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ABSTRACT

This paper evaluates the application of two well-known asymmetric stochastic volatility (ASV) models to option price forecasting and dynamic delta hedging. They are specified in discrete time in contrast to the classical stochastic volatility (SV) models used in option pricing. There is some related literature, but little is known about the empirical implications of volatility asymmetry on option pricing. The objectives of this paper are to estimate ASV option pricing models using a Bayesian approach unknown in this type of literature, and to investigate the effect of volatility asymmetry on option pricing for different size equity sectors and periods of volatility. Using the S&P MidCap 400 and S&P 500 European call option quotes, results show that volatility asymmetry benefits the accuracy of option price forecasting and hedging cost effectiveness in the large-cap equity sector. However, asymmetric SV models do not improve the option price forecasting and dynamic hedging in the mid-cap equity sector.

JEL-Classifications: C22; C51; C58

Keywords: Delta Hedging; Option Pricing; Stochastic Volatility; Volatility Asymmetry

1. Introduction

Options and other derivatives provide investors with an insurance against a loss or a gain above a certain level. The option price depends crucially on a precise forecast of the underlying asset volatility, which is known to be time-varying over the time to maturity. Therefore, the assumption of constant volatility of the Black-Scholes model is unarguably unrealistic. On the other hand, stochastic volatility models, such as the Heston model, have been an important part of the financial derivatives literature for a few decades. They incorporate two sources of uncertainty: one over the underlying returns whose diffusion term might be that of a CIR model, and the other over its volatility. In addition, the shape of the returns innovation distribution, such as the size of its tails and skewness, should not be disregarded in an efficient risk management practice. In this paper, we assume that the standardized returns innovations follow either a normal distribution or a Student-t distribution.

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One important stylized fact in option pricing is the volatility "smirk", which consists of higher market prices for in-the-money call prices (and out-of-the-money put prices) than those calculated from the Black-Scholes model. The volatility "smirk" is often modelled with continuous time SV models which allow for a negative correlation between the return level and its volatility, known as the leverage effect (see, e.g., Bakshi et al., 1997; Nandi, 1998; Bates, 2000; Chernov Ghysels, 2000; Pan, 2002; Jones, 2003). According to Christoffersen et al. (2009), the leverage effect is important for option pricing because it increases the probability of a large loss and, consequently, the value of in-the-money call prices and out-of-the-money put options. It leads returns to have a negative skew distribution and produces asymmetric volatility "smirks" (Renault, 1997). The different responses of the volatility to negative and positive past returns have been actively investigated in the literature (see Black, 1976; Christie, 1982; Engle Ng, 1993, among others), and the main conclusions report that the asymmetric response of the volatility has extreme relevance for asset pricing, portfolio selection and risk management. For instance, Duan (1995) and Heston Nandi (2000) show that an option contract may be mispriced if the volatility asymmetry is not accurately specified.[1]

The literature on financial econometrics proposes several discrete time volatility models that incorporate asymmetry into the volatility specifications. These can be classified into two main families. The first family is composed of asymmetric GARCH models, such as the E-GARCH by Nelson (1991) and the GJR-GARCH by Glosten et al. (1993). The second family consists of SV models, where the volatility is a latent process and consequently non-observable, as it depends on information from the past and on a random innovation term. It has been shown that SV models provide more flexibility in simultaneously capturing the high kurtosis of financial returns and the high persistence of volatility (Carnero et al., 2004). Several asymmetric SV models co-exist in the literature. Taylor (1994) and Harvey Shephard (1996) propose the asymmetric autoregressive stochastic volatility (AARSV) model, whose asymmetric volatility is defined by the correlation between the disturbances of the asset returns and its volatility. Some applications and extensions of this model can be found in Asai McAleer (2011), McAleer (2005), Yu et al. (2006), Asai (2008), Tsiotas (2012) and Yu (2012). On the other hand, Breidt (1996) and So et al. (2002) propose the threshold stochastic volatility (TSV) model, where the volatility level depends on whether past returns are positive or negative (see also Asai McAleer, 2006). More recently, Mao et al. (2015) propose a general ASV model which nests all the aforementioned parametric SV models. In the nonparametric setup, Yu (2012) proposes an ASV model, where the volatility asymmetry is time varying and depends nonparametrically on the type of news arrived to the market. This proposal is in spirit similar to that by Wu Xiao (2002), but it differs from the latter because it does not assume an additive functional form for the volatility asymmetry and uses different nonparametric methods.

In this paper, we study extensions of the AARSV and TSV for option pricing. Several studies have worked on the valuation of options with GARCH volatility, such as Christoffersen Jacobs (2004) and references therein. The advantage is that volatility is observed one step ahead and therefore is easy to estimate, but the implications of using GARCH models for option valuation are not obvious. Note that

[1]Leverage implies volatility asymmetry, but not all types of volatility asymmetry imply leverage. Hereafter, we refer to the broader concept of volatility asymmetry.

option pricing is often based on SV models. Furthermore, as Heston Nandi (2000) mention, not all GARCH specifications have closed-form solutions for option pricing and have to be evaluated through simulation, which is computationally intensive. Some studies that use simulation for option pricing are Bakshi et al. (1997), Benzoni (2002), Yu et al. (2006) and Stentoft (2011). On the other hand, Badescu et al. (2016) study the use of the basic symmetric autoregressive SV (ARSV) for option pricing and hedging and conclude that it outperforms standard GARCH models. Jiang van der Sluis (1999) also show that allowing for stochastic volatility can reduce the pricing errors.

In the present paper, SV models are estimated using the Just Another Gibbs Sampler (JAGS) algorithm developed by Plummer (2003). This software is publicly available and user-friendly, but since it is based on a single-move Gibbs sampling algorithm, it is not simulation efficient (see Meyer Yu, 2000; Yu, 2005, 2012, for similarities to the BUGS implementation of SV models). Yet, Meyer Yu (2000) and Yu (2005) find that the simulation-efficiency is less of a problem for ASV models than for the basic SV model. Our simulation study in Section 3 shows that the JAGS produces reliable parameter estimates of all the models in the paper.

Our results show that considering volatility asymmetry in SV models does not affect the accuracy of option price forecasts and dynamic delta hedging for mid-cap equity options. However, it benefits the accuracy of option price forecasting and hedging cost effectiveness in the large-cap equity sector, especially for short maturities and during highly volatile periods, such as the one including the global financial crisis. In this case, the type of volatility asymmetry that fares the best is the one that is incorporated in the TSV model. When the aim is hedging, conclusions depend on moneyness and market volatility. For periods of low volatility, the AARSV is the model with the smallest hedging effective costs for in-the-money and at-the-money options, whereas for out-of-the-money options the TSV is better. Regarding periods of high volatility, the models that provide the lowest effective costs are the basic SV and the TSV models, especially for short maturities. As the time to maturity increases, the basic SV model with Gaussian errors is the favoured one, regardless of the moneyness.

