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Keywords: Consumption smoothing, Long-run/Short-run estimation, Non-stationary Dynamic Factor Models, Kalman filter, Principal components, Resilience, Risk sharing.

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Abstract

In this paper, we analyze and compare the finite sample properties of alternative factor extraction procedures in the context of non-stationary Dynamic Factor Models (DFMs). On top of considering procedures already available in the literature, we extend the hybrid method based on the combination of principal components and Kalman filter and smoothing algorithms to non-stationary models. We show that, unless the idiosyncratic noise is non-stationary, procedures based on extracting the factors using the non-stationary original series work better than those based on differenced variables. The results are illustrated in an empirical application fitting non-stationary DFM to aggregate GDP and consumption of the set of 21 OECD industrialized countries. The goal is to check international risk sharing is a short or long-run issue.

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1 Introduction

Dynamic Factor Models (DFMs) were first introduced in economics by Geweke (1977) and Sargent and Sims (1977) with the aim of extracting the underlying common factors in a system of time series. In macroeconomics, these common factors are useful for building indicators and to predict key variables of the economy, among many other applications. Recently, econometricians have to deal with data sets consisting of hundreds of series, making the use of large dimensional DFMs very attractive in practice; see Breitung and Eickmeier (2006), Bai and Ng (2008), Stock and Watson (2011), Breitung and Choi (2013) and Bai and Wang (2016) for reviews of the existing literature.

It is well known that macroeconomic time series are frequently non-stationary and cointegrated. The connection between cointegration and common factors is analyzed by Stock and Watson (1988), Johansen (1991), Vahid and Engle (1993), Escribano and Peña (1994), Gonzalo and Granger (1995), Bai (2004), Bai and Ng (2004), Moon and Perron (2004), Banerjee et al. (2014a,b) and Barigozzi et al. (2016, 2017), among others. Although differencing has advantages in univariate time series to deal with non-stationarity, it should be made with great care when dealing with multivariate systems; see Box and Tiao (1977). It is well known that when differencing a cointegrated system, the long-run information, crucial to understand co-movements between the variables, is lost. Canova (1998) qualifies the detrending issue as "delicate and controversial" and compares the properties of the cyclical components of a system of seven real macroeconomic series obtained using seven univariate and three multivariate techniques. He concludes that the properties of the extracted business cycles vary widely across detrending methods. Sims (2012) claims that "when cointegration may be present, simply getting rid of the non-stationarity by differencing individual series so that they are all stationary throws away vast amounts of information and may distort inference". Consequently, the number of works dealing with non-stationary and possibly cointegrated DFMs is increasing. In the context of non-stationary systems, Bai (2004) proposes factor extraction implementing Principal Components (PC) to data in levels and derives the rates of convergence and limiting distributions of the estimated common trends and loading weights when the idiosyncratic components are stationary; see Engel et al. (2015) for an application to exchange rates. However, Barigozzi et al. (2016, 2017) point out that stationarity of the idiosyncratic components would produce an amount of cointegration for the observed system that it is not observed in the systems that are standard in the DFMs literature as, for example, those of Stock and Watson (2012) and Forni et al. (2009). The idiosyncratic component in those datasets is likely to be non-stationary and, consequently, an estimation strategy robust to the assumption that some of the idiosyncratic components are non-stationary should be preferred. Alternatively, PC can be implemented to first difference data. Then, the estimated factors can be either obtained by integration of their estimated first differences as proposed by Bai and Ng (2004) or by projecting the original system onto the space spanned by the estimated loading as proposed by Barigozzi et al. (2016).[1] Bai and Ng (2004) prove the consistency of PC factor estimates when they are obtained from first differenced

[1] In this paper we focus on DFMs without deterministic components. In this case, both approaches are equivalent.

data using the "differencing and recumulating" method; see Greenway-McGrevy et al. (2016) who obtain recumulated factors in the context of exchange rates. Additionally, in their Monte Carlo analysis, they evaluate and compare the finite sample properties of both PC procedures and show that the non-stationary common factors can be properly recovered by both approaches when the idiosyncratic components are stationary. However, when the idiosyncratic components are non-stationary, PC cannot be directly implemented to the original data as proposed by Bai (2004) and it is convenient to use the "differencing and recumulating" method proposed by Bai and Ng (2004). Finally, Choi (2016) extends the Generalized PC estimator (GPCE) to the case of unit roots in the common factors, deriving the asymptotic distribution of the common factors and factor loadings. He shows that the GPCE is more efficient than the traditional PC estimator. Although consistent, PC based approaches have a major limitation in that they are not exploiting in any way the dynamic nature of the factors, nor the serial and cross-sectional dependence, or the heterocedasticity of the idiosyncratic components. Consequently, they are not efficient.[2]

Instead of implementing PC procedures, factor extraction can be carried out using two-step Kalman Smoothing (2SKS) techniques based on combining PC factor extraction and a Kalman Smoother. The main advantage of the 2SKS comes from the flexibility of the Kalman filter to explicitly model the factor and idiosyncratic dynamics. In the stationary case, Doz et al. (2011, 2012) show that 2SKS outperforms PC in terms of the precision of the factor estimates and derive its asymptotic properties; see also Poncela and Ruiz (2016). 2SKS has been implemented to non-stationary systems by Seong et al. (2013) in a low-dimensional setting and in Quah and Sargent (1993) in a large but finite cross-sectional dimension case with orthogonal idiosyncratic components.

The contributions of this paper are twofold. First, we extend the analysis of Bai and Ng (2004) comparing the factors extracted using PC implemented to the original non-stationary system with those obtained by "differencing and recumulating". In the case of a single factor, we consider a wide range of structures of the idiosyncratic noises, including heteroscedasticity and temporal and/or cross-sectional dependences. We also consider systems with two factors with the factors being either both non-stationary or one stationary and the other non-stationary. With respect to the idiosyncratic components, we consider cases in which all of them are either stationary or non-stationary and cases in which some of them are stationary and others are not. We also include in the comparison the GPCE proposed by Choi (2016). Finally, we compare PC and 2SKS factor extraction. We analyze the performance of the 2SKS procedure when extracting factors using the first differenced data and estimating the original factors by recumulating. Furthermore, we propose a new 2SKS procedure which can be implemented to the original non-stationary system.[3]

[2]Other authors dealing with non-stationary DFMs are Eickmeier (2009), who analyzes the comovements and heterogeneity in the euro area by fitting a non-stationary DFM similar to Bai and Ng (2004), augmented with a structural factor setup from Forni and Reichlin (1998). Also, Bai and Ng (2010) extend the results of Bai and Ng (2004), and Forni et al. (2014) who evaluate the role of news shocks in generating the business cycle. In this paper, we focus on non-stationary DFMs based on time domain. For non-stationary DFMs based on frequency domain, see Eichler et al. (2011).

[3]In independent work, Barigozzi and Luciani (2017) also propose a generalization of Doz et al. (2011, 2012) to

The second contribution of this paper is an empirical application in which all factor extraction procedures are implemented to a non-stationary system of aggregate output and consumption variables of 21 OECD industrialized countries. International risk sharing focus on cross-border mechanisms to smooth consumption when a country is hit by a negative output shock. The goal is to check international risk sharing is a short or long-run issue. This is helpful to check if GDP fluctuations are directly passed to consumption on the contrary, can be at least partially cross-border smoothed (and therefore not totally passed to consumption). The use of possible non-stationary DFMs allows us to distinguish between long-run and short-run issues in consumption smoothing through international risk sharing. As far as we know, this is the first time that non-stationary DFMs are used in this context.

The rest of this paper is structured as follows. Section 2 describes the DFM and the factor extraction procedures considered. Section 3 presents the results of Monte Carlo experiments. Section 4 contains the empirical application to measure risk sharing. Finally, Section 5 concludes.

2 Factor extraction algorithms

In this section we introduce notation and describe the DFM considered. Furthermore, the PC and 2SKS factor extraction procedures are described.

