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SmartDriver: An assistant for reducing stress and improve the fuel consumption

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Abstract

The stress, safety and fuel consumption are variables that are strongly related. If the stress is high, the driver is more likely to make mistakes and have accidents. In addition, he or she will make decisions at short notice. The acceleration and deceleration increases, minimizing the use of energy generated by the engine. However, the stress can be reduced if we provide information about the environment in advance. In this paper, we propose a driving assistant which issues tips to the driver in order to improve the stress level. These tips are based on speed. The solution estimates the optimal average speed for each road section. In addition, the solution provides a slowdown profile when the user is close to a stress area. The objective is the initial vehicle speed minimizes the stress level and the sharp acceleration (positive and negative). In addition, the system employs gamification tools to encourage the driver to follow the recommendations. On the other hand, the proposal provides information about the driver and the road state in an anonymous way in order to improve the management of the city traffic. The proposal is run on an Android device and the driver stress is estimated using non-intrusives sensors and telemetry from the vehicle.

1 Introduction

Most of traffic accidents are due to human errors. In [National Highway Traffic Safety Administration, 2008] risk factors of traffic accidents are categorized as follows: human factors (92%), vehicle factors (2.6%), road/environmental factors (2.6%), and others (2.8%). Fatigue and stress are the cause of many accidents.

Stress can be defined as a change from a calm state to an excitation state in order to preserve the integrity of the person. Most stressors are intellectual, emotional, and perceptual. There are two types of stress: eustress and distress. Eustress is a good stress that improves performance and motivates. The stress is also classified as "eustress" when it leads us to a favorable state. In opposition, if the stress is negative and causes a degradation of performance, it is called "distress". This type of stress is due to an increase in the workload. This

increase may be due to multiple causes as: deceleration lane, a call, traffic density, etc.

There are many proposals to detect stress and measure the workload [Healy and Picard, 2000] [Healy and Picard, 2005]. Most of them are based on physiological features such as electromyogram, electrocardiogram, respiration, and skin conductance. In [Healy and Picard, 2000] the authors propose to use pattern recognition techniques to detect stress in automobile drivers. They employed four physiological sensor signals: electromyogram, galvanic skin, and respiration through chest cavity expansion. They were able to detect the stress by 86.6% (combining all the features). The success rate was 62.2% using a single sensor. Reference [Zhai and Barreto, 2006] presented an algorithm based on support vector machines to classify the affective states between "stress" and "relaxed". They monitored users using the following non-invasive and non-intrusive sensors: Galvanic Skin Response, Blood Volume Pulse, Pupil Diameter and Skin Temperature. In this case the percentage of success was even a 90.10%. Reference [Rani et al., 2002] proposed a real-time method for stress detection based on heart rate variability (HRV) using Fourier and wavelet analysis. [Ji et al., 2004] presented a probabilistic model for detecting fatigue based on visual characterize such as eyelid movement, gaze movement, head movement, and facial expression. This method was extended to detect "Nervous" and "Confused" affective states. This work demonstrates the suitability of Bayesian Network for information fusion and estimation of the stress level.

The main problem in this research topic is that there are no solutions to reduce the driver stress. The majority of the researchers are focused on detecting the stress level of the driver. On the other hand, most of the proposals are only validated through simulators. In this paper, we propose a system that makes recommendations in order to reduce the driving workload. These tips are constructed taking into account:

- Driver habits
- Vehicle Telemetry
- Road Information
- Driver Profile

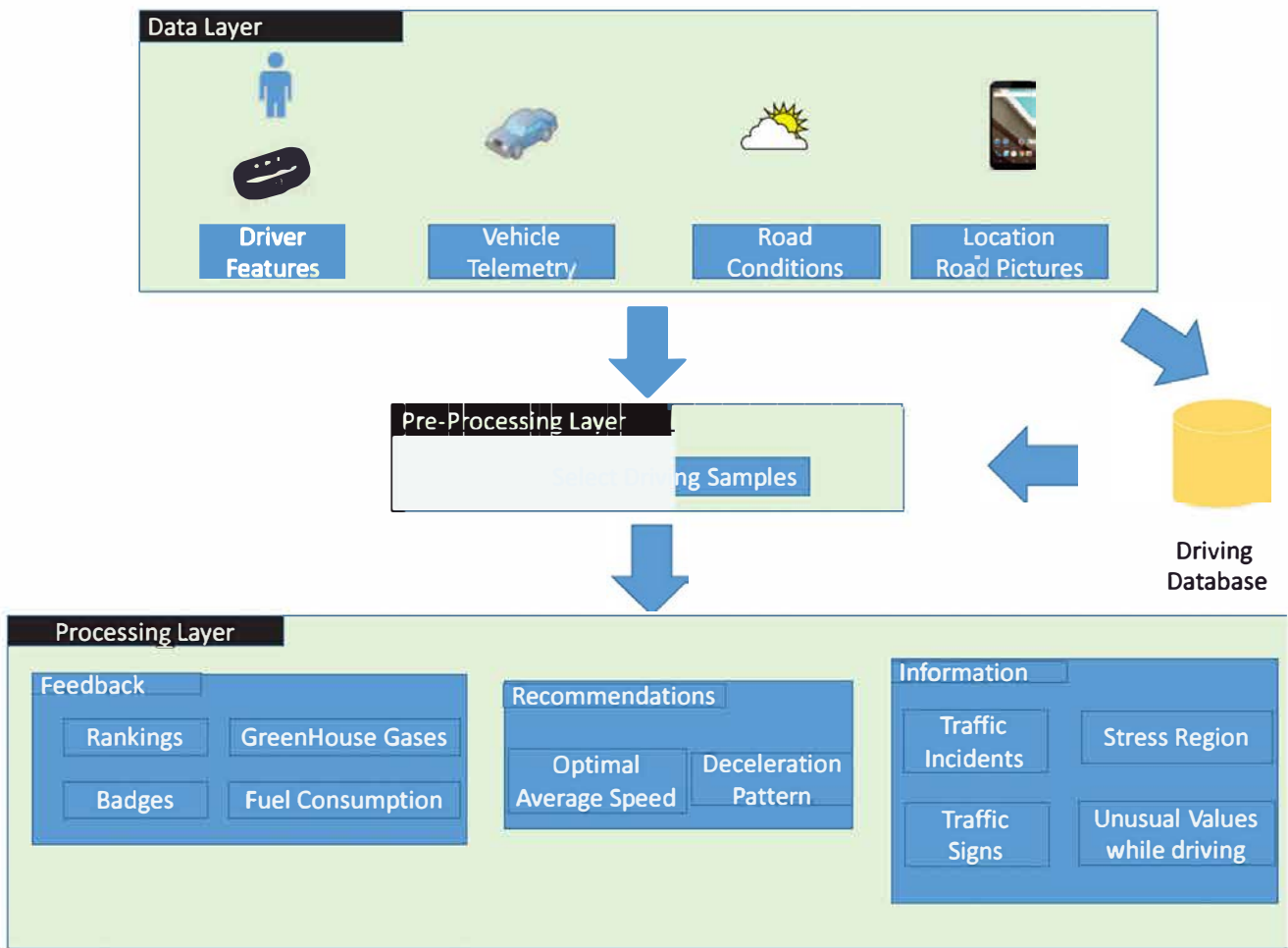


Fig. 1. Schema of the driving assistant.

2 SmartDriver Assistant

There is currently a large number of sensors (fixed and mobile) which allow us to monitor all user activity. The information captured is employed to build recommendations that reduce pollution caused by vehicles and improve the safety.

Figure 1 captures a schema of the proposal. The objective of the recommendations is smooth driving. This type of driving improves safety and reduces stress because the driver has more time to make decisions. In this case, the risk of accidents decreases. On the other hand, fuel consumption and the emission of gaseous pollutants also improves because the power produced by the engine is used. Then, we are going to describe the elements of the solution.

2.1 Data Acquisition System

We need a lot of information in order to make recommendations that will improve safety and fuel consumption. This information can be classified into three groups: driver features, life style tracking, and driving tracking.

Driver features

They affect both fuel consumption and safety. In the previous section, we saw how the appearance of fatigue and stress occurs gradually and is strongly related to the driver.

The age, gender, driving experience and personality are factors that determine the beginning of fatigue and stress. In previous research studies, many authors have highlighted that older drivers experience fatigue before young drivers.

On the other hand, the women stop driving when they feel sleep. However, men are still driving. Personality is another element that influence. Extroverted people are more likely to

fall asleep. On the other hand, people uninhibited make more mistakes during the driving without being drowsy and the fuel consumption is higher. In our case, we are going to take into account the driving style (aggressive, normal or efficient). The driver profile can be obtained objectively analyzing the vehicle telemetry.