This paper contributes to the current literature on option pricing with stochastic volatility in several ways. First, we propose a novel Bayesian approach in this literature to estimate SV models. This approach is open for public access, user-friendly and easy to implement. Second, we explore the effect of two types of volatility asymmetry in forecasting option prices and dynamic delta hedging, and compare their performances with that of the basic ARSV model. Finally, we use the S&P 500 and S&P MidCap 40 series of daily returns to measure the effect of volatility asymmetry in two different size equity sectors and in two periods which correspond to different levels of market risk. This is of extreme relevance for investors, financial institutions and firms dealing with options, and to our knowledge, it has not been studied before.

The paper is organized as follows. In Section 2, we introduce the stochastic volatility models for option pricing. Section 3 motivates the potential effects of volatility asymmetry on option pricing, and it presents the MCMC estimation methodology and the results from a simulation study to analyze the properties of the estimator in finite samples. In Section 4, all volatily models are estimated from the S&P 400 (mid-cap equity) and S&P 500 (big-cap equity) series. These parameter estimates of the SV models are used to forecast option prices, and the three model option price forecasts are compared with the model confidence set procedure. In addition, the delta hedging distributions of all models are compared visually. Finally, we provide conclusions and discuss further lines of research in Section 5.

2. Asset return models for option pricing

In the option pricing literature, the autoregressive stochastic volatility model is often expressed in terms of stochastic differential equations. The popularity of this model lies in its flexibility in accommodating many of the empirical features of the time series of returns, such as high kurtosis and volatility persistence. The log-difference of the asset price $S(t)$ which evolves continuously over time and its instantaneous volatility, $\sigma(t)$, are expressed as

$$\begin{aligned} d\ln S(t) =& (r+\lambda\sigma^2(t))dt + \sigma(t)dW_1(t) \\ d\ln\sigma^2(t) =& \left(\alpha + \beta\ln\sigma^2(t)\right)dt + \sigma_\eta dW_2(t), \end{aligned} \tag{1}$$

where σ_η is the volatility of the logarithm of the variance process, and $W_1(t)$ and $W_2(t)$ are two standard Brownian motion processes. Model (1) accommodates volatility asymmetry when the two Brownian motions are correlated. Note that this model assumes no dividends.

In order to facilitate the estimation, model (1) is often discretized. One possible approximation is the Euler-Maruyama scheme, which leads to the following discrete version of model (1):

$$\begin{aligned} y_t &= r + \lambda\sigma_t^2 + \sigma_t\epsilon_t \\ \ln\sigma_t^2 &= \mu + \phi(\ln\sigma_{t-1}^2 - \mu) + m(\epsilon_{t-1}) + \eta_{t-1}, \end{aligned} \tag{2}$$

where the continuously compounded return process is given by $y_t = \ln S(t) - \ln S(t-1)$, the price $S(t)$ is observed discretely within an equally spaced grid and $\epsilon_t = W_1(t+1) - W_1(t)$ is an identically distributed process with mean zero and variance one. In the log-variance equation, $\phi = 1+\beta$ is the persistence parameter, $\mu = \alpha/(1-\phi)$ is a scale parameter related to the marginal variance of y_t and $\eta_t = \sigma_\eta(W_2(t+1) - W_2(t))$ (see Yu, 2005) for a similar discretization. Function $m(\cdot)$ represents the volatility asymmetry that depends on ϵ_{t-1} and may be discontinuous. The disturbance, η_t, is assumed to be Gaussian with mean zero, variance σ_η^2 and independent of ϵ_t.

Model (2) nests several autoregressive stochastic volatility models used in option pricing. Depending on the shape of function $m(\cdot)$, the ones studied in this paper are:

(i) ARSV-OP: $m \equiv 0$,

(ii) AARSV-OP: $m(\epsilon_{t-1}) = \gamma_1\epsilon_{t-1}$, and

(iii) TSV-OP: $m(\epsilon_{t-1}) = \delta I(\epsilon_{t-1} < 0)$.

The AARSV-OP model is based on the ASV model of Taylor (1994) and Harvey Shephard (1996), while TSV-OP is similar to the model in So et al. (2002), where a scalar changes the value of the current volatility depending on the sign of past returns. $I(\cdot)$ is an indicator function which takes value one when the argument is true and zero otherwise.

Often the distribution of financial returns is leptokurtic; see Table 2. Therefore, we also assume that ϵ_t may follow a Student-t distribution with k degrees of freedom (see, for example, Liesenfeld Jung, 2000; Asai, 2008, who also consider a Student-t distribution for standardized returns).

3. Estimation of volatility asymmetry in option pricing

3.1. Motivation

The price of an European option depends on its initial asset price, moneyness, time to maturity and volatility of the underlying asset. The first four are fixed numbers, but volatility changes with time and leverage.

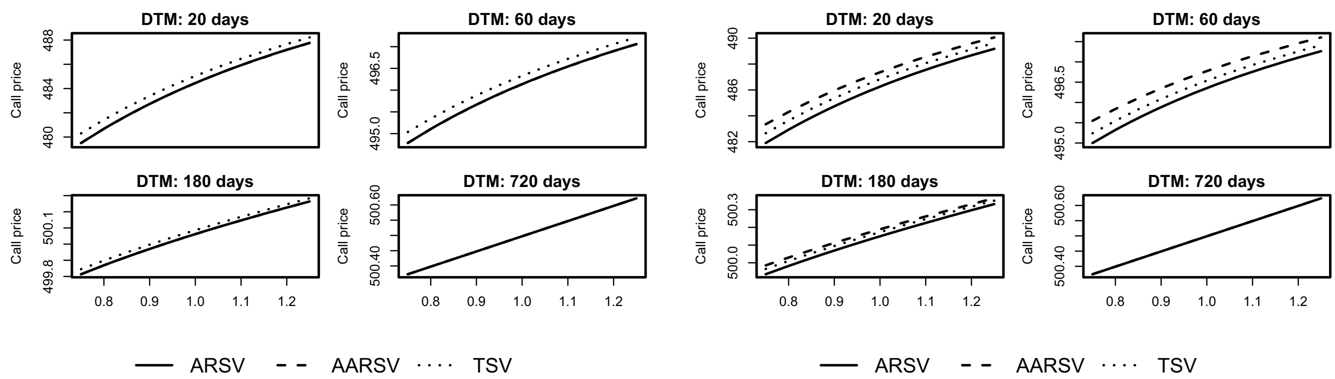
Simulations of 10000 European call option prices are performed to study their sensitivity to volatility asymmetry, which is often caused by the existence of leverage. Figure 1 shows the European call option prices with an initial asset price, $S_0 = 1000$; different moneyness, $S_0/X = 0.75$, 0.8, 0.85, 0.9, 0.95, 1, 1.05, 1.1, 1.2, 1.25 (from deep out-of-the-money to deep in-the-money); and different days to maturity from 20 to 720 days, with a risk-free interest rate of 1% per annum and with volatilities obtained with the ARSV-OP, AARSV-OP and TSV-OP models. The parameter values chosen for these models are: $r = 0$, $\lambda = 0$, $\mu = 0, \phi = 0.98, \delta = 0.1$ and $\gamma_1 = -0.02$; -0.5.

