problems of the network under high unbalanced operation.

I. INTRODUCTION

Due to the ever-increasing presence of renewable-based Distributed Generation (DG) such a Photovoltaic (PV) facilities, as well as Electric Vehicles (EVs) and energy storage units [1], Low Voltage (LV) distribution systems are becoming active, therefore some energy policies and actions have to be carried out to guarantee an optimal operation. The presence of these Distributed Energy Resources (DERs) could lead the system to a state with technical risks such as over-loading due to currents surpassing the limits of the cable lines and over-voltages due to a high amount of energy injected in the system. In these situations, Demand Response (DR) becomes a well suited solution to efficiently integrate those DERs.

Through different DR mechanisms based on certain market price signals or monetary incentives [2], a certain amount of load demand flexibility could be activated to return the system to the normal operation condition. Moreover, DR programs can be implemented in different timescales, being day-ahead, the time horizon in which the most common ones are implemented. For instance, in this time timescale, are implemented demand bidding/buyback programs as well as load curtailment/shifting programs [3].

DR has been widely researched by the scientific community in the last years and recent related works can be found in the literature. The authors in [4] proposed a genetic algorithm-based two stage stochastic optimisation procedure to determine the scheduling of shiftable appliances without considering LV networks. In [5] is presented an optimal DR program for rural areas in day ahead basis considering battery-based storage systems. In [6] is proposed an optimal planning for the day ahead to provide DR services considering PV units, energy storage and load shifting adopting as well an economic point of view. The authors in [7] proposed a DR scheduling under different price-based DR programs with the objective of minimise the customers electricity consumption cost. Nonetheless, the mentioned works do not examine the technical network contingencies adopting instead a economic point of view. Additionally, the majority of them only consider balanced operation conditions which is not the reality.

Therefore, in this paper is proposed a methodology to determine the amount of flexibility that could be activated for the day-ahead to solve the technical problems that arise in LV distribution networks with high penetration of PV-based DG. A modified three-phase optimal power flow including a DR mechanism is formulated to obtain the load flexibility. The resulting non-linear programming problem is solved in a sequence fashion, splitting the whole problem in optimisation blocks and updating the demand forecast according to the load flexibility.

II. SYSTEM MODELLING

LV distribution systems can be represented as a graph [8] with $b = |V|$ nodes (or buses) and $N = |E|$ edges (or lines), being $V = \{V_1, \dots, V_N\}$ the set of nodes of the system and $E = \{e_1, \dots, e_m\}$ the set of edges. Since LV distribution systems are operated in radial configuration, the considered graph has a tree topology. Moreover, as loads are connected in single phase mode, the system operation is usually in unbalanced conditions. Taking this into account, each individual line is defined as a four-wires line section, so the edge e_m is a four-tuple of the three phases plus the neutral cable $e_m = \{l_{k,j}^a, l_{k,j}^b, l_{k,j}^c, l_{k,j}^n\}$.

Regarding the notation of the paper, all the magnitudes expressed in lower case are in per unit values [9] using as the

the reference power S_B the rating of the Secondary Substation transformer, meanwhile the reference voltage V_B it set to the nominal line-to-line voltage (400V).

A. Power Flow Modelling

Following the formulation in [10] the complex magnitudes are separated into their real and imaginary, the components such as phase voltage $\bar{u}_k^p = u_k^{p,re} + j u_k^{p,im}$, current injection $\bar{i}_k^p = i_k^{p,re} + j i_k^{p,im}$ and current flow $\bar{i}_{kj}^p = i_{kj}^{p,re} + j i_{kj}^{p,im}$ where $p \in \{a, c, b\}$, k and j are the sending and receiving nodes of the line. Note that a Kron's reduction is applied assuming that the network distribution system is well-grounded so only the principal phases are taking into account. The state variables of the system are determined by solving the non-linear equation that constitute the power mismatch between the specified power injections (sp) and the calculated power injections (cal). $F(\mathcal{X}) = 0 : \mathbb{R}^n \rightarrow \mathbb{R}^n$. This non-linear problem is solved using IPOPT solver (*Interior Point OPTimizer*) [11] within the Python environment Pyomo [12]. The mismatch function is divided into active and reactive power parts.

