This is a postprint version of the following published document:

Amaris, H., Molina, Y. P., Alonso, M., Luyo, J. E. (2018). Loss Allocation in Distribution Networks Based on Aumann-Shapley. *IEEE Transactions on Power Systems*, 33(6), pp. 6656-6666.

DOI: https://doi.org/10.1109/TPWRS.2018.2844740

Loss Allocation in Distribution Networks Based on Aumann–Shapley

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***Abstract*—This paper outlines a procedure for loss allocation in both radial and meshed distribution networks with distributed generation that could be regulated in various ways. The method is analytically developed based on the theory of electrical circuits combined with game theory based on Aumann–Shapley, which guarantees both the electrical principles and the fair axioms of game theory. The proposed method obtains unitary participation coefficients for each network user based on the currents demanded/injected by each user and the network topology. The proposed allocation method based on Aumann–Shapley has been compared with other traditional allocation methods, is adaptable to distribution networks, and shows great potential and ease of implementation. Moreover, it can be applied to any kind of distribution network (radial or meshed) with distributed energy resources.**

***Index Terms*—Distributed energy resources, distributed generation, game theory, meshed networks.**

I. Introduction

ENERGY losses in electrical networks can be determined based on the difference between the energy generated at power stations and the energy measured at consumers' installations. In recent years, the difference between these values has increased, which entails an increase in electrical network losses. According to data from the 2014 World Bank Database [1], it can be observed that during the past decade, the average value of world energy losses has been slowly decreasing (8.85% in 2000, 8.47% in 2007 and 8.26% in 2014). These losses translate into a loss of energy efficiency in electrical networks and result in an increase in electricity prices for end users, who are the ones who must cover the network losses [2]. As such, it is imperative to apply measures that will help reduce network losses, since this improves the efficiency of electrical systems and the useful life of electrical infrastructures and at the same time translates into economic benefits for users.

Losses can be classified as technical losses due to dissipation of energy in lines and transformers and non-technical losses, which correspond to energy that is supplied but is not measured in client installations (because of errors in the meters or acts of electricity fraud).

Several publications in the scientific community propose measures for reducing the total values of losses [3], such as controlling the tap position of substation transformers or using power inverters for the regulation of reactive power.

In general, the method for reducing losses is by applying global control solutions at the Distribution System Operator (DSO) centers [4]. However, DSOs do not have any knowledge about the users (consumers, generators, prosumers) who are causing the increase in network losses. Knowing the locations of the customers who are responsible for the network losses will allow the DSO to apply specific local actions in those customer installations in order to reduce the network losses. These actions can be based on Demand Response strategies [5].

The research proposal sets out in this article is focused on allocating distribution network losses among network users, which is not a trivial process, as indicated in [6] and [7]. This problem has been studied extensively in the scientific literature and has been primarily applied to transmission networks, where the greatest interest lies in sharing the costs of the network among network users for economic purposes and electricity markets [8]. Currently, loss allocation in distribution networks are usually considered through electricity tariffs. In many countries, tariff designs are based on volumetric charges where each customer pays for the kilowatt-hours consumed without including the costs that each user inflicts on the grid. In general, there is no single accepted loss allocation method that can be applied in different countries. Moreover, in the majority of countries, network losses are hidden within the fixed and variable electricity fee. Very few countries, such as Hungary, include a loss distribution component in their electricity bill [9].

Differences in the distribution tariff schemes can be established even within one country. This is the case in Sweden, where each DSO is free to use the tariff design they prefer as long as that methodology is nondiscriminatory and objective [10].

It is not easy to compare the loss allocation methodologies that are applied in different countries due to the great diversity of tariff schemes and the lack of transparency about the methodology for calculating tariffs that each country uses [9]. However, some general conclusions can be highlighted:

- The majority of the distribution tariff schemes are based on the pro rata method, which is the simplest method but does not consider the network loss caused by each user.

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- Distributed generation (DG) is not required to pay distribution charges, as is the case in Germany, the Netherlands, Norway, Belgium, Poland and Portugal. The exception occurs in Great Britain, where embedded generators can be allocated negative charges if they are providing a benefit to the network [10].
- Losses due to reactive power flow through lines are not considered in the network losses.

The main difficulty for selecting the distribution allocation method is that network losses are non-linear functions of the injection power and power demand, which complicates the identification of the exact contribution of each network user to the total network losses.

As introduced in [11], "fair" distribution is difficult to achieve because almost all allocation methods entail a degree of arbitrariness.

The goal of a method for allocating losses is to share the total cost (losses) of the network among network users (both generators and consumers), in a fair way that reflects the real contribution of each user to the total network losses. Traditionally, methods of loss allocation have been applied to transmission networks, covering a wide range of methodologies with different levels of complexity. The simplest method is the pro rata method, where the cost is allocated to a client in proportion to their power connected to the network [12]. This method presents the problem of unfair distribution because it does not take into account the locations of the network clients, and as a consequence, it does not consider their contributions to the total losses.

For this reason, other methods based on circuits that consider the locations of users in the transmission networks have been proposed [13], [14]. However, the application of these methods to the distribution networks is not immediate, as the role of the swing/slang node is different in both networks.

Another proposal is the method of Marginal Loss Coefficients (MLC), which is based on distributing total line losses among the units connected to the network according to coefficients that consider the contribution of each unit (generator or consumer) to the total network losses [15]. These coefficients (MLCs) express the variation in the total active losses, L, due to the marginal variation in active power P_i and reactive power Q_i in each network node i. However, the application of MLC produces a nodal distribution loss that is approximately double that of the total network losses. As such, it is necessary to apply a reconciliation factor so that the total allocation loss to all users equals the total network loss.

The original proposal by [15] was intended for transmission networks, in which 50% of the total losses were allocated to the swing node. The main problems of applying this method to distribution networks are the following:

- In distribution networks, the lines are short, with a high R/X ratio, so the Newton-Raphson load flow solution methods might not converge.
- The computational complexity is high, as it is necessary to operate with the Jacobian matrix of the load flow.
- The resulting allocation does not correspond to a direct result of the formulation because it is necessary to apply the reconciliation and thus contains a factor of arbitrariness.
- It does not consider the contribution of reactive power in PV generation nodes.

