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Bachelor Thesis

Global Trajectory Optimisation of a Space-Based Very-Long-Baseline Interferometry Mission

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Creativity is born from anguish, just like the day is born form the dark night. It is in crisis that inventiveness is born, as well as discoveries made and big strategies. He who overcomes crisis, overcomes himself, without getting overcome.

Albert Einstein

Preface

This project was composed between September 20^{th} and June 22^{th} by a last year student pursuing a Bachelor's degree in Mechanical Engineering. The theme of the project is General Trajectory Optimisation of a Space Based Very-Long-Baseline Interferometry Mission. The project aims to develop an optimal solution to a space mission in which three spacecraft orbiting around Earth take measurements of diverse points of interest around the stellar sphere. This includes the developing of a solver's code as well as the study of the different approaches concerning manoeuvring.

Reading guide

The report conforms to the scientific standard of citing the sources used throughout. The report follows the Harvard citation method, where sources are listed in the text as [Surname, Year]. This citation refers to the bibliography at the end of the report, where books are listed with author, year, title, edition and publisher. Websites are listed with author, title, date and URL. The bibliography is alphabetically ordered.

Figures, tables and equations are numbered according to chapter and order of appearance therein. Abbreviations are used throughout the references as Fig. Sect. and Eq. referring to Figure, Section and Equation respectively.

Nomenclature

Many "standard" symbols in astrodynamics have been in use for centuries. This report attempts to keep many of these common symbols, giving alternatives wherever appropriate, and try to stay consistent throughout it.

First derivative	dot	$\dot{r}$
Second derivative	double dot	$\ddot{r}$
Vector	angular symbol	$\vec{r}$
Unit vector	hat	$\hat{r}$
Matrix	bold	$\mathbf{r}$

Abbreviations

Abbreviation	Meaning
COE	*Canonical Orbital Elements*
CPS	*Chemical Propulsion System*
ESA	*European Space Agency*
GMAT	*General Mission Analysis Tool*
GTOC	*General trajectory Optimisation Competition*
IAU	*International Astronomical Union*
LEO	*Low Earth Orbit*
LTS	*Low Thrust System*
MOEA	*Multi-Objective Evolutionary Algorithm*
NASA	*National Aeronautics and Space Administration*
NSGA	*Nondominated Sorting Genetic Algorithm*
O(Operator)	*Computational complexity*
RAAN	*Right Ascension of the Ascending Node*
RADN	*Right Ascension of the Descending Node*
SOI	*Sphere Of Influence*
VLBI	*Very Long Baseline interferometry*

Symbol list

The following list of symbols is alphabetical-lowercase, then uppercase; Arabic, then Greek letters:

Symbol	Explanation	Unit
a	*Semimajor axis*	*[km]*
a_P	*Acceleration due to perturbations*	$[km/s^2]$
d	*Day*	*day*
e	*Eccentricity*	*[]*
g_0	*Standard acceleration due to gravity at Earth's surface*	$[km/s^2]$
h	*Spacecraft altitude*	*[km]*
h	*Hour*	*[hour]*
h_{max}	*Maximum height of observation triangle*	*[km]*
h_{mid}	*Intermediate height value for 3 repeated observations*	*[km]*
h_{min}	*Minimum heigh of observation triangle*	*[km]*
$\vec{h}$	*Orbital momentum vector*	$[km^2/s]$
h_{pM}	*Flyby altitude at closest approach to the Moon*	*[km]*
i	*Inclination*	*[deg]*
$\hat{i_m}$	*Normal to observation and node-line direction plane*	*[km]*
$\hat{k}$	*Normal unitary vector, points to k direction in IJK frame*	*[km]*
l	*True longitude*	*[deg]*
m	*General mass*	*[kg]*
m	*Number of Lunar revolutions for resonant orbits*	*[revol]*
min	*Minute*	*[min]*
mo	*Month*	*[month]*
n	*Number of spacecraft revolutions for resonant orbits*	*[rev]*
n	*Mean motion*	$[s^{-1}]$
$\vec{n}$	*Node-line vector, points to ascending node*	*[km]*
$\vec{n}$	*Source measurement direction*	*[km]*
$\hat{n}$	*Node-line vector linked to node-line spacecraft*	*[km]*
p	*Semiparameter, semi-latus rectum*	*[km]*
$\vec{r}$	*Position vector*	*[km]*
r_{pM}	*Periapsis radius on a flyby respect to the Moon*	*[km]*
s	*Seconds*	*[s]*
$\vec{s}$	*Measurement plane normal direction*	*[km]*
t	*General time*	*[s]*
$\hat{t_d}$	*Trust direction*	*[km]*
u	*Argument of latitude*	*[deg]*
$\hat{u}$	*Unitary direction after applied flyby*	*[km]*
$\vec{v}$	*Velocity vector*	*[km/s]*
$\vec{v}_\infty$	*Hyperbolic infinity velocity*	*[km/s]*

Symbol	Explanation	Unit
$\vec{F}$	*General force*	*[N]*
B	*Laplace-Runge-Lenz vector*	*[$s^2/km3$]*
E	*Eccentric anomaly*	*[deg]*
G	*Gravitational constant*	*[$m^3/kg/s^2$]*
GEN	*Number of generations in Genetic Algorithm*	*[km]*
J	*Performance index*	*[km]*
JD	*Julian date*	*[JD]*
M	*Mean anomaly*	*[deg]*
M	*Number of objectives in Genetic Algorithm*	*[]*
MJD	*Modified julian date*	*[MJD]*
N	*Population size*	*[]*
I_{SP}	*Specific impulse of propulsion system*	*[s]*
P	*Repeat-observation weighting factor*	*[]*
Q	*Offspring population in Genetic Algorithm*	*[]*
R_E	*Earth radius*	*[km]*
R_M	*Moon radius*	*[km]*
T	*Orbital period*	*[s]*
$\mathbf{T}$	*Coordinate transformation matrix*	*[s]*
S	*Preferable selection value*	*[]*
$\vec{T}$	*General thrust vector*	*[N]*
α	*Right ascension, measured positively to the east*	*[deg]*
α	*Modulus of obtained velocity in a flyby*	*[km/s]*
α'	*Analogous right ascension in the Ecliptic frame, longuitude*	*[deg]*
β	*Cone angle anfter applied flyby*	*[deg]*
γ	*Flight path angle*	*[deg]*
δ	*Declination of a source*	*[deg]*
δ'	*Analogous declination in the Ecliptic frame, latitude*	*[deg]*
δ_t	*Turn angle in a flyby*	*[deg]*
ε	*Computational tolerance*	*[]*
ϵ	*Ecliptic plane obliquity*	*[deg]*
θ	*Angular direction determination after flyby*	*[rad]*
μ	*Standard gravitational parameter of Earth*	*[km^3/s^2]*
μ_M	*Standard gravitational parameter of Luna*	*[km^3/s^2]*
ν	*True anomaly*	*[rad]*
ξ	*Specific mechanical energy*	*[kJ/kg]*
ϖ	*Periapsis longitude*	*[deg]*
τ_G	*Geocentric reference frame*	*[]*
τ_P	*Perifocal reference frame*	*[]*
ω	*Argument of periapsis*	*[deg]*
Δv	*Delta-v, increment in velocity*	*[km/s]*
Υ	*Vernal equinox*	*[]*
Ω	*Righ ascension of the ascending node*	*[deg]*

Contents

Abstract

GTOC is an event in which the best aerospace engineers and mathematicians worldwide challenge themselves to solve a complex problem that cannot be solved with standard optimisation tools. The 8^{th} edition of GTOC is based on **orbital mechanics** with an important weight on **optimisation**. The description of the problem presents a three spacecraft configuration capable of taking measurements on the celestial sphere with VLBI instruments on board. The final objective stands for the maximisation of a *performance index* which depends in the course of action to measure different sources during a maximum mission time given.

In this thesis, is suggested a method to obtain an optimal result. Besides, the complexity of the mission demands a simplified and optimal approach to the problem as there are not methods available tu fully solve it.

The approach presented support itself on a *fuel-efficient* concept, using the propulsion systems on board only when is strictly necessary. The importance of *gravity assists* during the mission subjects the problem to reach the Moon with the available spacecraft as many times as possible. The obtention of resonant orbits after a *flyby* gives the opportunity to multiple **fuel-free orbital transfers** while measurements are taken between flybys.

The final optimal trajectory is determined using a MOEA which optimise the solution for each resonant orbit obtaining the maximum value of the *performance index* minimising *time* for each flyby in order to explore the maximum amount of possible resonant orbits.

Chapter 1

Introduction and motivation

The project General Trajectory Optimisation of a Space-Based Very-Long-Baseline Interferometry Mission is driven by the Aerospace Department of Universidad Carlos III de Madrid. The project's objective is to find an optimal solution to a specific orbital mechanics problem inside Earth's gravitational sphere of influence —SOI—, as well as to develop a code with a multi-purpose utility for space missions with satellites inside the Earth-Moon system. In addition, it is expected that the project will also be a research tool to any user interested in orbital mechanics, as well as spacecraft and planet motion around Earth with the different effects applied by the propulsion systems available, highlighting the complexity and mathematical methods used in celestial mechanics.

The development of this project is going to give a global view of the problem, the hypothesis taken, and the procedure to develop the solver's code using MATLAB. Furthermore, it is going to take into account the possibility of being customized and improved by others who share an interest in this field of physics. To achieve this, all procedures are going to be defined and explained thoroughly.

This chapter exposes the orbital mechanics theory needed and used throughout the project as well as provides a thorough description of the mission, its parameters, constraints and modelling approximations.

1.1 Problem description

Space missions can display lots of ways to be accomplished due to the unconstrained and non-frictional motion of celestial objects. For instance, one of their most important features regarding orbital mechanics is its optimisation, which has a huge impact on mission's costs and final results obtained.

The problem to solve is based on the Global Trajectory Optimisation Competition —GTOC—. GTOC was born in the Advanced Concepts Team of the European Space Agency —ESA—in 2005, this one is an event taking place every one-two years over roughly one month during which the best aerospace engineers and mathematicians world wide challenge themselves to solve a "nearly-impossible" problem of interplanetary trajectory design. Each problem's edition is designed by the winners of the previous one and over the years, the various problem statements and solutions returned, have formed a formidable database of experiences, solutions and challenges for the scientific community.

This project's aim is to solve the 8^{th} edition of the problem which was realised in 2015. GTOC 8^{th} edition's problem was defined by the Jet Propulsion Laboratory of the California institute of Technology [Petropoulos, 2015] and its main characteristics are:

- Global optimisation over a large design space with many local optima
- Unusual objective functions or constraints; which mean that no existing methods nor software can likely fully solve the problem

Highlighting one of the main aspects of the mission, the necessity of a constant implication of human resources to solve the problem is remarkable as no general methods are available to fully achieve a solution.

The mission follows the high-resolution mapping of radio sources in the universe using space-based Very-Long-Baseline Interferometry —VLBI—. This will be accomplished by three spacecraft flying around Earth in Low Earth Orbit —LEO —.

This VLBI system works using the instrumentation of the three spacecraft which can orient the plane defined by them towards each radio source. The goal is to take a number of observations in order to maximize a *performance index*, which depends on the source direction and is loosely related to the efficacy of the plane formed to obtain the measurement. Repeated observations of a source are rewarded extra if the observing triangle planes are of sufficiently different sizes.

1.2 Orbital mechanics

To achieve a solution and fully understand the mission, the use of orbital mechanics is thoughtfully employed.

Astrodynamics or orbital mechanics is the application of ballistics and celestial mechanics in practical problems concerning the motion of spacecraft. The application of this field of physics is the core of space mission design and, eventually, of the project.

To determine the orbit of an object in motion around a celestial body it is necessary to define its position and velocity vectors in a determined reference frame. Those vectors are compounded by 3 elements (one for each coordinate) creating a problem with 6 variables.

To obtain those parameters, the classical calculus procedures of Newton and Kepler's laws are applied to gravitational forces. It is necessary to obtain an orbit of an object (or spacecraft) m in relative motion to a celestial body m_c, the force of attraction between both bodies it is defined by Newton's Universal Gravitational Law [G. Mengali, 2013, Vallado, 2001]:

$$\vec{F} = -G\frac{m_c m}{r^2}\hat{r}$$

Where
G Universal gravitational constant
r Position vector

In astrodynamics, the simplification of this formula with the **standard gravitational constant** is used $\mu = G\ m_c$. This parameter is unique in each celestial body and its values are determined and well known for all the planets in the Solar System.

If this variable change is applied and it is known that $\hat{r} = \frac{\vec{r}}{\|r\|}$, the following equation is obtained:

$$\vec{F} = -\mu\frac{m}{r^3}\vec{r} \tag{1.2.1}$$

At that juncture, Newton's Second Law is applied, taking into account the fact that acceleration can be expressed in terms of position and time with the second derivative, obtaining the following differential equation:

$$-\mu\frac{m}{r^3}\vec{r} = m\frac{d^2\vec{r}}{dt^2} \tag{1.2.2}$$

In addition, some other considerations determined related to simplification are added, thus a complete Keplerian motion can govern the mission. Those are:

- The unique gravitational attraction between one primary body (Earth in this case) and all secondary bodies (the spacecraft)
- The perfect sphericity of celestial bodies so the gradient of gravity along the same radius is zero
- The null interaction of gravity and any kind of perturbation from any other bodies outside the Earth's SOI

With the addition of these simplifications, the equation which mathematically describes the *two body problem* is obtained:

$$\ddot{r} + \mu\frac{r}{r^3}\hat{r} = 0 \tag{1.2.3}$$

Before commencing to integer the motion equation to determine the relative trajectory of a spacecraft, it is necessary to take into account two mechanical constants which characterize the motion: the specific mechanical energy, ξ and the orbital momentum vector h.

$$\xi = \frac{v^2}{2} - \frac{\mu}{r} \tag{1.2.4}$$

$$h = \vec{r} \times \vec{v} \tag{1.2.5}$$

The values of these elements for a body in orbit (without any external perturbation) are constant, and they define a priori the equation of the trajectory. Particularly, the possible trajectories which can be described in Keplerian motion are represented from *conic equations*.

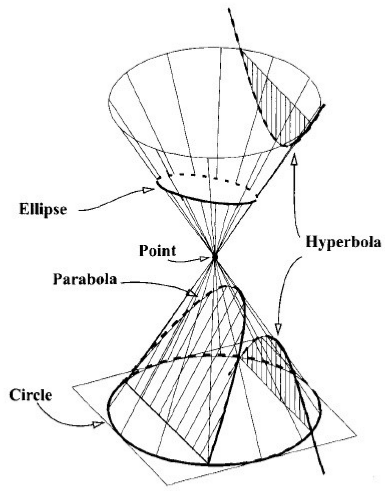


Figure 1.2.1: Different conic sections possible [Vallado, 2001][Figure 1-3]

Historically, the demonstration that two points of mass move along their barycentre following a trajectory described by a conic it was proposed in 1687 by Newton in his *"Philosophiae Naturalis Principia Mathematica"*[Newton, 1687].

1.2.1 Orbital elements

There are two main state representations to determine the orbit of a body; Cartesian and Keplerian representations. Although there are also many other ways to define orbital parameters they are less employed and they do not concern the objective of this project.

