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**Trajectory Optimization For Space
Debris Removal In The
Sun-Synchronous Orbit**

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Abstract

The problem proposed in the ninth edition of the Global Trajectory Optimization Contest (GTOC9) asked for the design of an optimal set of missions able to remove 123 orbiting space debris pieces from Sun Synchronous orbits minimizing the cost. Two different possible solutions have been proposed, depending on the propulsion system which drives the spacecraft, chemical or electrical engines. The aim was to compare both solutions in order to find out which option is more suitable.

An heuristic optimization, performed by a genetic algorithm, was presented to find a preliminary solution for this complex problem. This algorithm is developed to provide a rendez-vous sequence of debris minimizing the velocity increment, which is directly related to the propellant mass consumed, required to carry out the transfers between debris pieces, computed by a Lambert's Problem solver.

In the case of the spacecraft with chemical engine, a set of 10 missions, having its longest sequence 15 debris, was found as a solution. For the electric propulsion one, just 4 missions were needed to de-orbit the 123 debris pieces considered. Hence, the proposed algorithm is proven to be an efficient tool to assess the feasibility and optimality of future mission for de-orbiting debris.

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Nomenclature

Abbreviations

*MJD*₂₀₀₀ Modified Julian Date 2000

ADR Active Debris Removal

GTOC9 Global Trajectory Optimization Contest 9

IGO Inter-Governmental Organization

ISS International Space Station

JPL Jet Propulsion Laboratory

LEO Low Earth Orbit

MILP Mixed-Integer Linear Problem

MINLP Mixed-Integer Non-Linear Problem

NASA National Aeronautics and Space Agency

NEA Near Earth Asteroid

NLP Non-Linear Programming

NSGA Non-dominated Sorting Genetic Algorithm

NUDT National University of Defense Technology

OCP Optimal Control Problem

OST Outer Space Treaty

RAAN Right Ascension of Ascending Node

RADAR Radio Direction and Ranging

RCS Radar Cross-Section

SSO Sun Synchronous Orbit

TOF Time of Flight

TSP Travelling Salesmen Problem

Symbols

x_{child} Gen of child

x_{parent} Gen of parent

α Coefficient of cost function

ΔV Velocity Increment

$\dot{\Omega}$ Precession rate of Right Ascension of Ascending Node

$\dot{\omega}$ Precession rate of Argument of Perigee

$\frac{T}{m}$ Thrust to mass ratio

Z Integer Number

μ Standard gravitational constant

Ω Right Ascension of the Ascending Node

\Re Real Number

\mathbf{p} Vector of Static Parameters

\mathbf{u} Control Vector

\mathbf{y} State Vector

θ True Anomaly

a	Semimajor Axis
d	Index of debris
E	Eccentric Anomaly
e	Eccentricity
i	Inclination
I_{sp}	Specific Impulse
J	Cost Function
J_2	Coefficient of Earth Oblateness perturbation
M	Mean Anomaly
m_{dry}	Dry Mass
m_{in}	Initial Mass
n	Mean Motion
r	Position of debris
r_{eq}	Radii of the Earth
$T_{de-orbit}$	Time of de-orbiting operations
v	Velocity of debris
w	Argument of Perigee
z	Optimizing Variables of NLP
t	time

Chapter 1

Introduction

1.1 Motivation

In the past decades, technological advances and customer needs have resulted in an increase in space assets. However, the orbits are getting collapsed in the recent years due to inoperative satellites and other space debris. The increase of space debris is currently considered as one of the main tasks for future sustainability of both space activities and access. The risk of debris collisions with operating satellites could impede future space missions.[1][2]

From 1960's, approximately 5000 launches have placed around 6000 satellites into orbit. Just a little percentage is currently operational. In other words, around the 85 % of space objects are uncontrolled debris[3]. In total, there are more than 22000 objects larger than 10 cm and 500000 pieces of debris larger than 1 cm travelling at speeds up to $17,5 \frac{mille}{h}$, which is enough to damage an operative satellite or even a spacecraft.[4]

Space debris mitigation guidelines have been adopted at international level. For example, the guidelines imposed by NASA, known as flight rules, specify when the expected proximity of a piece of debris increases the probability of a collision enough that evasive action or other precautions to ensure the safety of the crew are needed [4]. But these indications might not be enough to solve this problem. This is why the development of debris removal missions is a real need nowadays.

These missions also might be necessary to clean up certain target space regions where debris collisions are more hazardous for commercial or human missions. As an illustration, the most populated region of space is located at low orbits, close to the poles of the Earth. It has been calculated that, at least between 5 and 10 debris items should be removed each year to

assure that space flight on current conditions will remain possible in the future. For example, to understand the risk represented by Space debris, it is enough to think that the ISS has to perform from time to time collision avoidance manoeuvres and more than 80 windows of the Space Shuttle has been replaced along its lifetime [3]. An exaggerated illustration of the space debris situation is shown in figure 1.1.

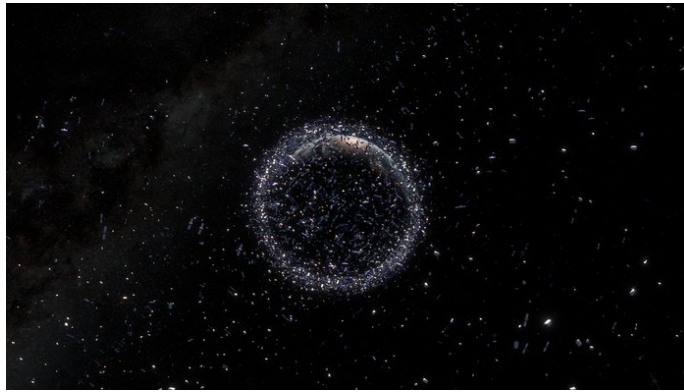


Figure 1.1: Space Debris
[7]

When designing these future missions to de-orbit space debris, trajectory optimization plays a very important role as it can save a significant amount of propellant, which is one of the major costs related to space missions due to two main reasons: the price of the own fuel and the increase in initial mass of the spacecraft, which significantly raises the cost of the launch.

Early spacecraft used chemical rocket engines to travel between orbits. They provide high thrust, which is usually approximated as impulsive (high ΔV) because they are able to produce large thrust for a relatively short period of time. Nevertheless, in the last years, aeronautical engineering developments have allowed to use electric engines for spacecrafts. Taking advantage of plasma properties, they provide continuous low-thrust along the transfer, which is used to follow spiral shape trajectories. Due to this fact, it is much more complicated to optimize the transfers for low-thrust engines.

[5]Advanced numerical methods for trajectory optimization have been developed parallel to the improvements in space exploration and digital computers. Trajectory optimization is a very complex task as a significant number of combinations must be handled until reaching the optimal solution. Initially, this kind of problems were resolved by application of optimal control techniques. That is the reason why optimization problems are typically

reformulated as Optimal Control Problems, whose resolution initially was only efficient for simple problems. Nowadays, due to the developments mentioned before, much more complicated problems can be resolved. Nevertheless, there is a long way to go in this field, as the optimization techniques can keep on improving.

This dissertation is focused on the preliminary design of an optimum set of missions with the purpose of removing some pieces of debris from Sun-Synchronous orbits. Such a complex problem was proposed such an international competition, creating an scenario of 123 debris. Through an Heuristic genetic algorithm, the most suitable sequence of debris is going to be optimally chosen, which will result in a reduction of the velocity increment needed to change the orbit. In other words, it will allow the lowest possible fuel consumption and thus, the cost of the mission will be reduced.

1.2 State of the Art

In this section, a review of the state of art of the two main fields related to this project will be provided : *De-orbiting Techniques* and *Tools for trajectory design*.

1.2.1 De-Orbiting techniques

Dealing with Space Debris removal, a lot of innovations are happening in the recent years due to the increase in the population of debris at low orbits. The tendency is that unless a solution is found, the population will keep increasing even without sending new objects to the space due to Kessler Effect. It consists on the theory of a NASA scientist, Donald J.Kessler, which states a self-cascading collision of Space Debris. These collisions between two debris would generate more debris that then collides with other objects, raising a lot the possibility of a new satellite to be bombarded. [6] Appart from the new policies of prevention for new launches, Active debris removal (ADR) is necessary to stabilise the growth of the debris population. ADR consists on sending missions to remove previously specified pieces of debris. It can be more efficient in terms of collision prevention if the following principles are applied to select the pieces of debris[7]:

- They should have high mass, as they are potentially more dangerous.
- They should have high collision probabilities, being in a densely populated region.

- They should be in high altitudes, where the orbital lifetime of the possible fragments is longer.

To estimate these facts, and to have an idea of future environment, several modelling programs were developed from 70s, which were also able to predict future fragmentations. NASA's last improvements allowed to implement curve fit methods, and was able to predict the position in terms of orbital elements(semimajor axis, excentricity, etc) and also the size of the piece of debris. [3]

A complete ADR operation includes the guidance to the piece of debris, the rendezvous and the capture and removal (de-orbit) activities. As these items are not operating at the moment of the removal, it is quite complicated to determine its position, as they are not emitting telemetry data anymore and the use of RADARs is required to determine the position from the ground. Approaching to the target object is a really complicated operation in which the avoidance of collisions must be assured. In the final part of the approach, relative navigation methods are used together with a camera able to determine the exact altitude. At the time of de-orbiting the target, several methods can be used. They can be classified in Controlled and Non-Controlled re-entry to the atmosphere. [8] [9]

[8][9]Controlled entry methods consist on sending powerful thrusters that catch the target by means of a robotic arm,space tethers or even using a net. Once they are attached, the thrusters de-orbit the target after a set of maneuvers and entry to the atmosphere together. Also, space tethers can be used as a harpoon that catches the debris item and stores it in the collection module.

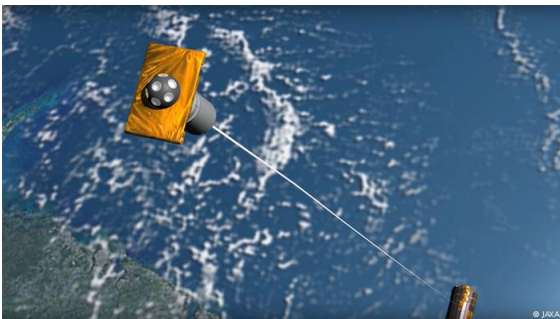


Figure 1.2: Space Tether
[10]



Figure 1.3: Robotic Arm
[10]

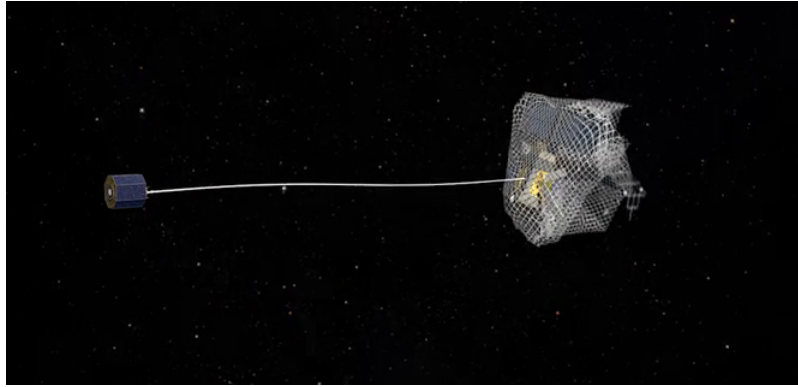


Figure 1.4: Net
[10]

[8][9] Non-Controlled entry methods force an 'uncontrolled' de-orbiting of the target. The most developed ways to do it are the following:

- Solid Propulsion De-Orbitation Kit. Solid rocket motors are attached to the surface of the target to push it to lower orbits in order to force a natural re-entry due to atmospheric drag effect.
- Drag augmentation sail. It consists on placing a sail on the surface of the target. It increases the size of the debris, which causes an increase in atmospheric drag that accelerates the orbital descent.
- Irradiation with ion engine. A plasma engine is encharged of irradiate the target with an electron beam. This constant bombardment of ions causes a momentum transfer to the debris item which accelerates the orbital descent. This method does not require a direct contact with the target.
- Expanding Foams. This method is based on the same principle that the Sail one. The foam is stuck onto the target, and it expands isotropically due to space conditions (vacuum and microgravity) resulting in a spherical form that covers the target debris, increasing the area-to-mass ratio and thus, increasing the atmospheric drag, decelerating the debris until it re-enters into the atmosphere. [FOAM-Space Debris Removal].