The differences in option pricing from the different models appear in Figure 1. They corroborate the importance of using the right form of volatility asymmetry for short and medium maturities. In fact, the differences in option prices between the ARSV-OP and the AARSV-OP models are negligible when γ_1 is small because they are nearly the same model; however, these differences are important for $\gamma_1 = -0.5$. When δ and γ_1 are large enough, the option prices obtained with the AARSV-OP and the TSV-OP models are different from those of the ARSV-OP. Only for very large maturities do all models perform similarly.

3.2. Bayesian estimation

Several methods are available to estimate SV models. From a frequentist viewpoint, we find importance sampling techniques in Shephard Pitt (1997); Durbin Koopman (1997) and Richard Zhang (2007); simulated maximum likelihood in Danielsson (1994) and Sandmann Koopman (1998); and approximate maximum likelihood in Fridman Harris (1998). Although MCMC methods often require more computational time than the previous methods, they are considered one of the most efficient for SV models; see, for instance, the single-move Metropolis-Hastings algorithm by Jacquier et al. (2004) and the multi-move algorithms by Kim et al. (1998) and Omori et al. (2007). Thew computing functions of all aforementioned models are rarely freely available, and their implementation requires a high level of programming competence. We overcome this issue with the JAGS algorithm (Plummer, 2003), an MCMC technique that is publicly available in the R package called *R2jags* (Su Yajima, 2015). JAGS

Figure 1. Simulated European call option prices from the ARSV-OP, AARSV-OP and TSV-OP models. The x-axis represents the moneyness: out-of-the-money for values lower than 1 and in-the-money for values higher than 1.



(a) $\gamma_1 = -0.02$ (b) $\gamma_1 = -0.5$

is a recent alternative to OpenBugs, which is used in Meyer Yu (2000); Yu (2005) and Yu (2012) for SV models.

We use the prior distribution specifications of Meyer Yu (2000) and Yu (2005). Futhermore, we assume that $r \sim N(0, 10)$, $\lambda \sim U(0, 1)$, $\delta \sim N(0.05, 10)$ and $k \sim \Gamma(2, 0.1)$, where Γ refers to the gamma distribution. All priors are assumed to be independent. Also, we choose a burn-in period of 10000 and a follow-up of 30000 iterations with three independent chains.

In order to check the reliability of the estimation method, we simulate 3000 observations from the ARSV-OP, AARSV-OP and TSV-OP models. We replicate the experiment 200 times and obtain the Monte Carlo mean and standard deviation of the parameter estimates. Table 1 reports the simulation results and the fixed true values of the parameters. We observe that the MCMC estimator produces accurate estimates of all models' parameters, even for innovations with fat tails.

4. Empirical application

In this section, we compare the AARSV-OP and TSV-OP models with the benchmark ARSV-OP model for option price forecasting and dynamic delta hedging.

4.1. Data

All models are fitted to two different series of daily returns: the S&P MidCap 400 and the S&P 500, downloaded from the OptionMetrics and CRSP databases, respectively. These series reflect the risk and properties of two different market segments. The S&P MidCap 400 consists of medium-sized companies from the S&P and Dow Jones indices, while the S&P 500 is a stock market index based on the 500 companies with the largest capitalization from the NYSE and NASDAQ. These indices are used as

Table 1
Finite sample properties of an MCMC estimator

	$\epsilon_t \sim$Normal			$\epsilon_t \sim$Student-t		
	true value	**mean**	**std. dev.**	**true value**	**mean**	**std. dev.**
			ARSV-OP			
r	0.000	-0.001	(0.020)	0.000	0.000	(0.024)
λ	0.100	0.101	(0.019)	0.100	0.101	(0.025)
$\mu(1-\phi)$	0.000	0.000	(0.005)	0.000	0.001	(0.005)
ϕ	0.980	0.978	(0.005)	0.980	0.977	(0.005)
σ_η^2	0.050	0.052	(0.008)	0.050	0.051	(0.011)
k	–	–	–	5.000	5.297	(0.641)
			AARSV-OP			
r	0.000	-0.004	(0.020)	0.000	-0.004	(0.022)
λ	0.100	0.109	(0.019)	0.100	0.106	(0.024)
$\mu(1-\phi)$	0.000	0.008	(0.005)	0.000	0.010	(0.005)
ϕ	0.980	0.984	(0.004)	0.980	0.982	(0.005)
γ_1	-0.080	-0.077	(0.015)	-0.080	-0.081	(0.014)
σ_η^2	0.050	0.048	(0.007)	0.050	0.052	(0.010)
k	–	–	–	5.000	5.313	(0.650)
			TSV-OP			
r	0.000	-0.003	(0.051)	0.000	-0.006	(0.059)
λ	0.100	0.102	(0.011)	0.100	0.101	(0.014)
$\mu(1-\phi)$	0.000	0.003	(0.019)	0.000	0.004	(0.019)
α	0.070	0.076	(0.032)	0.070	0.074	(0.036)
ϕ	0.980	0.977	(0.006)	0.980	0.977	(0.006)
σ_η^2	0.050	0.054	(0.010)	0.050	0.054	(0.011)
k	–	–	–	5.000	5.300	(0.632)

proxies for two different sectors: the mid-cap and large-cap equity sectors. Furthermore, empirical results are obtained from two different out-of-sample periods.

The in-sample sets are used to train the models (i.e., to estimate their parameters). The first out-of-sample period (02/01/2008–31/12/2010), used for one-step-ahead forecasting, includes the global financial crisis. The second out-of-sample period, denoted as the calm period, runs from 2014 until 2017 for the S&P 500 index and from 2010 until 2012 for the S&P MidCap 400 index. The exact periods for the in-sample and out-of sample periods can be found in Table A.1 of the appendix.

Table 2 reports the summary statistics of the two series of returns and their in-sample and out-of-osample periods. We observe that the statistical properties of both series of returns are quite similar for the calm and crisis periods, except for the skewness, which in some cases is positive. The excess of kurtosis is statistically significant at relevant levels of significance, and the skewness is not statistically different from zero.[2] Note that the in-sample sets of the calm period include observations from the

[2]Note that the p-values of the tests of kurtosis and skewness have been obtained using the procedure by Premaratne Bera (2017), which allows us to test kurtosis in the presence of asymmetry and skewness in the presence of excess of kurtosis.

global financial crisis, which explains the high kurtosis of the data in these periods.

Table 2
Descriptive statistics

Sample moments of daily SP&400 MidCap and S&P500 returns. Symbols ***, ** and * mean statistically significant at the 1%, 5% and 10% significance levels, respectively.

