Proceedings Paper:

Selivanov, A.A. orcid.org/0000-0001-5075-7229, Fradkov, A.L. and Fridman, E. (2011) Adaptive synchronization of nonlinear networks with delayed couplings under incomplete control and incomplete measurements. In: IFAC Proceedings Volumes. 18th IFAC World Congress, 28 Aug - 02 Sep 2011, Milano, Italy. Elsevier , pp. 1249-1254. ISBN 9783902661937

https://doi.org/10.3182/20110828-6-it-1002.02510

Adaptive synchronization of nonlinear networks with delayed couplings under incomplete control and incomplete measurements ⋆

Anton A. Selivanov * **Alexander L. Fradkov** *,**,*** **Emilia Fridman** ****

* *Department of Theoretical Cybernetics, St.Petersburg State University, Universitetsky prospekt, 28, Peterhof, 198504, St. Petersburg, Russia, (e-mail: antonselivanov@gmail.com).*
** *Institute for Problems of Mechanical Engineering, Russian Academy of Sciences, 61 Bolshoy ave V.O., St. Petersburg, 199178, Russia, fax: +7(812)321-4771, (e-mail: fradkov@mail.ru).*
*** *National Research University of Information Technologies, Mechanics and Optics, St. Petersburg, Russia*
**** *Department of Electrical Engineering-Systems, Tel Aviv University, Tel-Aviv 69978, Israel, (e-mail: emilia@eng.tau.ac.il).*

Abstract: Passification based adaptive synchronization method for decentralized control of dynamical networks proposed in (I. A. Dzhunusov and A. L. Fradkov. Adaptive Synchronization of a Network of Interconnected Nonlinear Lur'e Systems. Automation and Remote Control, 2009, Vol. 70, No. 7, pp. 1190-1205) is extended to the networks with delayed couplings. In the contrast to the existing papers the case of incomplete control and incomplete measurements is examined (both number of inputs and the number of outputs are less than the number of the state variables). Delay independent synchronization conditions are provided. The solution is based on passification in combination with using Lyapunov-Krasovskii functional.

1. INTRODUCTION

Adaptive synchronization in the networks of dynamical systems has attracted a growing interest during recent years, see Lü et al. [2004], Lu and Chen [2005], Li and Chen [2006], Yao et al. [2006], Zhou et al. [2006], Zhong et al. [2007], Lellis et al. [2009], Das and Lewis [2010]. It is motivated by a broad area of potential applications: formation control, cooperative control, control of power networks, communication networks, production networks, etc. Most of existing works, e.g. Lu and Chen [2005], Yao et al. [2006], Zhou et al. [2006], Zhong et al. [2007] and others are dealing with full state feedback and linear interconnections. Such system models are restrictive for applications. In some papers observer-based synchronization of networks is proposed, e.g. Yoshioka and Namerikawa [2008]. However, using observers leads to doubling the order of the overall system and therefore to increase of its complexity. In addition, existing papers consider fully controlled systems, where the number of controls is equal to the number of the state variables and each control variable enters the corresponding state equation, e.g. Das and Lewis [2010].

More simple solutions could be provided by static output feedback. An adaptive output feedback synchronization method for decentralized control of dynamical networks was proposed in Dzhunusov and Fradkov [2009]. In this paper the method is extended to the networks with delayed couplings.

Networks described by models with delays were intensively studied recently both in control and in physics literature Wang and Cheng [2009], Lu et al. [2008], Hua et al. [2007], Li et al. [2004], Mensour and Longtin [1998], Pyragas [1998]. However, adaptive output feedback synchronization algorithms for networks with delayed couplings are still to be investigated. Our approach is based on combination of passification method, see Andrievskii and Fradkov [2006] with using Lyapunov-Krasovskii functional.

Problem statement and assumptions are formulated in Section 2. In Section 3 the necessary preliminaries are given. Main results are presented in Section 4. In Section 5 an example and simulation results are described.