$$F(\mathcal{X}) = 0 = \begin{bmatrix} F_p(\mathcal{X}) \\ F_q(\mathcal{X}) \end{bmatrix} = \begin{bmatrix} p_{i,k}^{p,sp} + p_{i,k}^{p,cal} \\ q_{i,k}^{p,sp} + q_{i,k}^{p,cal} \end{bmatrix} \quad (1)$$

where \mathcal{X} is the state variable, indicated in expression (2) that correspond with the real and imaginary phase voltages $\{u_k^{p,re}, u_k^{p,im}\}$, $\forall k \in V \forall p \in \{a, b, c\}$ where b is the number of distribution system's buses.

$$\mathcal{X} = \begin{bmatrix} [u_1^{a,re} u_1^{b,re} u_1^{c,re} \dots u_N^{a,re} u_N^{b,re} u_N^{c,re}]^T \\ [u_1^{a,im} u_1^{b,im} u_1^{c,im} \dots u_N^{a,im} u_N^{b,im} u_N^{c,im}]^T \end{bmatrix} \quad (2)$$

The calculated (cal) complex power injection in each node k and phase p is given by expression (3). Working in the above equation, it is possible to find an expression for the power injected as a function of the line parameters and the voltage.

$$\bar{s}_{i,k}^{p,cal} = \bar{u}_k^p (\bar{i}_{i,k}^p)^* = \bar{u}_k^p \sum_{j \in \Omega_k} \sum_{p' \in \{a,b,c\}} (\bar{y}_{kj}^{pp'}) (\bar{u}_j^{p'})^* \quad (3)$$

The active and reactive power injections are obtained by extracting the complex power injection in real and imaginary parts as described in (4)

$$\begin{aligned} p_{i,k}^{p,cal} &= \text{Re}\{\bar{s}_{i,k}^{p,cal}\} = u_k^{p,re} i_{i,k}^{p,re} + u_k^{p,im} i_{i,k}^{p,im} \\ q_{i,k}^{p,cal} &= \text{Im}\{\bar{s}_{i,k}^{p,cal}\} = u_k^{p,im} i_{i,k}^{p,im} - u_k^{p,re} i_{i,k}^{p,re} \end{aligned} \quad (4)$$

The specified (sp) power injections are determined by the power balance between the positive power injected coming from the DG units, and the negative power injection (consumption) due to the customers demand, as indicated in (5).

$$\bar{s}_{i,k}^{p,sp} = \bar{s}_{g,k}^p - \bar{s}_{d,k}^p \quad (5)$$

Also the specified complex power injections $\bar{s}_{i,k}^{p,sp}$ is separated in active and reactive power injections indicated (6).

$$\begin{aligned} p_{i,k}^{p,sp} &= p_{g,k}^{p,sp} - p_{d,k}^{p,sp} \\ q_{i,k}^{p,sp} &= q_{g,k}^{p,sp} - q_{d,k}^{p,sp} \end{aligned} \quad (6)$$

The active and reactive power specified in each bus depends on the considered bus type. To solve the power flow problem, one of the buses is chosen to be the *slack* (denoted with k^*) and their power injections are equal to the net demand of the system including the losses. This slack node corresponds to the LV side of the secondary substation, which the voltage is fixed to the nominal value, the same as the reference voltage so $\bar{u}_k^p = 1 \angle 0$ pu. The rest of the nodes $\forall k \in V/k^*$ are considered as PQ which phase voltage \bar{u}_k^p is a variable and their power injections depend on whether there is connected customers with DG or not. The active power generation coming from the DG units are forecasted (FC) by means of the irradiation conditions meanwhile the load demand of each customer is also forecasted using the available data via smart meter. The forecasting procedure is explained in section II-E.

B. Distributed Generation Modelling

In this paper is considered that DG is based 100% on PV panels, in a way that every customer belonging to the set of DG nodes of the network (Ω_{DG}) have a PV facility. The variable power output of each PV facility is indicated in expression (7) for each phase p and node $k \in \Omega_{DG}$ based on [13] and depends on the weather conditions such a solar irradiance G and ambient temperature T_{amb} with has to be forecasted since these values are subject to uncertainty [14]. The resolution of the power generation is 10-min so to make the notation consistent with that time-steps the variables are expressed with t in the subindex, which take values in the daily timeframe work $t \in \{0, 10, \dots, 1440\}$ min. It is considered that PV inverters work with unitary power factor so the reactive power generated its zero $q_{g,k,t}^{p,sp} = 0$

$$p_{g,k,t}^{p,sp} = \frac{P_{PV_k}}{S_B} \left(\frac{\hat{G}_t}{1000} [1 + \gamma (T_{cell,t} + 25)] \right) \quad (7)$$

where:

