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Domestic, Vertical, and Horizontal Multinationals: A General Equilibrium Approach using the "Knowledge Capital Model"

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Abstract

One of the key factors behind the growth in global trade in recent decades is an increase in intermediate input as a result of the development of vertical production networks (Feensta, 1998). It is widely recognized that the formation of production networks is due to the expansion of multinational enterprises' (MNEs) activities. MNEs have been differentiated into two types according to their production structure: horizontal and vertical foreign direct investment (FDI). In this paper, we extend the model presented by Zhang and Markusen (1999) to include horizontal and vertical FDI in a model with traded intermediates, using numerical general equilibrium analysis. The simulation results show that horizontal MNEs are more likely to exist when countries are similar in size and in relative factor endowments. Vertical MNEs are more likely to exist when countries differ in relative factor endowments, and trade costs are positive. From the results of the simulation, lower trade costs of final goods and differences in factor intensity are conditions for attracting vertical MNEs.

Keywords: Foreign Direct Investment, Knowledge-Capital Model
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1. **Introduction**

One of the key factors behind the growth of global trade in recent decades is an increase in intermediate input as a result of the development of vertical production networks (Feensta, 1998). Manufacturing goods are no longer produced in a single country. Production processes are subdivided into several stages, in which respective countries specialize in producing parts and components. Many countries are involved in vertical production networks of producing just a single final good for consumers.

It is widely recognized that the production networks have formed due to the expansion of multinational enterprises' (MNEs) activities. Multinational enterprises have been differentiated into two types according to their production structure: horizontal FDI and vertical FDI. However, a new type of FDI which diverges from the vertical one has been proposed in the context of the recent expansion of more complex multinational activities; it is called export-platform FDI. Horizontal FDI maintain affiliates in home and host countries with the headquarters located in the home country, while vertical and export-platform FDI install affiliates in host countries with the headquarters located in the home country. The difference between vertical and export-platform FDI is where their products are sold: vertical FDI seek to sell their products in both the home and host country, while export-platform FDI seek to sell in a third market through the affiliates in the host country (Ekholm et al., 2007 and Matsuura and Hayakawa, 2008).

Theoretical research on MNEs has been conducted since the early 1960s (Hymer, 1976), but it developed dramatically from the mid 1980s as a result of the "new" trade theory. There are two important theoretical models of MNEs: one was presented by Helpman (1984) and the other by Markusen (1984). Helpman's model treats vertical MNEs with monopolistic competition and without trade costs. On the other hand, Markusen's model treats horizontal MNEs with one factor, assuming firm-level scale economy. Markusen (1997) combines horizontal and vertical motives in a model, so the model allows two types of MNE to exist at the same time. This is called the "knowledge capital" model. Zhang and Markusen (1999) extended the model to consider vertical MNEs that supply intermediate inputs to a final production plant in a host country. While their models were constructed in a two-region framework, Ekholm et al. (2007) extended the model into a three-region framework to include export-platform FDI. Matsuura and Hayakawa (2008) pointed out that recent explorations of FDI theories have shifted from the two-region setting to the three-region setting (for example, Yeaple, 2003 and Grossman et al., 2006).

Ekholm et al. (2007) and other models in the three-regional framework assume that skilled-labor-intensive intermediates are produced only at home, and the host country imports intermediate products and assembles final goods, combining intermediates and unskilled labor. However, those models do not adequately explain observed facts where some kinds of intermediate goods are produced in the host country. Our final goal is to extend Ekholm et al. (2007) to treat the procurement of intermediates from the host country in view of the present situation. We start from the simple model in the two-region framework in preparation for further extension. In this paper, we extend Zhang and Markusen (1999) to include horizontal and vertical FDI in the model with traded intermediates. There are no studies which treat vertical and horizontal FDI with traded intermediates at once, although more evolved models which treat vertical, horizontal and export-platform FDI with traded intermediates, such as

Ekholm et al. (2007), do exist. This paper serves to bridge the gap between Zhang and Markusen (1999) and Ekholm et al. (2007) in theoretical studies of FDI.

The remainder of this paper is organized as follows. The next section introduces the assumptions of the model and model structure. Section three provides the numerical general equilibrium model, then section four presents the simulation results. Finally, our conclusions and future extension are presented in section five.

2. The Model

2.1 Assumptions of the Model

The following three assumptions are borrowed from Markusen (2002:129):

1. *Fragmentation: the location of knowledge based assets may be fragmented from production. Any incremental cost of supplying services of the asset to a single foreign plant versus the cost to a single domestic plant is small.*
2. *Skilled-labor intensity: knowledge-based assets are skilled labor intensive relative to final production.*
3. *Jointness: the services of knowledge based assets are (at least partially) joint ("public") inputs into multiple production facilities. The added cost of a second plant is small compared to the cost of establishing a firm with local plant.*

Fragmentation and skilled-labor intensity motivate vertical MNEs, while jointness is associated with horizontal MNEs. It should be noted that fragmentation and jointness are not the same thing. Fragmentation can be interpreted as service provided by skilled labor, such as manager service. Manager skill can be transferred easily by shifting a manager from home to host, but cannot be simultaneously used in both because a manager can only be in one place at any one time. On the other hand, jointness, which can be represented by a blueprint, can easily be shared among plants without reducing the services provided in other locations.

2.2 Model Structure

There are two identical countries, denoted by *i* and *j*, producing two final goods using two factors, unskilled labor *L* and skilled labor *S*. *L* and *S* are required in both sectors and are mobile between sectors, but are internationally immobile.

Y is produced with *L* and *S* using a Cobb-Douglas type constant return to scale technology and under perfect competition. *Y* will be used as numeraire and so its price is set to unity. The production function for *Y* is:

$$Y_i = (S_i^Y)^{\alpha}(L_i^Y)^{1-\alpha}, \tag{1}$$

where S_i^Y and L_i^Y are skilled and unskilled labor used in the *Y* sector in country *i*. Subscripts *i* and *j* will respectively be used to denote countries 1 and 2. Marginal products of *S* and *L* in *Y* production are:

$$p_i^S = \alpha\left(\frac{S_i^Y}{L_i^Y}\right)^{\alpha-1} \text{ and } p_i^L = (1-\alpha)\left(\frac{S_i^Y}{L_i^Y}\right)^{\alpha}, \tag{2}$$

where p_i^S and p_i^L are wages for skilled and unskilled labor, respectively.