2.1 Dynamic Factor Model

We consider the following static DFM where the unobserved common factors, F_t, and the idiosyncratic noises, ε_t, follow potentially non-stationary VAR(1) processes:

$$Y_t = PF_t + \varepsilon_t, \tag{1}$$

$$F_t = \Phi F_{t-1} + \eta_t, \tag{2}$$

$$\varepsilon_t = \Gamma \varepsilon_{t-1} + a_t, \tag{3}$$

where $Y_t = (y_{1t}, \dots, y_{Nt})'$ and $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{Nt})'$ are $N \times 1$ vectors of the variables observed at time t and idiosyncratic noises respectively. The common factors, $F_t = (F_{1t}, \dots, F_{rt})'$ and the factor disturbances, $\eta_t = (\eta_{1t}, \dots, \eta_{rt})'$, are $r \times 1$ vectors, with r ($r < N$) being the number of common factors which is assumed to be known. The $N \times 1$ vector of idiosyncratic disturbances, a_t, is distributed independently from the factor disturbances, η_t, for all leads and lags. Furthermore, η_t and a_t, are assumed to be Gaussian white noises with positive definite covariance matrices $\Sigma_\eta = diag(\sigma^2_{\eta_1}, ..., \sigma^2_{\eta_r})P'P/N = I_r$ and Σ_a, respectively. $P = (p_1, \dots, p_N)'$, is the $N \times r$ matrix of factor loadings, where, $p_i = (p_{i1}, \dots, p_{ir})$ is an $1 \times r$ vector. For identification, we assume that $P'P/N = I_r$. Finally, $\Phi = diag(\phi_1, \dots, \phi_r)$ and $\Gamma = diag(\gamma_1, ..., \gamma_N)$ are $r \times r$ and $N \times N$ matrices containing the autoregressive parameters of the factors and idiosyncratic components, respectively, which can be equal to one; see, for example, Stock and Watson (1989)

the non-stationary case. They show empirically that the 2SKS extraction is more efficient than integrating the PC estimator of the first differences of the factors. However, they do not consider the comparison with recumulating the 2SKS not with PC implemented to the original non-stationary data.

and Barigozzi and Luciani (2017) for static DFMs for non-stationary data. Note that according to economic theory, there is full aggrement that some factors (related with, for example, technology shocks) have permanent effects while others (such as monetary policy shocks) have only transitory effects. Furthermore, there is also arguments to assume non-stationary idiosyncratic components. Barigozzi et al. (2016, 2017) point out that stationarity of the idiosyncratic components would produce an amount of cointegration for the observed system that it is not consistent with that observed in the systems that are standard in the DFMs literature as, for example, those of Stock and Watson (2002) and Forni et al. (2009). The idiosyncratic component in those datasets is likely to be non-stationary. The implausibility of a stationary idiosyncratic component is also confirmed empirically by Barigozzi et al. (2016) in a large macroeconomic system of quarterly series describing the US economy with about half of the estimated idiosyncratic components found to be non-stationary according to the test proposed by Bai and Ng (2004).

The DFM in equations (1) to (3) is not identified. To solve the identification problem and uniquely define the factors, a normalization is necessary. In the context of PC factor extraction, it is common to impose the restriction $P'P/N = I_r$ and $F'F$ being diagonal, where $F = (F_1, \ldots, F_T)$ is the $r \times T$ matrix of common factors; see, for example, Bai and Wang (2014) and Barigozzi et al. (2016).

2.2 PC factor extraction

The most popular procedures for factor extraction in large datasets are based on the PC procedure. The distinctive feature of PC is that it allows a consistent factor extraction without assuming any particular error distribution and specifications of the factors and idiosyncratic noises further than the cross-correlation of the latter being weak and the variability of the common factors being not too small.[4] Furthermore, PC is computationally simple which explains its wide implementation among practitioners when dealing with very large systems of economic variables.

PC factor extraction separates the common component, PF_t, from the idiosyncratic component, ε_t, through cross-sectional averages of Y_t in such a way that when N and T tend to infinity, the effect of the idiosyncratic component converges to zero remaining only the effects associated to the common factors. The PC estimators of P and F_t, are obtained as the solution to the following least squares problem

$$\min_{F_1,\ldots,F_T,P} V_r(P,F) \tag{4}$$

subject to $P'P/N = I_r$ and $F'F$ being diagonal where $V_r(P,F) = \frac{1}{NT}\sum_{t=1}^{T}(Y_t - PF_t)'(Y_t - PF_t)$. The solution to (4) is obtained by setting $\hat{P}^{PCL}$ equal to $\sqrt{N}$ times the eigenvectors corresponding to the r largest eigenvalues of YY' where $Y = (Y_1, \ldots, Y_T)$ is a $N \times T$ matrix of observable. The corresponding PC estimator of F using data in levels is given by

$$\hat{F}^{PCL} = N^{-1}\hat{P}^{PCL\prime}Y. \tag{5}$$

[4] Onatski (2012) considers a DFM in which the explanatory power of the factors does not strongly dominate

Alternatively, when the common factors are $I(0)$, Bai and Ng (2002) give the restriction $FF'/T = I_r$ with $P'P$ being diagonal, such that, the estimator of the matrix of common factors, $\hat{F}^{PCL}$, is the $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $Y'Y$, with estimated factor loadings, $\hat{P}^{PCL} = Y\hat{F}^{PCL'}/T$. When the common factors are $I(1)$, Bai (2004) proposes to use the restriction $FF'/T^2 = I_r$ with $P'P$ being diagonal. In this case, $\hat{F}^{PCL}$, is the T times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $Y'Y$ and $\hat{P}^{PCL} = Y\hat{F}^{PCL'}/T^2$. The difference is only computational, these latest restrictions are less costly when $N > T$, while that $P'P/N = I_r$ with FF' being diagonal are less costly when $N < T$.

In the context of stationary systems, if the common factors are pervasive and the serial and cross-sectional correlation of the idiosyncratic components is weak, Bai (2003) proves the consistency of $\hat{F}^{PCL}$, $\hat{P}^{PCL}$ and the common component, deriving their asymptotic distributions when N and T tend simultaneously to infinity, allowing for heteroscedasticity in both the temporal and cross-sectional dimensions; see also Bai and Ng (2002) and Stock and Watson (2002). Additionally, Bai (2004) extends the asymptotic results when F_t is $I(1)$ and ε_t is $I(0)$. When the idiosyncratic components are $I(1)$, Bai and Ng (2008) show that PC factor extraction implemented to data in levels yields inconsistent estimates of the common factors.

In order to obtain more efficient estimates of F_t and P relative to the PC factor extraction, Choi (2016) proposes a GPCE implemented to the original non-stationary system. Using the standardization $FF'/T^2 = I_r$, the feasible estimator of the factor space spanned by F_t, denoted by $\hat{F}_t^{GPCL}$, is T times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $Y'\hat{\Sigma}_\varepsilon^{-1}Y$ where $\hat{\Sigma}_\varepsilon^{-1/2} = \text{diag}(\hat{\sigma}_1, \ldots, \hat{\sigma}_N)$ with $\hat{\sigma}_i^2 = \sum_{t=1}^{T} \hat{\varepsilon}_{it}^2/T$, and $\hat{\varepsilon}_{it}^2$ are obtained after implementing the PC estimator of F_t proposed by Bai (2004) as in equation (5). The corresponding weights are given by $\hat{P}^{GPCL} = T^{-2}Y\hat{F}^{GPCL'}$; see Choi and Hwang (2012) for an application to forecasting the Korean inflation. Choi (2016) shows that the GLS version of the PC estimator is asymptotically equivalent to the original PC estimator.

Alternatively, instead of extracting the factors implementing PC to the original data, Bai and Ng (2004) propose differencing the data in a univariate fashion and extract the factors from the following differenced model

$$\Delta Y_t = P\Delta F_t + \Delta\varepsilon_t, \tag{6}$$

$$\Delta F_t = \Phi\Delta F_{t-1} + \Delta\eta_t, \tag{7}$$

$$\Delta\varepsilon_t = (\Gamma - I)\varepsilon_{t-1} + a_t, \tag{8}$$

where $\Delta = (1 - L)$ with L being the lag operator such that $LY_t = Y_{t-1}$. The weights are estimated as $\sqrt{N}$ times the first r normalized eigenvectors of the $N \times N$ sample covariance matrix of ΔY_t and denoted by $\hat{P}^{PCD}$. The corresponding estimated factors are given by

$$\hat{f}_t = N^{-1}\hat{P}^{PCD'}\Delta Y_t, \quad t = 2, \ldots, T. \tag{9}$$

Once the factors are extracted from the first differenced variables, the estimated factors can

the explanatory power of the idiosyncratic noises.

be obtained either by integration of their estimated first differences as proposed by Bai and Ng (2004) or by projecting the original system onto the space spanned by the estimated loading as proposed by Barigozzi et al. (2016). The "differencing and recumulating" estimated factor is given by

$$\hat{F}_t^{PCD} = \sum_{s=2}^{t} \hat{f}_s, \quad t = 2, \dots, T. \tag{10}$$

Note that assuming $Y_0 = 0$, the estimated differenced factor at time 1 is given by $\hat{f}_1 = N^{-1}\hat{P}^{PCD\prime}Y_1$ and, consequently, the estimated recumulated factor coincides with the projected factor which is given by

$$\hat{F}_t^{PCD} = N^{-1}\hat{P}^{PCD\prime}Y_t \quad t = 1, \dots, T. \tag{11}$$

Bai and Ng (2004) and Barigozzi et al. (2016) show that $\hat{F}_t^{PCD}$ is a consistent estimator for a rotation of F_t up to a level shift regardless of whether the idiosyncratic component, ε_t, is $I(0)$ or $I(1)$. Note that the factor estimators proposed by Bai and Ng (2004) and Barigozzi et al. (2016) are asymptotically equivalent with some finite sample differences when there are deterministic trends in the DFMs.