Life Tracking

Driver activity affects his or her stress level, fatigue and health. It is also frequent that drivers worsen the driving style from the point of view of fuel consumption when they have not rested enough. The proposed solution monitors the following variables:

- **Sleeping Time:** Many research works conclude sleep has a strong relationship with the traffic accidents and the tiredness. If driver has not slept enough hours, the solution must adapt the recommendations to maximize safety. For example, warn the user in advance when he or she is approaching a crossroads.
- **Awakened Time:** Sleep-related factors such as sleep deficit, sleep fragmentation and sleep apnea also increase accident risk. In [Young et al., 1997], the authors studied the effects of the sleep-disordered in the probability of accidents. The conclusions were that you people with apneas are more likely to have accidents
- **Working time:** If the driver is sitting many hours at the same location, it could indicate that he or she is in the workplace. This is another variable related to fatigue. The trip from home to work is different that return trip from the point of view of safety.

Driving Tracking

The parameters that are monitored can be classified into three groups: driver, vehicle, and environment.

Driver:

- **Heart Rate Variability:** Stress, fatigue and sleepiness has a great impact on the automatic nervous system. HRV signals are employed as an indicator of ANS neuropathy for normal, fatigued and drowsy states because the ANS is influenced by the sympathetic nervous system and parasympathetic nervous system. This indicator is not intrusive. A high variability means the driver has stress. In opposition, a low variability could be since the driver is tired or asleep.
- **Skin Temperature:** It changes due to the activity of the central and peripheral nervous system. The emotional nervous excitement affects to the sweat glands. If the driver is relaxing, sweats will decrease.

- **Acceleration and deceleration:** The acceleration (positive and negative) may indicate the presence of stress or fatigue. The cause of sudden accelerations could be the driver wants to arrive early to the destination, while a sharp slowdown may mean that he or she was asleep.
- **Standard deviation of vehicle speed:** Maximize the driving at constant speed has a positive effect on fuel consumption. In this situation, there is no acceleration resistance force. Therefore, the tractive force required to move the vehicle will be less. On the other hand, if the driver has to be changing the speed constantly, the probability of making driving mistakes increases as well as the stress level.
- **Positive Kinetic Energy (PKE)** measures the aggressiveness of driving and depends on the frequency and intensity of positive accelerations [Firth and Cenek, 2012]. A low value means that the driver is not stressed and drives smoothly. An unusual high value may indicate that driver are driving in an area that requires special attention such as acceleration lanes or roundabouts. It is calculated using the following equation:

$$PKE = \frac{\sum(v_i - v_{i-1})^2}{d} \quad (1)$$

where v is the vehicle speed (m/s) and d is the trip distance (meters) between v_i and v_{i-1} .

- **Time:** Fuel consumption is increased at rush hours. The driver has to accelerate and slow down more frequently. In addition, the engine is switched on during more time. This situation causes stress, increasing the accidents risk. On the other hand, night driving maximizes the likelihoods of sleep despite he or she has previously slept. This is due to the sleep cycle.
- **Traffic state:** When traffic is heavy, the stress level increases. In these cases, the tips must be adapted in order to avoid acceleration and deceleration.
- **Weather conditions:** The number of vehicles on the road grows when the weather conditions are bad, increasing the likelihood of traffic incidents. Moreover, the roll coefficient changes. Therefore, the advice have to take into account that factor. In addition, many studies highlight that when it is hot, the

fatigue appears before. On the other hand, cognitive capacity of the driver worsens when it is cold.

- Traffic signs: there are a great number of traffic signs which force the user to stop or decrease speed. Therefore, they cause stress. In addition, sometimes the visibility conditions are not good and the driver cannot take decisions in advance. The result is an increase in fuel consumption due to sharp slow-downs and even traffic accidents.
- Slope: energy demand depends on this variable. If the slope is ascending, the tractive force required to move the vehicle will increase. On the contrary, if the slope of the road is down it will act as a force that helps to move the vehicle.

Devices

We employ the following devices to monitor the driver and the vehicle:

Mobile Device: Current mobile devices have a large number of sensors that allow us to obtain information about the user and the environment. In this work, we use the camera to detect traffic signs. It is also used to take a picture when the vehicle is located in a stress area. On the other hand, GPS is used to obtain the vehicle location and the vehicle telemetry: speed, acceleration, deceleration, the percentage of time driving at constant speed, etc. The Internet connection is employed to obtain information about the road state and weather conditions. Also, it allows to send data (most stressed users, traffic incidents and unusual telemetry values) that can be used by third parties to improve the management of traffic.