2. PROBLEM STATEMENT

Consider a dynamical network consisting of N identical nodes described by n-dimensional nonlinear dynamical equations with delays:

⋆ The work was supported by the Russian Foundation for Basic Research (RFBR project 11- 08-01218) and Russian Federal Program Cadres (contract 16.740.11.0042).

$$\dot{x}_i(t) = Ax_i(t) + h(x_i(t), t) + \sigma \sum_{j=1}^{N} \alpha_{ij}(x_j(t) - x_i(t)) + \sigma \sum_{j=1}^{N} \beta_{ij}(x_j(t-\tau) - x_i(t-\tau)) + bu_i(t),$$
$$y_i(t) = Cx_i(t), \qquad i = 1, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of the node i, $Ax_i(t)$ is the linear part of the node dynamics with $A \in \mathbb{R}^{n\times n}$ and $h\colon \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is a continuously differential nonlinear function, σ is the coupling strength, $\tau > 0$ is the coupled delay, $\alpha = (\alpha_{ij})$, $\beta = (\beta_{ij}) \in \mathbb{R}^{N\times N}$ are the coupling matrices. With no extra restrictions it is assumed that

$$\sum_{j=1}^{N} \alpha_{ij} = 0, \sum_{j=1}^{N} \beta_{ij} = 0,$$

for all $i = 1, \ldots, N$. The matrices α, β represent the coupling strength and the underlying topology for non-delayed and delayed configuration respectively, $b \in \mathbb{R}^{n\times 1}$ is the control matrix, $u_i(t) \in \mathbb{R}$ is the control action, $y_i(t) \in \mathbb{R}^l$ is the measurement vector, $C \in \mathbb{R}^{l\times n}$ is the observation matrix.

Let $C([-\tau, 0], \mathbb{R}^n)$ be the Banach space of continuous functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with the norm $\|\phi\|_C = \sup_{-\tau\le z\le 0} \|\phi(z)\|$. For the functional differential equation (1), its initial conditions are given by $x_i(t) = \varphi_i(t) \in C([-\tau, 0], \mathbb{R}^n)$.

The problem is to design decentralized adaptive controllers generating signals $u_i(t)$ based on measured signals $y_i(t)$ to synchronize the network (1) to a given so called synchronous solution $\bar{x}(t)$, i.e. to ensure the control goal

$$lim_{t\to\infty} \|x_i(t) - \bar{x}(t)\| = 0, \qquad i = 1, 2, \ldots, N. \tag{2}$$

The vector-function $\bar{x}(t)$ is the solution of the leader (drive) system

$$\begin{aligned} \dot{\bar{x}}(t) &= A\bar{x}(t) + h(\bar{x}(t), t) + b\bar{u}(t) \\ \bar{y}(t) &= C\bar{x}(t), \end{aligned} \tag{3}$$

where $\bar{u}(t)$ is the given input function.

3. AUXILIARY RESULTS

3.1 Decentralized speed gradient algorithms

Consider a system consisting of N interconnected subsystems, dynamics of each being described by the following equation:

$$\dot{x}_i = F_i(x_i, \theta_i, t) + f_i(x, \theta, t), \qquad i = 1, \ldots, N, \tag{4}$$

where $x_i \in \mathbb{R}^{n_i}$ – state vector, $\theta_i \in \mathbb{R}^{m_i}$ – vector of inputs (tunable parameters) of subsystem, $x = (x_1, \ldots, x_N)^{\mathrm{T}} \in \mathbb{R}^n$, $\theta = (\theta_1, \ldots, \theta_N)^{\mathrm{T}} \in \mathbb{R}^m$ – aggregate state and input vectors of the system, $n = \sum_{i=1}^{N} n_i$, $m = \sum_{i=1}^{N} m_i$. Vector-function $F_i(\cdot)$ describes local dynamics of a subsystem, and vectors $f_i(\cdot)$ describe interconnection between subsystems.

Let $Q_i(x_i, t) \ge 0, i = 1, \ldots, N$ be local goal functions and let the control goal be:

$$\lim_{t\to\infty} Q_i(x_i, t) = 0, \qquad i = 1, \ldots, N. \tag{5}$$

For all $i = 1, \ldots, N$ we assume existence of smooth vector functions $x_i^*(t)$ such that $Q_i(x_i^*(t), t) \equiv 0$, i.e. $x_i^* = \mathrm{argmin}_{x_i} Q_i(x_i, t)$. Decentralized speed-gradient algorithm is introduced as follows (see Fradkov [2007]):

$$\dot{\theta}_i = -\Gamma_i \nabla_{\theta_i} \omega_i(x_i, \theta_i, t), \qquad i = 1, \ldots, N, \tag{6}$$

where

$$\omega_i(x_i, \theta_i, t) = \frac{\partial Q_i}{\partial t} + \nabla_{x_i} Q_i(x_i, t)^{\mathrm{T}} F_i(x_i, \theta_i, t),$$

$\Gamma_i = \Gamma_i^{\mathrm{T}} > 0 - m_i \times m_i$ - matrix.

3.2 Passification lemma

In order to formulate the passification lemma we need to introduce several definitions.