Good *X* is produced with increasing returns to scale technology by imperfectly competitive Cournot firms. *X* is produced in two stages. In the first stage, the intermediate product *M* is produced only in country *i* using skilled labor *S* alone. In the second stage, *X* is assembled using unskilled labor and intermediate inputs *M*. There are both firm-level and plant-level scale economies. There are free entry and exit of the firms, and entering firms choose their "type." There are six firm types, which are defined as follows:

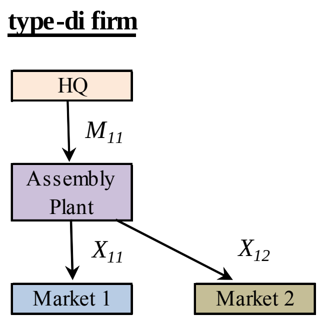
Type d_i: National firms that maintain a single plant, with headquarters in country *i*. Type-d_i firms produce *M* and *X* in country *i*. Some *X* may or may not be exported to country *j*.

Type h_i: Horizontal MNEs that maintain plants in both countries, with headquarters located in country *i*. Type-h_i firms produce *M* in country *i*, some of which is shipped to an assembly plant in country *j*. *X* is produced in both countries. Some of *X* may or may not be exported to country *i*.

Type v_i: Vertical MNEs that maintain a single plant, with headquarters in country *i*. Type-v_i firms produce *M* in country *i,* which is then shipped to an assembly plant in country *j*. Some *X* may or may not be exported to country *i*.

Figure 1 shows an image of each type of firm in the case when *i* = 1. In each pattern, the headquarters of the firm is located in country 1.

Figure 1: Firm type



The model allows domestic and multinational firms to arise endogenously. The term "regime" will denote the set of firm types active in an equilibrium.

There are additional assumptions regarding factor-intensity from the view points of activities and firm types. The factor-intensity assumption in terms of activities is as follows:

[headquarters only] > [integrated *X*] > [plant only] > [*Y*].

In terms of firm types, the assumption is:

[type-h firms] > [type-v and type-d firms].

Superscripts ($k = d, v, h$) will be used to designate a variable as referring to domestic firms, vertical MNEs, and horizontal MNEs, respectively. N_i^k will indicate the number of type-k firms active in an equilibrium in country i. The cost structure of industry *X* is as follows:

θ^S Unit input requirement for factor *S*

θ^L Unit input requirement for factor L

θ^M Unit input requirement for intermediate input M

τ^X Units of L required to ship one unit of X. This is paid by the exporting country.

τ^M Units of L required to ship one unit of M. This is paid by the exporting country.

G Plant-specific fixed cost in units of L required for the fixed costs of an X assembly plant, incurred in country *i* for type-d firms and type-h firms. Also, in country *j* for type-h and type-v firms, G is the same for any plant regardless of the type of firm and country.

F Firm-specific fixed cost in units of S required for the fixed costs of an X assembly plant, incurred in country *i* regardless of the firm type, and in country *j* for type-h and type-v firms. $F^k_{ij(i=j)}$ will be the skilled-labor requirement in the home or parent country, while $F^k_{ij(i\neq j)}$ will be the skilled-labor requirement in the foreign or host country.

Markusen (2002:135) makes three other assumptions regarding fixed cost as follows. First, he assumes that skilled-labor requirements for a type-h firm are greater than (but less than double) the skilled-labor requirements of a type-d firm. This is the jointness assumption. Second, the additional skilled-labor requirements of a type-h firm over a type-d firm are incurred partly in the home country and partly in the host country. The last assumption is that managerial and coordination activities require some additional skilled labor in the parent country for a type-h firm. For a firm based in country *i*, the following relationship exists:

Jointness: $$2F^d_{ij(i=j)} > \sum_j F^h_{ij} > F^d_{ij(i=j)} \text{ and } F^h_{ij(i=j)} > F^d_{ij(i=j)}.$$

Fragmentation is not perfect in that some costs are incurred in order to transfer technology. Therefore, type-v firms have higher skilled-labor requirements than type-d firms, but less than type-h firms:

Fragmentation: $\sum_j F_{ij}^h > \sum_j F_{ij}^v > F_{ij(i=j)}^d$.

A specific example used in our numerical model is described below. The values are:

G=2, $F_{ij(i=j)}^d$=11, $F_{ij(i=j)}^h$=12 and $F_{ij(i\neq j)}^h$=4, $F_{ij(i=j)}^v$=9 and $F_{ij(i\neq j)}^v$=3.

Total fixed cost requirements for firms are:

	type-d1	type-h1	type-v1	type-d2	type-h2	type-v2
L_1	2	2	--	--	2	--
S_1	11	12	9	--	4	3
L_2	--	2	2	2	2	2
S_2	--	4	3	11	12	9

The total fixed costs of type-d, type-h and type-v are 13, 20 and 14, respectively.

Next, the production costs of each type of firm are introduced.

Type-d firms

Type-d firms produce three products: $X_{ij(i=j)}^d$, $X_{ij(i\neq j)}^d$, and $M_{ij(i=j)}^d$. The skilled-labor requirements for type-d firms in country *i* are given by:

$$S_{ij(i=j)}^{d} = F_{ij(i=j)}^{d} + \theta^{S} M_{ij(i=j)}^{d} \text{ and } M_{ij(i=j)}^{d} = \theta^{M} \sum_{j} X_{ij}^{d}. \tag{3}$$

The unskilled-labor requirements in country *i* are:

$$L_{ij(i=j)}^{d} = G + \theta^{L} \sum_{j} X_{ij}^{d} + \tau^{X} X_{ij(i\neq j)}^{d}. \tag{4}$$

Therefore, the cost function of type-d firms is given by:

$$\begin{aligned} & p_i^S S_{ij(i=j)}^{d} + p_i^L L_{ij(i=j)}^{d} \\ & = \left(p_i^S \theta^S \theta^M + p_i^L \theta^L\right) X_{ij(i=j)}^{d} + \left\{p_i^S \theta^S \theta^M + p_i^L (\theta^L + \tau^X)\right\} X_{ij(i\neq j)}^{d} \\ & \quad + \left(p_i^S F_{ij(i=j)}^{d} + p_i^L G\right). \end{aligned} \tag{5}$$

Type-h firms

Type-h firms produce four products: $X_{ij(i=j)}^{h}$, $X_{ij(i\neq j)}^{h}$, $M_{ij(i=j)}^{h}$, and $M_{ij(i\neq j)}^{h}$.

