THE ECONOMICS OF NOISE POLLUTION

by

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Abstract

This thesis investigates the problems posed by the existence of noise pollution with the use of economic concepts. The analysis is conducted at various levels of abstraction and includes the opening up of some new fields of investigation, as well as tidying up and bringing together some previous work. The thesis is divided into six chapters. Chapter one introduces the subject and indicates the approach that is going to be taken in the subsequent chapters. Chapter two analyses the consequences for an optimum town of pollution such as noise. The necessary and sufficient conditions for an optimum are obtained and discussed. There are some comparative static results and the question of decentralisation is examined. Finally some simulation results are presented. The work in this chapter is perhaps least specific to noise pollution and would apply to any spatially distributed non-accumulating pollution. Chapter three examines the measurement of noise costs to households and compares two different approaches to the problem. Chapter four discusses the control of noise levels in the context of the economic analysis of Externalities and Public Bads, and emphasises some of the difficulties in obtaining optimum noise charges. Chapter five summarises the existing empirical work and adds some new results.

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This list is certainly incomplete. For one thing it does not include many people who indicated valuable references to me. I apologise for all names omitted that should not have been. All errors that remain are of course my own.

Chapter 1.

1.1. Scope

This thesis is concerned with an economic analysis of noise pollution[1]. This means that it concentrates on those aspects of noise pollution to which a professional economist may have something to contribute. Thus the coverage of material is essentially selective, and perhaps it is best to begin by stating those matters which are relatively neglected and explaining, in so far as we can why they are so.

First, it will appear to any acoustical engineer that the issues related to the measurement of noise and the construction of noise indices reflecting subjective evaluations of noise nuisance have been rather cursorily dealt with. The reason for this is simple - as economists we have rather little to say about them. The measurement of noise is essentially a physical problem, and as far as the construction of subjective indices is concerned, we are interested in evaluating whether such indices can be treated as cardinal economic quantities or whether they have to be transformed in some way to be converted into economic indicators. In Chapter 5 we discuss some of the evidence on this question with regard to indicators of aircraft noise. Unfortunately the price data required to do this is not available for other measures of noise nuisance.

Second, much of the discussion of the economics of noise pollution is concerned with aircraft noise. When we discuss the theory of the measurement of noise costs (Chapter 3) we illustrate the kind of problems involved with examples from the costing of aircraft noise, and the empirical work on this (Chapter 5) is drawn entirely from aircraft noise. We do consider other forms of noise nuisance when the control of noise levels is discussed (Chapter 4) and when the problems of environmental pollution such as noise nuisance are considered within the context of optimum towns. This bias towards aircraft noise arises from the way that noise economics has developed. Although, in overall terms, urban traffic noise is probably the most serious noise problem in the environment today[2], the suddenness with which jet aircraft noise became a major social problem (over the last decade and a half) has drawn more resources and attention to its measurement and control. As we state in the thesis, many of the techniques developed to measure the costs of aircraft noise carry over to the measurement of traffic noise and there is already some work in progress along these lines. The optimal control of surface traffic noise may, however, differ from that of air traffic noise and the relevant issues are discussed in chapter 4.

Third, we say very little about the physiological and psychological consequences of noise pollution, regarding which a considerable amount has been written[3]. Although a number of physiological effects are observed on individuals exposed to sustained traffic and industrial noise, it has not been established that such effects are damaging. This point is made with regard to the effects of noise on work by Kryter (1966) who states that, "Numerous laboratory and industrial studies have been made in attempts to show that noise has an adverse effect on the performance of physical and mental work. By and large, the results of these studies show that noise per se has little or no adverse effect upon performance provided that work does not require auditory communication of some sort. These results were found even in environments where the noise levels were such that near-daily exposure over several years would cause some permanent deafness".[4] Arguing on more general lines, but with regard to traffic noise the O.E.C.D. report (1971) states that, "There is at present no conclusive evidence that exposure to urban traffic noise under normal conditions produces harmful effects", and that, "one possible important - but as yet unproven - effect of traffic noise concerns the hastening of age-induced hearing loss (prebycusis)". Thus we may conclude that while research in this field is important, there is not enough evidence on the effects of noise on productivity or physical health to warrant either of these being considered as one of the unfortunate consequences of noise in an economic analysis.

In a recent publication Abey-Wickrama et alia (1969) have attempted to establish a positive association between the incidence of mental illness and exposure to high levels of aircraft noise, by doing a test of association of admissions to mental hospitals for two similar groups of households, one in the close vicinity of Heathrow Airport and the other in a quiet area nearby. They find a positive association for certain classes of individuals, and conclude that the matter warrants further investigation. It appears that in this study the control group was defined with respect to broad categories, namely socioeconomic status, age, sex, marital status, population density and migration rates. The incidence of psychiatric illness, however, can vary considerably across other classifications such as profession or occupation, occupation of husband, ethnic background and immigrant status, and similar factors. Until a finer classification and further study is done these results can only leave us with a vague sense of disquiet (sic) about high levels of aircraft noise. Given that these findings are confirmed a decision would have to be made as to whether the land area affected should be zoned for non-residential purposes or whether individuals should be informed about the possible consequences of living close to airports and allowed to make up their own minds. In our analysis, however, we do not consider any such deleterious effects.

1.2. The Development of Noise Pollution

Noise as a source of annoyance has been around for a long time. Being a passing experience, it leaves no trace. This, combined with the fact that the measurement of noise, as it is relevant to human perception, is a twentieth century development, has meant that as far as we know there is no precise documentation of noise levels before 1930.[5] Nevertheless there are a number of literary references to the excessive noise levels in the streets going back to the eighteenth century and urban historians record the use of straw on streets outside hospitals or the homes of rich people to reduce the noise of the clatter of horses hooves[6]. We do not know, however, whether the urban environment was noisier then than it is now. To be sure the growth of interest can be partly explained by the development of measuring techniques, but the causation goes the other way as well; an increased concern about the noise in our environment has led to suitable measures. This increased concern is undoubtedly due to increased material living standards among the population,and as a witness to this, it is the richer countries of the world that are more concerned about noise.

The earliest full survey of noise is probably the one undertaken in New York City between November 1929 and May 1930 by the New York Noise Abatement Commission. This showed that traffic noise was responsible for 36 per cent of all noise complaints, public transportation for 16 per cent, radios for 12 per cent, collections and deliveries for 9 per cent, whistles and bells for 8 per cent, construction for 7.5 per cent and miscellaneous sources for 11.5 per cent. These figures are quoted by Anthrop (1973) who points out, interestingly, that the measured noise levels in New York then were not very different from those of today, and that smaller cities such as Philadelphia now have noise levels closer to those of New York than they did forty years ago.

In the post war years most advanced countries have conducted some survey of traffic and neighbourbood noise with a view to enacting appropriate controlling legislation. One survey, which perhaps deserves special mention is the Wilson Report (1963) which reported on surveys in 1948 and 1961. This showed that the percentage of people disturbed by noise external to the home rose from 23 per cent in 1948 to 50 per cent in 1961[7]. This report also singled out traffic noise as the major noise irritant. The reports on other surveys in advanced industrial countries along these lines and the existing controlling legislation is summarised in the O.E.C.D. report (1971). With the advent of jet aircraft in 1958 a new, identifiable and compact source of noise nuisance emerged. This has received considerable attention since then. K.D. Kryter developed a scale referred to as Perceived Noise Level (PNL) which measures aircraft noise while taking account of the subjective response of individuals to its 'noisiness'. This scale was then used by McKennell in England to develop an index of aircraft noise that took account of the PNL as well as number of aircraft flying over head and correlated a linear combination of these factors with subjective noise annoyance responses. This index is known as the Noise and Number Index (NNI). Similar scales were developed in the U.S., France and Germany. Recently the (PNL) has been refined somewhat to produce a new scale called EPNL (Effective Perceived Noise Level) and this has been used in conjunction with a measure of aircraft numbers that takes account of the time of flight to produce an index called the Noise Exposure Forecast. (NEF).