	S&P 500				S&P400 MidCap			
	In-sample		Out-of-sample		In-sample		Out-of-sample	
	crisis	calm	crisis	calm	crisis	calm	crisis	calm
Mean	0.000	0.019	-0.197	0.040	0.043	0.021	0.007	0.000
Median	0.040	0.069	-0.002	0.024	0.050	0.075	0.060	-0.009
Maximum	5.790	10.642	10.642	2.359	5.970	15.242	9.962	2.691
Minimum	-6.044	-9.688	-9.688	-3.647	-7.331	-10.443	-11.526	-3.009
Variance	1.257	1.912	6.486	0.639	1.279	2.088	4.302	0.744
Skewness	0.041	-0.265	0.024	-0.456	-0.168	0.041	-0.367	-0.258
(p-value)	(0.798)	(0.516)	(0.957)	(0.218)	(0.254)	(0.949)	(0.248)	(0.173)
Kurtosis	5.994***	12.098***	6.523***	5.586***	5.743***	15.353***	7.185***	3.593***
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.040)
Jarque-Bera	742.8	6922	127.2	84.87	987.6	13000	588.9	7.028
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.030)

Option quotes are downloaded from OptionMetrics to compare with our option price forecasts. They contain the closing bid/ask European call option prices from the S&P MidCap 400 and S&P500 indices during the previously denoted crisis and calm periods. The theoretical option price is set equal to the midpoint of the best closing bid price and best closing offer price for the option. The data are collected for Wednesdays, and contracts with maturities lower than six days are excluded, following the practice in Christoffersen Jacobs (2004). The maturity is calculated in calendar days, and weekends are therefore treated as regular trading days. The data features may be found in Table 3.

Table 3 reports the number of in-the-money, at-the-money and out-of-the-money option contracts on the S&P MidCap 400 and S&P500 indices for the four sample periods. We observe that for the two indices and calm periods the number of out-of-the-money contracts is always greater than the number of contracts in-the-money for all the days to maturity considered. This is expected since the lure of out-of-the-money options is that they are less expensive than in-the-money options because there is a lower probability that the stock will exceed the strike price for the out-of-the-money option. Moreover, out-of-the money options offer great leverage opportunities. This together with lower costs make them quite attractive for traders. On the other hand, during the crisis period the number of in-the-money contracts exceeds the number of out-of-the-money contracts for short and long maturities. These options have intrinsic value, are often used to hedge or partially hedge certain positions, and are usually traded at a high volume. Finally, and for the S&P 500 index, the total number of contracts is much higher in the calm period than in the crisis period, perhaps due to the risk aversion of the investor who may move from the stock to the bond market. This is not the case for the options in the S&P MidCap 400 index. The total number of contracts is higher during the crisis period than during the calm period, and the majority of these are in-the-money or at-the-money contracts. This is expected since this index is composed of medium-sized companies, whose assets can more likely suffer from devaluation during

Table 3
S&P MidCap 400 and S&P 500 index call option data summary

The statistics are separated by days to maturity (DTM) and moneyness (S/X). num corresponds to the number of contracts and μ_{price} is the average of best bid and best ask used to estimate an average price.

S/X	< 0.94	$0.94 - 0.97$	$0.97 - 1.00$	$1.00 - 1.03$	$1.03 - 1.06$	≥ 1.06	Total
S&P MidCap 400 Index – Period: crisis							
DTM ≤ 60							
num	4849	704	643	624	591	5691	13102
μ_{price}	1.25	7.97	14.85	26.53	41.18	158.41	41.7
60 < DTM ≤ 180							
num	4420	573	517	493	490	4332	10825
μ_{price}	6.18	25.15	35.08	46.59	59.36	164.3	56.11
DTM > 180							
num	3153	404	381	372	361	2473	7144
μ_{price}	19.05	50.61	61.03	71.44	82.92	152.11	72.86
S&P MidCap 400 Index – Period: calm							
DTM ≤ 60							
num	1286	608	670	664	571	5028	8827
μ_{price}	1.46	6.36	13.88	28.33	46.95	197.32	49.05
60 < DTM ≤ 180							
num	1242	446	449	454	418	4793	7802
μ_{price}	10.17	25.28	37.14	51.69	67.39	224.28	69.33
DTM > 180							
num	958	367	348	339	299	3164	5475
μ_{price}	26.47	47.25	59.2	73.18	87.3	205.54	83.16
S&P 500 Index – Period: crisis							
DTM ≤ 60							
num	15146	2510	2461	2278	2088	23581	48064
μ_{price}	1.4	8.83	18.32	36.57	60.68	279.41	67.54
60 < DTM ≤ 180							
num	10792	1410	1437	1268	1182	12483	28572
μ_{price}	5.66	30.39	45.96	64.38	84.4	305.44	89.37
multicolumn3l**DTM > 180**							
num	7893	577	607	556	508	5566	15707
μ_{price}	18.44	78.48	94.33	113.07	131.92	337.92	129.03
S&P 500 Index – Period: calm							
DTM ≤ 60							
num	21672	14302	14557	13786	12971	129225	206513
μ_{price}	0.37	2.89	14.42	47.75	94.33	459.38	103.19
60 < DTM ≤ 180							
num	14353	7160	6999	6730	6287	73378	114907
μ_{price}	2.56	14.73	38.3	73.93	115.29	545.21	131.67
DTM > 180							
num	7133	1585	1468	1483	1359	28081	41109
μ_{price}	13.95	54.38	84.91	118.26	153.89	737.95	193.89

volatile periods, requiring more hedging in the investment.

4.2. Option price forecasting

Option price forecasting needs parameter estimates of model (2) from the in-sample dataset and the volatility forecast from the MCMC procedure, $\{\hat{\sigma}_t^{MCMC}\}_{t=0}^{T}$. As in Yu et al. (2006), a European call option price with initial price S_0, strike price X, days to maturity T and risk-free interest rate r_t can be calculated by simulation with the following steps:

1. Initialize
   - $h_0 = \ln(\sigma_0^2)$, where σ_0 is calculated as the standard deviation of the asset returns from the

training sample;

- $\epsilon_0 = (y_0 - r_0 - \hat{\lambda}\sigma_0^2)/\hat{\sigma}_0^{MCMC}$, where y_0 is the last return of the training sample; and
- η_0, a Gaussian variable with variance $\hat{\sigma}_\eta^2/365$ for daily data.

2. Generate $\hat{h}_t$ for each model:

$$\hat{h}_t = \hat{\mu} + \hat{\phi}(\hat{h}_{t-1} - \hat{\mu}) + m(\hat{\epsilon}_{t-1}) + \eta_t.$$

3. Calculate the residuals $\hat{\epsilon}_t = (y_t - r_t - \hat{\lambda}\sigma_t^2)/\hat{\sigma}_t^{MCMC}$ with $\sigma_t^2 = exp(\hat{h}_t)$, where y_t is the asset return at time t.

4. Repeat Steps 2-3 to calculate $\omega_+^2 = \frac{1}{T/365}\sum_{s=t}^{T}\hat{\sigma}_t$, where η_t is a Gaussian random variable with variance $\frac{1}{365}\hat{\sigma}_\eta^2$. Repeat the same steps with $-\eta_t$ to calculate ω_-^2.