Microsoft Band: This wereable is cross-platform and provides an SDK for Windows Phone, Android, and IOS. It allows us to estimate the stress level using multiple sensors:

- Optical heart rate monitor
- Three-axis accelerometer
- Gyrometer
- GPS
- Microphone
- Ambient light sensor
- Galvanic skin response sensors
- UV sensor
- Skin temperature sensor
- Capacitive sensor

2.2 Retrieving Driving Samples

The degree of stress is not the same for all users, even if they are driving under equal conditions. As mentioned above, there are a multitude of factors such as age, gender, and driving experience that affect stress. We have to take into account all these factors to improve the accuracy of recommendations. Tips have to be personalized for each driver. E.g.: an old driver feels more stress than a young when they drive at high speed. In this case, we propose to build a cluster considering the following driver features and the road state:

- Age
- Driving Experience
- Profile of the driver (Aggressive| Normal| Efficient)
- Gender (Male| Female)
- Weather
- Traffic

2.3 Stress region

On the road there are areas in which increase the driver workload. The causes can be: curves, traffic signs, roundabouts, crossings, acceleration lanes, departure lanes and road topology. In this section, we propose an algorithm to reduce the stress level in this type of regions. The proposal estimates an optimal slowdown pattern when drivers are approaching to a stress area. The objective is that the driver come into the stress region driving at a speed that minimizes workload.

Prediction of driving workload

A multi-layer perceptron (MLP) [Rumelhart et al., 1985] is employed in order to predict the driver workload. It is based on the results obtained by other drivers with similar characteristics. This algorithm is an artificial neural network that has multiple layers and whose main advantage is to allow non-linearly-separable problems. Neural networks were proposed in the 1940, when Warren McCulloch (a psychiatrist and neuroanatomist) and Walter Pitts (a mathematician) explored the computational capabilities of networks made of very simple neurons [Widrow and Hoff, 1960]. Later, in 1943, [Kennedy and Eberhart] introduced the perceptron, the simplest form of a neural network. The perceptron consists of a single neuron with adjustable synaptic weights and a threshold activation function. This proposal guaranteed the convergence only if the problem was linearly separable due to the basic properties of the perceptron which separate entries into two outputs.

The basic MLP structure consists of an input layer, output layer and one or more hidden layers. The number of layers determines the kind of problem that we can solve. The single layer perceptron is limited to calculating a single line of separation between classes. On the other hand, a three layer perceptron can produce arbitrarily shaped decision regions. The single layer perceptron is limited to calculating a single line of separation between classes. On the other hand, a three layer perceptron can produce arbitrarily shaped decision regions (Kolmogorov theorem), and are capable of separating any classes. Each layer has a set of neurons. The number of neurons depends on the type of problem to be solved. The neurons are connected with other neurons using weighted connections. Figure 2 captures the neuronal network to estimate the driving workload. Figure 3 shows a schema of the

