On the sustainability of a monocentric city : lower transport costs from new transport facilities

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On the sustainability of a monocentric city: Lower transport costs from new transport facilities

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Abstract

This paper proposes a general equilibrium model of a monocentric city based on Fujita and Krugman (1995). Two rates of transport costs per distance and for the same good are introduced. The model assumes that lower transport costs are available at a few points on a line. These lower costs represent new transport facilities, such as high-speed motorways and railways. Findings is that new transport facilities connecting the city and hinterlands strengthen the lock-in effects, which describes whether a city remains where it is forever after being created. Furthermore, the effect intensifies with better agricultural technologies and a larger population in the economy. The relationship between indirect utility and population size has an inverted U-shape, even if new transport facilities are used. However, the population size that maximizes indirect utility is smaller than that found in Fujita and Krugman (1995).

Keywords: Urban system, Monopolistic competition, Transport facilities

JEL classification: R12, F12, O14

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1 Introduction

New transport facilities such as high-speed motorway and railways connect points on a continuous space, providing a better transport service than in the case of ordinary transportation. Users decide whether to use a new transport facility based on its quality and location, including the entry and exist points of the high-speed motorways and stations. As the result, there are multiple transportation routes. For example, users' goods may pass their final destination on trains or high-spead motorways, but then return to the same route to reach their destination using local streets after exiting the high-speed motorway or train station. Thus, by introducing new transport facilities, geographic distances can differ from route distances, based on the lowest transport costs.

Building railroads or highways is regarded as a policy measure to change the spread of economic activity. The location of new transport facilities changes location advantages. Routes that do not run directly between an origin and destination may be chosen because they provide a better (e.g., quicker and/or cheaper) transport service. Thus, an area around a new transport facility may enjoy lower transport costs than those areas between two points of new transport facilities do.

After industrial agglomeration occurs, policymakers may choose to support rural areas or to narrow the gap between the core region and the periphery. This paper examines such cases. We clarify the impact of new transport facilities that connect two points of hinterlands or connect the city and its hinterland, as well as the impact of these facilities on the relocation of industries. As a result, we determine which options work best in certain situations.

New transport facilities mean cheaper transport routes are chosen. Fujita and Mori (1996) introduced two port cities in an urban model of new economic geography. This paper is similar to that of Fujita and Mori (1996). In Fujita and Mori (1996), port cities connect a point on a river bank with the opposite side of the river bank. However, Fujita and Mori (1996) uses only one transport cost per distance for a product, which makes clear the impact of hub effect. In this paper, two rates of transport costs per distance for the same good are introduced. Thus, Fujita and Mori (1996) consider that a hub, such as a port city, provides a gateway to additional demand. Here, the proposed model considers new transport facilities with lower transport costs that provide better access to the city or to its hinterland.

Our purpose is to clarify how new transport facilities that connect points on a line, offering lower transport costs, affect sustainability of a monocentric city. Under a monocentric equilibrium, new transport facilities make a qualitative difference to the city. Thus, we examine two cases: (1) two points with new transport facilities in the hinterland are not connected to the city by new transport facilities; and (2) two points with new transport facilities in the hinterland are connected to the city by new transport facilities. For our purpose, we simply add lower transport costs between the points on a line to Fujita and Krugman (1995) and Fujita, Krugman and Venables (1999; Chapter 9). Because we focus on the relative location of the city, as in Fujita and Krugman (1995), rather than the absolute location, as in Behrens (2007), we also examine the size and the shape of a hinterland. With regard to the emergence of new city, we use a numerical analysis to examine whether it is profitable for a manufacturing firm to deviate from the monocentric city.

The remainder of this paper is organized as follows. Section 2 introduces the proposed model. Then, the case where new transport facilities connect two points in the hinterland is analyzed in Section 3. The case where new transport facilities connect the city and two points the hinterland is analyzed in Section 4. Lastly, Section 5 concludes the paper.

2 The model

The underlying structure of this paper's model is closely related to that of the models in Fujita and Krugman (1995), Fujita and Mori (1996), Fujita and Mori (1997), Mori (1997) and Fujita, Krugman and Venables (1999). Hence, we only briefly describe its formal structure.

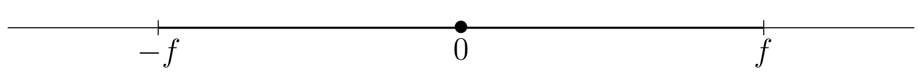


Figure 1: Monocentric spatial structure

Imagine a long, narrow economy, in which the domain is represented by a boundless, one dimensional location space, X, along which lies land of homogeneous quality, with one unit of land per unit distance. The economy has an agricultural sector and a manufacturing sector, which supply an agricultural good and a continuum of differentiated manufactured goods, respectively, to consumers (see Figure 1). The agricultural good production is subject to Leontief technology, using labor and land in a fixed proportion. Land use in the agricultural sector implies that it is necessarily dispersed in space, $[-f, 0) \cup (0, f] \in X$. The production activity of the manufacturing industry exhibits scale economies, using labor only. We assume that the manufacturing industries are concentrated at a point (a city), $0 \in X$.

The economy has a continuum of homogeneous workers with a given size, N. Each worker is endowed with a unit of labor, and is free to choose both the location and the sector. Consumers consist of workers and landlords. All landlords are attached to their land, and consume the entire revenue generated from their land.

There are two types of transport systems: (1) traditional transport systems can ship an agricultural good or manufactured goods between any locations; (2) new transport facilities can ship an agricultural good or manufactured goods between given fixed intervals only such as high-speed motorways or railways. As in Fujita and Krugman (1995), goods melt away at a constant proportional rate per unit distance in any transport system. If one unit of an agricultural good or manufactured goods is shipped a distance d by traditional transportation, $exp(-\tau^A d)$ or $exp(-\tau^M d)$ units arrive. However, if one unit of an agricultural good or manufactured goods is shipped a distance d only via the new transport facilities, $exp(-\tau^{TA} d)$ or $exp(-\tau^{TM} d)$ units arrive. We assume that the rate of melting away is smaller when using the new system: $\tau^A > \tau^{TA}$ and $\tau^M > \tau^{TM}$.

Every consumer shares the same Cobb-Douglas utility tastes:

$$U = A^{1-\mu} M^{\mu}, \quad M = \left[\int_0^n m(i)^{\rho} di \right]^{1/\rho}$$

where $0 < \rho < 1$. The intensity of the preference for varieties in manufactured goods is expressed as ρ and the elasticity of substitution between any two varieties is expressed as $\sigma \equiv 1/(1-\rho)$.

Given nominal wage rates w, and a set of prices, p^A and p^M for each variety i of manufactured goods, the budget constraint of a consumer is $p^A A + \int_0^n p^M(i) m(i) di = w$. Utility maximization subject to this budget constraint yields the following demand functions:

$$A = (1-\mu) w^A / p^A$$
$$m(i) = \mu w^M p^M(i)^{-\sigma} G^{\sigma-1} \quad \text{for} \quad i \in [0, n]$$

where G is the price index for manufactured goods given by

$$G = \left[\int_0^n p^M(i)^{-(\sigma-1)} di \right]^{-1/(\sigma-1)},$$

where w^A is the nominal wage rate of the agricultural sector and w^M is the nominal wage rate of the manufacturing sector. Hence, the indirect utility function is as follows:

$$U = (1-\mu)^{1-\mu} \mu^{\mu} Y G^{-\mu} p^{A^{-(1-\mu)}}.$$

One unit of an agricultural good is produced using c^A units of labor and one unit of land. The production technology used by manufacturers is the same as in typical NEG models (Fujita, Krugman and Venables, 1999), such that producing quantity $q(i)$ of any variety requires labor input l, given by $l = F + c^M q(i)$ where F and c are the fixed and marginal labor requirements, respectively.

We assume that all manufacturing firms are in a single city, located at site $r = 0$. Agricultural production extends around the city. We express the f.o.b. price of an agricultural good at each $r \in X$ as $p^A(r)$, the f.o.b. price of a variety of manufactured goods at r as $p^M(r)$, the nominal wage rate of the agricultural sector at each r as $w^A(r)$ and the nominal wage rate of the manufacturing sector at each r as $w^M(r)$.

We assume that $c^M = \rho$ and $F = \mu/\sigma$ to normalize the units of output $q(i)$ and the size n. Thus, expressing the number of manufacturing workers as L^M, the number of firms and the number of varieties become $n = L^M/\mu$ as Fujita, Krugman and Venables (1999). Furthermore, the optimal f.o.b. price is obtained as $p^M(r) = w^M(r)$. We choose manufactured goods in the city as the numéraire. Thus, we set $p^M(0) = w^M(0) = 1$.

In what follows, we first assume that all manufacturing firms are located within the city. Then, we derive the condition in which no manufacturing firms deviate from the city.

3 New transport facilities connecting two points of hinterlands

In this section, we focus on the case where new transport facilities connect two points within the hinterland or outside the hinterland, but the facilities are not connected to the city. We suppose that the points are located at $\bar{r} \in X$ and $-\bar{r} \in X$. An agricultural good is produced and exported to the city using only traditional transportation outside the city. Thus, expressing the delivered price of an agricultural good at the city as $p^A \equiv p^A(0)$, we obtain the f.o.b. price of an agricultural good at location $r \in X$: $p^A(r) = p^A e^{-\tau^A |r|}$, as in Fujita and Kruguman (1995).

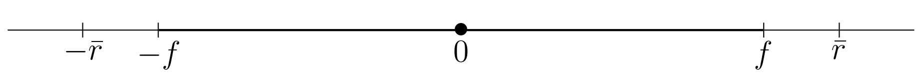


Figure 2: Monocentric spatial structure and the point of new transport facilities outside the hinterland

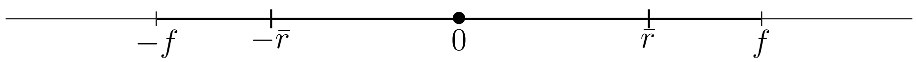


Figure 3: Monocentric spatial structure and the point of new transport facilities within the hinterland

Then, setting the land rents to 0 at the frontier $f \in X$ yields the nominal wage rate of agricultural workers at the frontier: $w^A(f) = p^A e^{-\tau^A f}/c^A$. Because manufactured goods are produced in the city and exported to the hinterland using only traditional transportation, we have the price index $G(r) = (L^M/\mu)^{-1/(\sigma-1)} e^{\tau^M |r|}$, as in Fujita and Krugman (1995). Because an agricultural good is supplied from the hinterland to the city by traditional transportation, the supply of food to the city is $S^A = 2\mu \int_0^f e^{-\tau^A |s|} ds$. Thus, using the full employment condition, which yields the city population $L^M = N - 2c^A f$, the same market clearing condition of an agricultural good in the city is obtained as Fujita and Krugman (1995). The equality between the real wage rates of an agricultural worker at the frontier and the real wage rates of a manufacturing worker in the city yields $p^A = c^A e^{\mu(\tau^A + \tau^M) f}$, which enables us to determine the equilibrium p^A and f with the market clearing condition of an agricultural good in the city.

We use the market potential function of Fujita, Krugman and Venables (1999): $\Omega(r) \equiv \omega^M(r)^\sigma / \omega^A(r)^\sigma$ where $\omega^M(r)$ and $\omega^A(r)$ express the real wage rate of manufacturing workers and that of agricultural workers, respectively, at location r. Since the equality between the real wage rate of agricultural workers at location r and the real wage rate of manufacturing workers in the city yields $w^A(r) = G(r)^\mu p^A(r)^{1-\mu}$, we obtain:

$$\Omega(r) = w^M(r)^\sigma e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]|r|} \quad (1)$$

By introducing new transport facilities, the difference between Fujita and Krugman (1995) and the model in this subsection is only in the nominal wage rate of manufacturing workers $w^M(r)$ at large r, as shown in Appendix A. That is, there is no difference between Fujita and Krugman (1995) and this model in terms of the nominal wage rate of manufacturing workers in the city and around the city. Thus, solving $\partial\Omega(0)/\partial r < 0$, a necessary condition for a monocentric city to be possible becomes $(1-\mu)\tau^A - (1+\rho)\mu\tau^M < 0$, as in Fujita and Krugman (1995).

The difference between Fujita and Krugman (1995) and the model in this subsection becomes clear in the market potential function shown in Figure 4.[1] The slopes of the

[1]Figure 4 is constructed using the following set of parameters: $c^A = 0.5$, $\sigma = 4$, $\mu = 0.5$, $\tau^A = 0.8$,

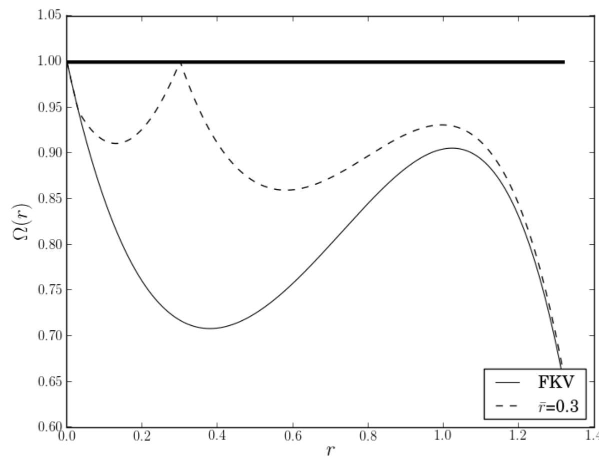


Figure 4: Market potential functions: FKV and the case with new transport facilities connecting two points in the hinterland

market potential functions around the city, which show the necessary condition for the existence of a monocentric city, are the same in both curves in the figure. However, a new city can emerge at the point of new transport facilities, $r = 0.3$, even if a new city does not emerge in the case of Fujita, Krugman and Venables (1999). Note that a market potential function has a cusp around $r = 0.3$, such as the case after the bifurcation in Fujita and Mori (1997), and the cusp implies that the lock-in effect works at that point. In other words, new transport facilities create a shadow around the point where they are located, similarly to the agglomeration shadow around a city center.

When the point of new transport facilities is located between the city and the frontier, the gap between the two market potential functions shows the impact of the new transport facilities. Subtracting the nominal wages without new transport facilities from those with new transport facilities, denoted as $W_1(r)$ and focusing on the area between the city and the location of the new transport facilities, we obtain $\partial W_1(\bar{r})/\partial r > 0$, as in Appendix A. Likewise, subtracting the nominal wages without new transport facilities from those with new transport facilities, denoted as $W_2(r)$, and focusing on the area between the location of the new transport facilities and the frontier, we obtain $\partial W_2(r)/\partial r < 0$, if the distance between the city and the frontier is large enough, otherwise we obtain $\partial W_2(r)/\partial r > 0$, as in Appendix A. If the market potential function without new transport facilities is almost

$\tau^M = 1$, $\tau^{TA} = 0.08$, and $\tau^{TM} = 0.1$. The value of f is calculated as $f = 1.32126227386$ by the numerical verification method.

flat around $\bar{r}$, we can say that the market potential function with the new transport facilities has a cusp at the point where the new transport facilities exist from the result we obtained on $W_1(\bar{r})$ and $W_2(r)$. [2]

Since the new transport facilities do not connect the city and the hinterland in this section, we can focus on using the new transport facilities to transport manufactured goods. We obtain that the nominal wage rates at $r = \bar{r}$ increase as the transport costs of manufactured goods decrease because of the new transport facilities, as shown in Appendix A. Thus, we find that *the market potential function at* $r = \bar{r}$ *shifts upward after lowering the transport costs by means of the new transport facilities, which will support the emergence of a new city at* $r = \bar{r}$*.*

If the location of the new transport facilities shifts slightly towards the frontier, the value of the market potential function on the city side of the area where new transport facilities are used decreases. In contrast, the value of the market potential function on the frontier side of the area increases if the distance between the city and the frontier is large enough as shown in Appendix A. *The value of the market potential function increases as we get closer to the point of the new transport facility, if the distance between the city and the frontier is large enough.* If the distance between the city and the frontier is short, locating between the location of the new transport facilities and the frontier is not as attractive, even if the new transport facility is closer.

Furthermore, when the new transport facilities are located outside the frontier, the value of the market potential function in the area where the new transport facilities are used increases as the distance between the new transport facilities and the frontier decreases, as shown in Appendix A. Thus, we find that *we do not need to have the new transport facilities outside the frontier to increase the value of the market potential function, because the choice to locate the new transport facilities at the frontier provides a higher value of the market potential function.*

[2]Since the initial condition of each location on the emergence of a new city in the case without new transport facilities is not the same in the hinterland, a before and after comparison of the impact of the new transport facilities is not enough to assess whether manufacturing firms relocate or not. In other words, we need to focus on the initial condition and the impact of the new transport facilities at the same time.