5. Compute C^+ using the Black-Scholes call option formula,

$$\begin{aligned} d_1 =& \frac{1}{\omega_+^2\sqrt{\frac{T}{365}}} \ln(S_0/X) + (\bar{r} + \frac{\omega_+^2}{2})\frac{T}{365} \\ d_2 =& d_1 - \omega_+\sqrt{\frac{T}{365}} \\ C^+ =& S_0 N(d_1) - X e^{-\bar{r}\frac{T}{365}} N(d_2)), \end{aligned}$$

where $\bar{r}$ is the average value of the daily risk free $r_1, \ldots, r_T$.

6. Similarly, compute C^- using ω_-^2.

7. Calculate the average $C_T = (C^+ + C^-)/2$.

8. Repeat Steps 1-5 M times to obtain a sequence of C_T values.

9. Calculate the estimated option price as the mean of C_T.

Remark 1. We use the antithetic variable technique to reduce the variance of C_T (see Broadie Detemple, 2004, for a survey on valuation methods to price options).

We run Yu et al. (2006)'s algorithm with our set of three volatility forecasters for each option price. The theoretical call price is the average between the best bid and best offer. The loss function used in the model confidence set (MCS) algorithm of Hansen et al. (2011) is the forecast squared error (FSE), where the error is the difference between the forecasted price and the theoretical price of the call. The Superior Model Sets of these call price forecasts with confidence levels of 90% are obtained with the R package named *MCS* (Catania Bernardi, 2017), and results are displayed in Table 4. In particular, for each category of days to maturity and moneyness, we report the set of models that contains the best model (rank = 1), in the sense that it has the minimum mean forecast squared error (MFSE), together with any other model(s) with the same MFSE at the 1% significance level. The p-value of each test is also reported.

For all maturities, we observe that the ARSV-OP model with normal errors is the most accurate call option price forecaster for the S&P 500 and S&P MidCap 400 indices during the calm period. This suggests that in periods of low market volatility, the volatility asymmetry does not contribute to option price forecasting. This is also the case for options on the S&P MidCap 400 index during the crisis period, where the ARSV-OP model is the best forecaster. Results are different during the crisis period for the S&P 500 options. In this case, the best forecaster is always the TSV-OP model with Gaussian errors for short maturities. As the number of days to maturity increase, we observe that the ARSV-OP model with Student-t errors becomes the preferred forecaster.

Note that medium-sized companies have a different capital structure than large companies. Often these firms use internal funds in the form of retained earnings before turning to external sources. When retained earnings are not enough, firms first seek out sources of debt before they use more costly external equity. Moreover, several authors find that the leverage is positively correlated with firm size and negatively correlated with profitability (Michaelas et al., 1999; Esperança et al., 2003; Sayilgan et al., 2006), which may be the reason why volatility asymmetry seems unimportant for calls on the S&P MidCap 400 index. Finally, if the leverage is negatively correlated with firm profitability, this might also explain why in the crisis period the TSV-OP model gains relevance as a forecaster of the call option prices on the S&P 500 index. Park (2016) also show that using volatility asymmetry has a strong effect on the valuation of VIX futures and options. The period of his analysis also includes the last financial crisis.

4.3. Dynamic delta hedging

In practice, financial investors hedge their positions to the underlying sources of risk to cover for possible future losses. *Dynamic delta hedging* is a common hedging strategy consisting in reacting to changes in the underlying price *delta*, which measures the sensitivity of the option price to the underlying asset price (Broadie Detemple, 2004). In this strategy, the portfolio is rebalanced each day by buying and selling shares of the underlying asset to keep the delta risk hedged. Each of our models will have a different hedging strategy depending on their volatility forecast as well as a different hedging cost. We define the effective cost as the discounted hedging cost minus the money obtained from selling the call (Giovanni et al., 2008). Thus, a strategy with a small cost will be preferred. Figures 2–5 show the distribution of the relative difference between the effective costs of several forecasters and the effective cost of the ARSV-OP model with Gaussian errors; i.e., the distribution of [3]

$$\frac{\text{effective cost (model) - effective cost (ARSV-OP(N))}}{\text{effective cost (ARSV-OP(N))}}.$$

Figures 2 and 4 correspond to the results of the S&P MidCap 400 and S&P 500 call hedging during the crisis period, respectively. The left column plots correspond to in-the-money call prices, the middle

[3]Hereafter, figures do not include "OP" in the model names due to lack of space.

Table 4
Superior Model Set from the forecast of European call options on the S&P MidCap 400 and S&P 500 indices

S/X	< 0.94				0.94 − 0.97				0.97 − 1.00			
	crisis		calm		crisis		calm		crisis		calm	
	SP400	SP500	SP400	SP500	SP400	SP500	SP400	SP500	SP400	SP500	SP400	SP500
DTM ≤ 60												
ARSV-OP(N)	2		1	1	2		1	1	2		1	1
AARSV-OP(N)												
TSV-OP(N)		1				1				1		
ARSV-OP(T)	1				1				1			
AARSV-OP(T)												
TSV-OP(T)												
p-value	0.29	0.00	0.00	0.00	0.87	0.00	0.00	0.00	0.61	0.00	0.00	0.00
60 < DTM ≤ 180												
ARSV-OP(N)	2		1	1	2		1	1	2		1	1
AARSV-OP(N)												
TSV-OP(N)		2				1				1		
ARSV-OP(T)	1	1			1	2			1	2		
AARSV-OP(T)												
TSV-OP(T)												
p-value	0.73	0.10	0.00	0.00	0.72	0.08	0.00	0.00	0.34	0.18	0.00	0.00
DTM > 180												
ARSV-OP(N)	1		1	1	1		1	1	1		1	1
AARSV-OP(N)												
TSV-OP(N)												
ARSV-OP(T)	2	1			2	1			2	1		
AARSV-OP(T)												
TSV-OP(T)												
p-value	0.03	0.00	0.00	0.00	0.36	0.00	0.00	0.00	0.50	0.00	0.00	0.00

5000 bootstrap samples using the SE loss function to measure discrepancy are used for a confidence level of 90%. Letters N and T in brackets refer to the normal and Student-t distributions of the error term.

column to at-the-money call prices and the right column to out-of-the-money call prices. The first row corresponds to short maturities, the middle row to medium maturities and the bottom row to large maturities. We observe that results are coherent with those of the call price forecasting. The ARSV-OP(N) and ARSV-OP(T) models perform similarly for the S&P MidCap 400 and S&P 500 indices, providing the smallest effective cost for all maturities and moneyness. Furthermore, for calls on the S&P 500 index, the TSV-OP(N) model is also a competitor, performing similarly to the other previous models.