On one hand, Cartesian elements are position and velocity vectors of the secondary body relative to the primary body (from each one's centre of mass). Those two three-dimensional vectors at a concrete instant of time give all the information needed to describe orbital motion. Moreover, it is necessary to define a reference frame (fixed of relative, inertial or not inertial) in order to maintain concordance.

On the other hand, Keplerian elements describe motion with a series of canonical orbital elements —COE—. This state representation has the same amount of components needed to fully define orbits. It is more useful in terms of precision and direct interpretation of values than the Cartesian state. Furthermore the Keplerian state describes its own orbital reference frame although it does require a previously defined thee-axis reference frame since, to describe all Keplerian elements, an earlier Cartesian position and velocity vectors must have been provided.

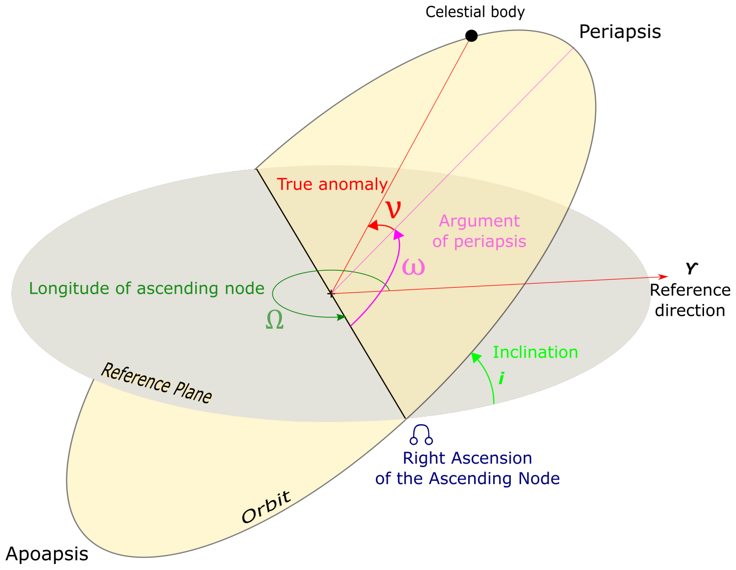


Figure 1.2.2: Orbital elements

The general expressions of Keplerian elements for elliptic orbits are:

$$\xi = -\frac{\mu}{2a} \qquad \tan\gamma = \frac{e\ \sin\nu}{1+e\ \cos\nu}$$
$$p = \frac{h^2}{\mu} = a(1-e^2) \qquad \cos i = \frac{\hat{k}\cdot\vec{h}}{h}$$
$$e = \frac{B}{\mu} = \sqrt{1+\frac{2\xi h^2}{\mu^2}} \qquad \cos\Omega = \hat{i}\cdot\vec{n}$$
$$\cos\omega = \hat{n}\cdot\vec{e}$$
$$r = \frac{p}{1+e\cos\nu} \qquad \cos\nu = \hat{e}\cdot\vec{r}$$

For equatorial elliptical orbits it does not exist Ω and ω since $\vec{n}$ is not defined (the equatorial plane coincide with the orbital plane). In this case, the *periapsis longitude*, ϖ is added governed by the Equatorial-Geocentric reference frame.

Orbits can also be circular and equatorial. Also in this case Ω and ω are not defined, but is also none the periapsis (so the *periapsis longitude* ϖ it cannot being used either). In this case the position of the spacecraft is defined by the *true longitude*, l which is the angle in the direction of motion respect x-axis.

As a last peculiarity concerning these orbits, it can be obtained a circular inclinated orbit in which Ω does exist but ω is not defined (in fact, neither it does the periapsis). As a consequence, the angle *argument of latitude*, u is used.
The above-mentioned singularities are described by the following formulae:

$$\cos\varpi = \hat{i}\cdot\hat{e} \qquad \cos l = \hat{i}\cdot\hat{r} \qquad \cos u = \hat{n}\cdot\hat{r}$$

1.2.2 Kepler's Equation and Kepler's Problem

Throughout history, celestial movement study has been an important topic full of problems to solve by astronomers. However, it is not going to be until the XVII century when Astronomy would accurately define celestial movement and position.

With the contribution of researchers such as Copernicus, Kepler, Galileo and Newton at that epoch, it was possible to measure the position at an instant of a celestial body using mathematics and observations —accurate to about 0.033° which is remarkable for the epoch—.

Moreover, there was an orbital problem did not have a solution: to know a celestial body's position and velocity after some time from the measurement taken. Although Isaac Newton (1642-1727) produced much of the mathematics required to solve the orbital problem, Kepler determined how to relate mean and true anomalies in the orbit to time.

Kepler's Equation allows for the determination of the relation of the time and angular displacement within an orbit, introducing the concepts of ***Mean anomaly*** and ***Eccentric anomaly***. Angles referred in Fig. 1.2.4. On the basis of Kepler's second and third laws, this equation— *Kepler's Equation*— relates time of flight from periapsis to the eccentric anomaly, semimajor axis, and eccentricity. Kepler introduces notation for the *mean anomaly, M* as:

$$M = E - e \sin E = \sqrt{\frac{\mu}{a^3}}(t - T) \tag{1.2.6}$$

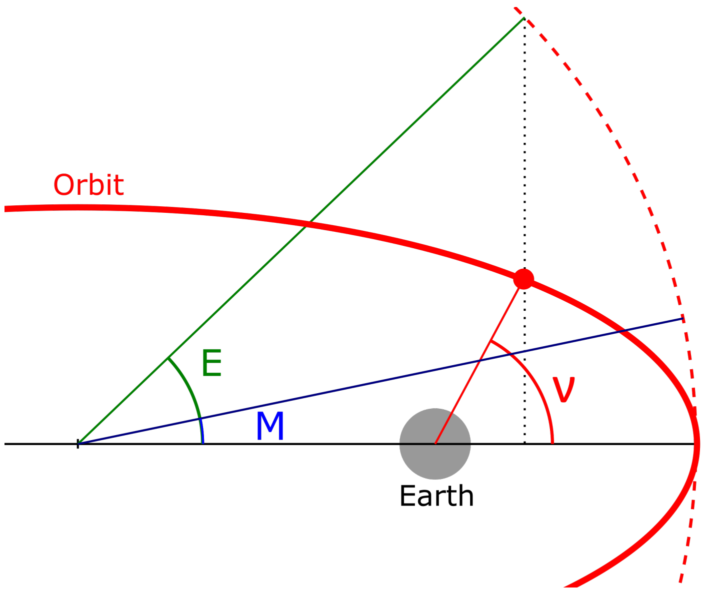


Figure 1.2.3: Visual representation of the True Anomaly with the Mean and Eccentric Anomaly as defined by Kepler

Kepler also introduces the *mean motion, n*, notation (or mean angular velocity) and the relation between *eccentric anomaly* and *true anomaly* (for elliptic orbits which is the case of study):

$$n = \sqrt{\frac{\mu}{a^3}} \tag{1.2.7}$$

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2} \tag{1.2.8}$$

Actually, two classes of problems arise from Kepler's equation: the time to travel between two known points on any type of orbit and the location of a celestial body after a certain amount of time. Kepler's equation captures the first problem although the second one leads to what is known as ***Kepler's problem*** or more generally, ***propagation***. This problem needs of an iterative method as Newton-Raphson to eventually converge into a valid solution [Battin, 1999].

The following iterative process is used to solve Kepler's equation to calculate the eccentric anomaly:

$$E_0 = M \tag{1.2.9}$$

$$E_{i+1} = E_i - \frac{M - E_i + e \sin E_i}{1 - e \cos E_i} \tag{1.2.10}$$

Eq. (1.2.10) is repeated until the condition $\mid E_{i+1} - E_i \mid \leq \varepsilon$ is satisfied where ε is the tolerance applied. Thereupon $E_{i+1} = E$.

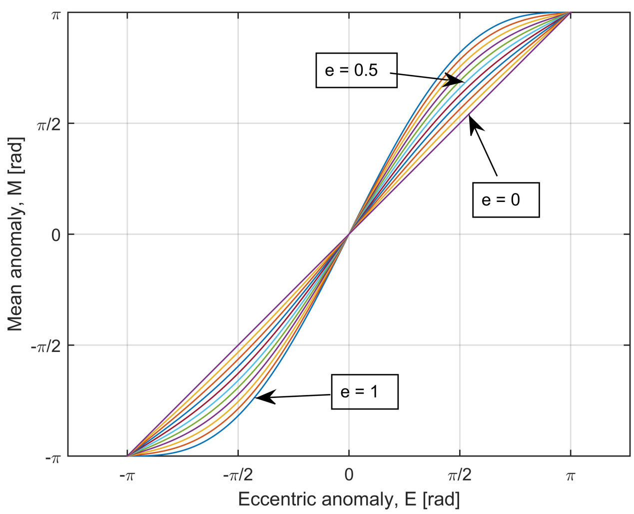


Figure 1.2.4: **Mean Anomaly vs Eccentric Anomaly:** Although the difference between the eccentric and mean anomaly is not great, as the eccentricity increases, the differences become larger.

1.2.3 Reference frames

One of the first requirements for describing an orbit is to define a suitable reference system. There are many different approaches to describe a reference frame in space. The choice rests on the most convenient approach to work with orbital elements and on the best visualisation way to interpret numerical results.

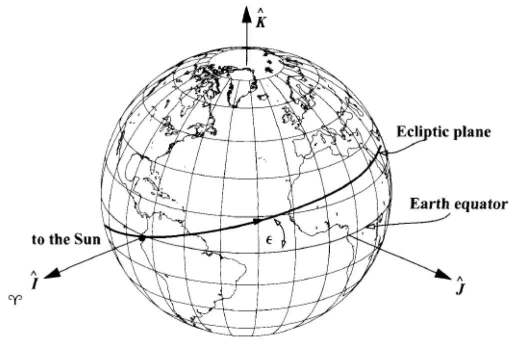


Figure 1.2.5: Equatorial and ecliptic planes in *IJK* reference frame [Vallado, 2001][Figure 3-7]

The Earth and its orbit around the Sun form the basis for celestial coordinate systems. The ***ecliptic*** plane —which defines the plane of Earth's mean orbit about the Sun—, its obliquity ϵ and the concept of ***vernal equinox***, which occurs at the ascending node of the Sun as viewed from Earth, are the foundations of this systems. It is well noted the direction of the vernal equinox, designated by Υ and often referred to as the *first point of Aries*.

Coordinate systems are divided into interplanetary systems, Earth-based systems and Satellite-based systems. Nonetheless, as the mission concerning this project it is exclusively inside Earth's SOI, interplanetary reference frames will not be chosen nor used.

The origin of Earth-based systems may be at the Earth's centre or at a site on the Earth's surface. Despite there are many reference frames originated in Earth, during this project and to avoid confusion only the ***Earth Mean Ecliptic and Equinox of J2000 frame*** is going to be used. Some of the most important coordinate systems are presented as follows:

The *Geocentric Equatorial Coordinate System* —also named *IJK*—, has its foundations in the Earth's equator. The I axis pointing towards the vernal equinox; the J axis is 90°to the east in the equatorial plane and the K axis extends through the North Pole. It is one of the most common systems in astrodynamics, but can also be potentially confusing for being a "pseudo" Newtonian inertial system referred to the equator and equinox at a particular epoch.

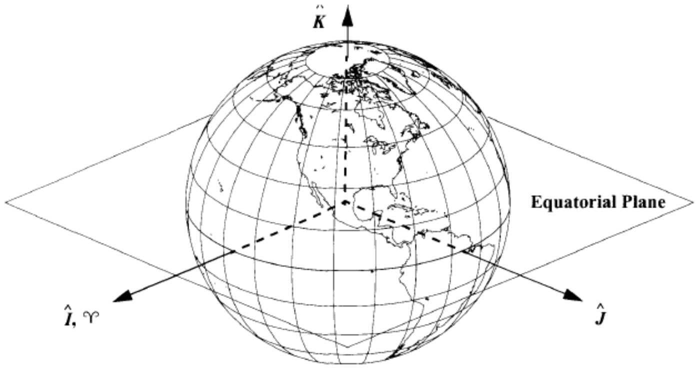


Figure 1.2.6: *IJK* reference frame with axis and equatorial plane [Vallado, 2001][Figure 3-11]

As mentioned, to maintain an order in the project a single reference Earth-based frame it is going to be used to display the results regarding Cartesian coordinates. It happens to be the ***Earth Mean Ecliptic and Equinox of J2000 frame***. This reference frame it is a quasi-inertial variation of the *Geocentric Equatorial Coordinate System* —with no variations concerning changes in the equinox and equator planes during the mission duration— and only has a difference between them: its plane of reference is the ecliptic plane which orbital components are further explained and defined in Sect. 1.3.1.

Although many Satellite-based systems exist, they all have their bases in the plane of the satellite's orbit. Keplerian orbital elements are used to describe these objects' locations.
Throughout the project, three similar Satellite-based reference frames are going to be used:

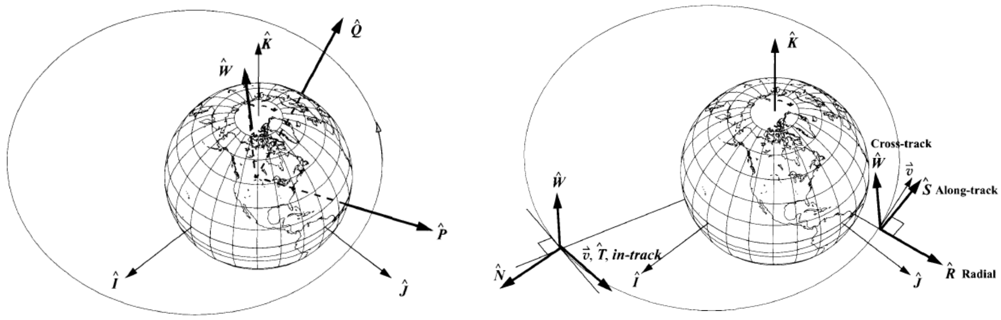
1. The ***Perifocal Coordinate System, PQW***. In this system the fundamental plane is the satellite orbit having its origin at the centre of the Earth. The P axis points towards the perigee, the Q axis 90° from P in the direction of the satellite motion and W axis is normal to the orbit. This system is used to describe orbits with a well-defined eccentricity and Keplerian elements.

2. The ***Satellite Radial System, RSW*** sometimes called *Gaussian coordinate system* and is sometimes given the letters *RTN* (radial, transverse, normal) or ***LVLH*** (local vertical, local horizontal). The R axis always points from the Earth's centre along the radius vector toward

the satellite as it moves through the orbit. The S axis point in the direction of the velocity vector (but not necessarily parallel to) and it is *perpendicular* to the radius vector.

3. The ***Satellite Normal System, NTW*** the T axis is tangential to the orbit and always point towards the posigrade velocity vector. The N axis lies in the orbital plane, normal to the velocity vector and the W axis is normal to the orbital plane.

These last two systems have characteristic interest while thrusting and in perturbation analysis. The differences between these systems are shown in the figure 1.2.7, noting special interest in the difference in *in-track* or *tangential* along the T axis.