Undoubtedly the most sophisticated measurement of noise costs has been done on aircraft noise, using the above measures. These measurements have so far been used to assess the noise costs of prospective airport sites but not, as yet, to evaluate the benefits of noise reduction in aircraft engines. It is to be hoped that as the economic techniques become better understood, it will be possible to conduct such evaluations, and, furthermore to use the techniques to evaluate alternative proposals for surface traffic reduction.

1.3. The Economists Role with Regard to Noise Pollution

We started out by stating the limitations of an economic analysis of noise pollution. We now consider the contribution that an economist can make in a field such as this.

One line of approach, and the one taken in this thesis, is that an economist can establish the principles by which the consequences of noise are valued so that they can be compared with other social costs. To do this he uses a numeraire - money as it happens, although it could be Stradivariuses - and he defines carefully the alternative assumptions upon which this evaluation can be made. He also assesses the applicability of the tools of the economic analysis to the problem of noise pollution and evaluates the policy lines for dealing with problems of noise in the light of his assessment. This kind of exercise is addressed to the professional or would-be professional economist and in it one must regard the theory and its application as both being under review.

The other approach that an economist can take is to set himself up as a champion for the 'economic' viewpoint. In this he will present the arguments <u>for</u> an evaluation of noise costs and policies towards noise by the application of the principles of economic theory, recognizing that in an essentially multidisciplinary field, the engineer, psychologist, geographer and aviation expert are all doing the same for those aspects of the problem that their speciality entails. The engineer, for example is always at pains to point out that the essentials of the noise problem lie in designing quieter engines, or ones that take-off vertically and that costing noise is an imprecise and effete exercise. The psychologist sees the noise problem as a matter of "perception" and unquantifiable in cost terms - a view he shares with the planner who sees the soul of man at stake. In this motley crowd the economist too has his caricature - too awful to mention - and the end result of most multidisciplinary projects is a function of the relative personal weights of the professional representatives as well as the relevance of their subjects. Thus although the contribution of each group to the decision making process, and the interactions of the groups is an interesting/and important field of study, this thesis has little to contribute to it.

1.4. The Development of the thesis

We begin our analysis by a review of some of the current developments in urban welfare economics. The problems of environmental externalities such as noise and air pollution have some bearing on the results obtained hitherto and in chapter 2 we discuss these externalities in the context of highly simplified models of towns. Some of the basic non-convexities that arise when such externalities are considered are discussed and we outline some of the difficulties in obtaining an optimum by competitive decentralisation.

Although the model is highly simplified it gives us some indication of the important trade offs in obtaining the optimum town. By parameterising the simple model that we construct and taking plausible ranges of parametric values, we obtain some numerical results of the various trade-offs available.

Having considered the problem of noise pollution in a simplified but general equilibrium framework we next consider in Chapter 3, the measurement of noise costs in the context within which they might be practically evaluated. In this section we outline two alternative approaches to costing noise and evaluate them in the light of the available evidence. Chapter 4 deals with optimum control of noise, the analytical tools involved and problems inherent therein. Chapter 5 discusses some of the empirical evidence regarding the measurement of aircraft noise costs, including the appropriateness of the noise measurement indices, and in Chapter 6 we try and bring together what overall conclusions emerge from the thesis.

The bias in this study is evidently towards the theoretical and normative aspects of the problem.

Chapters 2 and 4 are concerned with making optimal decisions with respect to certain welfare criteria, and are clearly normative in their content. Chapter 3 is concerned with noise cost measurement and along with the relevant empirical evidence in Chapter 5, the analysis here is descriptive in so far as it is concerned with measuring what <u>is</u>. However, at the same time we devote some attention to the welfare implications of the different costs measured. Regarding these biases we can only plead personal preference - doubtless others would have done it differently.

FOOTNOTES TO CHAPTER 1

1. A purist might complain at the use of the term noise pollution - pollution being defined in the S.O.E.D. as a physical impurity. However, the usage of the term in this context is common and well understood. Another caveat that should be added at the start is that noise pollution refers in this context to outdoor noises. Factory and household noise in so far as it does not affect people outside does not confer external disbenefit and therefore is beyond the scope of an economic treatment of noise as a pollutant.

2. See for example the O.E.C.D. report (1971) or the Noise Advisory Council (1972) (b).

3. For a survey listing much of this work see Branch (1971)

4. Kryter (1966) p.1332

5. This date refers to the noise survey carried out in New York See Anthrop (1973), Chapter 5.

6. For an interesting review of anecdotal evidence on noise in the Victorian cities the reader is referred to Dyos and Woolf (1973)

7. If the New York finding that surface noise has not increased over this sort of period holds for London, then the increased percentage of people reflects only an increased awareness of noise.

CHAPTER 2.

The Optimum Structure of Towns with Environmental Pollution such as Noise.

2. 1 INTRODUCTION.

In this chapter we consider the problem of forms of pollution such as noise in relation to the optimal planning of towns. This brings together two problems that have so far been considered separately in the literature, but which are clearly related. The pollution that is generated by the activities of production and consumption has an effect on the optimal location of the residential areas relative to the work areas, and the size of the town and the location of the individuals in it effects the optimal control of the pollution level. It is with these issues that we are concerned, when the characteristics of the pollution are similar to those of noise - i.e. concentrated in the town centres and spreading out over the residential areas.

The economic literature that deals with the welfare aspects of urban location is not vast. Perhaps one of the first attempts in this field was by Solow and Vickrey(1971). In this paper they analysed, in the simplest possible setting, the optimal allocation of a given urban land area between the generation of traffic and the carrying of traffic. Theyobtained a characterisation of the optimal proportion of land that should be allocated to the carrying of traffic as a function of the distance from the city centre, and showed that if a competitive market was used to allocate land in the absence of congestion tolls, it would tend to overallocate land for the use of traffic, especially near the centre. In a subsequent paper, Mirlees (1972), discussed the problem of the optimum town more fully. Individual preferences were represented by a cardinal utility function, the arguments of which were a consumption good, the distance from the city centre, and living area occupied. Production was organised at the centre, with households located around it. For a fixed population and a fixed amount of output, he maximised the sum of utilities to obtain the necessary and sufficient conditions for a maximum, and analysed what could be said from this about the distribution of the population around the town centre, and the distribution of utility over the population. In general, it is not possible to say whether the land area occupied by a household increases with the distance from a city centre. This depends on how the marginal utility of an increase in area changes as the distance from the centre increases. With regard to the distribution of utility Mirlees showed that in general it is not optimal for identical individuals (i.e. with the same preferences) to be treated identically. The extent of the inequality that is optimal depends on the particular cardinalisation that is chosen for the utility function and on the form of the utility function. However, the forms of the utility function that yield equal utility are rather restrictive and even in these cases Stern (1973) has shown that a small change in the structure of the problem leads to an unequal distribution of utility being optimal again. The impact of the cardinalisation chosen on the optimal distribution of utility is now a well known problem. For a utility function U(.) we may define a transformation of it, viz.