Regarding the calm period, the effective cost of the S&P MidCap 400 index is plotted in Figure 5. We observe that the results are also consistent with those of the forecasting session: the ARSV-OP(N) model provides the lowest effective cost for all moneyness at medium and long maturities. For short maturities, the AARSV-OP(T) and AARSV-OP(T) models provide smaller effective costs than ARSV-OP(N). The scenario changes substantially for calls on the S&P 500 index during the calm period, see Figure 3. For

Table 4
Superior Model Set from the forecast of European call options on the S&P MidCap 400 and S&P 500 indices (continued)

S/X	1.00 − 1.03				1.03 − 1.06				≥ 1.06			
	crisis		calm		crisis		calm		crisis		calm	
	SP400	SP500	SP400	SP500	SP400	SP500	SP400	SP500	SP400	SP500	SP400	SP500
DTM ≤ 60												
ARSV-OP(N)	2		1	1	2		1	1	2		1	1
AARSV-OP(N)												
TSV-OP(N)		1				1				1		
ARSV-OP(T)	1				1				1			
AARSV-OP(T)												
TSV-OP(T)												
p-value	0.67	0.00	0.00	0.00	0.87	0.01	0.00	0.00	0.96	0.00	0.00	0.00
60 < DTM ≤ 180												
ARSV-OP(N)	2		1	1	2		1	1	2		1	1
AARSV-OP(N)												
TSV-OP(N)		1				1				2		
ARSV-OP(T)	1	2			1	2			1	1		
AARSV-OP(T)												
TSV-OP(T)												
p-value	0.67	0.45	0.00	0.00	0.83	0.22	0.00	0.00	0.55	0.35	0.00	0.00
DTM > 180												
ARSV-OP(N)	1		1	1	1		1	1	1		1	1
AARSV-OP(N)												
TSV-OP(N)												
ARSV-OP(T)	2	1			2	1			2	1		
AARSV-OP(T)												
TSV-OP(T)												
p-value	0.67	0.00	0.00	0.00	0.71	0.00	0.00	0.00	1.00	0.00	0.00	0.00

5000 bootstrap samples using the SE loss function to measure discrepancy are used for a confidence level of 90%. Letters N and T in brackets refer to the normal and Student-t distributions of the error term.

in-the-money calls, the AARSV-OP(N) and AARSV-OP(T) models generally provide a lower effective cost than the basic ARSV-OP(N), regardless the days to the maturity. The lowest effective cost of at-the-money calls is provided by the AARSV-OP models for short and medium maturities. For long maturities, ARSV-OP(N) is the model which generally provides the lowest effective cost. Finally, for out-of-the-money calls, we observe that the smallest effective cost is given by the TSV-OP model.

Analyzing the volatility forecasts in the calm period for the S&P 500 index, we observe that the TSV-OP model provides the highest volatility forecasts, while the AARSV-OP model provides the lowest. Furthermore, it is known that high volatility increases the price of the out-of-the-money call options and decreases the price of the in-the-money call options. Given that the effective cost is the discounted hedging cost minus the money obtained from selling the call, the model that provides higher (lower) values for the volatility (for a constant hedging cost) leads to a smaller effective cost for out-of-the-money (in-the-money) call options. The evidence on the volatility can explain the results obtained for the S&P 500 index and the calm period.

5. Conclusions

This paper studies the effect of volatility asymmetry in option price forecasting and dynamic delta hedging under stochastic volatility. It is well-known that stochastic volatility models are very flexible and able to capture the main empirical features of financial returns; however, its estimation is complex and time consuming. This paper uses an MCMC approach, which is easy to implement and publicly available, making these models a real alternative for any user, trader or investor interested in option pricing.

Our empirical analysis shows evidence of accuracy improvement in option price forecasting when including volatility asymmetry in stochastic volatility models; especially during volatile market periods, in large-cap equity sectors and for short maturities. Regarding the mid-cap equity sector, the volatility asymmetry does not seem to be a determinating factor in option price forecasting. This supports recent results in Park (2016), who find that volatility asymmetry is important in the pricing of put options and short-term futures on the VIX. We have only considered call prices, and therefore it can be of interest to extend our research to analyze the differences in the effect of volatility asymmetry between call and put option prices forecasts.

Lastly, we report some interesting features of dynamic delta hedging. Specifically, the minimum effective cost of hedging is not affected by the volatility asymmetry on the mid-cap equity but it is affected by the volatility asymmetry on the big-cap equity. For this equity sector, models which incorporate volatility asymmetry provide smaller effective costs. These results depend on the option moneyness and the volatility period.

Figure 2. Distribution of effective cost relative to the simplest model for several maturities and moneyness for the S&P 400 index during the crisis period.

