A Mathematical Representation of "Excitement" in Games: A Contribution to the Theory of Game Systems

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June 2008

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Researchers have long believed the concept of "excitement" in games to be subjective and difficult to measure. This paper presents the development of a mathematically computable index that measures the concept from the viewpoint of an audience and from that of a player. One of the key aspects of the index is the differential of the probability of "winning" before and after one specific "play" in a given game. The index makes a large contribution to the study of games and enables researchers to compare and analyze the "excitement" of various games. It may be applied in many fields, especially the area of welfare economics, and applications may range from those related to allocative efficiency to axioms of justice and equity.

Keywords: game, game system, excitement

JEL classification: C69, D63

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Mathematical Representations of "Excitement" in Games : A Contribution to the Theory of Game Systems ♦

Satoru KUMAGAI*

Abstract

Researchers have long believed the concept of "excitement" in games to be subjective and difficult to measure. This paper presents the development of a mathematically computable index that measures the concept from the viewpoint of an audience and from that of a player. One of the key aspects of the index is the differential of the probability of "winning" before and after one specific "play" in a given game. If the probability of winning becomes very positive or negative by that play, then the audience will feel the game to be "exciting." The index makes a large contribution to the study of games and enables researchers to compare and analyze the "excitement" of various games. It may be applied in many fields, especially the area of welfare economics, and applications may range from those related to allocative efficiency to axioms of justice and equity.

♦ This paper is the complete version of Kumagai (2007), which focused only on only the index of "excitement" in games from the viewpoint of an audience.

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1. Introduction

Researchers have long believed the concept of "excitement" in games to be subjective and difficult to measure. It is true that "excitement" may differ significantly from person to person even though all are playing or watching the same game. The concept of "excitement" involves various factors such as having fun seeing or doing something fantastic or promoting a favorite sports team.

In this paper, "excitement" in games is refined through the development of a mathematically computable index that measures the concept from the viewpoint of the audience and then from that of the player. A key concept of the mathematical index is that the core element of "excitement" is the differential of the probability of "winning" before and after one specific "play" in a match[1]. If the probability of winning becomes very positive or very negative by that play, the audience feels the game to be "exciting." However, this may not be the only measure of "excitement", and the key concept may have problems. These problems and an alternative concept are explored later. The work presented here may provide a breakthrough in the development of an objective index of "excitement" from the viewpoint of a player, which may be the most important viewpoint from which to analyze the "excitement" of games.

This paper is constructed as follows: Section 2 contains a refinement of the concept of "excitement" for mathematical formulation. The first mathematical representation of "excitement" in games based on the above mentioned key concept is presented in Section 3. In Section 4, problems with the key concept are discussed, and

[1] The author first considered this idea in 1997. A similar concept may be found at http://live.protrade.com/, but it is not clear who originally developed the idea. The concept of "excitement" in games may be the "folk theorem" of game theory.

an alternative concept of "excitement" is proposed. Section 5 includes discussion of the difficulties in dealing with the concept of "excitement" from the viewpoint of a player. An index that measures the concept from the viewpoint of a rational player is then developed. Conclusions are presented in the final section.

2. Refining the Concept of "Excitement"

2.1 Two Factors of Excitement

Although there are various sources of "excitement" in games, such sources may be categorized into two groups:

The first group of sources includes doing something fun such as running, chasing after something, jumping, or shooting for example. This group contains both physical and virtual forms. For example, the "excitement" of shooting can be both physical (using real guns) and virtual (as in a computer game). Hereafter, this group of "excitement" will be termed the "fun factor".

The second group of sources includes seeking success or winning the game. This group of "excitement" arises from a formal system of rules (termed a "game system"). The "excitement" of chess comes primarily from the game system of chess, not from physically moving a knight or a pawn. This source of "excitement" will be termed the "game system factor".

In constructing an objective index of “excitement” in games, it is crucial to consider only the “game system factor”[2]. The “fun factor” is simply too subjective to be analyzed numerically.