(a) PQW reference frame

(b) NTW and RSW reference frames

Figure 1.2.7: Satellite-based systems used throughout the project [Vallado, 2001][Figures 3-14; 3-15]

1.2.4 Propulsion

On board each spacecraft is an impulsive Chemical Propulsion System —CPS— and a continuous Low Thrust System —LTS—. Both systems have their main difference in their thrust magnitude applied and fuel efficiency. While CPS applies a high magnitude of thrust instantaneously (expressed directly as a Δv) with a low efficiency, LTS employs a very low thrust magnitude in exchange for a very high fuel efficiency.

The losses of mass for each propulsion system are determined by the following expressions:

$$m_f = m_0 \cdot \exp(-\frac{\Delta v}{g_0\ I_{SP}}) \tag{1.2.11}$$

$$\frac{dm}{dt} = -\frac{T}{g_0\ I_{SP}} \tag{1.2.12}$$

On behalf of low thrust, to solve orbital mechanics without perturbations (such as solar radiation or gravitational forces of other nearby celestial bodies) or any type of delta-v being applied through time, it is possible and accurate using Keplerian orbit mechanics. Moreover, the addition of a continuous acceleration in time complicates the dynamics and solving methods available.

The system of motion's equations during the application of continuous thrust to solve is:

$$\begin{cases} \dfrac{d\vec{r}}{dt} = \vec{v} \\ \\ \dfrac{d\vec{v}}{dt} = -\dfrac{\mu}{r^3}\vec{r} + \vec{a}_P + \dfrac{\vec{T}}{m} \\ \\ \dfrac{dm}{dt} = -\dfrac{T}{g_0\ I_{SP}} \end{cases} \tag{1.2.13}$$

These equations have an analytical solution when the initial and final orbits satisfy certain conditions such as initial and final circular orbits and similar cases with non-complex geometries.

To obtain an acceptable approximation as a first approach, is desirable to simplify the equations than to compute a differential and complex algorithm (which could enlarge the elapsed computational time needed to solve the full mission). Starting from the two body problem, the differential equations defining spacecraft orbit raising use the following assumptions [G. Mengali, 2013]:

- The force of thrust is constant and always in the plain of motion
- The vehicle has a fixed propellant mass flow rate
- The vehicle's acceleration is due solely to the force of thrust and an inverse-square, central gravitational field that is spherically symmetrical

Starting from the two body problem equations (Eq. 1.2.3), adding the differential time and integer the equation, an approach to the solution can be achieved:

$$\Delta v = \int_{t_0}^{t_f} \left(\frac{\vec{t}}{m} - \frac{\mu}{r^3}\hat{r} + \vec{a}_P \right) dt \tag{1.2.14}$$

As already stated, all perturbation forces are neglected ($\vec{a}_P \equiv 0$) so, the remaining elements which provide delta-v along the trajectory are the ***thrust applied*** and the ***gravitational forces***. The following equation from (Eq. 1.2.14) is obtained:

$$\Delta v = \int_{t_0}^{t_f} \left(\frac{\vec{t}}{m} - \frac{\mu}{r^3}\hat{r} \right) dt \tag{1.2.15}$$

Nevertheless, the *mass flow rate* has to be taken into account into this delta-v equation to obtain optimal results. Further analysis can express this equation into a series of differential equations which can provide the numerical solution searched.

$$\begin{aligned}
\dot{x} &= v_x & \ddot{x} &= -\mu\frac{x}{r^3} + \frac{T_x}{m} & \dot{m} &= -\frac{T}{I_{SP}\ g_0} \\
\dot{y} &= v_y & \ddot{y} &= -\mu\frac{y}{r^3} + \frac{T_y}{m} & T &= \sqrt{T_x^2 + T_y^2 + T_z^2} \\
\dot{z} &= v_z & \ddot{z} &= -\mu\frac{z}{r^3} + \frac{T_z}{m} & r &= \sqrt{r_x^2 + r_y^2 + r_z^2}
\end{aligned} \tag{1.2.16}$$

1.3 Problem parameters and constraints

Once the fundamental theory is explained, the parameters which govern the problem can be exposed and properly recognised.

Orbital mechanics can be incredibly complex if the dynamics are not simplified so, to solve the problem the next simplifications will be taken into account:

- The Sun and Moon's gravity are excluded
- Earth is modelled as a point of mass
- The Moon is assumed to follow a conic orbit around Earth
- Flybys of the Moon are to be modelled as patched conics

During the mission, a predefined set of physical constants and conversions to stablish tangible and constant parameters along the problem is going to be used:

Parameter	Value	Unit
Gravitational parameter of Earth μ	398600.4329	*[km^3/s^2]*
Gravitational parameter of Luna μ_M	4902.8006	*[km^3/s^2]*
Earth radius	6378.14	*[km]*
Luna radius	1737.5	*[km]*
Standard acceleration due to gravity	$9.80665 \cdot 10^{-3}$	*[km/s^2]*
Day	86400	*[s]*
Year	365.25	*[days]*

With concern to dynamics, the spacecraft and the Moon orbiting Earth are governed by the following formulae from Eq. (1.2.3):

$$\ddot{x} + \frac{x}{r^3} = 0 \qquad \ddot{y} + \frac{y}{r^3} = 0 \qquad \ddot{z} + \frac{z}{r^3} = 0$$

Where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1 - e^2)}{1 + e\ \cos\nu}$$

The mission is also restricted to the following statements:

- Each spacecraft can use its CPS only once, and the chemical system must be used before the LTS of the spacecraft is first used
- The mission begins when the first propulsion system in a spacecraft is used
- The spacecraft are not required to perform their impulses at the same time as each other
- The mission must start between 58849.0 MJD and 5888.0 which corresponds to the complete month of January 2020
- The mission must end within three years from the start date
- The time at which the last observation is made marks the end of the mission
- The spacecraft range to Earth must obey the following at all times:

$$6578.15\ km \leq r \leq 10\ 000\ 000\ km$$

- For a patched-conic Moon flyby to occur, the spacecraft geocentric position must match the Moon's geocentric position within 1 km

It is remarkable to state that the direction of delta-v and thrust applied along the entire mission is unconstrained.

1.3.1 Initial Orbits

Spacecraft: The three spacecraft are initially located in a 400-km altitude circular orbit around Earth in the ecliptic plane as shown in Fig. 1.3.1

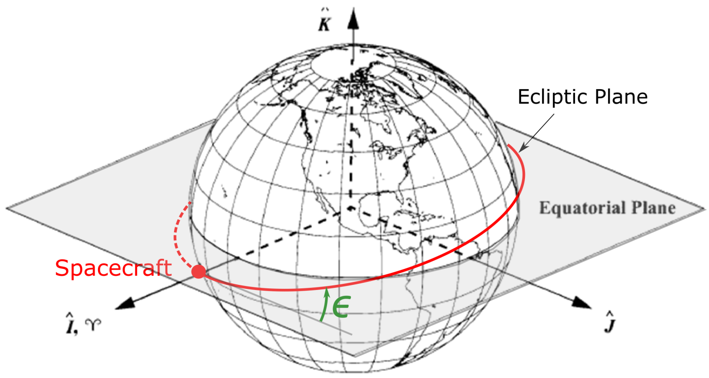


Figure 1.3.1: Initial spacecraft conditions at t_0 in IJK reference frame

$$
\begin{aligned}
t_0 &= 58849.0 \; MJD \\
x(t_0) &= R_E + 400 \; km \\
a(t_0) &= x(t_0) \\
e(t_0) &= 0 \\
i(t_0) &= 0 \; deg
\end{aligned}
$$

Notice that all three spacecraft lie together on the same point on the positive x-axis at t_0 and at a distance of 400 km from the centre of the Earth, in a circular orbit with the inclination of the ecliptic plane. The ecliptic plane is in a state of constant but inappreciable variation, the mean value at the realisation date of this project is about 23.4°. The currently variation is decreasing 0.0013° per hundred years due to planetary perturbations.

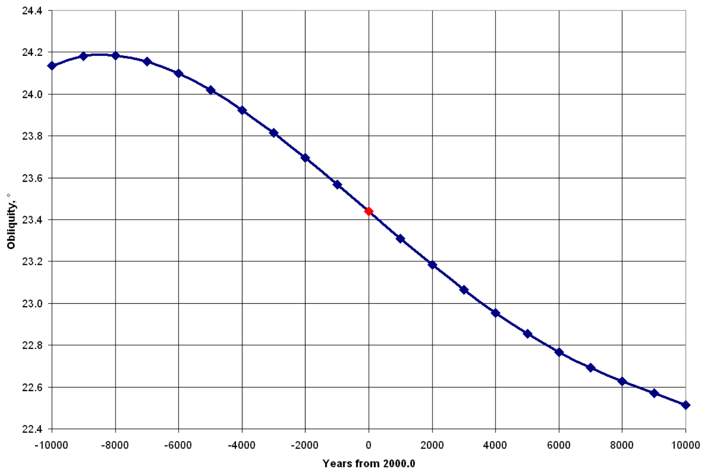


Figure 1.3.2: Obliquity of the ecliptic for 20,000 years [Laskar, 1986][Table 8]

Each spacecraft has on-board two propulsion systems; a CPS and a LTS with the following characteristics:

$$I_{SP_{CPS}} = 450\ s$$
$$I_{SP_{LTS}} = 5000\ s$$
$$\Delta v_{CPS} = 3\ km/s$$
$$T_{LTS} \leq 0.1\ N$$

Which mean that the Specific Impulse of the CPS and LTS are, respectively, 450 and 5000 seconds. The maximum delta-v applicable by the CPS is 3 km/s and the LTS can apply a thrust up to 0.1 N.

The spacecraft configuration constraint the use of the LTS. In order to use the low thrust, the CPS needs to previously have been used and jettisoned of the spacecraft, then, no further use is available.

Each spacecraft has an initial mass of 4000 kg and a minimum permissible mass of 1890 kg. The initial mass is the mass before any propulsion system has been employed and the remaining mass is the quantity of oxidiser and liquid fuel that the propulsion systems need to work.

Moon: To stablish Lunar position at mission's starting time it will be necessary to calculate some of its canonical Keplerian orbit elements, starting off the following given data:

Orbit element	Value	Unit
Semimajor axis μ	383500.0	*[km]*
Eccentricity μ_M	0.04986	*[]*
Inclination	5.2586	*[deg]*
RAAN	98.0954	*[deg]*
Arg. Periapsis	69.3903	*[deg]*
Mean anomaly	164.35025	*[deg]*

To obtain the actual position of the Moon in its orbit it is necessary to solve Kepler's problem (Sect. 1.2.2).

1.3.2 Time

The main purpose of time is to define with precision the moment of a phenomenon. Time is a fundamental dimension in almost every branch of science. In Astrodynamics, time is especially critical because objects move so far so quickly. There are copiously time systems to define ***epochs*** and ***dates*** based in the Sun and Earth motion or in the star positions however, concerning the project, only one system is going to be used.

Time units are given by the initial data in *Modified Julian Date* —MJD— and there are going to be converted into seconds during the implementation of the code to achieve a better precision.

The *Julian date, JD* is the interval of time measured in days from January 1, 4713 B.C. 12:00 stablishing the duration of *Julian year* as 365.25 days. To find the Julian date from a known date and time within the period March 1, 1900 to February 28, 2100 the next algorithm could be used:

$$JD = 367 \cdot yr - INT\left\{\frac{7\left[yr + INT\left(\frac{mo+9}{12}\right)\right]}{4}\right\} + INT\left(\frac{275mo}{9}\right) + d + 1721013.5 + \frac{\frac{\left(\frac{s}{60} + min\right)}{60} + h}{24} \tag{1.3.1}$$

Where the year, month, day, hour, minute and second are known; thus, the INT relation denotes real truncation. The values of this system are typically quite large so, instead of using this system it is recommended by the International Astronomical Union — IAU — to use a ***Modified Julian***

Date, MJD commonly calculated as follows:

$$MJD = JD - 2400000.5 \tag{1.3.2}$$

This system reduces the size of the date and it can reduce potential confusion because it begins each day at midnight instead of noon. What is more, the current case of the initial time given in this mission corresponds directly to January 1, 2020 00:00.

1.3.3 Patched-conic flyby

Lunar flybys are modelled using the patched-conics approximation and neglecting the time spent inside the Moon's sphere of influence. The gravity assist —flyby— occurs at time t_G when the spacecraft geocentric position equals the Moon's geocentric position to within 1 km; the spacecraft geocentric velocity undergoes a discontinuous change in such a way that the outgoing and incoming hyperbolic excess velocity relative to the Moon have the same magnitude and are separated by the turn angle δ_t. Specifically:

$$\begin{aligned}
\vec{x}(t_{G-}) &= \vec{x}_M(t_{G-}) \\
\vec{x}(t_{G+}) &= \vec{x}_M(t_{G+}) \\
\vec{v}_{\infty G-} &= \vec{v}(t_{G-}) - \vec{v}_M(t_{G-}) \\
\vec{v}_{\infty G+} &= \vec{v}(t_{G+}) - \vec{v}_M(t_{G+}) \\
|\vec{v}_{\infty G+}| &= |\vec{v}_{\infty G-}| = v_\infty \\
\vec{v}_{\infty G+} \cdot \vec{v}_{\infty G-} &= v_\infty^2 \ \cos\delta_t \\
\sin\left(\frac{\delta_t}{2}\right) &= \frac{\mu_M/(R_M + h_{pM})}{v_\infty^2 + \mu_M/(R_M + h_{pM})}
\end{aligned}$$

This is subjected to the timing and altitude constraints:

$$t_{G+} = t_{G-} \qquad h_{pM} \geq 50\ km \qquad v_\infty \geq 0.25\ km/s \qquad h_{pM} = r_{pM} - R_M$$

For computational purposes, the equality condition on the flyby position can be relaxed up to 1 km. And similarly the tolerance on the velocity condition is 1 m/s:

$$|\vec{x}(t_G) - \vec{x}_M(t_G)| \leq 1\ km \qquad\qquad |\vec{v}_{\infty G+}| - |\vec{v}_{\infty G-}| \leq 1\ m/s$$

1.3.4 Observation plane orientation

All along the mission, the three spacecraft create a triangular plane joining the points of space they are occupying at each instant. The plane normal is tracing the direction of the observation that can be taken. The normal — $\vec{n}$ — of the observing triangle is given by:

$$\vec{n} = \pm(\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1) \tag{1.3.3}$$

Where $\vec{r}_i$ is the position vector of the i^{th} spacecraft relative to the Earth, and a choice of sign is available. The degenerate case where the cross product is zero cannot be used to make an observation. The direction of a source is given by the vector $\vec{s}$:

$$\vec{s} = \cos\delta\cos\alpha\ \hat{x}\ + \cos\delta\sin\alpha\ \hat{y}\ + \sin\delta\ \hat{z} \tag{1.3.4}$$

Where α is the right ascension of the source, and δ the declination. Which have to be properly analysed in order to work in the proposed reference frame as listed in Sect. 1.2.3.