The skilled-labor requirements for type-h firms in country *i* are given by:

$$S_{ij(i=j)}^{h} = F_{ij(i=j)}^{h} + \theta^{S} \sum_{j} M_{ij}^{h} \text{ and } M_{ij}^{h} = \theta^{M} X_{ij}^{h}. \tag{6}$$

The unskilled-labor requirements in country *i* are:

$$L_{ij(i=j)}^{h} = G + \theta^{L} X_{ij(i=j)}^{h} + \tau^{M} M_{ij(i\neq j)}^{h}. \tag{7}$$

The skilled-labor requirements in country *j* are:

$$S_{ij(i\neq j)}^{h} = F_{ij(i\neq j)}^{h}. \tag{8}$$

The unskilled-labor requirements in country *j* are:

$$L_{ij(i\neq j)}^{h} = G + \theta^{L} X_{ij(i\neq j)}^{h}. \tag{9}$$

Therefore, the cost function of type-h firms is given by:

$$\begin{aligned}
&\sum_{j}\left(p_j^S S_{ij}^h + p_j^L L_{ij}^h\right) \\
&= \left(p_i^S \theta^S \theta^M + p_i^L \theta^L\right) X_{ij(i=j)}^h + \left\{\left(p_i^S \theta^S + p_i^L \tau^M\right)\theta^M + p_j^L \theta^L\right\} X_{ij(i\neq j)}^h \\
&\quad + \sum_{j}\left(p_j^S F_{ij}^h + p_j^L G\right).
\end{aligned} \tag{10}$$

Type-v firms

Type-v firms produce three products: $X_{ij(i=j)}^{v}$, $X_{ij(i\neq j)}^{v}$, and $M_{ij(i=j)}^{v}$. The skilled-labor requirements for type-v firms in country *i* are given by:

$$S_{ij(i=j)}^{v} = F_{ij(i=j)}^{v} + \theta^{S} M_{ij(i\neq j)}^{v} \text{ and } M_{ij(i\neq j)}^{v} = \theta^{M} \sum_{j} X_{ij}^{v}. \tag{11}$$

The unskilled-labor requirements in country *i* are:

$$L_{ij(i=j)}^{v} = \tau^{M} M_{ij(i\neq j)}^{v}. \tag{12}$$

The skilled-labor requirements in country *j* are:

$$S^v_{ij(i\neq j)} = F^v_{ij(i\neq j)}. \tag{13}$$

The unskilled-labor requirements in country *j* are:

$$L^v_{ij(i\neq j)} = G + \theta^L \textstyle\sum_j X^v_{ij} + \tau^X X^v_{ij(i=j)}. \tag{14}$$

Therefore, the cost function of type-v firms is given by:

$$\begin{aligned}
&\textstyle\sum_j \left(p^S_j S^v_{ij} + p^L_j L^v_{ij}\right) \\
&= \left\{\left(p^S_i \theta^S + p^L_i \tau^M\right)\theta^M + p^L_j(\theta^L + \tau^X)\right\} X^v_{ij(i=j)} \\
&\quad + \left\{\left(p^S_i \theta^S + p^L_i \tau^M\right)\theta^M + p^L_j \theta^L\right\} X^v_{ij(i\neq j)} + \textstyle\sum_j \left(p^S_j F^v_{ij} + p^L_j G\right).
\end{aligned} \tag{15}$$

Let $\bar{S}_i$ and $\bar{L}_i$ denote total factor endowments in country *i*. The factor market equilibrium can be defined by:

$$\bar{S}_i = S^Y_i + N^d_i S^d_{ij(i=j)} + \textstyle\sum_j \left(N^h_j S^h_{ji} + N^v_j S^v_{ji}\right), \tag{16}$$

$$\bar{L}_i = L^Y_i + N^d_i L^d_{ij(i=j)} + \textstyle\sum_j \left(N^h_j L^h_{ji} + N^v_j L^v_{ji}\right). \tag{17}$$

In an equilibrium, the *X* sector makes no profit, so country *i*'s national income denoted by Q_i is:

$$Q_i = p_i^S \overline{S_i} + p_i^L \overline{L_i}. \tag{18}$$

Let X_i^C and Y_i^C denote the consumptions of *X* and *Y* in country *i*. The utility of a representative consumer in each country is assumed to be defined by the following Cobb-Douglas type function:

$$U_i = \left(X_i^C\right)^{\beta}\left(Y_i^C\right)^{1-\beta} \tag{19}$$

where,

$$X_i^C = \sum_j \left(N_j^d X_{ji}^d + N_j^h X_{ji}^h + N_j^v X_{ji}^v\right) \text{ and } Y_i^C = Y_i.$$

Maximizing utility subject to the income constraint, we obtain the first-order conditions that give demands for *X* and *Y*:

$$p_i^X X_i^C = \beta Q_i \text{ and } Y_i^C = (1-\beta) Q_i \tag{20}$$

An equilibrium in the *X* sector is determined by the pricing equation (marginal revenue equals marginal cost) and free entry conditions. The proportional markup of price over marginal cost is denoted by ε_{ij}^k. This can be read as the markup of a type-*k* firm in country *j*. The pricing equations of each type of firm are:

$$p_i^X\left(1-\varepsilon_{ij(i=j)}^d\right) \le p_i^S \theta^S \theta^M + p_i^L \theta^L, \tag{21}$$

$$p_i^X\left(1-\varepsilon_{ij(i\neq j)}^d\right) \leq p_i^S\theta^S\theta^M + p_i^L(\theta^L+\tau^X), \tag{22}$$

$$p_i^X\left(1-\varepsilon_{ij(i=j)}^h\right) \leq p_i^S\theta^S\theta^M + p_i^L\theta^L, \tag{23}$$

$$p_i^X\left(1-\varepsilon_{ij(i\neq j)}^h\right) \leq p_i^S\theta^S\theta^M + p_i^L\tau^M\theta^M + p_j^L\theta^L, \tag{24}$$

$$p_i^X\left(1-\varepsilon_{ij(i=j)}^v\right) \leq p_i^S\theta^S\theta^M + p_i^L\tau^M\theta^M + p_j^L(\theta^L+\tau^X), \tag{25}$$

$$p_i^X\left(1-\varepsilon_{ij(i\neq j)}^v\right) \leq p_i^S\theta^S\theta^M + p_i^L\tau^M\theta^M + p_j^L\theta^L. \tag{26}$$

The optimal markup in a Cournot model with homogenous products is given by the firm's share divided by the Marshallian price elasticity of demand in that market. Since Marshallian elasticity of demand is -1 in this model with Cobb-Douglas demand, a firm's markup can be rewritten as:

$$\varepsilon_{ij}^k = \frac{p_j^X X_{ij}^k}{\beta Q_j}. \tag{27}$$

Substituting the markup equations shown above into the pricing equations gives the following expressions for demand or output in terms of price:

$$X_{ij(i=j)}^d \geq \beta Q_i\left[\frac{p_i^X - p_i^S\theta^S\theta^M - p_i^L\theta^L}{\left(p_i^X\right)^2}\right], \tag{28}$$

$$X_{ij(i\neq j)}^d \geq \beta Q_j\left[\frac{p_j^X - p_i^S\theta^S\theta^M - p_i^L(\theta^L+\tau^X)}{\left(p_j^X\right)^2}\right], \tag{29}$$

$$X^h_{ij(i=j)} \geq \beta Q_i \left[\frac{p^X_i - p^S_i \theta^S \theta^M - p^L_i \theta^L}{\left(p^X_i\right)^2}\right], \tag{30}$$

$$X^h_{ij(i\neq j)} \geq \beta Q_j \left[\frac{p^X_j - p^S_i \theta^S \theta^M - p^L_i \tau^M \theta^M - p^L_j \theta^L}{\left(p^X_j\right)^2}\right], \tag{31}$$

$$X^v_{ij(i=j)} \geq \beta Q_i \left[\frac{p^X_i - p^S_i \theta^S \theta^M - p^L_i \tau^M \theta^M - p^L_j (\theta^L + \tau^X)}{\left(p^X_i\right)^2}\right], \tag{32}$$

$$X^v_{ij(i\neq j)} \geq \beta Q_j \left[\frac{p^X_j - p^S_i \theta^S \theta^M - p^L_i \tau^M \theta^M - p^L_j \theta^L}{\left(p^X_j\right)^2}\right]. \tag{33}$$