$$\Phi(\cdot,\sigma) = \frac{1}{1-\sigma} U(.)^{\frac{1}{1-\sigma}}$$

Such a transform is isoelastic, with the elasticity of Φ with respect to U being $1/1-\sigma$. Then as σ is increased the marginal utility of the arguments of the utility function decrease more rapidly, and any inequality is penalised further in deriving the optimal conditions. In the limiting case as $\sigma \to \infty$ we obtain the Rawlsian welfare function:

$$\Phi(\,.\,,\infty) = \inf.\ U(.)\,,$$

and in this case we get, in the Mirrlees framework, an equal level of utility for all individuals as being optimal. The reason why utility inequalities can be optimal in this context is that individuals are indivisible and that any one person can only be resident in one place. If we could smear people over space, the concavity of the utility function would automatically lead to an equal distribution of utility. The force of this important point can be seen by considering the following simple problem:

There are two individuals who can each either be located at a distance of one mile from their place of work or four miles from their place of work. The concave utility function is

$$U = (A - x)^{\frac{1}{2}} + C^{\frac{1}{2}} \qquad A > 0,\ x > 0\ ,\ C > 0$$

Where x is the distance in miles from the place of work, C is the amount of the consumption good and A is a constant. If there are 8 units of the consumption good to allocate and we let A equal 5[1], then it is clear that total utility is maximised when consumption is equally allocated; i.e.

$$U_1 = 2 + 2 = 4$$
$$U_2 = 1 + 2 = 3$$

Of course the two individuals could be made to change places periodically so as to equate the total utility that each received. This, however, could only be considered worthwhile if equality had some separate desirability, over and above that expressed in the utility function.

In view of the above discussion there is certainly some appeal for the use of the Rawlsian Welfare function in this context, for it generates an equal distribution of utility when there are good reasons for thinking that equality is desirable. It is not, however, overwhelmingly clear that other cardinalisations with desirable properties should not be considered, especially in attempting to examine the <u>extent</u> of inequality that is optimal for different values of σ. This task has been attempted by Dixit (1973) within the confines of a model where production was organised centrally and the major consideration was a trade-off between economies of scale in production and diseconomies of congestion in commuter transport. Obtaining numerical solutions for plausible parameter values he showed that the extent of inequality can become quite large for values of σ that are thought to be reasonable in other contexts.

The model as outlined above was extended by Mirrlees to deal with the issue of congestion in commuter transport, and congestion in residential location. Regarding congestion in commuter transport, he derived the optimality condition when there was a trade-off between an increase in land use for roads to the centre, which would lead to a reduction in congestion costs and a fall in the land area available for residential location. These optimality conditions can be obtained by a competitive realisation where there is a land rent function, relating land rents to the distance from the city centre, a distribution of incomes over the population, with each individual <u>not</u> knowing in advance what his position in that distribution will be, and a commuter subsidy function, fixing a subsidy to all residents at distance r from the centre as $q(r)$. Each household takes this information as given and maximises its utility subject to its budget constraint. With the appropriate functions and distribution, the desired optimum solution can be obtained. This is indeed an interesting result, for it establishes the possibility of competitive decentralisation when some of the usual conditions for such a decentralisation are not satisfied. (i.e. we have a continuum of commodities and individuals and a non convex consumption set, and normally these conditions do not allow a competitive decentralisation.)

Finally Mirrlees introduces the problem of the optimum size of a town when one part of a fixed population has to be located in t identical towns and the rest is located in the countryside. Production in the towns takes place, initially, under conditions of increasing returns to scale with labour as the only factor, and production in the country takes place at a constant average product of labour. A simple rule can be derived for the optimum size of the towns, with and without externalities. This rule suggests that the optimum size of the town is probably one with locally increasing returns to scale when there are externalities and certainly one with locally increasing returns to scale when externalities are not considered. It is also possible that the rule would not define the optimum size of the town uniquely, and in that case global comparisons are necessary. The reason for considering increasing returns to scale is apparent - it provides the only reason for the existence of towns in this framework. If the marginal product of labour were less than the average product, then dispersed production would save on transportation costs, possibly provide more living space for each household, and cause no loss of output. This point has been made clearly by Starrett (1972) who establishes the optimality of increasing returns to labour in determining the town size under rather general conditions.

The above discussion of the importance of increasing returns is conducted within the context of only one factor of production - labour. When there is more than one factor of production involved then there is the possibility of considering increasing returns to labour alone (and diminishing returns to the other factors) or diminishing returns to each of the factors and increasing returns to scale. Both would suffice to provide a rationale for the existence of centralised production, but would have different implications for town size. In his recent paper Dixit[2] examines the optimum size of a town under various plausible assumptions regarding the parameters, when output depends on the amount of labour and the size of the central business district. Both factors are assumed to have diminishing returns, but there are increasing returns to scale overall. Within this framework Dixit obtains the optimum town size as a function of the economies of scale present, and concludes that a town size of over a million is difficult to justify. When choosing plausible values of the elasticities of output with respect to labour and land, he admits that his choice of parameters suggests too low a density of workers in the central business district but argues that with different parameters he "could not study economies of scale to any significant extent without running into increasing returns to labour alone." This suggests, however, that increasing returns to labour alone is perhaps the more typical case.

2. 2 The Problem of Pollution in Optimum Towns.

In this chapter we work very much in the simple framework outlined by Mirlees and developed by others to consider the problems of pollution in the optimal construction of towns. The pollution in this context can be considered as a byproduct of the production process. It emanates at the centre and spreads out over the residential areas, decreasing and eventually disappearing as the distance from the centre gets large enough. Thus noises, industrial waste, and air pollution could be typical examples. This byproduct can be reduced, but doing so involves allocating resources to removing it. In Section 2.3 we consider a town of fixed size and define the necessary conditions relating to the optimum allocation of individuals and the optimum level of pollution for the utilitarian and Rawlsian Welfare functions. We also investigate the conditions under which a unique optimum will exist and conclude that such uniqueness is extremely improbable, even when very special forms of the utility function are considered. From this it appears that the decentralisation proposed by Mirlees will not carry through straightforwardly to the case where there is a spatial context to the pollution. In some special circumstances decentralisation to a local optimum is feasible, as long as the government checks independently for the local concavity of the sum of the marginal opportunity costs of pollution in terms of the consumption, with respect to the level of pollution. In this section we also examine in the context of an additive utility function the allocation of resources to individuals as a function of the distance from the city centre, and the relationship between the residential positions and the work centre as a function of the level of pollution, and the size of the work force.

In Section 2.4 the optimum size of the town is considered, when the population is to be allocated in a large number of similar towns. The optimum town size is compared with that which would emerge under a competitive structure, with no control on pollution. The conditions under which the optimum town is smaller than the competitive town are stated. The decentralisation of the optimum town when the size is a variable is of course even more difficult than that of a town of fixed size and in general one would expect not to be able to attain such a decentralisation.

In Section 2.5, a parametric representation is set up and the optimum town size obtained for various parameter values. This is compared to the size that would emerge under alternative competitive conditions. Section 2.6 offers some general conclusions.

It will be clear to anyone who has examined the literature in this field that the models used are a grotesque simplication of the manifold reasons for the existence of towns. Some of these reasons, especially those relating to the social and trading aspects of towns, are hardly examined by any of the models. The other problems of urban organisations, such as commuter and residential congestion, the trade off between travel time and living space, between economies in production and diseconomies in congestion, between economies of production and costs of cleaning up the pollution, and <u>others</u>, are usually examined in isolation from each other. The explanations that can be given for this partial treatment are, (a) that to consider a problem in isolation highlights those features that are special to itself and (b) that the study of urban welfare economics is still in its infancy and more complex models must follow simple ones. Nevertheless it remains true that the results obtained must be interpreted with caution. Specifically the numerical results regarding the trade-offs examined here exaggerate the choices available, and when all the margins of adjustment are considered, the effects of pollution on the urban environment will not be so marked as is suggested here.