Definition 1. A linear system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t)$ with the transfer matrix $W(\lambda) = C(\lambda I - A)^{-1}B$, where $u(t), y(t) \in \mathbb{R}^l$ and $\lambda \in \mathbb{C}$ is called *minimum-phase* if the polynomial $\varphi(\lambda) = \det(\lambda I - A)\det W(\lambda)$ is Hurwitz. The system is called *hyper-minimum-phase* if it is minimum-phase and the matrix $CB = lim_{\lambda\to\infty}\lambda W(\lambda)$ is symmetric and positive definite.

We will need the passification lemma in the following form (see Fradkov [1976], Fradkov [2003]).

Lemma 1. (Passification lemma). Let the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$, $g \in \mathbb{R}^{l\times m}$ be given and the full-rank condition $rank(B) = m$ holds. Then for existence of a positive-definite $n \times n$-matrix $P = P^{\mathrm{T}} > 0$ and $l \times m$-matrix θ_* such that

$$PA_* + A_*^{\mathrm{T}} P < 0, PB = C^{\mathrm{T}} g, A_* = A - B\theta_*^{\mathrm{T}} C \tag{7}$$

it is necessary and sufficient, that the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= g^{\mathrm{T}} Cx(t) \end{aligned} \tag{8}$$

is hyper-minimum-phase.

Corollary 1. Suppose there exists $g \in \mathbb{R}^{l\times m}$ such that $g^{\mathrm{T}} C(\lambda I - A)^{-1}B$ is hyper-minimum-phase. Then there exist $P > 0$, θ_*, $\varepsilon > 0$ such that

$$PA_* + A_*^{\mathrm{T}} P < -\varepsilon I, PB = C^{\mathrm{T}} g, A_* = A - B\theta_*^{\mathrm{T}} C. \tag{9}$$

Remark 1. If the system (8) is hyper-minimum-phase then there exists θ_* such that a control law $u = \theta_*^{\mathrm{T}} y + v$, where v is a new control signal, makes the system (8) *strictly passive*, i.e. there exist a nonnegative scalar function $V(x)$ and a scalar function $\mu(x)$, where $\mu(x) > 0$ for $x \neq 0$, such that

$$V(x) \le V(x_0) + \int_0^t [v(t)^{\mathrm{T}} y(t) - \mu(x(t))]\, dt \tag{10}$$

for any solution of the system (8) satisfying $x(0) = x_0$.

3.3 Barbalat's lemma

We will need the Barbalat's lemma in the following form (see Popov [1973]):

Lemma 2. If $f(t)$ is a uniformly continuous function such that $f(t) \ge 0$ for all $t \ge 0$ and $\int_0^\infty f(t)dt < \infty$ then $f(t) \to 0$ while $t \to \infty$.

4. MAIN RESULTS

4.1 Adaptive controller structure

We will look for a control law in the following form:

$$u_i(t) = -\theta_i(t)^{\mathrm{T}}(y_i(t) - \bar{y}(t)) + \bar{u}(t), \tag{11}$$

where $\theta_i(t) \in \mathbb{R}^l$ is the vector of tunable parameters. Such choice of a control law is motivated by the following idea. If the difference between output of a subsystem and the leader system is not zero, we take a control proportional to this difference. The more difference value is the more control action we should apply to make it closer to zero. If the difference is zero, then we apply the same control action to the subsystem as we apply to the leader system.

Denote $e_i(t) = x_i(t) - \bar{x}(t)$ and apply the speed gradient algorithm with the local goal function $Q_i(e_i, t) = \frac{1}{2} e_i(t)^{\mathrm{T}} P e_i(t)$, where $P \in \mathbb{R}^{n\times n}$ is a positive definite matrix that will be determined later. Using (1),(3) derive an equation for $e_i(t)$:

$$\begin{aligned}&\dot{e}_i(t) = Ae_i(t) + h(x_i(t),t) - h(\bar{x}(t),t) + \sigma \sum_{j=1}^{N} \alpha_{ij} e_j(t) + \\ &\sigma \sum_{j=1}^{N} \beta_{ij} e_j(t-\tau) + b\left(u_i(t) - \bar{u}(t)\right), \\ &y_i(t) - \bar{y}(t) = Ce_i(t), \qquad i = 1, \dots, N.\end{aligned} \tag{12}$$