There are three zero profit conditions, corresponding to the three types of firms. Zero profit conditions can be given as the requirement that markup revenues are less than or equal to fixed costs:

$$\textstyle\sum_j p^X_j \varepsilon^d_{ij} X^d_{ij} \leq p^S_i F^h_{ij(i=j)} + p^L_i G, \tag{34}$$

$$\textstyle\sum_j p^X_j \varepsilon^h_{ij} X^h_{ij} \leq \sum_j \left(p^S_j F^h_{ij} + p^L_j G\right), \tag{35}$$

$$\textstyle\sum_j p^X_j \varepsilon^v_{ij} X^v_{ij} \leq \sum_j p^S_j F^v_{ij} + p^L_j G. \tag{36}$$

Using equations (28) through (33), the zero profit conditions (34) through (36) can be rewritten as:

$$\left(p^X_i - p^S_i \theta^S \theta^M - p^L_i \theta^L\right) X^d_{ij(i=j)}$$

$$+\{p_j^X - p_i^S\theta^S\theta^M - p_i^L(\theta^L + \tau^X)\}X_{ij(i\neq j)}^d$$
$$\leq p_i^S F_{ij(i=j)}^h + p_i^L G, \tag{37}$$

$$\left(p_i^X - p_i^S\theta^S\theta^M - p_i^L\theta^L\right)X_{ij(i=j)}^h$$
$$+\left(p_j^X - p_i^S\theta^S\theta^M - p_i^L\tau^M\theta^M - p_j^L\theta^L\right)X_{ij(i\neq j)}^h$$
$$\leq \textstyle\sum_j\left(p_j^S F_{ij}^h + p_j^L G\right), \tag{38}$$

$$\{p_i^X - p_i^S\theta^S\theta^M - p_i^L\tau^M\theta^M - p_j^L(\theta^L + \tau^X)\}X_{ij(i=j)}^v$$
$$+\left(p_j^X - p_i^S\theta^S\theta^M - p_i^L\tau^M\theta^M - p_j^L\theta^L\right)X_{ij(i\neq j)}^v$$
$$\leq \textstyle\sum_j p_j^S F_{ij}^v + p_j^L G. \tag{39}$$

To summarize the *X* sector in the model, the twelve inequalities (28) through (33) are associated with the twelve output levels, and the six inequalities (37) through (39) are associated with the number of firms in each regime. Factor prices can be derived from factor-market-clearing conditions (16) and (17). Goods prices are obtained by equation (20).

3. The Numerical General Equilibrium Model

Markusen (2002) pointed out two difficulties in solving the model by comparative statics: one difficulty is the “many dimensions of the model” and the other is the “many inequalities of the model”. In this paper, we formulate the model as a complementarily problem following Markusen (2002). The program code for the general algebraic modeling system (GAMS) is given in the Appendix[1].

Table 1 shows the value used in the calibration of our model to the center of the Edgeworth box, where only type-h firms are active due to the high trade cost of 20%. Viewed in the column-wise direction, the table shows the input structure, while viewed in the row-wise direction, the table shows the output distributions. A zero column sum implies that the zero profit conditions are satisfied and a zero row sum indicates that the market-clearing conditions are satisfied. Positive entries are receipts, while negative entries are payments. All activity levels are one initially, except type-h activities. There are five type-h firms (2.5 for each type-h firm) at the initial point, so the markup is 20%. The fixed costs of other firm types are defined earlier in this paper. θ^S, θ^L and θ^M are exogenously determined as 1.0, 0.875 and 0.125.

Table 1 Calibration of the model at the center of the Edgeworth box

	Y1	Y2	X11	X12	X22	X21	N1	N2	U1	U2	CONS1	CONS2	ENT1	ENT2	Rowsum
CY1	100								-100						0
CY2		100								-100					0
CX1			50			50			-100						0
CX2				50	50					-100					0
FC1							20						-20		0
FC2								20						-20	0
L1	-80		-35		-35		-2	-2			154				0
S1	-20		-5		-5		-12	-4			46				0
L2				-35		-35	-2	-2				154			0
S2				-5		-5	-4	-12				46			0
UTIL1									200		-200				0
UTIL2										200		-200			0
MK11			-10										10		0
MK12				-10									10		0
MK22					-10									10	0
MK21						-10								10	0
Colsum	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

CY	Price of good Y	Y	Output of Y
CX	Consumer price of X	X	Output of X by type-h firm
FC	Price of fixed cost	N	Output of fixed cost for type-h firm
L	Price of unskilled-labor	U	Welfare
S	Price of skilled-labor	CONS	Income of representative consumer
UTIL	Price of a unit of utility	ENT	Income of the owner of type-h firms
MK	Markup		

Source: Markusen (2002), *Multinational Firms and the Theory of International Trade*, Massachusetts: MIT press, p. 161.

[1] Note that some solutions might not be found when one runs the presented program, which solves the model 361 times. Such kind of error occurs when the choice of initial values of variables becomes inadequate under certain conditions. One may solve the model individually by setting other initial values to recover the lost solutions.

The elasticity of substitution Y is derived by calibration of the model, using the values in Table 1.

4. Simulation Results

Figures 2-5 present world Edgeworth boxes, where the vertical dimension is the total world endowment of S (skilled-labor) and the horizontal axis is the total world endowment of L (unskilled-labor). In the Edgeworth boxes, division of the world factor endowment between two countries is shown with country 1 measured from the southwest (SW) corner and country 2 measured from the northeast (NE) corner. The model is repeatedly solved for each cell 361 times, altering the distribution of factor endowments. Each cell of Figures 2-5 represents the equilibrium regime and the numbers inside the cell show which type of firm is active in the regime. Table 2 presents the values we used to show which type of firm is active. For example, if the value in the cell is 101, it shows that the domestic and horizontal firms of country 1 are active. Also, number 110.01 shows that the domestic and vertical firms of country 1 and domestic firms of country 2 are active. Figures 2-5 are gradation-coded according to the active firm type.