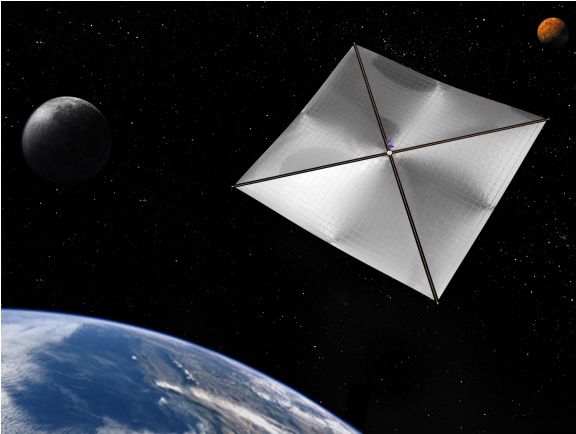


Figure 1.5: Drag Augmentation Sail
[10]



Figure 1.6: Irradiation with ion engine
[10]



Figure 1.7: Expanding Foams
[1]

1.2.2 Trajectory design tools

Focusing now in the design of multi-rendezvous spacecraft trajectories, it is very remarkable the challenge that has supposed for engineers due to the multiple factors which must be optimized. They include the decisions of the items to be reached, the sequence followed to visit all of them and the design of the trajectory shape to go from one to the other.[11]

A proper optimization of such a mission demands the visit of the maximum population possible, reducing as much as possible both the time required to do it and the propellant consumed. Basically, this is equivalent to reduce as much as possible the ΔV needed to carry

out the transfer. Typically, the optimum sequence is obtained by an Heuristic optimization algorithm. This solution will be used later on as a first approach for a Deterministic optimization algorithm that will obtain a finer solution. Currently, Heuristic algorithms are highly developed for these trajectory optimization problems. There are several types, and a couple of examples will be given.

Students from Kyoto University [12] solved a multi-objective space debris removal problem based on a TSP problem (Travelling Salesmen Problem). It consists on the optimization of the order of the cities to be visited in order to minimize the total path distance travelled. In the case of the debris removal problem, two objectives wanted to be optimized, the Radar Cross-section (RCS) and the ΔV . As it is preferred to remove the biggest items, it is required to maximize the RCS of the debris and as usual, minimize the velocity increment needed to perform the operation. For this purpose, a multi-objective Evolutionary Algorithm combined with Lambert's Problem was used to find the most suitable solution. Analyzing the solutions obtained, they observed a tradeoff between the total RCS and ΔV . If trajectories achieved the maximum RCS, the minimum velocity increment was never achieved.

Another option to solve this kind of problems is the Series Method algorithm. At the beginning, it was developed to design interplanetary science mission to tour near-Earth asteroids (NEAs), getting successful results. By this method, the sequence of the debris visited is constructed in series, choosing each following item based on the minimum ΔV to reach it. It is similar to a computer chess program, which only cares about one move ahead when choosing the next move. If needed, it can be specified the first debris to be visited. Anyway, the algorithm will choose the first target by trying each one as the first when making the itineraries, choosing then the first target that maximizes the total number of debris pieces visited. This method does not require too much computationally expense. [13]

1.3 Socio-economic Aspect and Regulatory Framework

The importance of a proper conservation of outer space environment has gained international recognition from a technical, legal and policy perspective. As the implementation of active debris removal measures is very influenced by the applicable regulations, it is very important that other legal, regulatory, financial, safety and strategic challenges are also addressed. Nevertheless, it already exists a legal framework for outer space activities.

[14]It consists of five international treaties: the 1967 Outer Space Treaty (OST), the 1968 Rescue Agreement, the 1972 Liability Convention, the 1975 Registration Convention and the

1979 Moon Agreement. They are the base for the base of national space legislation of more than 20 countries, and a tendency to adopt set of measures at international level that comes from concepts stated in these earlier treaties.

The most important one is OST, as it was the base for the following Treaties and contains the basic principles for space activities. It is considered to contain principles of international law, which designates that outer space and celestial bodies as a global common asset. Its use and exploration is considered as a right for all humankind. This means the use of the outer space must be free for all the states, does not matter their economic or technological situation. There is no need to ask for permission from any entity. Accessibility in terms of carrying out space activities should be preserved in both short and long term perspective, as the dependency on space is expected to grow in the future. That is the reason why it is very important to assure the sustainability of the activities and the proper preservation of outer space. Some limitations appear in OST that states that the operations carried out must be conducted with due regard to the interests of other states that are carrying out operations too, avoiding the 'harmful contamination'. It consists on avoiding the operations that may cause harmful interferences with other ones of other states. [14]

These limitations to protect the environment come from the common interest to access outer space, but they don not cover regulations for Active Debris Removal.

There are a lot of deficiencies related to ADR, but the most important ones are the following: [14]

- The legal framework does not clarify whether an object is considered space debris or not. It could be questionable which criteria to define an object as space debris should be applicable.
- The jurisdiction and control over the space objects shall be retained by the state that launched the object. Due to this, ADR can only be used to remove an item under by the owner state or under its permission. This may lead to situations in which the way to proceed is not clear, for example:
 - If the state owner does not perform the removal neither allows a third party to do it.
 - If the owner is unknown because the space object has not been registered.
- The liability regime is not completed for ADR operations. According to Liability Convention, in outer space, the liability is fault-based. Therefore, if a private party carries out an ADR operation and damages an operative object of a third entity, the liability is attributed to the launching state of the removed object and not to the ADR

conductor. This means that costs incurred should be carried by the launching state. It is questionable if the standard for fault liability should be the same as for carrying out an ADR operation.

- There is no way to change or transfer the ownership of space objects according to the existing treaties. This means that once a state has launched a space object, even if it has been thereafter sold to another third, the original launching entity remains liable for all the possible damages caused by such space object, although at the moment of the accident it may not have any control over the object.

Only a few transfers have taken place along the history, but taking into account the vast development of space activities, the legal issues related to the transfer of ownership will gain more importance.

There is a need to complete the regulatory framework about ADR operations, as it has been explained, but what already exists is a set of essential prerequisites to conduct an ADR operation: [15]

- A cost effective technique must be developed.
- Proper policies to protect the parties involved.
- Available and willing target for removal.
- Enough inversion to finance the operation.
- Capability to perform the operation assuring the safety of the public on the ground, at sea and travelling by air.

Taking into account the deficiencies of the current regulations of space and the needings to perform ADR operations, it is quite clear that a proper regulatory framework should start by covering at least the following aspects:

- Creation of an inter-governmental organization (IGO) to foster the development of the technology of ADR and to perform the operations on a commercial basis. The international agreement, base of such an organization, should contain a clear definition of space debris and a provision under which the states that have space objects registered authorise the removal of such items.
- Commercial basis for space debris removal operations for each government. This way, each country could create a tax related to space debris generation on final users.
- Include an assured removal clause that requires the operator to take out an insurance policy to cover costs of removal in case of accident or malfunctioning [15]

1.4 Goals

The main goal of this dissertation focuses on the selection of a sequence of debris for removal in Sun-Synchronous Orbits (SSO) region reducing as much as possible the fuel consumption, or even using low-thrust engines. As mentioned before, the scenario is based on the one proposed for the GTOC9 competition. The approach will consist on using a novel shape-based technique to model the trajectory which will consist on a multi-objective optimization at the beginning and single-objective which focuses only in the fuel consumption. Results are expected to provide a preliminary assessment for future space debris removal missions. Seeking this major goal, some intermediate objectives must be achieved:

- Exhaustive revision of basic orbital dynamics, including shape-based methods for the representation of the trajectory.
- Literature review on Optimal Control Theory and Optimal Control Methods regarding both low-thrust and chemical propulsion. In particular, genetic algorithms and direct collocation techniques.
- Characterization of the debris population to remove.
- Solve the proposed Debris removal problem using an heuristic algorithm.
 - Elaborate the necessary routines for the optimization algorithm.
 - Perform simulations obtaining trade-off between travelled time and propellant mass consumed.
- Comparison of the results with existing ones from other GTOC9 participants.

1.5 Outline of the document

This dissertation paper is organized as follows:

In Chapter 2, different optimization techniques are explained. A review of Optimal Control Theory is done, together with a comparison between Deterministic and Heuristic optimization methods. A deeply explanation of genetic algorithms is provided.

In Chapter 3, an study of the debris population based on the model proposed by GTOC9 is performed. The steps to follow when designing the removal missions are detailed, together

with the explanation of the differences between chemical and electric propulsion engines performances.

In Chapter 4, the different algorithms used to solve the problem are proposed. An explanation of the differences between the two algorithms implemented will be provided in order to help to understand why the changes are done and what is the final objective. Results obtained for each algorithm will be delivered in Chapter 5.

In Chapter 6 the conclusions obtained from the results are explained, focusing on the main aspects that can be improved in order to reach better solutions in future studies.

Chapter 2

Spacecraft Trajectory Optimization Techniques

In this section, a variety of methods to solve this kind of problems will be presented based on the surveys wrote by T.Betts [5] and Anil V.Rao[16].

2.1 Optimal Control Problem

The deveopment of numerical methods to solve trajectory optimization problems has been parallel to improvements in space exploration. These problems are based on Optimal Control Problem (OCP).

Optimal Control is a subject focused on determining the inputs to a dynamical system that optimizes, minimizing or maximizing, a certain objective function. The dynamics of the system are defined by a set of ordinary differential equations written in explicit form:

$$\dot{\mathbf{y}} = f[\mathbf{y}(t), \mathbf{u}(t), \mathbf{p}, t] \quad (2.1)$$

where \mathbf{y} is the state vector, \mathbf{u} is the control variables vector, \mathbf{p} is the vector of static parameters and t is the time.

The initial and terminal conditions are given by

$$\psi_{0l} \leq \psi[\mathbf{y}(t_0), u(t_0), p, t_0] \leq \psi_{0u} \quad (2.2)$$

$$\psi_{fl} \leq \psi[\mathbf{y}(t_f), u(t_f), p, t_f] \leq \psi_{fu} \quad (2.3)$$

where t_0 and t_f are the initial and terminal time, respectively.

In addition, the solution must satisfy algebraic path constraints of the form:

$$g_l \leq g[y(t), u(t), p, t] \leq g_u \quad (2.4)$$

as well as simple bounds for state vector and control variables.

$$y_l \leq y(t) \leq y_u \quad (2.5)$$

$$u_l \leq u(t) \leq u_u \quad (2.6)$$

These parameters, state and control variables can be either continuous or discrete. If all of them are continuous ($y(t) \in \mathbb{R}^m$ and $u(t) \in \mathbb{R}^n$), the problem dealt is a classic OCP. On the other hand, if any of the variables is discrete ($y(t) \in Z^m$ or $u(t) \in Z^n$), the problem becomes an hybrid-OCP, which is the type of problem faced in this project.

Once the OCP is defined, a method must be applied to minimize a cost function J , which is the objective of the problem.

$$\mathbf{J} = \phi[\mathbf{y}(t_0), t_0, \mathbf{y}(t_f), t_f; \mathbf{p}] + \int_{t_0}^{t_f} L[\mathbf{y}(t), \mathbf{u}(t), t; \mathbf{p}] dt \quad (2.7)$$

The numerical approaches typically used to solve Optimal Control Problems use to fall into two main groups: Direct and Indirect Methods.

Just a general overview about Indirect Methods will be provided, as this kind of methodology is out of the scope of the main work of this dissertation.

In an Indirect Method, the *calculus of variations* is used to determine the first order optimality conditions of the OCP . It is focused on determining functions that optimize a function of a function. This action is also known as *functional optimization*. The conditions obtained are derived from a Hamiltonian System.

The main advantages of Indirect Methods are the great accuracy of the solution obtained, which satisfies the first-order optimality conditions. On the other hand, there exist several disadvantages too, as the small radii of convergence and the need of a initial guess for the costate.

2.1.1 Direct Methods

Direct methods are fundamentally different to Indirect Methods. They adjust directly the state and control variables by parametrizing them, in order to transform the infinite Optimal Control Problem into a finite dimensional non-linear optimization problem. It is very

convenient as this kind of problems can be solved by Non-Linear Programming techniques, also known as NLP. [17].

Also, there exist some special cases of hybrid-OCP which can be reformulated as Mixed-Integer Non-linear Optimization problems by using direct methods.

The most widely used direct methods are Single Shooting, Multiple Shooting and Collocation.

- Direct Single Shooting. It is the most basic one and it consists on a control parametrization method with a relatively small number of NLP variables. Most successful applications of this method are for launch and orbit transfers.
- Direct Multiple Shooting. The trajectory is divided in time steps, and Single Shooting method is applied in each of the intervals. This method is an improvement because errors due to the unknowns initial conditions are reduced, as the integration is performed in much smaller time intervals.
- Direct Collocation Method. It is considered as one of the most powerful and suitable methods to solve optimal control problems. It consists on a state and control parametrization where both variables are approximated by a specified functional form. The most common collocation forms are *local and global collocation*. The first one is carried out in a similar way than methods that divide the time span in several sub-intervals, which must be continuous. They used to lead to a large sparse. In the case of *global collocation methods*, the number of segments is fixed. What varies is the degree of the polynomial.

2.2 Mixed-Integer Non Linear Optimization

As mentioned before, it is possible to transform optimal control problems into non-linear optimization or non-linear programming problems. That is why it is very useful to dominate the resolution of this kind of problems.

Non-Linear Programming looks forward to minimize the cost function by calculating increments in the vector of control variables, while observing all constraints. In order to make it possible, at least first gradient information must be available for the cost function.