4 New transport facilities connecting the city and two points in the hinterland

In this section, we focus on the case where the new transport facilities connect the city and two points in the hinterland. We suppose that the point is located at $\bar{r} \in X$ and $-\bar{r} \in X$.

For simplicity, we suppose that the impact of the new transport facilities is the same on the agricultural good and the manufactured goods, such that $\tau^{TA}/\tau^{A} = \tau^{TM}/\tau^{M}$. To derive the lowest transport costs for manufactured goods sent from the city, solving $-\tau^{A} r = -\tau^{TA}\bar{r} - \tau^{A}(r - \bar{r})$, we obtain:

$$T^{M}_{r0} = \begin{cases} \tau^{M}|r| & \text{if } 0 < |r| < b^{+}_{M} \\ \tau^{TM}\bar{r} + \tau^{M}(\bar{r} - |r|) & \text{if } b^{+}_{M} < |r| < \bar{r} \\ \tau^{TM}\bar{r} + \tau^{M}(|r| - \bar{r}) & \text{if } \bar{r} < |r| \end{cases} \tag{2}$$

where $b^{+}_{M} \equiv \frac{\tau^{TM}/\tau^{M}+1}{2}\bar{r}$.[3] The threshold $b^{+}_{M} \in r$, which shows whether the new transport facilities are used, exists between the city and the location of the new transport facilities. The transport costs of an agricultural good from r to the city are expressed as T^{A}_{r0}. Since $\tau^{TA}/\tau^{A} = \tau^{TM}/\tau^{M}$, we have $b^{+}_{M} = b^{+}_{A}$. Furthermore, we find that the users of the new transport facilities in this case are located in $b^{+}_{M} < |r|$.

Expressing the price of the agricultural good in the city as $p^{A} \equiv p^{A}(0)$ and minimizing the agricultural transport costs, we obtain the agricultural price at r:

$$p^{A}(r) = p^{A} e^{-T^{A}_{r0}} \tag{3}$$

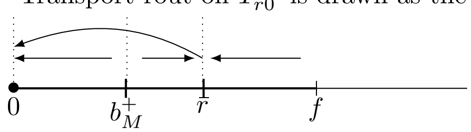
Similarly, we obtain the price index of manufactured goods as follows:

$$G(r) = \left(\frac{L_M}{\mu}\right)^{1/(1-\sigma)} e^{T^{M}_{r0}} \tag{4}$$

Since $\omega^{A}(r) = \omega^{M}(0)$ which means that the real wage rate of agricultural workers currently prevailing at each r is the same as the real wage rate of manufacturing workers in the center, we obtain the nominal wage rate of agricultural workers at each r:

$$w^{A}(r) = e^{\mu T^{M}_{r0} - (1-\mu) T^{A}_{r0}} \tag{5}$$

[3] Transport rout on T^{M}_{r0} is drawn as the following figure:



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Given the location of the closest frontier from the center, f^{min}, which is the smallest r such that rent becomes zero, $R(r) = max\{p^A(r) - c^A w^A(r), 0\} = 0$, the equality of the real wage rate of the frontier farmer and a worker in the city yields the price of an agricultural good in the city center:

$$p^A = \begin{cases} c^A e^{\mu(\tau^M + \tau^A) f^{min}} & \text{if } 0 < f^{min} < b_A^+ \text{: case 1} \\ c^A e^{\mu[(\tau^{TA} + \tau^{TM})\bar{r} + (\tau^A + \tau^M)(f^{min} - \bar{r})]} & \text{if } \bar{r} < f^{min} \text{: case 2} \end{cases} \tag{6}$$

Note that (6) is a strictly increasing function of f^{min}.

Then, given the location of the closest frontier to the city, we can examine the characteristics of land rent. If $\mu\tau^M - (1-\mu)\tau^A > 0$,[4] from (3) and (5), we obtain $p^A(r)' < 0$ and $w^A(r)' < 0$ if $r \in (0, b_A^+) \vee \bar{r} < r$, but $p^A(r)' > 0$ and $w^A(r)' > 0$ if $r \in (b_A^+, \bar{r})$. Thus, we obtain $R(r)' > 0$ if $r \in (b_A^+, \bar{r})$, but $R(r)' < 0$ if $r \in (0, b_A^+) \vee \bar{r} < r$.

Now, we derive the domain of arable lands from the condition of $R(r)' \gtreqless 0$ and $R(r) = 0$, with given f^{min}:

$$r_A = \{[-f^{min}, 0), (0, f^{min}]\} \quad \text{if } 0 < f^{min} \leq r_{s1} \text{ or } r_{s2} \leq f^{min} \tag{7}$$

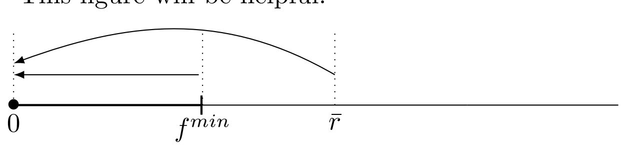
$$r_A = \{[-f^{max}, -f^{mid}], [-f^{min}, 0), (0, f^{min}], [f^{mid}, f^{max}]\} \quad \text{if } r_{s1} < f^{min} < b_A^+ \tag{8}$$

where $f^{mid} \equiv r_{s1} + \bar{r} - f^{min}$, $f^{max} \equiv f^{min} + \bar{r} - r_{s1}$, $r_{s1} \equiv \left(\frac{\tau^{TM} + \tau^{TA}}{\tau^M + \tau^A}\right)\bar{r} = \frac{\tau^{TM}}{\tau^M}\bar{r} = \frac{\tau^{TA}}{\tau^A}\bar{r}$, $r_{s2} \equiv b_A^+ + \bar{r} - r_{s1} = b_A^- + \bar{r}$ and $b_A^- \equiv \bar{r}\frac{1 - \tau^{TA}/\tau^A}{2}$.

In other words, the hinterland region occurs around $r = \bar{r}$ if $r_{s1} \leq f^{min} < b_A^+$, otherwise no hinterland region emerges. Note that the price in (6) becomes the same among the six frontiers that emerge when $r_{s1} \leq f^{min} < b_A^+$, as in (8). Using the conditions in (8), we can explain why the hinterland regions emerge. As a thought experiment, we consider that the location of $\bar{r}$ is far from the city center and then decreasing gradually. The new transport facilities are not used when $r_{s1} > f^{min} \Leftrightarrow \tau^{TA}\bar{r} > \tau^A f^{min}$ because of the significant distance between the city and the location of the new transport facilities. Then, shifting the new transport facilities toward the city, the new transport facilities can be used for the first time when $\tau^{TA}\bar{r} = \tau^A f^{min}$, because the transport costs of sending goods to the city are the same between $\bar{r}$ and f^{min}.[5] Then, locating the new transport facilities much

[4] This condition holds when the necessary condition for the existence of monocentric city in Fujita and Krugman (1995) is satisfied.

[5] This figure will be helpful:



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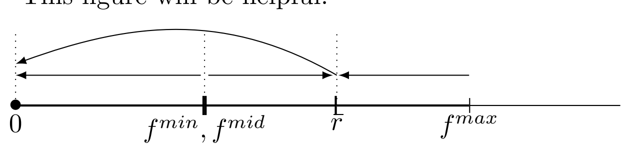
closer to the city, condition $r_{s1} < f^{min}$ is satisfied, which means the additional transport costs in hinterland regions can be covered under $\tau^{TA}\bar{r} < \tau^{A} f^{min}$.

After a hinterland region emerges, condition $f^{min} < b_A^+$, which corresponds to $\tau^A f^{min} < \tau^{TA}\bar{r} + \tau^A(\bar{r} - f^{min})$, is satisfied. The condition implies that the transport costs using the new transport facilities from the frontier located closest to the city among six frontiers are larger than the transport costs when using traditional transportation from the frontier. After locating $\bar{r}$ closer still, the locations f^{min} and f^{mid} provide the same transport costs to the city. Thus, we obtain $f^{min} = f^{mid}$ if $f^{min} = b_A^+$. The transport costs from f^{max} to the city become the same as the transport costs from $f^{min} = f^{mid}$ to the city. [6] Under this condition, f^{max} in the case of hinterland regions changes to f^{min} when the hinterland regions dissolve into a continuous hinterland. This is why the discontinuity of f^{min} in the conditions of (7) and (8) exists.

The shift to a continuous hinterland can be seen from the condition in (7). The condition $r_{s2} = f^{min}$ corresponds to $(2\bar{r} - f^{min})\tau^A = \tau^A(f^{min} - \bar{r}) + \tau^{TA}\bar{r}$. The breaking point at which a continuous hinterland separates into hinterland regions and a remaining area occurs at $2\bar{r} - f^{min} \in X$, which is located between $\bar{r}$ and the city. In other words, the distance between the breaking point and the city is $2\bar{r} - f^{min}$. The distance from the breaking point to the new transport facilities and the distance from the frontier to the new transport facilities are both $f^{min} - \bar{r}$. Thus, the condition $r_{s2} = f^{min}$ means that the transport costs by traditional transportation from the breaking point to the city are the same as: (1) the sum of the transport costs by traditional transportation from the breaking point to the new transport facilities and those using the new transport facilities from the location of the new transport facilities to the city: or (2) the sum of the transport costs by traditional transportation from the frontier to the new transport facilities and those using the new transport facilities from the facilities to the city. If $r_{s2} \leq f^{min}$, the transport costs from the breaking point to the city are lower than the transport costs from the frontier to the city. That is, land rent at the breaking point is positive if land rents at the frontier is 0.

Figure 5[7] shows how to determine the size and shape of arable lands when $r_{s1} <$

[6] This figure will be helpful:



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[7] Figure 5 is constructed using the following parameters: $\bar{r} = 1.6$, $c^A = 0.5$, $\mu = 0.5$, $\tau^A = 0.8$, $\tau^M = 1.0$, $\tau^{TA} = 0.6$, $\tau^{TM} = 0.75$, $N = 4.36$ and $p^A = 1.61405$. The value of f^{min} is calculated as

$f^{min} < b_A^+$. The dotted and bold curves in the figure show $p^A(r)$ and $c^A w^A(r)$, respectively. Both curves kink twice where there is no difference between using the traditional or new transport systems, and where the new transport facilities exist. The vertical line in the figure shows the location of three frontiers in $r > 0$. Thus, the area between the two vertical lines and $p^A(r) > c^A w^A(r)$ can be a hinterland region. The shape of $p^A(r)$ is simply affected by the transport costs of an agricultural good.

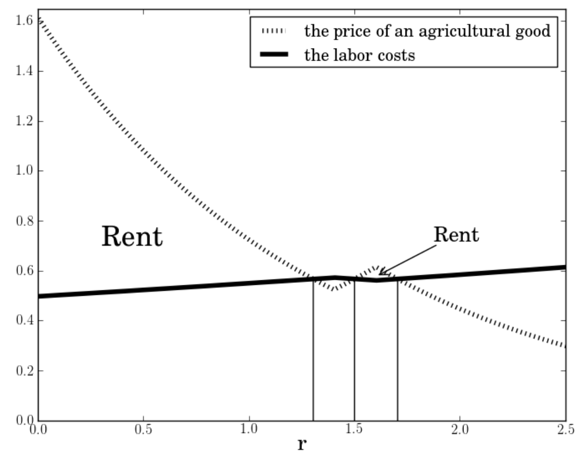

Figure 5: Determining the size and shape of arable lands

From (3), (7) and (8), the supply of agricultural goods to the city becomes:

$$S_A = \begin{cases} \frac{2\mu}{\tau^A}\left(1 - e^{-\tau^A f^{min}}\right) \\ \qquad \text{if } 0 < f^{min} \leq r_{s1} \\ \frac{2\mu}{\tau^A}\left\{1 - e^{-\tau^A f^{min}} + 2e^{-\tau^{TA}\bar{r}}\left[1 - e^{-\tau^A(f^{min} - r_{s1})}\right]\right\} \\ \qquad \text{if } r_{s1} < f^{min} < b_A^+ \\ \frac{2\mu}{\tau^A}\left\{1 - e^{-\tau^A b_A^+} + e^{-\tau^{TA}\bar{r}}\left[1 - e^{-\tau^A(\bar{r} - b_A^+)} + 1 - e^{-\tau^A(f^{min} - \bar{r})}\right]\right\} \\ \qquad \text{if } r_{s2} \leq f^{min} \end{cases} \tag{9}$$

From (7) and (8), the labor in the city becomes:

$$L_M = \begin{cases} N - 2c^A f^{min} & \text{if } 0 < f^{min} \leq r_{s1} \text{ or } r_{s2} \leq f^{min} \\ N - 2c^A\left(3f^{min} - 2r_{s1}\right) & \text{if } r_{s1} < f^{min} < b_A^+ \end{cases} \tag{10}$$

$f^{min} = 1.30210707971$ by the numerical verification method.

Because the demand of an agricultural good in the city is $D_A = (1-\mu)w^M L^M / p^A$ from (9) and (10), the market clearing condition for an agricultural good yields the price of an agricultural good in the city:

$$
p^A = \begin{cases} \frac{(1-\mu)(N-2c^A f^{min})\tau^A}{2\mu\left(1-e^{-\tau^A f^{min}}\right)} & \text{if } 0 < f^{min} \leq r_{s1}\text{: case I} \\ \frac{(1-\mu)\left[N-2c^A\left(3f^{min}-2r_{s1}\right)\right]\tau^A}{2\mu\left\{1-e^{-\tau^A f^{min}}+2e^{-\tau^{TA}\bar{r}}\left[1-e^{-\tau^A(f^{min}-r_{s1})}\right]\right\}} & \text{if } r_{s1} < f^{min} < b_A^+\text{: case II} \\ \frac{(1-\mu)(N-2c^A f^{min})\tau^A}{2\mu\left\{1-e^{-\tau^A b_A^+}+e^{-\tau^{TA}\bar{r}}\left[1-e^{-\tau^A(\bar{r}-b_A^+)}+1-e^{-\tau^A(f^{min}-\bar{r})}\right]\right\}} & \text{if } r_{s2} \leq f^{min}\text{: case III} \end{cases} \tag{11}
$$

Note that (11) is a strictly decreasing function of f^{min}.

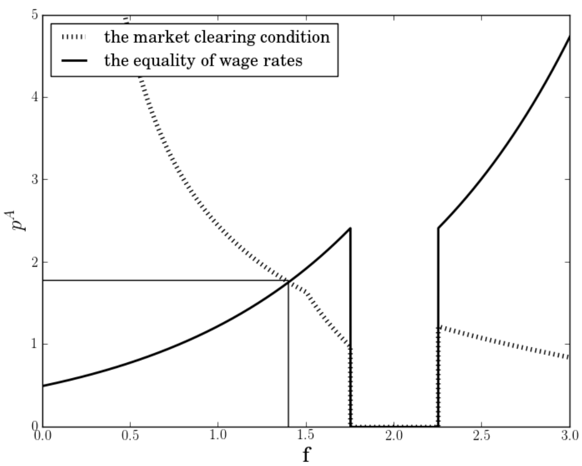

Figure 6: Determining the equilibrium of p^A and f^{min} under Fujita, Krugman and Venables (1999; Chapter 9)

The values of f^{min} and p^A are derived from the equality of p^A in case I of (11) and that in case 1 of (6) if $0 < f^{min} < r_{s1}$, as shown in Figure 6[8]; from case II of (11) and case 1 of (6) if $r_{s1} < f^{min} < b_A^+$, as in Figure 7[9]; and case III of (11) and case 2 of (6) if $r_{s2} \leq f^{min}$, as in Figure 8.[10] In the first case, new transport facilities are not used, because the new transport facilities are too far from the city. In the second case, the

[8]Figure 6 is constructed using the following set of parameters: $c^A = 0.5$, $\mu = 0.5$, $\tau^A = 0.8$, $\tau^M = 1.0$, $\tau^{TA} = 0.6$, $\tau^{TM} = 0.75$, and $\bar{r} = 2$.

[9]Figure 7 is constructed using the following set of parameters: $c^A = 0.5$, $\mu = 0.5$, $\tau^A = 0.8$, $\tau^M = 1.0$, $\tau^{TA} = 0.6$, $\tau^{TM} = 0.75$, and $\bar{r} = 1.6$.