Table 2 Values for the firm type

	Country 1	Country 2
Domestic	100	10
Horizontal	1	0.1
Vertical	0.01	0.001

Figure 2 shows the equilibrium regime at 20% transportation cost of final good X and 20% of intermediate input M. This is the base case of our simulation analyses. The values shown in Table 1 are used to solve the model at the center of the Edgeworth box. The figure is read as follows: The center of the Edgeworth box, where countries are similar in size and in relative endowment, shows there are only type-h firms. The number 0.01 at the top-left corner of the figure means that there are only type-v1 firms, where 95% of world skilled-labor endowment and 5% unskilled-labor endowment are in country 1. At the edges of the box are the regions in which only type-v firms are active in each equilibrium. This means that type-v firms are active when countries differ in relative factor endowment.

Figure 2 Equilibrium regime in the base case ($\tau^X = 0.2, \tau^M = 0.2$)

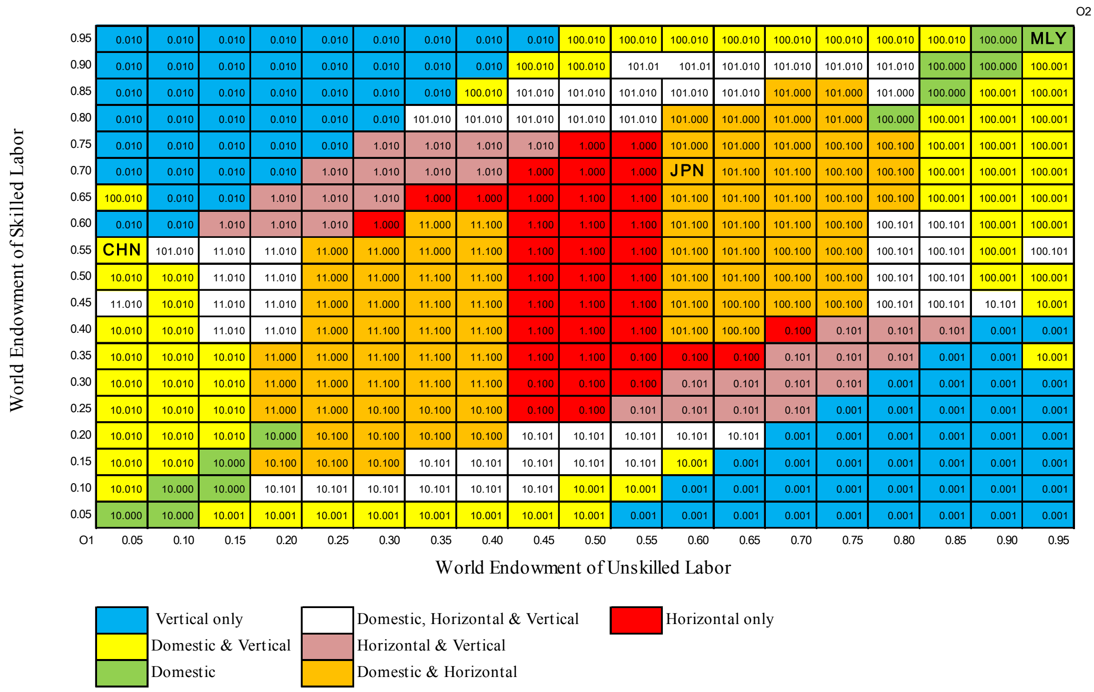


Figure 3 is the equilibrium regime when the trade costs of intermediate goods *M* are lowered from 20% to 1%. Figure 3 shows that type-h firms become a lot more important than in the base case.

Figure 3 Equilibrium regime when intermediate trade costs are lowered

$$(\tau^X = 0.2, \tau^M = 0.01)$$

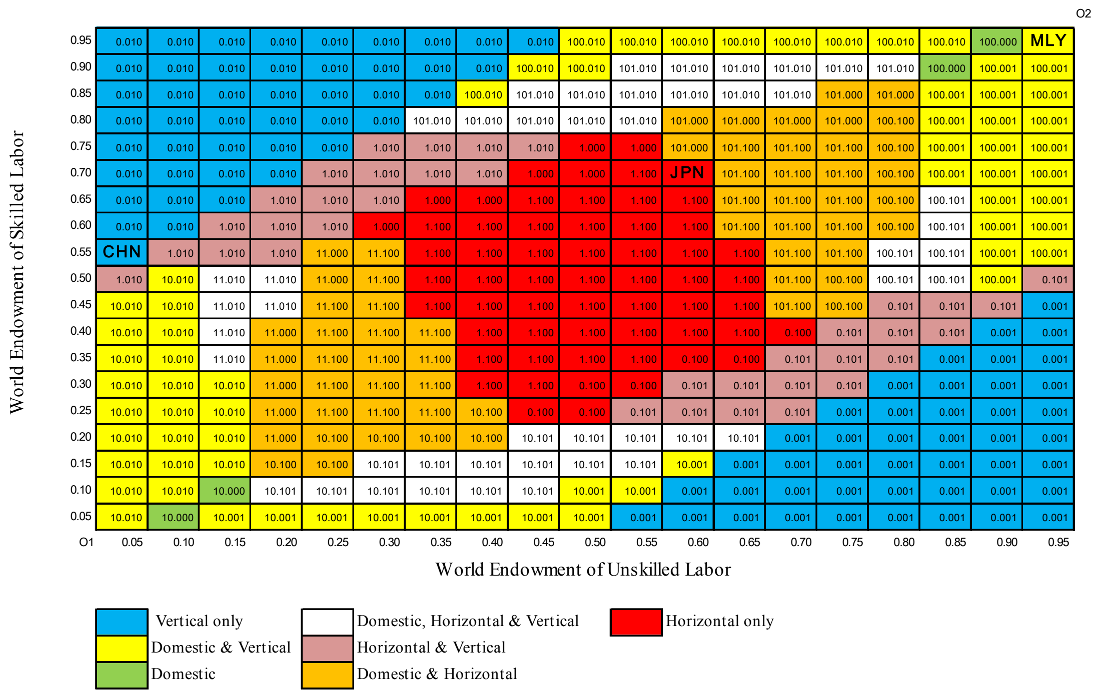


Figure 4 is the equilibrium regime when the trade costs of final goods are lower than the base case. The result shows that multinational firms are going to be vertical firms (type-v).