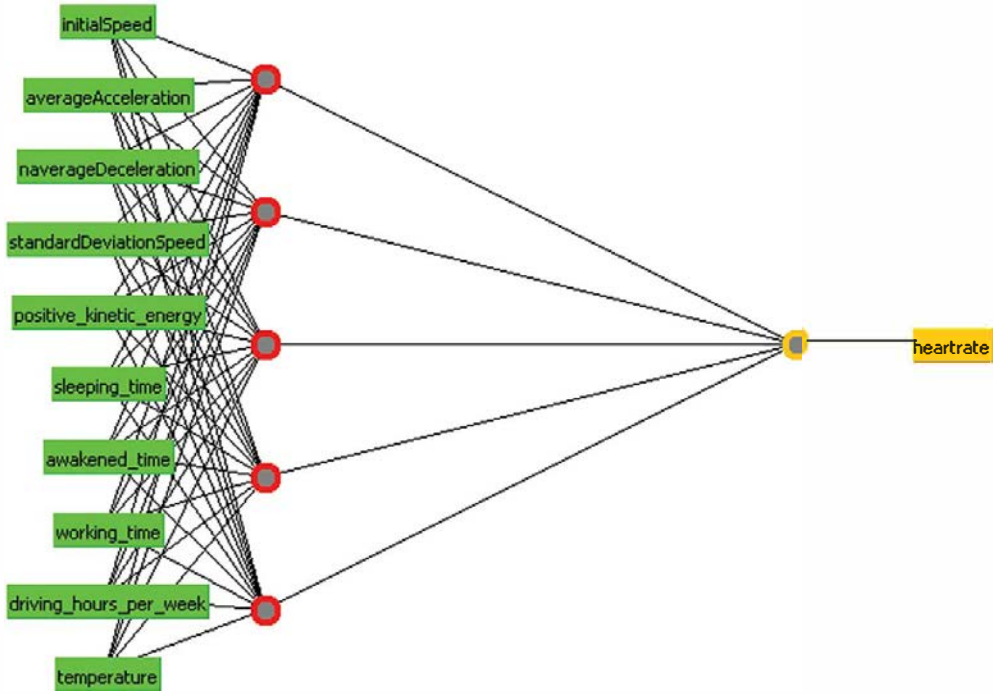


Fig. 2. Multilayer Perceptron to estimate the heart rate

solution. The proposal estimate the optimal initial speed based on the previous driver behavior using the neural network. A deceleration profile is then calculated if the driver must slow down.

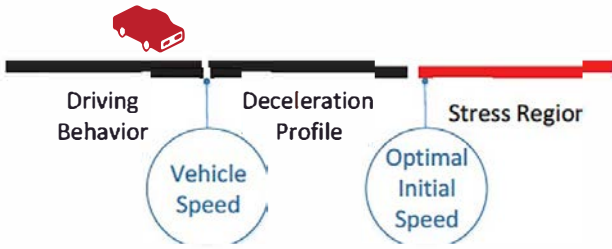


Fig. 3. Proposal to reduce driver stress in stress areas.

2.4 Optimal Average Speed

Speed is a factor that is closely related to demand attention from the driver. When the speed is high, the driver has less time to make decisions. This causes an increase in the stress level and the likelihood of traffic accidents. In this paper we propose an algorithm to estimate the optimal speed.

On the roads, we can find road signs which recommend a speed. These tips are only based on the road topology (slope, angle, and the road width). Its aim is to prevent the vehicle

go out from the road. However, the optimal speed from the point of view of stress and safety is a dynamic value. It changes depending on age, gender, physical activity, level of fatigue, driver skill, traffic and weather conditions, etc.

Particle Swarm Optimization (PSO) is used to estimate the average speed for each section road. It was proposed by Kennedy and Eberhart en 1995 [Kennedy and Eberhart, 1995]. It is an evolutionary algorithm based on the social behavior of bird flocks. The PSO algorithm maintains multiple potential solutions at one time. Each solution is represented by a particle in the search space. It has the following elements:

- Position: In our case is the recommended speed.
- Pbest: This is the best position on the current particle (speed that minimizes the heart rate)
- Gbest: It is the best position among all particles
- Speed: Is calculated using equation 2. It determines what will be the next speed of the particle.

$$v_i(t+1) = wv_i(t) + c1r1 * (Pbest(t) - x_i) + c2r2 * (Gbest(t) - x_i(t)) \quad (2)$$

where $v_i(t)$ is the particle's velocity at time t , w is the inertia weight, $x_i(t)$ is the particle's position at time t , $Pbest$ is the particle's individual best solution as of time t , and $gBest(t)$ is

the swarm's best solution as of time t , $c1$ and $c2$ are two positive constants, and $r1$ and $r2$ are random values in the range [0-1]

The particles "fly" or "swarm" through the search space to find the minimum value. During each iteration of the algorithm, they are evaluated by an objective function to determine its fitness. Next position is calculated by equation 3

$$x_i(t + 1) = x_i(t) + v_i(t + 1) \quad (3)$$

where $x_i(t)$ is the current particle's position and $v_i(t + 1)$ is the new velocity.

The definition of the fitness function is very important in this type of algorithms. In our case is a multi-layered perceptron (MLP). The input variable are the following: Average Speed, number of high accelerations (positive and negative), standard deviation of vehicle speed, positive kinetic energy, sleeping time, awakened time, working time, and temperature. The challenge is to minimize the stress level. Therefore, the output is the heart rate. Fitness Function is trained using driving samples obtained by other drivers under similar conditions.

2.4 Providing feedback

As we saw in previous works, it is important to encourage and motivate the driver in order to he or she follows the recommendations of the assistant. We employ techniques of gamification to achieve this goal. Gamification can be defined as the use of game design elements in non-game contexts such as learning environments. The idea is to use concepts from games like: the challenge, the competitiveness and progression in order to motivate the user for improving the driving style. In our case, the system provides the following feedback when the driver completes the trip:

Driving style ranking: the position is obtained using a fuzzy logic system. This system evaluates the driving from the point of view of safety and energy saving. The output depends on variables such as: acceleration, deceleration, positive kinetic energy, etc. If the user is driving smoothly, he or she will get a high score. A smooth driving influences positively both fuel consumption and safety. We employ fuzzy logic because it allows us to simulate the human knowledge when carrying out certain tasks such as driving. The objective is to model the behavior of an efficient and safe driver.