2.2 Two Viewpoints of Excitement

It is also useful to separate two viewpoints of “excitement” in games: the viewpoint of the “player” and that of the “audience.” The “excitement” of games from the viewpoint of the player is much more complex than that of the audience. So, this paper first concentrates on the latter and then proceeds to the former. Consider the following points:

First, players of a game put much effort into playing the game. Thus, the “excitement” of games from the viewpoint of the player might be well analyzed using a “cost-benefit” type of analysis. Conversely, audience “excitement” can be analyzed without considering audience “efforts” in watching the game.

Second, player “excitement” tends to contain more of the “fun factor” and may be more objective than audience “excitement”. For example, football can be exciting without any rules because kicking and chasing a ball is in itself “fun” and “exciting”.

Third, player “excitement” tends to be strongly influenced by the result of the game, that is, by who wins and who loses. An “exciting match” is “not exciting” at all for the loser. On the other hand, audience “excitement” tends to be more subjective and

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[2] Huizinga (1955) and Caillois (1961) both analyzed “play as fun”. However, little research had been conducted on this factor prior to that of Salen and Zimmerman (2004).

“process oriented.” “Excitement” of a member of the audience who is a fanatic fan of a player or team can be quite different from that of others. Therefore, for purposes of this study, a hypothetical “neutral” audience that does not care about who wins (evaluates the process of the match as neutral) will be used as sole evaluator of “excitement” in games.

3. “Excitement” of Games from the Viewpoint of a Neutral Audience

Figures 1 and 2 show the movement of the probability of winning for hypothetical baseball game scores seen in Tables 1 and 2. The probability of winning before the game is 0.5 because it is unknown which team will win, and both hypothetically have an equal chance of winning. After the game, the probability of winning for a team converges to 1 or 0 (the team has either won or lost). The probability of winning during the game moves with the score.

Table 1: Baseball Game I

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Total*
N	9	0	0	6	0	0	0	0	0	15
B	0	0	0	0	0	0	0	0	0	0

Table 2: Baseball Game II

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Total*
N	0	0	0	0	0	0	0	0	1	1
B	0	0	0	0	0	0	0	0	2x	2

Figure 1: Movement of the Probability of Winning (Game I)

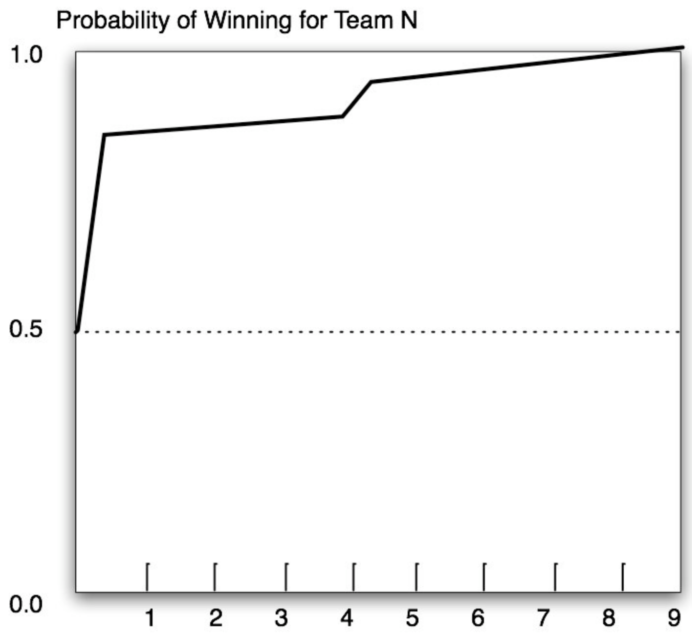
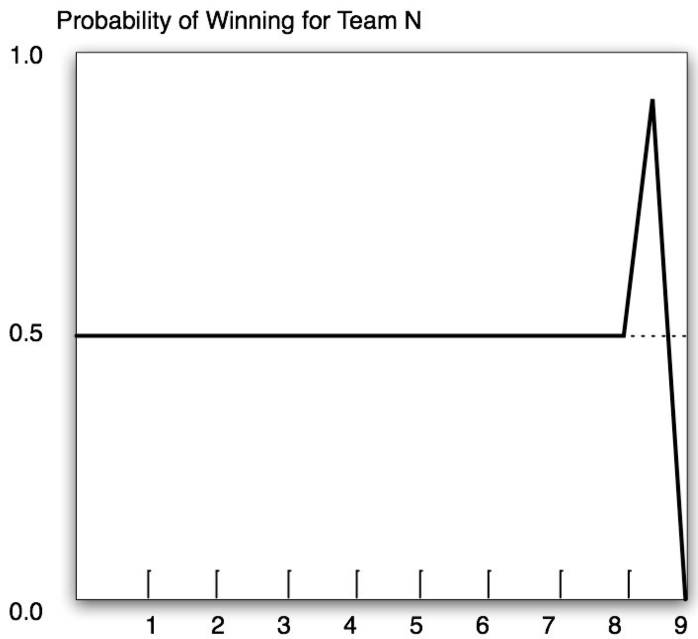