The same authors proposed the method of Direct Loss Coefficients (DLC) [15], the goal of which is to obtain a direct relationship between line losses, L, and the power injection, avoiding the reconciliation process of the MLC method. This method proposes using a Taylor series expansion around the operation point to obtain the relation of losses, L, with nodal voltages: module (U) and phase (θ). Its problems include the computational complexity being high, as it is necessary to calculate the Hessian matrix, and the result of the allocation of losses containing inherent imprecisions due to the truncation error of the Taylor expansion.

The method based on impedance matrices was originally proposed for transmission networks by [16]. It is based on calculating the losses as the sum of nodal network losses, knowing the admittance matrix of the circuit and the solution of the load flow at a specific operation point. This method allocates 50% of the losses to generators and 50% to loads. However, this formulation cannot be applied in distribution networks.

Several studies related to loss allocation in distribution networks have been published in recent years. Some offer high computational complexity, which hinders their application to real-time situations [17]. Others are based on logarithmic schemes [18], which provide different results for the same problem when they are compared to other allocation methods [19]. The fact that different loss allocation methods provide different results for the same network means that some methods are not providing an accurate solution.

The Branch Current Decomposition (BCDLA) method was proposed by [19], and it is developed specifically for radial distribution networks. This method only requires the information of the network topology and the solution of the load flow. However, this methodology cannot be applied to weakly meshed distribution networks.

This article sets outs a method for loss allocation in distribution networks based on game theory. More specifically, it is based on Aumann-Shapley theory and circuit laws, offering an analytical solution that is easy to implement and results in a fair share among participants, such as: consumers, prosumers, Distributed Energy Resources (DER). These two aspects allow the loss allocation proposal to be applied in distribution networks (radial or meshed) where the real-time operation is needed.

The main contributions of the proposed method are as follows:

- The proposal of a new formula for allocating losses among participants (consumers, prosumers, DGs and DER) based on Aumann-Shapley in the distribution networks. The result is an analytical solution that is easy to implement in distribution networks (radial and meshed), offering low complexity.
- It considers the real and imaginary components of the current injections, so it identifies and quantifies the individual contribution of each user in terms of active and reactive power losses. This offers the advantage of differentiating

the participation of the DER that is operating in voltage control mode from another that works in power control mode.

- Losses allocated to DER and loads are independent, even when they are in the same bus.
- It considers circuit laws and at the same time has desirable characteristics in terms of economic coherence, because it is based on circuit laws in combination with Aumann-Shapley.
- The methodology offers consistency with different load levels, simultaneously allowing great accuracy and fair billing to the different types of users connected to the distribution network, such as consumers, prosumers and DER.

II. Methods Based on Game Theory

The problem of allocating losses among network users can be considered a cooperative game, which consists of sharing total network losses among a group of agents that either reduce or cause them. The goal of the organization is to find an efficient and fair allocation procedure, that is, with none of the participants favored at the expense of others and without anyone coming out of it harmed. Cooperative game theory has been applied to loss allocation in transmission networks [20]. To do this, generators, consumers and, in general, users of the electrical network are modeled as rational agents or players interested in forming groups and coalitions to obtain the maximum benefit in the final result. The solution provided by game theory is fair, efficient and stable [21]. Within this discipline, the most used methods are based on the Shapley value and the Aumann-Shapley method, described below.

A. Methods of Loss Allocation Based on the Shapley Value

The Shapley value method finds an expected marginal contribution to each player in the game with respect to a uniform distribution on the set of all permutations in the entry order of players to analyze all possible combinations in the game. The cost, the profit, the benefits or the participation of each agent is calculated when it is the first, second, third and so on. The average value of the incremental costs calculated in each permutation determines the cost that corresponds to each agent. Thus, the influence of the player entry order on the cost allocation is eliminated [22].

The Shapley value can be interpreted as being the average value of the incremental costs of including the agent. It considers all sub-coalitions that do not contain this particular agent, including the empty sub-coalition. Assuming that the probabilities of the occurrences of sub-coalitions of several sizes are the same, the allocation is formally defined through the analytical expression (1).

The Shapley value is based on the concepts of fairness and efficiency [21]:

- *Fairness*: Each player must receive an allocation according to their global contribution to the game.
- *Efficiency*: The sum of the distribution of losses among the participants must coincide with total network losses.

The Shapley value is defined by:

$$\Phi_i(v) = \sum_{S, i \in S} \frac{(|S|-1)!\,(n-|S|)!}{n!}\left[v(S) - v(S - \{i\})\right] \tag{1}$$

where:

- S: Coalition, defined as the group of agents that unite to obtain greater benefits than they would have if they went alone
- $|S|$: the number of elements of coalition S
- $v(S)$: value of the coalition
- i: player i-th
- n: number of players

Thus, the **Shapley value** of a player is defined as the average value of the allocations in all possible orders of player incorporation in the coalitions [23].

B. Methods of Loss Allocation by Aumann–Shapley

Robert J. Aumann and Lloyd Shapley proposed the Aumann-Shapley method to solve the problem of cost allocation by addressing smaller units of the same participant [24]. The idea of allocation by the Aumann-Shapley method consists of dividing the contribution of each player into infinitesimal parts. The number of permutations increases, so applying the previous Shapley method, [25], would complicate it.

However, working with incremental, infinitesimal units shows that it is possible to obtain a closed, direct solution without having to perform any permutation of players in the coalitions.

Now, the incremental cost of an infinitesimal agent can be approximated to its marginal cost. Furthermore, the number of permutations allows for the assumption that all of them have the same uniform probability of appearing, respecting the proportion of the size of the agents.

The Aumann-Shapley solution [26] is defined in the following equation:

$$\pi_k = \int_0^1 \frac{\partial S_{LOSS}(\lambda I)}{\partial I_k} d\lambda \tag{2}$$

where S_{LOSS} is the complex electrical losses, I_k is the injection current at node k, π_k represents the unitary Aumann-Shapley participation of element k, and λ is the integration variable, which ranges from 0-1. The losses associated with this agent are defined by:

$$L_k = I_k \pi_k \tag{3}$$

where L_k is the allocation of losses to agent k.

It can be noted that Aumann-Shapley calculates the average value of each player's incremental cost when it grows uniformly from zero to its current value. The parameters vary constantly in the interval [0,1], so the same proportion of division is applied in all the agents.

The Aumann-Shapley solution is fair and economically efficient since it uses information about marginal costs and fulfills the following axioms of game theory:

- *Symmetry*: two players that have the same contribution in the total network losses receive the same unitary participation.
- *Efficiency*: The sum of all losses allocated to each individual player coincides with the total network losses.
- *Additivity*: The sum of the losses allocated to a player that decides to play two games separately is equal to the loss allocated to this player when both games are played at the same time.
- *Monotonicity*: If network losses increase/diminish, losses allocated to players increase/diminish.