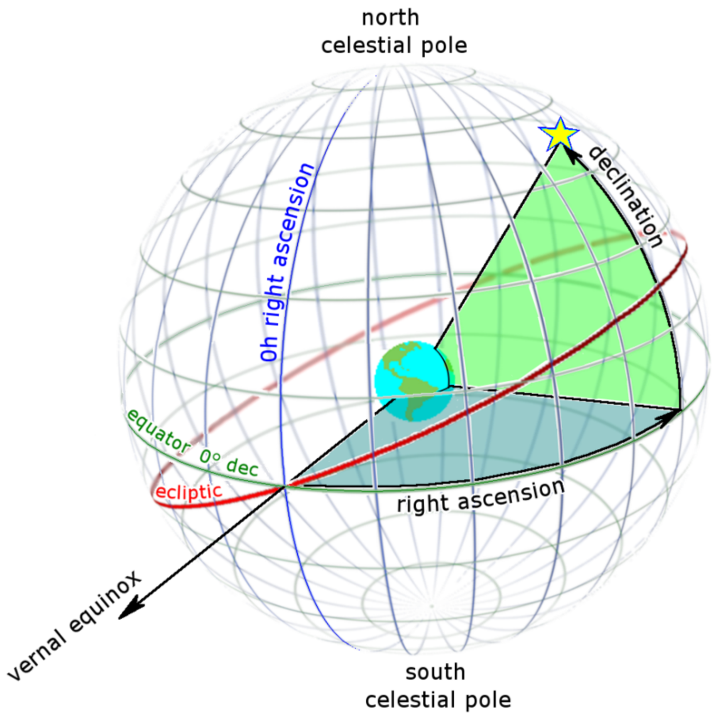


Figure 1.3.3: Right ascension and declination and its differences between equatorial and ecliptic planes

An observation can be taken when the vectors $\vec{n}$ and $\vec{s}$ are aligned. On addition, the alignment tolerance is 0.1°.

1.3.5 Performance index

The project is going to be driven by the optimisation of all the mission sequence, subsequently that will be converted into one specific value, the ***Performance Index, J***. That value determines the performance of all the measurements taken along the mission in concordance with GTOC description.
The *performance index* function it is defined by the following expression:

$$J = Ph(0.2 + \cos^2 \delta) \tag{1.3.5}$$

In this formulae, h denotes the smallest of the three altitudes of the observing triangle and must satisfy:

$$h \leq 10000\ km$$

An altitude of a triangle is the perpendicular distance from a vertex to the opposite side (or its extension). δ is the declination of the source being observed . P is a weighting factor for repeated observations that takes different values according to the next statements:

- If an observation is the first observation taken of a source: $P = 1$
- If an observation is the second observation taken of a previously observed source: $P = 3$ if $\frac{h_{max}}{h_{min}} \geq 3$, otherwise $P = 1$.
- If an observation is the third observation taken of a previously observed source: $P = 6$ if $\frac{h_{max}}{h_{min}} \geq 3$ and $\frac{h_{mid}}{h_{min}} \geq 3$, else $P = 3$ if $\frac{h_{max}}{h_{min}} \geq 3$ and the second observation of the source had a weight of $P = 1$, otherwise $P = 1$.
- If an observation is the fourth or greater observation taken of a previously observed source: $P = 0$.

Looking back at the performance index canonical equation (Eq. 1.3.5), it is recommended to find the canonical parameters which define the solution to the function, elements as position, velocity or delta-v.

Thus, as the declination and altitude are functions of $\vec{r}(t)$, and P it is defined by the order of measurements which is again a function of $\vec{r}(t)$; it can be confirmed that:

$$J = f(\vec{r}(t))$$

To optimise the problem it is necessary to take into account the position of the radio sources and try to obtain the maximum *performance index* with the same amount of measurements. To achieve this, it is mandatory to take 2-3 measurements of the same radio source if the time between measurements it is not greater than **15 days**, which is the minimum possible time between measurements.

The project's aims to obtain a value of this performance index in order to, subsequently, add the solver's code into a *Multi-Objective Genetic Algorithm* —MOEA—code [Deb et al., 2002] to ensure the optimal trajectory to follow. As the initial possibilities are unlimited, this further study will be attached to a series of flyby manoeuvres which are explained in advance in Chapter 3.

Chapter 2

Mission structure

Once the mission is fully described and the mathematical theories exposed, the initial decisions to change the trajectories can be taken.

This chapter introduces the solving parameters of the mission, gives the premise of the initial manoeuvres chosen, the solution approach taken and the final configuration searched.

The initial considerations to be taken care about expose an infinite number of possible trajectories, this situation gives an outstanding importance to the first manoeuvres applied, those of which have to be fully analysed in order to find the optimal solution.

2.1 Initial conditions

At the point where the mission starts, an infinite range of possibilities takes place hence the decision to apply a delta-v at a specific time instant has to be chosen.

To approach the mission in the most efficient and easy way possible is critical to take into consideration the capabilities this project's author owns. GTOC is a problem solved by teams with the best Astrophysicist and Mathematicians of the world, usually professionals working for space agencies as ESA or NASA. For instance, the combination of three spacecraft orbiting at different independent orbits and taking measurements surpass a highly advanced knowledge in the topic. On that account **several simplifications** are determined.

In order to reduce the search field and to approach an optimum and much simpler solution, the problem is approached from the following standard conditions:

Two of the spacecraft —**node-line spacecraft**— form the same fixed direction so the resultant triangular plane formed is exclusively in function of the position of the other spacecraft —**out-of-plane spacecraft**—. That fixed direction is going to lay in the ecliptic plane, formed by two spacecraft orbiting in the same circular orbit, furthermore they are phased 180° the one from the other to achieve the maximum value of h in the performance index function (Fig. 2.1.1).

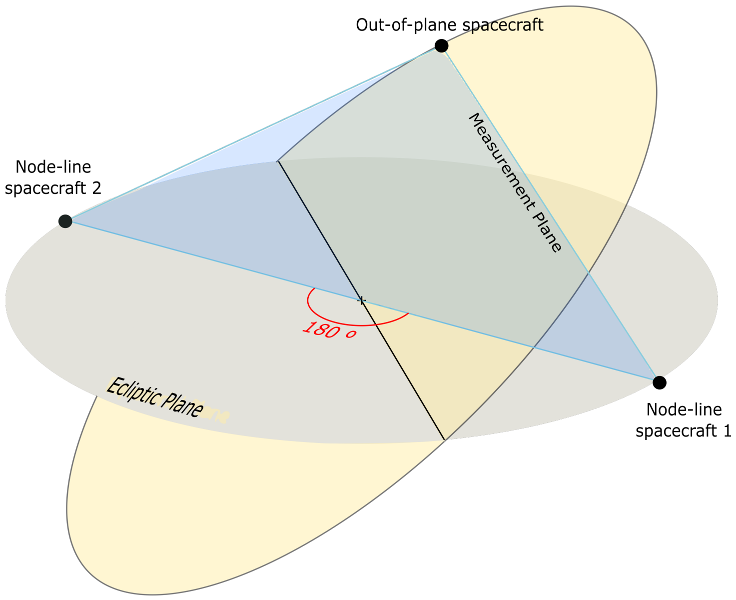


Figure 2.1.1: Standard conditions searched and measurement plane attitude

To reach those conditions with the optimum time and fuel consumptions, a deep study is necessary. The mission is primarily subdued to time and fuel restrictions, therefore *Lunar-flybys* are an excellent choice of **fuel-free orbital transfers** and, taking into account the fact that the LTS on board the spacecraft has a relatively small thrust, achieving a transfer from LEO to the Moon for the *out-of-plane spacecraft* is a major trajectory to be done.

Regarding the *node-line-spacecraft*, both need to raise their apogees and achieve a circular orbit as fast as possible, nevertheless this can be accomplished with their propulsion systems on board.

Therefore, the first objective to be accomplished is the optimal raising of spacecraft's apoapsis. To generate a function to solve the *performance index* the first manoeuvres using the CPS and LTS until standard conditions are met are fixed so the solver will start and point its tendency to a solution near the global maximum zone. Up to this point, it is necessary to define the first burns the spacecraft are going to perform. This is determinant in the mission since it is going to greatly define the final value of the *performance index*, hence they have to be cautiously chosen.

2.2 Manoeuvring

2.2.1 Initial considerations for lunar-flyby

Regarding the out-of-plane spacecraft, another range of possibilities needs to be solved to find the most optimal case. To encounter the Moon at this point, the CPS alone can be used, but maybe the 3 km/s of delta-v on board are not enough to reach the Moon. In this case it will be necessary to use both propulsion systems to reach it [Mingotti et al., 2009].

Another possibility is to use the LTS alone with its 0.1 N of thrust. Eventually the spacecraft will encounter the Moon but the use of the low-thrust from LEO might not rise the orbit's altitude fast enough to consider this manoeuvre optimal in terms of elapsed time.
It is necessary to solve each possibility exhaustively in order to find the best manoeuvre.

CPS approach

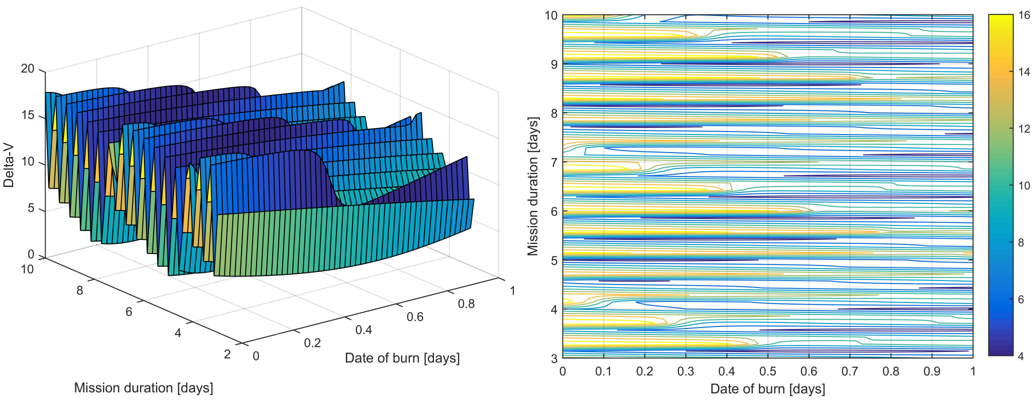
The objective is to reach the designated target (the Moon) using a single impulsive burn of 3 km/s or less. These are the initial conditions of a famous and useful algorithm in orbital mechanics called *"Lambert's Problem"* [Avanzini, 2008]. The solution to this geometric and time-dominated problem is achieved by an iterative method like *Newton-Raphson's*.

First of all, if the delta-v available for the CPS is applied, it is required to know if there is enough fuel inside our spacecraft. As the propulsion system used in this approach is the impulsive one, the mass loss is calculated due to equation (1.2.11).

$$m_f = 4000\ kg \cdot \exp\left(-\frac{3\ \frac{km}{s}}{9.80665 \cdot 10^{-3} \frac{km}{s^2} \cdot 450\ s}\right) = 2026.851\ kg > 1890\ kg$$

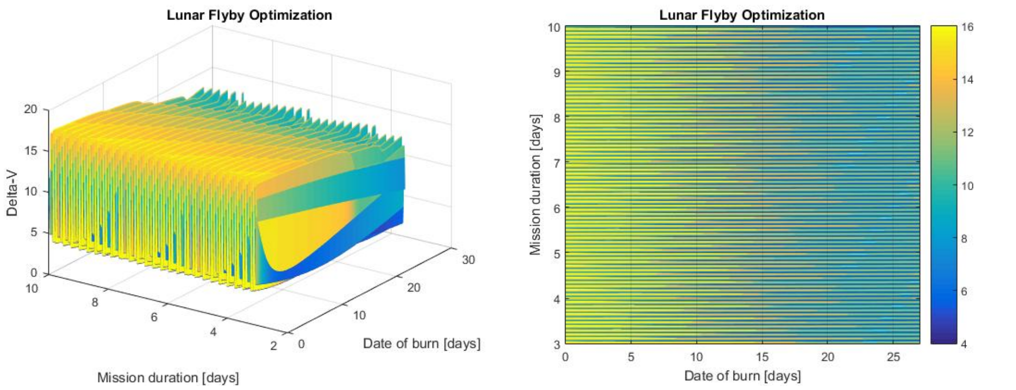
The initial conditions of the trajectory searched are two bodies moving at variable different speeds therefore, the time of departure and the time of arrival are two important variables to take into account as well as the delta-v needed. If these three variables are combined into a chart it will be obtained what is commonly called a *"pork-chop plot"*, which gives a way to visualize and to optimise the necessary delta-v.

After the proper code development to solve the trajectory, the results show that, unfortunately, for a complete lunar period —27.14 days—, there is not enough delta-v in the CPS to perform a flyby with a single burn. As shown in figures 2.2.2a and 2.2.2b and for a better precision figures 2.2.1a and 2.2.1b; it is necessary at least some more delta-v to reach the Moon in a single impulsive burn since the minimum delta-v needed it is a slightly higher value of 3 km/s. If this approach case is preferred, it will be needed to use the LTS once the CPS is first fired.



(a) 3D plot (b) Pork-chop plot

Figure 2.2.1: CPS approach to the Moon for a day



(a) 3D plot (b) Pork-chop plot

Figure 2.2.2: CPS approach to the Moon for a complete Lunar period

LTS Circular orbit phasing

For all manoeuvres discussed so far, it has been assumed that there has been use of impulsive thrust. Although *impulsive* models work thriving with most manoeuvres, some energetic burns may require more time to be applied. Those set of manoeuvres are denominated low-thrust transfers [Genta and Maffione, 2016].

It is possible to change a spacecraft's orbital inclination and semi-major axis using low-thrust, many-revolution manoeuvres, however this type of trajectories have limited techniques to be solved since their mathematical complexity. Low-thrust trajectories have attracted a lot of work, and studies showing how to make them optimal and continuous are especially relevant. A few of the most popular of those studies are: [Edelbaum, 1965], who has published many papers discussing on changes in semimajor axis and inclination; [Alfano and Thorne, 1994, Wiesel and Alfano, 1985], who studied the analytical solution for the nearly optimal case of a continuous, tangential, orbit transfer using low thrust and many revolutions.

To apply this technique to the mission, as a first approach, only the LTS is going to be used to raise spacecraft's orbit to the Moon. As mentioned before, there is a problem with mathematical complexity and a method to find an analytical solution to the equations of motion.

However, to notice what choice is most efficient in this transfer, the need to be exhaustively precise is not required; an orbital simulation software can be used to simulate the trajectory. NASA's General Mission Analysis Tool — GMAT —can simulate mission's parameters and, erasing all the perturbation effects possible to generate Keplerian orbits as similar as they can be, GMAT provides the opportunity to generate a continuous thrust trajectory without the necessity of solving the motion equations directly. Fig. 2.2.3 shows the trajectory obtained with the simulator.

In the simulation, the time needed to perform the manoeuvre is approximately 8 years, achieving an almost circular trajectory. To check this, the procedures given by *"Propulsion Requirements for Controllable Satellites"* are applied [Edelbaum, 1961] creating a MATLAB code to obtain an accurate solution. Despite GMAT gives an excellent way to visualize data, Edelbaum's procedure should achieve analogous results. After implementing in MATLAB Edelbaum's algorithms, a comparison between the simulator and this method can be done.

It is important to take into account the fact that the procedure described by Edelbaum it is purely design for circular low thrust orbits transfers; this means that the initial and final orbits must be circular, having the possibility of a change in inclination and height.