Figure 2: Movement of the Probability of Winning (Game II)



A possible mathematical formulation of “excitement” from the viewpoint of the neutral audience for one specific “play” or “move” at time t in a match, X_t, is as follows:

$$X_t = | p_t - p_{t-1} | \qquad \text{... (1)}$$

where p_t is the probability of winning for a team at time t.

Thus, the index of the “excitement” of the game, G, is constructed by summing X_t from the beginning of the game to the end of the game:

$$G = \sum_{t=1}^{T} | p_t - p_{t-1} | \quad \text{... (2)}$$

where p_t is the probability of winning for a team at time t, and T is the time of the end of the game.

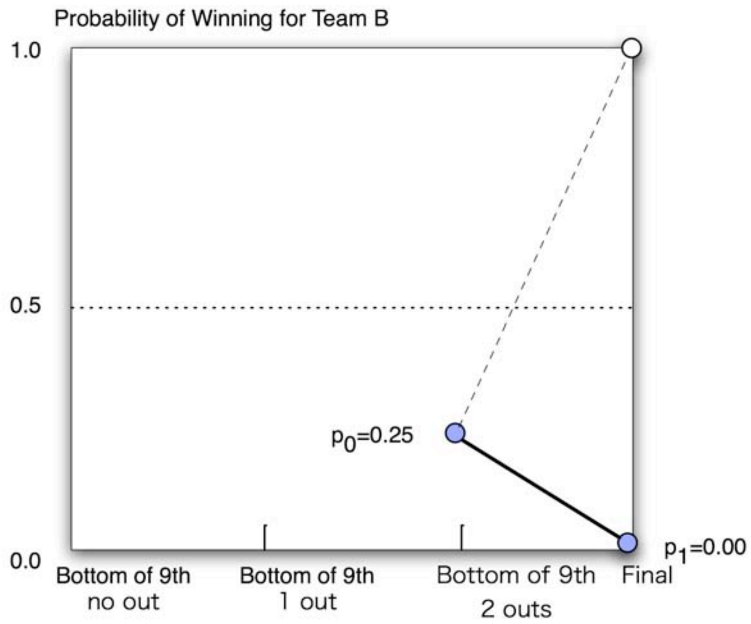
4. Problems of an *Ex Post* Index and Advantages of an *Ex Ante* Index

According to index G, Game I takes the value of 0.5 and Game II takes the value of approximately 1.5. Therefore, index G indicates that Game II should be more exciting than Game I. This quantitative result would seem to make intuitive sense, but such is not always the case. Consider the following situation that could arise in a baseball game:

- It is the bottom of the 9th inning.
- The score is N 1 – 0 B, and team B is at bat.
- Bases are loaded with two outs.