2.3 Two-step Kalman Smoother

The 2SKS procedure was proposed by Doz et al. (2011) for stationary DFMs. Therefore, 2SKS can be implemented to ΔY_t. The 2SKS factor extraction procedure is based on combining PC and Kalman Smoother techniques. First, the common factors and factor loadings are estimated using PC obtaining $\hat{P}^{PCD}$ and $\hat{f}_t$ and the corresponding idiosyncratic and factor residuals, $\hat{\varepsilon} = Y - \hat{P}^{PCD}\hat{f}$ and $u_t = \hat{f}_t - \hat{\Phi}\hat{f}_{t-1}$ where $\hat{\Phi}$ is the ordinary least squares (OLS) estimator of the regression of $\hat{f}_t$ on $\hat{f}_{t-1}$. These residuals are used to estimate the covariance matrices $\hat{\Psi} = \text{diag}\left(\hat{\Sigma}_\varepsilon\right)$ where $\hat{\Sigma}_\varepsilon = \hat{\varepsilon}\hat{\varepsilon}'/T$ with $\hat{\varepsilon} = (\hat{\varepsilon}_1, \dots, \hat{\varepsilon}_T)'$ is an $N \times T$ matrix and $\hat{\Sigma}_\eta = uu'/T$ where $u = (u_1, \dots, u_T)$ is an $r \times T$ matrix. Assuming that $f_0 \sim N(0, \Sigma_f)$, the unconditional covariance of the factors can be estimated as $\text{vec}\left(\hat{\Sigma}_f\right) = \left(I_{r^2} - \hat{\Phi} \otimes \hat{\Phi}\right)^{-1} \text{vec}\left(\hat{\Sigma}_\eta\right)$. After writting the DFM in equations (6) to (8) in state-space form, with the system matrices substituted by $\hat{P}$, $\hat{\Psi}$, $\hat{\Phi}$, $\hat{\Sigma}_\eta$ and $\hat{\Sigma}_f$, the Kalman smoother is run to obtain an updated estimation of the factors denoted by $\hat{f}_t^{KS}$. Finally, estimates of the common factors, $\hat{F}_t^{KSD}$, are obtained by recumulating analogously to equation (10).

Doz et al. (2011) prove the consistency of $\hat{f}_t^{KS}$ when N and T are large considering assumptions slightly different than those in Bai and Ng (2002), Stock and Watson (2002) and Bai (2003) but with a similar role. The 2SKS works well in finite samples obtaining more accurate factor estimates of $f_t = \Delta F_t$ even in the presence of cross-sectional heteroscedasticity in the idiosyncratic noises, see Doz et al. (2011). Finally, Doz et al. (2012) propose iterating the 2SKS procedure until convergence is achieved in terms of two consecutive log-likelihood values.

[5] Barigozzi and Luciani (2017) propose an alternative extension in which, in order to isolate common trends and stationary factors, they use a nonparametric approach which identifies the common trends as those linear combinations of the factors obtained by the leading eigenvectors of a transformation of the long-run covariance matrix as proposed by Peña and Poncela (2006), Pan and Yao (2008), Lam et al. (2011) and Zhang et al. (2016).

Considering the possibility of non-stationary common factors, we propose to extend the 2SKS algorithm as follows[5]

1. Obtain PC estimates of P and F_t with data in levels given by expression (5). Compute the idiosyncratic residuals $\hat{\varepsilon} = Y - \hat{P}^{PCL}\hat{F}^{PCL}$.

2. For each estimated idiosyncratic component, $\widehat{\varepsilon}_i$, $i = 1, ..., N$, following Bai and Ng (2004), test the null hypothesis $H_0 : \gamma_i = 1$.

   (a) If H_0 is rejected, then $\gamma_i = 0$ and $\widehat{a}_i = \widehat{\varepsilon}_i$. Compute $\widehat{\sigma}^2_{a_i} = \sum_{t=1}^{T} \widehat{a}^2_{it}/T$.

   (b) If H_0 is not rejected, then $\gamma_i = 1$ and $\widehat{a}_{it} = \Delta\widehat{\varepsilon}_{it}$. Compute $\widehat{\sigma}^2_{a_i} = \sum_{t=1}^{T} \widehat{a}^2_{it}/T$.the covariance matrix of the idiosyncratic terms, , as in the.

3. For each estimated factor, $\hat{F}^{PCL}_{jt}$, $j = 1, ..., N$, carry out the Augmented Dickey Fuller (ADF) test.

   (a) If the null hypothesis of a unit root is rejected, obtain the OLS estimate of the autoregressive coefficient, $\hat{\phi}_j$, the residuals $u_{jt} = \hat{F}^{PCL}_{jt} - \hat{\phi}\hat{F}^{PCL}_{jt-1}$ and the sample variance of the factor disturbance, $\hat{\sigma}^2_{\eta_j} = \sum_{t=1}^{T} u^2_{jt}/T$. The initial state of the factor is assumed to have zero mean and variance estimated by $\hat{\sigma}^2_{F_j} = \hat{\sigma}^2_{\eta_j}/(1 - \hat{\phi}^2_j)$.

   (b) If the null hypothesis is not rejected, then $\hat{\phi}_j = 1$ and the residuals are computed as $u_{jt} = \Delta\hat{F}^{PCL}_{jt}$. Calculate the variance of the factor residuals, $\hat{\sigma}^2_{\eta j} = \sum_{t=1}^{T} \Delta\hat{F}^{PCL^2}_{jt}/T$. Assume a diffuse prior for the initial factor with mean zero and variance $\hat{\sigma}^2_{F_j} = \kappa$, where κ is a large constant that empirically performs well (for instance $\kappa = 10^7$); see Harvey and Phillips (1979), Burridge and Wallis (1985) and Harvey (1989).[6]

4. Obtain $\hat{\Phi} = diag(\hat{\phi}_1, \ldots, \hat{\phi}_r)$, $\hat{\Sigma}_\eta = diag(\hat{\sigma}^2_{\eta_1}, \ldots, \hat{\sigma}^2_{\eta_r})$ and $\widehat{\hat{\Psi}} = diag(\hat{\sigma}^2_{a_1}, \ldots, \hat{\sigma}^2_{a_N})$ and use them together with $\hat{P}^{PCL}$ in the KS to obtain the estimated common factors $\hat{F}^{KSL}$.

3 Finite sample performance

In this section, we carry out Monte Carlo experiments in order to study the performance of the factor extraction procedures described in the previous section. The experiments are based on $R = 500$ replicas generated by the DFM in equations (1)-(3) with sample sizes $T = (100, 500)$ and $N = (12, 50, 200)$. The factor loadings are generated once as $P \sim U[0, 1]$ and the autoregressive matrix of the idiosyncratic components is diagonal, $\Gamma = \gamma I$, with $\gamma = (-0.8, 0, 0.7, 1)$.[7] We consider three specifications of dependence of the idiosyncratic noises: a) homoscedastic and cross-sectionally uncorrelated, with $\Sigma_a = \sigma^2_a I$ where $\sigma^2_a = (0.1, 1, 10)$; b) heteroscedastic and cross-sectionally uncorrelated with the variances generated by $\sigma^2_{a_i} \sim U[0.05, 0.15]$, $\sigma^2_{a_i} \sim U[0.5, 1.5]$

[6] Koopman (1997) gives an exact solution for the initialization of the Kalman filter and smoothing for state space models with diffuse initial conditions.