- P_{PV_k} is the peak power output of the PV installation (W) under Standard Test Conditions (STC) which means under a solar irradiation $G = 1000 \text{ W/m}^2$ as well as ambient temperature of $T_{amb} = 25^\circ\text{C}$.
- \hat{G}_t is the solar irradiance (W/m^2) that receive the PV panel of the customer k on the inclined plane. The hat indicates that this variable is forecasted.
- γ is the power-temperature coefficient ($\%/^\circ\text{C}$) of the PV panel of the customer k .
- T_{cell} is the temperature ($^\circ\text{C}$) of the panel surface witch depends on the ambient temperature $\hat{T}_{amb,t}$ and the solar irradiance G and the Nominal Operating Cell Temperature (NOCT) ($^\circ\text{C}$) (8)

$$T_{cell,t} = \hat{T}_{amb,t} + \hat{G}_t \left(\frac{\text{NOCT} - 20}{800} \right) \quad (8)$$

C. Demand Modelling

Real load data with 10-min resolution coming from smart meters are used in this paper with residential load patterns. To account for the voltage dependency of the demand, is considered a load ZIP model (9) [15] for all the nodes with customers $\forall k \in \Omega_c$ and for all phases $\forall p \in \{a, b, c\}$.

$$\begin{aligned} p_{d,k,t}^{p,sp} &= \hat{p}_{d,k,t}^p \left[c_{p,k}^{p,1} \left(u_{k,t}^p \right)^2 + c_{p,k}^{p,2} \left(u_{k,t}^p \right) + c_{p,k}^{p,3} \right] \\ q_{d,k,t}^{p,sp} &= \hat{q}_{d,k,t}^p \left[c_{q,k}^{p,1} \left(u_{k,t}^p \right)^2 + c_{q,k}^{p,2} \left(u_{k,t}^p \right) + c_{q,k}^{p,3} \right] \end{aligned} \quad (9)$$

where:

- $u_{k,t}^p$ is the phase voltage magnitude in pu.
- $\hat{p}_{d,k,t}^p = \hat{P}_{d,k,t}^p / S_B$ is the active demand forecasted in pu.
- $\hat{q}_{d,k,t}^p = \hat{Q}_{d,k,t}^p / S_B$ is the reactive demand forecasted in pu.
- $c_{p,k}^{p,1}, c_{p,k}^{p,2}, c_{p,k}^{p,3}, c_{q,k}^{p,1}, c_{q,k}^{p,2}, c_{q,k}^{p,3} \in (0, 1)$ are the active and reactive power demand sensibility coefficients that determine the voltage dependence of the customer k connected to phase p . These coefficients have to be forecasted along the demand.

D. Demand Response Modelling

A load shifting mechanism is considered for managing the flexibility of the load demand of the customers participating in the DR program. This mechanism consists of reducing or increasing a certain quantity of power demand $\Delta p_{d,k,t}^p$ at certain time instant t to provide load demand flexibility and so to reduce overloading in the cables and over voltages due to the presence of DG in the distribution system at the request of the DSO.

From the point of view of the DR, two types of customers are considered depending on if they participate in the program or not, therefore a set of customers participating in the DR program is defined $\tilde{\Omega}_c \subset V$ and also a set of Non-DR customers is defined as $\Omega_c \subset V$.

For the DR customers, the formulation for the specified power injection has to be modified, and include the load shift $\Delta p_{d,k,t}^p$ as it is indicated in (10). Note that the term for the power generation $p_{g,k,t}^{p,sp}$ will be present if the customer has a PV facility connected to the grid.

$$p_{i,k,t}^{p,sp} = p_{g,k,t}^{p,sp} - \left(p_{d,k,t}^p + \Delta p_{d,k,t}^p \right), \forall k \in \tilde{\Omega}_c \quad (10)$$

The DR mechanism is subject to certain operational constraints related to the availability of the DR customers to provide such load demand shift. First, the load shift has to guarantee a minimum power supply $p_{d,k,t}^{p,min}$, defined for each DR customer by a percentage β_k of the power contracted $p_{ctd,k} = P_{ctd,k} / S_B$ as indicated in (11).