2.3 An Optimum Town of Fixed Size

2.3.1 The Optimising problem and conditions defining the optimum.

In this section we consider a town with a fixed number of individuals with identical tastes. Production is organized at the centre and consists of producing a consumption good and a byproduct called pollution. Some of the town's individuals can be allocated to the removal of pollution. Thus the production assumptions may be written down as,

$$y = f(n_1) \qquad f'' > 0 \quad f' > 0 \quad y \geq 0 \quad n_1 \geq 0 \tag{1}$$

$$Z = f(n_1) - g(n_2) \quad g' > 0 \quad f(N) \geq Z \geq 0 \tag{2}$$

$$n_1 + n_2 = N \qquad N > 0 \tag{3}$$

where y is the quantity of the consumption good, n_1 the amount of labour used in its production, Z the net quantity of pollution, and n_2 the amount of labour used in pollution removal. N is the total amount of labour. The units of measurement of pollution are so chosen that one unit of output produces one unit of 'gross' pollution as a byproduct[3]. There are assumed to be increasing returns to labour in production, but no assumption is made as yet regarding returns to labour in pollution removal. The households utility depends on the quantity of the consumption good, c, that it is allocated, the distance from the centre at which it resides, x, and the level of pollution, q, that it suffers. q will depend upon the net quantity of pollution produced, z, and the distance from the centre at which the household is located. Thus we may write:

$$U = U(c, x, q(z,x)) \quad U_c > 0, \quad U_x < 0 \quad U_q < 0 \tag{4}$$

We do not consider here the amount of living space occupied as a variable. This can easily be incorporated into the analysis but is held constant so as to concentrate on the other issues. In our analysis we will consider utility to be an increasing function of c and a decreasing function of q and x. The function q will be assumed to be an increasing function of z and a decreasing function of x .

To consider the optimisation as simply as possible we assume that households are located on a line with the town at the centre. Again it is possible to examine the case when households are located in the two dimensional Euclidean space with the centre of the town as its origin. As long as x can then be interpreted as the distance from the centre, implying that pollution spreads itself 'symmetrically' around the town, nothing is gained by doing so. The consideration of geographic and climatic factors that would lead to an asymmetric distribution of pollution in the neighbourhood of the town is not the concern of this analysis. When we come to consider some numerical values for the optimum town then it is important whether the town is treated as monocentric or as long and narrow. At that stage we will consider both shapes. Therefore, taking the additive welfare function initially, we may write the maximisation as

$$\max \quad 2\int_x U(\, c(.),\, x,\, q(x, Z)\,)\, h(x)\, dx \tag{5}$$

Where U is a strictly concave function of c, x, and q, increasing in c and decreasing in x and q, and where q is an increasing function of Z and a decreasing function of x. All the relevant partial derivatives are assumed to exist. By the assumption that each household occupies a fixed amount of space, the function h(x) takes the values $\frac{2}{N}$ or 0, it taking the former value if the value of x for which it is evaluated is assumed to be 'occupied'. The constraints on the maximisation are

$$\frac{N}{2}\int_x c(x)\;\; h(x)\, dx \;=\; y \tag{6}$$

$$y \;=\; F(N,\, Z) \tag{7}$$

Equation (7) is an explicit form of the relationship between y, N and Z that is obtained from equations (1) to (3) when n_1 and n_2 are eliminated. It is clear that the necessary conditions for a maximum to the above require that consumption be so allocated that the marginal utility of consumption is the same for all households, no matter where they are located [4]. If this were not so, then it would always be possible to increase the sum of utilities by marginally reallocating consumption. Furthermore the marginal product of Z should be equal to the sum of the marginal rates of substitution between the consumption good and Z. This is the familiar condition for determining the optimum quantity of a Public 'bad'. Thus we may write the necessary optimum conditions as,

$$\text{For all } x: \; h(x) > o, \qquad U_c = \lambda \tag{8}$$

$$\frac{N}{2} \cdot \int_x - (U_z \,/\, U_c)\, h(x) \geq F_z \tag{9}$$

With equality holding in (9) when $o < Z < f(N)$.

These necessary conditions will define one or more sets of functions $h(x)$, $c(x)$, and one or more sets of values of Y and Z. We define {P} as the set of points x where $h(x)$, as obtained from the necessary conditions, is positive. Then if, the utility function is continuous in x it is clear that all individuals who are located at extreme points of this set {P} must have equal utility [5]. For, consider two households at x_1 and x_2 and suppose that $U(x_1) > U(x_2)$. (We supress the other arguments of the utility functions as they are held constant.) By the definition of continuity, $\forall\ \varepsilon > o\ \exists\ \partial > o : |x - x_1| < \partial \rightarrow |U(x) - U(x_1)| < \varepsilon$. If we let $\varepsilon = U(x_1) - U(x_2)$ then we know that there is an x, sufficiently close to x_1 where the difference in utilities between the two individuals would be smaller than with one of them located at x_2. Since x_1 is an extreme point of {P}, it follows that there is a point close enough to x_1 that is unoccupied. Hence overall utility can be increased by relocating individuals.

To say anything more about the location of the individuals, it is necessary to make some further assumptions about the utility function. If the utility function is concave in x and c then the population density is non zero over a continuous range of values of x. To see this suppose that the population density is positive for $x \leq x_0$ and $x \geq x_{00}$ and zero for $x_{00} > x > x_0$. We may write the utility function as:

$$U(c,x,\ q(x,Z)) \quad = \quad V(c,x,Z) \tag{10}$$

For given Z, we have by the concavity assumption $\forall \quad 0<\lambda<1$:

$$V\ (\lambda x_0 \ + \ (1-\lambda)\ x_{00},\ \ \lambda c_0 + (1-\lambda)\ c_{00},\ Z) \geq (\lambda V(x_0,\ c_0,\ Z) \ + \ (1-\lambda)\ V(x_{00},\ c_{00},\ Z) \tag{11}$$

$$V\ ((1-\lambda)\ x_0 + \lambda\ x_{00},\ (1-\lambda)\ c_0 + \lambda\ c_{00},\ \ Z) \geq (1-\lambda)V(x_0,\ c_0,\ Z) \ + \ \lambda V(x_{00},\ c_{00},\ Z) \tag{12}$$

Adding:

$$V\ (\lambda x_0 + (1-\lambda)x_{00},\ \lambda c_0 + (1-\lambda)c_{00},\ Z) \ + \ V((1-\lambda)x_0 + \lambda x_{00},\ (1-\lambda)c_0 + \lambda c_{00},\ Z) \geq V(x_0,\ c_0,\ Z) + V(x_{00},\ c_{00},\ Z) \tag{13}$$

Hence by redistributing c and relocating the households previously at x_0 and x_{00} so that they are closer together, we may increase the overall utility.

The concavity of the utility function in c and x requires, in addition to what has already been assumed regarding the utility function, that the marginal disutility of living further out does not rise as net pollution increases ($U_{xq} \leq 0$) and that the level of net pollution declines less rapidly as the distance from the centre increases. ($q_{xx} > 0$). While there is some evidence to suggest that the latter might be the case for noise and for some forms of pollution, [6] there is no reason to suppose that $U_{xq} \lesseqgtr 0$. As a special case we may consider the utility function that is additively separable between c and the other arguments of the function. In that case $U_{xq} = 0$, and the requirement that the marginal utility of consumption be constant (equation (8)) is satisfied when all individuals have equal consumption, and the distribution of utilities for a concave utility function in c and x may be represented as shown in diagram 1, and 2. When the utility function is additively separable, however, the population density can be non zero even if the utility function is not concave in x. All that is required then, is that utility as a function of x be single peaked.