Several researchers have used Aumann-Shapley for cost distribution in transmission networks [22], [27], sharing 50% of the total network losses among the slack bus and generators and the other 50% among consumers. In distribution networks, this formulation is not acceptable because the slack node in these networks is the connection point to the grid and consequently has a loss allocation that is assumed to be zero.

The method proposed in this article solves the problem of loss allocation in distribution networks via Aumann-Shapley, identifying the individual participation of the power components (active and reactive) of the loads and DER in each node and each branch of the network, considering each component as an independent agent. Moreover, the properties of the Aumann-Shapley method guarantee equitable allocations.

III. Loss Allocation Among Participants

Complex power losses, S_{LOSS} (active, reactive) of a network with N nodes can be obtained from the following equation:

$$S_{LOSS} = \sum_{i=1}^{N} U_i I_i^* \tag{4}$$

where U_i is the voltage in the i-th node and the injection current to the node is I_i. Taking node 1 as the slack/swing node and the voltage reference, (4) can be expressed as shown in the following equation:

$$S_{LOSS} = U_1 I_1^* + \sum_{i=2}^{N} U_i I_i^* \tag{5}$$

considering that the nodal current I_1 is related to the nodal currents $\{2, \ldots, N\}$, as follows:

$$I_1 = -\sum_{i=2}^{N} I_i \tag{6}$$

and substituting in (5), results in the following expression:

$$S_{LOSS} = \sum_{i=2}^{N} (U_i - U_1) I_i^* \tag{7}$$

The nodal injection current can be calculated from the nodal voltages and the network admittance matrix:

$$I_i = \sum_{j=1}^{N} Y_{(i,j)} U_j \tag{8}$$

where the voltages can be referred to the swing node, obtaining:

$$I_i = \sum_{j=2}^{N} Y_{(i,j)} (U_j - U_1) \tag{9}$$

Solving the voltage vector U_i, $i \in \{2, \ldots, N\}$, the following expression is obtained:

$$U_i = U_1 + \sum_{j=2}^{N} Z_{(i,j)} I_j \tag{10}$$

where matrix Z represents the impedance matrix of the circuit without including node 1. Keep in mind that Z does not correspond to *Z-bus* [16], so it can be applied to any distribution network, avoiding the problem of the inversion of the singular admittance matrix.

A. Loss Allocation Due to the Real Component of the Current

This section explains the allocation of losses (active, reactive) among participants who are connected to distribution network nodes $k \in \{2, \ldots, N\}$. The distribution will be done considering the nodal injection currents, expressed in Cartesian components, that correspond to the consumption/generation by agents connected to node k.

The unitary participation (2) of the active component of the nodal current in bus k is:

$$\pi_{loss,k}^{re} = \int_0^1 \frac{\partial S_{LOSS}(\lambda I)}{\partial I_k^{re}} d\lambda \tag{11}$$

On the basis of (7), it can be expressed as:

$$\pi_{loss,k}^{re} = \int_0^1 \frac{\partial}{\partial I_k^{re}} \left(\sum_{i=2}^{N} (U_i - U_1)(\lambda I_i^*) \right) d\lambda \tag{12}$$

Keeping in mind (10) and decomposing the nodal current at node k in its real and imaginary components, the partial derivatives of the voltage with respect to the current can be obtained as:

$$\frac{\partial U_k}{\partial I_k^{re}} = \frac{\partial}{\partial I_k^{re}} \left(U_1 + \sum_{j=2}^{N} Z_{(i,j)} I_j \right) = Z_{(k,i)} \tag{13}$$

Substituting in (12):

$$\pi_{loss,k}^{re} = \int_0^1 2 \sum_{j=2}^{N} Z_{(j,k)} \left(\lambda I_j^{re} \right) d\lambda \tag{14}$$

results in:

$$\pi_{loss,k}^{re} = \sum_{j=2}^{N} Z_{(j,k)} I_j^{re} \tag{15}$$

Finally, the total losses (3) due to the active component of the nodal current k can be calculated by the following expression:

$$L_k^{re} = \pi_{loss,k}^{re} I_k^{re} = I_k^{re} \sum_{j=2}^{N} Z_{(j,k)} I_j^{re} \tag{16}$$

B. Loss Allocation Due to the Imaginary Component of the Current

Likewise, the unitary participation of the reactive component of the current in bus k is:

$$\pi_{loss,k}^{im} = \int_0^1 \frac{\partial S_{LOSS}(\lambda I)}{\partial I_k^{im}} d\lambda \quad (17)$$

Keeping in mind (7) and substituting the partial derivatives of the voltage with respect to the current:

$$\pi_{loss,k}^{im} = \int_0^1 2 \sum_{j=2}^{N} Z_{(j,k)} \left(\lambda I_j^{im}\right) d\lambda \quad (18)$$

integrating:

$$\pi_{loss,k}^{im} = \sum_{j=2}^{N} Z_{(j,k)} I_j^{im} \quad (19)$$

Therefore, the expression for total losses due to the imaginary component of the nodal current k is obtained:

$$L_k^{im} = \pi_{loss,k}^{im} I_k^{im} = I_k^{im} \sum_{j=2}^{N} Z_{(j,k)} I_j^{im} \quad (20)$$

C. Complex Loss Allocation

Once the contributions of losses due to the active and reactive components of the nodal currents are known, the total nodal loss based on the sum of both individual contributions L_k^{re} and L_k^{im} can be obtained:

$$L_k = L_k^{re} + L_k^{im} = \sum_{j=2}^{N} Z_{(j,k)} \left(I_k^{re} I_j^{re} + I_k^{im} I_j^{im}\right) \quad (21)$$

It is important to highlight that there is a connection between the active and reactive power in line losses. That is, the flow of active (reactive) power through the lines will produce not only active (reactive) losses but also reactive (active) losses. As such, the flow of active power influences reactive losses like the flow of reactive power influences active losses. This connection cannot be ignored in the process of allocating losses [28].

D. Loss Allocation at Nodes With Load and Generation

At those network nodes where both loads and generators are connected, it will be necessary to distribute the loss allocated to the node among the different agents that are connected to it.