This chapter is focused in Mixed-Integer Non Linear problems (MINLP), as they are the problems which are dealt in this project. A MINLP problem has the following form:

Determine the vector of decision variables z that minimize the cost function.

$$\text{Minimize} \quad f(z) \tag{2.8}$$

$$\text{subject to} \quad g(z) = 0 \tag{2.9}$$

$$h(z) \leq 0 \tag{2.10}$$

where $g(z) \in \mathfrak{R}^m$ refers to the equality constraints and $h(z) \in \mathfrak{R}^p$ refers to the inequality ones. The z variables can be either real ($z \in \mathfrak{R}^n$) or integer ($z \in Z^n$)

Numerical methods to solve NLP's are divided into two categories: *Deterministic*, also known as *Gradient-Based methods*, and *Heuristic methods*. Heuristic methods will be more deeply discussed, as the work carried out to solve the problem was performed by these methods.

2.2.1 Deterministic Methods

A general review of the Deterministic Methods whose use is more extended is delivered in this section, based on the paper by Pietro Belloti. [18]

In general, the integrality constraints are solved using some kind of tree-search strategy. Deterministic methods fall into two categories: *single-tree* and *multitree methods*. To perform such methods, it is necessary to take into account the following assumptions:

1. The set X is a bounded polyhedral set.
2. The constraint functions are twice continuously differentiable convex functions.

The methods are the following:

- Multitree Methods. They consist on decomposing the MINLP into an alternating sequence of NLP subproblems and MINLP relaxations. The three main methods are the following:
 - *Outer approximation*. Used to create a *Mixed-Integer Linear Problem* (MILP).
 - *Generalized Benders decomposition*. Developed before outer approximation, it has the advantage that these methods involve only the integer variables and one objective variable.
 - *Extended cutting-plane method*. They are a variation of the outer approximation that linearizes all functions at the solution of the MILP master problem.

- Single-Tree Methods. They consist on hybrid approaches that use outer approximation properties but require only a single MILP tree to be searched. They have a double advantage:
 - The needing of resolving related MILP master problems is avoided.
 - A tree whose nodes can be warm-started by reusing basis data from parent nodes is searched.
- Branch-and-Bound. The basic concepts of this methods underpin both classes of methods mentioned and it constitutes a good example of a single-tree method. The algorithm starts by solving the NLP relaxation, the root node, defined only by a set of bounds on the integer variables. Branching consists on dividing the feasible region into subsets such that every solution to MINLP is feasible in one of the subsets. Then, it is necessary to prun the subproblems that do not fullfill the optimality and feasibility conditions.

2.2.2 Heuristic Methods

This kind of methods differ fundamentally from a gradient-based method. They are global techinques, which means that the solution found will be globally optimal. These methods are very useful as they are able to work with discrete variables. Problems that deal with this kind of variables are able to determine wether a candidate solution is actually a solution only by comparison with other candidates.

There exist several Heuristic Methods, such us *Genetic Algorithms*, *Simulated Annealing*, *Tabu Search* or *Montecarlo Methods*, whose basic notion is to choose random values for the unknown problem variables. Another particularity of this kind of methods is that they allow the resolution of multi-objective problems. Generic Algoritmhs are the ones that will be deeply discussed, as they are used to solve the problem of this project, more concretely a Non-Dominated Sorting Genetic Algorithm.

A genetic algorithm is an evolutionary approach that emulates on a computer evolutionary processes based on genetics. As mentioned before, the algorithm chooses an initial population of possible solutions, each of them having a particular fitness that reflects its quality.

Genetics algorithms basically consist on five steps: encoding, fitness, selection, crossover and mutation.

- Encoding Mechanism. It provides a way to represent the set of variables to optimize

in the problem. For example, it must be determined whether the variables randomly chosen represent integer or real values.

- Fitness Function. It provides a mechanism to evaluate the quality of each population of possible solutions according to the objective function to be optimized.
- Selection Mechanism. At this step, the more fit solutions are chosen to survive to the next generation. The selection is carried out by tourney.
- Crossover mechanism. It combines the survival populations. It results in a child string that has a different fitness (not necessary higher) from either of its parents. They combine in this way:

$$\begin{aligned}x_{child1} &= x_{parent1} + \frac{1}{2}(x_{parent1} - x_{parent2}) \\x_{child2} &= x_{parent2} + \frac{1}{2}(x_{parent1} - x_{parent2})\end{aligned}$$

where x_{child} corresponds to the gen of the child and x_{parent} corresponds to the gen of the parent.

- Mutation Mechanism. At this stage, the childs are mutated according to a previously selected factor. Then, the populations resultant are evaluated again and the process is repeated. An scheme explaining the process followed by genetic algorithms is shown in figure 2.1.

To conclude this chapter, it is remarkable to say that unfortunately, as this kind of algorithms do not exploit gradient information, they are not computationally competitive with deterministic methods. Nevertheless, the solution obtained by a heuristic method can be used as a really accurate initial guess for gradient-based methods.

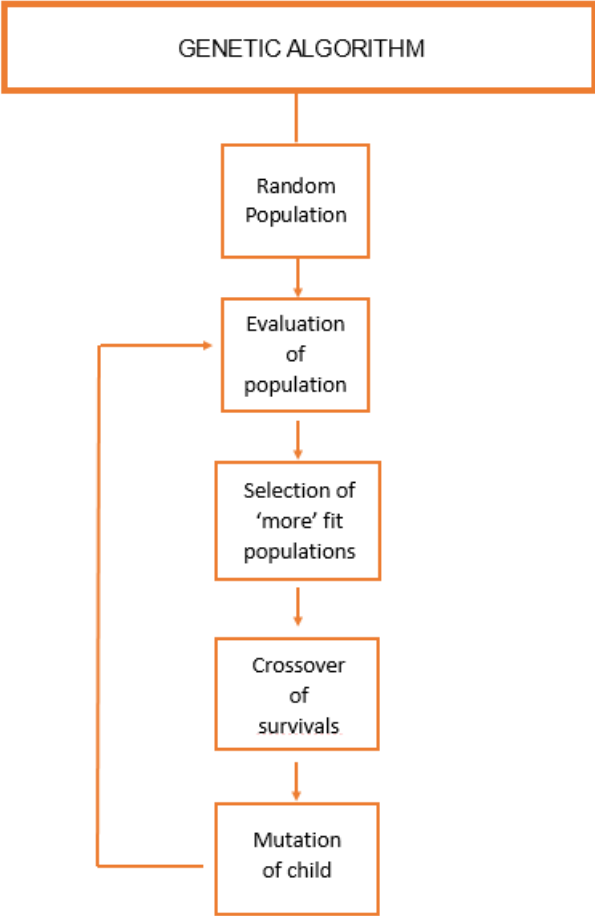


Figure 2.1: Genetic Algorithm scheme

Chapter 3

Modelling

In this chapter, the scenario proposed in GTOC9 will be studied. The objective is to properly know the behavior of the space debris items to be removed, and their exact position at any time. In order to assure a better understanding, a review of Orbital Elements nomenclature will be given.

Also, the missions proposed to solve the problem will be detailed, indicating the transfer strategy that will be followed by the spacecrafts and the requirements of each launching, related to the moment when it has to be carried out and the mass restrictions . For the same mass restrictions, the differences that would cause the use of chemical or electric propulsion engines will be explained.

3.1 Population of debris. GTOC9

GTOC9 problem consists on the removal of 123 pieces of debris from the Sun-synchronous LEO's. These orbits are characterized by allowing a satellite to pass over a point of the Earth's surface at the same local solar time. The position of such debris were provided in terms of *Orbital Elements* for a given reference time. These orbital elements define the orbit and the position of the debris piece on it.

[19]To define an orbit in the plane requires two parameters, *semimajor axis* \mathbf{a} and *eccentricity* \mathbf{e} . Describing the orientation of the orbit in three dimension requires three additional parameters, known as Euler angles. The angle between the x-axis and the vector joining the origin of coordinates with the ascending node (point where the orbit crosses the equatorial XY plane from below) is the first Euler angle, called *right ascension of the ascending node*

Ω . The second parameter is the *inclination* i , and it is the angle between the normal vector of the plane of the orbit \mathbf{h} and the Z-axis, or what is the same, the angle between the planes of the orbit and XY. The third Euler angle locates the perigee, and it is called *argument of perigee* ω . It is the angle between the vector joining the origin and the ascending node. Lastly, to locate a point on the orbit, the *true anomaly* θ is the parameter used. All these orbital elements are represented in figure 3.1.

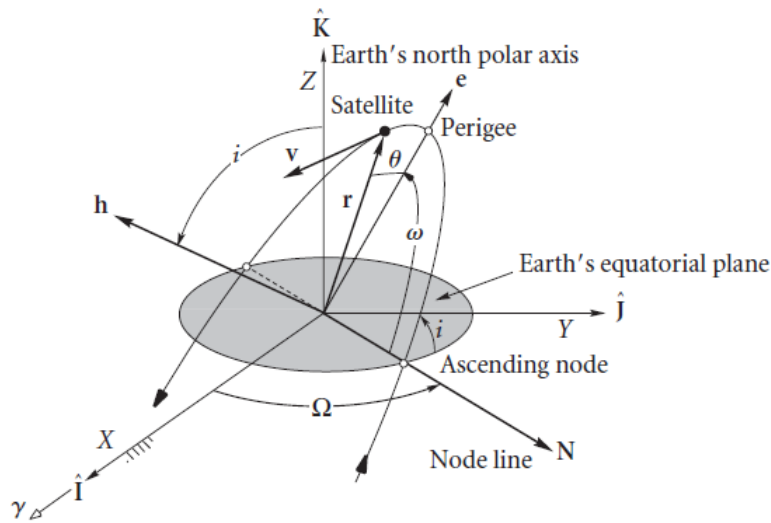


Figure 3.1: Orbital Elements

Figure 3.2 represents how the scenerario looks like, showing the position of the debris at reference time together with the orbits path that they follow in cartesian coordinates. Orbital elements can be transformed into cartesian coordinates easily.

A preliminary study of the population has been performed. This allows to locate the most crowded orbital region, where it is more convenient to remove debris pieces. To do it, semimajor axis, eccentricity and inclination parameters has been classified, as they are the parameters that remain constant.

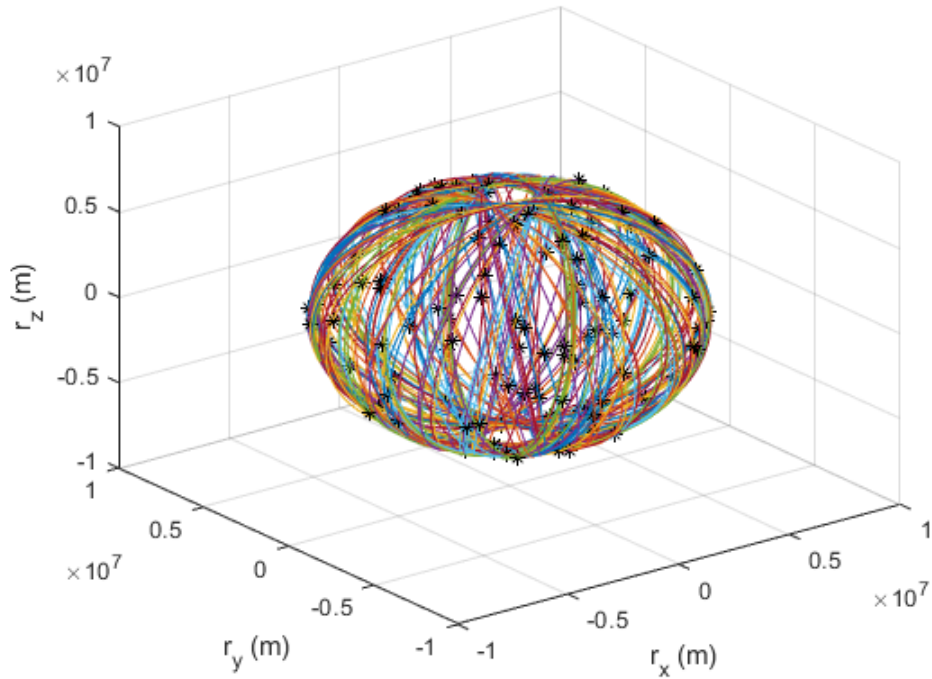


Figure 3.2: Debris Population

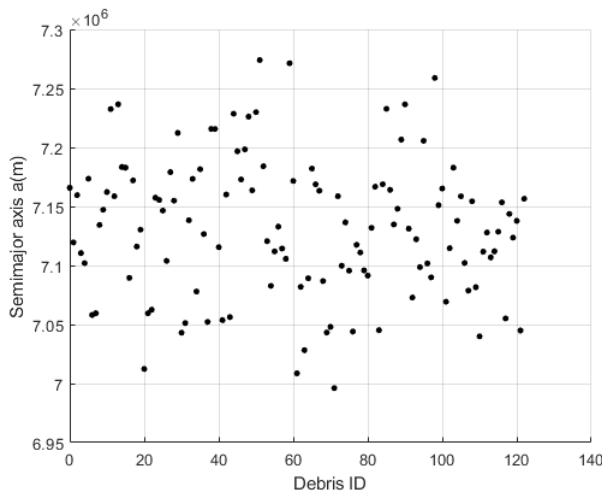


Figure 3.3: Semimajor axis of each debris

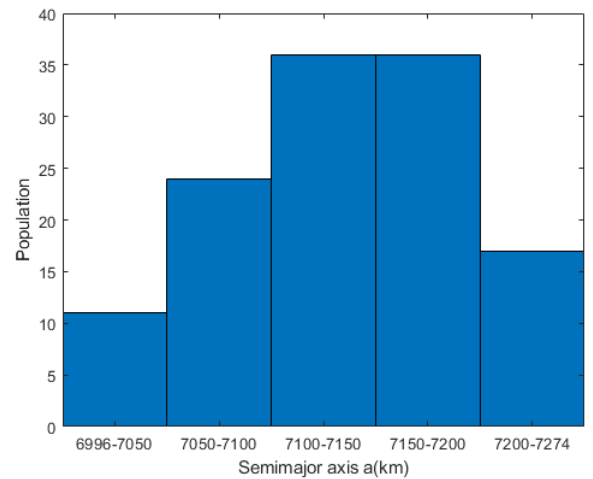


Figure 3.4: Classification of semimajor axis

As it can be observed, figure 3.3 represents the semimajor axis of each of the items. It is shown together with figure 3.4, in which the data obtained is agruped to indicate how many debris pieces are in each region. The same analysis was performed for e and i .