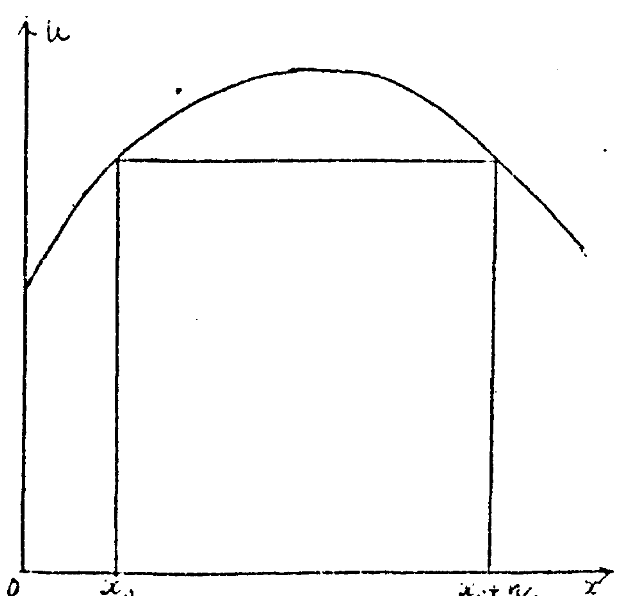


Diagram 1.

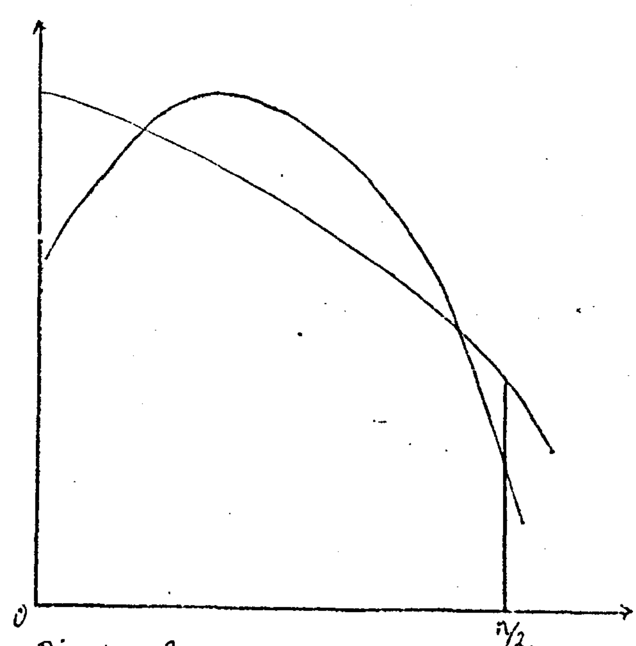


Diagram 2.

The area that is occupied is simply the peak of the utility function. Since each individual uses a fixed length, we may choose the units of x such that he occupies a unit length, and the points x_0 and $x_0 + n/2$ are chosen either side of the peak, subject to the constraint that $U(x_0) = U(x_0 + n/2)$. This constraint applies as long as $x_0 \neq 0$. If the residential area starts at the town centre as in diagram 2, then $U(x_0) \geq U(x_0 + n/2)$.

So far we have only consider/ed the necessary conditions for the maximisation of an additive welfare function. In the case of the Rawlsian welfare criterion, we wish to choose c(x) and Z, so as to maximise utility, subject to utility being independent of x (i.e.constant for all persons). The necessary condition relating to the choice of Z, for given N will still be that the sum of the marginal rates of substitution between c and Z (the marginal opportunity cost of c in terms of Z in consumption) be equal to the marginal product of Z, (for non zero Z and non zero antipollution). For if that condition were not satisfied, it would be possible to raise everyones utility by marginally changing Z and reallocating the change in output so as to keep all utilities equal. The choice of the function c(x) and the location of individuals can be looked at in terms of the indifference points between c and x. If for any given set of indifference points that represent a given level of utility we choose the combinations of c and x, so as to minimise the use of the consumption good to locate n/2 individuals, then we will obtain the necessary conditions for an optimum, for this procedure will allow us to choose the highest indifference curve for a given level of output. It seems intuitively clear that if the utility function is continuous, all individuals located by the above procedure such that they have no 'neighbours' on at least one side must have equal consumption. For if they did not, we may consider two such households located at x_0 and x_{00} where the consumption of the former c_0, is greater than that of the latter c_{00}. It would then be possible to move the individual at x_o sufficiently close to x_{oo}, such that he obtained the same utility as before, but used less consumption resources. Hence the original allocation could not have been optimal. Formally we may state this proposition as,

Let the set of points at which individuals are located be $\{P^1\}$. If the utility function is continuous in x and c and increasing in c then a necessary condition for maximising a Rawlsian welfare function is that consumption at all extreme points be equal.

Proof.

Let x_o and x_{oo} be two extreme points and let $c_o > c_{oo}$. If we can show that there exists a $c^* < c_o$, such that for some unoccupied x $U(x, c^*) = U(x_o, c_o)$ then we have established our proposition.
By continuity in the neighbourhood of x_{oo} we have that
$\forall \ \varepsilon > o \ \exists \ \delta \ : \ | x - x_{oo} | < \delta \Rightarrow | U(c_{oo}, x) - U(c_{oo}, x_{oo}) | < \varepsilon$
Since x_{oo} is an extreme point, given ε, there exists an unoccupied x:

either $\quad 0 < U(c_{oo}, x) - U(c_{oo}, x_{oo}) < \varepsilon \qquad (14)$

or $\quad 0 < U(c_{oo}, x_{oo}) - U(c_{oo}, x) < \varepsilon \qquad (15)$

In case (14) $U(c_{oo}, x) > U(c_{oo}, x_{oo})$ and therefore there exists a $c^* < c_{oo} < c_o : U(c^*, x) = U(c_{oo}, x_{oo})$. In case (15) $\varepsilon > U(c_{oo}, x_{oo}) - U(c_{oo}, x) > 0$ and therefore by making ε small enough we can make the difference between $U(c_{oo}, x_{oo})$ and $U(c_{oo}, x)$ sufficiently small so that, since U is increasing in c, there exists a c^*, $c_o > c^* > c_{oo}$ and

$$U(c_{oo}, x_{oo}) = U(c^*, x). \qquad (16)$$

In the case where U is concave in x the indifference curves between c and x may be drawn as shown in diagrams 3 and 4. The location of the individuals will be continuous in both cases, with consumption falling initially with distance and then rising in diagram 3 and rising continuously in diagram 4.