[10]Figure 8 is constructed using the following set of parameters: $c^A = 0.5$, $\mu = 0.5$, $\tau^A = 0.8$, $\tau^M = 1.0$, $\tau^{TA} = 0.6$, $\tau^{TM} = 0.75$, and $\bar{r} = 1$.

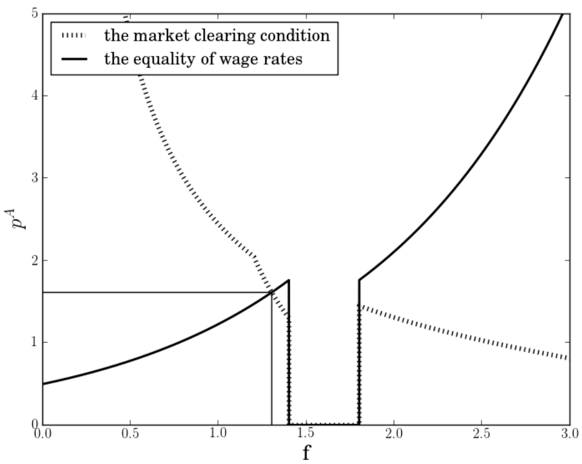

Figure 7: Determining the equilibrium p^A and f^{min} in the case with hinterland regions

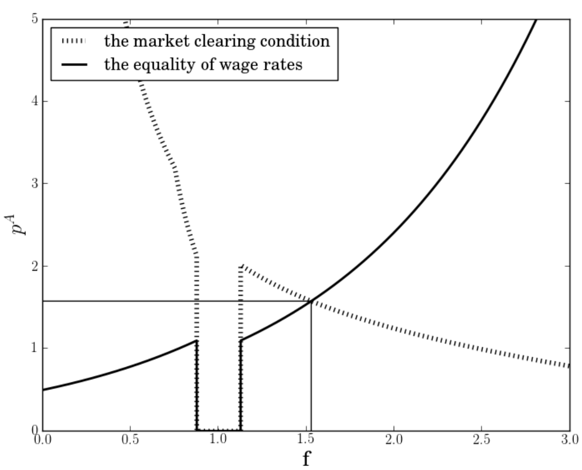

Figure 8: Determining the equilibrium p^A and f^{min} in the case without hinterland regions

new transport facilities are used and the hinterland regions emerge. In the last case, the new transport facilities, but hinterland regions do not emerge because the new transport facilities are close to the city.

The first case can be used to determine the impact of the new transport facilities. Comparing case I of (11) with case II by using a simple calculation, we find that the curve of case II of (11) is lower than the curve in case I of (11). Comparing case I of (11) and case III of (11) by using a simple calculation, we find that the curve of case III of (11) is lower than the curve of case I of (11). Likewise, comparing case 1 of (6) with case 2 of (6), we find that the curve of case 2 of (6) is lower than the curve of case 1 of (6). Thus, *both p^A and f^{min} decrease from the existence of new transport facilities connecting the city and two points in the hinterland when hinterland regions emerge. However, when hinterland regions do not emerge, p^A decrease from the existence of new transport facilities connecting the city and two points in the hinterland*. In the latter case, the impact of the new transport facilities on f^{min} is ambiguous.

Next, we examine the effect of a marginal increase in each parameter on the major variables of the monocentric equilibrium, as shown in Appendix B. Table 1 summarizes the results. The parameters μ, c^A, τ^M, and N affected similarly as in Fujita and Krugman (1995). That is, the impact of τ^A has been changed by introducing the new transport facilities.

The new parameters are $\bar{r}$ and τ^{TA}. The case with hinterland regions is simple. If the transport costs from the new transport facilities decrease, the size of arable land increases, the population in the city decreases, and the price of an agricultural good decreases. If the location of the new transport facilities shifts away from the city, the size of arable land decreases, the population in the city increases, and the price of an agricultural good increases. The case without hinterland regions is not as simple because the value of μ changes the result. If the value of μ is large, a decrease in the transport costs from the new transport facilities or an increase in the distance between the city and the new transport facilities results in a larger hinterland and a smaller population in the city. Otherwise, a decrease in the transport costs from the new transport facilities or an increase in the distance between the city and the new transport facilities has the opposite impact on the size of the hinterland and the city population. However, if the new transport facilities are close to the frontier, more distance between the city and the new transport facilities causes a larger hinterland as in Appendix B. A larger value of μ means that, in contrast to the market clearing condition, the equality of the real wage rates between manufacturing workers in the city and workers on the frontier is relatively important as a determinants

Table 1: Effect of a marginal increase in each parameter on the monocentric equilibrium

	With hinterland regions					No hinterland regions		
	f^{min}	Hinterland regions	f^{min}+Hinterland regions	L^M	p^A	f^{min}	L^M	p^A
$\bar{r}$	$+$	$-$[2]	$-$	$+$	$+$	$\pm$[8]	$\mp$[11]	[14]
μ	$-$	$-$	$-$	$+$	$\pm$[5]	$-$	$+$	$\pm$[15]
c^A	$-$	$-$	$-$	$-$[4]	$\pm$[6]	$-$	$-$[4]	$\pm$[16]
τ^A	$\pm$[1]	$+$[3]	$+$[3]	$-$[3]	$\pm$[7]	$\pm$[9]	$\mp$[12]	$\pm$[17]
τ^M	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$+$
τ^{TA}	$+$	$-$[2]	$-$	$+$	$+$	$\pm$[10]	$\mp$[13]	$+$
N	$+$	$+$	$+$	$+$[4]	$+$	$+$	$+$[4]	$+$

[1]$+$ if $\frac{2\mu c^A f^{min}}{1-\mu} e^{\mu(\tau^M+\tau^A)f^{min}} \left[\mu(1-3e^{-\tau^A f^{min}}+2e^{-\tau^{TA}\bar{r}})+3\right] + 6c^A f^{min} < N,$ otherwise $-$.

[2]$-$ if the necessary condition of the monocentric city holds.

[3]if N is large.

[4]if τ^M is large.

[5]$+$ if $2c^A\left(3\mu f^{min} e^{\mu(\tau^M+\tau^A)f^{min}-\tau^A f^{min}} + 6f^{min} - 3\mu f^{min} - 2c^A \bar{r}\tau^{TA}/\tau^A\right) > N$, otherwise $-$.

[6]$+$ if $\frac{\partial Z}{\partial f^{min}} \frac{2c^A}{(1-\mu)(\tau^M+\tau^A)\tau^A} > N$ where $\frac{\partial Z}{\partial f^{min}} = \mu(\tau^M+\tau^A)e^{\mu(\tau^M+\tau^A)f^{min}}(1+2e^{-\tau^{TA}\bar{r}} - 3e^{-\tau^A f^{min}}) + 3\tau^A e^{\mu(\tau^M+\tau^A)f^{min}} e^{-\tau^A f^{min}} + 3\frac{1-\mu}{\mu}\tau^A > 0$, otherwise $-$.

[7]$+$ if $N > \frac{3\tau^M f^{min} c^A}{\tau^A+\tau^M}\left(\frac{\mu}{1-\mu}e^{\mu(\tau^M+\tau^A)f^{min}}+1\right)$, otherwise $-$.

[8]$+$ if $\frac{\tau^A\left[e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - e^{-\tau^A\bar{r}-\tau^A(f^{min}-\bar{r})}\right] - \tau^{TA}\left[2e^{-\tau^{TA}\bar{r}} - e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]}{(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]} < \mu$, otherwise $-$.

[9]$+$ if $\frac{2\mu^2 c^A(f^{min}-\bar{r})}{1-\mu} e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}$
$\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right] + 2c^A f^{min} < N$, otherwise $-$.

[10]$+$ if $\mu < \frac{2e^{-\tau^{TA}\bar{r}} - e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}}{(1+\tau^M/\tau^A)\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]}$, otherwise $-$.

[11]$-$ if $\frac{\tau^A\left[e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - e^{-\tau^A\bar{r}-\tau^A(f^{min}-\bar{r})}\right] - \tau^{TA}\left[2e^{-\tau^{TA}\bar{r}} - e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]}{(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]} < \mu$, otherwise $-$.

[12]$-$ if $\frac{2\mu^2 c^A(f^{min}-\bar{r})}{1-\mu} e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}$
$\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right] + 2c^A f^{min} < N$, otherwise $+$.

[13]$-$ if $\mu < \frac{2e^{-\tau^T A} - e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}}{(1+\tau^M/\tau^A)\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]}$, otherwise $+$.

[14]The result is ambiguous.

[15]$+$ if $2\mu\tau^A\left[\left(\frac{\tau^{TA}+\tau^{TM}}{\tau^A+\tau^M}\right)\bar{r} + f^{min} - \bar{r}\right]\left[p^A e^{-\tau^A f^{min}+(\tau^A-\tau^{TA})\bar{r}} + c^A\frac{1-\mu}{\mu}\right] > N$, otherwise $-$.

[16]$+$ if $\frac{\mu^2}{1-\mu}\frac{c^A}{\tau^A}p^A + \frac{\mu}{1-\mu}\frac{c^A}{\tau^A+\tau^M}e^{\mu[(\tau^{TA}+\tau^{TM})\bar{r}+(\tau^A+\tau^M)(f^{min}-\bar{r})]} + \frac{c^A}{\tau^A+\tau^M} > N$, otherwise $-$.

[17]$+$ if $N > \frac{2c^A(\tau^A\bar{r}+\tau^M f^{min})}{\tau^A+\tau^M}$, otherwise $-$.

of the locations of the frontiers.

Examining the relationships between real wage rates in manufacturing sector and the location of the frontier, as shown in Appendix C, we find that *the relationship between the population size in the economy and the real wages in the city has an inverted $\mathcal{U}$-shape, under the no-black-hole condition, as in Fujita and Krugman (1995).* In other words, the scale economies of the population N dominate when N is small, but the scale diseconomies of N dominate when N is large.

Furthermore, by using the new transport facilities, the critical population level N, such that $\partial\omega(0)/\partial N = 0$, with or without hinterland regions, becomes smaller than the critical population level of Fujita and Krugman (1995), as explained in Appendix C. In other words, *the new transport facilities connecting the city and the hinterland decrease the size of the population in the economy which maximizes indirect utility.*

Using the lowest transport costs from r to s, $s \neq r$[11]:

$$T_{rs} = \begin{cases} \tau^M|r-s| & \text{if } \bar{r} < r \text{ and } b_M^- < s \\ \tau^{TM}\bar{r} + \tau^M s + \tau^M(r-\bar{r}) & \text{if } \bar{r} < r \text{ and } 0 < s < b_M^- \\ \tau^M(r-\bar{r}) + \tau^{TM}\bar{r} + \tau^M|s| & \text{if } \bar{r} < r \text{ and } -b_M^+ < s < 0 \\ \tau^M(r-\bar{r}) + 2\tau^{TM}\bar{r} + \tau^M(\bar{r}-|s|) & \text{if } \bar{r} < r \text{ and } -\bar{r} < s < -b_M^+ \\ \tau^M(r-\bar{r}) + 2\tau^{TM}\bar{r} + \tau^M(|s|-\bar{r}) & \text{if } \bar{r} < r \text{ and } s < -\bar{r} \\ \tau^M|r-s| & \text{if } b_M^+ < r < \bar{r} \text{ and } r - b_M^+ < s \\ \tau^M(\bar{r}-r) + \tau^{TM}\bar{r} + \tau^M s & \text{if } b_M^+ < r < \bar{r} \text{ and } 0 < s < r - b_M^+ \\ \tau^M(\bar{r}-r) + \tau^{TM}\bar{r} + \tau^M|s| & \text{if } b_M^+ < r < \bar{r} \text{ and } -b_M^+ < s < 0 \\ \tau^M(\bar{r}-r) + 2\tau^{TM}\bar{r} + \tau^M(\bar{r}-|s|) & \text{if } b_M^+ < r < \bar{r} \text{ and } -\bar{r} < s < -b_M^+ \\ \tau^M(\bar{r}-r) + 2\tau^{TM}\bar{r} + \tau^M(|s|-\bar{r}) & \text{if } b_M^+ < r < \bar{r} \text{ and } s < -\bar{r} \\ \tau^M|r-s| & \text{if } b_M^- < r < b_M^+ \text{ and } 0 < s \\ \tau^M r + \tau^M|s| & \text{if } 0 < r < b_M^+ \text{ and} -b_M^+ < s < 0 \\ \tau^M r + \tau^{TM}\bar{r} + \tau^M(\bar{r}+s) & \text{if } 0 < r < b_M^+ \text{ and } -\bar{r} < s < -b_M^+ \\ \tau^M r + \tau^{TM}\bar{r} + \tau^M(-s-\bar{r}) & \text{if } 0 < r < b_M^+ \text{ and } s < -\bar{r} \\ \tau^M|r-s| & \text{if } 0 < r < b_M^- \text{ and } 0 < s < r + b_M^+ \\ \tau^M r + \tau^{TM}\bar{r} + \tau^M|\bar{r}-s| & \text{if } 0 < r < b_M^- \text{ and } r + b_M^+ < s \end{cases} \quad (12)$$

[11]The derivation process of T_{rs} is in Appendix D.

the market potential function can be expressed as:

$$\Omega(r) = w^M(r)^\sigma e^{-\sigma[\mu T_{r0}^M - (1-\mu)T_{r0}^A]} \tag{13}$$

where

$$w^M(r)^\sigma = \left(Y(0)e^{-(\sigma-1)T_{r0}^M} G(0)^{\sigma-1} + \int_{r_A} Y(s)e^{-(\sigma-1)T_{rs}} G(s)^{\sigma-1}\,\mathrm{d}s \right) \tag{14}$$

$$Y(s) = \begin{cases} w^M(s)L^M & \text{if } s = 0 \text{ thus } Y(0) = L^M \\ p^A(s) & \text{if } s \neq 0 \end{cases} \tag{15}$$

More details on the components of $w^M(r)$ can be found in Appendix E.

Solving $\partial\Omega(0)/\partial r < 0$, the necessary condition for sustaining a monocentric equilibrium is derived, as follows:

$$(1-\mu)\tau^A - (\rho+1)\mu\tau^M < 0 \qquad \text{if} \quad 0 < f^{min} \leq r_{s1} \tag{16}$$

$$(1-\mu)\tau^A - (\rho+1)\mu\tau^M - \frac{\rho(1-\mu)\tau^M}{1 + \frac{1-e^{-\tau^A f^{min}}}{2e^{-\tau^{TA}\bar{r}}\left[1-e^{-\tau^A(f^{min}-r_{s1})}\right]}} < 0 \tag{17}$$
$$\text{if} \quad r_{s1} < f^{min} < b_A^+$$

$$(1-\mu)\tau^A - (\rho+1)\mu\tau^M - \frac{\rho(1-\mu)\tau^M}{1 + \frac{1-e^{-\tau^A b_A^+}}{e^{-\tau^{TA}\bar{r}}\left[2-e^{-\tau^A(\bar{r}-b_A^+)}-e^{-\tau^A(f^{min}-\bar{r})}\right]}} < 0 \tag{18}$$
$$\text{if} \quad r_{s2} \leq f^{min}$$

Since the first and the second terms of (17) or (18) are the same as in (16) and the last term of (17) or (18) is negative, we find that *the existence of the new transport facilities connecting the city and a point in the hinterland makes the lock-in effect stronger* [12] *than in an economy without new transport facilities.* Furthermore, it is possible to sustain a monocentric city by connecting the city and a point in the hinterland with new transport facilities, even if a monocentric city is not sustainable without new transport facilities.

In Fujita and Krugman (1995), the first and the second terms are explained as a wage-pull towards the fringe and a demand-pull of city workers towards the center, respectively. The additional new term shows a decrease in demand from the hinterland when a manufacturing firm moves a short distance away from the city.

[12] The strength of lock-in effect is measured as $\Omega(r)'$, as in Fujita, Krugman and Venables (1999, p.164).

In the cases with and without hinterland regions, the new term becomes smaller with a decrease in c^A or an increase in N, which we derive using a , shown in Appendix F. This is because the expansion of the hinterland increases the demand from the hinterland. Thus, *the lock-in effects become stronger with better agricultural technology and a larger population in the economy.* Note that agricultural technology and the population size in the economy do not affect the lock-in effect in the case without new transport facilities, as shown in (16).