Considering that two agents in node k are connected (load and generation), it can be said that S_{Lk} is the demand for the load connected at node k and that S_{Gk} is the power of the DG unit located at the same node (see Fig. 1). The resulting complex power injection at node k will be $S_k = P_k + jQ_k$ and corresponds to the sum of the power demanded by the load and the power injected by the DG source. The nodal injection current I_k will be the sum of the injection current due to the generation and the load as:

$$I_k = I_{Gk} + I_{Lk} \quad (22)$$

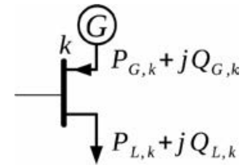

Fig. 1. Node k with load and generation.

Applying the same nodal unitary participation ($\pi_{loss,k}^{re}$, $\pi_{loss,k}^{im}$) to the generation and the load, the following holds:

- Losses associated with the DG: Losses owing to the real component of the current injected by the generator are $L_{Gk}^{re} = \pi_{loss,k}^{re} I_{Gk}^{re}$, and those that correspond to the imaginary component are $L_{Gk}^{im} = \pi_{loss,k}^{im} I_{Gk}^{im}$, hence, the resulting total loss for the DG source will be:

$$L_{Gk} = L_{Gk}^{re} + L_{Gk}^{im} \quad (23)$$

- Losses associated with the load: Likewise, losses owing to the real load current are $L_{Lk}^{re} = \pi_{loss,k}^{re} I_{Lk}^{re}$, and the contribution owing to the imaginary component is $L_{Lk}^{im} = \pi_{loss,k}^{im} I_{Lk}^{im}$, so the total loss allocated to the load is:

$$L_{Lk} = L_{Lk}^{re} + L_{Lk}^{im} \quad (24)$$

Note that at node k, it holds that the total loss associated with node k (21) is the sum of the losses allocated to the generation and the load:

$$L_k = L_{Gk} + L_{Lk} \quad (25)$$

IV. Branch Loss Allocation Among Participants

In this case, branch losses are allocated among participants, (26) where U_l is the voltage drop of the l branch and z_l is the branch series impedance:

$$S_{loss,l} = U_l I_l^* = z_l I_l I_l^* \quad (26)$$

The branch current I_l is defined as a linear combination of nodal injection currents, that is:

$$I_l = \alpha_{l,2} I_2 + \ldots + \alpha_{l,N} I_N = \sum_{j=2}^{N} \alpha_{(l,j)} I_j \quad (27)$$

Consequently, complex power losses associated with branch l (26) can be expressed according to the nodal currents:

$$S_{loss,l} = z_l \left(\sum_{j=2}^{N} \alpha_{(l,j)} I_j\right) \left(\sum_{j=2}^{N} \alpha_{(l,j)} I_j\right)^* \quad (28)$$

A. Branch Loss Allocation Due to the Real Component of Nodal Currents

The active unitary participation associated with branch l due to the nodal current in bus k is expressed as:

$$\pi_{loss,l,k}^{re} = \int_0^1 \frac{\partial S_{LOSS}(\lambda I)}{\partial I_k^{re}} d\lambda \quad (29)$$

Deriving (28) with respect to I_k^{re} allows the unitary participation to be expressed as:

$$\pi_{loss,l,k}^{re} = \frac{1}{2} z_l \left[\alpha_{(l,k)} \sum_{j=2}^{N} \alpha_{(l,j)}^{*} I_j^{*} + \alpha_{(l,k)}^{*} \sum_{j=2}^{N} \alpha_{(l,j)} I_j \right] \tag{30}$$

B. Branch Loss Allocation Due to the Imaginary Component of Nodal Currents

In the same way, for the imaginary component of the current:

$$\pi_{loss,l,k}^{im} = \int_0^1 \frac{\partial S_{LOSS}(\lambda I)}{\partial I_k^{im}} d\lambda \tag{31}$$

Deriving (28) with respect to I_k^{im}, the unitary participation is expressed as:

$$\pi_{loss,l,k}^{im} = j\frac{1}{2} z_l \left[\alpha_{(l,k)} \sum_{j=2}^{N} \alpha_{(l,j)}^{*} I_j^{*} - \alpha_{(l,k)}^{*} \sum_{j=2}^{N} \alpha_{(l,j)} I_j \right] \tag{32}$$

C. Complex Branch Loss Allocation

Once the active and reactive unitary participation coefficients for branch l are known, the branch loss allocation can be obtained by (3). Equations (30) and (32) depend on the coefficients α, which are directly calculated by using the nodal voltages and branch impedance, according to (33):

$$I_l = \frac{1}{z_l} (U_k - U_m) = \frac{1}{z_l} \sum_{j=2}^{N} \left(Z_{(k,j)} - Z_{(m,j)} \right) I_j \tag{33}$$

Equating the previous equation to (27), it can be deduced that coefficients α depend only on the network topology as:

$$\alpha_{(l,i)} = \frac{1}{z_l} \left(Z_{(k,i)} - Z_{(m,i)} \right) \tag{34}$$

Consequently, the loss allocation for l branch due to the i-th nodal current is obtained by the following expression:

$$L_{loss,l,i} = \pi_{loss,l,i}^{re} I_i^{re} + \pi_{loss,l,i}^{im} I_i^{im} \tag{35}$$

which only depends on the impedance branch and nodal currents.

V. Features of the Loss Allocation Formulation

A. Algorithm and Computational Complexity

The proposed loss allocation formulation offers two possibilities, to compute the loss allocation among network users (consumer, DG, DER) and/or to compute the branch loss allocation among users. Both possibilities are shown in the pseudo-code detailed in Algorithm 1.

Notably, the aforementioned method provides an analytical solution, requiring only the nodal currents from each agent, and the data of the network. From a complexity point of view, the loss allocation algorithm requires $3(N-1)$ multiplications and $2(N-1)$ additions, N being the number of nodes of the network, which leads to an order of complexity $\mathcal{O}(1)$. In contrast, Shapley value-based allocation methods requires 2^n computations, where n is the number of players. Methods based on derivatives [15] rely on computations of Jacobian and Hessian matrices which increase the order complexity when compared to the proposed method.

If the currents (real and imaginary components) are not available, it is necessary to first compute them, by applying load flow or state estimation algorithms. The impedance matrix (without the slack bus) is computed only once for a fixed topology. Whenever the topology changes, the impedance matrix has to be updated.