$$p_{d,k,t}^p + \Delta p_{d,k,t}^p \geq \beta_k \cdot p_{ctd,k} \quad (11)$$

Second, in terms of energy, the load shift works as an energy management tool where the energy resulting from the negative

load shifting (demand reduction) has to be equal to the energy resulting from the positive load shifting (load increase), that way the load shifting is referred to the variation in time of the use of the appliances in a certain time framework as indicated in (12).

$$\sum_{t \in \{t_0, \dots, t_n\}} \Delta p_{d,k,t}^p = 0 \quad (12)$$

Finally, the load shift is also bounded by an operational limit related to the power contracted of each customer by a percentage α_k as indicated in (13).

$$-\alpha_k \cdot p_{ctd,k} \leq \Delta p_{d,k,t}^p \leq \alpha_k \cdot p_{ctd,k} \quad (13)$$

E. Short-Term Forecasting

In this paper, the demand of each customer as well as the PV Generation coming from each PV facility connected to the network are forecasted for the day ahead with 10-time resolution. The load data used to perform that short-term forecasting was gathered from the model [16] which data comes from a Advanced Metering Infrastructure (AMI). A widely used forecasting technique is the ARIMA model [17] that combine Auto-Regressive models (AR) with Moving-Average (MA) applied for non-stationary time series by using integrated moving average. Since Load demand and PV generation (solar irradiation and temperature) are not-stationary processes, ARIMA model is chosen to perform the forecasting [18]. An ARIMA(p_a, d_a, q_a) process at time $t + \tau$ (τ period in the future) can be expressed as is indicated in (14)

$$\hat{y}_{t+\tau} = \delta + \sum_{i=1}^{p_a+d_a} \phi_i y_{t+\tau-i} + \varepsilon_{t+\tau} - \sum_{j=1}^{q_a} \theta_j \varepsilon_{t+\tau-j} \quad (14)$$

In the above expression, δ its a constant term that depends on the mean value of the time series $\delta = \mu_y (1 - \phi_0)$, ε_t is a white noise process and $y_{t+\tau}$ is the time series that represent the variables to be forecasted such a load demand $P_{d,k,t}^p$ and $Q_{d,k,t}^p$ as well as solar irradiance G_t and ambient temperature $T_{amb,t}$. The value of the time series forecasted in time τ is $\hat{y}_{t+\tau}$ (i.e. the time series variables to be forecasted such a ambient temperature T_{amb} or solar irradiance G). The degree of the model (p_a, d_a, q_a) is defined by p_a , which represents the order of the auto-regressive part, d_a which denotes the degree of the first differencing involved and q_a which denotes the order of the moving average. The degree of the model as well as the coefficients have to be fitted using the STATS MODELS [19]. The later, is an open-source tool widely used in statistical studies. Different ARIMA models have been compared using the AIC statistic (*Akaike Information Criteria*) which definition its indicated in the expression (15).

$$AIC = 2K - 2\ln(L) \quad (15)$$

Where K is the the number of parameters of the model and L is the value of the maximum likelihood function of the ARIMA model. The AIC statistic quantify the goodness of

the fit as well as the simplicity of the model. Between two models, the one with the lowest AIC value is the better to be fitted [20].

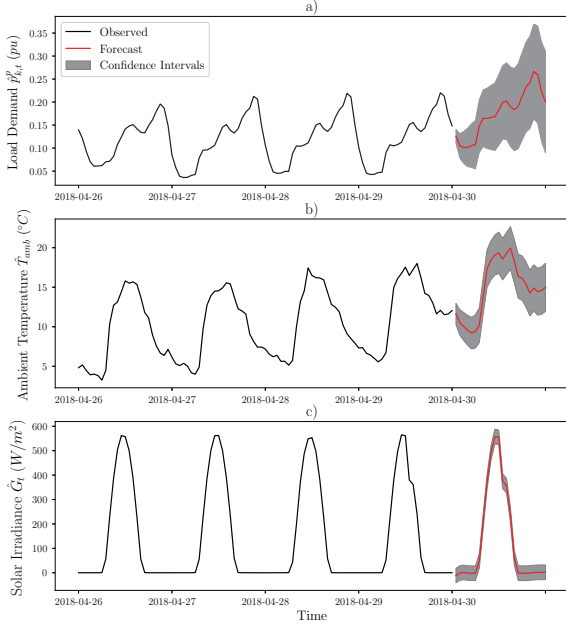


Fig. 1. Forecasted variables

Samples of the carried out forecasting are showed in Fig 1 for load demand $\hat{p}_{d,t}$ in the upper plot a), for ambient temperature $\hat{T}_{amb,t}$ in the middle plot b) and for solar irradiance \hat{G}_t in the lower plot c). Weather data has been collected from several weather stations in the Madrid area provided by AEMET (Agencia Estatal de Meteorología) [21].