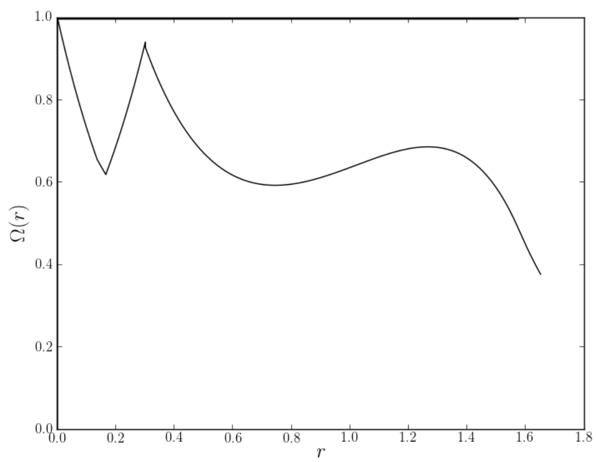

Figure 9: Market potential function when new transport facilities connect the city and the hinterland

Next, we compare the market potential functions for the case when the new transport facilities are located within the city and outside the city and for the case when the facilities are only outside the city. Figure 9 [13] illustrates the market potential function when the new transport facilities lie inside the city. The value of parameters are the same as in Figure 4. However, we obtain different values of f^{min}, which is shown as the bold line on the horizontal axis of Figure 4 and Figure 9 with $\Omega(r) = 1$. Comparing Figure 4 and Figure 9, we find that (1) the hinterlands expand by connecting the city and the hinterland with new transport facilities and (2) the value of the market potential function at $r = 0.2$ in Figure 4 is almost 0.75, but becomes about 0.6 in Figure 9, which suggests that the lock-in effect becomes stronger after connecting the city and the hinterland with

[13]Figure 9 is constructed using the following set of parameters: $c^A = 0.5$, $\sigma = 4$, $\mu = 0.5$, $\tau^A = 0.8$, $\tau^M = 1$, $\tau^{TA} = 0.08$, and $\tau^{TM} = 0.1$ as used for constructing Figure 4. The value of f^{min} is calculated by the numerical verification method.

new transport facilities.

Both Figure 4 and Figure 9 have a cusp in the hinterland. However, Figure 4 shows that monocentric equilibrium may not be sustained if some manufacturing firms move at $\bar{r} = 0.3$, whereas Figure 9 shows that monocentric equilibrium is sustained. Under our parameters, we find that new transport facilities connecting only points in the hinterland support the emergence of a new city more than new transport facilities connecting the city and the hinterland.

5 Conclusion

In this paper, two transport costs for the same goods are introduced to the model of Fujita and Krugman (1995), which is a general equilibrium model of a city as a point on a line. We examined the conditions under which all manufacturing firms agglomerate in a city and the comparative statics of the monocentric equilibrium.

Of the two transport costs, one is lower than the other. Furthermore, the lower transport cost is available only at a few points on a line, whereas the higher transport cost has no restriction on its usage on a line. We suppose that the lower transport costs of sending between points represent new transport facilities. Then, we supposed two cases of connections to the new transport facilities. In the first case, points in the hinterland are on each side of the city and the same distance from the city. Goods are sent from one point to the other. In the second case, we set the lower transport costs to send goods to and from one point in the hinterland to the city, and also to and from the other point in the hinterland to the city.

In the first case, we find that it is better to locate the new transport facilities at the frontier than outside the frontier for the emergence of an additional city. Furthermore, the lower transport costs offered by the new transport facilities shift the market potential function at the point of the new transport facilities upward. Thus, an additional city may emerge at the new transport facilities if the transport costs of the facilities are low enough.

In the second case, we find that the location of the new transport facilities determines the size and the shape of the arable land. If the new transport facilities are not located near the city, hinterland regions may emerge. The lock-in effects from the existence of the new transport facilities become stronger in the second case than in the first case. [14] In the

[14] As IDE-GSM, a multiple-region NEG model with modal choice is used to derive the impact of lowering transport costs numerically. Some regions have transport hubs such as airports and stations in IDE-GSM,

second case, the lock-in effects become stronger with better agricultural technologies and a larger population in the economy. This result may provide an explain on the history of Chicago which became a megalopolis after the railroads (Cronon 1991). As in Fujita and Krugman (1995), we found that an inverse U-shape relationships exists between indirect utility and the population, even if new transport facilities exists. However, the critical level of the population that maximizes indirect utility is smaller in the second case.

Comparing the two cases, the first case supports a rural area more by the emergence of new city containing the manufacturing sector because the connection between the city and the hinterland via the new transport facilities intensifies the lock-in effect. This makes the monocentric city sustainable and impedes the impact of better access at the new transport facilities for the emergence of a new city. A larger population in the economy and labor-saving agricultural technology intensify this tendency. However, new transport facilities are used to send an agricultural good in the second case, even if a firm in the manufacturing sector does not emerge in the hinterland. However, new transport facilities are not used until a firm starts operating around the new transport facility.

As an extension of this paper, it is natural to examine the emergence of new cities or the emergence of a port city because Fujita and Krugman (1995), as are the studies of Fujita and Mori (1996, 1997), Fujita, Krugman and Mori (1999), and Mori (1997). Another way to extend this model is to introduce realistic transport costs, such as increasing returns to scale in transport sector (Mori 2012).

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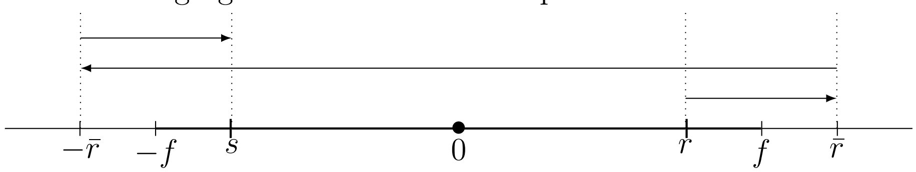
whereas others do not. Thus, our results show that by lowering transport costs, stronger lock-in effect in the city with new transport facilities may emerge, and may sustain firms in the city.

Appendix A Nominal wage rates when two new transport facilities exist

Depending on the relationship between the location of the frontier and the new transport facilities, we have three cases on the nominal wage rate of manufacturing workers.

The four route choices are important. The first and second route is the case when the new transport facilities are located outside the frontier, whereas the third and the last route are the cases when the facilities are located inside the frontier.

The following figure is useful for the explanation of the first and second cases.

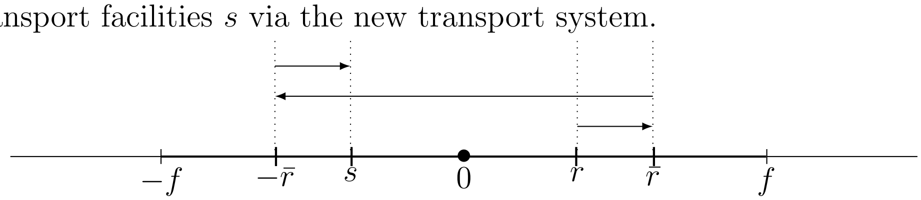


Expressing the location of dispatching goods as $r \in X$ and the destination as $s \in X$, the first case is when transport routes are limited to the direct link between r and s, whereas the second case is when goods are transported from r to $\bar{r}$, from $\bar{r}$ to $-\bar{r}$, and from $-\bar{r}$ to s.

First, the condition when new transport facilities are never used in any r is derived under $f < \bar{r}$ by solving $\tau^M(\bar{r}-r)+2\tau^{TM}\bar{r}+\tau^M(\bar{r}+s) > \tau^M(r-s) \Leftrightarrow s > r-\bar{r}-\bar{r}\tau^{TM}/\tau^M$, where $r > 0$ and $s < 0$. Since $-f < s < 0$, we set $-f > r - \bar{r} - \bar{r}\tau^{TM}/\tau^M \Leftrightarrow r < \bar{r} + \bar{r}\tau^{TM}/\tau^M - f$ and then, since $0 < r < f$, we set $f < \bar{r} + \bar{r}\tau^{TM}/\tau^M - f \Leftrightarrow f < (1+\tau^{TM}/\tau^M)\bar{r}/2$. Thus, new transport facilities are not used when $f < (1+\tau^{TM}/\tau^M)\bar{r}/2$.

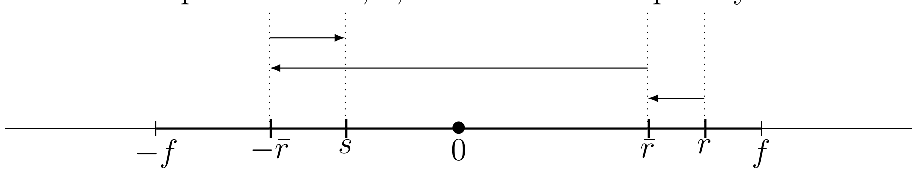
Second, new transport facilities may be used depending on r under $(1+\tau^{TM}/\tau^M)\bar{r}/2 < f < \bar{r}$. Since new transport facilities are used if $s + \bar{r}(1 + \tau^{TM}/\tau^M) < r$ and not used if $s + \bar{r}(1 + \tau^{TM}/\tau^M) > r$, new transport facilities are not used if $0 < r < (1 + \frac{\tau^{TM}}{\tau^M})\bar{r} - f$: otherwise the facilities are used.

Third, the transport route is such that goods are transported from a point inside the location of the new transport facilities, r, to a point inside the location of the new transport facilities s via the new transport system.



Under $f < \bar{r}$, solving $\tau^M(\bar{r}-r)+2\tau^{TM}\bar{r}+\tau^M(\bar{r}+s) < \tau^M(r-s) \Leftrightarrow s < r-\bar{r}-\bar{r}\tau^{TM}/\tau^M$, where $r > 0$ and $s < 0$, setting $\bar{r} < r - \bar{r} - \bar{r}\tau^{TM}/\tau^M$ yields $r > \bar{r}\tau^{TM}/\tau^M$, as when the route in the above figure is used instead of the direct link between r and s.

Last, the following figure explains the case when goods are transported from a point outside the location of the new transport facilities, r, to a point inside the other location of the new transport facilities, s, via the new transport system.



Solving $\tau^M(r-s) > \tau^M(r-\bar{r}) + 2\tau^{TM}\bar{r} + \tau^M(\bar{r}+s)$ yields $s < -\bar{r}\tau^{TM}/\tau^M$, as when the route in the above figure is used instead of the direct link between r and s.

The first case is when $0 < f < (1+\frac{\tau^{TM}}{\tau^M})\bar{r}/2$. In other words, the location of the new transport facilities is far from the frontier. Since nobody use the new transport facilities, we obtain the same nominal wage rate for manufacturing workers as in Fujita and Krugman (1995), as follows:

$$\left[w^M(r)\right]^\sigma = Y(0)e^{-(\sigma-1)\tau^M r}G(0)^{\sigma-1} + \int_{-f}^{0} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds$$
$$+ \int_{0}^{r} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds + \int_{r}^{f} Y(s)e^{-(\sigma-1)\tau^M(s-r)}G(s)^{\sigma-1}ds \quad (19)$$

The second case is when $(1+\frac{\tau^{TM}}{\tau^M})\bar{r}/2 < f < \bar{r}$. Here, the new transport facilities are located outside the frontiers, but they are used if manufacturing firms are located near the frontier. Otherwise, manufacturing firms use only traditional transportation. These manufacturing firms increase when the transport costs of the new transport facilities become much lower than the costs of traditional transport. If $0 < r < (1+\frac{\tau^{TM}}{\tau^M})\bar{r} - f$, the nominal wage rate of manufacturing workers becomes:

$$\left[w^M(r)\right]^\sigma = Y(0)e^{-(\sigma-1)\tau^M r}G(0)^{\sigma-1} + \int_{-f}^{0} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds$$
$$+ \int_{0}^{r} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds + \int_{r}^{f} Y(s)e^{-(\sigma-1)\tau^M(s-r)}G(s)^{\sigma-1}ds$$

whereas, if $(1+\frac{\tau^{TM}}{\tau^{M}})\bar{r}-f<r<f$, we obtain

$$\begin{aligned}\left[w^M(r)\right]^\sigma &= Y(0)e^{-(\sigma-1)\tau^M r}G(0)^{\sigma-1} \\ &\quad + \int_{-f}^{r-\bar{r}(1+\tau^{TM}/\tau^M)} Y(s)e^{-(\sigma-1)[\tau^M(\bar{r}-r)+\tau^{TM}2\bar{r}+\tau^M(\bar{r}+s)]}G(s)^{\sigma-1}ds \\ &\quad + \int_{r-\bar{r}(1+\tau^{TM}/\tau^M)}^{r} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds \\ &\quad + \int_{r}^{f} Y(s)e^{-(\sigma-1)\tau^M(s-r)}G(s)^{\sigma-1}ds \end{aligned} \tag{20}$$

The third and fourth cases are when $\bar{r}<f$, which means that the new transport facilities are located between the city and the frontier. Depending on the location of the manufacturing firms, we have three types of firms: (1) firms close to the city use traditional transportation; (2) firms near the city and near the new transport facilities and (3) firms near the frontier and near the new transport facilities use both traditional and the new transport facilities. The last two types differ in terms of the direction of transportation. If $0<r<\frac{\tau^{TM}}{\tau^{M}}\bar{r}$, the nominal wage rate becomes:

$$\begin{aligned}\left[w^M(r)\right]^\sigma &= Y(0)e^{-(\sigma-1)\tau^M r}G(0)^{\sigma-1} + \int_{-f}^{0} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds \\ &\quad + \int_{0}^{r} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds + \int_{r}^{f} Y(s)e^{-(\sigma-1)\tau^M(s-r)}G(s)^{\sigma-1}ds\end{aligned}$$

If $\frac{\tau^{TM}}{\tau^{M}}\bar{r}<r<\bar{r}$, the nominal wage rate becomes:

$$\begin{aligned}\left[w^M(r)\right]^\sigma &= Y(0)e^{-(\sigma-1)\tau^M r}G(0)^{\sigma-1} + \int_{-f}^{-\bar{r}} Y(s)e^{-(\sigma-1)[\tau^M(\bar{r}-r)+\tau^{TM}2\bar{r}+\tau^M(-s-\bar{r})]}G(s)^{\sigma-1}ds \\ &\quad + \int_{-\bar{r}}^{r-\bar{r}(1+\tau^{TM}/\tau^M)} Y(s)e^{-(\sigma-1)[\tau^M(\bar{r}-r)+\tau^{TM}2\bar{r}+\tau^M(\bar{r}+s)]}G(s)^{\sigma-1}ds \\ &\quad + \int_{r-\bar{r}(1+\tau^{TM}/\tau^M)}^{r} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds \\ &\quad + \int_{r}^{f} Y(s)e^{-(\sigma-1)\tau^M(s-r)}G(s)^{\sigma-1}ds\end{aligned} \tag{21}$$

If $\bar{r} < r < f$, the nominal wage rate becomes:

$$\begin{aligned}
\left[w^M(r)\right]^\sigma &= Y(0)e^{-(\sigma-1)\tau^M r}G(0)^{\sigma-1} \\
&+ \int_{-f}^{-\bar{r}} Y(s)e^{-(\sigma-1)[\tau^M(r-\bar{r})+\tau^{TM}2\bar{r}+\tau^M(-s-\bar{r})]}G(s)^{\sigma-1}ds \\
&+ \int_{-\bar{r}}^{-\frac{\tau^{TM}}{\tau^M}\bar{r}} Y(s)e^{-(\sigma-1)[\tau^{TM}(2\bar{r})+\tau^M(r+s)]}G(s)^{\sigma-1}ds \\
&+ \int_{-\frac{\tau^{TM}}{\tau^M}\bar{r}}^{0} Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds \\
&+ \int_0^r Y(s)e^{-(\sigma-1)\tau^M(r-s)}G(s)^{\sigma-1}ds + \int_r^f Y(s)e^{-(\sigma-1)\tau^M(s-r)}G(s)^{\sigma-1}ds \quad (22)
\end{aligned}$$

Note that we obtain the same nominal wage rates for manufacturing firms around the city as in Fujita and Krugman(1995).

From (20),

$$\begin{aligned}
\frac{\partial[w^M(r)]^\sigma}{\partial\bar{r}} &= -\frac{\tau^A(1+\tau^{TM}/\tau^M)}{2}\frac{1-\mu}{1-e^{-\tau^A f}} \\
&\times\Big\{e^{[-(\sigma-1)\tau^M+\tau^A]r-\tau^A(1+\tau^{TM}/\tau^M)\bar{r}} \\
&\quad+\frac{2(\sigma-1)\tau^M}{2(\sigma-1)\tau^M-\tau^A}e^{(\sigma-1)\tau^M r-2(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+[2(\sigma-1)\tau^M-\tau^A]f}\Big\} < 0 \quad (23)
\end{aligned}$$

Thus, we obtain $\partial\Omega(r)/\partial\bar{r} < 0$ when $(1+\frac{\tau^{TM}}{\tau^M})\bar{r} - f < r < f$ and $f < \bar{r}$.