The procedure's solutions are presented as follows:

$$Initial\ orbit\ altitude = 400\ km$$
$$Initial\ orbit\ inclination = 0^\circ$$
$$Initial\ orbit\ velocity = 7.668\ km/s$$

$$Final\ orbit\ altitude = 377121.86\ km$$
$$Final\ orbit\ inclination = 0^\circ$$
$$Final\ orbit\ velocity = 1.019\ km/s$$

$$I_{SP} = 5000\ s$$
$$T_{LTS} = 0.1\ N$$
$$m_0 = 4000\ kg$$
$$m_f = 3492.75\ kg$$
$$t_{LTS} = 3078.268\ days$$

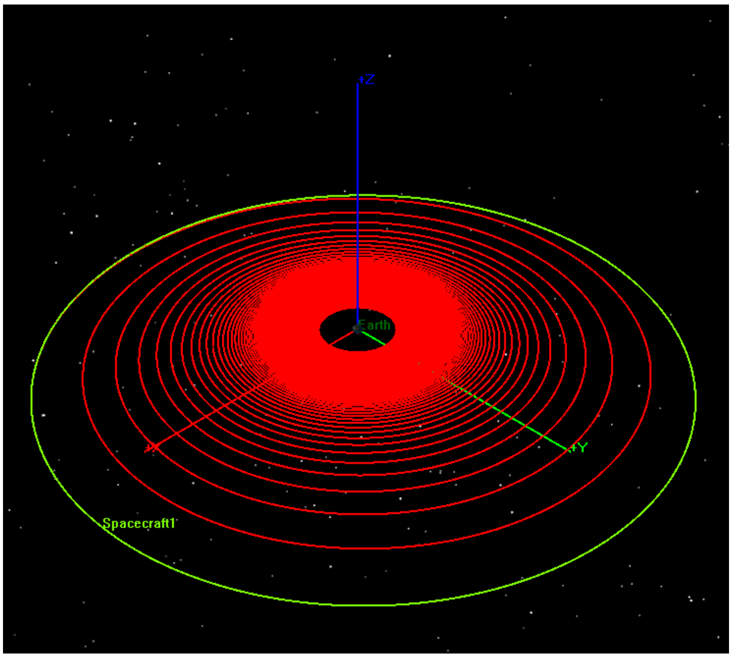


Figure 2.2.3: Trajectory of a continuous LTS burn along the posigrade vector until Moon approach

As it can be seen, it is possible to accomplish the flyby with the LTS and with minimum fuel consumption however, the low magnitude of thrust the propulsion system is capable of provide greatly enlarges the time needed to achieve the lunar orbit. This time of around 8.4 $years$ is far greater than the time provided for the mission (3 $years$). Unfortunately, it is not possible to achieve the flyby with this alternative either.

2.2.2 LEO-Moon Manoeuvres

In the previous sections, it has been proved that both propulsion systems onboard are insufficient by themselves alone to accomplish a lunar flyby. The best option to optimise a flyby trajectory is the initial raise of the apogees from LEO with the CPS and, right after the impulse is performed, the activation of the LTS until the spacecraft reaches the Moon.

The LTS's low thrust magnitude provides a greater performance the lower the gravitational force acting on the spacecraft hence, as gravity decreases with the square value of the distance from the source (Eq. 1.2.3), the higher the orbit the best performance is obtained. For instance, the CPS will apply its 3 km/s of delta-v to raise the apoapsis creating a highly eccentric orbit to, subsequently, approach the Moon with the LTS.

LTS to Moon insertion

As previously stated, the use of a continuous low thrust propulsion system to achieve a rendezvous manoeuvre to encounter another body in space when the departure orbit is not circular, necessarily adds the inclusion of a number of constraints in the trajectories and manoeuvres. For the use of this system with a non-constrained manoeuvre, the equation system to solve will not have analytical solution and the problem would be approached with other methods that can compute the solution in the best way possible. Taking into consideration the fact that the final destination is the Moon and, a Lunar-flyby occurs when the geocentric position of the spacecraft and the Moon match into 1 km of distance or less, the accuracy of the method to use has to be quite high.

The continuous thrust equations listed (Eq. 1.2.16) have analytical solutions when the initial and final orbits satisfy certain conditions, comparatively to Edelbaum's method. However, the initial conditions are adverse to the process of finding a solution to the equations since after the delta-v applied with the CPS, high elliptical orbits with high eccentricities are obtained. This complicates the process and adds the requirement to use complex computational algorithms to accomplish an accurate solution.

As a consequence, to obtain the correct flight path towards the Moon it is necessary to integer the equation system throughout a multiple-revolution

travel in the direction of the spacecraft's posigrade velocity vector, adjusting the thrust magnitude until a Lunar rendezvous occurs.
The solution to this trajectory is thoroughly discussed in Sec. 3.3.

2.3 Standard conditions and final trajectory

Once an initial Lunar-flyby is achieved and two of the spacecraft share the same orbit on the ecliptic plane phased 180° the one from the other, the following steps in the mission are going to be determined by the use of all gravity assists. As far as possible, the LTS system will be unused in order to save fuel and the main "propulsion" system will be the gravitational assistance of the Moon.

To accomplish a high number of gravity assists, resonant orbits between the out-of-plane spacecraft and the Moon are going to be searched. With this approach it is possible to return to the Moon for another flyby after a *"n" revolution* journey. Sampling the resultant orbit after the flyby with its planes formed in the search of *radio sources* will provide a *performance index* value.

Each possible orbit will define a value for the performance index. In order to obtain its maximum value for each manoeuvre, the LTS can be fired to reach some radio sources in the vicinity as well to adjust the harmonic orbit to the Moon. As the LTS is a very-high-efficient propulsion system (sacrificing thrust in order to gain that efficiency) it can be used exhaustively through the mission without the concern of a lack of fuel.
The resultant orbits chosen depend on two principal parameters:

- The maximum **performance index value** the trajectory is able to obtain
- The minimum **elapsed trajectory time** between flybys

The amount of radio sources given to perform the measurements are scattered along the celestial sphere in an relatively homogeneous way so there is not a cloud of sources considerably close by to aim to. Furthermore, the sources which give a high value of J (declination near $\pm 90°$) are much less abundant than the others.

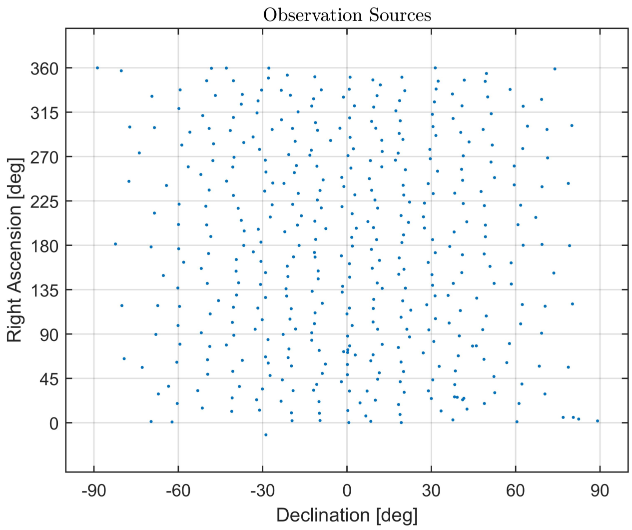


Figure 2.3.1: **Right Ascension vs Declination** of all radio sources given

As referred in Fig. 2.3.1, for a near-polar ($\approx \pm 90°$) value of the declination the valid positions for measurement are distributed more widely and reach a wider variety of trajectories than the sources with a near-equatorial ($\approx 0°$) declination, which are subjected to timing conditions in order to adjust the correct right ascension for a declination given.

Consequently, the presence of the weighting factor P changes the perspective of the mission. Although the more observations are taken, the higher the value of J will be; a proper spacecraft distribution taking into account P will acquire a higher performance value with a lesser number of observations. In addition, once the spacecraft is positioned after firing their propulsion systems, every orbital change using the LTS will be very time-wasteful.

To achieve the best trajectories, a *Multi Objective Genetic Algorithm* is used. This algorithm gives the best trajectories for the two objective functions (performance index and elapsed time). As the trajectories obtained are subjected to resonance with the Moon, after a certain period of time it will be possible to acquire another flight path by a gravity assist. The best solution for each flyby will give a succession of possible measurements until the mission time expires, which is finally all translated into a final value of the performance index.

Chapter 3

Code development

In this final chapter the results are displayed and the complete algorithm is explained as well as the known errors and possible improvements to achieve a better value of J.

Throughout the development of the project there have been many steps until the final solver's code was achieved. The first approach to orbital mechanics with MATLAB needs to be truly organized and ensure a wide vision of the mission. However, the main issue regarding project's management is the amount of different functions necessary to solve a specific manoeuvre hence, visualizing a trajectory or defining the resultant orbit after an impulsive manoeuvre (things with the lowest solving difficulty at first), are hard-working tasks which have to be exhaustively revised in order to obtain a correct result in all case scenarios.

3.1 Code structure

To implement a MATLAB function to solve J it is necessary to insert $\Delta\vec{v}(t)$ and the time when the measurements are taken. An important issue with the performance index solver is the insertion of all the Lunar-flybys, however using the patched conics technique, the flybys can be simplified as another $\Delta\vec{v}(t)$ with the determination of the instant when the geocentric position of a spacecraft and the Moon is less than 1 km. J is defined as function of:

$$J = f(\Delta\vec{v}(t), \vec{r}(t))$$

In order to achieve the final objective, the development of the code starts by the smaller and basic functions to determine orbital elements which further implementation will be elemental. It is also well noted that each manoeuvre applied has a different configuration in terms of code, indeed, all function development is general and easily modified to further implementation in the code.

The mission has some prefixed orbits that need to be determined for some circumstances, hence in order to ensure a consistent calculation, a set of primordial functions have to be generated:

1. Orbit determination and propagation (bold expressions refer to functions as defined in the code):
   - COE from position and velocity vectors (**rv2coe**)
   - Position and velocity vectors from COE (**coe2rv**)
   - Kepler's problem solution (**kepler**)
   - Direct Kepler's problem solution (**keplerdirect**)
2. Reference frame change:
   - Right ascension and declination from position vector (**ijk2radec**)
   - Position vector from right ascension and declination (**radec2ijk**)
   - Change of reference frame from Ecliptic to Equatorial (**ec2eq**)
   - Change of reference frame form Equatorial to Ecliptic (**eq2ec**)
3. Time conversions:
   - Change seconds to MJD (**sec2mjd**)
   - Change MJD to seconds (**mjd2sec**)

These functions mainly allow propagation between the spacecraft and the Moon, to determine positions with right ascension and declination angles, transform the values between the different reference frames and angles, and subject the problem to MJD time system. Throughout the code, the creation of more complex functions by the abovementioned prime functions is thoroughly extended.

To ensure a better comprehension of the code sequence, a brief summary of the whole code is presented:

With the initial conditions of the mission, the primordial functions and propulsive manoeuvres the ***standard conditions*** are met and a flyby can be applied. However, before applying the flyby, radio source data has to be transformed into useful information for the code in order to achieve the exact coordinates to perform the measurements: this is done through function

observations. Afterwards all data gathered is collected into ***measurement*** function, which obtains the final velocity conditions after the flyby and the possible sources that can be measured. At that time the function ***performance_index*** explores all the search field of possible combinations of measurements and determines the maximum value of J thus, including this function with the genetic algorithm ***nsga_2***, the final function ***global_trajectory_optimisation*** obtains the optimal trajectory searched.

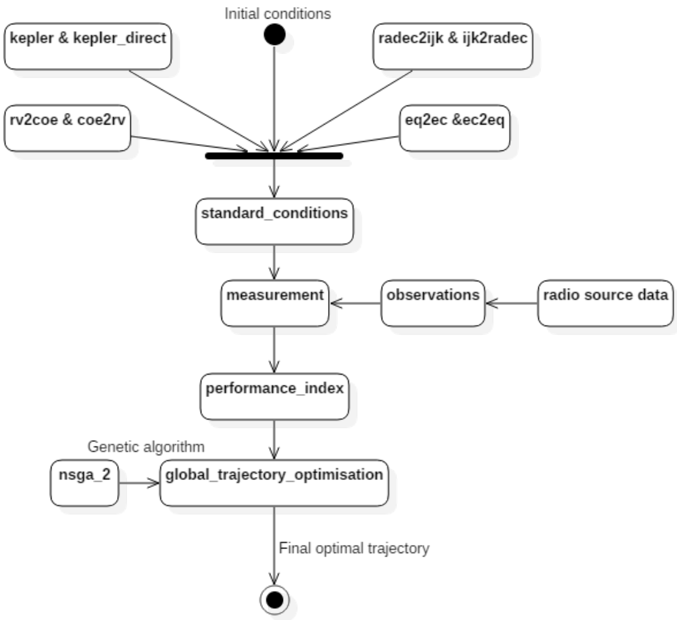


Figure 3.1.1: Activity diagram of code functions

3.2 Spacecraft and Moon initial orbits

Before starting to build up the code for the diverse burns, is necessary to determine the initial spacecraft and Moon orbits with their Keplerian and Cartesian elements. With the initial data provided almost all orbital elements of the spacecraft are determined. For the initial time t_0 it is known their orbital inclination (ecliptic obliquity, ϵ), position ($\vec{r}$), argument of latitude (u) and $RAAN$ (Ω).

The only parameter left to know is the velocity, which modulus can be acknowledge with the specific mechanical energy formulae (Eq. 1.2.4):

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = 7.6685\ \frac{km}{s}$$

Subsequently the vectorial components are easily defined due to the counter clockwise direction of the spacecraft $\hat{v}_{t_0} = [0, 1, 0]$. Once the position and velocity vectors are determined, it is possible to obtain all COE through function "*rv2coe*".

As discussed and concerning Moon's initial position, its determination is linked to Kepler's equation solution (Eq. 1.2.6). Though it, the *eccentric anomaly* (and subsequently the *true anomaly*) can be solved. With the initial data provided (Sect. 1.3.1) all COE are obtained and as consequence, through function "*coe2rv*", its position and velocity vectors.

3.3 Manoeuvring to Standard Conditions

CPS

Regarding each spacecraft impulsive burns, all of them perform similar manoeuvres to achieve the standard conditions. The CPS of two of the spacecraft is applied at the ascending node of the Moon-Spacecraft orbit system. Furthermore, the other spacecraft ignite their CPS at the descending node of the system (true anomaly's 180° further) to achieve orbits with large enough distances between spacecraft's positions during the mission and also to obtain a common point with the Moon in which perform a flyby (Fig. 3.3.1).

Applying impulses at these points, a correct alignment with Lunar orbit is ensured from the starting point. Hence it is possible for the LTS to align the inclinations of the spacecraft with the Moon, as well as the opportunity to encounter the Moon at the *RAAN* or *RADN* without the necessity to change the inclination of the transfer orbit.