Algorithm 1: Distribution Loss Allocation Aumann-Shapley

Input: Network Topology Data:
- N number of Nodes
- N_p number of Users (Consumers, DG, D-STATCOM, etc.)
- N_b number of Branches
- Z Network impedance matrix (without the slack bus)

Input: Users Nodal Currents Matrix: $I_{j,u}$ (N x N_p)

```
I_j^re ← Σ_u I_{j,u}^re          ▷total nodal real current
I_j^im ← Σ_u I_{j,u}^im          ▷total nodal imag current
if is Node Loss Allocation required then
    call Node_Loss_Allocation ()
if is Branch Loss Allocation required then
    call Branch_Loss_Allocation ()
```

Algorithm 2: `Node_Loss_Allocation ()`

```
for node = 2, N do
    Compute (15)          ▷real nodal unitary participation
    Compute (19)          ▷imag nodal unitary participation
for node = 2, N do
    for user = 1, N_p do
    Compute (21)          ▷complex loss allocation
return
```

Algorithm 3: `Branch_Loss_Allocation ()`

```
for branch = 1, N_b do
    for node = 2, N do
        Compute (34)          ▷branch coefficients
    Compute (30)          ▷real branch unitary participation
    Compute (32)          ▷imag branch unitary participation
for branch = 1, N_b do
    for user = 1, N_p do
        Compute (35)          ▷branch loss allocation to user
return
```

B. Assumptions

The main assumptions considered are the following:

- The active and reactive currents of users independently participate in the allocation of losses.
- Distribution network losses are allocated among network participants (customers, prosumers, DER units).
- Loss allocations at nodes with load and generation will be considered different agents.

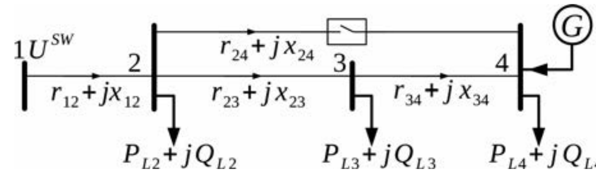

Fig. 2. Four-node network with generation and three loads.

C. Application

It must be highlighted that the computational complexity of the proposed formula depends only on the number of nodes. It is not affected by the number of existing loops in the network or by the number of network users. Consequently, it can be easily applied to large-scale networks (radial or/and meshed).

The proposed loss allocation method can be applied to distribution networks with many DERs, such as: converters, distributed energy storage devices and reactive power compensators. Once the DER nodal currents are available (by measurement or power flow solutions) the proposed loss allocation method will assign the corresponding losses to the DER owner using only the currents (real, imaginary) and the network topology.

The formulation is designed to grid connected distribution networks. When microgrids are operating in grid-connected mode, the proposed loss allocation method can be directly applied. However, the operation mode of microgrids in islanded mode requires a different formulation in order to maintain the distinction among the slack node and the other nodes when disconnecting from the main grid.

D. Consistency for Different Load Levels

The formulation offers consistency for different load levels, which can be proved mathematically as follows:

Considering a fixed network topology and knowing the nodal currents (real, imag), complex losses can be obtained by (21).

If all users increase/reduce their current in the same factor ρ, such as, $\widehat{I_k^{re}} = \rho I_k^{re}$ and $\widehat{I_k^{im}} = \rho I_k^{im}$, then losses allocated to each user increase/reduce their value proportionally:

$$\widehat{L_k} = \sum_{j=2}^{N} Z_{(j,k)} \left(\widehat{I_k^{re}}\,\widehat{I_j^{re}} + \widehat{I_k^{im}}\,\widehat{I_j^{im}}\right) = \rho^2 L_k \qquad (36)$$

which means that the share of losses among all network users for different loads levels is determined proportionally by the factor ρ^2, whatever their original current value.

E. Comparison With Other Loss Allocation Methods Based on Game Theory

In terms of equity, the proposed method guarantees an equitable solution by complying with the axioms of symmetry, efficiency, additivity and monotonicity, which are inherent when the Aumann-Shapley method is applied. Compared with the Shapley methods [25], the Aumann-Shapley method offers the advantage of guaranteeing isonomy with respect to the size of agents.

The proposed method is designed specifically for distribution networks with DER, and it assumes that 100% of the losses are the responsibility of the network users, such as: DER, customers and prosumers. Therefore, every individual user is treated as a complex current/voltage source, and the analytical solution obtained starts from (2). It considers the real and imaginary components of the current injections and then identifies and quantifies the individual contribution into active and reactive power losses. Moreover, this complex loss allocation formulation considers the real current component contribution to reactive losses, as well as the imaginary current component contribution to active losses.

However, the method presented in [29] has been designed for transmission networks and involves two steps; in the first step, the generators are modeled as current sources and the loads as constant admittances and in the second step, the generators are modeled as constant admittances and the loads as current sources. In this way, it is possible to allocate losses by 50% for loads and 50% for generators, which is one of the assumptions of [29].

VI. Case Studies

In this section, the proposed method will be applied to two examples of distribution networks, and the validity of the method will be verified by comparing it with other existing methods.

A. Four-Node Distribution Network With Distributed Generation

Fig. 2 shows a four-node distribution network with a tie-line switch (between node 2 and node 4), which can be operated in radial form (switch open) or meshed form (switch closed). Node 1 is the swing node with a voltage $U_{\text{sw}} = 1.01$ [p.u]. At nodes 2, 3 and 4, three equal loads (PQ) are connected with power demand $S_L = 0.5 + j0.3$ [p.u]. At node 4, a DG source is connected and can operate in two modes of operation. The four branch impedances are equal to a value $z_l = 0.02 + j0.01$ [p.u].

This four-node network will be used as a network type to explain the procedure for executing the proposed method by means of the following cases:

- case a) radial network (switch open): DG behaves like node PQ, it only injects $P = 1.0$ [p.u], ($Q = 0$ [p.u])
- case b) radial network (switch open): DG behaves like node PV, $P = 1.0$ [p.u], $U_4 = 1.01$ [p.u]

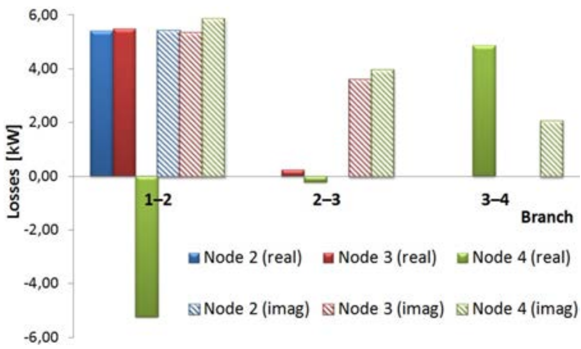

Fig. 3. Branch losses case a: radial network, (solid) nodal active current contribution, (hatched) nodal reactive current contribution.