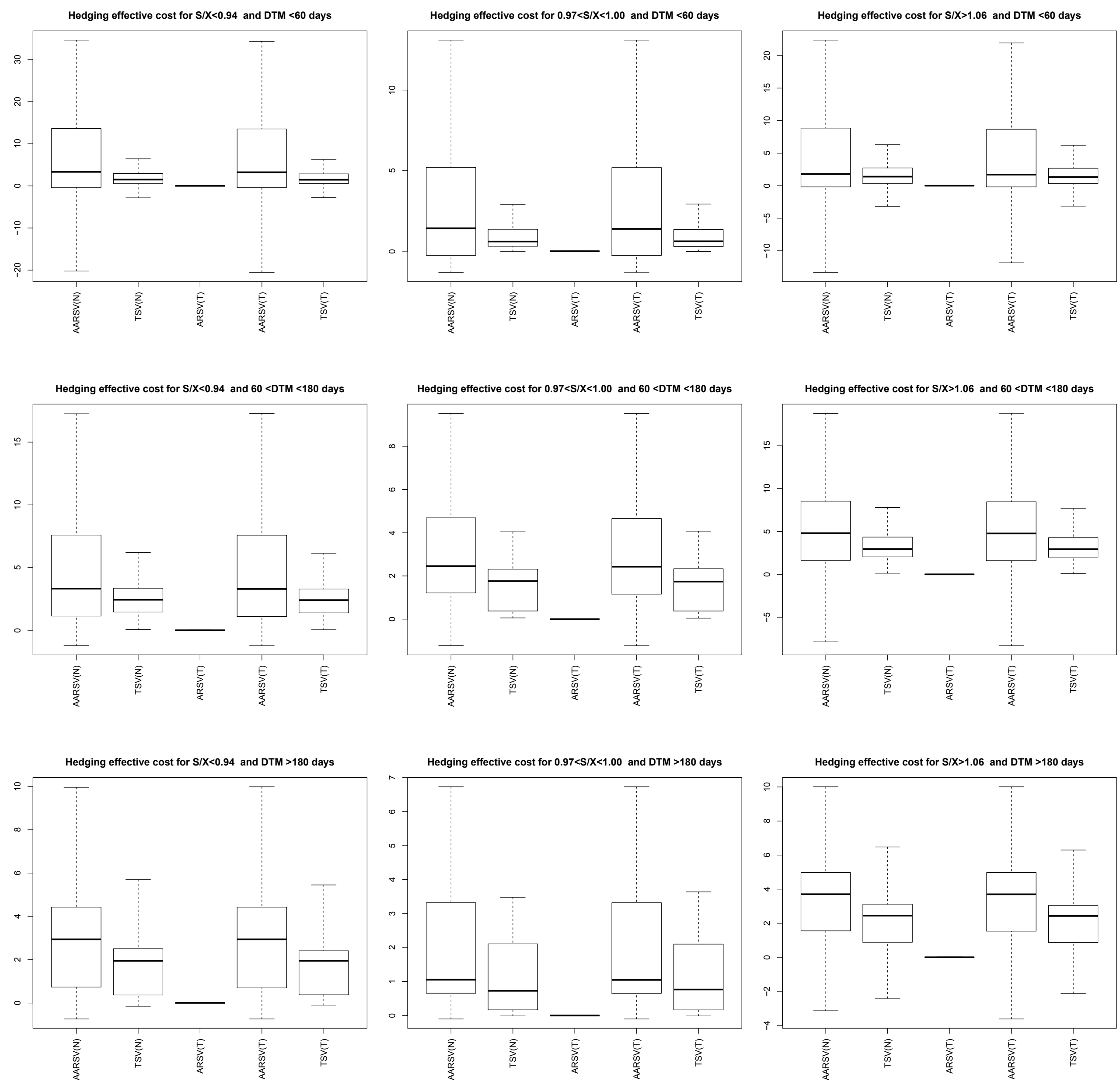


Figure 3. Distribution of the dynamic hedging relative to the simplest model for several maturities and moneyness for the S&P 400 index during the calm period.

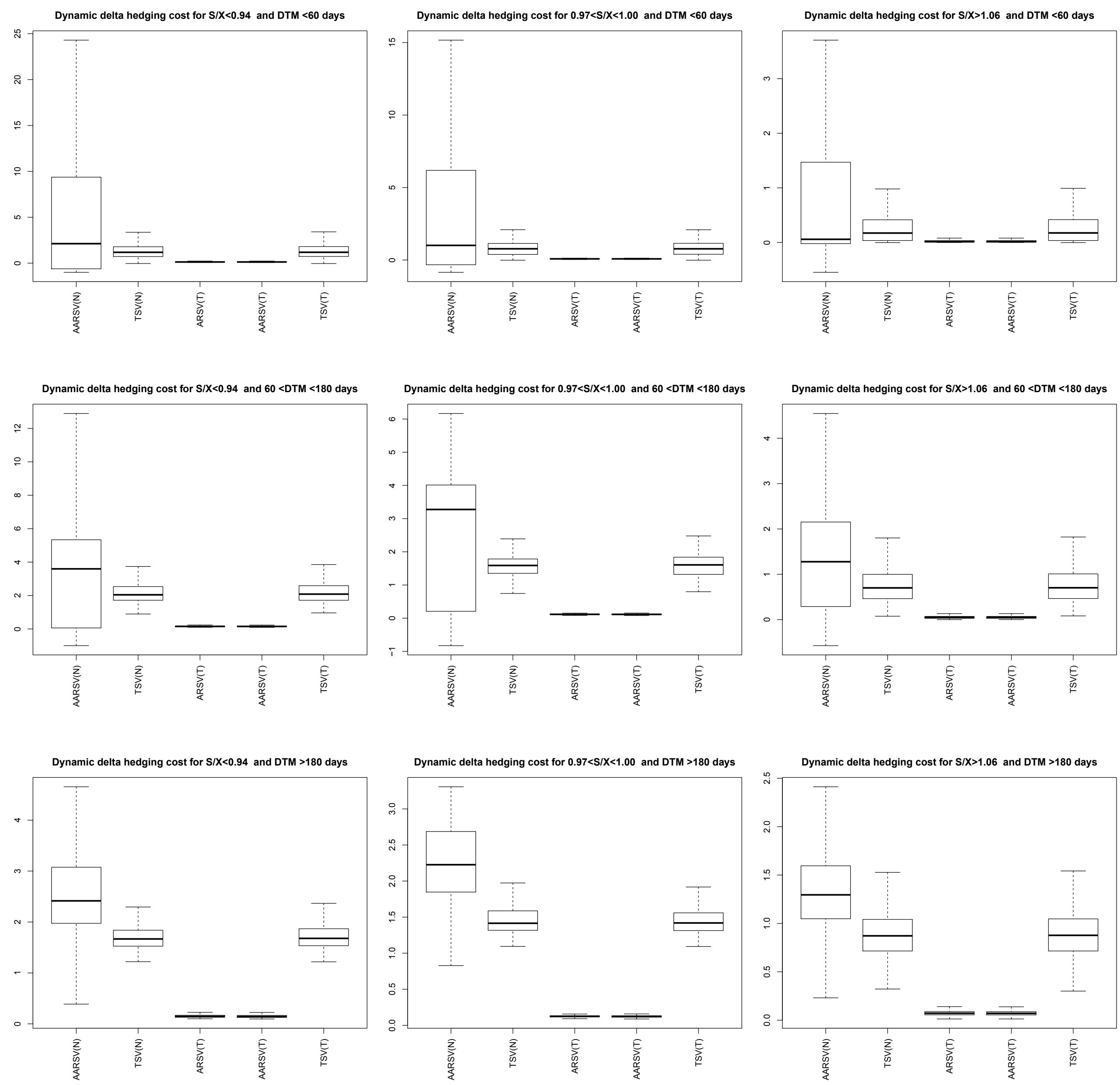


Figure 4. Distribution of effective cost relative to the simplest model for several maturities and moneyness for the S&P 500 index during the crisis period.