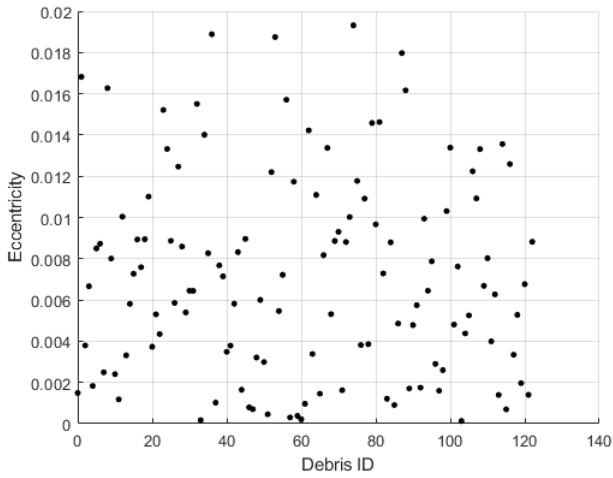


Figure 3.5: Eccentricity of each debris

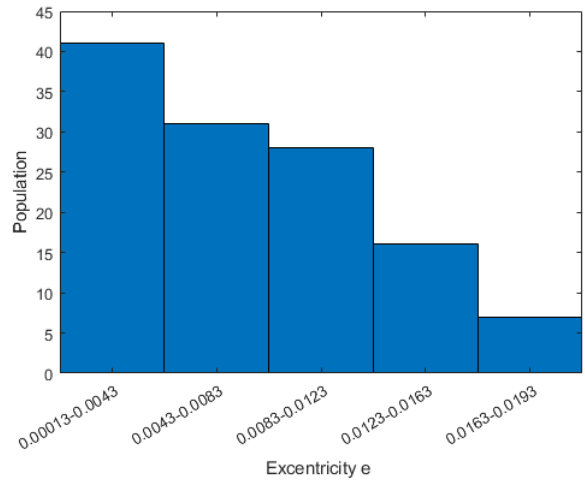


Figure 3.6: Classification of eccentricity

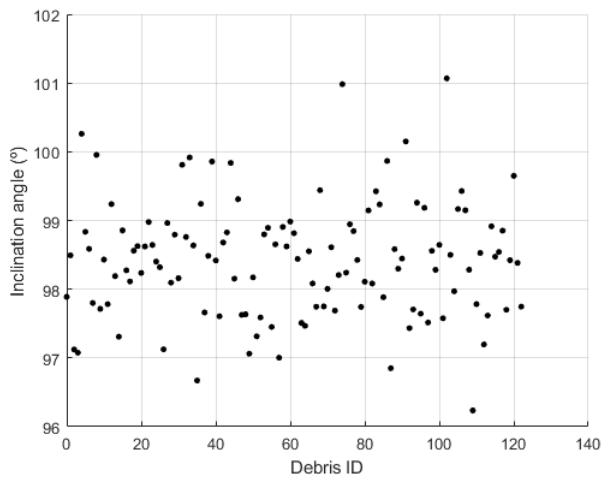


Figure 3.7: Inclination of each debris

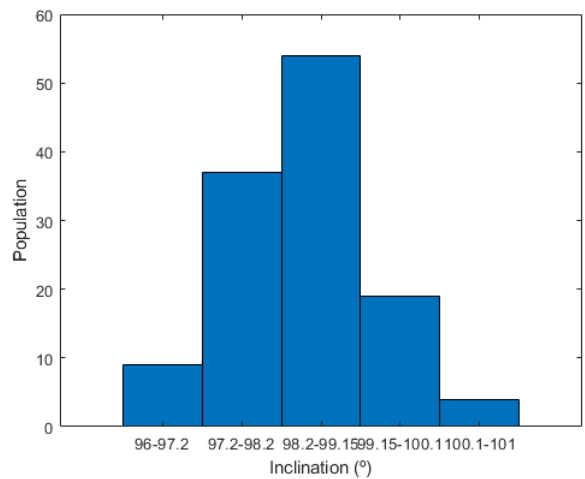


Figure 3.8: Classification of inclination

According to the results obtained from the study, it is possible to conclude that the most populated orbital region is defined by:

- Semimajor axis in [7100-7200] km
- Eccentricity in [0.0013-0.0083]
- Inclination in [97.2-99.15]^o

Space debris items are in constant movement. Dynamics of space debris are affected by perturbations such as drag, solar radiation, the Earth's gravitational potential and other planetary gravitational effects.

It is the Earth's gravitational potential the main influence to the mean motion of the objects, defined by this equation. [20]

$$n = \sqrt{\frac{\mu}{a^3}} \quad (3.1)$$

where, $\mu = 398600.4418 \frac{km^3}{s^2}$, the standard gravitational constant for the Earth.

The other main contribution to the dynamics is the oblateness perturbation (related to harmonics term J_2). It affects to the precession rates of the argument of the perigee and the right ascension of the ascending node. As mentioned before, semimajor axis, eccentricity and inclination remains constant.

$$\dot{\Omega} = -\frac{3}{2}J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i \quad (3.2)$$

$$\dot{w} = \frac{3}{4}J_2 \left(\frac{r_{eq}}{p}\right)^2 n(5 \cos^2 i - 1) \quad (3.3)$$

where $p = a(1 - e^2)$ and $r_{eq} = 6378137$ m, being the semilatus rectus and the radii of the Earth, respectively.

Taking into account the dynamics, the ephemerides of the debris can be described with the following equations:

$$\Omega - \Omega_0 = \dot{\Omega}(t - t_0) \quad (3.4)$$

$$w - w_0 = \dot{w}(t - t_0) \quad (3.5)$$

$$M - M_0 = n(t - t_0) \quad (3.6)$$

It is possible to derive the true anomaly θ from the mean anomaly from the following relation with the eccentric anomaly E .

$$\tan \frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{\theta}{2} \quad (3.7)$$

The eccentric anomaly is computed by the Kepler's equation from the mean anomaly:

$$E - e \sin E = M \quad (3.8)$$

3.2 Mission design

3.2.1 Transfer strategy

Missions proposed to solve the problem are defined as a sequence of rendez-vous manoeuvres from one debris to other to perform de-orbiting activities.

At the beginning of each mission, it is considered that the spacecraft is released by the launcher directly for the rendez-vous with the first debris of the sequence. This means that launching operations are not taken into account for the mission design. Once the rendez-vous with the target is performed, a de-orbit operation is performed to the object. The re-entry of the debris object is carried out in a non-controlled manner, but it is not specified which method of removal is used. An orbiting debris is considered as removed if its position and velocity at some epoch coincides with the spacecraft position and velocity vector and if the spacecraft remains in proximity of the debris for five days performing de-orbiting operations. These de-orbiting manoeuvres are considered to not cause any change in the mass of the spacecraft. Once the target is de-orbited, at the end of the five days, the chaser starts performing a manoeuvre towards the next space debris item indicated in the mission debris sequence, and so on. In this project, the phases of the mission are considered to start at the beginning of the removal operations of one debris item and to finish at the rendez-vous instant with the next target. The duration of the powered transfer from one debris piece to other is represented as *Time of Flight TOF*. A scheme of the transfer strategy is shown in figure 3.9.

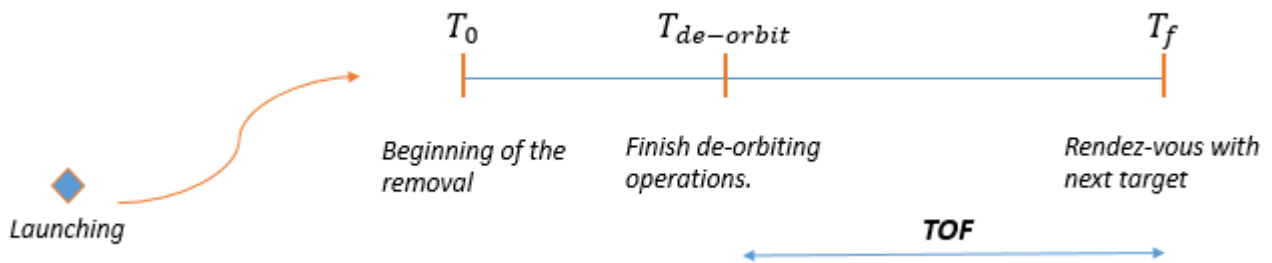


Figure 3.9: Scheme of transfer strategy

The cost of the whole set of n missions is defined by the following cost function:

$$J = \sum_{i=1}^n [\alpha(m_0 - m_{dry})^2] \quad (3.9)$$

where $\alpha = 2.0 * 10^{-6} \frac{M.EUR}{kg^2}$.

As it can be observed, the total cost just depends on the number of launchings and the initial mass, as the dry mass is set to be $m_{dry} = 2000 \text{ kg}$.

First, it is important to remark that the time necessary to carry out each of the missions does not influence the final cost. That is the reason why it is very important to take care of the position of the plane of the orbits. Choosing consecutive debris whose plane orbits are close enough will result in a fundamental decrease of the fuel needed to perform the transfer. As the time of flight does not affect to the cost, it is possible to spend larger amounts of time for the transfers in order to allow the plane orbits to get closer. Significant movement of plane orbits takes several days. It is also important to take into account the position of the debris item into the orbit. The period of the orbits of the space debris considered is quite small, taking only up to 100 minutes a whole revolution. That is why, to choose the most suitable time of flight between two space debris objects, it is convenient to take into account the situation of the plane orbits and the position of the debris piece inside the orbit.

3.2.2 Trajectory definition

To compute the trajectory path followed by the spacecraft from an initial debris item d_i at t_i to another debris d_f at time t_f , a *Lambert's Problem* solver is used. Knowing the time of flight required to travel from one debris to the other, and the position of position of both of them, the solver is expected to find the trajectory. Such trajectory is determined by the position of the debris and its velocity at that position v_t . Once this velocity is known, it is necessary to calculate the increment of velocity. This increment has two components:

$$\begin{aligned} |\Delta V_1| &= |V_{t_i} - V_i| \\ |\Delta V_2| &= |V_f - V_{t_f}| \end{aligned} \Rightarrow |\Delta V| = |\Delta V_1| + |\Delta V_2| \quad (3.10)$$

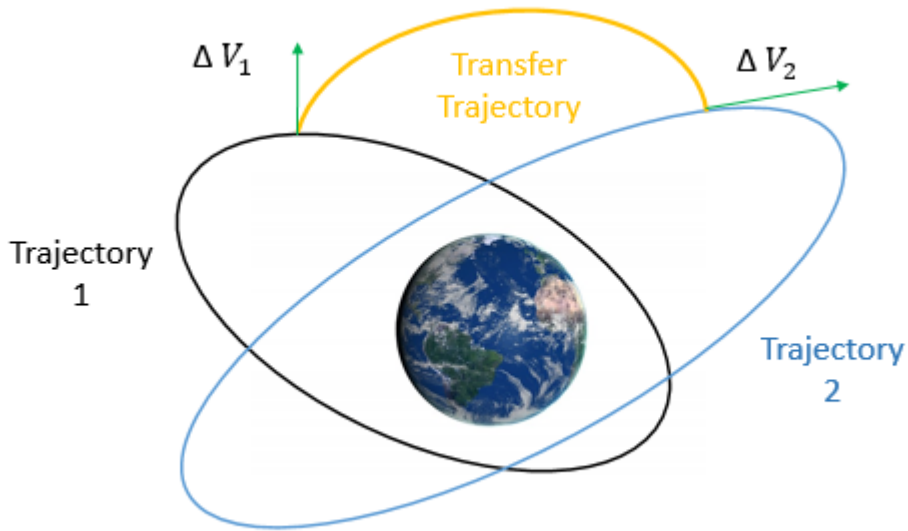


Figure 3.10: Trajectory to travel between to points

Figure 3.10 represents the two components of the velocity increment. As it can be observed, ΔV_1 is the increment needed to leave the initial orbit and start the travel. When the spacecraft arrives to the objective, it needs a second increment ΔV_2 to leave the transfer orbit and remain in the orbit of the objective.