To obtain an equatorial high-eccentric orbit rising at its maximum the apogee with $\Delta v = 3\ km/s$ it is necessary to apply that delta-v in the following unitary vector on NTW satellite-based coordinates:

$$\Delta\vec{v}_{NTW} = [3, 0, 0]\ \ km/s$$

That unitary vector transformed to the *Earth Mean Ecliptic and Equinox of J2000 frame* will be:

$$\Delta\hat{v}_{xyz_1} = [-0.9900, -0.1408, 0]$$
$$\Delta\hat{v}_{xyz_{2-3}} = [0.3650, -0.9309, 0]$$

Therefore, the delta-v applied to archive the orbit is:

$$\Delta\vec{v}_{xyz_1} = [-3.9900, -3.1408, 0] \;\; km/s \qquad at \qquad t_1 = 1513.2927 \; s$$

$$\Delta\vec{v}_{xyz_{2-3}} = [3.3650, -3.9309, 0] \;\; km/s \qquad at \qquad t_2 = 2776.8140 \; s$$

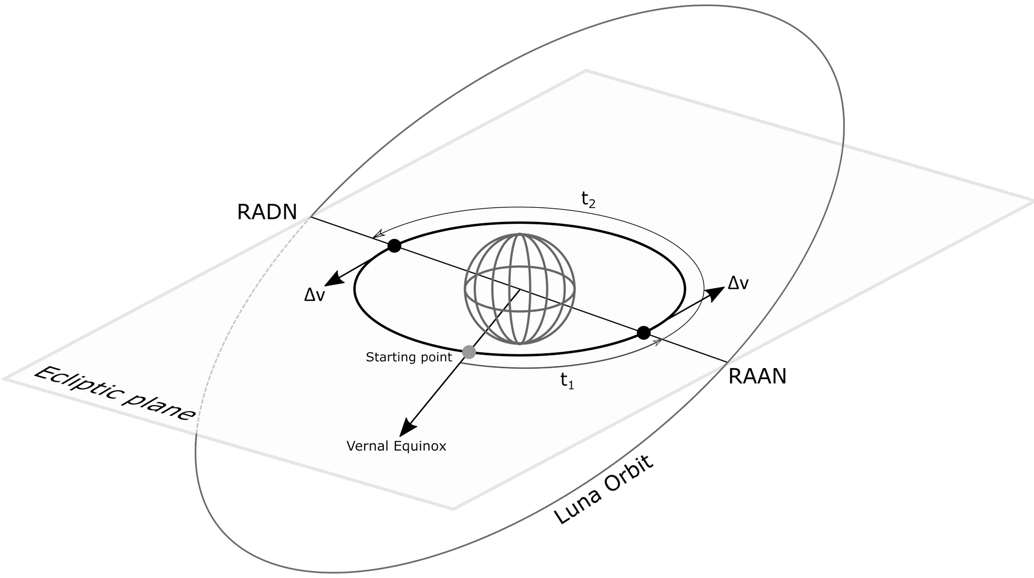


Figure 3.3.1: Initial visualisation of first CPS burns

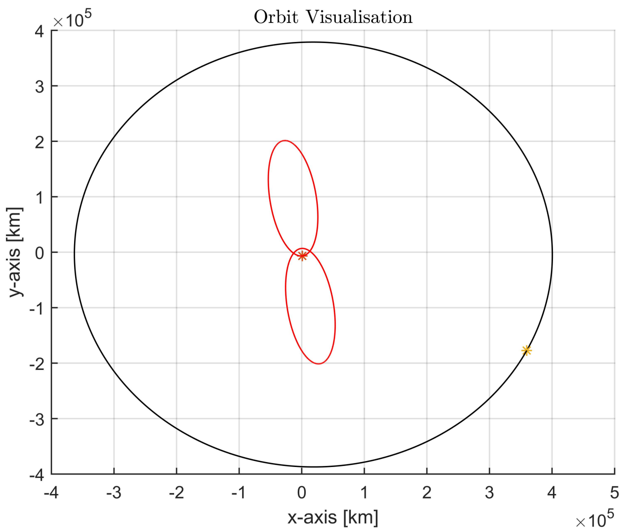


Figure 3.3.2: Final orbits achieved for a CPS burn of 3 km/s along the posigrade vector

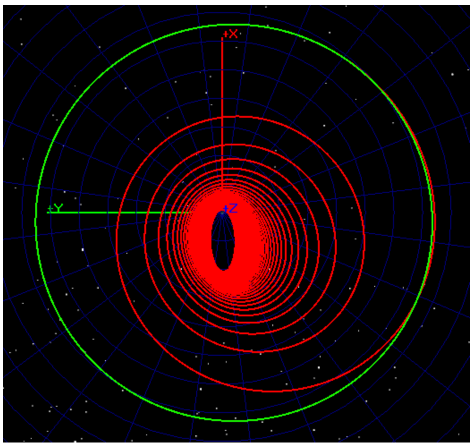
LTS

The objective searched is the elaboration of a computational algorithm which can solve low-thrust equations (Eq. 1.2.16) with some initial conditions given. During the mission the position of the spacecraft $\vec{r}$ is known along all trajectories but thrust has to be fixed in modulus and direction during the integration time chosen. Thus, with the initial conditions of thrust and integration time given, the algorithm calculates the final velocity and position vectors reached with the total mass loss of the flight.

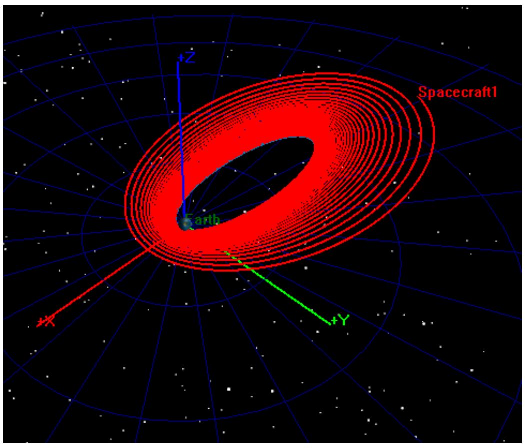
However, the exact calculation of these kind of procedures **goes far beyond the aim of this project** hence the LTS trajectories and results necessary to achieve standard conditions are simulated with GMAT.

The two node-line spacecraft carry out a different LTS burn than the out-of-plane spacecraft. Regarding the node-line spacecraft after their CPS application, they try to circularise their orbits using the LTS at its maximum thrusting magnitude (0.1 N) towards $\hat{t_d} = [0.7071, 0.7071, 0]$ direction in the $\boldsymbol{LVLH}$ Satellite Radial System (Sect. 1.2.3) until they achieve the minimum value of eccentricity possible. This performs a trajectory as described in Fig. 3.3.3a.

On behalf of the out-of-plane spacecraft the same procedure is applied with the exception of the thrusting direction. To achieve an intersection with the Moon, maximum magnitude of low thrust is applied along the direction $\hat{t_d} = [1, 0, 0]$ in the $\boldsymbol{NTW}$ Satellite Normal System. The described trajectory is referenced in Fig. 3.3.3b.



(a) Node-line spacecraft trajectory



(b) Out-of-plane spacecraft trajectory

Figure 3.3.3: Simulation of both LTS trajectories the spacecraft have to perform in order to achieve standard conditions

After this manoeuvres, the estimated time of arrival to standard conditions for all spacecraft is 212.25 days thus the first flyby occurs at 58061.25 MJD and the mission ends at 59944.75 MJD.

3.4 Flyby

Once the flight path approaches the geocentric position of the Moon at the designated time given, flyby occurs. As stated, patched-conics approximation is considered in this kind of events, which represents that all the change in velocity and inclination during the time spent inside the SOI of the Moon is going to be applied instantaneously in a point.

The patched conics method simplifies the real parameters of a gravity assist simulating the targeted mass as a point in space until the spacecraft is reasonably close by to enter its SOI. The hyperbolic orbit during the fight time inside the SOI is also simplified hence the flyby occurs instantly in a point within a relatively close geocentric distance between the two objects. The result will be an instantaneous change of velocity according to the conditions listed in Sect. 1.3.3.

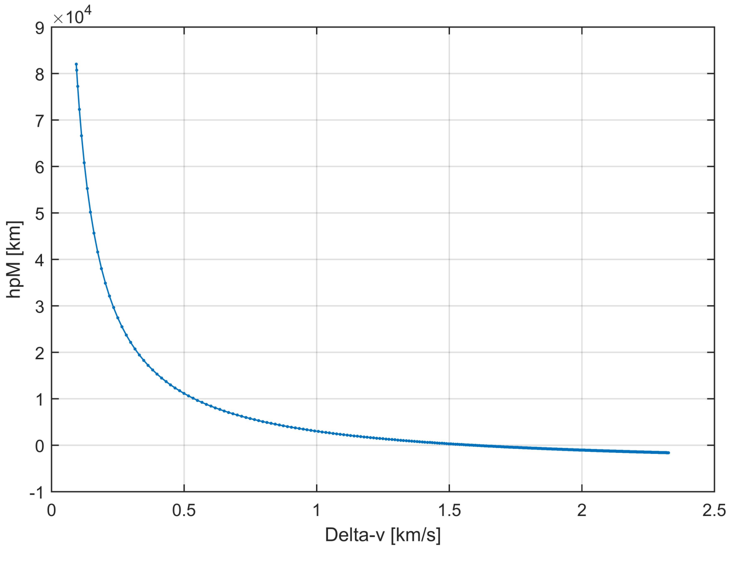


Figure 3.4.1: **Lunar periapsis heigh vs delta-v** obtained in a flyby

The variables to take into consideration are the spacecraft and Lunar velocity vectors (which are prefixed at the moment of the gravity assist by the performed trajectory) and the final velocity targeted. To solve the procedure, the final velocity obtained after the flyby has to be determined. If the final velocity is fixed, the value of height at lunar periapsis it will be determined. The possibility of an unacceptable value of final velocity can de given ($h_{pM} \leq 50\ km$), in which case it will be necessary to change the spacecraft's approach trajectory to the Moon; or, if the velocity vector obtained has an approximate enough value from the targeted one, take it as valid and use the LTS afterwards (out of lunar influence) to change the orbital parameters to the desired ones. However, one of the aims for the selected approach is to, as far as possible, do not use the LTS after the standard conditions are met.

Using the equations provided [Battin, 1999], it is possible to obtain a direct relation between delta-v gained and height at lunar periapsis for some fixed initial conditions given as shown in Fig. 3.4.1. As it can be seen, the lower the lunar periapsis height, the higher the delta-v obtained. For instance, this displays the direct relation between the final velocity targeted and the value of h_{pM} needed. A gravity assist of this kind provides a great advantage in both delta-v and velocity direction change, which can be used in direct changes of orbital inclination.

In conclusion, it is possible to acquire very high values of delta-v if a flyby takes place although no fuel is burned during the procedure. However, even knowing the optimal parameters to achieve the maximum delta-v in the flyby, the conditions to acquire a desired final velocity are not given. To achieve this it is necessary to develop an algorithm that, given the **initial velocity vectors** (spacecraft and Moon's), could solve the patched conics to obtain a specific value of h_{pM} to achieve a desired **final velocity**.

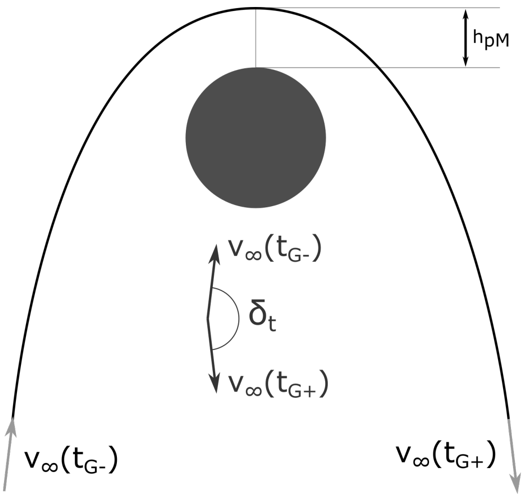
The first constraint for flybys is to obtain a series of resonant orbits with the Moon, hence there is no need to use the LTS on board and a instantaneous change of velocity can be applied every n revolution journey. Since the resultant orbit can make the system take measurements, a value of J is attached to it, subsequently this allow to study all the possible post-flyby orbits and choose the maximum value of J for the configuration.

In all the possible orbits that can be obtained, seven variables describe the problem: $\vec{r}_{SP}, \vec{v}_{SP}, \vec{r}_M, \vec{v}_M, \theta, m, n$. The initial position and velocity of the spacecraft and the Moon are unconditionally related to the approach trajectory taken during the first manoeuvres. Variables m and n define the number of revolutions performed by the Moon and the spacecraft, respectively, before returning to the same position and allow another flyby. Those four variables ($\vec{v}_{SP}, \vec{v}_M, m, n$) determine an angle β in the flyby plane which creates a cone of possible directions; and a modulus $\alpha = \|\vec{v}(t_{G+})\|$. These two elements are assembled by the following algorithm:

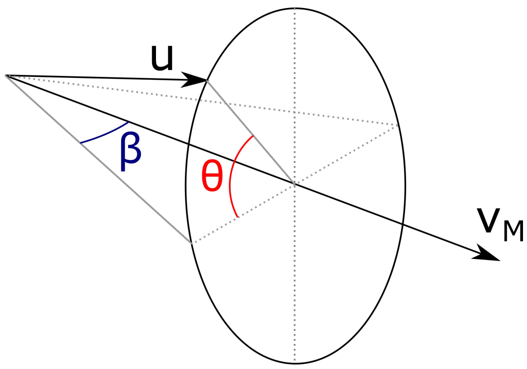
$$T_{SP} = \frac{T_M\ m}{n}$$
$$a_{SP} = \left(\frac{\mu\ T_{SP}^2}{4\pi^2}\right)^{1/3}$$
$$\alpha = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a_{SP}}\right)}$$
$$\vec{v}_\infty = \vec{v}_{SP} - \vec{v}_M$$
$$\beta = \arccos\left(\frac{\alpha^2 + v_M^2 - v_\infty^2}{2\ \alpha\ v_M}\right)$$

There is a last variable to be defined: angle θ completely determines the direction given by the flyby travelling along the circumference described by β (Fig. 3.4.2).

For instance, the final velocity obtained $\vec{v}(t_{G+}) = \hat{u}\cdot\alpha$ has to meet patched conics requirements and the value of the Lunar periapsis height cannot be below 50 km.



(a) Relation among $\vec{v}_\infty$ vectors with h_{pM} and δ_t



(b) Vector $\hat{u}$ with angles β and θ

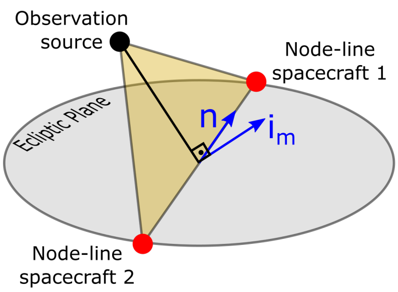
Figure 3.4.2: Flyby mathematical description

3.5 Measurements

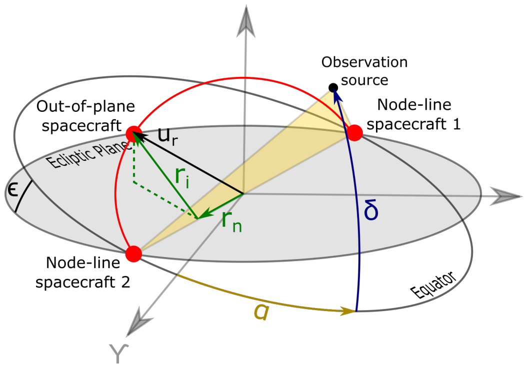
Source measurement is subjected to spacecraft position which can be transposed to right ascension and declination. However, the problem arises when a study of possible measurements is done taking into account all three spacecraft movement, which is constantly variable. Therefore, there is another simplification to be applied to study the problem in a plausible way:

- The constant motion of the node-line spacecraft is fixed

To obtain an optimal measurement trajectory, each radio source is plotted independently in function of the right ascension and declination by which can be measured. Sources are defined by right ascension and declination, since the problem is study in the *Earth Mean Ecliptic and Equinox of J2000 frame* those angles are transposed to their analogous (longitude, α' and latitude, β') into the ecliptic frame. As a result, the node line $\hat{n}$ where the two spacecraft lay is defined. The node line and source analysed form a plane which normal vector defines the direction $\hat{i_m}$, simultaneously, those two vectors define the trajectory out-of-plane spacecraft has to perform in order to continuously measure the source (Fig. 3.5.1).