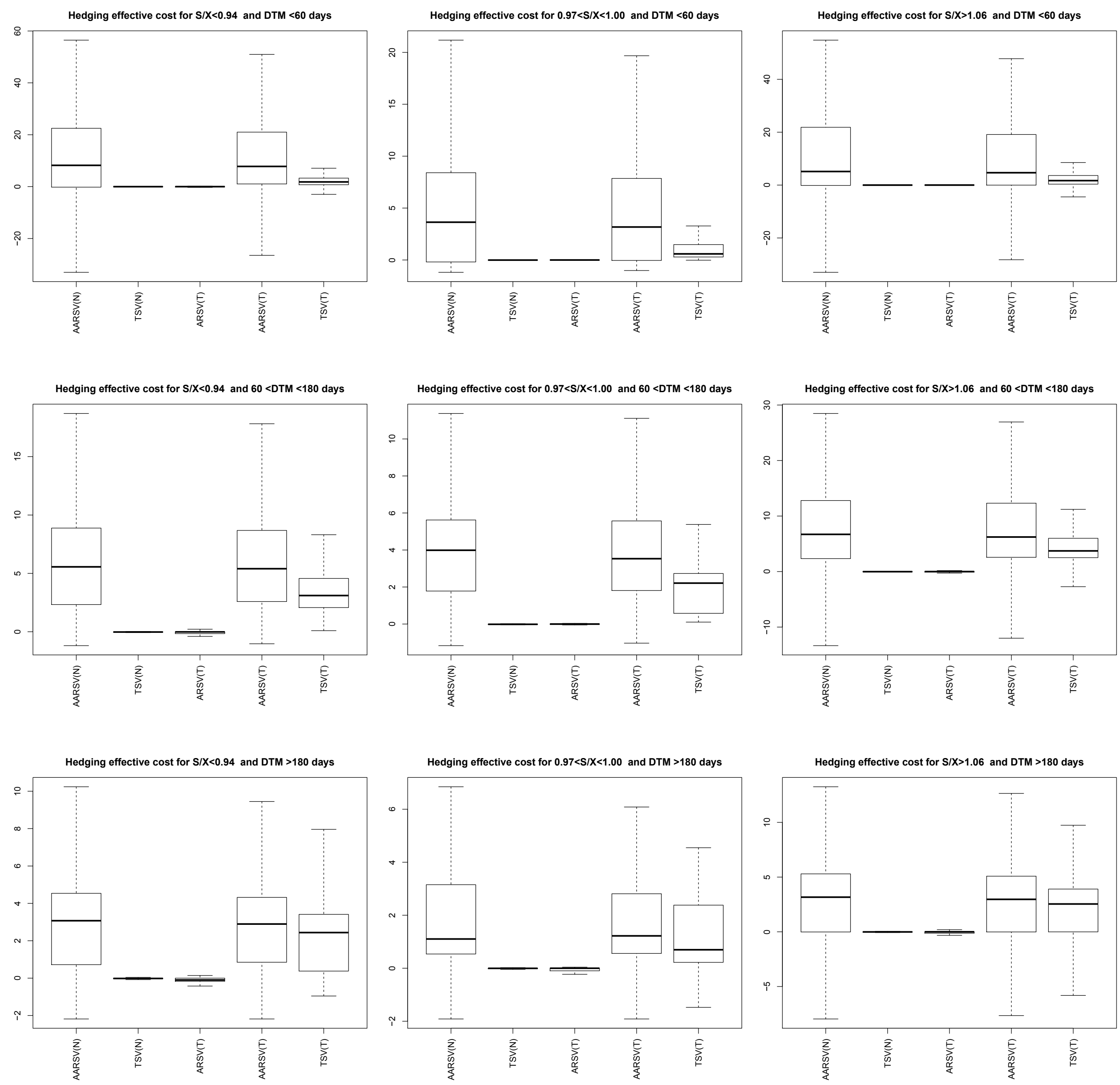
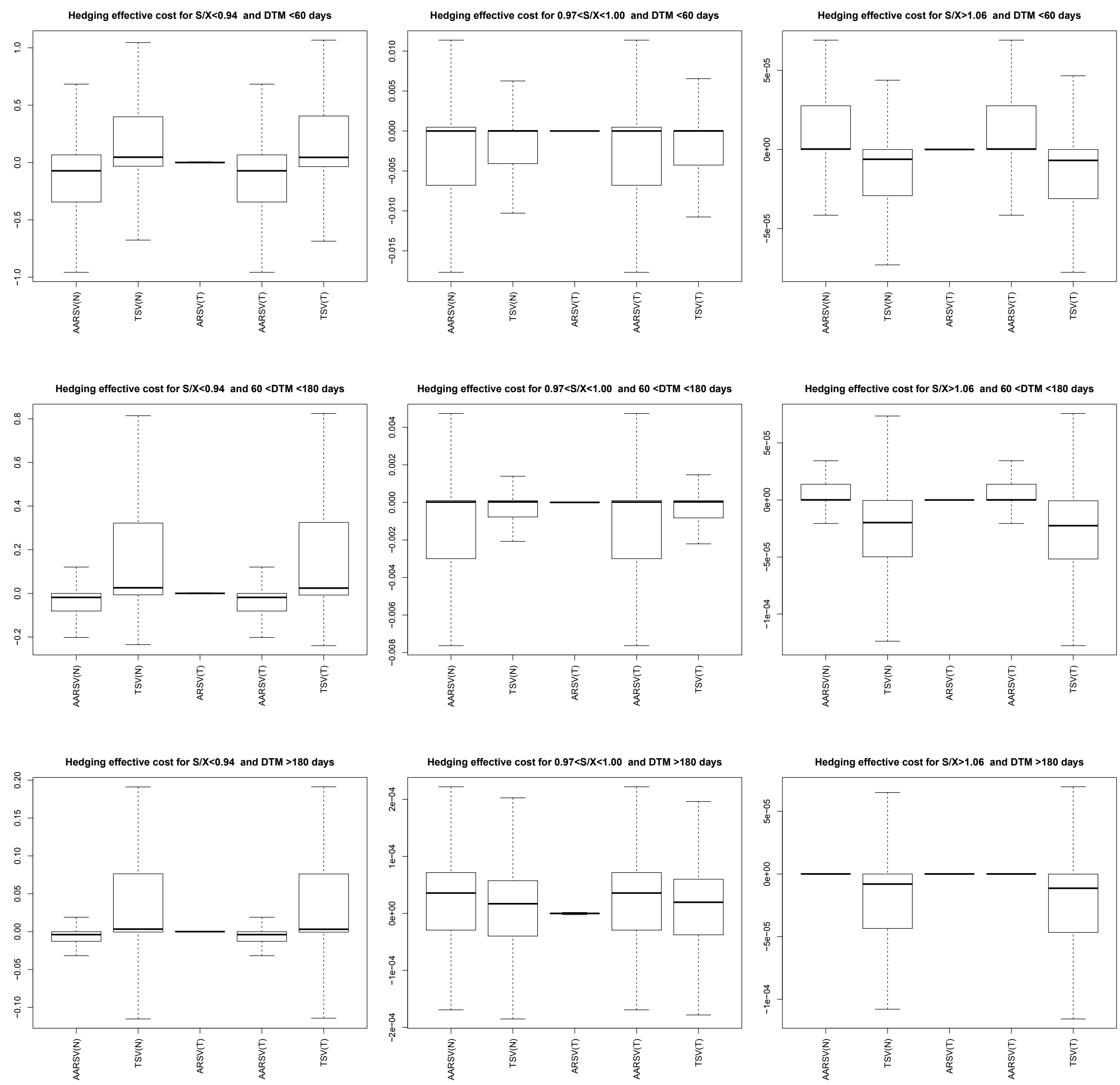


Figure 5. Distribution of effective cost relative to the simplest model for several maturities and moneyness for the S&P 500 index during the calm period.



Appendix

Table A.1
Description of sample periods.

	In-sample		Out-of-sample	
	crisis	calm	crisis	calm
S&P500	31/01/1996–31/12/2007	02/01/2002–31/12/2013	02/01/2008–31/12/2010	02/01/2014–30/01/2017
S&P400 MidCap	8/02/1996–31/12/2007	06/02/1998–31/12/2009	02/01/2008–31/12/2010	04/01/2010–31/12/2012

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