[7] Alternatively, we generate artificial systems by model M1 where the temporal dependence of the idiosyncratic errors is $\gamma = \text{diag}(-0.8I_{N/2}, 1I_{N/2})$ and $\gamma = \text{diag}(0I_{N/2}, 0.7I_{N/2})$. The results are very similar to those when all idiosyncratic errors have the same dependence with $\gamma = -0.8$ and $\gamma = 0$, respectively. It seems that the results

and $\sigma^2_{a_i} \sim U[5,15]$; c) homoscedastic and cross-sectionally correlated with weak cross-correlation generated following Kapetanios (2010) as $\Sigma^{1/2}\varepsilon_t$ where $\Sigma = [\sigma_{i,j}], \sigma_{i,j} = \sigma_{j,i} \sim U(-0.1, 0.1)$ for $|i-j| \leq 5$ for $i, j = 1, \ldots N$. Finally, with respect to the unobserved factors, we consider four different data generating processes (DGPs). The first DGP, denoted as model 1 (M1), has $r = 1$, $\Phi = 1$ and $\sigma^2_\eta = 1$ so that the factor is given by a random walk. The second and third models (M2 and M3) introduce a second random walk with $r = 2$ and $\Phi = I$ while $\Sigma_\eta = I$ (M2) and $\Sigma_\eta = diag(1, 5)$ (M3). Finally, the fourth model considered (M4) also has two factors but one is stationary while the other is not. In particular, in model M4, $\Sigma_\eta = I$ and $\Phi = diag(1, 0.5)$.

For each DGP considered, the common factors are estimated using the procedures described in Section 2 obtaining $\hat{F}^{PCD}_t$ and $\hat{F}^{KSD}_t$, based on "differencing and recumulating", and $\hat{F}^{PCL}_t$, $\hat{F}^{GPCE}_t$ and $\hat{F}^{KSL}_t$, based on data in levels.[8] Following Bai (2004), the performance of the factor extraction procedures is evaluated by computing the sample correlation between the true factor, F_t, and a rotation of the estimated factors, $\hat{\delta}'_j \widehat{F}^{(j)}_t$, estimated by the following regression

$$F_{jt} = \hat{\delta}'_j \widehat{F}^{(j)}_t + \widehat{\nu}_t.$$

Figure 1 plots the Box-plots of the sample correlations between the true and rotated estimated factors obtained through the Monte Carlo replicates when the systems are generated by the M1 model with homoscedastic idiosyncratic errors with $\sigma^2_a = 10$ when the temporal and cross-sectional dimensions are $(N, T) = (12, 50), (12, 100), (50, 100), (200, 100)$ and $(200, 500)$. Several conclusions can be obtained from Figure 1. First, all procedures based on differencing and recumulating are similar among them. The same can be said about the procedures based on extracting factors directly from the data in levels. Second, regardless of N and T, the correlations of the "differencing and recumulating" PC procedure can be rather low when the temporal dependence of the idiosyncratic component is negative. Furthermore, using the "differencing and recumulating" estimator implemented with the 2SKS procedure generates even smaller correlations, mainly when $\gamma = -0.8$. Note that, when the serial dependence of the idiosyncratic components is such that $\gamma < 0.5$, the variance of the differenced idiosyncratic component, $\sigma^2_{\Delta\varepsilon}$, is larger than the corresponding variance of the original component, σ^2_ε; see, for example, Corona et al. (2016). Consequently, the performance of the procedures using data in first differences deteriorates in this case. However, if $\gamma \geq 0.5$, then $\sigma^2_{\Delta\varepsilon} < \sigma^2_\varepsilon$ and, consequently, the procedures based on "differencing and recumulating" may have an advantage. Third, if the idiosyncratic noises are white noise, the 2SKS procedures implemented to raw data generate correlations which are always close to 1. Note that the two-step procedure proposed in this paper does a remarkably good job. Only when the cross-sectional and temporal dimensions are very large, the procedures based on first differences estimate factors with correlations close to one. Fourth, if the dependence of the idiosyncratic noises is positive, differencing or extracting the factors using the original non-stationary system yields similar correlations. Only when N

are driven by the smallest temporal dependence among the idiosyncratic noises. These results are available upon request.

[8]Note that, in the context of the DFM considered in this paper, the Monte Carlo results for the procedure proposed by Barigozzi et al. (2016) (BLL) are almost identical to those obtained by the procedure proposed by Bai and Ng (2004).

and T are relatively small, differencing performs worse. Finally, when the idiosyncratic errors are non-stationary, i.e. $\gamma = 1$, extracting the factors using differenced or original data yields similar moderate correlations. Only when N is very large, we observe the result established by the asymptotic theory with the procedures based on "differencing and recumulating" having correlations close to one while the non-consistent procedures based on original non-stationary data having smaller correlations.

The Box-plots plotted in Figure 1, help to understand the role of the dynamic dependence of the idiosyncratic noises on the performance of the alternative factor extraction procedures considered. In order to evaluate the effect of the variance of the disturbance of the idiosyncratic noises, Figure 2 plots the Box-plots of the correlations of the common factor estimates and the simulated ones for model M1 with $\gamma = -0.8$ and the same dimensions considered above and $\sigma_a^2 = 0.1$, 1 and 10. Note that if σ_a^2 is small, then all procedures have correlations close to 1 regardless of the cross-sectional and temporal dimensions and whether they are based on first differences or original data. The deterioration of the procedures based on "differencing and recumulating" is already observed for $\sigma_a^2 = 1$ with the exception of very large N and T. Finally, in Figure 3, we study the role of the variance of the idiosyncratic noises when $\gamma = 1$. In this case, it is clearly better to take first differences to the original series. The performance of the procedures based on extracting factors from the original data is only reasonable when $\sigma_a^2 = 0.1$.

To evaluate the precision of the factor estimates and summarizing the results, we carry out a response surface analysis by regressing the sample correlation averages on the cross-sectional and temporal dimensions, N and T, and the temporal dependence and variance of the idiosyncratic noises, γ and σ_a^2, for model M1 with homoscedastic, heteroscedastic and cross-correlated idiosyncratic noises. In the case of heteroscedastic idiosyncratic errors, the value of σ_a^2 considered as regressor is the expected value of the variances for each idiosyncratic noise. The regression parameter estimates together with the corresponding standard errors and adjusted R^2 are reported in Table 1. First, we can observe that the average correlation of the procedures based on "differencing and recumulating" is clearly smaller than that of the procedures implemented to original data. As above, we also observe that the correlations are similar among methods based on first differences and among methods based on original systems. Second, it is also clear that the correlations between the true factors and the rotated estimates obtained using procedures based on differenced data increase with γ, the temporal dependence of the idiosyncratic noise. This result could be expected given that, as explained above, when $\gamma < 0.5$, the variance of the differenced idiosyncratic component, $\sigma_{\Delta\varepsilon}^2$, is larger than the corresponding original variance, σ_ε^2, and, consequently, the recovery of the common factors is less precise. Furthermore, note that the increase in the correlations between true and rotated extracted factors is larger for 2KSD than for the PCD procedure, as expected given the flexibility of the Kalman filter to explicitly model the idiosyncratic dynamics. However, the correlations decrease with γ when the factor extraction procedures are implemented to original data. Third, increasing σ_a^2 negatively affects factor extraction for all procedures. However, for the same reasons explained above, the effect of σ_a^2 is less important if the factors are extracted using original non-stationary observations than when they are extracted using first-differenced data. Finally, Table 1 shows that the results are almost the same regardless of the particular specifications of the idiosyncratic components. It is remarkable that, for the particular specifications of the heteroscedasticity considered in this paper, the correlations between the true and rotated estimated factors obtained when the PCL and GPCE procedures are implemented are very similar.

Finally, we consider the three models with two factors. Figure 4 plots the Box-plots of the correlations across the Monte Carlo experiments between the true and rotated estimated common factors through the Monte Carlo experiments for models M2, M3 and M4 (by rows) with $\sigma_a^2 = 10$ and $\gamma = -0.8$. In each case, we consider homoscedastic, heteroscedastic and cross-correlated idiosyncratic errors (by columns) The cross-sectional and temporal dimensions are $N = 50$ and $T = 100$. First of all, as far as the two factors are non-stationary, models M2 and M3, we can observe the same patterns as those described for the case of one single factor. However, when one factor is a random walk and the second factor is stationary, model M4, none of the procedures estimate this factor adequately.the results are drastically deteriorated when extracting the stationary common factor.[9] Finally, Figure 5 plots the Box-plots of the correlations across Monte Carlo replicates when the idiosyncratic noise is $I(1)$ and $\sigma_a^2 = 1$. As expected, we can observe that the common factors are better extracted when we use first-differenced data.