Then $\omega_i(e_i, \theta_i, t) = e_i(t)^{\mathrm{T}} P[Ae_i(t) + h(x_i(t),t) - h(\bar{x}(t),t) - b\theta_i(t)^{\mathrm{T}}(y_i(t) - \bar{y}(t))]$ and $\nabla_{\theta_i}\omega_i(e_i,\theta_i,t) = -e_i(t)^{\mathrm{T}} Pb(y_i(t) - \bar{y}(t))$. According to the speed gradient algorithm the adaptive law is chosen as $\dot{\theta}_i = -\gamma_i \nabla_{\theta_i}\omega_i(e_i,\theta_i,t)$, but this algorithm is not realizable because $e_i(t)$ in $\omega_i(e_i,\theta_i,t)$ is not available. Suppose there exists a vector $g \in \mathbb{R}^l$ such that $Pb = C^{\mathrm{T}} g$. Then $\dot{\theta}_i = -\gamma_i \nabla_{\theta_i}\omega_i(e_i,\theta_i,t) = \gamma_i e_i(t)^{\mathrm{T}} Pb(y_i(t) - \bar{y}) = \gamma_i (Ce_i(t))^{\mathrm{T}} g(y_i(t) - \bar{y}) = \gamma_i (y_i(t) - \bar{y})^{\mathrm{T}} g(y_i(t) - \bar{y})$. We derive a *realizable* control law:

$$\begin{aligned}u_i(t) =& -\theta_i(t)^{\mathrm{T}}(y_i(t) - \bar{y}(t)) + \bar{u}(t), \\ \dot{\theta}_i =& \gamma_i (y_i(t) - \bar{y})^{\mathrm{T}} g(y_i(t) - \bar{y}),\end{aligned} \tag{13}$$

where $\gamma_i \in \mathbb{R}$ is an arbitrary chosen positive constant.

4.2 Synchronization conditions for Lipschitz nonlinearity

Theorem 1. Let $h(x_i(t),t)$ be Lipschitz continuous, i.e. there exists a Lipschitz constant η satisfying $\|h(x_i(t),t) - h(\bar{x}(t),t)\| \le \eta \|e_i(t)\|$ for $1 \le i \le N$. Suppose that

$$\frac{\varepsilon}{2} I > P\left(\eta I + \sigma \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2}\right)$$

for $i = 1, 2, \dots, N$, where ε, P are from (9).

Then the adaptive controller (13) ensures achievement of the goal (2) and boundedness of functions $\theta_i(t)$ on $[0,\infty)$ for all solutions of the closed-loop system (1),(13) with bounded initial conditions $\|\bar{\varphi}(t)\|_C < \zeta$, $\|\varphi_i(t)\|_C < \zeta$ and any $\gamma_i > 0$.

Proof. Since $\frac{\varepsilon}{2} I > P\left(\eta I + \sigma \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2}\right)$, there exists $\delta > 0$ such that

$$\left(\frac{\varepsilon}{2} - \delta\right) I > P\left(\eta I + \sigma \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2}\right).$$

Select a Lyapunov-Krasovskii functional

$$\begin{aligned}V(t) =& \frac{1}{2}\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P e_i(t) + \\ & \frac{1}{2\gamma_i}\sum_{i=1}^{N} (\theta_i(t) - \theta_*)^{\mathrm{T}}(\theta_i(t) - \theta_*) + \\ & \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i(s)^{\mathrm{T}} H_i e_i(s) ds,\end{aligned} \tag{14}$$

where $H_i = \delta I + \sigma P \sum_{j=1}^{N} \frac{|\beta_{ji}|}{2}$ is a positive definite matrix. Note that as soon as initial conditions are bounded $V(0)$ is bounded too. Differentiating the function $V(t)$ we obtain

$$\begin{aligned}&\dot{V}(t) = \sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P[Ae_i(t) + h(x_i(t),t) - h(\bar{x}(t),t) + \\ &\sigma \sum_{j=1}^{N} \alpha_{ij} e_j(t) + \sigma \sum_{j=1}^{N} \beta_{ij} e_j(t-\tau)] - \\ &\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} Pb\theta_i(t)^{\mathrm{T}} Ce_i(t) + \\ &\sum_{i=1}^{N} (\theta_i(t) - \theta_*)^{\mathrm{T}} e_i(t)^{\mathrm{T}} C^{\mathrm{T}} g Ce_i(t) + \\ &\sum_{i=1}^{N} [e_i(t)^{\mathrm{T}} H_i e_i(t) - e_i(t-\tau)^{\mathrm{T}} H_i e_i(t-\tau)] = \\ &\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P(A - b\theta_*^{\mathrm{T}} C) e_i(t) + \\ &\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P(h(x_i(t),t) - h(\bar{x}(t),t)) + \\ &\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P\sigma \left[\sum_{j=1}^{N} \alpha_{ij} e_j(t) + \sum_{j=1}^{N} \beta_{ij} e_j(t-\tau)\right] + \\ &\sum_{i=1}^{N} [e_i(t)^{\mathrm{T}} H_i e_i(t) - e_i(t-\tau)^{\mathrm{T}} H_i e_i(t-\tau)].\end{aligned} \tag{15}$$