III. LOAD DEMAND FLEXIBILITY

The flexibility methodology proposed in this paper is based on incorporating the DR mechanism described as a control variable into an unbalanced three-phase Optimal Power Flow (OPF). This results in a non-linear programming problem (NLP) consisting of minimise the total load shift necessary to meet the technical limits of the system along the day ahead time framework $t \in T = \{t_0, \dots, t_n\}$ as is indicated in the objective function (16) subject to the limits of the demand flexibility mechanism (11)-(13), as well as the constraints related to the statutory limits of current and voltage (17).

Minimise:

$$OF = \sum_{t \in T} \sum_{p \in \{a,b,c\}} \sum_{k \in \hat{\Omega}_c} (\Delta p_{d,k,t}^p)^2 \quad (16)$$

Subject to:

$$\begin{aligned} (i_{kj,t}^{p,re})^2 + (i_{kj,t}^{p,im})^2 &\leq (i_{kj}^{max,p})^2 \\ (u^{min})^2 &\leq (u_{k,t}^{p,re})^2 + (u_{k,t}^{p,im})^2 \leq (u^{max})^2 \end{aligned} \quad (17)$$

IV. METHODOLOGY

The optimisation process is carried out in a rolling-window way as illustrated in Fig. 2 with a certain frequency along the time framework denoted with t_k . In this paper, that frequency is set up to be 6 hours, $k = 360$ min, which implies that 4 (i.e. $n/k = 4$) optimisation blocks has to be solved.

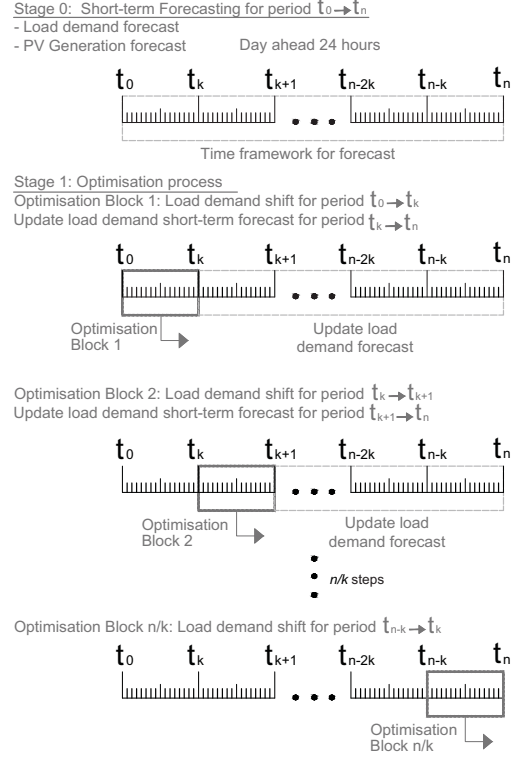


Fig. 2. Rolling window based optimisation procedure

Previous to everything, in stage 0 (Fig. 2) the short-term forecast with 10-min resolution explained in section II-E is carried out for the demand and PV generation to the day ahead, that represent the initial time framework for forecast in the interval $t_0 \rightarrow t_n$ with $n = 1440$ min.

For t_0 , the load shift from each DR customer is obtained, by solving the non-linear optimisation problem for the first 6-hours block ($t_0 \rightarrow t_k$). Then, the demand forecast is updated for the new forecasting time-framework ($t_k \rightarrow t_n$) considering the new values of demand of the first block $p_{d,k,t}^p = p_{d,k,t}^p + \Delta p_{d,k,t}^p$. After that, in t_k the optimisation problem is solved again with the new demand forecast to obtain the load shift for the current block. Following that, the load demand forecast for the new forecast time framework ($t_{k+1} \rightarrow t_n$) is updated and the optimisation process continues to the next block ($t_{k+1} \rightarrow t_{k+2}$). The process continues until all of the n/k optimisation blocks have been solved and the load shifts for the complete 24 hours are obtained.