They conclude that if ε_t is stationary, with autoregressive parameters smaller than 0.5 while F_t is non-stationary, then overdifferencing the idiosyncratic components may introduce distortions on the determination of the number of factors given that the relation between the variances of the common and idiosyncratic components is modified with the variances of ΔF_t decreasing and the variances of $\Delta\varepsilon_t$ increasing in relation to the variance of F_t and ε_t, respectively. Recall as well, that some procedures do not yield consistent estimates when the idiosyncratic noises are $I(1)$

4 Empirical analysis

International or cross-border risk sharing focuses on the smoothing of consumption when a country is hit by a negative output shock. In an ideal world of perfect risk sharing, consumption should be insured. However, in practice, risk sharing is far from being full or complete and a percentage of GDP shocks are passed into consumption and are not smoothed. In a time series context, risk sharing has been traditionally addressed in the literature as a short-run issue and, consequently, analyzed within the context of stationary models. Nevertheless, more recently, some authors question this view and bring in the long-run perspective to the problem, although the results are not conclusive. For instance, Becker and Hoffmann (2006) and Pierucci and Ventura (2010) analyse risk sharing within a cointegration context. Artis and Hoffmann (2008, 2012) argue that risk sharing has increased at lower frequencies and relate their results to the permanent income hypothesis. On the contrary, Leibrecht and Scharler (2008) using cointegration techniques and vector error correction models found that while consumption risk sharing in the short-run was around 30%, only accounts for a 10% in the long-run. As regards

[9] The results are similar even if the cross-sectional and temporal dimensions are increased.

factor models, Del Negro (2002) implement a stationary DFM to disentangle movements in US state output and consumption due to national, regional or state-specific factors. Very recently, for capital flows, Byrne and Fiess (2016) apply non-stationary factor models to analyze the common and idiosyncratic elements in emerging markets' capital inflows.

The economic interpretation of the common factor analysis in our model should be as follows. If there is full risk sharing, idiosyncratic consumption and output cannot share a common factor since these two variables should be orthogonal in an ideal case of complete risk sharing where, under certain assumptions, domestic consumption should be a constant fraction of the aggregate world output. Hence, lack of complete full risk sharing should be detected through commonalities between domestic output and consumption. If we can find non-stationary common factors among the series of output and GDP we could conclude that there is no risk sharing in the long-run.

Our sample covers the following 21 industrialized OECD countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DEN), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Ireland (IRL), Italy (ITA), Japan (JPN), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR) and United States (USA). The data are annual observations of Gross Domestic Product (GDP) and Consumption (C) from National Accounts and cover the time span 1960-2014 with $N = 42$ and $T = 55$. The main source of data is AMECO, the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs (DG ECFIN), which provides harmonized statistics on all of the variables required to perform the analysis. The nominal GDP and C have been transformed in Purchasing Power Standard (PPS) units by dividing the nominal aggregates by the appropriate PPS exchange rate reported by AMECO. To compute per-capita variables, the real aggregates expressed in PPS are divided by the population taken from the OECD Statistics. We build the aggregate GDP and C for the set of countries included in the analysis. To build the aggregates, we use weighted averages in order to reflect the importance of each country in the group of economies. Then, starting from the real indicators computed for each country in PPS, we followed the weighting procedure described in Beyer et al. (2001), where the aggregation is performed directly on growth rates (first difference of logs) but using time-varying weights of countries that are given by their relative share in real GDP, in levels. The aggregate GDP and consumption growth rates are integrated to get the log of the aggregate variables. As initial condition for the aggregated GDP (consumption), we aggregate the levels of real GDP (consumption) and take logs. To define the idiosyncratic variables or gaps in log levels we subtract the log of the aggregate from the log level of a specific country. The resulting gap could be interpret as the log of the percentage of a particular country GDP (consumption) over the aggregate variable. (see Giannone and Reichlin, 2006, for the same interpretation).

Unit root tests are performed for the GDP and consumption gaps for all countries and, overall, we can consider that the series are $I(1)$. In order to determine the number of common factors, we implement the procedure proposed by Onatski (2010) and choose $r = 5$ regardless whether it is implemented to data in levels or first differences; see Corona et al. (2016) for a comparison on alternative procedures to determine the number of common factors in non-stationary DFMs.

Since, we do not know if the idiosyncratic errors are stationary or not, we differentiate the data and extract 5 common factors using PCFD. Then, we recumulate the extracted common factors and the specific components. We use PANIC to check if the idiosyncratic errors are non-stationary. We performed individual tests for each idiosyncratic error and the pooled test proposed by Bai and Ng (2004) where the pooled statistic of the log of the pvalues (p_i) of the individual tests follows a standard normal distribution

$$P = \frac{-2\sum_{i=1}^{N} \log p_i - 2N}{\sqrt{4N}}. \tag{12}$$

Pooled tests could not be used in the original data because of strong cross correlation due to the common factors but they can be used in the specific components since this strong cross correlation has been removed after extracting the common factors. Both the individual tests over the idiosyncratic components as well as the pooled test (the p statistic was 0.19) indicate the idiosyncratic components are non-stationary. In this case, we have to choose any of the methods to extract the common factors that work with the data in first differences, since if the errors are non-stationary the procedures that work with the data in levels do not yield to consistent estimates. This was reflected in our simulations by the low correlations between the generated common factors and the estimated ones.

The rationale for finding that the idiosyncratic errors are non-stationary should be as follows. A large part of the commonality has been removed when generating the data as the variables that enter into the model are already deviations from the aggregate. This aggregate might proxy world comovements. Nevertheless, there are still strong correlations in the data that we remove through the common factors. If what it is left is non-stationary, as it might seem the case, it means that there are persistent movements that are generated internally and not shared among countries or due to interactions with third countries, as it might happen with the U.S. and Mexico. Another way of looking at this result is as follows: if after removing r_1 non-stationary common factors, what is left is stationary, it means that we should find $2N - r_1$ cointegrating relations among the data. This is not the case and, therefore, we conclude that in our model after removing r common factors (r_1 being non-stationary), what is left is non-stationary as well.

We proceed using PCFD to recover the common factors and the factor loadings. As mentioned before, we applied the "differencing and recumulating" method suggested by Bai and Ng (2004), although any method that works with the data in first differences could be used as well. We test how many of the common factors are non-stationary. The extracted sample factors in first differences are orthogonal as this condition is imposed for identifcation purposes, however the recumulated common factors do not need to be orthogonal. Therefore, we test how many of the common factors are non-stationary using the variant of the test for common trends of Stock and Watson (1988) proposed by Bai and Ng (2004). Basically, the test consist of deciding how many of the eigenvalues of the first order autoregressive matrix, after correcting for serial correlation in the residuals are close enough to 1. The estimated eigenvalues are 0.66, 0.83, 0.90, 0.91 and 1.02. We cannot reject the null hypothesis of 5 common trends, even though the fifth eigenvalue is only 0.66. Since T is not so large, we can conclude that there are 5 common factors in the data and, at least, 4 of them are non-stationary factors.

The next step is to decide if the factor loadings are different from zero and if we find a loadings different from zero associated to GDP and consumption for the same country. Since the factor loading matrix is the same for the model in first differences than for the model in levels, and in the model in levels the idiosyncratic errors are I(1), we perform inference about the factor loadings using the factor model in first differences (the asymptotic distribution of the loadings is given in Bai, 2003).

We analyze the factor loadings for the first common factor (see Figure 6) related to GDP series. The factor loadings could be considered different from zero for all countries but Australia, Canada, Denmark, UK and Switzerland. It gives positive weight to the Anglo-Saxon countries (USA, CAN, GBR, NZL and AUS) although it can be only considered different from zero for US and New Zealand while the weights have the opposite sign for the rest of European countries (other than the United Kingdom) and Japan. Within the last set, the highest, in absolute value, are given to Greece, Portugal, Spain followed by Japan. Curious enough, Greece, Portugal, Spain (jointly with Ireland and Italy that also have significant factor loadings of the same sign) constitute the so called PIIGS group, peripheral European countries where risk sharing has collapsed during the last recession and subsequent sovereign debt crisis faced. Kalemli-Ozcan et al. (2014) point out that the governments of these countries did not save during the expansionary phases of the business cycle and were not able to borrow on the international markets during the crisis due to the high levels of outstanding public debt. Ireland is also included in this set although its case is slightly different, with government deficits related to banking failures (see Kalemli-Ozcan et al., 2014). This might be the reason why Ireland is included in this group instead of within the Anglo-Saxon countries. Japan has experienced a long lasting recession and sluggish output growth since the early 1990s. We check the results through other estimation methods. No matter the estimation method, the factor loadings in domestic or idiosyncratic consumption seem to follow very closely those of idiosyncratic output, indicating lack of risk sharing. This interpretation should be in accordance with Becker and Hoffmann (2006) and Pierucci and Ventura (2010).