Using the inequality $2x^{\mathrm{T}} y \le x^{\mathrm{T}} Qx + y^{\mathrm{T}} Q^{-1} y$ it is easy to show that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_{ij} e_i(t)^{\mathrm{T}} P e_j(t) \le \\ &\sum_{i=1}^{N}\left[e_i(t)^{\mathrm{T}} P e_i(t) \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}|}{2}\right]\end{aligned} \tag{16}$$

and

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ij} e_i(t)^{\mathrm{T}} P e_j(t-\tau) \le \\ &\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P e_i(t) \sum_{j=1}^{N} \frac{|\beta_{ij}|}{2} + \\ &\sum_{i=1}^{N}\left[e_i(t-\tau)^{\mathrm{T}} P e_i(t-\tau) \sum_{j=1}^{N} \frac{|\beta_{ji}|}{2}\right].\end{aligned} \tag{17}$$

Then we can conclude that

$$\dot{V}(t) \leq \sum_{i=1}^{N} e_i(t)^{\mathrm{T}}(-\frac{\varepsilon}{2}I + \eta P + \sigma P \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}|}{2} + H_i)e_i(t) + \sum_{i=1}^{N} e_i(t-\tau)^{\mathrm{T}}(\sigma P \sum_{j=1}^{N} \frac{|\beta_{ji}|}{2} - H_i)e_i(t-\tau) =$$

$$\sum_{i=1}^{N} e_i(t)^{\mathrm{T}}[(\delta - \frac{\varepsilon}{2})I + \eta P + \sigma P \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2}]e_i(t) - \delta \sum_{i=1}^{N} e_i(t-\tau)^{\mathrm{T}} e_i(t-\tau) < -\delta \sum_{i=1}^{N} \|e_i(t-\tau)\|^2. \tag{18}$$

$$V(t) = V(0) + \int_0^t \dot{V}(s)ds \leq V(0) - \delta \int_0^t \sum_{i=1}^{N} \|e_i(s-\tau)\|^2 ds$$

Since $\dot{V} \leq 0$, $V(t)$ is bounded. Therefore $\int_0^\infty \sum_{i=1}^{N} \|e_i(s-\tau)\|^2 ds < \infty$. Using Barbalat's lemma we conclude that $e_i(t) \to 0$ while $t \to \infty$ for $i = 1, 2, \dots, N$. That is, the zero solution of the closed-loop system (12),(13) is asymptotically stable and $x_i(t) - \bar{x}(t) \to 0$ while $t \to \infty$ for $i = 1, 2, ., N$.

It is obvious that if $\theta_i(t) \to \infty$ then $V \to \infty$. But as it was shown $V(t) \leq V(0)$. That proves that $\theta_i(t)$ are uniformly bounded. □

4.3 Synchronization under matching conditions

The synchronization conditions formulated in Theorem 1 require that the Lipschitz constant η of $h(x,t)$ is sufficiently small which imposes strong restrictions on the nonlinearity $h(x,t)$. For structured nonlinearities this restriction may be relaxed. In order to formulate further result we need to introduce the following definition.

Definition 2. For given $G \in \mathbb{R}^l$ a function $f\colon \mathbb{R}^l \to \mathbb{R}$ is called G-monotonically decreasing if for any $x, y \in \mathbb{R}^l$ the following condition holds: $(x-y)^{\mathrm{T}} G(f(x) - f(y)) \leq 0$.

Note that if $l = 1$ and $G > 0$ then this definition coincides with the classical definition of a monotonically decreasing function.

Theorem 2. Suppose there exists $h_0(x,t)\colon \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}$ such that $h(x,t) = bh_0(x,t)$. Suppose that

$$\varepsilon I > \sigma P \sum_{j=1}^{N} (|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|)$$

for $i = 1, 2, \dots, N$, where ε, P are from (9). If for any t the function $h_0(x,t)$ is a g-monotonically decreasing function, where g is from (9), then the adaptive controller (13) ensures achievement of the goal (2) and boundedness of functions $\theta_i(t)$ on $[0, \infty)$ for all solutions of the closed-loop system (1),(13) with bounded initial conditions $\|\bar{\varphi}(t)\|_C < \zeta$, $\|\varphi_i(t)\|_C < \zeta$ and any $\gamma_i > 0$.