(a) $\hat{n}$ and $\hat{i_m}$ definition



(b) Spacecraft measurement trajectory

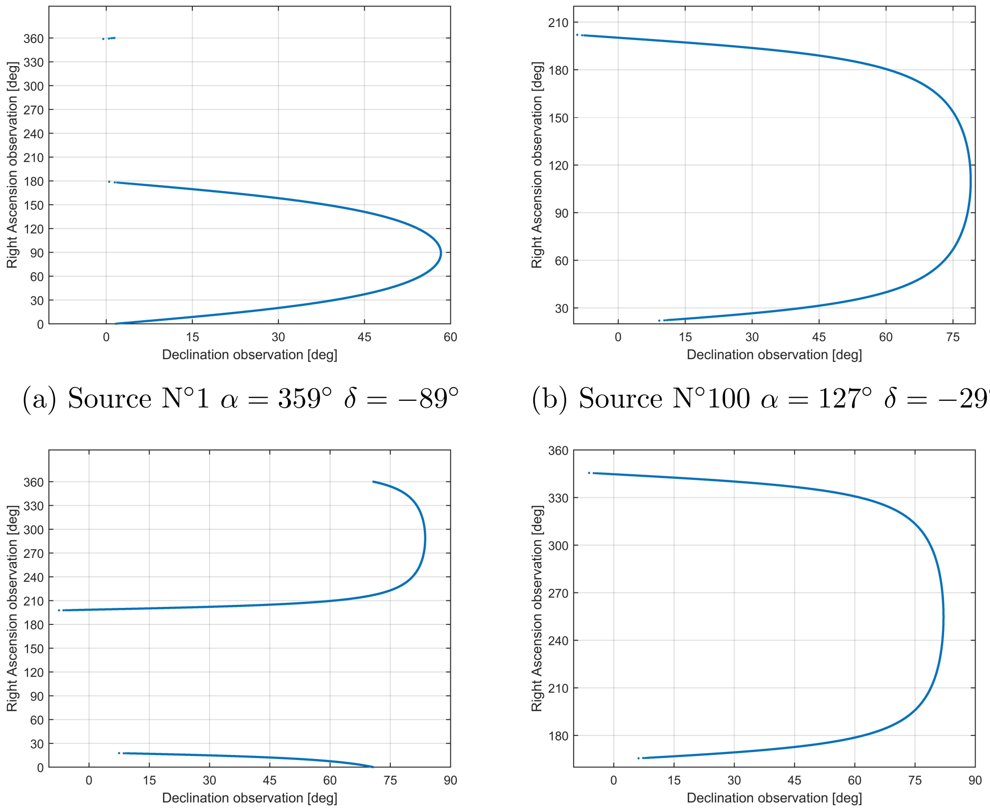
Figure 3.5.1: Reference frames and for initial study of source measurement. Note that unitary vectors which build up the frame are represented in blue while the measurement trajectory is represented in red

As previously stated, in order to obtain a pure geometric configuration in which sources could be measured, the $\hat{n}$ and $\hat{i}_m$ frame is used to relate the right ascension and the declination of the out-of-plane spacecraft with the possible measurement planes of each source. The obtention of this data is followed by the next algorithm:

$$\vec{n} = [-\sin(\alpha'), \cos(\alpha'), 0]$$
$$\vec{i}_m = [-\sin(\delta')\cos(\alpha'), -\sin(\delta')\sin(\alpha'), \cos(\delta')]$$
$$\vec{u_r} = r_i \cdot \vec{i}_m + r_n \cdot \vec{n}$$

Where:
$r_i^2 + r_n^2 = 1$
α' longitude of the source in the ecliptic frame
δ' latitude of the source in the ecliptic frame

Subjected to $\hat{n}$ and $\hat{i_m}$, each observation source has its unique output trajectory (Fig. 3.5.2). The data obtained for each source is saved into a **".xlsx"** file to improve computational time which it will be called further in the code.



(a) Source N°1 $\alpha = 359°$ $\delta = -89°$

(b) Source N°100 $\alpha = 127°$ $\delta = -29°$

(c) Source N°201 $\alpha = 116°$ $\delta = 1°$

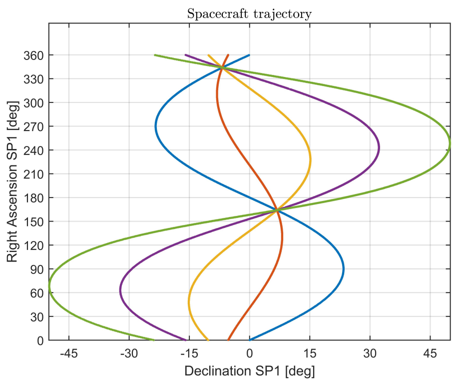
(d) Source N°420 $\alpha = 69°$ $\delta = 3°$

Figure 3.5.2: Measurement trajectories for different radio sources

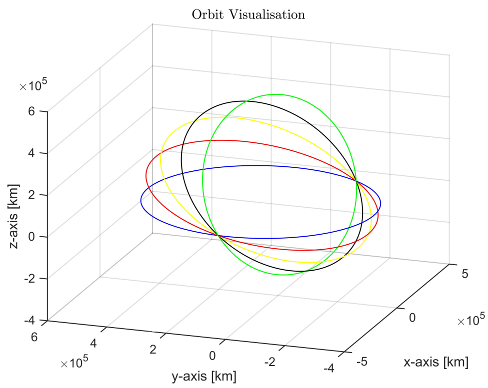
When a possible flyby trajectory is calculated, the resultant orbit is rendered subdued to right ascension and declination (Fig. 3.5.3a) hence the data from sources and spacecraft trajectory can be compared. As a result, two functions are compared:

1. Out-of-plane spacecraft trajectory

2. Source measurement trajectory



(a) Right ascension and declination



(b) Orbit visualisation in Ecliptic Frame

Figure 3.5.3: **Right ascension vs declination of Out-of-plane spacecraft:** Possible orbits obtained after a flyby. Note the obliquity change due to the ecliptic and right ascension-declination different frames

For each observation source, the intersection between both data —figures 3.5.2 and 3.5.3a— is searched. Physically, each intersecting point implies a possible measurement for the designated source at the time given. As a result, This approach allows to know how many, where and when sources can be measured for a designated flight trajectory.

Finally, $r_{\vec{SP}}$ and $v_{\vec{M}}$ do not have direct use in the final trajectory determination but its inclusion in the algorithm ensures the propagation through the trajectory in order to measure observation times as well as the guarantee that the flyby conditions are met at the designated flight times:

$$r_{\vec{SP}}(t_{flyby}) = r_{\vec{M}}(t_{flyby})$$
$$r_{\vec{SP}}(n \cdot T_{SP}) = r_{\vec{M}}(m \cdot T_M)$$

In conclusion, the application of the abovementioned studies into a MATLAB algorithm, including the input of $\vec{r}_{SP}, \vec{v}_{SP}, \vec{r}_M, \vec{v}_M, \theta, m, n$ the final velocity vector after the gravity assist is obtained as well as instants and positions of the out-of-plane spacecraft to perform possible measurements.

3.6 Genetic Algorithm Optimisation

Due to the flyby and measurement research, the calculations of all parameters needed are obtained and the direct determination of J is concluded.

Since the initial manoeuvres are fixed, so are positions and velocities of the spacecraft and the Moon until the first flyby conditions are met. This allows the MATLAB function less inputs to work with from the beginning, reducing the necessary inputs to m, n, θ.

To ensure the optimum parameters from each resonant orbit, a *Multi Objective Genetic Algorithm* is applied to the function in order to reach the maximum value the mission has to offer.

A MOEA is a type of local search that mimics evolution by taking a population of strings, which encode possible solutions, and combines them based on a fitness function to produce individuals that are more fit. The algorithm used to optimise the mission is NSGA-II [Deb et al., 2002]. NSGA-II is a fast and elitist multi-objective evolutionary algorithm that optimise two or more functions using nondominated sorting and sharing. This algorithm works with a multi-objective optimisation purpose in order to find the global maximum or minimum of a set of objective functions.

Unlike in single-objective optimisation, there are two aims in multiobjective optimisation:

- Converge to the Pareto-optimal set
- Maintenance of diversity in solutions of the Pareto-optimal set

In the proposed NSGA-II a list of population members N is created. This population evolves and mutates through a set of generations using mathematical methods that mimic natural evolution processes. Thought generations, the best members of the population are accepted to be part of the next generation while the rest are replaced by offsprings from the best solutions —elitism—.

The set of multiple objectives in a problem gives rise to a set of optimal solutions (largely known as Pareto-optimal solutions) instead of a single optimal solution. Without any further information, it cannot be stated that any of this Pareto-optimal solutions are better than the other. NSGA-II employs a nondominated sorting method, comparing each solution with every other solution in the population to find its dominated value.

Firstly, the algorithm creates a random population P_0. This population is sorted based on the nondomination and each solution is assigned a fitness rank equal to its nondomination level. Afterwards, the usual binary tournament selection, recombination and mutation operators are used in order to create an offspring population Q_0 of size N. The actual population $R_0 = P_0 + Q_0$ of size 2 N is sorted according to nondomination. Since the function uses elitism, the best sets of solutions pass to the next generation until the population N is complete and the rest solutions are rejected.

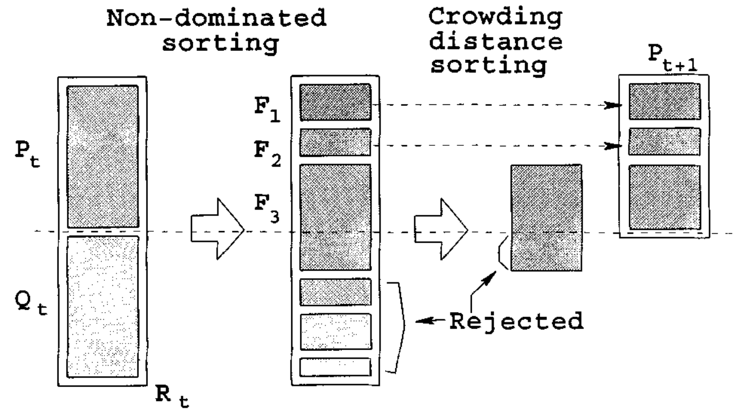


Figure 3.6.1: NSGA-II procedure throughout generations
[Deb et al., 2002][Figure 2]

With the new population P_t the algorithm's main loop uses selection, crossover and mutation to create a new population Q_t of size N. Once more, the nondomination sorting fit and ranks each solution and therefore a crowding distance sorting evaluates the density of solutions in order to obtain a diverse variety of solutions as possible P_{t+1} (Fig. 3.6.1).

The computational complexity of the algorithm is defined as $O(MN^3)$ (where M is the number of objectives and N is the population size). In order to obtain a consistent result for each trajectory a population $N \in [20, 50]$ along $GEN \in [5, 50]$ is executed for each flyby trajectory.

When the genetic algorithm presents its multiple results, they are evaluated and the most convenient solution is chosen.

3.7 Result overview

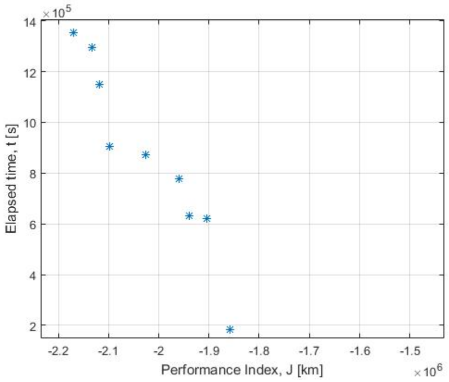
Throughout the conformation of the mission a series of functions have been created. All of them were assembled in a sequence which grants an optimal trajectory for each flyby. After proper simulations the first results are finally achieved.

The solution process defines an array of N elements for each objective function and for number of decision variables (or inputs). That population is the result of a number of evolutionary processes through generations inside Pareto-optimal set of solutions. To obtain a preference for one of the solutions, ***nsga_2*** provides a value of preferable selection $S \in [0, 1]$ that points out which solution has the better values in relation with the objective functions.

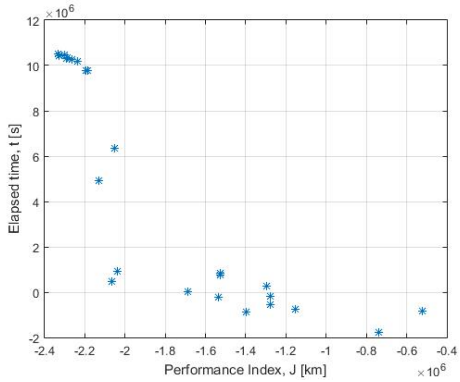
Through the simulations, many different values of J are obtained. The flybys are constrained by the mission **final date** (59944.25 MJD) thus the flyby sequences go on until the time is depleted. The highest value calculated is $J = 26.732 \cdot 10^6$ km and is linked to the following flight sequence:

$Flyby$ #	t $[MJD]$	m	n	θ $[rad]$	ΔJ $[km \cdot 10^6]$	J $[km \cdot 10^6]$
1	58061.250	2	3	5.943	1.554	1.554
2	58115.961	1	2	0.731	1.084	2.638
3	58143.316	3	2	1.534	1.814	4.452
4	58225.383	3	2	5.229	1.973	6.425
5	58307.449	3	3	5.594	2.401	8.827
6	58389.516	2	2	5.577	1.689	10.501
7	58444.227	2	3	2.932	1.755	12.271
8	58498.938	1	2	1.208	0.917	13.319
9	58526.293	1	2	0.383	0.918	14.107
10	58553.649	2	3	0.534	1.592	15.700
11	58608.360	2	3	0.975	1.754	17.453
12	58663.071	1	2	1.426	0.872	18.325
13	58690.426	3	3	3.937	2.365	20.690
14	58772.693	2	1	0.867	0.876	21.566
15	58854.764	2	3	5.176	1.739	23.306
16	58882.115	3	3	2.376	2.164	25.470
17	59936.127	2	3	5.805	1.817	**27.287**

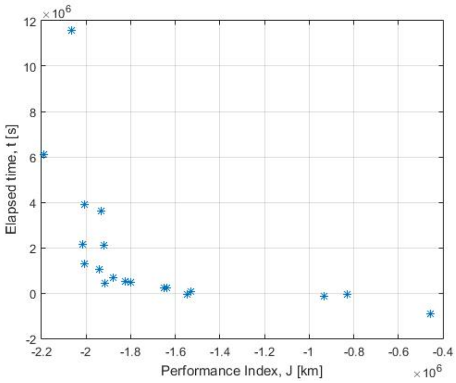
Achieving the Pareto-optimal front makes impossible to make any one population individual better off without making at least one other worse off, hence the selected results are extracted from this front for each performed flyby. The selection of a specific population is done with the higher value of S, which indicate the higher nondominated solution of the front.