Figure 4 Equilibrium regime when the trade costs of final goods are lowered

$$(\tau^X = 0.01, \tau^M = 0.2)$$

O2

World Endowment of Skilled Labor	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.95	100.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	100.010	100.010	100.010	100.010	100.010	100.010	100.010	100.010	MLY
0.90	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	10.010	100.010	100.010	100.010	100.010	100.010	100.010	100.001	100.000	100.001
0.85	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	10.010	110.010	100.010	100.010	100.010	100.010	100.010	100.010	100.001	100.001	100.001
0.80	0.010	0.010	0.010	0.010	0.010	0.010	0.010	10.010	10.010	10.010	110.010	100.010	100.010	100.010	100.010	100.001	100.001	100.001	100.001
0.75	0.010	0.010	0.010	0.010	0.010	0.010	10.010	10.010	10.010	10.010	110.010	100.010	100.010	100.010	100.001	100.001	100.001	100.001	100.001
0.70	0.010	0.010	0.010	0.010	0.010	10.010	10.010	10.010	10.010	10.010	110.010	JPN	100.010	100.001	100.001	100.001	100.001	100.001	100.001
0.65	0.010	0.010	0.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.010	110.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.60	0.010	0.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.55	CHN	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.50	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.000	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.45	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	0.001
0.40	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	0.001	0.001
0.35	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.001	110.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	0.001	0.001	0.001
0.30	10.010	10.010	10.010	10.010	10.010	10.010	10.001	110.001	110.001	100.001	100.001	100.001	100.001	100.001	0.001	0.001	0.001	0.001	0.001
0.25	10.010	10.010	10.010	10.010	10.010	10.001	10.001	10.001	110.001	100.001	100.001	100.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001
0.20	10.010	10.010	10.010	10.010	10.001	10.001	10.001	10.001	110.001	100.001	100.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.15	10.010	10.010	10.010	10.001	10.001	10.001	10.001	10.001	10.001	110.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.10	10.010	10.000	10.001	10.001	10.001	10.001	10.001	10.001	10.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.05	10.000	10.001	10.001	10.001	10.001	10.001	10.001	10.001	10.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	10.001
O1	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95

World Endowment of Unskilled Labor

Vertical only
Domestic & Vertical
Domestic
Domestic, Horizontal & Vertical
Horizontal & Vertical
Domestic & Horizontal
Horizontal only

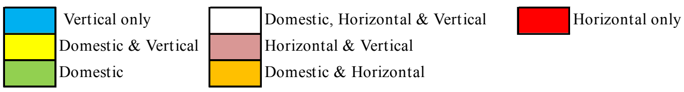
Finally, the case where the trade costs of both goods are lowered is examined. The result shows that Figure 4 and Figure 5 are almost the same. This means that τ_X is crucial for determining the operational pattern of firms.

Figure 5 Equilibrium when trade costs of both goods are lowered

$$(\tau^X = 0.01, \tau^M = 0.01)$$

World Endowment of Skilled Labor (rows) / World Endowment of Unskilled Labor (columns); O1 (origin, bottom left), O2 (top right)

	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.95	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	100.010	100.010	100.010	100.010	100.010	100.010	100.010	100.010	MLY
0.90	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	100.010	100.010	100.010	100.010	100.010	100.010	100.010	100.001	100.001	100.001
0.85	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	10.010	100.010	100.010	100.010	100.010	100.010	100.010	100.010	100.001	100.001	100.001
0.80	0.010	0.010	0.010	0.010	0.010	0.010	0.010	10.010	10.010	100.010	100.010	100.010	100.010	100.010	100.010	100.001	100.001	100.001	100.001
0.75	0.010	0.010	0.010	0.010	0.010	0.010	10.010	10.010	10.010	110.010	100.010	100.010	100.010	100.010	100.001	100.001	100.001	100.001	100.001
0.70	0.010	0.010	0.010	0.010	0.010	10.010	10.010	10.010	10.010	10.010	100.010	JPN	100.010	100.001	100.001	100.001	100.001	100.001	100.001
0.65	0.010	0.010	0.010	0.010	10.010	10.010	10.010	10.010	10.010	10.010	110.010	100.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.60	0.010	0.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.55	CHN	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.50	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.000	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001
0.45	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	0.001
0.40	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.010	110.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	100.001	0.001	0.001
0.35	10.010	10.010	10.010	10.010	10.010	10.010	10.010	10.001	110.001	100.001	100.001	100.001	100.001	100.001	100.001	0.001	0.001	0.001	0.001
0.30	10.010	10.010	10.010	10.010	10.010	10.010	10.001	10.001	10.001	100.001	100.001	100.001	100.001	100.001	0.001	0.001	0.001	0.001	0.001
0.25	10.010	10.010	10.010	10.010	10.010	10.001	10.001	10.001	10.001	110.001	100.001	100.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001
0.20	10.010	10.010	10.010	10.010	10.001	10.001	10.001	10.001	10.001	10.001	100.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.15	10.010	10.010	10.010	10.001	10.001	10.001	10.001	10.001	10.001	10.001	100.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.10	10.010	10.010	10.001	10.001	10.001	10.001	10.001	10.001	10.001	10.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.05	10.010	10.001	10.001	10.001	10.001	10.001	10.001	10.001	10.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001



Based on the above analyses, it can be said that horizontal MNEs are more likely to exist when countries are similar in size and in relative factor endowments. Vertical MNEs are more likely to exist when countries differ in relative factor endowments, and trade costs are positive. From the results of the simulation, lower trade costs of final goods and differences in factor intensity are the conditions for attracting vertical MNEs.

Overall, we have obtained some idea from the simulation analyses, but in order gain a deeper insight, we pick up three cells and examine those. We label these three cells in the box as CHN (China), JPN (Japan), and MLY (Malaysia) according to their factor endowments relative to the United States. Note that the factor endowment of the United States is measured from the southwest corner, while that of the other labeled countries is measured from the northeast corner. The location of the labeled country is determined by the share of factor endowment. For example, the location of Japan is upper-right from the center, since Japan has a 40% share of unskilled labor and 30% share of skilled labor if there are only two countries, Japan and the United States, in the world. The locations of the other countries are determined in the same manner.

The value for the case of the US and China (CHN) in Figure 2 is 10.010, and the number means that type-v1 and -d2 firms arise in the equilibrium regime. Other than Figure 2, only type-v1 firms are active with lowering transportation cost for final or intermediate goods. In other words, type-d1 firms are crowded out by type-v1 firms. Type-v1 firms install their affiliates in China aiming at abundant unskilled labor. If a country like China with abundant unskilled labor wants to keep domestic firms (type d*j*), then the transportation cost for both goods needs to be high.

The value for the JPN cell in Figure 2 is 101.1, which means that there are type-d1, -h1, and -h2 firms. Type-d1 firms are crowded out and only type-h firms arise if the transportation cost for intermediate goods is lowered. On the other hand, if the transportation cost for final goods is lowered, then type-d1, -v1, and -d2 firms would be active.

MLY is located at the top-right corner of the figure and represents a small economy. From the fact that type-v1 firms arise only in Figures 3 and 5, it seems that lowering the trade costs of intermediate goods is crucial to whether the country can host affiliates for a small economy.