As said before, the time of flight between two debris within the sequence must be wisely chosen, as it is desired that the planes of the debris' orbits, which move due to the previously mentioned Earth oblateness effect, are as close as possible. In figure 3.11, the behavior of the velocity increment ΔV required to travel between two debris as a function of the time of flight is represented.

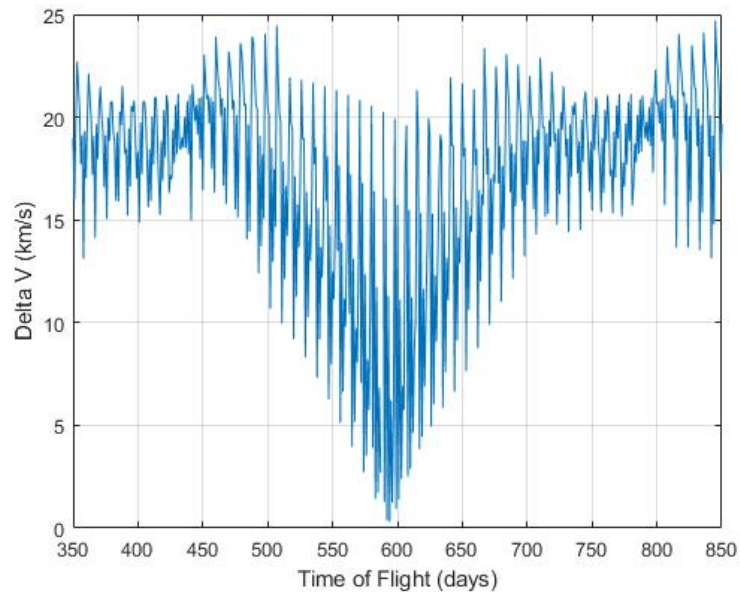


Figure 3.11: Velocity increment vs TOF

As it can be observed, for a time lapse of one year, the velocity increment required to travel between one debris to the next target varies very much. To check that the reason of this variation is the proximity of the planes, the evolution of the RAAN (Ω) variable along the same time lapse must be studied.

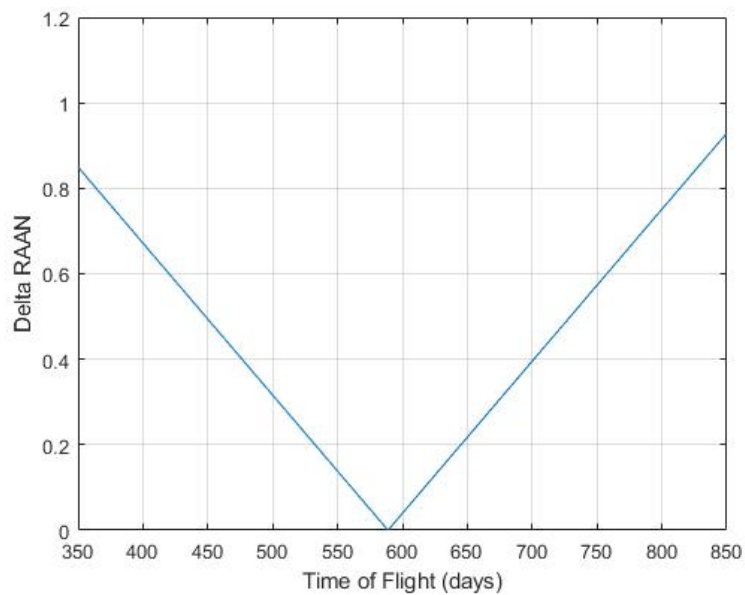


Figure 3.12: RAAN difference vs TOF

Figure 3.12 shows that, as expected, the minimum values of $\Delta\Omega$ are reached at the same time than minimum ΔV values. Right ascension of ascending node is the only variable that affects to the proximity of the planes, as the inclination does not change with time.

The other fact that affects ΔV is the position inside the own orbit of the debris. The oscillations that can be appreciated in figure 3.11 are due to this effect. The time lapse that must be taken into account is reduced to the period of the debris, as previously said. Such velocity increment is represented in the next figure.

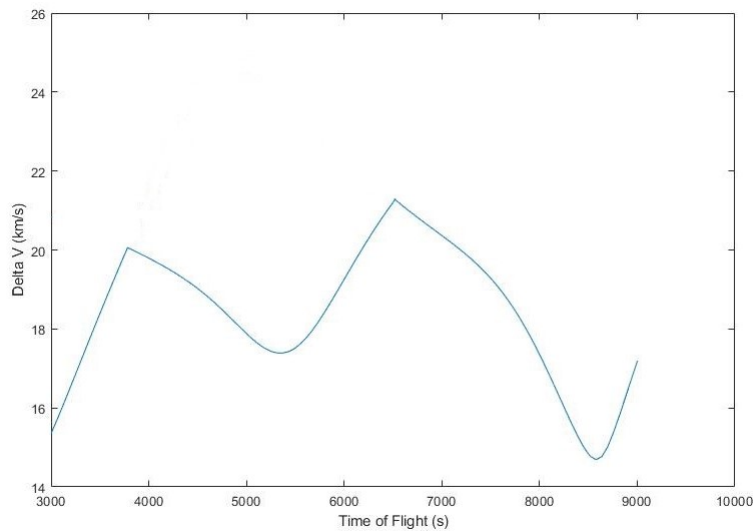


Figure 3.13: Velocity increment vs TOF (period)

The sudden changes in velocity increment comes from the change of direction of the trajectory. The path followed must be in the same direction than the motion caused by gravitational potential, even the shortest path is in the opposite way, as it requires more thrust to travel opposite to the mean motion of the orbit than going over a longer distance. The peaks in ΔV are because of the change from the shorter to the longer path, or vice versa.

The ideal case to perform a transfer with minimum velocity increment requirements occurs when the orbits are in the same plane (inclination and RAAN of both orbits are equal), and one of the items is placed at the perigee of the orbit at the beginning of the transfer and the other in the apogee at the arrival moment. In such case, the transfer to travel from one to the other is known as *Hohmann transfer*.

To choose the best time of flight for the trajectory, first the time that allows the most suitable plane position must be located. Once this time interval is found, it must be chosen the time

for which the position of the items in the orbit requires a lower ΔV .

3.2.3 Engine selection

The maximum propellant mass allowed to carry is $m_p = 5000 \text{ kg}$. This is equivalent to limitation in the maximum velocity increment ΔV allowed. This relation is given by Tsiolkovsky equation:

$$\frac{m_f}{m_i} = \exp\left(-\frac{\Delta V}{I_{sp}g}\right) \quad (3.11)$$

where I_{sp} and g are the specific impulse of the engine and the gravity acceleration, respectively.

The specific impulse is a property which depends on the propulsive system. That is the reason why, at this stage, it is very important to choose properly the type of engine that is going to drive the spacecraft. Chemical propulsion engines used to have lower specific impulse than electric propulsion ones. The reason is that the specific impulse of a chemical rocket engine is limited by the maximum temperature achieved during the reaction, which is subjected to the wall resistance to temperature. In this particular case, the chemical rocket engine has a specific impulse $I_{sp} = 340$ seconds. On the other hand, the low-thrust engine has a specific impulse $I_{sp} = 3000$ seconds, almost ten times greater. According to Tsiolkovsky equation, for the same mass ratio, an electric propulsion engine will allow a total velocity increment much greater than the one allowed by a chemical one. This fact results in an increment of the number of spaces debris items that can be de-orbited in a single mission.

Electric propulsion engines are fundamentally different from chemical propulsion ones. Instead of providing huge instantaneous impulses, as in the case of chemical rocket engines, low-thrust is applied continuously along the whole powered transfer. This fact results in spiral shaped trajectories, which are computed by different methods. Anyway, it is considered that a transfer computed by a Lambert's Problem solver can be performed by a low-thrust powered spacecraft if the ratio $\frac{T}{m} = \frac{\Delta V}{T_{OF}}$ is lower than $10^{-4}[\frac{m}{s^{-2}}]$.

A solution for both type of engines will be computed in this project.

Chapter 4

Solution Approach

In this chapter, the algorithm used to compute the most suitable solution is going to be explained.

The aim of such algorithm is to perform a *multiple mission optimization*. The main task is to divide the debris population into several debris missions, choosing the rendez-vous sequence for each mission which reduces as much as possible the objective function. As time spent in the missions is not important, a single-objective optimization is going to be performed. The only objective to be minimized is the cost function, which depends directly on the velocity increment required to carry out the transfers.

$$J = \Delta V = \sum_{i=1}^n [\Delta V_i]$$

being n the number of manoeuvres of the rendez-vous sequence.

In other words, the sequence chosen for each mission must reduce ΔV as much as possible. An heuristic genetic algorithm, known as *NSGA II*, which is a non-dominating sorting genetic algorithm, is used for such purposes.

This algorithm works with discrete variables. The design variables to be optimized are the starting epoch t_1 of the de-orbiting operations in the first debris, and the debris identification ordered in function of the visiting sequence.

$$\mathbf{x} = [t_1, d_1, d_2, \dots, d_{final}]$$

where t_1 is a real variable, while d are integer variables.

The debris must be visited just once to perform the de-orbiting operations. That is the reason why the algorithm must assure that there are no repeated debris in the sequence. In

order to it, a vector with the real list of debris items is created inside the objective function, and the design variables optimized by the algorithm are actually the indices of such vector. Once one debris is visited, it is deleted from the vector with the list of debris. An example is given in figure 4.1.

Initial Debris List	[1 2 3 4 5]	
Genetic Algorithm Input	[3 2 3 1 1]	
Transfer 1	[1 2 3 4 5]	Position 3
Transfer 2	[1 2 4 5]	Position 2
Transfer 3	[3 4 5]	Position 3
Transfer 4	[3 4]	Position 1
Transfer 5	[5]	Position 1

Figure 4.1: Mechanism to not consider already visited debris

It is very important to define proper limits for the vector of design variables, which constitutes the indices for the vector with the real list of debris, as previously mentioned. The upper limit of each variable is the number of remaining debris which have not been removed yet, and the lower limit is 1.

Once the design variables vector is optimized, it must be post-processed in order to obtain the real sequence of debris.

The design variable vector is used directly as the chromosome of an individual. The fitness assignment process is straightforward and only binary elitist tournament selection is supported to prevent the loss of good solutions. Arithmetic or intermediate crossover is the one supported by the version used. The values defining the genetic algorithm are delivered in table 4.1:

Population size	50
Maximum Generations	150
Mutation Fraction	$\frac{2}{nvar}$
Crossover Fraction	$\frac{2}{nvar}$

Table 4.1: Values defining the genetic algorithm

where *nvar* is the number of variables to be optimized.

The number of debris removed in a mission is found by trial and error. Several trials are made until the minimized velocity increment obtained by the algorithm fullfills the maximum allowable by the engine used, according to equation 3.11.

The objective function of the algorithm to be optimized calculates the transfers between debris which requires the minimum velocity increment for a set of times of flight. As mentioned in the previous chapter, the velocity increment is calculated for a range of times of flight which allow the change of the proximity of the planes of the orbits, but also for a range of time that takes into account the change of the position of the debris item into the orbit.

A Lambert's problem solver takes part on the objective function as orbital targeting algorithm, which allows the computation of the velocity increment. It is very remarkable the fact that this solver does not take into account the Earth oblateness effect in the computations. Such solver needs the following inputs:

- Position of the initial debris d_i at the beginning of the transfer.
- Position of the target debris d_f at the end of the transfer.
- Time of flight of the transfer.
- Direction of the transfer (long or short path)
- Number of complete revolutions of the trajectory.

By these inputs, the solver is able to compute the variables that define the transfer. These outputs are the velocities of the orbits at the positions of d_i and d_f .

The algorithm choses the best sequence of visiting to remove several debris. De-orbiting operations are performed to each debris just once and take 5 days ($T_{de-orbit}$). That is the reason why, the algorithm has been designed in a way that is able to fullfill such requirement, rejecting the debris after visting them, assuring that in the sequence there will not appear any repeated debris.

A scheme of the objective function for chemical propulsion rocket engines is delivered in algorithm 1.

In the case of computing the solution for a low-thrust engine, a different criteria is followed. As it as been already mentioned, the solution obtained will be considered as suitable for a spacecraft with electric propulsion engine if $\left(\frac{T}{m}\right) = \frac{\Delta V}{T_{OF}} < 5 \cdot 10^{-4}$. That condition must be imposed in the objective funtion, as shown in the scheme of algorithm 2.