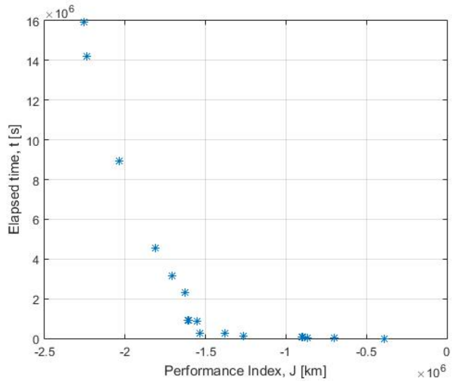
(a) $N = 20 \;\; GEN = 50$



(b) $N = 50 \;\; GEN = 5$



(c) $N = 20 \;\; GEN = 10$



(d) $N = 30 \;\; GEN = 20$

Figure 3.7.1: Pareto-optimal fronts for different populations and generations. Negative values of time are neglected since they do not have physical sense. Note that negative values of J are due to NSGA-II methodology to maximise objective functions.

3.8 Future modifications and implementations

GTOC problems are solved by teams of professional astrophysicists worldwide due to their difficulty and complexity, consequently the work done by a single engineering student is not enough to fully solve one of them.

The approach given simplifies the complexity of the problem and gives an excellent solution, however other design alternatives could provide improved results. Modifications can be applied throughout the whole mission (from the first manoeuvres chosen to the final optimisations after flybys) due to the unconstrained motion of the spacecraft.

The following content points out some of the possible improvements of the mission:

Optimal LEO-Standard conditions trajectory: The approach taken to achieve standard conditions is subdued to the utilisation of the LTS after the CPS, this provides a wide range of possibilities. The followed approach is optimum enough, however the low thrust equations need to be properly solved through propagation and a suitable method to encounter the Moon and achieve high-altitude circular orbits for the node-line spacecraft is necessary.
Moreover, a possible accuracy method for LTS manoeuvring is given as follows:

To approach a precise and realistic simulation a new constraint to the numerical integration process can be added:

Keplerian trajectories in space are conics, it can be determined that, the change of angular separation between the two position vectors calculated in every step (initial and final) is a parameter to highly take into account for a good thrusting precision. To establish a range for that parameter and, taking into account the computational time and precision needed, a good range of values will be between $0.1°$ and $1°$ of angular separation in cases of poor precision needed and, when the dynamics will be required to be smoother, the step time can be decreased as much as it is necessary to obtain the desired accuracy.

Motion-independent spacecraft: To accomplish a suitable approach to the measurement problem, both node-line spacecraft positions were determined by the right ascension of the sources and fixed throughout the out-of-plane spacecraft trajectory. Further implementations to the mission should take into consideration the constant motion and timing of the plane formed by all spacecraft between flybys.

LTS for each flyby trajectory: Few trials have been done to implement this approach of the mission, however, the time of realisation of this project was not enough to fully develop it.

With the complexity of the addition of another grade of freedom to the mission, the LTS on board the spacecraft can provide a better flight path if is ignited between flybys. This provides more freedom regarding source measurement and would polish the solution thanks to a differential change of trajectory mid-path. For each flyby, a value of maximum delta-v is calculated in order to reach more or/and better measurement conditions. This value of delta-v can be a objective function for the MOEA to minimise (Fig. 3.8.1). Therefore the equation of low thrust propulsion (Eq. 1.2.12) can analyse what happens if that delta-v can be applied before the next flyby occurs.

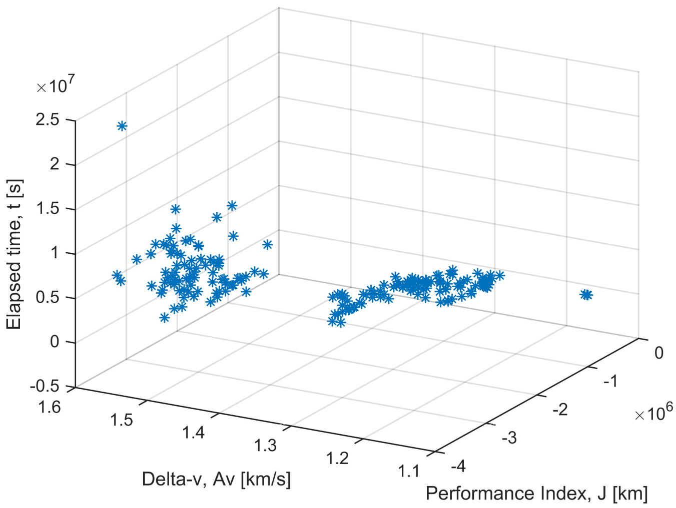


Figure 3.8.1: NSGA-II Pareto front for 3 objective functions $J, t, \Delta v$

3.9 Conclusions

With the realisation of this project a simplified and optimal solution to GTOC composed by a synergy of all learned concepts throughout its fulfillment has been provided. Although finding a better solution than the competition winners was a major objective, a feasible alternative for the author's capabilities has been achieved.

With multitude challenging problems that have been proposed throughout its realisation (*Lambert's problem, Low-thrust manoeuvring, patched conics flybys...*), this project has certainly been didactic for its author.

Starting from an initial theoretical approach to orbital mechanics and optimisation procedures, different sets of possible solutions have been evaluated in order to optimise the mission. From the theoretical analysis, a subsequent MATLAB code has been developed with specific functions to solve Keplerian orbital mechanics and the addition of mission parameters and constraints. The final value of J acquired is $27.287 \cdot 10^6$ which, in the final competition rankings will achieve the 12^{th} rank out of 18 teams presented [GTOC 8 results]. With GTOC high stands, the result is consistent and positive.

Many of the hypothesis taken have discharged large work to finally reach a non-optimal approach of the mission, hence they have been discarded. Nevertheless, their study has been a challenge which has enhance the knowledge of the problem until the final manoeuvre sequence chosen was obtained.

Personal statement: Working on this project has been a rewarding experience for me. Work in a complex field such as orbital mechanics and integrating every function needed into a generic numerical computing environment as MATLAB is certainly challenging. Starting from a blank script on day 1 to seeing how everything built up and started to form part of something bigger (with many effort and long sleepless nights however) was really pleasing.

I am glad I chose this project as my Bachelor's Thesis because since I started working on it, I have not stopped learning new and interesting things from this passionate field.

Appendix A

Conversion from Position and Velocity to Canonical Orbital Elements

Subsequently, the conversion method from the all eleven orbital elements to the geocentric position and the velocity is described:

1. Inputs:
    - Spacecraft position $\vec{r}[km]$
    - Spacecraft velocity $\vec{v}[km/s]$
2. Outputs:
    - Semi-latus rectum p $[km]$
    - Semimajor axis a $[km]$
    - Eccentricity e $[km]$
    - Inclination i $[rad]$
    - Longitude of the ascending node Ω $[rad]$
    - Argument of the periapsis ω $[rad]$
    - True anomaly ν $[rad]$
    - Mean anomaly M $[rad]$
    - Argument of latitude u $[rad]$
    - True longitude l $[rad]$
    - Longitude of periapsis ϖ $[rad]$

3. Calculation sequence:

Firstly, the orbital momentum $\vec{h}$ and the eccentricity vector $\vec{e}$ are calculated:

$$\vec{h} = \vec{r} \times \vec{v}$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}$$

Coming up next, semimajor axis, eccentricity and semi-latus rectum are defined:

$$a = \frac{v^2}{2} - \frac{\mu}{r} \qquad p = \frac{h^2}{\mu} \qquad e = \|\vec{e}\|$$

Inclination is calculated with $\vec{k}$ (vector perpendicular to the orbital plane):

$$i = \arccos\left(\frac{\vec{h} \cdot \hat{k}}{h}\right)$$

Subsequently, the true anomaly is calculated as:

$$\nu = \begin{cases} \arccos \dfrac{\vec{e} \cdot \vec{r}}{e\ r}, & \text{if } \vec{r} \cdot \vec{v} \geq 0 \\ 2\pi - \arccos \dfrac{\vec{e} \cdot \vec{r}}{e\ r}, & \text{if } \vec{r} \cdot \vec{v} \leq 0 \end{cases}$$

Up to this junction, the eccentric and mean anomalies are defined as:

$$E = 2 \arctan \frac{\tan \frac{\nu}{2}}{\sqrt{\frac{1+e}{1-e}}}$$

$$M = E - e \sin E$$

With $\vec{n}$, the right ascension of the ascending node and the longitude of the periapsis are calculated:

$$\Omega = \begin{cases} \arccos \dfrac{\vec{n} \cdot \hat{\imath}}{n}, & \text{if } \vec{n} \cdot \hat{\jmath} \geq 0 \\ 2\pi - \arccos \dfrac{\vec{n} \cdot \hat{\imath}}{n}, & \text{if } \vec{n} \cdot \hat{\jmath} < 0 \end{cases}$$

$$\omega = \begin{cases} \arccos \dfrac{\vec{n} \cdot \vec{e}}{n\ e}, & \text{if } \vec{e} \cdot \hat{k} \geq 0 \\ 2\pi - \arccos \dfrac{\vec{n} \cdot \vec{e}}{n\ e}, & \text{if } \vec{e} \cdot \hat{k} < 0 \end{cases}$$

If certain types of orbits are obtained (circular equatorial, circular inclined, elliptical equatorial) the argument of latitude, true longitude and longitude of periapsis need to be calculated:

$$\begin{aligned} u &= \arccos \frac{\vec{r} \cdot \vec{n}}{r}, \qquad &\text{if } i \neq 0 \ \&\ e = 0 \\ l &= \arccos \frac{\vec{r} \cdot \vec{n}}{r}, \qquad &\text{if } i = 0 \ \&\ e = 0 \\ \varpi &= \arccos \frac{\vec{e} \cdot \vec{n}}{e}, \qquad &\text{if } i = 0 \ \&\ e \neq 0 \end{aligned}$$

Appendix B

Conversion from Canonical Orbital Elements to Position and Velocity

Subsequently, the conversion method from the geocentric position and the velocity to the eleven canonical orbital elements is described:

1. Inputs:
   - Semi-latus rectum p $[km]$
   - Semimajor axis a $[km]$
   - Eccentricity e $[km]$
   - Inclination i $[rad]$
   - Longitude of the ascending node Ω $[rad]$
   - Argument of the periapsis ω $[rad]$
   - True anomaly ν $[rad]$
   - Mean anomaly M $[rad]$
   - Argument of latitude u $[rad]$
   - True longitude l $[rad]$
   - Longitude of periapsis ϖ $[rad]$
2. Outputs:
   - Spacecraft position $\vec{r}[km]$
   - Spacecraft velocity $\vec{v}[km/s]$

3. Calculation sequence:

It is convenient to use the *perifocal coordinate system* in which the position vector has the following expression:

$$\vec{r} = r\cos\nu\hat{P} + r\sin\nu\hat{Q}$$

Where the distance r is given, as a function of the orbital parameters, from the conic polar equation:

$$r = \frac{p}{1 + e\cos\nu} \qquad with \qquad p = a(1 - e^2) \tag{B.0.1}$$

Hence, the components of $\vec{r}$ in the *Perifocal coordinate system* are:

$$[\vec{r}]_{\tau_P} = r\begin{bmatrix}\cos\nu\\ \sin\nu\\ 0\end{bmatrix} = \frac{a(1-e^2)}{1+e\cos\nu}\begin{bmatrix}\cos\nu\\ \sin\nu\\ 0\end{bmatrix}$$

On behalf of $\vec{v}$ it is remarkable to take into account that the vectors $\hat{P}$ and $\hat{Q}$ are constant in time thus the Perifocal reference frame can be assumed as an inertial system:

$$\frac{d\hat{P}}{dt} \equiv \frac{d\hat{Q}}{dt} = 0$$

Deriving respect to time the above expression of $\vec{r}$ given (Eq. B.0.1) and applying the corresponding mathematical simplifications the following expression is obtained:

$$[\vec{v}]_{\tau_P} = \sqrt{\frac{\mu}{p}}\begin{bmatrix}-\sin\nu\\ e+\cos\nu\\ 0\end{bmatrix}$$

Note that the components of $\vec{r}$ and $\vec{v}$ calculated depend exclusively of the elements a, e and ν. That is owe to the fact that the abovementioned expressions are referred to a Perifocal reference frame.

The components of $\vec{r}$ and $\vec{v}$ respect to a *Geocentric-Equatorial system* or its analogous depend of all orbital elements. Therefore:

$$[\vec{r}]_{\tau_G} = [x\ y\ z]^T$$
$$[\vec{v}]_{\tau_G} = [v_x\ v_y\ v_z]^T$$

$$[\vec{r}]_{\tau_G} = [\mathbf{T}(i,\Omega,\omega)]^T[\vec{r}]_{\tau_G}$$
$$[\vec{v}]_{\tau_G} = [\mathbf{T}(i,\Omega,\omega)]^T[\vec{v}]_{\tau_G}$$

Developing $[\mathbf{T}(i,\Omega,\omega)]$ matrix the final expressions for $\vec{r}$ and $\vec{v}$ are obtained ($s = \sin$, $c = \cos$):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{p}{1+\cos\nu} \begin{bmatrix} c\nu(c\Omega c\omega - s\Omega s\omega ci) - s\nu(c\Omega s\omega + s\Omega c\omega ci) \\ c\nu(s\Omega c\omega + c\Omega s\omega ci) - s\nu(s\Omega s\omega - c\Omega c\omega ci) \\ c\nu s\omega si + s\nu c\omega si \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \sqrt{\frac{\mu}{p}} \begin{bmatrix} -s\nu(c\Omega c\omega - s\Omega s\omega ci) - (e + c\nu)(c\Omega s\omega + s\Omega c\omega ci) \\ -s\nu(s\Omega c\omega + c\Omega s\omega ci) - (e + c\nu)(s\Omega s\omega - c\Omega c\omega ci) \\ -s\nu s\omega si + (e + c\nu)c\omega si \end{bmatrix}$$

Appendix C

Project's costs

The project is fully solved with purely computing material thus a decent computer, which can easily hold the computational calculus of MATLAB, is adequate. On behalf of human resources, approximately one year is the time for a last year student to complete the work done. The following table displays the breakdown of costs:

Materials	Computer	699.00 €
Software	Complete MATLAB r2016a license	1 729.00 €
	Microsoft Office 2016 1 year license	69.99 €
Human Resources	Annual salary of Entry-level Aerospace Engineer	36 448.22 €
	Social security (28.3%)	10 314.85 €
Total project's costs		**49 261.06 €**

Information links of project's costs (URLs only available in electronic version):

- Commercial MATLAB license
- Microsoft Office 2016
- Entry- level experienced Aerospace Engineer in the UK
- Social security payments
- Computer

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