5. Concluding Remarks and Further Extension

In this paper, we examined which type of firm arises as a function of a country's characteristics by extending the model presented by Zhang and Markusen (1999) using numerical general equilibrium analysis. The simulation results revealed that horizontal MNEs are more likely to exist when countries are similar in size and in relative factor endowments. Vertical MNEs are more likely to exist when countries differ in relative factor endowments, and trade costs are positive. Based on the results of the simulation, lower trade costs of final goods and differences in factor intensity are the condition for attracting vertical MNEs.

This study is not a pioneering work in the field of FDI theory, but will function as a bridge between traditional FDI under a two-region setting and the more recent FDI under a three-region framework.

So far, FDI theories under the three-region framework assume that only the home country produces intermediate goods. However, nowadays host countries produce some kinds of intermediate goods. The production networks for hard disk drives (HDD) are a good example to understand the present production networks among North and South, such as in the Asian region. We can observe a clear division of production of HDDs: relatively factor-intensive countries such as Japan and Taiwan produce parts and components, while unskilled-labor-intensive countries such as China assemble components. Also, even within the parts and components production of HDDs, sophisticated division of production has been occurring. In the production of HDDs, there are several key parts and components which greatly impact the quality of the final product and require much R&D investment, such as GMR heads, media, and spindle motors. The media, for example, requires high technology as a whole, but the levels of technology required in each stage are slightly different from each other. As the first stage to produce media, a blank is cut from the material, and then the substrate is obtained by polishing the blank. Finally, the media is produced by spraying a magnetic layer onto the substrate. In the Asian region, the first step is done mainly in Japan, then the second stage in Malaysia and the final stage in China. Hummels and Uchida (2010) calculated this phenomenon quantitatively by using a vertical specialization index and showed that most of the countries in the Asian region except China engage more in parts and components production than in assembly in the vertical specialization chain. Ozeki (2010) also showed, using Surveys on Overseas' Business Activities of Japanese multinational firms, that Japan's affiliates procure more than 50% of intermediate inputs locally, followed by home (Japan) at 30%.

In view of the present situation in East Asia, in a future study we will extend Ekholm et al. (2007) to treat the procurement of intermediate goods from the host country, based on the model and program developed in this paper.