From (21),

$$\begin{aligned}
\frac{\partial[w^M(r)]^\sigma}{\partial\bar{r}} &= -\frac{1}{2}\frac{1-\mu}{1-e^{-\tau^A f}}e^{(\sigma-1)\tau^M r} \\
&\times\Big\{[2(\sigma-1)\tau^{TM}]e^{-2(\sigma-1)\tau^{TM}\bar{r}}\left(\frac{2(\sigma-1)\tau^M}{2(\sigma-1)\tau^M-\tau^A}e^{-\tau^A\bar{r}} - e^{-\tau^A f}\right) \\
&\quad+\frac{2(\sigma-1)\tau^A\tau^M}{2(\sigma-1)\tau^M-\tau^A}e^{-[2(\sigma-1)\tau^{TM}+\tau^A]\bar{r}} \\
&\quad+\frac{2\tau^A(1+\tau^{TM}/\tau^M)[(\sigma-1)\tau^M-\tau^A]}{2(\sigma-1)\tau^M-\tau^A}e^{[-2(\sigma-1)\tau^M+\tau^A]r}e^{-\tau^A(1+\tau^{TM}/\tau^M)\bar{r}}\Big\} \quad (24)
\end{aligned}$$

yields $\partial[w^M(r)]^\sigma/\partial\bar{r} < 0$ under $\frac{\tau^{TM}}{\tau^M}\bar{r} < r < \bar{r}$ if $(\sigma-1)\tau^M - \tau^A > 0$. Thus, we obtain

$\partial\Omega(r)/\partial\bar{r} < 0$ under $\frac{\tau^{TM}}{\tau^M}\bar{r} < r < \bar{r}$ if $(\sigma-1)\tau^M - \tau^A > 0$. Likewise, from (22),

$$\begin{aligned}\frac{\partial[w^M(r)]^\sigma}{\partial\bar{r}} &= \frac{1-\mu}{2(1-e^{-\tau^A f})}e^{-(\sigma-1)\tau^M r} \\ &\times \left(e^{2(\sigma-1)(\tau^M-\tau^{TM})\bar{r}}\left\{\frac{2(\sigma-1)\tau^M[2(\sigma-1)(\tau^M-\tau^{TM})-\tau^A]}{2(\sigma-1)\tau^M-\tau^A}e^{-\tau^A\bar{r}}\right.\right. \\ &\left.-2(\sigma-1)(\tau^M-\tau^{TM})e^{-\tau^A f}\right\} \\ &\left.+\frac{\tau^A\tau^{TM}}{\tau^M}\frac{2[(\sigma-1)\tau^M-\tau^A]}{2(\sigma-1)\tau^M-\tau^A}e^{-\tau^A\tau^{TM}\bar{r}/\tau^M}\right) \qquad (25)\end{aligned}$$

becomes at least positive if the first term in braces becomes positive. The first term in braces becomes positive if

$$f > log\left(\frac{\tau^M[2(\sigma-1)(\tau^M-\tau^{TM})-\tau^A]}{(\tau^M-\tau^A)[2(\sigma-1)\tau^M-\tau^A]}\right) + \bar{r} \qquad (26)$$

holds. Thus, $\partial[w^M(r)]^\sigma/\partial\bar{r} > 0$ under $\bar{r} < r < f$ if f is sufficiently large.

From (19) and (21), subtracting the nominal wage rates without new transport facilities from those with the new transport facilities, which is expressed as $W_1(r)$, yields

$$\begin{aligned}\frac{\partial W_1(\bar{r})}{\partial r} &= \frac{1}{2}(\sigma-1)(1-\mu)\tau^M\left[1+\frac{1}{1-e^{-\tau^A f}}\left(1-\frac{2[(\sigma-1)\tau^M-\tau^A]}{2(\sigma-1)\tau^M-\tau^A}e^{-\frac{\tau^A\tau^{TM}}{\tau^M}\bar{r}}\right)\right] \\ &+\frac{1}{2}\frac{1-\mu}{1-e^{-\tau^A f}}\tau^A e^{[-(\sigma-1)\tau^M-\tau^A\tau^{TA}/\tau^M]\bar{r}}\frac{2[(\sigma-1)\tau^M-\tau^A]}{2(\sigma-1)\tau^M-\tau^A} \\ &+\frac{1}{2}\frac{1-\mu}{1-e^{-\tau^A f}}(\sigma-1)\tau^M\frac{2(\sigma-1)\tau^M}{2(\sigma-1)\tau^M-\tau^A}e^{(\sigma-1)\tau^M\bar{r}-2(\sigma-1)\tau^{TM}\bar{r}}\left(e^{-\tau^A\bar{r}}-e^{-\tau^A f}\right) > 0 \qquad (27)\end{aligned}$$

Likewise, from (19) and (22), subtracting the nominal wage rates without new transport facilities from those with the new transport facilities, which is expressed as $W_2(r)$, yields

$$\begin{aligned}\frac{\partial W_2(r)}{\partial r} &= \frac{(\sigma-1)(1-\mu)}{2}\tau^A e^{-(\sigma-1)\tau^M r} \\ &\times\left(1-\frac{1}{1-e^{-\tau^A f}}\left\{e^{2(\sigma-1)(\tau^M-\tau^{TM})\bar{r}}\left[\frac{2(\sigma-1)\tau^M}{2(\sigma-1)\tau^M-\tau^A}e^{-\tau^A\bar{r}}-e^{-\tau^A f}\right]\right.\right. \\ &\left.\left.+1-\frac{2[(\sigma-1)\tau^M-\tau^A]}{2(\sigma-1)\tau^M-\tau^A}e^{-\frac{\tau^A\tau^{TM}}{\tau^M}\bar{r}}\right\}\right) \qquad (28)\end{aligned}$$

Thus, we obtain

$$\begin{aligned}&\frac{\partial W_2(r)}{\partial r} \gtrless 0 \Leftrightarrow \\ &f \lessgtr log\left(\frac{2(\sigma-1)\tau^M e^{[2(\sigma-1)(\tau^M-\tau^{TM})-\tau^A]\bar{r}}-2[(\sigma-1)\tau^M-\tau^A]e^{-\frac{\tau^A\tau^{TM}}{\tau^M}\bar{r}}}{[2(\sigma-1)\tau^M-\tau^A](e^{2(\sigma-1)(\tau^M-\tau^{TM})\bar{r}}-1)}\right)/\tau^A \qquad (29)\end{aligned}$$

Substituting $r = \bar{r}$ into (22), which is the case when $\bar{r} < f$, yields $\Omega(\bar{r}) = w(\bar{r})^\sigma e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\bar{r}}$, where

$$\begin{aligned} w(\bar{r})^\sigma =& \mu e^{-(\sigma-1)\tau^M \bar{r}} + \frac{1-\mu}{2(1-e^{-\tau^A f})} \\ &\times \left[e^{(\sigma-1)\tau^M \bar{r}} \left(\frac{2(\sigma-1)\tau^M}{2(\sigma-1)\tau^M - \tau^A} e^{-\tau^A \bar{r}} - e^{-\tau^A f} \right) \left(1 + e^{-2(\sigma-1)\tau^{TM}\bar{r}} \right) \right. \\ &\left. - \frac{2(\sigma-1)\tau^M}{2(\sigma-1)\tau^M - \tau^A} e^{-[(\sigma-1)\tau^M + \tau^A \tau^{TM}/\tau^M]\bar{r}} + \frac{2[(\sigma-1)\tau^M - \tau^A]}{2(\sigma-1)\tau^M - \tau^A} e^{-(\sigma-1)\tau^M \bar{r}} \right] \end{aligned} \tag{30}$$

Solving $\partial w(\bar{r})^\sigma / \partial \tau^{TM}$ yields $\partial w(\bar{r})^\sigma / \partial \tau^{TM} < 0 \iff \Lambda < 1$ if $2(\sigma-1)\tau^M - \tau^A > 0$ and $\partial w(\bar{r})^\sigma / \partial \tau^{TM} < 0 \iff \Lambda > 1$ if $2(\sigma-1)\tau^M - \tau^A < 0$ where

$$\Lambda \equiv \frac{\tau^A}{2(\sigma-1)\tau^M} \frac{e^{-[2(\sigma-1)\tau^M - \tau^A](1-\tau^{TM}/\tau^M)\bar{r}} - e^{-\tau^A(f-\bar{r})}}{1 - e^{-\tau^A(f-\bar{r})}}$$

Thus, we obtain $\partial w(\bar{r})^\sigma / \partial \tau^{TM} < 0$. Note that f is not affected by τ^{TM}.

Appendix B Derivation of the comparative analysis on the monocentric equilibrium

B.1 The case with hinterland regions

B.1.1 The impact on the location of frontiers

Rearranging the equality between case II of (11) and case 1 of (6), we obtain

$$\begin{aligned} Z \equiv& e^{\mu(\tau^M + \tau^A) f^{min}} \left(1 - 3e^{-\tau^A f^{min}} + 2e^{-\tau^{TA}\bar{r}} \right) \\ &- \frac{1-\mu}{2\mu} \left(\frac{N\tau^A}{c^A} - 6f^{min}\tau^A + 4\tau^{TA}\bar{r} \right) = 0 \end{aligned}$$

Then, we obtain:

$$\begin{aligned} \frac{\partial Z}{\partial f^{min}} =& \mu(\tau^M + \tau^A) e^{\mu(\tau^M + \tau^A) f^{min}} \left(1 + 2e^{-\tau^{TA}\bar{r}} - 3e^{-\tau^A f^{min}} \right) \\ &+ 3\tau^A e^{\mu(\tau^M + \tau^A) f^{min}} e^{-\tau^A f^{min}} + 3\frac{1-\mu}{\mu}\tau^A > 0 \end{aligned}$$

$$\frac{\partial Z}{\partial \bar{r}} = -2\tau^{TA} e^{\mu(\tau^M + \tau^A) f^{min} - \tau^{TA}\bar{r}} - \frac{2(1-\mu)}{\mu}\tau^{TA} < 0$$

$$\frac{\partial Z}{\partial \tau^{TA}} = -2\bar{r} e^{\mu(\tau^M + \tau^A) f^{min} - \tau^{TA}\bar{r}} - \frac{2(1-\mu)}{\mu}\bar{r} < 0$$

$$\frac{\partial Z}{\partial \mu} =(\tau^M + \tau^A) f^{min} e^{\mu(\tau^M+\tau^A) f^{min}} \left(1 - 3e^{-\tau^A f^{min}} + 2e^{-\tau^{TA}\bar{r}}\right) + \frac{1}{2\mu^2}\left(\frac{N\tau^A}{c^A} - 6 f^{min}\tau^A + 4\tau^{TA}\bar{r}\right) > 0$$

$$\frac{\partial Z}{\partial \tau^M} = \mu f^{min} e^{\mu(\tau^M+\tau^A) f^{min}} \left(1 - 3e^{-\tau^A f^{min}} + 2e^{-\tau^{TA}\bar{r}}\right) > 0$$

$$\frac{\partial Z}{\partial \tau^A} = f^{min} e^{\mu(\tau^M+\tau^A) f^{min}} \left[\mu\left(1 - 3e^{-\tau^A f^{min}} + 2e^{-\tau^{TA}\bar{r}}\right) + 3\right] + 3\frac{1-\mu}{\mu} f^{min} - \frac{1-\mu}{2\mu}\frac{N}{c^A} \gtrless 0$$

$$\Leftrightarrow \frac{2\mu c^A f^{min}}{1-\mu} e^{\mu(\tau^M+\tau^A) f^{min}} \left[\mu(1 - 3e^{-\tau^A f^{min}} + 2e^{-\tau^{TA}\bar{r}}) + 3\right] + 6c^A f^{min} \gtrless N$$

$$\frac{\partial Z}{\partial c^A} = \frac{1-\mu}{2\mu}\frac{N\tau^A}{{c^A}^2} > 0$$

$$\frac{\partial Z}{\partial N} = -\frac{1-\mu}{\mu}\frac{\tau^A}{c^A} < 0$$

Thus, combining the derived results with the implicit function theorem yields $\partial f^{min}/\partial \bar{r} > 0$, $\partial f^{min}/\partial \tau^{TA} > 0$, $\partial f^{min}/\partial \mu < 0$, $\partial f^{min}/\partial \tau^M < 0$, $\partial f^{min}/\partial c^A < 0$ and $\partial f^{min}/\partial N > 0$.

We obtain $\partial f^{min}/\partial \tau^A > 0$ if N is large, otherwise we obtain $\partial f^{min}/\partial \tau^A < 0$.

B.1.2 The impact on the size of hinterland regions

Since the size of hinterland regions, EL, is expressed as $2f^{min} - 2\tau^{TA}\bar{r}/\tau^A$, we obtain

$$\frac{\partial EL}{\partial \bar{r}} \gtrless 0 \Leftrightarrow -2\frac{\partial Z}{\partial \bar{r}} - 2\frac{\tau^{TA}}{\tau^A}\frac{\partial Z}{\partial f^{min}} = 2e^{\mu(\tau^M+\tau^A) f^{min}} \tau^{TA} \times \left\{ \frac{(1-\mu)\tau^{TA} - \mu\tau^{TM}}{\tau^{TA}} (1 + 2e^{-\tau^{TA}\bar{r}} - 3e^{-\tau^A f^{min}}) - 1 - \frac{1-\mu}{\mu} e^{-\mu(\tau^M+\tau^A) f^{min}} \right\} \gtrless 0$$

Since $(1-\mu)\tau^{TA} - \mu\tau^{TM} < 0$, under the necessary condition for the monocentric city, using the assumption $\tau^{TA}/\tau^A = \tau^{TM}/\tau^M$, we obtain $\partial EL/\partial \bar{r} < 0$.

Similarly,

$$\frac{\partial EL}{\partial \tau^{TA}} \gtrless 0 \Leftrightarrow -2\frac{\partial Z}{\partial \tau^{TA}} - 2\frac{\bar{r}}{\tau^A}\frac{\partial Z}{\partial f^{min}} = -2\bar{r} \times \left\{ \frac{1-\mu}{\mu} + e^{\mu(\tau^M+\tau^A) f^{min}} + (1 + 2e^{-\tau^{TA}\bar{r}} - 3e^{-\tau^A f^{min}}) e^{\mu(\tau^M+\tau^A) f^{min}} \left(-1 + \mu\frac{\tau^M + \tau^A}{\tau^A}\right) \right\} \gtrless 0$$

Since $(1-\mu)\tau^A - \mu\tau^M < 0$ under the necessary condition for the monocentric city, we obtain $\frac{\partial EL}{\partial \tau^{TA}} < 0$.

Since $EL = 2f^{min} - 2\tau^{TA}\bar{r}/\tau^A$, $\partial f^{min}/\partial\mu < 0$ and $\partial f^{min}/\partial\tau^M < 0$ imply $\partial EL/\partial\mu < 0$ and $\partial EL/\partial\tau^M < 0$ respectively.

Since $\partial EL/\partial\tau^A = 2\partial f^{min}/\partial\tau^A + 2\tau^{TA}\bar{r}/(\tau^A)^2$, as in the case of $\partial f^{min}/\partial\tau^A$, large N provides $\partial EL/\partial\tau^A > 0$.

We obtain $\partial EL/\partial c^A = 2\partial f^{min}/\partial c^A < 0$ and $\partial EL/\partial N = 2\partial f^{min}/\partial N > 0$.

B.1.3 The impact on the size of arable land

From $f^{min} + EL = 3f^{min} - 2\tau^{TA}\bar{r}/\tau^A$, we obtain $\partial(f^{min} + EL)/\partial\bar{r} \gtreqless 0 \Leftrightarrow -3\partial Z/\partial\bar{r} - 2\frac{\tau^{TA}}{\tau^A}\partial Z/\partial f^{min} \gtreqless 0$. Since

$$
\begin{aligned}
-3\frac{\partial Z}{\partial \bar{r}} - 2\frac{\tau^{TA}}{\tau^A}\frac{\partial Z}{\partial f^{min}} &= e^{\mu(\tau^A+\tau^M)f^{min}} \\
&\times \left\{-6[(1-\mu)\tau^{TA} - \mu\tau^{TM}](e^{-\tau^A f^{min}} - e^{-\tau^{TA}\bar{r}}) - 2\mu(\tau^{TA}+\tau^{TM})(1-e^{-\tau^{TA}\bar{r}})\right\} < 0
\end{aligned}
$$

we obtain $\partial(f^{min} + EL)/\partial\bar{r} < 0$.