Algorithm 1 Chemical propulsion

1: **function** OBJECTIVE FUNCTION FOR MISSION OPTIMIZATION(d, t, r, v, TOF) ▷ Where
d - debris, t-time, r-position, v-velocity, TOF-Time of Flight, n-number of maneouvres

2: Choose $t_{initial}$

3: **for** $i = 2$ to number of debris of the sequence **do**

4: Calculate TOF vector

5: **for** $j = 1$ to length of TOF vector **do**

6: Calculate $r_{d,i-1}$ and $v_{d,i-1}$ at $t_{i-1} + T_{de-orbit}$

7: Calculate $r_{d,i}$ and $v_{d,i}$ at $t_{i-1} + TOF_j$

8: Calculate ΔV_j by Lambert's Problem Solver

9: **end for**

10: Find $\Delta V_{min,i} = \min(\Delta V_j)$

11: Find $TOF_{\Delta V_{min,i}}$

12: $t_i = t_{i-1} + T_{de-orbit} + TOF_{\Delta V_{min}}$

13: **end for**

14: $\Delta V_{min} = \sum_{i=1}^n [\Delta V_{min,i}]$

15: **end function**

Algorithm 2 Electrical propulsion

1: **function** OBJECTIVE FUNCTION FOR MISSION OPTIMIZATION(d, t, r, v, TOF) ▷ Where
d - debris, t-time, r-position, v-velocity, TOF-Time of Flight, n-number of maneouvres

2: Choose $t_{initial}$

3: **for** $i = 2$ to number of debris of the sequence **do**

4: Calculate TOF vector

5: **for** $j = 1$ to length of TOF vector **do**

6: Calculate $r_{d,i-1}$ and $v_{d,i-1}$ at $t_{i-1} + T_{de-orbit}$

7: Calculate $r_{d,i}$ and $v_{d,i}$ at $t_{i-1} + TOF_j$

8: Calculate ΔV_j by Lambert's Problem Solver

9: Compute the ratio $\left(\frac{T}{m}\right)_j = \frac{\Delta V_j}{TOF_j}$

10: **end for**

11: Find $\left(\frac{T}{m}\right)_{min,i} = \min\left(\frac{T}{m}_j\right)$

12: Find $TOF_{\frac{T}{m}_{min,i}}$

13: Find $\Delta V_{min,i}\left(\left(\frac{T}{m}\right)_{min,i}\right)$

14: $t_i = t_{i-1} + T_{de-orbit} + TOF_{\Delta V_{min}}$

15: **if** $\left(\frac{T}{m}\right)_{min,i} > 10^{-4}$ **then**

16: Increment TOF and repeat the calculations.

17: **end if**

18: **end for**

19: $\Delta V_{min} = \sum_{i=1}^n [\Delta V_{min,i}\left(\left(\frac{T}{m}\right)_{min,i}\right)]$

20: **end function**

As it can be observed, the algorithm carries out computations until it fullfills the requirements. Then, the ΔV chosen is the one that reduces as much as possible the thrust to mass ratio, promoting as much as possible the use of a low-thrust engine.

The solution of the algorithm is the sequence of debris which minimizes the cost of a single mission. Then, it is necessary to implement it as many times as needed to design enough missions to remove all the 123 debris items. It must be moodified in each implementation, as the debris already removed cannot be considered for the following missions. Once a mission is finished, the following one cannot start until at least 30 days after the ending epoch of the previous mission.

Chapter 5

Results

To determine the maximum number of the debris, the algorithm proposed in this work was implemented several times until the solution reached fulfilled the mass requirements obtained from equation 3.11.

Two different solutions are provided, one for each type of the propulsion engines used.

The vector of the design variables optimized is provided together with the starting and ending epochs for each of the missions. Also, two tables indicating the time of flight and the velocity increment of each manoeuvre of the sequence are provided for each mission.

5.1 Chemical rocket engines

The specific impulse of the used chemical propulsion engine is $I_{sp} = 340$ seconds. According equation 3.11, the maximum velocity increment has been calculated assuming the dry mass of the spacecraft $m_{dry} = 2000$ kg and using the maximum propellant mass allowable. Thus, the initial mass is $m_i = 7000$ kg. The maximum velocity increment obtained is $\Delta V = 4.17 \frac{km}{s}$. As this solution will provide just an initial guess for further calculations, a mission which requires up to $\Delta V = 5.00 \frac{km}{s}$ are accepted as suitable.

According to it, a set of 10 rendez-vous missions has been designed. The rendez-vous sequences obtained are presented in table 5.1:

As it can be observed, the sequences becomes shorter with time. The reason why this happens is that in the first missions, the easier removable debris are visited because of their

Mission	Start Epoch(MJD_{2000})	End Epoch (MJD_{2000})	ΔV	# Debris	Sequence
1	23785.55	24531.47	4.551	15	44, 32, 40, 108, 123, 61, 87, 97, 96, 104, 37, 82, 39, 69, 46
2	24587.16	25522.94	4.497	15	75, 122, 5, 47, 60, 105, 18, 59, 63, 1, 78, 100, 51, 73, 68
3	25569.69	26475.47	4.988	14	27, 41, 64, 26, 79, 36, 2, 43, 17, 30, 22, 102, 113, 65
4	26508.57	27139.18	4.005	13	33, 80, 81, 83, 21, 42, 101, 94, 98, 119, 74, 70, 8
5	27398.72	27809.22	4.765	13	106, 16, 6, 13, 121, 54, 24, 58, 88, 72, 109, 10, 31
6	27839.62	28500.44	4.761	13	28, 34, 93, 55, 115, 25, 35, 12, 56, 99, 77
7	28726.56	29211.58	3.654	12	53, 111, 85, 76, 107, 90, 48, 11, 14, 86, 29, 52
8	29242.35	29998.04	4.446	12	89, 112, 110, 20, 103, 50, 114, 23, 91, 19, 66, 7
9	30092.91	30583.46	3.819	10	49, 57, 62, 9, 15, 4, 120, 92, 71, 95
10	30629.23	30919.52	3.745	6	67, 38, 116, 45, 3, 117

Table 5.1: Rendez-vous sequence chemical propulsion

Mission	$\Delta V \frac{m}{s}$
1	407.11, 564.55, 290.01, 639.85, 245.00, 201.50, 152.18, 312.64, 155.93, 396.58, 301.42, 258.86, 380.78, 233.37
2	475.37, 677.75, 208.82, 265.86, 209.60, 373.84, 199.89, 428.73, 377.94, 471.21, 137.83, 347.61, 136.19, 186.80
3	320.25, 283.03, 364.97, 343.61, 314.83, 1650.87, 605.49, 166.66, 287.85, 136.66, 222.65, 99.16, 192.60
4	271.35, 441.03, 237.12, 454.52, 163.62, 527.94, 394.62, 598.73, 132.63, 331.43, 301.53, 150.19
5	672.29, 629.38, 432.16, 577.01, 244.69, 655.73, 413.48, 224.87, 316.28, 172.43, 234.73, 192.32
6	247.77, 397.97, 314.43, 1036.51, 326.60, 197.88, 208.59, 460.36, 468.96, 645.08, 75.79, 113.95
7	297.70, 461.19, 372.23, 633.07, 328.02, 627.30, 213.83, 290.41, 102.10, 134.08, 193.96
8	479.84, 378.87, 412.71, 568.86, 584.64, 590.81, 222.66, 239.08, 363.85, 351.55, 253.16
9	526.25, 226.40, 583.20, 488.90, 323.29, 228.28, 752.63, 326.72, 363.58
10	353.97, 2035.95, 309.34, 379.95, 665.33

Table 5.2: ΔV of each maneouvre

situation, and in the following ones a greater velocity increment is needed to travel between them, being impossible to visit as many as before.

It is interesting to study each transfer of the missions separately. In order to do it, the velocity increment required to travel between each debris, and time of flight required have been calculated too. This data is delivered in tables 5.2 and 5.3.

To allow a better understanding of such amount of data, the most relevant information obtained from tables 5.2 and 5.3 is represented in figures 5.1 and 5.2. As it can be observed, the average, minimum and maximum ΔV and TOF are the data represented in figures 5.1 and 5.2.

It can be checked that the average ΔV is similar in every mission, approximately between 300 and 400 $\frac{m}{s}$, except in the last mission, in which this value raises to almost 800 $\frac{m}{s}$. The minimum and maximum values in the last mission are extreme, due to the fact that the remaining debris to de-orbit are the located in a very disperse way, making the transfers more difficult.

Mission	TOF (days)
1	10.10, 70.01, 10.06, 100.06, 90.06, 60.08, 10.10, 10.06, 40.06, 60.01, 30.06, 90.08, 10.10, 80.06
2	90.10, 60.03, 90.04, 10.08, 100.04, 50.01, 100.10, 50.01, 100.06, 100.01, 40.08, 20.06, 10.11, 40.04
3	80.08, 10.08, 50.06, 0.06, 100.08, 100.04, 100.04, 90.06, 90.08, 60.01, 50.08, 60.01, 50.10
4	10.06, 20.08, 20.01, 20.01, 40.05, 100.04, 80.01, 50.10, 60.01, 40.10, 30.06, 100.04
5	10.04, 30.11, 20.01, 30.01, 20.04, 80.04, 50.10, 10.01, 10.06, 10.04, 50.06, 30.04
6	40.08, 20.08, 60.06, 100.06, 50.06, 70.10, 40.04, 20.08, 10.08, 30.04, 100.10, 60.06
7	60.01, 40.01, 10.04, 10.08, 10.08, 0.06, 20.11, 40.08, 90.08, 70.06, 80.01
8	50.10, 10.01, 20.10, 40.08, 100.04, 90.06, 90.06, 90.08, 30.06, 90.04, 90.06
9	40.06, 40.01, 10.06, 70.08, 20.10, 50.06, 90.10, 70.01, 50.06
10	30.10, 0.08, 70.06, 100.01, 60.04

Table 5.3: TOF of each maneouvre

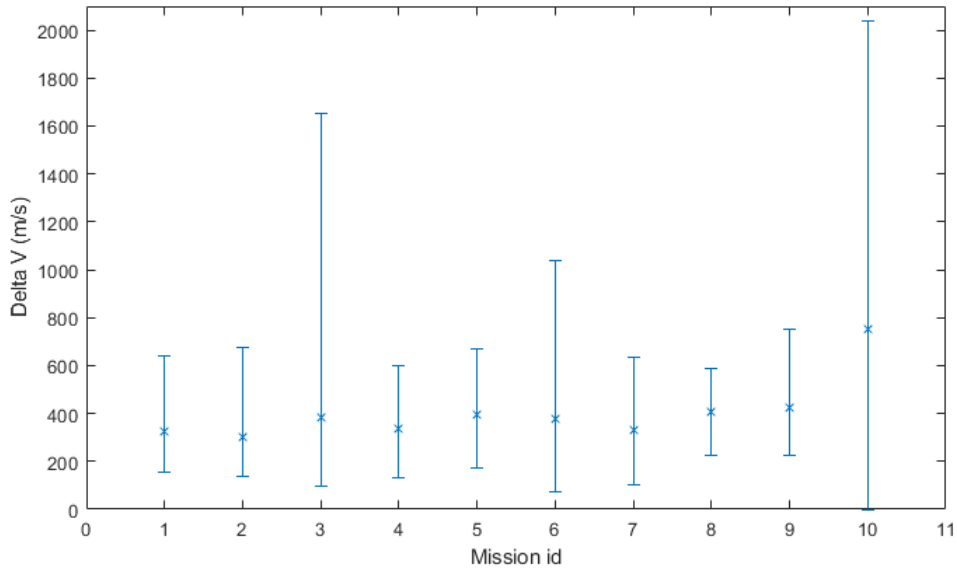


Figure 5.1: Average, maximum and minimum velocity increment per mission

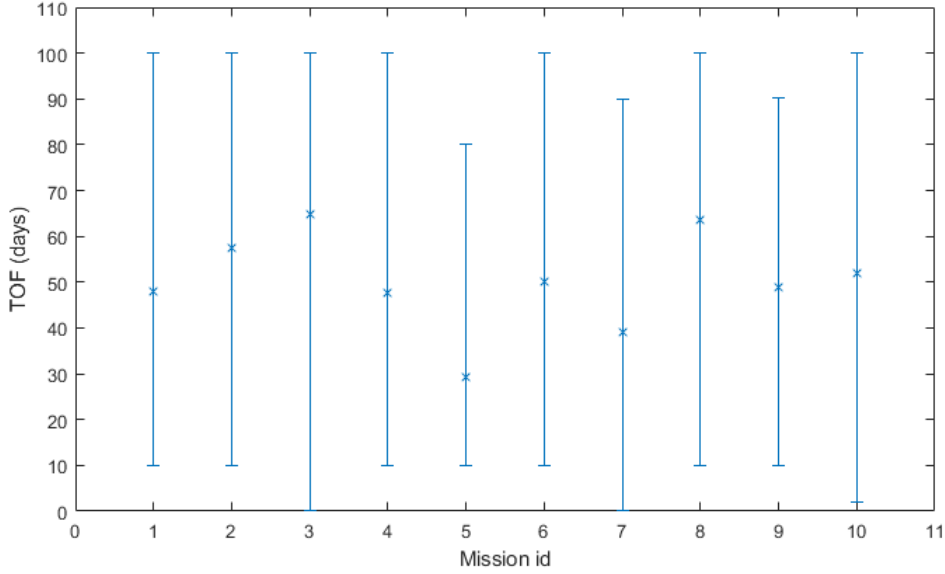


Figure 5.2: Average, maximum and minimum TOF per mission

In the case of time of flight, it can be observed that in each mission the time required to travel between debris is very irregular, which leads to think that the position of the plane of the orbits of the debris are very different, and for some transfers the time required is very short, while for others it is necessary to spend 100 days, which is the upper limit established for each transfer.