V. CASE STUDY

To implement the load demand flexibility estimation procedure presented in this paper, a real LV feeder with high pene-

tration of PV-based DG generation has been selected from the Research Project LVNS (*Low Voltage Network Solutions*) [22] showed in Fig. 3. The selected feeder consist of 23 customers connected in a single-phase configuration along the feeder in pairs being the normal operation of the network unbalanced with sporadic over-voltage and over-loading situations as well. Each customer has a PV facility with peak power of $P_{PV_k} = 4$ kW (which means that each facility has a compound of 16 panels with 250 W of peak power each one) The power contracted by each customer is $P_{ctd,k} \in \{3, 15\}$ kW. The network its headed by a Secondary substation transformer with a power rating of 800 kVA. The DR control parameters are selected to take the following values: $\beta_k \in (0.05, 0.1)$ and $\alpha_k \in (0.1, 0.5)$ which exhibit a real situation [16].

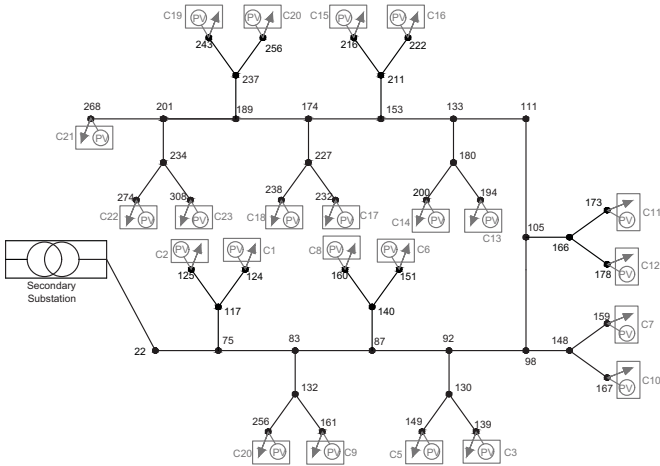


Fig. 3. LV Unbalanced feeder of the case study [22]

In Fig. 4 the results of the unbalanced power flow are shown as follows: in plot a) the total demand and the total PV generation, in plot b) the total power losses, in plot c) the currents of each phase of the line most loaded and in plot d) the phase voltage magnitude of the node with the maximum voltage. As it can be seen, the network operation conditions is unbalanced and the technical limits of the systems has been overtaken.

A. Methodology Results

After applying the load demand flexibility methodology, the results are showed in Fig 5 as follow: in plot a) the new load demand obtained in dashed line, in plot b) the new power losses of the system, in plot c) the total load flexibility obtained, in plot d) the new maximum phase current in dashed line, and in plot e) the new maximum phase voltage in dashed line. At first sight, it can be observed that load flexibility works shifting the demand from the early and late hours to the central hours of the day to absorb the power injected by the PV units. With this, the demand flexibility can return the system to normal operation conditions since voltage and current limits are respected. Finally, the DR mechanism works as an energy efficiency measure since reduce the peak of power losses.