Safe Driving Ranking: The position in this ranking depends on the degree of compliance with safety recommendations provided by the assistant.

Green House Gases and Fuel Consumption: Driver can compare the result with the value obtained in other similar journeys (road type, weather, and traffic) made in the last month.

Badges: They are an important element for many people in order to encourage them to save fuel and to drive safely. Achievements are a traditional gamification method used to

accomplish a certain behavior or to compare the performance of users. Achievements do not normally imply monetary compensation, but they are based on an emotional reward. It has a positive impact on the user to reach a pre-set of objectives and is based on the concept of "status". For this reason, we have incorporated the following achievements to our smart-driving assistant:

1. Complete a lap without accelerating sharply
2. Complete a lap without decelerating sharply
3. Complete a lap with a low value of PKE (Positive Kinetic Energy)
4. Complete a lap following the deceleration pattern provided by the assistant
5. Complete a lap driving at the average speed recommended
6. Get 5 points in the Driving Style/Safe Ranking
7. Get 7 points Ranking Driving Style/Safe Ranking
8. Get 10 points in the Driving Style/Safe Ranking
9. Position in the top 5 in the Driving Style ranking
10. Position in the top 5 in the Safe Driving Ranking
11. First position in the Driving Style Ranking
12. First position in the Safe Driving Ranking.

2.5 Information provided by the assistant

The driving assistant described in this paper is one of the elements of the HERMES project [HERMES, 2015]. The objective of this project is to integrate different agents and infrastructure elements of a Smart City in a cooperative and massive system that optimizes urban movements, minimizes the emission of pollutant gases, maximizes the well-being of the citizens and offers new opportunities for business on the Smart City.

In this context, the vehicles act as mobile sensors that collect data and send them to a central system. This central system labels the data semantically and analyzes them using artificial intelligence algorithms. The results are recommendations to improve the management of the city. For example, it is possible to predict the traffic or infer which is the most polluted area of the city. In addition, the HERMES project allows access to these information flows in real time following the philosophy of Linked Open Data. The aim is to build an ecosystem for business development in the Smart City. SmartDriver assistant provides the following information:

Traffic incidents: Its number has increased in recent years due to the rapid growth of the metropolitan population and the number of vehicles in circulation. A traffic incident is defined as a non-recurrent and unpredictable event that interrupts the flow of normal traffic by reducing the capacity of the road [Srinivasan et al., 2003]. Traffic incidents include events such as: accidents, disabled vehicles, bad weather conditions, rock falls, road works, and malfunction of traffic signs. The solution detects traffic accidents taking into account the usual driving profile of the driver and the real time telemetry.

Traffic signs: In regions where there are many traffic signs, the stress level and fuel consumption increases. If the driver know them in advance, he or she can adjust the speed, avoiding sudden downturns. The result is that the energy generated by the engine is maximized and the driving mistakes are reduced. In the literature there are many methods to detect traffic signs. In our case, we use an algorithm with three stages:

- Shape: We look for geometric figures which coincide with traffic signs.
- Color detection: We choose shapes which contain colors present at traffic signs
- Viola and Jones: This algorithm is applied on the shapes selected in the previous step.

Stress Region: There are road sections where driving is more difficult due to the visibility problems (curve), topology of the road (roundabout) or the activities that driver should make (deceleration lane, acceleration lane). Data Envelopment Analysis [Charnes et al., 1985] is used to estimate the efficiency of driving and the stress level in each area. Data envelopment analysis (DEA) is a linear programming methodology to estimate the efficiency of multiple decision making units (DMUs) when the production process presents a structure of multiple inputs and outputs. This method was proposed by Charnes, Cooper, and Rhodes [11]. In our proposal, each DMU represents a different driving samples obtained under similar conditions (time, weather, traffic, sleeping time, and road type) by the same driver. The aim is to detect the road points where the driver workload is high and fuel consumption increases. This algorithm compares different regions by assigning them an efficiency value. If it is low, it means that it is a stress area.

Unusual driving values: The driving assistant presented in this paper is continuously monitoring the driver behavior driver. When it detects an exceptional value, it is immediately sent to a central system (HERMES). These values can be: sudden accelerations (positive and negative), high speed, standard deviation of vehicle speed, heart rate, etc. This information can be used to improve traffic management and safety in a Smart City.

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