Assume the batting average of the hitter at bat to be .250. Further, for simplicity, assume that there is no base on balls, and that a single is enough for 2 runs (team B wins). Assume the hitter is then struck out and that the game is then over. Figure 3 shows the movement of the probability of winning for team B in this situation.

Figure 3 Movement of the Probability of Winning for Team B



In this situation, the probability of winning for Team B before the play (p_0) is 0.25 since the batting average of the hitter at bat equals the probability of winning for Team B. Obviously, the probability of winning for Team B after the play (p_1) is 0. Thus, the change in the probability of winning is $X = |0.25 - 0.00| = 0.25$.

Actually, the outcome of the game is difficult to know precisely. Thus, the value of X=0.25 seems to be reasonable post game (*ex post)*. This is one of the most exciting situations in baseball. X might be 0.75 if the hitter had at least hit a single or more.

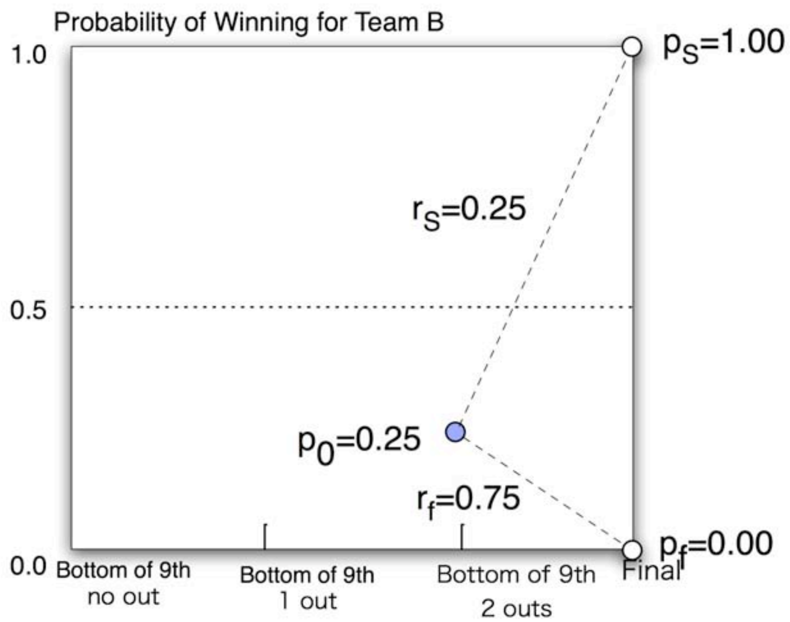
This analysis shows that the index of "excitement" (G) proposed in the previous section is inherently determined post game (*ex post).* The result is evaluated, not the situation that might lead to higher "excitement". Index G is thus reasonable for measuring the "excitement" of games after they have ended. This has a drawback. If a baseball game is like the one above, but with no run at all until the last inning, then the value of G for that game becomes quite small relative to a game with many runs. In this case, Index G does not seem to make intuitive sense relative to a "feeling" of "excitement".

To solve this issue, another index of "excitement" is proposed as follows:

$$X' = E(X) = r_s \mid p_s - p_0 \mid + r_f \mid p_f - p_0 \mid \quad \text{... (3)}$$

where r_s and r_f are the probabilities of a successful play and a failed play respectively, p_s and p_f are the probabilities of winning after the successful or failed play respectively, and p_0 is the probability of winning before the play (Figure 4).

Figure 4 Expected Movement of the Probability of Winning for Team B



This index incorporates the expected "excitement" of the situation. By allowing multiple outcomes and summing *X'*, another index of "excitement" for games may be constructed as follows:

$$G' = \sum_{t=1}^{T} X'_t$$

where $$X'_t = E_t(X_t) = \sum_{i=1}^{N} r_t^i \mid p_t^i - p_{t-1} \mid \qquad \ldots(4)$$

and $$p_{t-1} = \sum_{i=1}^{N} r_i^t p_t^i .$$

N is the number of possible outcomes, r_t^i is the probability of the realization of play i at time t, and p_t^i is the probability of winning after play i at time t.