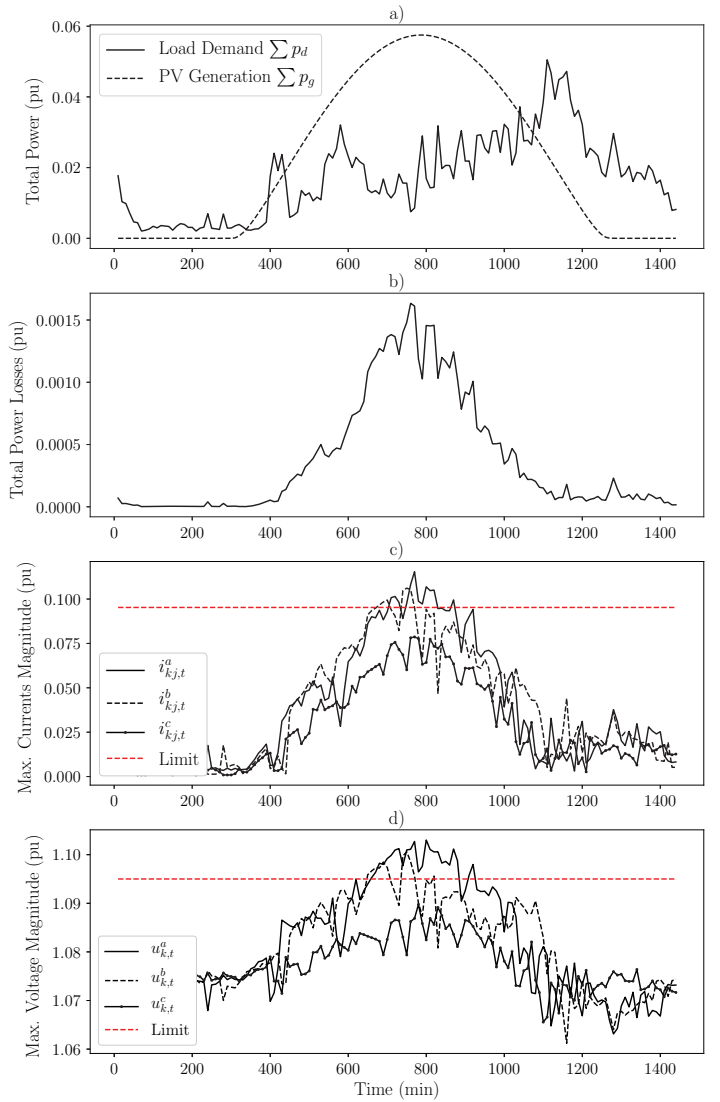


Fig. 4. Network state under normal operation

VI. CONCLUSIONS

In this paper, an optimisation based methodology for load demand flexibility management is proposed to solve the technical problems related to voltage and currents in unbalanced distribution systems with high penetration of PV-based DG units. The load demand flexibility methodology is formulated as an optimal power flow which results in a non-linear programming problem (NLP). The load demand as well as the weather conditions such as ambient temperature and solar irradiation have been forecasted for the day ahead. The optimisation problem is solved in a sequential way to take into account the changes in the initial forecast. The simulation results show the capability of the load shift mechanism to reduce the over-voltages and over-loading's of the system and even a reduction of the power losses has been achieved. Further research lines may include the consideration of energy storage

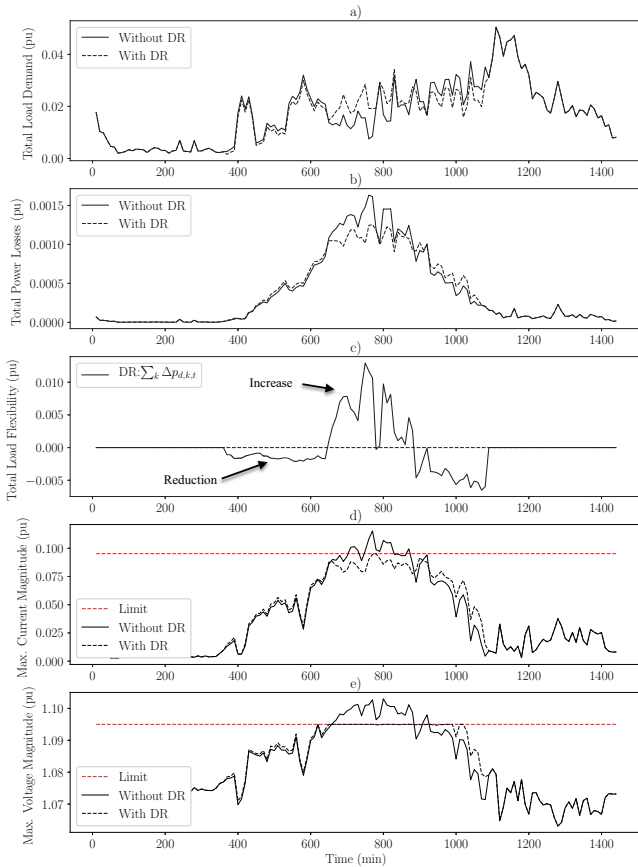


Fig. 5. Network state with load demand flexibility

(chemical such a batteries or thermal such as hot water) as well as modelling the temperature-dependant appliances of the customers.

ACKNOWLEDGMENT

The work done by Valentin Rigoni and Alireza Soroudi is supported by a research grant from Science Foundation Ireland (SFI) under the SFI Strategic Partnership Programme Grant No. SFI/15/SPP/E3125. The opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Science Foundation Ireland. Jose Angel Velasco and Hortensia Amaris are supported by the Spanish Ministry of Economy and Competitiveness through the National Program for Research under the project OSIRIS (RTC-2014-1556-3).