The second common factor gives the highest positive weight to New Zealand. On the negative side appears Japan. The next 2 common factors are devoted to separate Greece from other countries. Basically, the 3rd common factor separates Greece from Portugal and the 4th one to separates Greece from Ireland and Norway. The fifth common factor loads on several countries and has a difficult interpretation.

There are 21×5=105 loadings associated to each country for GDP and the same quantity associated to consumption. Only in 27 out of the 105 possible cases, factor loadings were significant for one of the variables (GDP or consumption) and not for the other (which could be an indication of risk sharing). However, we find that when a loading is significant for GDP for one country, it is usually significant and of the same sign for consumption for the same country.

5 Conclusions

In this study we examine the finite sample performance of alternative factor extraction procedures to estimate non-stationary common factors in the context of large DMFs. Furthermore, we extend the hybrid method from Doz et al. (2011) based on combining PC and Kalman smoothing, applying the technique to original non-stationary observations. We show that, when the idiosyncratic errors are non-stationary, the approaches based on estimating the common factors using non-stationary time series in levels do not perform well and that the procedures based on first differences should be used. This fact was pointed out by Bai and Ng (2008) for the basic PC estimator and we have checked that the same holds for the remaining methods that use data in levels. The empirical application shows that for a non-stationary system of 21 OECD industrialized economies, at least four common factors are non-stationary, such that, consumption and GDP share common trends. Furthermore, we apply PANIC to the estimated idiosyncratic errors, concluding that this component is non-stationary. Hence, these facts suggest the lack of full risk sharing both in the short and long-run.

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Tables and Figures

Table 1: Response surface analysis by regressing sample correlations averages on the sample size, serial correlation and the variance of the idiosyncratic disturbance. Standard errors between parenthesis.

Dependent variable: Sample correlation averages						
M1 with homoscedastic idiosyncratic errors						
Regressor	PCD	KSD	PCL	BLL	GPCE	KSL
Constant	0.8517	0.7901	0.9548	0.8523	0.9437	0.9611
	(0.0445)	(0.0591)	(0.0316)	(0.0444)	(0.0328)	(0.0315)
N	0.0002	0.0004	0.0001	0.0002	0.0001	0.0001
	(0.0002)	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
T	0.0003	0.0003	0.0000	0.0003	0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
γ	0.1331	0.2145	-0.1009	0.1329	-0.1093	-0.1062
	(0.0281)	(0.0374)	(0.0200)	(0.0281)	(0.0207)	(0.0199)
σ_a^2	-0.0296	-0.0353	-0.0100	-0.0102	-0.0296	-0.0090
	(0.0044)	(0.0058)	(0.0031)	(0.0044)	(0.0032)	(0.0031)
$\bar{R}^2$	0.5035	0.5026	0.3175	0.5031	0.3266	0.3215
M1 with heteresocedastic idiosyncratic errors						
	PCD	KSD	PCL	BLL	GPCE	KSL
Constant	0.8454	0.7879	0.9542	0.8459	0.9372	0.9618
	(0.0440)	(0.0580)	(0.0317)	(0.0440)	(0.0333)	(0.0314)
N	0.0003	0.0004	0.0001	0.0003	0.0001	0.0001
	(0.0002)	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
T	0.0003	0.0003	0.0000	0.0003	0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
γ	0.1309	0.2150	-0.0993	0.1306	-0.1086	-0.1043
	(0.0278)	(0.0367)	(0.0200)	(0.0278)	(0.0211)	(0.0198)
σ_a^2	-0.0322	-0.0367	-0.0108	-0.0103	-0.0321	-0.0091
	(0.0043)	(0.0057)	(0.0031)	(0.0043)	(0.0033)	(0.0031)
$\bar{R}^2$	0.5362	0.5236	0.3139	0.5358	0.3291	0.3162
M1 with cross-correlated idiosyncratic errors						
	PCD	KSD	PCL	BLL	GPCE	KSL
Constant	0.8537	0.7929	0.9538	0.8543	0.9453	0.9599
	(0.0439)	(0.0583)	(0.0316)	(0.0439)	(0.0327)	(0.0313)
N	0.0002	0.0003	0.0001	0.0002	0.0001	0.0001
	(0.0002)	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
T	0.0003	0.0003	0.0001	0.0003	0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
γ	0.1296	0.2116	-0.0986	0.1293	-0.1075	-0.1040
	(0.0278)	(0.0369)	(0.0199)	(0.0277)	(0.0206)	(0.0198)
σ_a^2	-0.0299	-0.0358	-0.0105	-0.0299	-0.0102	-0.0093
	(0.0043)	(0.0057)	(0.0031)	(0.0043)	(0.0032)	(0.0031)
$\bar{R}^2$	0.5097	0.5098	0.3166	0.5094	0.3260	0.3205

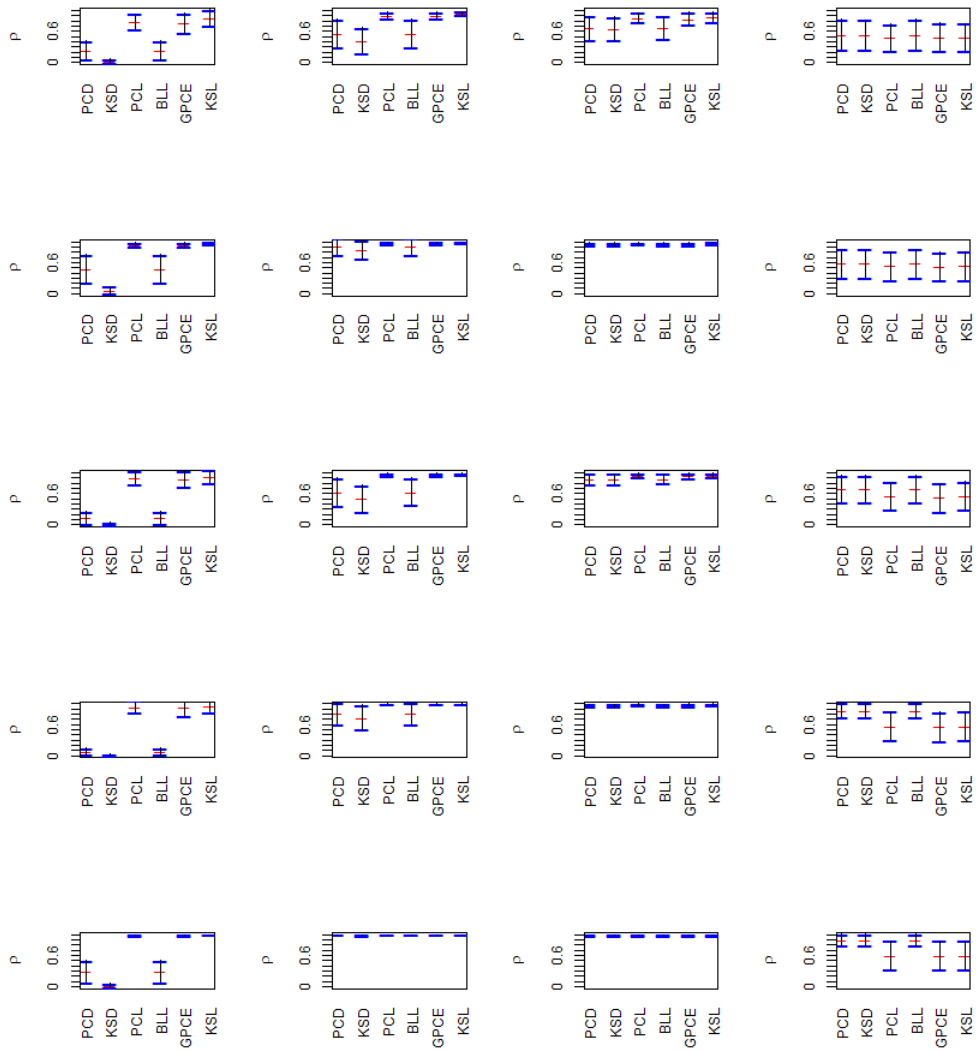

Figure 1: Sample correlations between $\{\hat{\delta}_j'\hat{F}_t^{PCD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{KSD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{PCL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{BLL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{GPCE}\}$ and $\{\hat{\delta}_j'\hat{F}_t^{KSL}\}$ with $\{F_t\}$. We consider the M1 model with homoscedasticity in idiosyncratic errors with $\sigma_a^2 = 10$. First row indicates $N = 12$ and $T = 50$; second row $N = 12$ and $T = 100$; third row $N = 50$ and $T = 100$; fourth row $N = 200$ and $T = 100$ and fifth row $N = 200$ and $T = 500$. The first column plots $\gamma = -0.8$, second column $\gamma = 0$, third column $\gamma = 0.7$ and fourth column $\gamma = 1$.