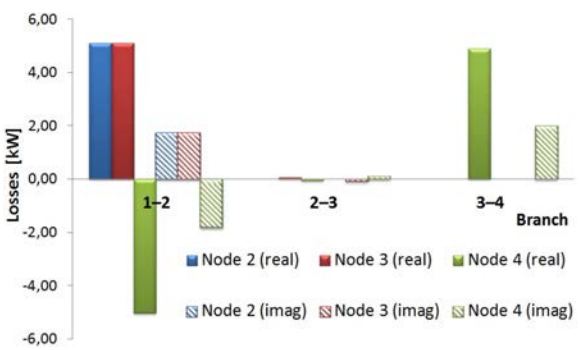

Fig. 4. Branch losses case b: radial network, (solid) nodal active current contribution, (hatched) nodal reactive current contribution.

- case c) meshed network (switch closed): DG behaves like node PQ, it only injects $P = 1.0$ [p.u], ($Q = 0$ [p.u])

1) Case a: In this case, the tie-line switch is open, and the network is operated in radial form. The DG connected at node 4 injects only active power, but loads at nodes 2, 3 and 4 demand both active and reactive power. It can be noted in Fig. 3 that total active power demand at node 4 (load and DG) increases losses through branch $3-4$, but its contribution to branch $1-2$ losses is negative, which means that the DG injection at node 4 is helping to reduce the losses in this branch $1-2$. The contribution of DG to branch $2-3$ losses is irrelevant, as it neither increases nor decreases losses in this branch. Because DG is injecting only active power, reactive power demand from loads 2, 3 and 4 must be totally provided by the grid. Consequently, this reactive power demand by load 4 increases losses at branches $1-2$, $2-3$ and $3-4$. The same occurs with the reactive power demand from loads 2 and 3.

2) Case b: In this case, the network is operated in radial form. It can be observed (Fig. 4) that in case b), there is a reduction in losses because the generator at node 4 operates as a node PV, injecting reactive power to maintain the nodal voltage at the set point ($U_4 = 1.01$ [pu]). In this case, reactive power generation by DG connected at node 4 slightly increases the losses at branch $3-4$, but it has a negative influence in branch $1-2$ losses.

3) Case c: This third case is similar to case a), with the exception that the tie-line switch between nodes 2 and 4 is closed, and consequently, the network is meshed with a loop. In Fig. 5, it can be seen that closing the switch increases the real losses of branches $2-3$ and $3-4$ due to the load demand connected at node 3. In case a), the load connected at node 3 increases losses only in the first branch $1-2$. Meanwhile, in this case, the load connected at node 3 is also increasing losses in branches $2-3$ and $3-4$. However, the load at node 3 is helping to reduce losses in branch $2-4$. In contrast, the power at node 4 (load and DG) is responsible for increasing the losses at branch $2-4$.

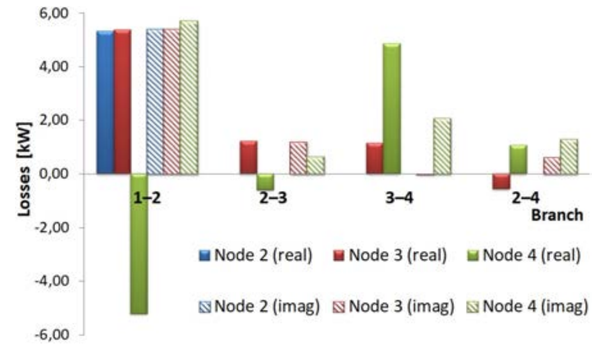

Fig. 5. Branch losses case c: meshed network, (solid) nodal active current contribution, (hatched) nodal reactive current contribution.

TABLE I
NODAL LOSS ALLOCATION [kW]

Node	Proposed			BCDLA			MLC			DLC		
	a	b	c	a	b	c	a	b	c	a	b	c
2	10.8	6.8	10.7	10.8	6.8	NA	11.0	7.0	10.8	11.1	7.0	11.0
3	14.6	6.8	14.3	14.6	6.8	NA	15.0	7.0	14.5	15.1	7.0	14.7
4	11.2	0.1	3.4	11.2	0.1	NA	10.7	NA	3.7	10.6	NA	3.6
Total	36.6	13.7	29.0	36.6	13.7	NA	36.6	NA	29.0	36.8	NA	29.3

4) Comparative Analysis: At this point, it is interesting to compare the results of the proposed loss allocation method based on Aumann-Shapley with those obtained when applying other methods, such as BCDLA, MLC and DLC. These results are shown in Table I.

- When dealing with radial networks and DGs operating in power control mode (case a), the four allocation methods produce quite similar results.
- Derivative methods such as MLC and DLC offer equivalent results, provided that generation is treated as node PQ, but they do not behave well with PV nodes in voltage support mode (case b).
- In meshed networks (case c), BCDLA cannot be applied.
- The proposed loss allocation methodology is the one that can be applied to meshed networks (case c), in contrast to BCDLA, and to voltage support buses (case b) in contrast to MLC and DLC.

B. 33-Node Distribution Network With Distributed Energy Resources

In this case, we use the IEEE 33-node distribution network proposed in [30], where the basic network has been modified to include DER units: DG and Distribution Static Synchronous

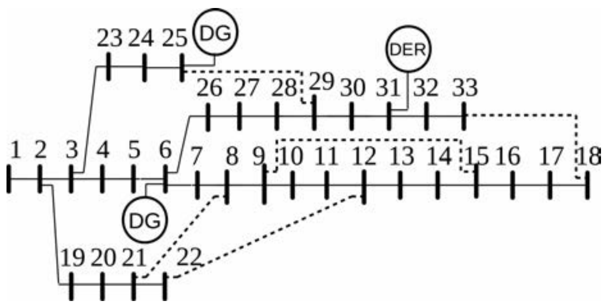

Fig. 6. IEEE-33 node distribution network with DER.