Call *G'* an *ex ante* index of "excitement" and *G* an *ex post* index of "excitement". *G'* captures "excitement" before the play while *G* captures "excitement" after the play.

5. "Excitement" of Games from the Viewpoint of a Rational Player

As stated in 2.2, there are two viewpoints of "excitement" in games. One is the "excitement" of games from the viewpoint of a player, and the other is that of an audience. The former, the focus of this section, is much more complex than the latter but much more important for analyzing the "excitement" of games. Here, the former concept is refined step by step to make it fit objective analysis.

5.1. Fun Factor vs. Game System Factor

As discussed in 2.1, there are two major sources of “excitement” in games: the “fun factor” and the “game system factor.” Player “excitement” tends to contain more of the “fun factor” than audience “excitement” and may be more subjective. In constructing an objective index of “excitement” in games from the viewpoint of the player, it is crucial to leave out the “fun factor” and consider only the “game system factor.”

However, this does not mean that the “fun factor” is not important for analyzing the “excitement” of games from the viewpoint of the player. The “fun factor” is quite important but beyond the scope of analysis using the objective indices developed here.

5.2. Process vs. Result

Though the “fun factor” may be removed, the concept of “excitement” from the viewpoint of the player still poses difficulties. As mentioned in 2.2, player “excitement” tends to be strongly influenced by the result of the game. In this paper, Player “excitement” emanating from the result of the game (who wins and who loses) is excluded from analysis. The result of the game matters as far as it affects the process of the game, but there are two reasons to exclude the result of the game from analysis here: First, player utility from the result of the game is simply too subjective to be analyzed numerically. For some players, the result of the game is almost all the utility they gain from it, and the process of the game doesn’t matter at all. For other players, the process of the game is quite important, and the results matter very little. This kind of subjectivity can be avoided by completely omitting utility from the result of the game.

Second, analyses of the result of games are handled quite well by other economic theories. Establishing yet another criteria for evaluating the result of games seems unnecessary. If the result needs to be evaluated in addition to the process of the game, a player utility may be calculated using another theory, multiplied by some weighting factor, and added to the "excitement" index developed here.

5.3. Dealing with Player "Effort"

After excluding the "fun factor" and the utility of the results of games from the analysis, there still remains a very important issue related to the "excitement" of the game from the viewpoint of the player. Players put much more effort into playing a game than the audience. Thus, "excitement" of games from the viewpoint of the player might be well analyzed using a "cost-benefit" type of analysis. How to deal with the "efforts" put into the game by players is the most crucial point of constructing the objective index of the "excitement" of games from the viewpoint of the player.

An *ex-post* index of "excitement" of games may be used as the benefit of a game, and the sum of the costs that is an increasing function of the "effort" the player puts into the game may be subtracted from this index.

$$I = G - \sum_{t=1}^{T} C(e_t) \qquad \ldots(5)$$

where I is the index of "excitement" from the viewpoint of the player, G is the *ex-post* index of "excitement" of games , C is the cost function, and e_t is player effort at time t.

This formulation has a significant problem. G should also be a function of e_t. The effort of the player is likely to change the consequence of the game and G as well. If not, the best way to maximize index *I* is to minimize effort or by putting no effort into the game. This does not make intuitive sense.

5.4. Are Player "Efforts" Really "Costs"?

A more fundamental question relative to *I* is whether or not player efforts are really costs. In other words, if *G* is the same for two games, is the game that a player puts "less effort" into more exciting for a player than one in which "more effort" is exerted? Intuitively, most people would answer "no". It seems that the amount of effort a player puts into a game is an important factor in determining how "exciting" a game is.

Thus, suppose that the sum of player "efforts" put into a game is itself an index of "excitement" of the game from the viewpoint of the player. Thus,

$$I' = \sum_{t=1}^{T} e_t \qquad \ldots(6)$$

where e_t is the level of player effort put into the game at time *t*.