We obtain $\partial(f^{min} + EL)/\partial\tau^{TA} \gtreqless 0 \Leftrightarrow -3\partial Z/\partial\tau^{TA} - 2\frac{\bar{r}}{\tau^A}\partial Z/\partial f^{min} \gtreqless 0$. Since

$$
\begin{aligned}
-3\frac{\partial Z}{\partial \tau^{TA}} - 2\frac{\bar{r}}{\tau^A}\frac{\partial Z}{\partial f^{min}} &= -2e^{\mu(\tau^A+\tau^M)f^{min}} \\
&\times \left\{3(e^{-\tau^A f^{min}} - e^{-\tau^{TA}\bar{r}}) + \mu\frac{\tau^M+\tau^A}{\tau^A}(1+2e^{-\tau^{TA}\bar{r}} - 3e^{-\tau^A f^{min}})\right\} < 0
\end{aligned}
$$

, we obtain $\partial(f^{min} + EL)/\partial\tau^{TA} < 0$.

We obtain $\partial(f^{min} + EL)/\partial\tau^A \gtreqless 0 \Leftrightarrow -3\partial Z/\partial\tau^{TA} - 2\frac{\bar{r}}{\tau^A}\partial Z/\partial f^{min} \gtreqless 0$.

We obtain $\partial(f^{min} + EL)/\partial\tau^A = -3\frac{\partial Z}{\partial\tau^A}/\frac{\partial Z}{\partial f^{min}} + 3\tau^{TA}\bar{r}/\tau^{A^2}$. Since $\partial Z/\partial\tau^A < 0$ under large N, we obtain $\partial(f^{min} + EL)/\partial\tau^A < 0$ if N is large.

From $\partial(f^{min} + EL)/\partial\mu = 3\partial f^{min}/\partial\mu$, $\partial(f^{min} + EL)/\partial c^A = 3\partial f^{min}/\partial c^A$, $\partial(f^{min} + EL)/\partial\tau^M = 3\partial f^{min}/\partial\tau^M$ and $\partial(f^{min}+EL)/\partial N = 3\partial f^{min}/\partial N$, We obtain $\partial f^{min}/\partial\mu < 0$, $\partial f^{min}/\partial c^A < 0$, $\partial f^{min}/\partial\tau^M < 0$ and $\partial f^{min}/\partial N > 0$ respectively.

B.1.4 The impact on the number of manufacturing workers

From $L^M = N - 2c^A(3f^{min} - 2\tau^{TA}\bar{r}/\tau^A)$, we obtain $\frac{\partial L_M}{\partial\bar{r}} \lesseqgtr 0 \Leftrightarrow -3\frac{\partial Z}{\partial\bar{r}} - 2\frac{\tau^{TA}}{\tau^A}\frac{\partial Z}{\partial f^{min}} \gtreqless 0$. Since $-3\partial Z/\partial\bar{r} - 2\frac{\tau^{TA}}{\tau^A}\partial Z/\partial f^{min} < 0$, we obtain that $\partial L_M/\partial\bar{r} > 0$.

Similarly, since $\frac{\partial L_M}{\partial\tau^{TA}} \lesseqgtr 0 \Leftrightarrow -3\frac{\partial Z}{\partial\tau^{TA}} - 2\frac{\bar{r}}{\tau^A}\frac{\partial Z}{\partial f^{min}} \gtreqless 0$. Since $-3\partial Z/\partial\tau^{TA} - 2\frac{\bar{r}}{\tau^A}\partial Z/\partial f^{min} < 0$, we obtain that $\partial L_M/\partial\tau^{TA} > 0$.

From $\partial L^M/\partial \tau^A = -6c^A \partial f^{min}/\partial \tau^A - 4c^A \tau^{TA} \bar{r}/{\tau^A}^2$ and $\partial f^{min}/\partial \tau^A > 0$ if N is large, we obtain $\partial L^M/\partial \tau^A < 0$ if N is large.

Since $\partial L^M/\partial \mu = -6c^A \partial f^{min}/\partial \mu$ and $\partial f^{min}/\partial \mu < 0$, we obtain $\partial L^M/\partial \mu > 0$.

Since $\partial L^M/\partial \tau^M = -6c^A \partial f^{min}/\partial \tau^M$ and $\partial f^{min}/\partial \tau^M < 0$, we obtain $\partial L^M/\partial \tau^M > 0$.

We obtain $\partial L^M/\partial N \gtreqless 0 \Leftrightarrow \partial Z/\partial f^{min} + 6c^A \partial Z/\partial N \gtreqless 0$. Thus, we find that large τ^M provides $\partial L^M/\partial N > 0$.

Since $\partial L^M/\partial c^A \gtreqless 0 \Leftrightarrow f^{min} \partial Z/\partial f^{min} + 2\tau^{TA}\bar{r}/3\tau^A - c^A \partial Z/\partial c^A \lesseqgtr 0$, large τ^M provides $\partial L^M/\partial c^A < 0$.

B.1.5 The impact on the price of an agricultural good

From $p^A = c^A e^{\mu(\tau^M+\tau^A)f^{min}}$, we obtain $\partial p^A/\partial f^{min} = \mu(\tau^M + \tau^A)c^A e^{\mu(\tau^M+\tau^A)f^{min}} > 0$. Thus, we find that $\partial p^A/\partial \bar{r}$, $\partial p^A/\partial \tau^{TA}$ and $\partial p^A/\partial N$ has the same sign as $\partial f^{min}/\partial \bar{r}$, $\partial f^{min}/\partial \tau^{TA}$ and $\partial f^{min}/\partial N$.

A simple calculation yields

$$
\begin{aligned}
&\frac{\partial p^A}{\partial \mu} = (\tau^M + \tau^A)c^A e^{\mu(\tau^M+\tau^A)f^{min}}(f^{min} + \mu\frac{\partial f^{min}}{\partial \mu}) \gtreqless 0 \\
&\Leftrightarrow 2c^A \left[3\mu f^{min} e^{\mu(\tau^M+\tau^A)f^{min} - \tau^A f^{min}} + 6f^{min} - 3\mu f^{min} - 2c^A \bar{r}\tau^{TA}/\tau^A\right] \gtreqless N
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial p^A}{\partial c^A} = e^{\mu(\tau^M+\tau^A)f^{min}}\left[1 + \mu(\tau^M + \tau^A)c^A \frac{\partial f^{min}}{\partial c^A}\right] \gtreqless 0 \\
&\Leftrightarrow \frac{\partial Z}{\partial f^{min}} \frac{2c^A}{(1-\mu)(\tau^M + \tau^A)\tau^A} \gtreqless N
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial p^A}{\partial \tau^A} = \mu c^A e^{\mu(\tau^M+\tau^A)f^{min}}\left[f^{min} + (\tau^M + \tau^A)\frac{\partial f^{min}}{\partial \tau^A}\right] \gtreqless 0 \\
&\Leftrightarrow N \gtreqless \frac{3\tau^M f^{min} c^A}{\tau^A + \tau^M}\left(\frac{\mu}{1-\mu} e^{\mu(\tau^M+\tau^A)f^{min}} + 1\right)
\end{aligned}
$$

A simple calculation yields

$$
\frac{\partial p^A}{\partial \tau^M} = \mu c^A e^{\mu(\tau^M+\tau^A)f^{min}}\left[f^{min} - (\tau^M + \tau^A)\frac{\partial Z}{\partial \tau^M}/\frac{\partial Z}{\partial f^{min}}\right]
$$

Since

$$
\frac{\partial Z}{\partial f^{min}} f^{min} - (\tau^M + \tau^A)\frac{\partial Z}{\partial \tau^M} = 3f^{min}\tau^A e^{\mu(\tau^M+\tau^A)f^{min} - \tau^A f^{min}} + 3\frac{1-\mu}{\mu}\tau^A f^{min} > 0
$$

we obtain $\partial p^A/\partial \tau^M > 0$.

B.2 The case without hinterland regions

B.2.1 The impact on the location of frontiers

Rearranging the equality between case III of (11) and case 2 of (6), we obtain

$$
\begin{aligned}
Y \equiv & e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]} \\
& \times \left(1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right) \\
& -\frac{1-\mu}{2\mu}\left(\frac{N}{c^A}-2f^{min}\right)\tau^A=0
\end{aligned}
$$

Then, we obtain:

$$
\begin{aligned}
\frac{\partial Y}{\partial f^{min}} = & \mu(\tau^A+\tau^M)e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]} \\
& \times \left(1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right) \\
& +\tau^A e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}e^{-\tau^A f^{min}+(\tau^A-\tau^{TA})\bar{r}} \\
& +\frac{1-\mu}{\mu}\tau^A>0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\partial Y}{\partial \bar{r}} \gtrless 0 \\
\Leftrightarrow & \frac{\tau^A\left[e^{-(\tau^{TA}+\tau^A)\bar{r}/2}-e^{-\tau^A\bar{r}-\tau^A(f^{min}-\bar{r})}\right]-\tau^{TA}\left[2e^{-\tau^{TA}\bar{r}}-e^{-(\tau^{TA}+\tau^A)\bar{r}/2-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}}\right]}{(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]} \\
& \gtrless \mu
\end{aligned}
$$

Since the sign of the first term of the numérater becomes negative if $2\tau^A/(3\tau^A-\tau^{TA}) < \bar{r}/f^{min}$, we obtain $\partial f^{min}/\partial \bar{r} > 0$ if $2\tau^A/(3\tau^A-\tau^{TA}) < \bar{r}/f^{min}$. We find that $\partial f^{min}/\partial \bar{r} > 0$ if μ is sufficiently large, otherwise $\partial f^{min}/\partial \bar{r} < 0$.

From the condition $\tau^{TM} = \tau^{TA}\tau^M/\tau^A$, we obtain

$$
\begin{aligned}
& \frac{\partial Y}{\partial \tau^{TA}} \gtrless 0 \\
\Leftrightarrow & \mu \gtrless \frac{2e^{-\tau^{TA}\bar{r}}-e^{-(\tau^{TA}+\tau^A)\bar{r}/2}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}}{(1+\tau^M/\tau^A)\left[1-2e^{-(\tau^{TA}+\tau^A)\bar{r}/2}+2e^{-\tau^{TA}\bar{r}}-e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right]}
\end{aligned}
$$

Thus, we find $\partial f^{min}/\partial \tau^{TA} > 0$ if μ is small, otherwise $\partial f^{min}/\partial \tau^{TA} < 0$.

A simple calculation yields

$$\frac{\partial Y}{\partial \mu} = [(\tau^M + \tau^A) f^{min} - (\tau^A - \tau^{TA} + \tau^M - \tau^{TM})\bar{r}]$$
$$\times e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}$$
$$\times \left(1 - 2e^{-(\tau^{TA}+\tau^A)\bar{r}/2} + 2e^{-\tau^{TA}\bar{r}} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right)$$
$$+\frac{1}{2\mu^2}\left(\frac{N}{c^A} - 2f^{min}\right)\tau^A > 0$$

Thus, we obtain $\partial f^{min}/\partial \mu < 0$.

A simple calculation yields

$$\frac{\partial Y}{\partial \tau^M} = \mu(f^{min} - \bar{r})e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}$$
$$\times \left(1 - 2e^{-(\tau^{TA}+\tau^A)\bar{r}/2} + 2e^{-\tau^{TA}\bar{r}} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right) > 0$$

Thus, we obtain $\partial f^{min}/\partial \tau^M < 0$.

A simple calculation yields

$$\frac{\partial Y}{\partial \tau^A} = \mu(f^{min} - \bar{r})e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}$$
$$\times \left(1 - 2e^{-(\tau^{TA}+\tau^A)\bar{r}/2} + 2e^{-\tau^{TA}\bar{r}} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right) + \frac{1-\mu}{\mu}f^{min} - \frac{(1-\mu)N}{2\mu c^A} \gtrless 0$$
$$\Leftrightarrow \frac{2\mu^2 c^A(f^{min} - \bar{r})}{1-\mu}e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TM})\bar{r}]}$$
$$\times \left[1 - 2e^{-(\tau^{TA}+\tau^A)\bar{r}/2} + 2e^{-\tau^{TA}\bar{r}} - e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})}\right] + 2c^A f^{min} \gtrless N$$

Thus, large N implies $\partial f^{min}/\partial \tau^A > 0$. Otherwise, we obtain $\partial f^{min}/\partial \tau^A < 0$.

Furthermore,

$$\frac{\partial Z}{\partial c^A} = \frac{\partial Y}{\partial c^A} = \frac{1-\mu}{2\mu}\frac{N\tau^A}{{c^A}^2} > 0$$
$$\frac{\partial Z}{\partial N} = \frac{\partial Y}{\partial N} = -\frac{1-\mu}{\mu}\frac{\tau^A}{c^A} < 0$$

yields $\partial f^{min}/\partial c^A < 0$ and $\partial f^{min}/\partial N > 0$ in both cases.

B.2.2 The impact on the number of manufacturing workers

From $L^M = N - 2c^A f^{min}$, since $\partial L^M/\partial f^{min} < 0$, the signs become opposite between $\partial f^{min}/\partial \bar{r}$ and $\partial L^M/\partial \bar{r}$.

From $\partial L^M/\partial f^{min} < 0$, the signs become opposite between $\partial f^{min}/\partial \tau^{TA}$ and $\partial L^M/\partial \tau^{TA}$.

From $L^M = N - 2c^A f^{min}$, we obtain $\partial L^M/\partial f^{min} < 0$. Thus, we obtain $\partial L^M/\partial \mu > 0$ and $\partial L^M/\partial \tau^M > 0$.

The large N implies $\partial L^M/\partial \tau^A < 0$. Otherwise, we obtain $\partial L^M/\partial \tau^A > 0$.

From $L^M = N - 2c^A f^{min}$, we obtain $\partial L^M/\partial N \gtreqless 0 \Leftrightarrow \partial Y/\partial f^{min} + 2c^A \partial Y/\partial N \gtreqless 0$. Since $f^{min} > \bar{r}$, we also find that large τ^M provides $\partial L^M/\partial N > 0$.

Furthermore, since $\partial L^M/\partial c^A \gtreqless 0 \Leftrightarrow -\partial f^{min} \partial Y/\partial f^{min} + c^A \partial Y/\partial c^A \gtreqless 0$, using $f^{min} > \bar{r}$, large τ^M provides $\partial L^M/\partial c^A < 0$.

B.2.3 The impact on the price of an agricultural good

From $p^A = c^A e^{\mu[(\tau^{TA}+\tau^{TM})\bar{r}+(\tau^A+\tau^M)(f^{min}-\bar{r})]}$, we obtain

$$\begin{aligned}\frac{\partial p^A}{\partial \mu} &= p^A \left[(\tau^{TA}+\tau^{TM})\bar{r} + (\tau^A+\tau^M)(f^{min}-\bar{r}) - \mu(\tau^A+\tau^M)\frac{\partial Y}{\partial \mu}\right] \gtreqless 0 \\ &\Leftrightarrow 2\mu\tau^A \left[\left(\frac{\tau^{TA}+\tau^{TM}}{\tau^A+\tau^M}\right)\bar{r} + f^{min} - \bar{r}\right]\left[p^A e^{-\tau^A f^{min}+(\tau^A-\tau^{TA})\bar{r}} + c^A \frac{1-\mu}{\mu}\right] \gtreqless N\end{aligned}$$

$$\begin{aligned}\frac{\partial p^A}{\partial \tau^{TA}} &= \mu p^A \left[(1+\tau^M/\tau^A)\bar{r} + (\tau^A+\tau^M)\frac{\partial f^{min}}{\partial \tau^{TA}}\right] \gtreqless 0 \\ &\Leftrightarrow \frac{\partial Y}{\partial f^{min}}\bar{r}(1+\tau^M/\tau^A) - (\tau^A+\tau^M)\frac{\partial Y}{\partial \tau^{TA}} \gtreqless 0\end{aligned}$$

Since

$$\begin{aligned}\frac{\partial Y}{\partial f^{min}}\bar{r}(1+\tau^M/\tau^A) &- (\tau^A+\tau^M)\frac{\partial Y}{\partial \tau^{TA}} \\ &= (\tau^A+\tau^M)\bar{r}\left\{\frac{1-\mu}{\mu} - e^{\mu[(\tau^M+\tau^A)f^{min}-(\tau^A-\tau^{TA}+\tau^M-\tau^{TA})\bar{r}]}\right. \\ &\times \left.\left[e^{-(\tau^{TA}+\tau^A)\bar{r}/2} - 2e^{-\tau^{TA}\bar{r}}\right]\right\} > 0\end{aligned}$$

we obtain $\partial p^A/\partial \tau^{TA} > 0$.