TABLE II
IEEE-33 Radial Distribution Network Loss Allocation

			Load Loss Assigment [kW]					
	Load		Without DG (case A)			With DG (case B)		
Node	**P[kW]**	**Q[kvar]**	**proposed**	**BCDLA**	**[31]**	**proposed**	**BCDLA**	**[31]**
2	100	60	0.31	0.31	0.31	0.06	0.06	0.07
3	90	40	1.51	1.51	1.63	0.13	0.13	0.23
4	120	80	3.25	3.25	3.25	0.27	0.27	0.51
5	60	30	1.97	1.97	2.10	0.08	0.08	0.25
6	60	20	2.73	2.73	3.23	-0.07	-0.06	0.28
7	200	100	10.34	10.34	11.20	0.35	0.35	1.59
8	200	100	11.49	11.49	12.35	1.35	1.35	2.58
9	60	20	3.53	3.53	4.12	0.62	0.62	1.04
10	60	20	3.88	3.88	4.50	0.91	0.91	1.39
11	45	30	3.46	3.46	3.37	0.95	0.95	1.15
12	60	35	4.56	4.56	4.62	1.29	1.29	1.60
13	60	35	4.96	4.96	4.98	1.63	1.63	1.92
14	120	80	10.56	10.56	10.12	3.66	3.66	4.05
15	60	10	4.24	4.24	5.35	1.35	1.35	2.01
16	60	20	4.7	4.70	5.37	1.61	1.61	2.11
17	60	20	4.81	4.81	5.48	1.70	1.70	2.21
18	90	40	7.67	7.67	8.18	2.80	2.80	3.37
19	90	40	0.3	0.30	0.32	0.08	0.09	0.11
20	90	40	0.58	0.58	0.60	0.36	0.36	0.38
21	90	40	0.63	0.63	0.65	0.41	0.41	0.43
22	90	40	0.67	0.67	0.69	0.45	0.46	0.47
23	90	50	1.92	1.92	1.95	0.20	0.20	0.26
24	420	200	11.25	11.25	11.71	1.08	1.08	1.03
25	420	200	12.58	12.58	13.03	0.07	0.07	0.02
26	60	25	2.98	2.98	3.39	0.03	0.03	0.42
27	60	25	3.14	3.14	3.59	0.11	0.11	0.80
28	60	20	3.53	3.53	4.38	0.29	0.29	1.21
29	120	70	9.11	9.11	10.17	1.55	1.55	3.42
30	200	600	37.86	37.86	22.55	14.64	14.64	21.75
31	150	70	11.87	11.87	13.98	1.78	1.97	4.68
32	210	100	16.89	16.89	19.77	2.69	2.69	6.60
33	60	40	5.38	5.38	5.75	1.08	1.08	2.30
			DG Loss Assigment [kW]					
	DG case B)		Without DG (case A)			With DG (case B)		
Node	**P[kW]**	**Q[kvar]**	**proposed**	**BCDLA**	**[31]**	**proposed**	**BCDLA**	**[31]**
6	2044	989.93				-0.04	-0.05	-13.77
25	695.87	521.9		0		-0.05	-0.05	-1.26
31	520.8	0				0.03	-0.16	-11.78

Compensator (D-STATCOM), as indicated in Fig. 6. The network is a 12.66 kV system, and it includes five looping branches (tie-lines) that allow the network to be operated in radial form (open switches) or meshed form (closed switches). The data for demand and generation are shown in Table II and coincide with the data indicated in [31].

The following cases of operation are considered:

- Case A: Radial network without DER.

TABLE III
IEEE-33 Tie-Lines

	S1	S2	S3	S4	S5
node from	8	9	12	18	25
node to	21	15	22	33	29

- Case B: Radial network with DGs.
- Case C: Meshed network with DGs and D-STATCOM.
- Case D: Meshed network with DGs, D-STATCOM and different load levels.

1) Case A: In this case, there is no DER unit connected to the network, and all the tie-lines are open. In case A network losses are 202.677 kW and all the losses allocated to nodes are positive (Table II), which indicates that users contribute to increasing them. The three methods (the proposed method, BCDLA and [31]) give similar results when the network is operated radially and there is no DER.

2) Case B: In case B, the network is radial, but there are three DGs units located at nodes 6, 25 and 31. In case B network losses diminish substantially (43.430 kW), so loss allocations to each user are, in general, reduced due to the participation of DGs. It is seen at node 6 (Table II), that both the load demand and the generation contribute to reduce losses and their loss allocation is negative. Similar results are obtained by the BCDLA. However, it must be noted that the method proposed by [31] gives higher losses to the DG compared to the losses allocated by the proposed method and by the BCDLA.

At node 25, negative losses (indicating gain) are allocated to the DG, but the allocation of losses to the load of that node is still positive.

The location of the third DG source at node 31, which injects only active power, has a positive influence in the network losses, which indicates that part of the power it injects produces losses in the network. It is important to note that the method proposed by [31] allocates higher negative losses to DG connected at node 31, showing an opposite behavior compared to BCDLA and with the proposed method in this paper.

3) Case C: This case corresponds to the 33-node meshed distribution network, which can include up to five loops (Table III). It must be highlighted that because the distribution network is meshed, the BCDLA and [31] allocation methods cannot be applied, making the proposed method the only one that can provide the loss allocation solution.

The load and production data coincide with the data used in case B with the exception of the DG located at node 31, which is replaced by a D-STATCOM of 500 kvar in order to prove how the proposed method behaves with reactive power compensators and meshed networks. DG data located at nodes 6 and 21 have the same values of case B.

Table IV shows the total network losses and the allocation results for the different meshed situations when there is only a single loop up to five loops (totally meshed). In general, as the number of loops increases, total network losses are reduced, and consequently, losses allocated to each participant are reduced.