Figure 2: Sample correlations between $\{\hat{\delta}_j' \hat{F}_t^{PCD}\}$, $\{\hat{\delta}_j' \hat{F}_t^{KSD}\}$, $\{\hat{\delta}_j' \hat{F}_t^{PCL}\}$, $\{\hat{\delta}_j' \hat{F}_t^{BLL}\}$, $\{\hat{\delta}_j' \hat{F}_t^{GPCE}\}$ and $\{\hat{\delta}_j' \hat{F}_t^{KSL}\}$ with $\{F_t\}$. We consider the M1 model with homoscedasticity in idiosyncratic errors with $\gamma = -0.8$. First row indicates $N = 12$ and $T = 50$; second row $N = 12$ and $T = 100$; third row $N = 50$ and $T = 100$; fourth row $N = 200$ and $T = 100$ and fifth row $N = 200$ and $T = 500$. The first column plots $\sigma_a^2 = 0.1$, second column $\sigma_a^2 = 1$, and third column $\sigma_a^2 = 10$.

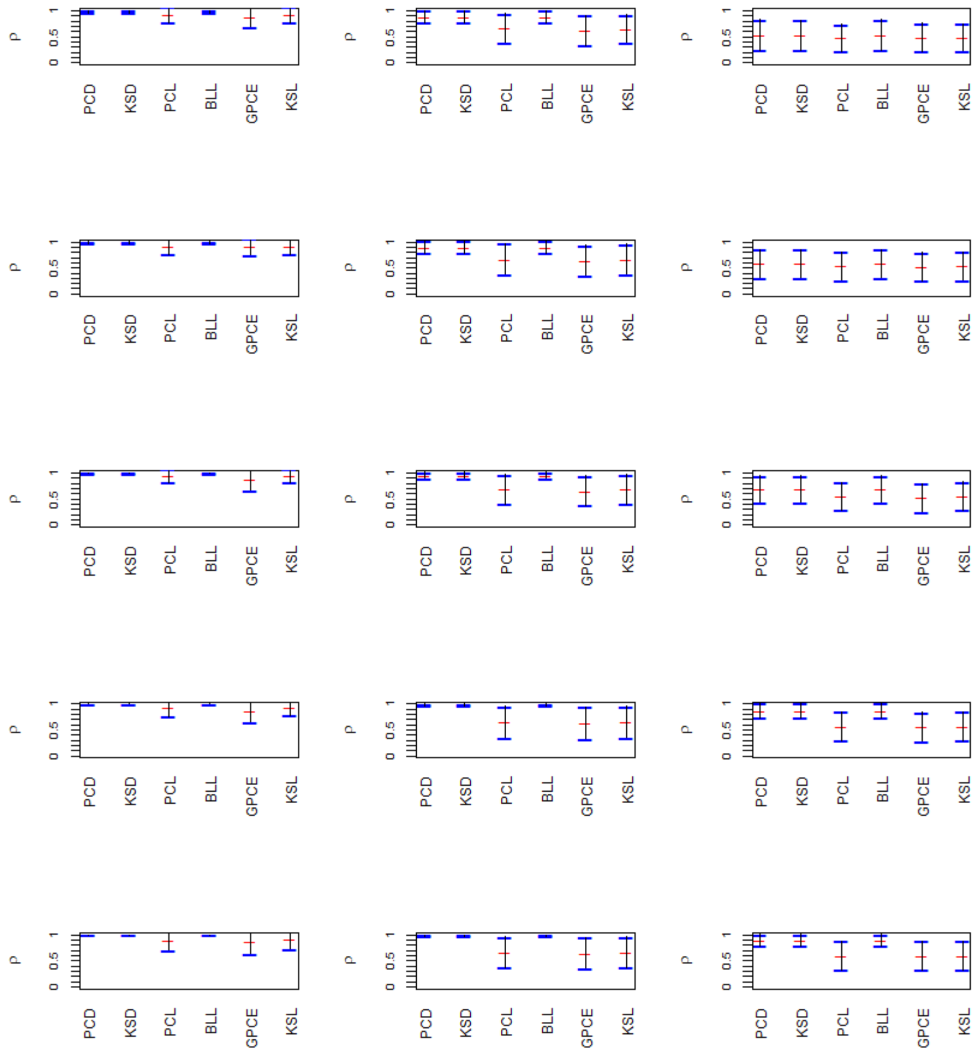

Figure 3: Sample correlations between $\{\hat{\delta}_j'\hat{F}_t^{PCD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{KSD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{PCL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{BLL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{GPCE}\}$ and $\{\hat{\delta}_j'\hat{F}_t^{KSL}\}$ with $\{F_t\}$. We consider the M1 model with homoscedasticity in idiosyncratic errors with $\gamma = 1$. First row indicates $N = 12$ and $T = 50$; second row $N = 12$ and $T = 100$; third row $N = 50$ and $T = 100$; fourth row $N = 200$ and $T = 100$ and fifth row $N = 200$ and $T = 500$. First column plots $\sigma_a^2 = 0.1$, second column $\sigma_a^2 = 1$ and third column $\sigma_a^2 = 10$.