A simple calculation yields

$$\begin{aligned}\frac{\partial p^A}{\partial \tau^M} &= \mu p^A \left[f^{min} - \bar{r} + (\tau^A+\tau^M)\frac{f^{min}}{\tau^M}\right] \gtreqless 0 \\ &\Leftrightarrow \frac{\partial Y}{\partial f^{min}}(f^{min}-\bar{r}) - (\tau^A+\tau^M)\frac{\partial Y}{\partial \tau^M} \gtreqless 0\end{aligned}$$

Since

$$\begin{aligned}&\frac{\partial Y}{\partial f^{min}}(f^{min}-\bar{r}) - (\tau^A+\tau^M)\frac{\partial Y}{\partial \tau^M} \\ &\quad = (f^{min}-\bar{r})\left(\tau^A e^{\mu[(\tau^{TA}+\tau^{TM})\bar{r}+(\tau^A+\tau^M)(f^{min}-\bar{r})]} e^{-\tau^A f^{min}+(\tau^A-\tau^{TA})\bar{r}} + \frac{1-\mu}{\mu}\tau^A\right) > 0\end{aligned}$$

we obtain $\partial p^A/\partial\tau^M > 0$.

A simple calculation yields

$$\begin{aligned}
\frac{\partial p^A}{\partial c^A} &= e^{\mu[(\tau^{TA}+\tau^{TM})\bar{r}+(\tau^A+\tau^M)(f^{min}-\bar{r})]}\left[1+\mu c^A(\tau^A+\tau^M)\frac{\partial f^{min}}{\partial c^A}\right] \gtreqless 0 \\
\Leftrightarrow & \frac{\partial Y}{\partial f^{min}} - \mu c^A(\tau^A+\tau^M)\frac{\partial Y}{\partial c^A} \gtreqless 0 \\
\Leftrightarrow & \frac{\mu^2}{1-\mu}\frac{c^A}{\tau^A}p^A + \frac{\mu}{1-\mu}\frac{c^A}{\tau^A+\tau^M}e^{\mu[(\tau^{TA}+\tau^{TM})\bar{r}+(\tau^A+\tau^M)(f^{min}-\bar{r})]} + \frac{c^A}{\tau^A+\tau^M} \gtreqless N
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial p^A}{\partial \tau^A} &= \mu p^A\left[f^{min}-\bar{r}+(\tau^A+\tau^M)\frac{\partial f^{min}}{\partial \tau^A}\right] \gtreqless 0 \\
\Leftrightarrow & \frac{\partial Y}{\partial f^{min}}(f^{min}-\bar{r}) - (\tau^A+\tau^M)\frac{\partial Y}{\partial \tau^A} \gtreqless 0 \\
\Leftrightarrow & N \gtreqless \frac{2c^A(\tau^A\bar{r}+\tau^M f^{min})}{\tau^A+\tau^M}
\end{aligned}$$

Since

$$\frac{\partial p^A}{\partial f^{min}} = \mu(\tau^A+\tau^M)p^A > 0$$

and $\partial f^{min}/\partial N > 0$, we obtain $\partial p^A/\partial N > 0$.

Appendix C The population and the real wage rates of the manufacturing sector in the city

The result in Fujita, Krugman and Venables (1999) is as follows:

$$\frac{\partial\omega(0)}{\partial f^{min}} = \omega(0)\frac{\mu(\tau^M+\tau^A)}{\sigma-1}\left(\frac{\mu-\rho}{1-\rho}+\frac{\tau^A}{\tau^M+\tau^A}\frac{e^{-\tau^A f^{min}}}{1-e^{-\tau^A f^{min}}}\right) \tag{31}$$

Similar relationships on f^{min} are derived in the case with hinterland regions:

$$\frac{\partial\omega(0)}{\partial f^{min}} = \omega(0)\frac{\mu(\tau^M+\tau^A)}{\sigma-1}\left(\frac{\mu-\rho}{1-\rho}+\frac{\tau^A}{\tau^M+\tau^A}\frac{e^{-\tau^A f^{min}}}{\frac{1+2e^{-\tau^{TA}\bar{r}}}{1+2e^{-\tau^{TA}\bar{r}+\tau^A r_{s1}}}-e^{-\tau^A f^{min}}}\right) \tag{32}$$

and in the case without hinterland regions:

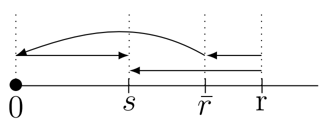
$$\frac{\partial\omega(0)}{\partial f^{min}} = \omega(0)\frac{\mu(\tau^M+\tau^A)}{\sigma-1}\left\{\frac{\mu-\rho}{1-\rho}+\frac{\tau^A}{\tau^M+\tau^A}\frac{e^{-\tau^A f^{min}}}{\frac{1-e^{-\tau^A b_A^+}+e^{-\tau^{TA}\bar{r}}[2-e^{-\tau^A(\bar{r}-b_A^+)}]}{e^{\bar{r}(\tau^A-\tau^{TA})}}-e^{-\tau^A f^{min}}}\right\} \tag{33}$$

Thus, introducing new transport facilities does not cause qualitative changes to the justification of Henderson's assumption of an inverted -U relationship between city size and the utility of the city's residents.

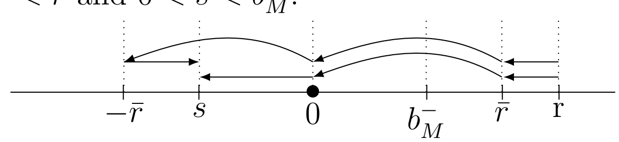
Comparing (31) and (32), or (31) and (33), we obtain the result on the critical population level. Since the relationship between the different parts of (31) and (32) is $(1+2e^{-\tau^{TA}\bar{r}})/(1+2e^{-\tau^{TA}\bar{r}+\tau^{A}r_{s1}}) < 1 \Leftrightarrow 1 < e^{\tau^{TA}\bar{r}}$, and also since the relationship between the different parts of (31) and (33) is $\frac{1-e^{-\tau^{A}b_A^{+}}+e^{-\tau^{TA}\bar{r}}[2-e^{-\tau^{A}(\bar{r}-b_A^{+})}]}{e^{\bar{r}(\tau^{A}-\tau^{TA})}} < 1 \Leftrightarrow e^{-\frac{\tau^{TA}+\tau^{A}}{2}\bar{r}}(e^{-\frac{\tau^{TA}+\tau^{A}}{2}\bar{r}}-2) < 1-e^{-\tau^{A}\bar{r}}$, the value of f^{min} such that $\partial\omega(0)/\partial f^{min}=0$ in (31) is larger than the value of f^{min} such that $\partial\omega(0)/\partial f^{min}=0$ in (32) or in (33).

Appendix D Derivation of the lowest transport costs when three new transport facilities exist

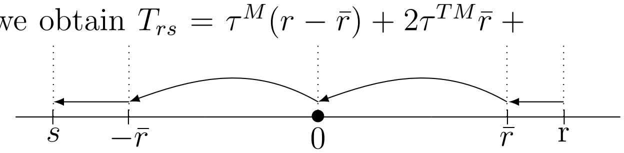
Here, we examine the lowest transport costs, T_{rs} under the existence of new transport facilities between $0 \in X$ and $\bar{r} \in X$, as well as between $-\bar{r} \in X$ and 0 with τ^{TM} instead of $\tau^{M} > \tau^{TM}$.

Focusing on the cases in the following figure: 

, setting $0 < s < \bar{r} < r$, since $\tau^{M}(\bar{r}-s) \gtrless \tau^{TM}\bar{r}+\tau^{M}s \Leftrightarrow b_M^{-} \gtrless s$, we obtain $T_{rs}=\tau^{M}(r-s)$ if $\bar{r}<r$ and $b_M^{-}<s$ and $T_{rs}=\tau^{TM}\bar{r}+\tau^{M}s+\tau^{M}(r-\bar{r})$ if $\bar{r}<r$ and $0<s<b_M^{-}$.

Focusing on the cases in the following figure: 

, setting $\bar{r}<r$ and $-\bar{r}<s<0$, since $\tau^{M}(r-\bar{r})+\tau^{TM}\bar{r}+\tau^{M}|s| \gtrless \tau^{M}(r-\bar{r})+2\tau^{TM}\bar{r}+\tau^{M}(\bar{r}-|s|) \Leftrightarrow -b_M^{+} \gtrless s$, we obtain $T_{rs}=\tau^{M}(r-\bar{r})+\tau^{TM}\bar{r}+\tau^{M}|s|$ if $\bar{r}<r$ and $-b_M^{+}<s<0$ and $T_{rs}=\tau^{M}(r-\bar{r})+2\tau^{TM}\bar{r}+\tau^{M}(\bar{r}-|s|)$ if $\bar{r}<r$ and $-\bar{r}<s<-b_M^{+}$.

Setting $\bar{r}<r$ and $s<-\bar{r}$, since $\tau^{TM}<\tau^{M}$, we obtain $T_{rs}=\tau^{M}(r-\bar{r})+2\tau^{TM}\bar{r}+\tau^{M}(|s|-\bar{r})$, which is depicted in the following figure. 

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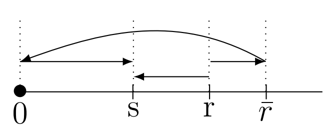
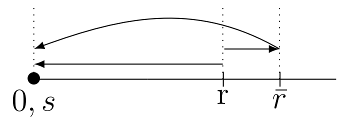
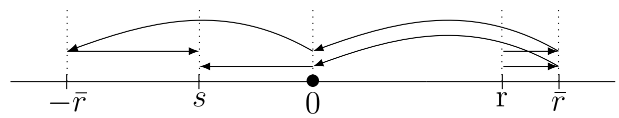
Focusing on the cases in the following figure: 

, setting $0 <$

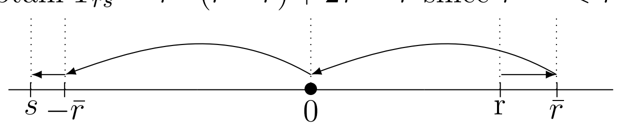
$s < r < \bar{r}$, we obtain $\tau^M(r-s) \gtrless \tau^M(\bar{r}-r) + \tau^{TM}\bar{r} + \tau^M s \Leftrightarrow r - b_M^+ \gtrless s$. Since $r - b_M^+ > s > 0 \Leftrightarrow r > b_M^+$, we obtain $T_{rs} = \tau^M(\bar{r}-r) + \tau^{TM}\bar{r} + \tau^M s$ if $b_M^+ < r < \bar{r}$ and $0 < s < r - b_M^+$, and $T_{rs} = \tau^M(r-s)$ if $r - b_M^+ < s < r < \bar{r}$.

Focusing on the cases in the following figure: 

, setting $s = 0$, we obtain $0 < r < \bar{r}$, $\tau^M r \gtrless \tau^M(\bar{r}-r) + \tau^{TM}\bar{r} \Leftrightarrow r \gtrless b_M^+$. This result is used for the initial setting of some of the following cases.

Focusing on the cases in the following figure: 

, setting $b_+^M < r < \bar{r}$ and $-\bar{r} < s < 0$, $\tau^M(\bar{r}-r) + \tau^{TM}\bar{r} + \tau^M|s| \gtrless \tau^M(\bar{r}-r) + 2\tau^{TM}\bar{r} + \tau^M(|s|-r) \Leftrightarrow -b_M^+ \gtrless s$. Thus, we obtain $T_{rs} = \tau^M(\bar{r}-r) + \tau^{TM}\bar{r} + \tau^M|s|$ if $b_+^M < r < \bar{r}$ and $-\bar{r} < s < -b_M^+$, and $T_{rs} = \tau^M(\bar{r}-r) + 2\tau^{TM}\bar{r} + \tau^M(|s|-r)$ if $b_M^+ < r < \bar{r}$ and $-b_M^+ < s < 0$.

Setting $b_M^+ < r < \bar{r}$ and $s < -\bar{r}$, we obtain $T_{rs} = \tau^M(\bar{r}-r) + 2\tau^{TM}\bar{r}$ since $\tau^{TM} < \tau^M$, which is depicted in the following figure: 

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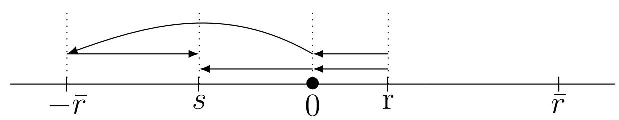
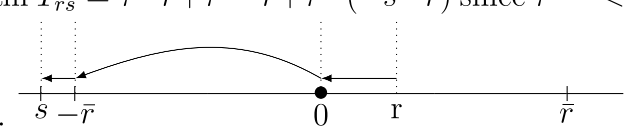
Focusing on the cases in the following figure: 

, setting $-\bar{r} < s < 0 < r < b_M^+$, $\tau^M r + \tau^M|s| \gtrless \tau^M r + \tau^{TM}\bar{r} + \tau^M(\bar{r}+s) \Leftrightarrow -b_M^+ \gtrless s$ yields $T_{rs} = \tau^M r + \tau^M|s|$ if $-b_M^+ < s < 0$ and $0 < r < b_M^+$, and $T_{rs} = \tau^M r + \tau^{TM}\bar{r} + \tau^M(\bar{r}+s)$ if $-\bar{r} < s < -b_M^+$ and $0 < r < b_M^+$.

Setting $0 < r < b_M^+$ and $s < -\bar{r}$, we obtain $T_{rs} = \tau^M r + \tau^{TM}\bar{r} + \tau^M(-s-\bar{r})$ since $\tau^{TM} < \tau^M$, which is depicted in the following figure: 

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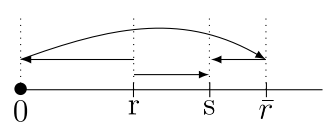
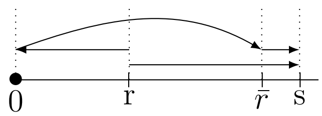
Focusing on the cases in the following figure: 

, setting $0 < r < s < \bar{r}$, we obtain $\tau^M(s-r) \gtrless \tau^M r + \tau^{TM}\bar{r} + \tau^M(\bar{r}-s) \Leftrightarrow s \gtrless r + b_M^+$. Since $\bar{r} > s > r + b_M^+ \Leftrightarrow b_M^- > r$, $T_{rs} = \tau^M(s-r)$ if $0 < r < s < r + b_M^+$, and $T_{rs} = \tau^M r + \tau^{TM}\bar{r} + \tau^M(\bar{r}-s)$ if $0 < r < b_M^-$ and $r + b_M^+ < s < \bar{r}$.

Focusing on the cases in the following figure: 

, setting $0 < r < \bar{r} < s$, $\tau^M(s-r) \gtrless \tau^M r + \tau^{TM}\bar{r} + \tau^M(s-\bar{r}) \Leftrightarrow b_M^- \gtrless r$. $T_{rs} = \tau^M(s-r)$ if $b_M^- < r < \bar{r} < s$ and $T_{rs} = \tau^M r + \tau^{TM}\bar{r} + \tau^M(s-\bar{r})$ if $0 < r < b_M^-$ and $\bar{r} < s$.

Summarizing the above results yields (12).