The more relevant results appear at nodes 6, 21, and 31 where there are DER units. It can be noted that the D-STATCOM

TABLE IV
IEEE-33 Meshed Distribution Network Loss Allocation

	Load loss assigment [kW]				
Node	**S5-S2-S3-S1-S4**	**S5-S2-S3-S1**	**S5-S2-S3**	**S5-S2**	**S5**
2	0.07	0.07	0.07	0.07	0.07
3	0.20	0.19	0.18	0.22	0.22
4	0.21	0.20	0.17	0.26	0.27
5	0.05	0.05	0.02	0.09	0.09
6	-0.08	-0.08	-0.11	-0.02	-0.01
7	-0.01	-0.01	-0.16	0.21	0.23
8	0.83	0.88	0.67	1.19	1.22
9	0.46	0.48	0.45	0.66	0.67
10	0.54	0.57	0.55	0.80	0.96
11	0.45	0.48	0.47	0.68	0.85
12	0.60	0.64	0.63	0.93	1.20
13	0.71	0.77	0.75	1.03	1.53
14	1.51	1.64	1.58	2.14	3.38
15	0.65	0.69	0.67	0.90	1.49
16	0.74	0.80	0.77	1.02	1.65
17	0.81	0.89	0.86	1.11	1.75
18	1.26	1.43	1.39	1.77	2.77
19	0.11	0.11	0.11	0.10	0.10
20	0.45	0.46	0.52	0.38	0.38
21	0.51	0.53	0.61	0.43	0.43
22	0.64	0.67	0.72	0.47	0.47
23	0.36	0.35	0.33	0.38	0.39
24	2.86	2.74	2.61	2.95	2.97
25	2.82	2.64	2.48	2.90	2.92
26	-0.03	-0.03	-0.06	0.03	0.03
27	0.05	0.04	0.01	0.10	0.10
28	0.30	0.28	0.25	0.33	0.33
29	0.99	0.92	0.87	1.01	1.01
30	3.80	3.22	3.02	3.42	3.44
31	1.93	1.77	1.70	1.87	1.88
32	2.86	2.62	2.52	2.77	2.78
33	0.86	0.77	0.74	0.81	0.82
	DER loss assigment [kW]				
Node	**S5-S2-S3-S1-S4**	**S5-S2-S3-S1**	**S5-S2-S3**	**S5-S2**	**S5**
6	3.07	3.05	4.37	1.00	0.84
25	-4.91	-4.51	-4.20	-4.96	-5.00
31	-0.94	-0.29	-0.17	-0.34	-0.35
	Network Losses [kW]				
Total	24.75	25.05	25.36	26.69	31.86

located at bus 31 reduces network losses for all situations from one loop to five loops. In contrast, DG located at bus 6 increases losses, and consequently, its loss allocation is always positive. According to the customers, it can be noted that customers near the D-STATCOM and DG located at bus 25 are allocated negative losses according to the increased number of loops, which means that their demand is reducing network losses.

4) Case D: In this situation, the topology that has been considered corresponds to the totally meshed network (five closed tie-lines) with the D-STATCOM located at node 31 and two DGs connected at nodes 6 and 21. The load and generation are varied from 10% to 100% of the data used in case B. A similar load demand variation is performed in [32]. Fig. 7 shows the loss allocation at the nodes located in the main feeder (from node 9 to node 18) for different load and generation percentages (referred in case B) and total network losses for each load variation is shown in Table V.

It is important to emphasize that for different load levels, the variation in losses assigned to every single user is the same. This means that all participants (loads, DG and D-STATCOM)

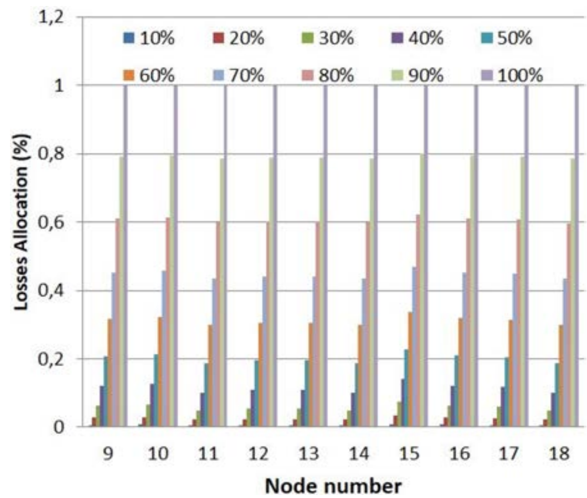

Fig. 7. Loss allocation for different load levels (IEEE-33 meshed distribution network with DER).

TABLE V
Network Losses for Different Load Levels

Level [%]	10	20	30	40	50	60	70	80	90	100
Total [kW]	0.35	1.42	3.20	4.95	6.69	9.04	12.02	15.63	19.87	24.75

are assigned the same proportion of losses for each load and generation percentage. This last case shows the consistency of the proposed formulation for different load levels.

5) Discussion of Results: From previous results, the following conclusions can be inferred:

- The solution provided by the proposed Aumann-Shapley methodology offers the characteristics of fairness, efficiency, and stability, which are required for the correct allocation of distribution losses among participants. Moreover, for radial distribution networks, the proposal solution behaves similarly to BCDLA [19], which confirms that the proposal provides accurate loss allocation solutions.
- It is demonstrated that the proposed formulation can be applied straightforwardly to meshed distribution networks, which is an important advantage compared to the BCDLA methodology [19]. The BCDLA methodology has been specifically formulated for radial distribution networks and cannot be applied to meshed distribution networks.
- Furthermore, the proposed loss allocation methodology satisfies the axioms of fairness and consistency for different loads levels. It has been shown that every single agent (customer, DER) is responsible for the network losses proportionally to the load level.
- It has been shown that the proposed formulation can be applied to distribution networks (radial and meshed) with DG and reactive power compensators such as D-STATCOM without requiring any modification or assumption in the formulation.

VII. Conclusion

This paper demonstrates the potential of the Aumann-Shapley game theory methodology to allocate losses among network users (consumers, generators, prosumers, etc.) for distribution networks.

The allocation of losses proposed in this study combines circuit theory with game theory through the analytical application of the Aumann-Shapley method in power distribution networks. The results obtained corroborate the influence of active and reactive components in the allocation of complex losses among different agents. For the distribution of losses among network users, it must be considered that the results of the allocation of negative active losses do not entail a cross-subsidy among participants because the Aumann-Shapley results reflect the participation of agents that reduce total network losses with benefits for all network users.

The proposed method of allocating losses can be directly applied for both radial and meshed networks with or without DG, offering a full range of applications to distribution networks. This is one of the advantages that the methodology offers relative to other methods of allocation, such as the BCDLA, which can be applied only to radial networks. Likewise, it is able to handle sources of DER operating in power control mode or voltage control mode, where methods based on the Jacobian matrix (MLC, DLC) cannot be directly applied.

The simplicity of the final formulation of the proposed method is noteworthy, as it permits its implementation in distribution networks where it is necessary to allocate the losses in real-time.

The methodology can be used to obtain the coefficients of losses of different clients for tariff purposes in Smart Grids. Additionally, it can be applied to Demand Response situations offering the possibility to select users that show a greater potential for reducing losses. With this knowledge, local actions can be focused on key clients to increase network efficiency, making it not necessary to implement central control actions in the DSO centers.

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