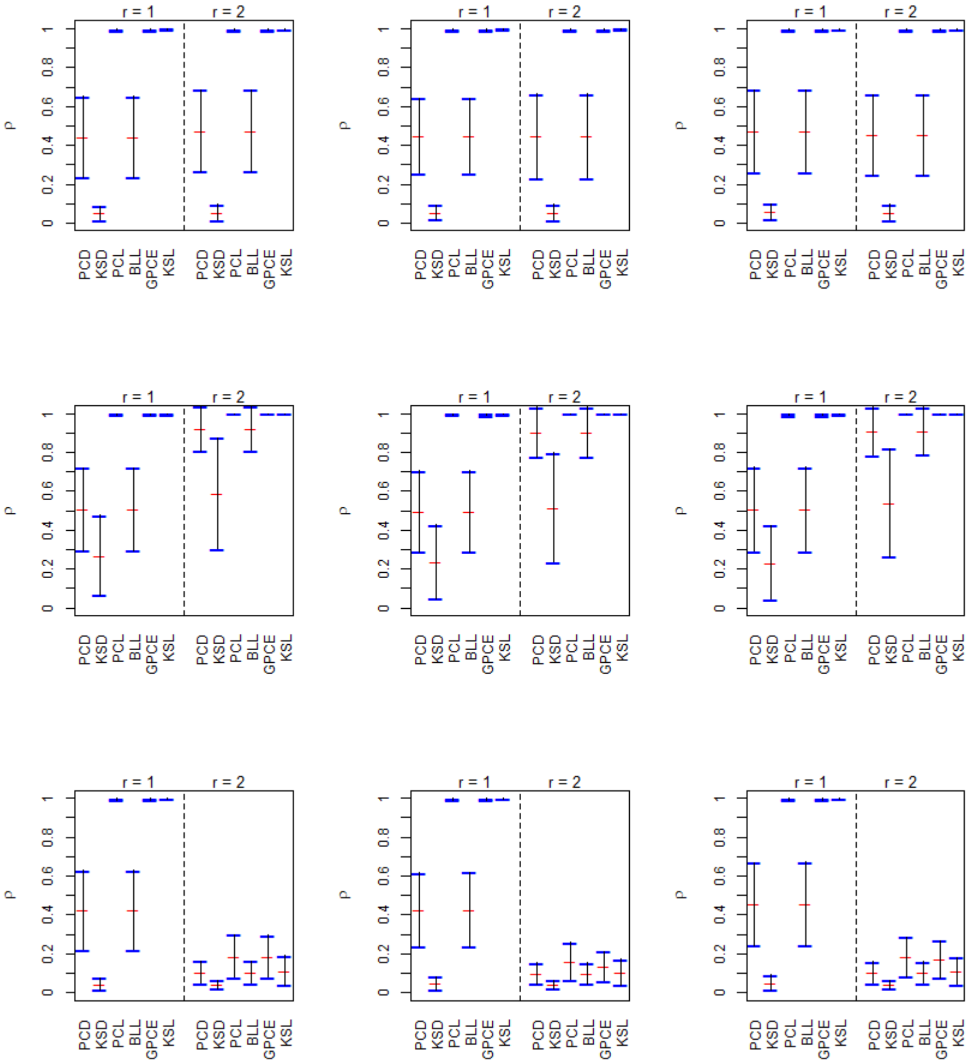

Figure 4: Sample correlations between $\{\hat{\delta}_j'\hat{F}_t^{PCD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{KSD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{PCL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{BLL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{GPCE}\}$ and $\{\hat{\delta}_j'\hat{F}_t^{KSL}\}$ with $\{F_t\}$. We consider the $N = 50$ and $T = 100$ with $\sigma_a^2 = 10$ and $\gamma = -0.8$. First row plots M2 model, second row M3 model and third row M4 model. First column indicates the homoscedasticity, second column heteroscedasticity and third column cross-sectionally correlated idiosyncratic errors.

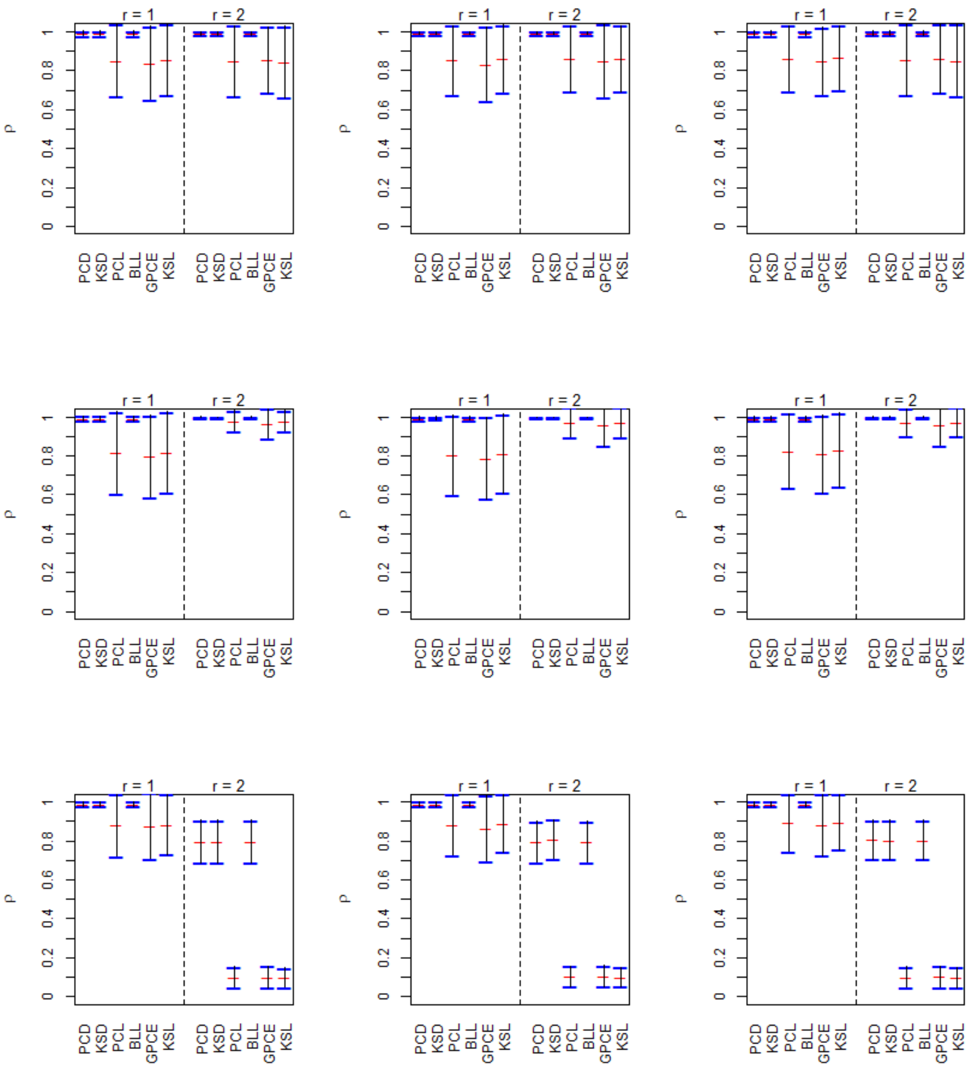

Figure 5: Sample correlations between $\{\hat{\delta}_j'\hat{F}_t^{PCD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{KSD}\}$, $\{\hat{\delta}_j'\hat{F}_t^{PCL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{BLL}\}$, $\{\hat{\delta}_j'\hat{F}_t^{GPCE}\}$ and $\{\hat{\delta}_j'\hat{F}_t^{KSL}\}$ with $\{F_t\}$. We consider the $N = 50$ and $T = 100$ with $\sigma_a^2 = 1$ and $\gamma = 1$. First row plots M2 model, second row M3 model and third row M4 model. First column indicates the homoscedasticity, second column heteroscedasticity and third column cross-sectionally correlated idiosyncratic errors.

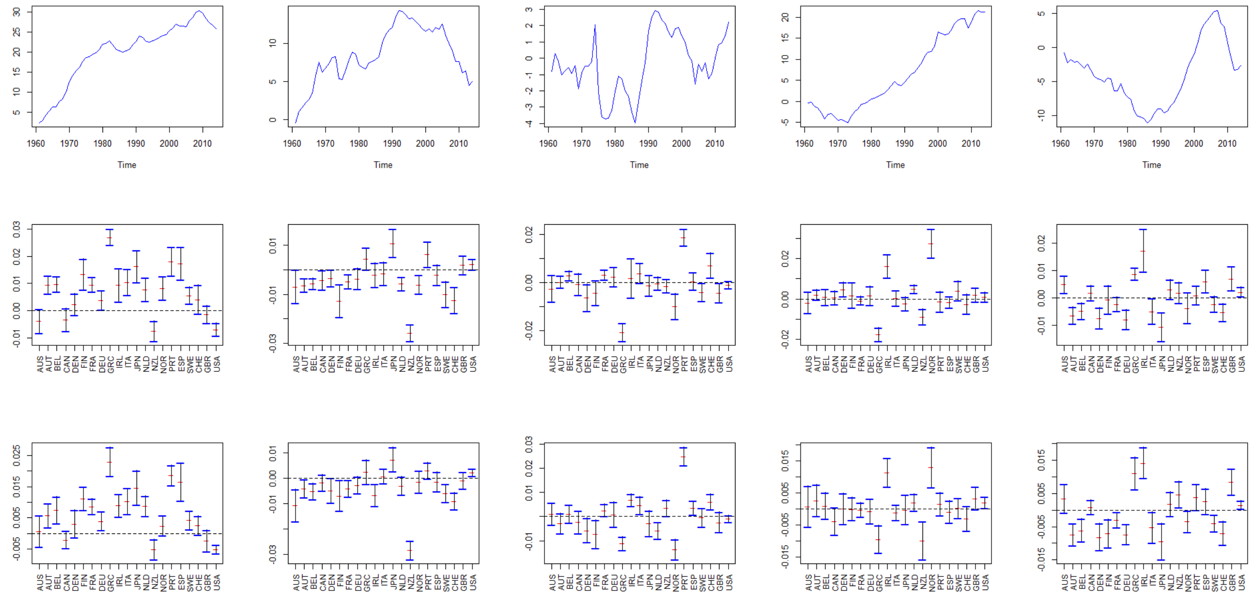

Figure 6: Top panel $\hat{F}_{jt}$, middle panel $\hat{p}_{ij}$ for GDP ($i = 1, \ldots 21$) and bottom panel $\hat{p}_{ij}$ for C ($i = 22, \ldots 42$) for $j = 1, \ldots, 5$.. All estimations are obtained using PCD.