5.5. Optimal "Effort" for A Rational Player

Although *I'* seems to capture some aspects of the "excitement" of a game, it has an inherent deficiency. It is heavily dependent on who the player is. For example, a child may put full effort into a game regardless of the process. In this case, *I'* has nothing to do with the game itself but just shows that the player is "excited." Thus, a typical player who is suitable for objective analysis must be presupposed.

Here, a "rational" player is assumed and measuring the "excitement" of games based on his/her "efforts" may proceed. The index of "excitement" of games from the viewpoint of a rational player is as follow:

$$I'' = \sum_{t=1}^{T} e_t^*$$

$$\text{where} \quad e_t^* = \underset{e_t \in [0,\bar{e}]}{\arg\max}\left(U(p_t(e_t)) - U(p_{t-1}) - C(e_t)\right) \qquad \ldots(7)$$

e_t^* is the optimum level of effort the player puts into the game at time *t*, considering the utility of gaining the probability of winning and the costs of the effort. $U(\bullet)$ is a utility function and $\frac{\partial U}{\partial p} > 0$, $p_t(\bullet)$ is the expected probability of winning at time *t*, which is a function of e_t assumed to be $\frac{\partial p}{\partial e} \geq 0$. $\bar{e}$ is the maximum effort the player can put into the game. A rational player puts much effort into the game if the level of his/her effort affects the difference in the probability of winning before and after that effort. This may be translated into his/her utility (Figure 5). Conversely, if the level of his/her effort does not have much effect on the probability of winning, the player may put little or no effort into the game (Figure 6). The truly exciting game relative to this index is a game in which a player who can obtain the best result with minimum effort still voluntarily puts forth large effort.

Figure 5: Optimal Level of Effort When the Play Matters Much

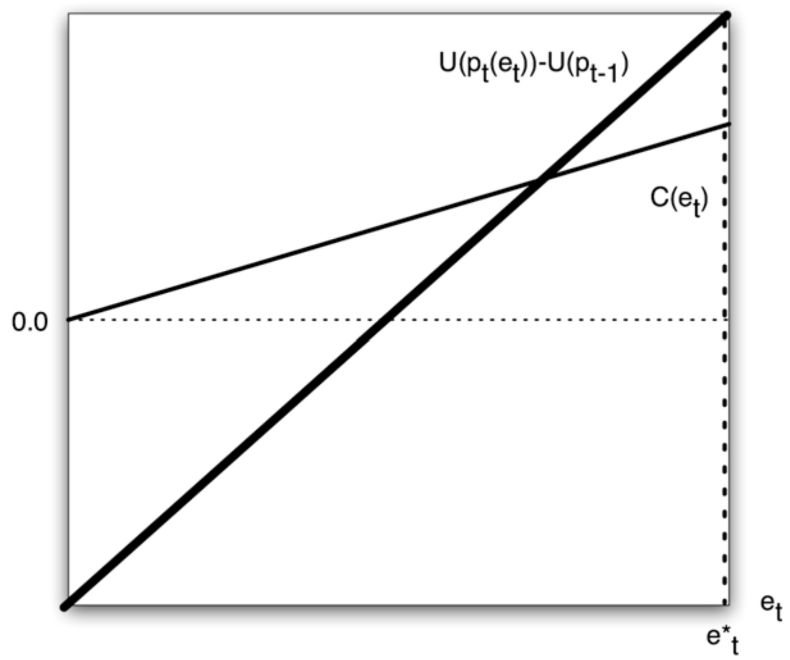
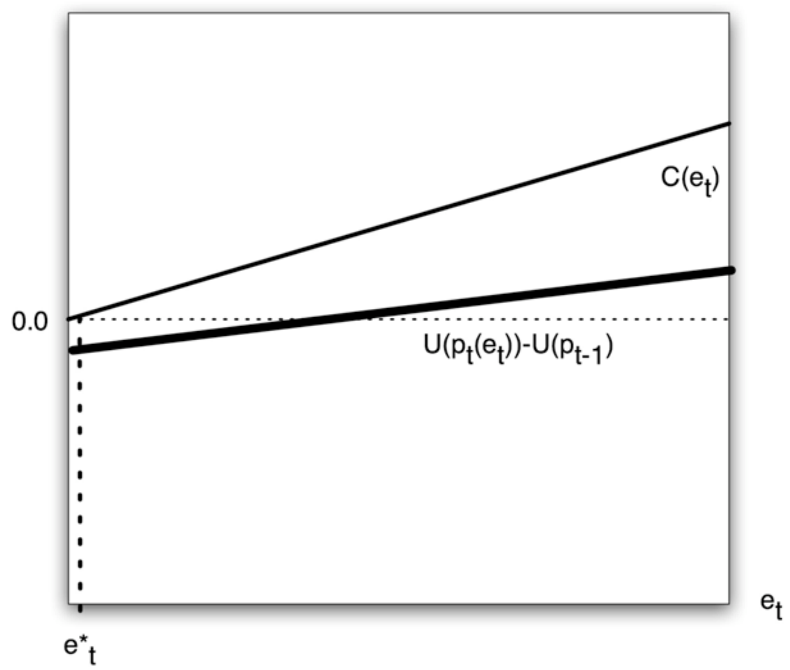