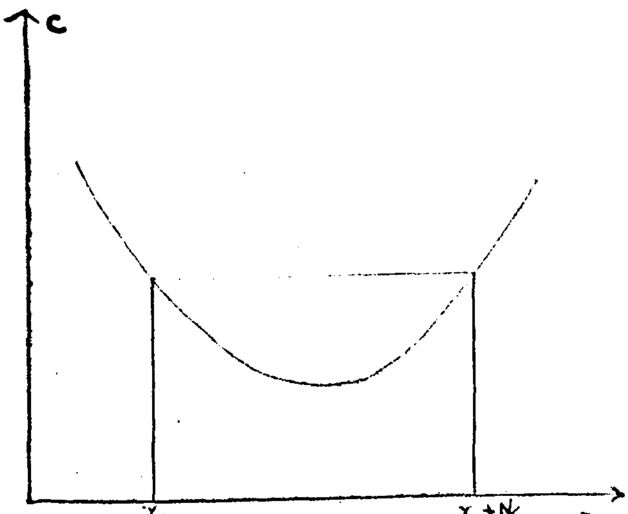


Diagram 3

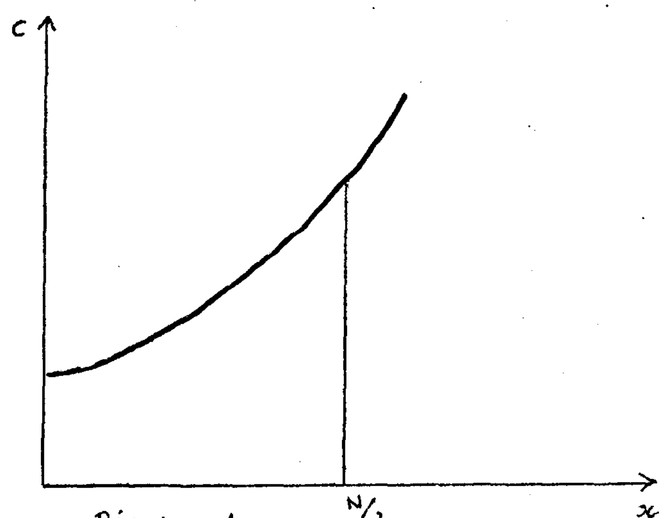


Diagram 4

So far in our analysis we have only considered the necessary conditions for an optimum to an additive utility welfare criterion and to a Rawlsian welfare criterion. We now examine with what additional conditions the necessary conditions derived above combine to represent a global optimum.

The sufficiency conditions can perhaps best be discussed with reference to diagram 5 below. This represents the production and welfare trade-offs between Y, the total quantity of the consumption good, and Z the net quantity of pollution. The production constraint is represented by the bold line. In general a positive output is possible with no pollution, the point Y_0 occuring when the maximum amount of labour is allocated to pollution removal. The welfare trade-offs between Y and Z represent the amount of extra Y the community needs, given a marginal increase in Z, to keep total welfare constant <u>when the location of the individuals is optimally chosen</u> for each Y and Z. It is clear that conditions (8) and (9) will be sufficient for a maximum if the production constraint and the function representing the welfare trade-off are both concave. We therefore have to investigate the conditions under which these functions will be concave.

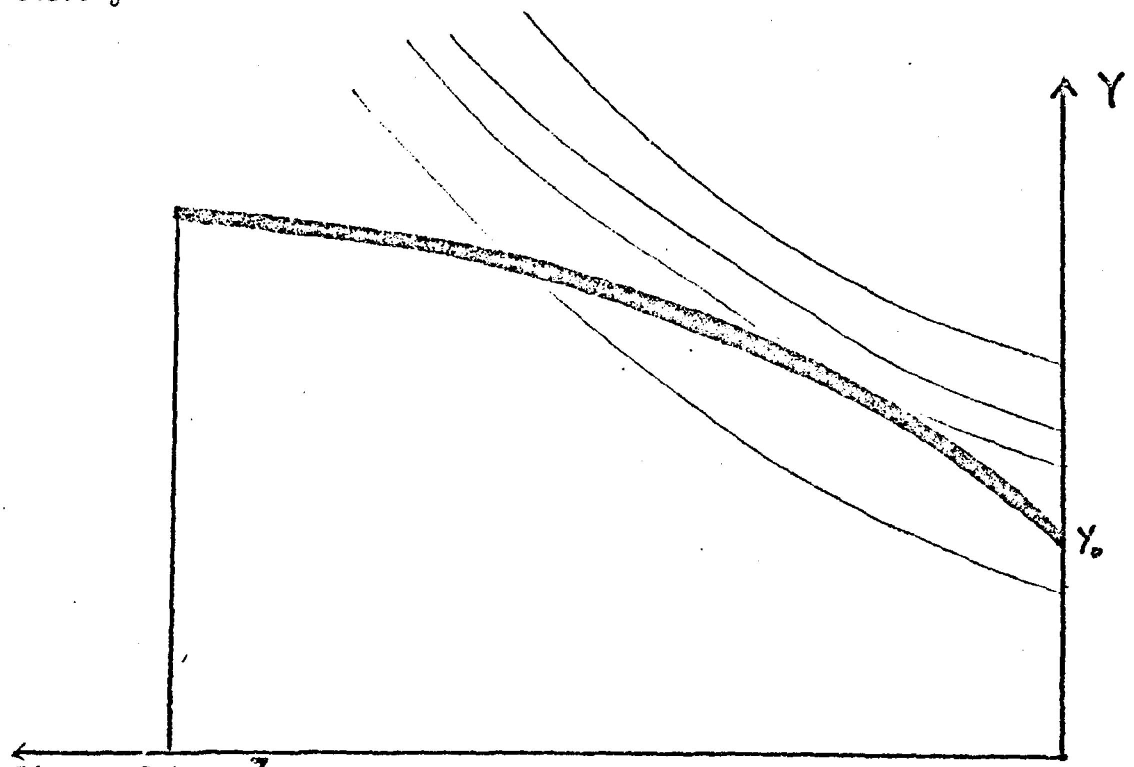


Diagram 5

<u>The Production Constraint</u>

With increasing returns in the production sector an essential assumption on the production side of the model, the concavity of F(N,Z) with respect to Z is of course not guaranteed. It is possible, however, to obtain necessary and sufficient conditions on the f(.) and g(.) functions for F(N,Z) to be concave in Z. Intuitively F(.) will be concave in Z if as the output level Y increases, the net pollution Z increases at an increasing rate. Now with increasing returns in production, as Y increases it will absorb less and less of the labour force per unit increase in Y. This will take labour away from the antipollution activity at a declining rate and the effect on the net level of pollution will depend on how the antipollution activity declines. If there are increasing returns in that activity then its output will decline at a declining rate (recall that labour is being withdrawn from it progressively more slowly) and consequently Z will increase at a decreasing rate. However if there are increasing returns in production and decreasing returns in pollution removal then the rate of increase of Z will depend on how slowly labour is being withdrawn from it as Y increases, relative to how fast its output declines as labour is transferred to producing Y. In this regard we may state the following condition:

Given equations (1) -- (3), the production constraint $Y = F(N,Z)$ will be concave in Z if and only if,

$$\frac{f''}{f'} < - \frac{g''}{g'} \qquad (17)$$

for all feasible values of n_1 and n_2. (One prime denotes the first derivative and two primes denotes the second derivative.)