Proof. Since $\frac{\varepsilon}{2} I > \sigma P \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2}$, there exists $\delta > 0$ such that

$$\left(\frac{\varepsilon}{2} - \delta\right) I > \sigma P \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2}.$$

Select a Lyapunov-Krasovskii function as in (14) with the same H_i. By making calculations similar to (15) one can derive the following inequality for $\dot{V}$:

$$\dot{V}(t) \leq \sum_{i=1}^{N} e_i(t)^{\mathrm{T}} \left((\delta - \frac{\varepsilon}{2})I + \sigma P \sum_{j=1}^{N} \frac{|\alpha_{ij}| + |\alpha_{ji}| + |\beta_{ij}| + |\beta_{ji}|}{2} \right) e_i(t) + \sum_{i=1}^{N} (y_i(t) - \bar{y}(t))^{\mathrm{T}} g(h_0(y_i(t), t) - h_0(\bar{y}(t), t)) - \delta \sum_{i=1}^{N} \|e_i(t-\tau)\|^2 < -\delta \sum_{i=1}^{N} \|e_i(t-\tau)\|^2. \tag{19}$$

Similarly to the end of the proof for Theorem 1 we conclude that $x_i(t) - \bar{x}(t) \to 0$ while $t \to \infty$ and $\theta_i(t)$ are uniformly bounded for $i = 1, 2, ., N$.□

Remark 2. The statements of Theorem 1 and Theorem 2 hold in case of time-varying time delay $\tau(t)$ if $\dot{\tau}(t)$ is small enough. The upper bound for $\dot{\tau}(t)$ can be found by applying the same Lyapunov-Krasovskii function (14). As soon as for time-varying delay the following inequality holds

$$\dot{V} \leq \sum_{i=1}^{N} e_i(t-\tau(t))^{\mathrm{T}} [\frac{\sigma}{2} P \sum_{j=1}^{N} |\beta_{ji}| \dot{\tau}(t) - \delta I(1 - \dot{\tau}(t))] e_i(t - \tau(t)) \tag{20}$$

all statements hold if $\tau(t)$ is such that

$$\sup_{t \geq 0} \dot{\tau}(t) < \frac{2\delta}{2\delta + \sigma \|P\| \max_{1 \leq i \leq N} \sum_{j=1}^{N} |\beta_{ji}|}.$$

Remark 3. Note that all obtained results are delay-independent.

5. EXAMPLE. NETWORK OF LORENZ SYSTEMS

Lorenz system is a well known example of nonlinear system possessing complex chaotic behavior. Let us apply our results to synchronize a network of three interconnected identical Lorenz subsystems with the leader subsystem. Denote by $\sigma_L = 10, r_L = 28, b_L = \frac{8}{3}$ the value of the parameters for which chaos was observed in the original work Lorenz [1963].

For numeric simulation we took the following values of the system parameters:

$$A = \begin{pmatrix} -\sigma_L & \sigma_L & 0 \\ r_L & -1 & 0 \\ 0 & 0 & -b_L \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, C = (0\ 1\ 0)$$

$$h(x,t) = \begin{pmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix},$$

and the following values of coupling parameters: $\sigma = 0.2$, $\tau = 1$ seconds,

$$\alpha = \begin{pmatrix} -1 & 1/2 & 1/2 \\ 1/3 & -1 & 2/3 \\ 2/3 & 1/3 & -1 \end{pmatrix}, \beta = \begin{pmatrix} -1 & 1/4 & 3/4 \\ 1/2 & -1 & 1/2 \\ 1/3 & 2/3 & -1 \end{pmatrix}.$$