Finally, the final cost according to equation 3.9, and considering $m_{dry} = 2000$ kg and $m_{in} = 7000$ kg fixed for all the missions is :

$$J = 500 \text{ Millions of Euros}$$

5.2 Electrical propulsion

The electric propulsion engine considered for these missions has a specific impulse $I_{sp}=3000$ seconds. As in the case of the chemical propulsion spacecraft, the initial mass is fixed to $m_i = 7000$ kg. Thus, according to equation 3.11, the maximum velocity increment is $\Delta V = 36.86 \frac{km}{s}$. Because of the same reason of the previous case, a mission will be considered as suitable for a total velocity increment up to $38 \frac{km}{s}$, as it would be corrected in a second optimization.

In the case of electric propulsion engines, there exists another factor to take into account

apart from the maximum velocity increment. It is the requirement which states that the thrust to mass ratio must respect the condition already mentioned, $\left(\frac{T}{m}\right) = \frac{\Delta V}{TOF} < 5 \cdot 10^{-4}$. The ratio of each transfer of each of the missions performed is delivered in table 5.4, to ensure that such requirement is fulfilled.

Mission	$\frac{\Delta V}{TOF} \left[\frac{m}{s^2}\right]$
1	1.55·10 ⁻⁴ , 2.45·10 ⁻⁵ , 2.44·10 ⁻⁴ , 1.04·10 ⁻⁴ , 7.52·10 ⁻⁵ , 7.26·10 ⁻⁵ , 2.28·10 ⁻⁵ , 5.91·10 ⁻⁵ , 4.59·10 ⁻⁵ , 6.27·10 ⁻⁵ , 8.52·10 ⁻⁵ , 3.38·10 ⁻⁵ , 3.22·10 ⁻⁵ , 1.28·10 ⁻⁴ , 2.04·10 ⁻⁵ , 4.71·10 ⁻⁵ , 1.20·10 ⁻⁴ , 2.90·10 ⁻⁴ , 1.76·10 ⁻⁴ , 1.26·10 ⁻⁴ , 4.96·10 ⁻⁵ , 2.54·10 ⁻⁵ , 2.04·10 ⁻⁵ , 2.71·10 ⁻⁵ , 1.38·10 ⁻⁴ , 5.05·10 ⁻⁵ , 5.22·10 ⁻⁵ , 3.30·10 ⁻⁵ , 1.98·10 ⁻⁵ , 7.64·10 ⁻⁵ , 1.17·10 ⁻⁴ , 4.34·10 ⁻⁵ , 3.14·10 ⁻⁵ , 3.15·10 ⁻⁵ , 3.51·10 ⁻⁵ , 7.07·10 ⁻⁵ , 1.82·10 ⁻⁴
2	1.74·10 ⁻⁴ , 9.16·10 ⁻⁵ , 4.20·10 ⁻⁴ , 1.71·10 ⁻⁵ , 2.46·10 ⁻⁵ , 1.78·10 ⁻⁴ , 8.17·10 ⁻⁵ , 1.62·10 ⁻⁵ , 7.77·10 ⁻⁵ , 1.86·10 ⁻⁴ , 2.31·10 ⁻⁴ , 4.71·10 ⁻⁵ , 1.53·10 ⁻⁴ , 5.60·10 ⁻⁵ , 5.51·10 ⁻⁵ , 1.66·10 ⁻⁵ , 2.79·10 ⁻⁴ , 5.98·10 ⁻⁵ , 3.36·10 ⁻⁵ , 2.60·10 ⁻⁵ , 9.39·10 ⁻⁵ , 2.99·10 ⁻⁵ , 8.37·10 ⁻⁵ , 6.32·10 ⁻⁵ , 4.55·10 ⁻⁵ , 5.17·10 ⁻⁵ , 4.41·10 ⁻⁵ , 1.73·10 ⁻⁴ , 3.49·10 ⁻⁵ , 4.29·10 ⁻⁵ , 7.47·10 ⁻⁶ , 8.23·10 ⁻⁵ , 6.00·10 ⁻⁵ , 5.57·10 ⁻⁵ , 1.64·10 ⁻⁵ , 2.77·10 ⁻⁵ , 9.13·10 ⁻⁵ , 3.74·10 ⁻⁵
3	3.34·10 ⁻⁵ , 7.64·10 ⁻⁵ , 3.85·10 ⁻⁵ , 6.78·10 ⁻⁵ , 6.01·10 ⁻⁵ , 7.08·10 ⁻⁵ , 8.96·10 ⁻⁵ , 2.70·10 ⁻⁵ , 3.77·10 ⁻⁵ , 1.39·10 ⁻⁵ , 1.95·10 ⁻⁵ , 5.95·10 ⁻⁵ , 2.29·10 ⁻⁵ , 1.95·10 ⁻⁵ , 3.15·10 ⁻⁵ , 6.11·10 ⁻⁵ , 6.97·10 ⁻⁵ , 3.45·10 ⁻⁵ , 7.47·10 ⁻⁵ , 4.00·10 ⁻⁵ , 3.54·10 ⁻⁵ , 1.23·10 ⁻⁴ , 1.76·10 ⁻⁵ , 4.11·10 ⁻⁵ , 2.32·10 ⁻⁵ , 3.40·10 ⁻⁵ , 2.76·10 ⁻⁵ , 5.05·10 ⁻⁵ , 1.39·10 ⁻⁵
4	3.29·10 ⁻⁵ , 1.17·10 ⁻⁴ , 1.41·10 ⁻⁴ , 4.70·10 ⁻⁵ , 6.05·10 ⁻⁵ , 3.87·10 ⁻⁴ , 4.33·10 ⁻⁵ , 4.95·10 ⁻⁴ , 1.06·10 ⁻⁴ , 2.47·10 ⁻⁵ , 1.04·10 ⁻⁴ , 1.08·10 ⁻⁵ , 3.56·10 ⁻⁵ , 5.62·10 ⁻⁴

Table 5.4: Thrust to mass ratio of each manoeuvre

The results obtained for this propulsion system conclude that 4 missions are enough to remove all the debris items considered. The rendez-vous sequences are delivered in table 5.5.

Mission	Start Epoch(MJD ₂₀₀₀)	End Epoch(MJD ₂₀₀₀)	ΔV (km/s)	# Debris	Sequence
1	21928.06	27100.19	27.94	39	68, 91, 113, 114, 29, 32, 7, 9, 62, 50, 37, 22, 39, 109, 49, 23, 48, 75, 60, 81, 24, 79, 4, 100, 59, 27, 83, 45, 57, 26, 106, 119, 35, 94, 107, 101, 98, 84, 19
2	29800.00	34372.37	28.85	39	78, 92, 46, 38, 63, 67, 112, 56, 66, 58, 2, 36, 54, 95, 55, 43, 61, 52, 21, 25, 12, 40, 123, 41, 111, 108, 42, 103, 90, 89, 82, 16, 31, 1, 44, 115, 120, 13, 64
3	42969.16	46785.87	12.35	30	17, 34, 96, 85, 97, 69, 33, 76, 105, 122, 14, 80, 77, 5, 116, 30, 72, 47, 53, 86, 65, 110, 73, 6, 74, 117, 99, 93, 71, 3
4	48084.05	49684.74	7.33	15	10, 28, 104, 11, 70, 51, 20, 15, 88, 121, 87, 18, 8, 118, 102

Table 5.5: Rendez-vous sequence electric propulsion

As it can be observed, the sequences are much longer than for chemical propulsion. The most remarkable fact is the duration of the missions. As it can be observed, the shorter one is the last one, as it has to remove a lower number of debris, but it still requires more than 4 years to be performed completely.

As in the previous configuration, the velocity increment and the time of flight required per transfer in each mission is delivered in tables 5.6 and 5.7.

Mission	$\Delta V \frac{m}{s}$
1	2673.18, 402.20, 2111.57, 898.77, 779.61, 564.88, 374.76, 789.16, 1021.91, 673.69, 922.07, 736.49, 467.62, 222.49, 443.47, 350.98, 773.61, 1035.76, 2497.79, 760.91, 869.35, 599.82, 350.96, 176.74, 375.04, 357.54, 741.51, 677.23, 512.65, 273.48, 396.37, 1014.09, 674.87, 434.61, 407.86, 424.58, 367.03, 786.09
2	903.09, 791.76, 7257.64, 296.22, 296.22, 318.72, 462.28, 353.47, 266.63, 604.38, 644.63, 601.98, 773.34, 660.35, 290.47, 619.20, 258.69, 483.63, 310.16, 493.33, 135.03, 406.22, 414.07, 940.124, 546.18, 393.05, 894.58, 724.44, 597.18, 604.47, 222.56, 109.81, 428.09, 466.64, 867.18, 254.69, 430.55, 473.33, 550.19
3	491.27, 528.50, 398.96, 761.61, 831.21, 428.78, 619.62, 349.94, 456.40, 215.89, 236.53, 514.87, 316.92, 320.20, 136.01, 528.40, 843.46, 417.80, 322.99, 656.13, 459.62, 320.18, 288.70, 675.27, 280.55, 264.56, 358.40, 131.16, 192.03
4	370.12, 1008.16, 609.78, 568.28, 669.51, 262.49, 732.71, 639.29, 172.00, 724.88, 93.55, 400.06, 291.77

Table 5.6: Velocity increment of each maneouvre

Mission	TOF (days)
1	200.08, 190.06, 100.06, 100.08, 120.04, 90.04, 190.06, 200.06, 200.01, 170.01, 170.08, 100.01, 160.06, 80.06, 40.01, 200.04, 190.06, 100.01, 100.03, 50.06, 80.08, 140.06, 160.06, 100.01, 160.10, 30.08, 170.04, 150.01, 180.01, 160.1, 60.01, 00.08, 160.04, 150.02, 140.06, 60.10, 50.08
2	60.04, 100.04, 200.08, 200.04, 150.08, 30.10, 50.06, 190.10, 90.04, 40.06, 30.06, 190.06, 50.10, 60.04, 130.06, 180.10, 20.03, 60.04, 170.04, 60.11, 50.08, 160.04, 130.04, 100.06, 100.08, 200.08, 190.10, 40.04, 200.06, 60.06, 170.04, 60.08, 90.05, 180.08, 180.08, 180.06, 60.04, 170.04
3	170.06, 80.01, 120.04, 130.01, 160.06, 70.10, 80.04, 150.01, 140.01, 180.08, 140.04, 100.10, 160.06, 190.06, 50.04, 100.06, 140.10, 140.08, 50.06, 190.01, 150.10, 30.10, 190.08, 190.01, 140.06, 90.11, 150.01, 30.08, 160.10
4	130.01, 100.06, 50.01, 140.01, 150.10, 200.06, 70.10, 170.04, 70.06, 80.06, 80.04, 100.06, 130.04, 60.04

Table 5.7: Time of flight of each maneouvre

The most relevant data was plotted and delivered in figures 5.3, 5.4 and 5.5. This time, two different plots are delivered for the average and extreme values of velocity increment to

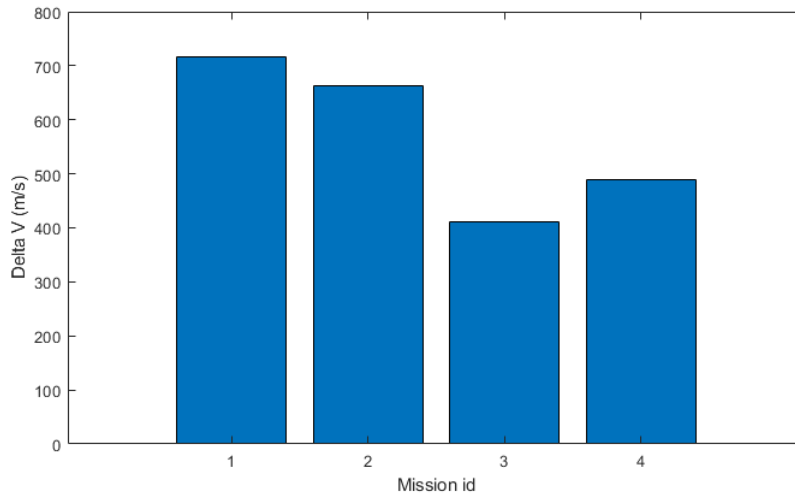


Figure 5.4: Average velocity increment per mission (low-thrust)

allow a better observation, as there exist a significant difference between the minimum and maximum values.