<u>Proof</u>

From equation (1) - (3) we obtain,

$$\Psi\ (Y - Z) + \phi\ (Y) - N = 0 \qquad (18)$$

where $$\Psi = g^{-1}(.) \quad \text{and} \quad \phi = f^{-1}(.) \qquad (19)$$

For concavity we require that $\partial^2 Z / \partial Y^2 > 0$. Differentiating Z implicitly with respect to Y [7];

$$\left.\frac{dZ}{dY}\right]_{n\,=\,\text{const.}} = \frac{\phi'}{\Psi'} + 1 \gtrless 0 \qquad (20)$$

Differentiating again,

$$sgn \frac{\partial^2 Z}{\partial y^2} = sgn \quad \Psi' \phi'' + \phi' \Psi'' \left(1 - \frac{dZ}{dY}\right) \tag{21}$$

substituting from (13)

$$sgn \frac{\partial^2 Z}{\partial y^2} = sgn \quad (\Psi')^2 \phi'' + (\phi')^2 \Psi'' \tag{22}$$

Since the first expression in brackets is negative (15) will be positive if:

$$(\Psi')^2 \phi'' < (\phi')^2 \Psi'' \tag{23}$$

or alternatively

$$\frac{\phi''}{(\phi')^2} < \frac{\Psi''}{(\Psi')^2} \tag{24}$$

working back to the inverted functions we have

$$\phi' = \frac{1}{f'} \quad \text{and } \phi'' = -\frac{1}{(f')^3} \cdot f'' \tag{25}$$

$$\text{and } \Psi' = \frac{1}{g'} \quad \text{and } \Psi'' = -\frac{1}{(g')^3} \cdot g'' \tag{26}$$

which yields

$$\frac{f''}{f'} < -\frac{g''}{g'} .$$

Condition (17) has to hold for all feasible values of n_1 and n_2. Since $Z \geq 0$, this implies that it has to hold for

$$n_1 > n_1^*$$

$n_2 < N - n_1^*$, *where n_1^* is the solution to*

$$f(n_1) - g(N - n_1) = 0 \tag{27}$$

Condition (11) has a natural interpretation. The terms f'' / f' and g'' / g' are measures of the coefficient of absolute risk aversion in the theory of risk.
Their properties are explained in Arrow (1965) but essentially they attempt to capture the degree of concavity or convexity of a function. In our context they may be interpreted as measuring the extent of decreasing or increasing returns to scale, and condition (10) says that Y will be concave in Z if and only if the <u>decreasing</u> returns to pollution removal, as measured by the 'absolute coefficient of returns' are greater than the increasing returns to production.

The Welfare Function.

The welfare function whose trade-off between Y and Z we are interested in, may be written as

$$\max_{\tilde{x}} \; 2\int_x U(c(x),\, x,\, q(x,Z))\, h(x)\, dx \tag{28}$$

$$\text{s.t. } N. \int_x c(x)\; h(x)\; dx \;=\; Y \tag{29}$$

We may refer to the maximum as $w\,(Y,\, Z)$. It is unfortunately true that even with the simplest form of utility function there are no reasonable conditions for the concavity of $w\,(.)$. This can be seen by considering the additively separable utility function, that has been discussed above,

$$U(c,\, x,\, q(x,\, Z)) \;=\; V(c) \;+\; R(x,\, q(x,\, Z)) \tag{30}$$

In this case it follows directly from (8) that $c(x)\; h(x) \;=\; \dfrac{Y}{N}\; h(x)$, and all individuals have equal consumption. If we further assume that $R(.)$ is single-peaked in x for all Z, the residential area can be expressed an interval of the real line. As stated above the concavity of R

in x is sufficient for this, although it is not necessary.

Therefore, considering an additively separable utility function that is single peaked in x, we may write ω (.) as [9],

$$\omega\,(Y,\ Z) \quad = \int_{x_o(Z)}^{x_o(Z)\,+\,N/2} \left\{ V\left(\frac{Y}{N}\right) + R\Big(x,\ q(x,\ Z)\Big)\right\} dx \qquad (31)$$

$x_o \geq 0.$

Where we have chosen the units of measurement of x, such that each individual occupies a unit length.

It may be readily verified that,

$\omega_Y \geq 0$ and $\omega_{YY} < 0$, and

$$\omega_Z = \int_{x_o(Z)}^{x_o(Z)\,+\,N/2} \left\{\frac{\partial}{\partial Z} R\Big(x,\ q(x,Z)\Big)\right\} dx + \left\{R\Big(x_o + N/2),\ q(x_o + N/2, Z)\Big) - R\Big(x_o,\ q(x_o,\ Z)\Big)\right\} \frac{dx_o}{dZ} \qquad (32)$$

Since the utility at all extreme points is equal this reduces to

$$\omega_Z \quad = \int_{x_o(Z)}^{x_o(Z)\,+\,N/2} \left\{\frac{\partial}{\partial Z} R\Big(x,\ q(x,\ Z)\Big)\right\} dx < \quad 0 \qquad (33)$$

and ω_{ZZ} is then given by,

$$\omega_{ZZ} \quad = \left\{\frac{\partial}{\partial Z} R\Big((x_o + N/2),\ q((x_o + N/2),\ Z)\Big) - \frac{\partial}{\partial Z} R\Big((x_o),\ q(x_o, Z)\Big)\right\}\frac{dx_o}{dZ} + \int_{x_o(Z)}^{x_o(Z)\,+\,N/2} \frac{\partial^2 R}{\partial Z^2}\ dx \qquad (34)$$

The second part of the RHS of (34) will have the sign given by,

$$sgn. \int_{x_0(Z)}^{x_0(Z)+N/2} \frac{\partial^2 R}{\partial Z^2} dx = sgn \left\{ R_{qq}(q_Z)^2 + R_q q_{ZZ} \right\} \qquad (35)$$

< 0 if $q_{ZZ} \geq 0$

undetermined otherwise

The first part of the RHS of (34) has the same sign as,

$$sgn. \left\{ \frac{\partial}{\partial Z} R(x_0 + N/2, Z) - \frac{\partial}{\partial Z} R(x_0, Z) \right\} \frac{dx_0}{dZ} = sgn. \left\{ \frac{\partial^2 R}{\partial x\, \partial Z} \cdot \frac{dx_0}{dZ} \right\} \qquad (36)$$

> 0 if $q_{xZ} \leq 0$ $R_{xq} \geq 0$

undetermined otherwise

For the last proposition we require the result that $\frac{dx_0}{dZ} > 0$ if $q_{xZ} \leq 0$.

This is shown in the next section when some comparative static observations are discussed.

It appears therefore that even in this simple case we cannot sign the expression for w_{ZZ}, and hence we cannot establish the concavity of the welfare function. One 'special' case when it will be concave is when the boundaries of the town are independent of Z, and $q_{ZZ} \geq 0$. However, it would be extremely fortuitious if the former were to be true.

For the reasons given above the necessary conditions (8) and (9) cannot guarantee a global maximum. The possibility of multiple equilibria is therefore inherent in the structure of the problem. This means that the welfare criterion has to be evaluated at each point $\{Y, Z\}$ where the necessary conditions are fulfilled and overall maximum chosen. A local concavity condition that is sufficient for a local maximum, however, can be obtained. From diagram 5 it is clear that at a local maximum the welfare indifference surface must be locally concave. The slope of this curve is given by,

$$\left.\frac{dy}{dZ}\right]_{\omega = const} = 2 \cdot \int_{x_0(Z)}^{x_0(Z) + N/2} - \frac{U_Z}{U_c}\, dx$$

Hence its rate of change is given by

$$\left.\frac{d^2y}{dZ^2}\right]_{\omega = const} = 2 \frac{\partial}{\partial Z} \int_{x_0(Z)}^{x_0(Z) + N/2} - \frac{U_Z}{U_c}\, dx. \tag{37}$$

In the case of the utility function (30) this requirement of local concavity may be expressed as:

$$2 \cdot \left[\left\{ \frac{R_Z\left((x_0 + N/2),\ Z \right)}{V_c} - \frac{R_Z\left(x_0\ , Z \right)}{V_c} \right\} \frac{dx_0}{dZ} + \int_{x_0}^{x_0 + N/2} \frac{R_{ZZ}}{V_c} \right] < 0, \tag{38}$$

evaluated at the relevant Y and Z. The first term within brackets is the 'marginal' change in the MRS between Z and c, due to an alteration <u>in the optimal location</u> of households when Z increases and the second term with the integral is the change in sum of the MRS between Z and c when Z increases and the locations are held constant. With a reasonably well behaved utility function and pollution distribution function,[11] the first term will be positive and the second negative. We require that the sum be negative.