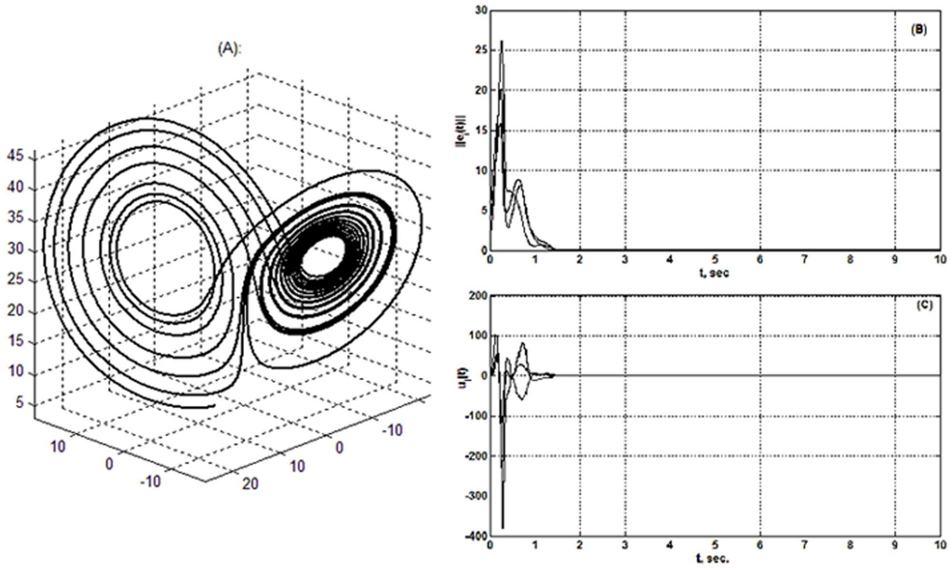

Fig. 1. Synchronization of three Lorenz systems with hyper-minimum-phase linear part. (A): Phase portrait of the leader subsystem, (B): $\|e_i\|$, (C): $u_i, i = 1, 2, 3$.

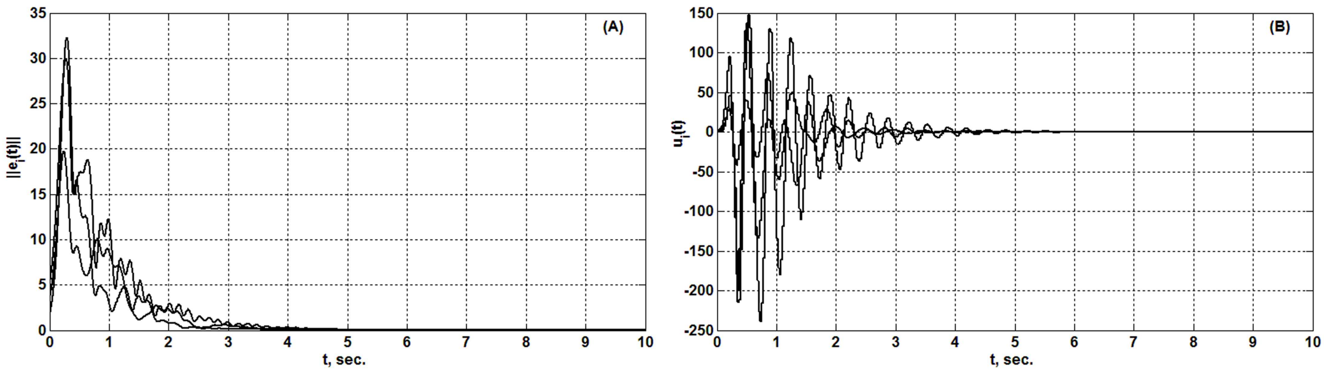

Fig. 2. Synchronization of three Lorenz systems with non hyper-minimum-phase linear part. (A): $\|e_i\|$, (B): $u_i, i = 1, 2, 3$.

Let us take $g = 1$. Then the transfer function of the linear system will have the following form $W(\lambda) = g^{\mathrm{T}} C(\lambda I - A)^{-1} b = \frac{\lambda+10}{\lambda^2+11\lambda-270}$. Therefore, the system is hyper-minimum-phase. By taking $\theta_* = -28$ and $P = I$, where I is an identity matrix, we ensure the conditions of Theorem 1. Hence, a decentralized adaptive controller (13) provides synchronization goal (2).

Phase portrait of the leader subsystem, $\|e_i(t)\|$, $u_i(t)$, $i = 1, 2, 3$ found by numeric simulations are shown in Fig. 1. It is easy to see that $\|e_i(t)\| \to 0$.

Now we would like to demonstrate that the conditions of Theorem 1 and Theorem 2 are not necessary, i.e. synchronization takes place for non hyper-minimum-phase system. Let us consider the same system but with $C = (1\ 0\ 0)$. The transfer function of the linear part is $W(\lambda) = g\frac{10}{\lambda^2+11\lambda-270}$. That is, for all g the linear system is not hyper-minimum-phase. And still according to the numeric simulations the control algorithm (13) with $\gamma = 1$, $g = 1$ will work. The results are presented in Fig. 2. We can see that $\|e_i(t)\| \to 0$, i.e. all three Lorenz systems are synchronized.