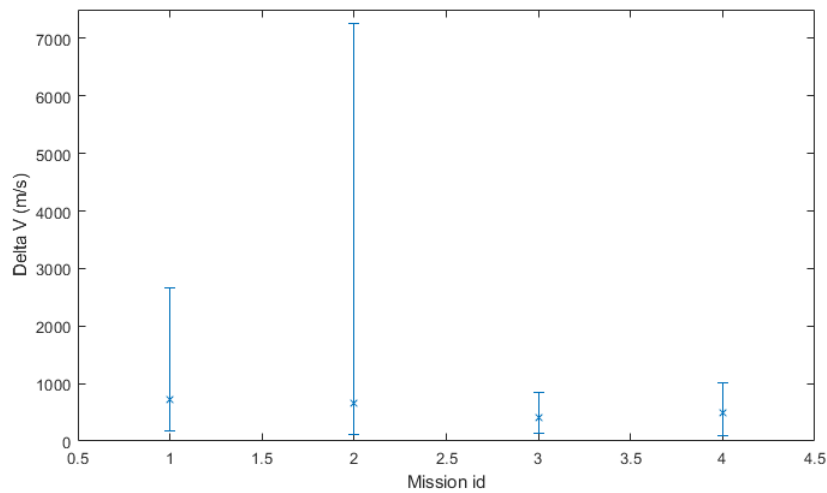


Figure 5.3: Average, maximum and minimum velocity increment per mission

As it can be observed, the two first missions require a greater velocity increment. This is due to the number of debris removed in the missions. It is very difficult that all the transfers can be performed in a favourable situation. The maximum velocity increment is very high in this missions, which leads to think that some of the transfers of these two missions require a high velocity increment. The most extreme case takes place in the second mission, whose

third transfer requires a velocity increment $\Delta V = 7257.64$ days.

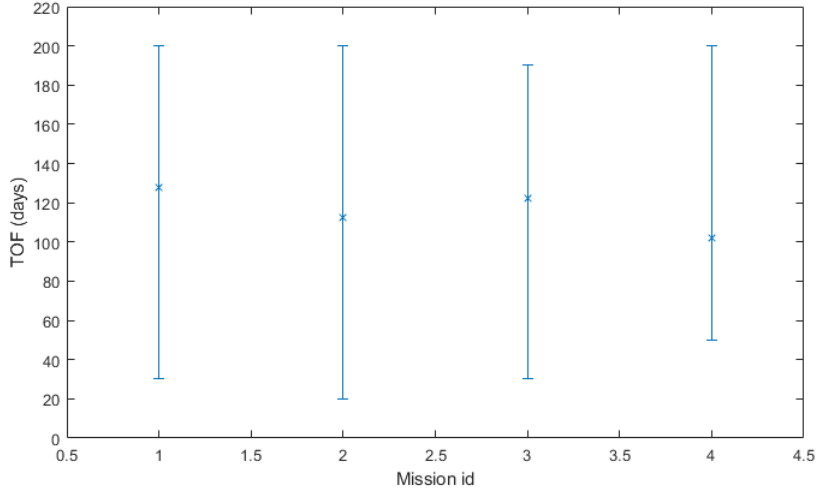


Figure 5.5: Average, maximum and minimum vtime of flight per mission

As it can be observed in figure 5.5, the average time of flight required lays between 100 and 130 days for the four missions. Also, it is remarkable the fact that the maximum and minimum values are very similar. This occurs because in each mission, the transfers chosen have a larger duration in order to allow the use of spacecrafts driven by an electric propulsion system .

Finally, the final cost according to equation 3.9, and considering $m_{dry} = 2000$ kg and $m_{in} = 7000$ kg fixed for all the missions is :

$$J = 200 \text{ Millions of Euros}$$

5.3 Comparison with GTOC9 participants results

To check the reliability of the solution obtained in this study, the results are going to be compared with the two best solutions submitted to GTOC9 contest. These works were delivered by *Jet Propulsion Laboratory* (JPL) Team [21] and the *National University of Defense Technology* (NUDT) Team[22]. [Poner referencias]. Their results are delivered in figures 5.7 and 5.8 at the end of the chapter.

Firs of all, it is important to remind that only the solution for a chemical propulsion spacecraft can be compared with the results submitted to the contest, as the participants only considered this kind of engines ($I_{sp} = 340$ seconds)

JPL designed a set of 10 missions, while NUDT requires 12 missions to remove all the debris. The solution delivered in this project reaches the objective of removing all the space debris pieces in 10 missions, as JPL did. Nevertheless, the length of the missions is quite different. In other words, they were able to remove more debris pieces in a single mission, as the maximum number of debris items de-orbited by a mission proposed in this project is 15, while JPL designed a mission which removed 21 debris and NUDT 17.

The order of removal is not the same, but that does not suppose any problem as the sequences proposed by each of the teams considered were very different too. Nevertheless, some of the debris removed in the two first missions coincide with the ones stated by NUDT.

The most significant different is the time required to perform the missions. Both JPL and NUDT have proposed missions that required between 200 and 300 days to be completed. On the other hand, the solution provided in this project propose missions which require a wide range of times to be performed, from 450 to 950 days, except in the case of the last mission, that only requires 290 days, as it de-orbits only 6 debris pieces.

This major difference in time makes really difficult to find similar sequences, as when they have finished the de-orbiting operations, the solution proposed in this project has performed only 4 missions. This makes that the position of planes of the orbits differ significantly, promoting the visit of different debris.

Comparing the ΔV required per transfer in each mission, the conclusion is that the results of obtained are greater than the ones obtained by NUDT. This team provides a plot that represent the average, maximum and minimum values (shown in figure), as in figure 5.1. The maximum average velocity increment that they required is the only one that lay in the bounds of our results. As it can be observed in figure 5.6 , the maximum and minimum ΔV required are lower too.

To sum up, their results are significantly more suitable, as they propose missions which require a lower velocity increment and can be performed in a shorter period of time. This is why they carried out more optimization process to define the missions. The results presented in this dissertation correspond to the first optimization performed by the two first ranked participants of the contest.

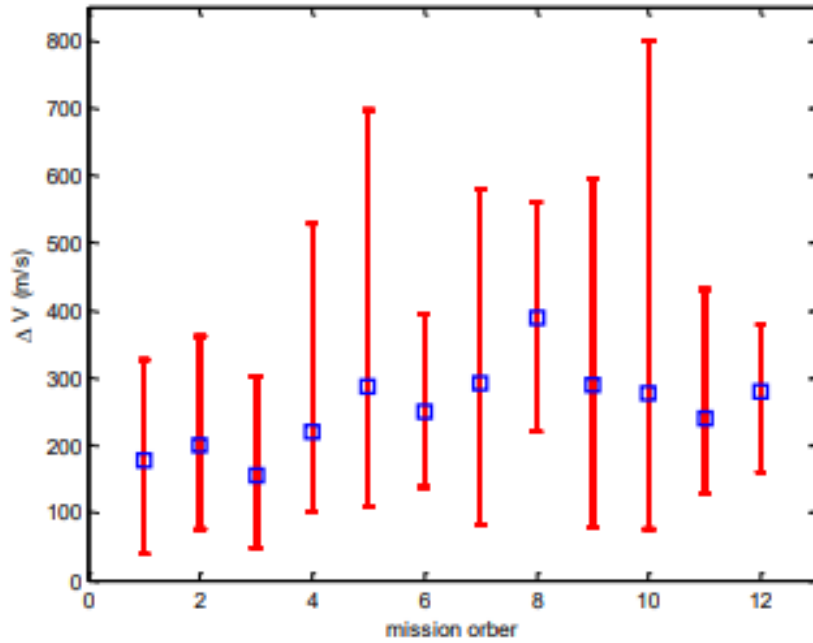


Figure 5.6: NUDT average, maximum and minimum velocity increment per mission [22]

Mission	Start MJD2000	End MJD2000	Launch Mass, kg	Number of objects	Debris ID
1	23557.18	23821.03	5665.38	14	23,55,79,113,25,20,27,117,121,50,95,102,38,97
2	23851.08	24024.53	4666.15	12	19,115,41,26,45,82,47,85,7,2,11,77
3	24057.47	24561.49	6589.58	21	72,107,61,10,28,3,64,66,31,90,73,87,57,35,69,65,8,43,71,4,29
4	24637.26	24916.44	5679.10	11	108,24,104,119,22,75,63,112,37,32,114
5	24946.47	25232.94	4906.59	14	84,59,98,1,40,51,36,67,62,99,54,122,76,15
6	25262.95	25455.15	5062.74	10	101,48,53,5,12,39,58,13,60,74
7	25485.20	25682.33	4082.33	10	49,9,70,93,105,46,88,118,18,91
8	25712.38	25915.53	3725.73	9	86,34,100,30,92,6,110,96,81
9	25946.06	26237.29	4897.35	12	33,68,116,106,14,52,120,80,16,94,83,89
10	26267.80	26416.00	3438.62	10	44,111,56,78,0,17,109,103,42,21

Figure 5.7: JPL results [21]

Mission Order	Start Epoch (MJD)	End Epoch (MJD)	Debris Number	Debris Removal Sequence
1	23517.00	23811.52	17	0, 115, 12, 67, 19, 48, 122, 7, 63, 61, 82, 107, 41, 11, 45, 85, 47
2	23893.80	24092.29	11	58, 28, 90, 51, 72, 69, 10, 66, 73, 64, 52
3	24122.30	24427.74	12	84, 86, 103, 16, 121, 92, 49, 23, 20, 54, 27, 36
4	24461.50	24660.15	10	8, 43, 9, 55, 95, 14, 102, 39, 113, 110
5	24785.00	24975.41	12	83, 75, 22, 35, 119, 24, 108, 37, 112, 104, 32, 114
6	25006.00	25198.32	9	118, 65, 74, 50, 94, 21, 97, 79, 120
7	25281.60	25454.87	10	62, 1, 40, 76, 89, 99, 15, 59, 98, 116
8	25555.40	25669.64	8	117, 91, 93, 70, 18, 105, 88, 46
9	25702.40	25860.22	9	5, 53, 33, 68, 71, 80, 57, 60, 106
10	25912.74	26055.85	8	2, 81, 96, 6, 100, 30, 34, 26
11	26087.53	26262.18	10	87, 29, 101, 31, 38, 25, 4, 77, 13, 3
12	26292.26	26381.58	7	44, 111, 56, 78, 17, 109, 42

Figure 5.8: National University of Defense Technology results
[22]

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this project, a very complex Mixed-Integer Nonlinear optimization problem has been solved. One of the reasons of its complexity is the deep combinatorial analysis that must be performed. In order to remove the 123 space debris items, a significant number of combinations must be carried out to determine the optimum rendez-vous sequence. Such is the difficulty of the problem, that it was proposed in an international competition.

Two alternative solutions have been delivered in this project, depending on the propulsion system used by the spacecraft performing the missions. It has been shown that using a spacecraft propelled by an electric propulsion engine allows to carry out missions able to remove a significantly greater number of debris, reducing the cost of the operation from 500 to 200 million Euros, as a lower number of missions is needed. The disadvantage of using this type of spacecraft is that the time needed to perform the transfer between debris is greater, being around 120 days on average, while for chemical propulsion it is typically between 40 and 70 days, which increases significantly the duration of the mission. Nevertheless, as the mitigation of Kessler effect would require the removal of just 5 to 10 space debris pieces per year, this option is still valid and preferable as it requires a lower inversion.

This solution constitutes just an initial estimation for further optimizations. The process followed has several limitations that do not assure an extremely accurate solution. For example, the Lambert's Problem Solver used for the calculations does not take into account the Earth oblateness effect. In the same way that J_2 perturbation affects to orbits of the debris, it also affects to the transfer orbit. As this is not taken into account, the resulting

transfer orbits and the velocity increment calculated from it are not completely accurate.

For simplicity, some variables were considered as fixed. For example, the time invested in de-orbiting the debris was fixed to 5 days, while the spacecraft could have remained more time in orbit in order to start the transfer to the next one in a more suitable situation.

The initial mass of the spacecraft is considered fixed to 7000 kg, in order to estimate the maximum velocity increment that the spacecraft can provide with the maximum mass of fuel allowable. For this first approach missions that required a little greater velocity increment than the maximum theoretically allowed have been accepted. This margin results in missions which require more propellant than the maximum which is able to carry the spacecraft, around 500 kg more. Obviously, this is not correct, but it is still acceptable as a first approach for further optimizations. Such inaccuracies will be corrected in the single mission optimization which should be performed by deterministic methods to reach a better solution, in which the propellant mass required will be adjusted to the real limits.

To sum up, it has been checked the efficacy of Heuristic methods, as the genetic algorithm used in this project, to solve in a quite accurate way nonlinear optimization problems as it do not need any initial guess and it is able to work with discrete variables.

6.2 Future work

The improvements that can be applied in the future are the following:

- Consideration at every stage of the process of the orbital perturbations due to the Earth oblateness.
- Use of more powerful computational devices. One of the advantages of Heuristic solvers is that they allow a parallel use of the sources of the computer, which means that the more powerful the computer, the bigger population can be considered and a better solution can be obtained in a faster way.
- The improvement and optimization of the existing Heuristic methods to assure an effectively solution of realistic problems.

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