Appendix E Components of market potential functions when transport facilities are connected to the city

A simple calculation yields

$$Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1} = \begin{cases} \mu e^{-(\sigma-1)\tau^M|r|} & \text{if } 0 < |r| < b_M^+ \\ \mu e^{-(\sigma-1)[\tau^{TM}\bar{r}+\tau^M(\bar{r}-|r|)]} & \text{if } b_A^+ < |r| < \bar{r} \\ \mu e^{-(\sigma-1)[\tau^{TM}\bar{r}+\tau^M(|r|-\bar{r})]} & \text{if } \bar{r} < |r| \end{cases} \tag{34}$$

and

$$Y(s)G(s)^{\sigma-1} = \begin{cases} \frac{p^A}{L^M}\mu e^{\tau^M(\sigma-1)|s|-\tau^A|s|} & \text{if } 0 < |s| < b_M^+ \\ \frac{p^A}{L^M}\mu e^{(\sigma-1)[\tau^{TM}\bar{r}+\tau^M(\bar{r}-|s|)]-\tau^{TA}\bar{r}-\tau^A(\bar{r}-|s|)} & \text{if } b_A^+ < |s| < \bar{r} \\ \frac{p^A}{L^M}\mu e^{(\sigma-1)[\tau^{TM}\bar{r}+\tau^M(|s|-\bar{r})]-\tau^{TA}\bar{r}-\tau^A(|s|-\bar{r})} & \text{if } \bar{r} < |s| \end{cases} \tag{35}$$

Then, from (12) and (35), we obtain, if $0 < r < b_M^-$,

$$\frac{Y(s)G(s)^{\sigma-1}e^{-(\sigma-1)T_{rs}}}{p^A\mu/L^M} = \begin{cases} e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } s < -\bar{r} \\ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } -\bar{r} \leq s \leq -b_M^+ \\ e^{-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } -b_M^+ < s < 0 \\ e^{-(\sigma-1)\tau^M r}e^{[2(\sigma-1)\tau^M-\tau^A]s} & \text{if } 0 < s < r \\ e^{(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } r \leq s \leq b_M^+ \\ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{[-2(\sigma-1)\tau^M+\tau^A]s} & \text{if } b_M^+ < s \leq r + b_M^+ \\ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } r + b_M^+ < s \leq \bar{r} \\ e^{(-\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } \bar{r} < s \end{cases} \tag{36}$$

Similarly, we obtain, if $b_M^- \leq r < b_M^+$,

$$\frac{Y(s)G(s)^{\sigma-1}e^{-(\sigma-1)T_{rs}}}{p^A\mu/L^M} = \begin{cases} e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } s < -\bar{r} \\ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } -\bar{r} \leq s \leq -b_M^+ \\ e^{-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } -b_M^+ < s < 0 \\ e^{-(\sigma-1)\tau^M r}e^{[2(\sigma-1)\tau^M-\tau^A]s} & \text{if } 0 < s \leq r \\ e^{(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } r < s < b_M^+ \\ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{[-2(\sigma-1)\tau^M+\tau^A]s} & \text{if } b_M^+ \leq s \leq \bar{r} \\ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } \bar{r} < s \end{cases} \tag{37}$$

Likewise, if $b_M^+ \leq r \leq \bar{r}$, we obtain

$$\frac{Y(s)G(s)^{\sigma-1}e^{-(\sigma-1)T_{rs}}}{p^A\mu/L^M} = \begin{cases} e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } s < -\bar{r} \\ e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } -\bar{r} \leq s \leq -b_M^+ \\ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } -b_M^+ < s < 0 \\ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } 0 < s < r - b_M^+ \\ e^{-(\sigma-1)\tau^M r}e^{[2\tau^M(\sigma-1)-\tau^A]s} & \text{if } r - b_M^+ \leq s < b_M^+ \\ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } b_M^+ \leq s \leq r \\ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{[-2(\sigma-1)\tau^M+\tau^A]s} & \text{if } r < s \leq \bar{r} \\ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } \bar{r} < s \end{cases} \tag{38}$$

Lastly, if $\bar{r} < r$, we obtain

$$\frac{Y(s)G(s)^{\sigma-1}e^{-(\sigma-1)T_{rs}}}{p^A\mu/L^M} = \begin{cases} e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } s < -\bar{r} \\ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } -\bar{r} \leq s \leq -b_M^+ \\ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } -b_M^+ < s < 0 \\ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } 0 < s < b_M^- \\ e^{-(\sigma-1)\tau^M r}e^{[2(\sigma-1)\tau^M-\tau^A]s} & \text{if } b_M^- \leq s < b_M^+ \\ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}e^{\tau^A s} & \text{if } b_M^+ \leq s \leq \bar{r} \\ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}e^{[2(\sigma-1)\tau^M-\tau^A]s} & \text{if } \bar{r} < s \leq r \\ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}e^{-\tau^A s} & \text{if } r < s \end{cases} \tag{39}$$

From (11), we obtain:

$$\frac{\mu p^A}{L_M} = \begin{cases} \frac{(1-\mu)\tau^A}{2\left(1-e^{-\tau^A f^{min}}\right)} & \text{if } f^{min} < r_{s1} \\ \frac{(1-\mu)\tau^A}{2\left\{1-e^{-\tau^A f^{min}}+2e^{-\tau^{TA}\bar{r}}\left[1-e^{-\tau^A (f^{min}-r_{s1})}\right]\right\}} & \text{if } r_{s1} < f^{min} < b_A^+ \\ \frac{(1-\mu)\tau^A}{2\left\{1-e^{-\tau^A b_A^+}+e^{-\tau^{TA}\bar{r}}\left[1-e^{-\tau^A (\bar{r}-b_A^+)}+1-e^{-\tau^A (f^{min}-\bar{r})}\right]\right\}} & \text{if } r_{s2} < f^{min} \end{cases} \tag{40}$$

Furthermore, we need to clarify the ranges of $s \in X$ that depend on f^{min}. For this purpose, we obtain $0 < r_{s1} < b_A^+/2 < b_M^- < b_A^+ = b_M^+ < \bar{r}$ if $0 < \tau^{TA}/\tau^A < 1/3$, whereas we obtain $0 < b_M^- \leq b_A^+/2 \leq r_{s1} < b_A^+ = b_M^+ < \bar{r}$ if $1/3 \leq \tau^{TA}/\tau^A < 1$. Thus, six cases emerge, depending on τ^{TA}/τ^A and/or f^{min}. From (8), (34), (36), (37), (38), (39), and (40), we can derive a part of the nominal wage rates of the manufacturing sector at $r \in X$ as follows.

First, in the case of $0 < \tau^{TA}/\tau^A < 1/3$ and $f^{min} < r_{s1}$, or $1/3 \leq \tau^{TA}/\tau^A < 1$ and $f^{min} < b_A^-$, if $0 < r \leq f^{min}$, we obtain:

$$\begin{aligned} \frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{0}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{(\sigma-1)\tau^M r}\int_{r}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \end{aligned} \tag{41}$$

If $f^{min} < r < b_M^+$, we obtain:

$$\begin{aligned} \frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \end{aligned} \tag{42}$$

If $b_M^+ \leq r < f^{min} + b_M^+$, we obtain:

$$\begin{aligned} \frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{r-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{r-b_M^+}^{f^{min}} e^{[2\tau^M(\sigma-1)-\tau^A]s}\,\mathrm{d}s \end{aligned} \tag{43}$$

If $f^{min} + b_M^+ \leq r \leq \bar{r}$, we obtain:

$$\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} = e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s + e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \tag{44}$$

If $\bar{r} < r$, we obtain:

$$\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} = e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s + e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \tag{45}$$

Second, in the case of $1/3 \leq \tau^{TA}/\tau^A < 1$ and $b_A^- < f^{min} < r_{s1}$, if $0 < r \leq f^{min}$, we obtain (41). If $f^{min} < r \leq b_M^+$, we obtain (42). If $b_M^+ < r \leq \bar{r}$, we obtain (43). If $\bar{r} < r$, we obtain:

$$\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} = e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s + e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{b_M^-} e^{-\tau^A s}\,\mathrm{d}s + e^{-(\sigma-1)\tau^M r}\int_{b_M^-}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \tag{46}$$

Third, in the case of $0 < \tau^{TA}/\tau^A < 1/3$ and $r_{s1} < f^{min} < b_A^+/2 < b_A^-$, if $0 < r \leq f^{min}$, we otain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{0}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{(\sigma-1)\tau^M r}\int_{r}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{(-\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s \end{aligned} \tag{47}$$

If $f^{min} < r \leq b_M^+ - f^{min}$, we obtain:

$$\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\
&+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{(-\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned} \tag{48}$$

If $b_M^+ - f^{min} < r \leq b_M^-$, we obtain:

$$\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{r+b_M^+} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\
&+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{r+b_M^+}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{(-\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned} \tag{49}$$

If $b_M^- < r \leq b_M^+$, we obtain:

$$\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} = e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s$$
$$+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s$$
$$+ e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s$$
$$+ e^{-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s$$
$$+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{f^{mid}}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s$$
$$+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s \tag{50}$$

If $b_M^+ < r \leq f^{min} + b_M^+$, we obtain:

$$\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} = e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s$$
$$+ e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s$$
$$+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s$$
$$+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{r-b_M^+} e^{-\tau^A s}\,\mathrm{d}s$$
$$+ e^{-(\sigma-1)\tau^M r}\int_{r-b_M^+}^{f^{min}} e^{[2\tau^M(\sigma-1)-\tau^A]s}\,\mathrm{d}s$$
$$+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s$$
$$+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s \tag{51}$$

If $f^{min} + b_M^+ < r \leq r_{s1} + \bar{r} - f^{min}$, we obtain:

$$
\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned}
\tag{52}
$$

If $r_{s1} + \bar{r} - f^{min} < r \leq \bar{r}$, we obtain:

$$
\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{r} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned}
\tag{53}
$$

If $\bar{r} < r \leq f^{min} + \bar{r} - r_{s1}$, we obtain:

$$\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{f^{mid}}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned} \tag{54}$$

If $f^{min} + \bar{r} - r_{s1} < r$, we obtain:

$$\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{f^{mid}}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s
\end{aligned} \tag{55}$$

Fourth, in the case of $0 < \tau^{TA}/\tau^A < 1/3$ and $r_{s1} < b_A^+/2 < f^{min}$, if $0 < r \leq b_M^+ - f^{min}$, we obtain (47). If $b_M^+ - f^{min} < r \leq f^{min}$, we obtain:

$$
\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{0}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\
&+ e^{(\sigma-1)\tau^M r}\int_{r}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{r+b_M^+} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\
&+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{r+b_M^+}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{(-\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned} \tag{56}
$$

If $f^{min} < r \leq b_M^-$, we obtain (49). If $b_M^- < r \leq b_M^+$, we obtain (50). If $b_M^+ < r \leq r_{s1} + \bar{r} - f^{min}$, we obtain (51). If $r_{s1} + \bar{r} - f^{min} < r \leq f^{min} + b_M^+$, we obtain:

$$
\begin{aligned}
\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{r-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\
&+ e^{-(\sigma-1)\tau^M r}\int_{r-b_M^+}^{f^{min}} e^{[2\tau^M(\sigma-1)-\tau^A]s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{r_{s1}+\bar{r}-f^{min}}^{r} e^{\tau^A s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\
&+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s
\end{aligned} \tag{57}
$$

If $f^{min} + b_M^+ < r \leq \bar{r}$, we obtain (53).

Fifth, in the case of $0 < \tau^{TA}/\tau^A < 1/3$ and $b_A^- < f^{min} < b_M^+$, or $1/3 \leq \tau^{TA}/\tau^A < 1$ and $r_{s1} < f^{min} < b_A^+$, if $0 < r \leq b_M^-$, we obtain (56). If $b_M^- < r \leq f^{min}$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{0}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{(\sigma-1)\tau^M r}\int_{r}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{f^{mid}}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{max}} e^{-\tau^A s}\,\mathrm{d}s\end{aligned} \tag{58}$$

If $f^{min} < r \leq b_M^+$, we obtain (50). If $b_M^+ < r \leq r_{s1} + \bar{r} - f^{min}$, we obtain (51). If $r_{s1} + \bar{r} - f^{min} < r \leq f^{min} + b_M^+$, we obtain (57). If $f^{min} + b_M^+ < r \leq \bar{r}$, we obtain (53). If $\bar{r} < r \leq f^{min} + \bar{r} - r_{s1}$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{b_M^-} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{b_M^-}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{f^{mid}}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r}^{f^{min}+\bar{r}-r_{s1}} e^{-\tau^A s}\,\mathrm{d}s\end{aligned} \tag{59}$$

If $f^{min} + \bar{r} - r_{s1} < r$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-f^{mid}} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{b_M^-} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{b_M^-}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{f^{mid}}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{min}+\bar{r}-r_{s1}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s\end{aligned} \tag{60}$$

Finally, in the case of $\bar{r} + b_A^- < f^{min}$, if $0 < r \leq b_M^-$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{-b_M^+}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{0}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{(\sigma-1)\tau^M r}\int_{r}^{b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{b_M^+}^{r+b_M^+} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\ &+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{r+b_M^+}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{(-\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s\end{aligned} \tag{61}$$

If $b_M^- < r \le b_M^+$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{(\tau^A-\tau^{TA})\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\tau^{TA}+\tau^A)\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{-b_M^+}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{0}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{(\sigma-1)\tau^M r}\int_{r}^{b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{b_M^+}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s\end{aligned} \tag{62}$$

If $b_M^+ < r \le \bar{r}$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-f^{min}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[-(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{-b_M^+}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^M+\tau^{TM})\bar{r}+(\sigma-1)\tau^M r}\int_{0}^{r-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{r-b_M^+}^{b_M^+} e^{[2\tau^M(\sigma-1)-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{b_M^+}^{r} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r}^{\bar{r}} e^{[-2(\sigma-1)\tau^M+\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s\end{aligned} \tag{63}$$

If $\bar{r} < r \leq f^{min}$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{min}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-b_M^+}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{b_M^-} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{b_M^-}^{b_M^+} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{b_M^+}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{r} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}+(\sigma-1)\tau^M r}\int_{r}^{f^{min}} e^{-\tau^A s}\,\mathrm{d}s\end{aligned} \tag{64}$$

If $f^{min} < r$, we obtain:

$$\begin{aligned}\frac{w^M(r)^\sigma - Y(0)e^{-(\sigma-1)T_{r0}}G(0)^{\sigma-1}}{p^A\mu/L_M} &= e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-f^{max}}^{-\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(-\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{-\bar{r}}^{-b_M^+} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{-b_M^+}^{0} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)(\tau^{TM}-\tau^M)\bar{r}-(\sigma-1)\tau^M r}\int_{0}^{b_M^-} e^{-\tau^A s}\,\mathrm{d}s \\ &+ e^{-(\sigma-1)\tau^M r}\int_{b_M^-}^{b_M^+} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}+\tau^M)-\tau^{TA}-\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{b_M^+}^{\bar{r}} e^{\tau^A s}\,\mathrm{d}s \\ &+ e^{[(\sigma-1)(\tau^{TM}-\tau^M)-\tau^{TA}+\tau^A]\bar{r}-(\sigma-1)\tau^M r}\int_{\bar{r}}^{f^{min}} e^{[2(\sigma-1)\tau^M-\tau^A]s}\,\mathrm{d}s\end{aligned} \tag{65}$$

Appendix F Derivation of the comparative analysis on the necessary condition for sustaining monocentric equilibrium

Expressing the last term of the left-hand side of (17) as C_2, we obtain

$$\frac{\partial C_2}{\partial f^{min}} = \frac{\rho(1-\mu)\tau^M}{\left\{1+\frac{1-e^{-\tau^A f^{min}}}{2e^{-\tau^{TA}\bar{r}}\left[1-e^{-\tau^A(f^{min}-r_{s1})}\right]}\right\}^2} \frac{\tau^A}{2e^{-\tau^A\bar{r}}\left[1-e^{-\tau^A(f^{min}-r_{s1})}\right]^2} \times e^{-\tau^A f^{min}}\left(1-e^{\frac{\tau^A+\tau^{TA}}{2}\bar{r}}\right) < 0 \qquad (66)$$

Likewise, expressing the last term of the left-hand side of (18) as C_3, we obtain

$$\frac{\partial C_3}{\partial f^{min}} = -\frac{\rho(1-\mu)\tau^M}{\left\{1+\frac{1-e^{-\tau^A b_A^+}}{e^{-\tau^{TA}\bar{r}}\left[2-e^{-\tau^A(\bar{r}-b_A^+)}-e^{-\tau^A(f^{min}-\bar{r})}\right]}\right\}^2} \times \frac{1-e^{-\tau^A b_A^+}}{\left\{e^{-\tau^{TA}\bar{r}}\left[2-e^{-\tau^A(\bar{r}-b_A^+)}-e^{-\tau^A(f^{min}-\bar{r})}\right]\right\}^2}\tau^A e^{-\tau^{TA}\bar{r}-\tau^A(f^{min}-\bar{r})} < 0 \qquad (67)$$

Since c^A and N do not appear in (17) and (18), from $\partial f^{min}/\partial c^A < 0$ and $\partial f^{min}/\partial N > 0$, we obtain $\partial C_2/\partial c^A > 0$, $\partial C_2/\partial N < 0$, $\partial C_3/\partial c^A > 0$ and $\partial C_3/\partial N < 0$.

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