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Appendix: Program Code

```
$TITLE A 2-COUNTRY 2-SECTOR OLIGOPOLY MODEL OF HORIZONTAL AND VERTICAL MNES

SETS
   I    /A,B/
   C     /1*19/;

ALIAS (I,J,II),(C,R,RR);

PARAMETERS
   SE(I)
/A   46
 B   46/

   LE(I)
/A 154
 B 154/

   SY0(I)
/A   20
 B   20/

   LY0(I)
/A   80
 B   80/

   XC0(I)
/A   80
 B   80/

   XCV0(I)
/A 100
 B 100/;

SCALARS
```

```
     THETA_S /1/
     THETA_M /.125/
     THETA_L /.875/
     TAU_X     /.20/
     TAU_M     /.01/
     G            /.80/
     UP          /0/;

PARAMETER
     FD(I)
/A 4.4
  B 4.4/;

TABLE
     FH(I,J)
        A     B
  A 4.8 1.6
  B 1.6 4.8;

TABLE
     FV(I,J)
        A     B
  A 3.6 1.2
  B 1.2 3.6;

PARAMETERS
     GSE,GLE,Y0(I),PX0(I),ALPHA(I),PHI(I),BETA(I),PSI(I);
GSE= SUM(I,SE(I));
GLE= SUM(I,LE(I));
Y0(I)= SY0(I)+LY0(I);
PX0(I)= XCV0(I)/XC0(I);
ALPHA(I)= SY0(I)/Y0(I);
PHI(I)= Y0(I)/(SY0(I)**ALPHA(I)*LY0(I)**(1-ALPHA(I)));
BETA(I)= XCV0(I)/(XCV0(I)+Y0(I));
PSI(I)= (XC0(I)+Y0(I))/(XC0(I)**BETA(I)*Y0(I)**(1-BETA(I)));
```

```
OPTION DECIMALS= 8;
DISPLAY
     GSE,GLE,Y0,PX0,ALPHA,PHI,BETA,PSI;

POSITIVE VARIABLES
     YC(I),Y(I),SY(I),LY(I),MU(I),XC(I),XD(I,J),XH(I,J),XV(I,J),
     ND(I),NH(I),NV(I),LAMBDA(I),PS(I),PL(I),PY,PX(I);

EQUATIONS
     EQMU(I),EQY(I),EQSY(I),EQLY(I),EQXD(I,J),EQXH(I,J),EQXV(I,J),
     EQND(I),EQNH(I),EQNV(I),EQLAMBDA(I),EQXC(I),EQYC(I),EQPS(I),EQPL(I),
*     EQPY,
     EQPX(I);

EQMU(I)..
     PHI(I)*SY(I)**ALPHA(I)*LY(I)**(1-ALPHA(I))-Y(I) =G= 0;
EQY(I)..
     MU(I)-PY =G= 0;
EQSY(I)..
     PS(I)-MU(I)*ALPHA(I)*PHI(I)*(SY(I)/LY(I))**(ALPHA(I)-1) =G= 0;
EQLY(I)..
     PL(I)-MU(I)*(1-ALPHA(I))*PHI(I)*(SY(I)/LY(I))**ALPHA(I) =G= 0;
EQXD(I,J)..
     XD(I,J)-BETA(J)*(PS(J)*SE(J)+PL(J)*LE(J))
     *(PX(J)-PS(I)*THETA_S*THETA_M-PL(I)*(THETA_L+TAU_X$(ORD(I) NE ORD(J))))
     /PX(J)**2 =G= 0;
EQXH(I,J)..
     XH(I,J)-BETA(J)*(PS(J)*SE(J)+PL(J)*LE(J))
     *(PX(J)-(PS(I)*THETA_S+PL(I)*TAU_M$(ORD(I) NE ORD(J)))*THETA_M
     -PL(J)*THETA_L)/PX(J)**2 =G= 0;
EQXV(I,J)..
     XV(I,J)-BETA(J)*(PS(J)*SE(J)+PL(J)*LE(J))
     *(PX(J)-(PS(I)*THETA_S+PL(I)*TAU_M)*THETA_M
     -SUM(II$(ORD(I) NE ORD(II)),PL(II))*(THETA_L+TAU_X$(ORD(I) EQ ORD(J))))
     /PX(J)**2 =G= 0;
EQND(I)..
```

```
    PS(I)*FD(I)+PL(I)*G
    -SUM(J,(PX(J)-PS(I)*THETA_S*THETA_M
    -PL(I)*(THETA_L+TAU_X$(ORD(I) NE ORD(J))))
    *XD(I,J)) =G= 0;
EQNH(I)..
    SUM(J,PS(J)*FH(I,J)+PL(J)*G)
    -SUM(J,(PX(J)-(PS(I)*THETA_S+PL(I)*TAU_M$(ORD(I) NE ORD(J)))*THETA_M
    -PL(J)*THETA_L)
    *XH(I,J)) =G= 0;
EQNV(I)..
    SUM(J,PS(J)*FV(I,J))+SUM(J$(ORD(I) NE ORD(J)),PL(J))*G
    -SUM(J,(PX(J)-(PS(I)*THETA_S+PL(I)*TAU_M)*THETA_M
    -SUM(II$(ORD(I) NE ORD(II)),PL(II))*(THETA_L+TAU_X$(ORD(I) EQ ORD(J))))
    *XV(I,J)) =G= 0;
EQLAMBDA(I)..
    PS(I)*SE(I)+PL(I)*LE(I)-PX(I)*XC(I)-PY*YC(I) =G= 0;
EQXC(I)..
    LAMBDA(I)*PX(I)-BETA(I)*PSI(I)*(XC(I)/YC(I))**(BETA(I)-1) =G= 0;
EQYC(I)..
    LAMBDA(I)*PY-(1-BETA(I))*PSI(I)*(XC(I)/YC(I))**BETA(I) =G= 0;
EQPS(I)..
    SE(I)-SY(I)
    -ND(I)*(THETA_S*THETA_M*SUM(J,XD(I,J))+FD(I))
    -NH(I)*THETA_S*THETA_M*SUM(J,XH(I,J))-SUM(J,NH(J)*FH(J,I))
    -NV(I)*THETA_S*THETA_M*SUM(J,XV(I,J))-SUM(J,NV(J)*FV(J,I)) =G= 0;
EQPL(I)..
    LE(I)-LY(I)
    -ND(I)*(SUM(J,(THETA_L+TAU_X$(ORD(I) NE ORD(J)))*XD(I,J))+G)
    -NH(I)*TAU_M*THETA_M*SUM(J$(ORD(I) NE ORD(J)),XH(I,J))
    -SUM(J,NH(J)*(THETA_L*XH(J,I)+G))
    -NV(I)*TAU_M*THETA_M*SUM(J,XV(I,J))
    -SUM(J,NV(J)*(SUM(II,(THETA_L+TAU_X$(ORD(J) EQ ORD(II)))
    *XV(J,II))+G)$(ORD(I) NE ORD(J))) =G= 0;
*EQPY..
*   SUM(I,Y(I))-SUM(I,YC(I)) =G= 0;
EQPX(I)..
```

```
     SUM(J,ND(J)*XD(J,I)+NH(J)*XH(J,I)+NV(J)*XV(J,I))-XC(I) =G= 0;

YC.LO(I)= 1.00E-10; Y.LO(I)= 1.00E-10; SY.LO(I)= 1.00E-10;
LY.LO(I)= 1.00E-10; MU.LO(I)= 1.00E-10; XC.LO(I)= 1.00E-10;
XD.LO(I,J)= 1.00E-10; XH.LO(I,J)= 1.00E-10; XV.LO(I,J)= 1.00E-10;
ND.LO(I)= 1.00E-10; NH.LO(I)= 1.00E-10; NV.LO(I)= 1.00E-10;
LAMBDA.LO(I)= 1.00E-10; PS.LO(I)= 1.00E-10; PL.LO(I)= 1.00E-10;
PX.LO(I)= 1.00E-10;

YC.L(I)= Y0(I);
Y.L(I)= Y0(I);
SY.L(I)= SY0(I);
LY.L(I)= LY0(I);
MU.L(I)= 1;
XC.L(I)= XC0(I);
XD.L(I,J)= 0;
XH.L(I,J)= XC0(I)/(2.5*2);
XV.L(I,J)= 0;
ND.L(I)= 0;
NH.L(I)= 2.5;
NV.L(I)= 0;
LAMBDA.L(I)= (XC0(I)+Y0(I))/(LE(I)+SE(I));
PS.L(I)= 1;
PL.L(I)= 1;
PX.L(I)= PX0(I);

PY.FX= 1;

MODEL MNE_OHVM
     /EQMU.MU,EQY.Y,EQSY.SY,EQLY.LY,EQXD.XD,EQXH.XH,EQXV.XV,
      EQND.ND,EQNH.NH,EQNV.NV,EQLAMBDA.LAMBDA,EQXC.XC,EQYC.YC,
      EQPS.PS,EQPL.PL,
*      EQPY.PY,
      EQPX.PX/;

OPTIONS
```

```
     ITERLIM= 1.00E+08,
     RESLIM= 1.00E+08,
     LIMROW= 0,
     LIMCOL= 0,
     SOLPRINT= OFF,
     MCP= PATH;

PARAMETERS
     NDA(R,C),NDB(R,C),NHA(R,C),NHB(R,C),NVA(R,C),NVB(R,C),REGIME(R,C);

LOOP(C,
LOOP(RR,
LOOP(R$((1-UP)$(ORD(R) EQ ORD(RR))+UP$(ORD(R) EQ CARD(R)-ORD(RR)+1)),

SE(I)= (GSE*(1-.05*ORD(R)))$(ORD(I) EQ 1)
     +(GSE*.05*ORD(R))$(ORD(I) EQ 2);
LE(I)= (GLE*.05*ORD(C))$(ORD(I) EQ 1)
     +(GLE*(1-.05*ORD(C)))$(ORD(I) EQ 2);

YC.L(I)= Y0(I);
Y.L(I)= Y0(I);
SY.L(I)= SY0(I);
LY.L(I)= LY0(I);
MU.L(I)= 1;
XC.L(I)= XC0(I);
XD.L(I,J)= 0;
XH.L(I,J)= XC0(I)/(2.5*2);
XV.L(I,J)= 0;
ND.L(I)= 0;
NH.L(I)= 2.5;
NV.L(I)= 0;
LAMBDA.L(I)= (XC0(I)+Y0(I))/(LE(I)+SE(I));
PS.L(I)= 1;
PL.L(I)= 1;
PX.L(I)= PX0(I);
```

```
SOLVE MNE_OHVM USING MCP;

NDA(R,C)$(ND.L("A") GE 1.00E-02)= 100;
NDB(R,C)$(ND.L("B") GE 1.00E-02)= 10;
NHA(R,C)$(NH.L("A") GE 1.00E-02)= 1;
NHB(R,C)$(NH.L("B") GE 1.00E-02)= .1;
NVA(R,C)$(NV.L("A") GE 1.00E-02)= .01;
NVB(R,C)$(NV.L("B") GE 1.00E-02)= .001;
REGIME(R,C)= NDA(R,C)+NDB(R,C)+NHA(R,C)+NHB(R,C)+NVA(R,C)+NVB(R,C);

);
);
IF(UP, UP=0; ELSE UP=1;);
);

DISPLAY
    REGIME;
```