Figure 6: Optimal Level of Effort When the Play Matters Little



5.6. Optimal "Effort" and the *Ex Ante* Index of "Excitement"

While *I''* seems to be one of the proper indices to measure the "excitement" of games from the viewpoint of the player, it has two large deficiencies in the analysis of games: First, it is hard to observe player efforts correctly in many cases. Second, the index is too complicated to be used in analyzing games without actually playing them because e_t^* must be calculated for every possible consequence of the game.

The solution for this problem is to use the *ex ante* index of "excitement" proposed in previous section as a proxy of *I''*. Comparing equations (4) and (7), it can be seen that they are quite similar. The following principles may be derived:

a) When X'_t is low, e_t^* is also low because any effort does not change p_t to any significant degree.

b) When X'_t is high, e_t^* is also high insofar as higher e_t increase the probability of the realization of plays that will increase the overall probability of winning.

Thus, the *ex ante* index of "excitement" (*G'*) is a good proxy of an effort-based index of the index of "excitement" from the viewpoint of the player (*I''*) (insofar as higher efforts increase the probability of winning).

6. Conclusion

An effort-based index of "excitement" from the viewpoint of a rational player was presented in this paper. Further, the *ex ante* index of "excitement" was shown to be a good proxy for this effort-based index. This may have large impact on the development of games and/or rules. Formerly, the development of “exciting” games seemed more an art, and it was hard to generalize the process of development. Now, this objective index of “excitement” from the viewpoint of the player may be used as a benchmark for developing more “exciting” games.

As an example, the validity of the following hypothetical principle in developing exciting games mathematically can be checked:

The probability of winning at the start of a game should be set to $\frac{1}{2}$.

By accumulating valid principles in developing exciting games, a “general theory” of game systems may be possible in the future. While it might appear to have relevance only to games, this research may well have a large impact on several fields of social science, especially the area of welfare economics. Traditionally, welfare economics has only considered the utility of goods distributed through some interaction of rational individuals. This research shows that the process of interaction among people may have utility when considered by itself, and it may be possible to analyze this utility mathematically.

A zero-sum game may not be zero-sum at all when “excitement” is considered. Therefore, the welfare of ***all*** people, even in a zero-sum restriction, can be improved by improving the game system or “institution.” The process of allocation may sometimes be more important than the allocation itself, and the formulations developed in this paper may provide the base for exploring such matters. Applications are possible in many fields ranging from allocative efficiency to axioms of justice and equity.

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