REFERENCES

[1] A. Soroudi, P. Siano, and A. Keane, "Optimal DR and ESS scheduling for distribution losses payments minimization under electricity price uncertainty," *IEEE Transactions on Smart Grid*, vol. 7, no. 1, pp. 261–272, Jan 2016.

[2] A. M. Carreiro, H. M. Jorge, and C. H. Antunes, "Energy management systems aggregators: A literature survey," *Renewable Sustainable Energy Reviews*, vol. 73, pp. 1160–1172, 2017.

[3] R. Jain, K. K. Ramakrishnan, and D. M. Chiu, "Identified Short and Real-Time Demand Response Opportunities and The Corresponding Requirements and Concise Systematization of The Conceived and Developed DR Programs," DREAM-GO projec, Tech. Rep. Deliverable D2.1-v3.0, March 2017.

[4] Y. Wang, H. Liang, and V. Dinavahi, "Two-stage stochastic demand response in smart grid considering random appliance usage patterns," *IET Generation, Transmission & Distribution*, vol. 12, no. 18, pp. 4163–4171, 2018.

[5] R. Morsali and R. Kowalczyk, "Demand response based day-ahead scheduling and battery sizing in microgrid management in rural areas," *IET Renewable Power Generation*, vol. 12, no. 14, pp. 1651–1658, 2018.

[6] J. A. Gomez-Herrera and M. F. Anjos, "Optimal collaborative demand-response planner for smart residential buildings," *Energy*, vol. 161, pp. 370–380, 2018.

[7] S. Nan, M. Zhou, and G. Li, "Optimal residential community demand response scheduling in smart grid," *Applied Energy*, vol. 120, pp. 1280–1289, 2018.

[8] D. Jungnickel, *Graphs, Networks and Algorithms*. Springer-Verlag, 2009.

[9] A. Gomez-Exposito, A. J. Conejo, and C. Canizares, *Electric Energy Systems: Analysis and Operation*. CRC Press, 2008.

[10] P. A. N. Garcia, J. Luiz, R. Pereira, S. Carneiro, M. Da Costa, and N. Martins, "Three-Phase Power Flow Calculations Using the Current Injection Method," *IEEE Transactions on Power Systems*, vol. 15, no. 2, 2000.

[11] A. Wächter and L. T. Biegler, "On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–27, 2006.

[12] W. E. Hart, J.-P. Watson, and D. L. Woodruff, "Pyomo: modeling and solving mathematical programs in python," *Mathematical Programming Computation*, vol. 3, no. 3, pp. 219–260, 2011.

[13] E. Lorenzo, G. Araujo, A. Cuevas, M. Egado, J. Miano, and R. Zilles, *Solar Electricity: Engineering of Photovoltaic Systems*. Progensa Editorial, 1994.

[14] A. Soroudi and T. Amraee, "Decision making under uncertainty in energy systems: State of the art," *Renewable and Sustainable Energy Reviews*, vol. 28, pp. 376–384, 2013.

[15] M. S. Hossain, H. M. M. Maruf, and B. Chowdhury, "Comparison of the zip load model and the exponential load model for cvr factor evaluation," in *2017 IEEE Power Energy Society General Meeting*, July 2017, pp. 1–5.

[16] E. McKenna and M. Thomson, "High-resolution stochastic integrated thermal-electrical domestic demand model," *Applied Energy*, vol. 165, pp. 445–461, 2016.

[17] R. Vazquez, H. Amaris, M. A. G. Lopez, J. I. Moreno, D. Olmeda, and J. Coca, "Assessment of an Adaptive Load Forecasting Methodology in a Smart Grid Demonstration Project," *Energies*, vol. 190, no. 10, pp. 25–27, 2017.

[18] D. C. Montgomery, C. L. Jennings, and M. Kulahci, *Introduction to Time Series Analysis and Forecasting*. Wiley, 2008.

[19] S. Seabold and J. Perktold, "Statsmodels: Econometric and statistical modeling with python," in *9th Python in Science Conference*, 2010.

[20] D. R. Burnham, K. P.; Anderson, *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach*. Springer-Verlag, 2002.

[21] "Agencia Estatal de Meteorologia (AEMET)," <http://www.aemet.es/en/serviciosclimaticos>, accessed on 14 August 2016.

[22] "Low Voltage Network Solutions (LVNS)," <https://www.enwl.co.uk/zero-carbon/smaller-projects/low-carbon-networks-fund/low-voltage-network-solutions/>.