Remark 4. Both hyper-minimum-phase and non hyper-minimum-phase systems were also simulated for other values of time delay τ ($\tau = 0.01, 0.1, 10, 100$ seconds). In all cases the plots were similar to each other with insignificant differences. Therefore the results are practically delay-independent and they are valid for strong delays.

6. CONCLUSION

The delay-independent synchronization conditions for delayed coupling networks consisting of nonlinear systems with incomplete measurement, incomplete control, incomplete information about system parameters are obtained. The design of the control algorithm providing synchronization is based on speed-gradient method, while derivation of sufficient conditions for synchronization is based on the Passification lemma.

Further research will be aimed at the extension of the obtained results to the systems with variable delay.

REFERENCES

B. R. Andrievskii and A. L. Fradkov. Method of Passification in Adaptive Control, Estimation, and Synchronization. *Automation and Remote Control*, 67(11):1699–1731, 2006.

A. Das and F.L. Lewis. Distributed Adaptive Control for Synchronization of Unknown Nonlinear Networked Systems. *Automatica*, 46(12):2014–2021, 2010.

I. A. Dzhunusov and A.L. Fradkov. Adaptive Synchronization of a Network of Interconnected Nonlinear Lur'e Systems. *Automation and Remote Control*, 70(7):1190–1205, 2009.

A. L. Fradkov. Quadratic lyapunov functions in adaptive stabilization problem of a linear dynamic plant. *Siberian Mathematical Journal*, 17(2):341–348, 1976.

A. L. Fradkov. Passification of nonsquare linear systems and feedback Yakubovich-Kalman-Popov Lemma. *Europ. J. Contr.*, (6):573–582, 2003.

A. L. Fradkov. *Cybernetical Physics: From Control of Chaos to Quantum Control*. Springer, Heidelberg, Germany, 2007.

C.C. Hua, Q.G. Wang, and X. Guan. Global adaptive synchronization of complex networks with nonlinear delay coupling interconnections. *Physics Letters A*, 368 (3-4):281–288, 2007.

P.De Lellis, M. di Bernardo, and F.Garofalo. Novel decentralized adaptive strategies for the synchronization of complex networks. *Automatica*, 45:1312–1318, 2009.

C. Li, H. Xu, X. Liao, and J. Yu. Synchronization in small-world oscillator networks with coupling delays. *Phys. A*, 335(3-4):359–364, 2004.

Z. Li and G. Chen. Global synchronization and asymptotic stability of complex dynamical networks. *IEEE Trans. Circuits Syst. II, Exp.Briefs*, 53(1):28–33, 2006.

E. N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20(2):130141, 1963.

J. Lu and G. Chen. A time-varying complex dynamical network model and its controlled synchronization criteria. *IEEE Trans. on Autom. Control*, 50(6):841–846, 2005.

J. Lü, X. Yu, and G. Chen. Chaos synchronization of general complex dynamical networks. *Phys. A*, 334(1-2):281–302, 2004.

J. Lu, J. Cao, and D. W. C. Ho. Adaptive Stabilization and Synchronization for Chaotic Lur'e Systems With Time-varying delay. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 55(5): 1347 – 1356, 2008.

B. Mensour and A. Longtin. Synchronization of delay-differential equations with application to private communication. *Phys. Lett. A*, 244(1-3):59–70, 1998.

V. M. Popov. *Hyperstability of Automatic Control Systems*. New York: Springer, 1973.

K. Pyragas. Synchronization of coupled time-delay systems: Analytical estimations. *Phys. Rev. E*, 58(3):3067–3071, 1998.

X. Wang and Y. Cheng. Synchronization of the Non-delayed and Delayed Coupling Complex Network Via Adaptive Controlling. *International Journal of Nonlinear Science*, 8(1):73–78, 2009.

J. Yao, D.J. Hill, Z.H. Guan, and H.O. Wang. Synchronization of complex dynamical networks with switching topology via adaptive control. *Proceedings of the 45th IEEE Conf. Dec. Control*, pages 2819–2824, 2006.

C. Yoshioka and T. Namerikawa. Observer-based Consensus Control Strategy for Multi-Agent System with Communication Time Delay. *Proceedings of IEEE MSC-2008, San-Antonio, USA*, pages 1037–1042, 2008.

W.-S. Zhong, G. M. Dimirovski, and J. Zhao. Decentralized Synchronization of an Uncertain Complex Dynamical Network. *Proc. 26th Am. Control Conf., New York, USA*, pages 1437–1442, 2007.

J. Zhou, J. Lu, and J. Lu. Adaptive synchronization of an uncertain complex dynamical network. *IEEE Trans. on Autom. Control*, 51(4):652–656, 2006.