2.3.2. Some Comparitive Static Observations

It is of some interest to ask how the allocation of consumption would vary across the town, and how the choice of residential locations would respond to changes in the net level of pollution produced. In the case of the additive welfare function, we consider the changes in consumption over a range of values of x when individuals are continuously located. If the function c(x), is piecewise differentiable and we consider it over one such differentiable range, then differentiating (8) with respect to x we obtain

$$\frac{dc}{dx} = - \frac{u_{cq}q_x + u_{cx}}{u_{cc}} \quad \text{for } x_a > x > x_b \tag{37}$$

Where x_a and x_b represent one range over which c(x) is differentiable. Whether or not c increases with x depends on the sign of the numerator, which in turn depends on the complementarity and substitutability relationships between the consumption good and net pollution suffered (c and q) and between the consumption good and distance from the centre (c and x). In general there is no reason to assume a particular relation between these goods and 'bads' and so it is not possible to make any general statement regarding how consumption varies with distance from the centre.

When the utility function is additively separable between c and the other arguments, then consumption is equal for all individuals. In diagrams (1) and (2) we showed the distribution of utility this would imply when the utility function was concave in x. It is interesting to note that in this case any increase in the town's population will lead individuals being located both further out and closer in, whenever this is possible (i.e. whenever $x_o \neq o$). Furthermore any increase in the level of pollution Z will not necessarily lead to the township moving further out (i.e. $\frac{\partial x_o}{\partial Z}$ may be of either sign).

This can easily be seen in diagram 6 below. At level of pollution Z_o the town is optimally located between x_o and $x_o + N/_2$. At an increased level of pollution $Z_o + \Delta Z_o$ the new utility curve will lie below the original curve, and be concave. However, it may be that the change in utility is greater closer to the centre than it is further out (case (a) in diagram 6), or it may be that the change is greater further out, than it is near the centre. (Case (b) in diagram 6). In the former case it is clear that the same population can be best located by shifting everyone a little further out. In the latter case, however, the reverse is true. Thus the result depends on the sign of the partial derivative U_{xZ}. By differentiating the utility function it is clear that $U_{xZ} > 0$ if $q_{xZ} \leq o$. This corresponds to case (a). However if

$U_{xZ} < 0$ then $q_{xZ} > o$, as in case (b).

Therefore for it to be optimal for the residential area to be moved further from the centre, the change in the distribution of pollution as a result of an increase in the level produced has to be such that areas further out are relatively no worse affected. Whether $q_{xZ} \lessgtr o$ is of course an empirical question.

In the case of the Rawlsian welfare function we note that the allocation of the consumption good is such as to hold utility constant. Thus where utility would increase with an increase in the distance from the centre, the consumption allocated declines with the distance, and where utility would decrease with

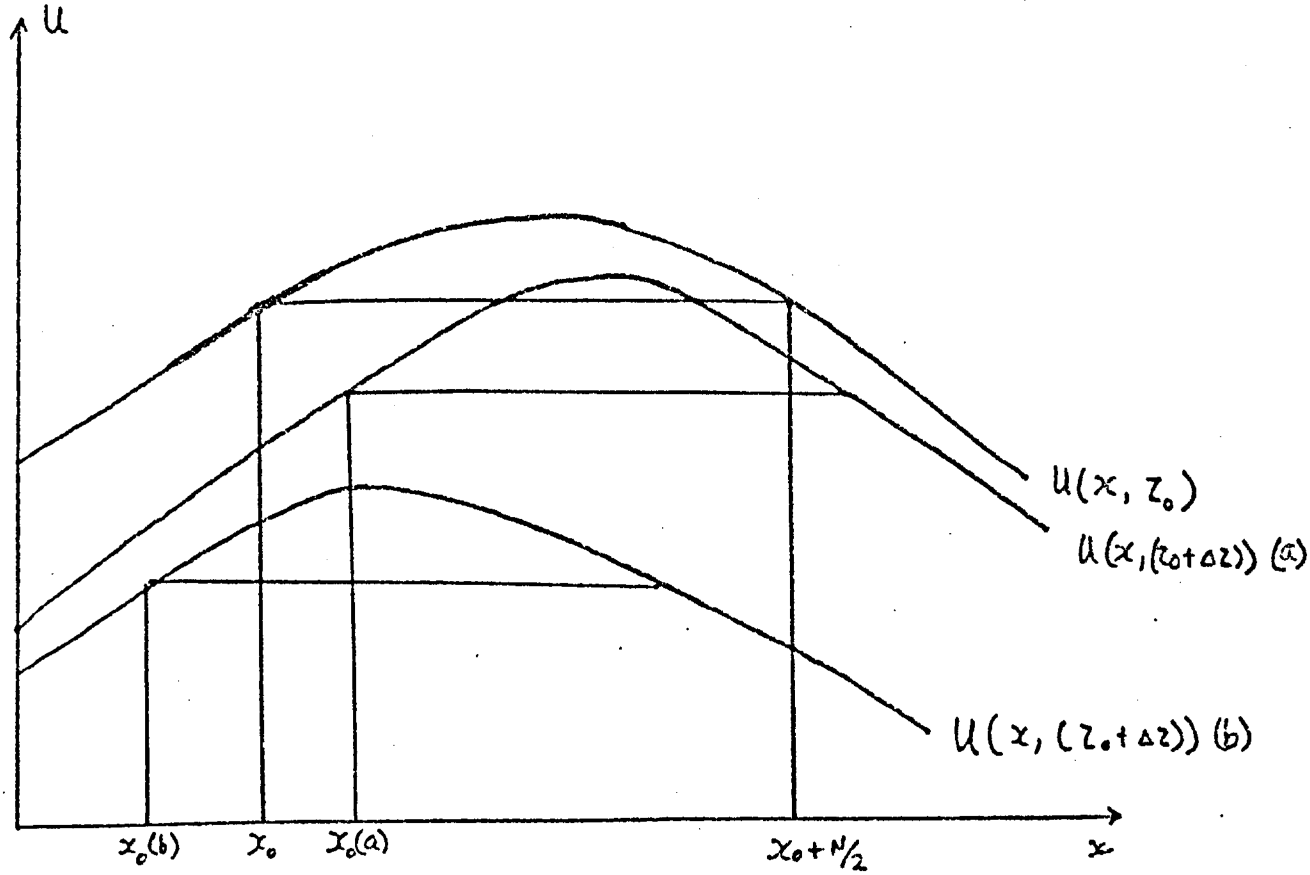


DIAGRAM (6)

distance from the centre, the consumption allocated increases. With insistence on complete equality of utility, the locations chosen will depend both on the level of pollution Z and the level of output Y. The latter will be the case even when the utility function is additively separable between c and x[12]. When the utility function is concave in c and x it is possible to analyse the effects of qualitative changes of x_0, the distance from the centre of the town that the residential district starts, with respect to Z and Y. In general this is rather involved and the answers appear to depend on third order partial derivatives of the utility function, which cannot reasonably be signed. However if the utility function is additively separable in c and x, then

$$\frac{\partial x_0}{\partial Z} \gtrless 0 \quad \text{as} \quad u_{xZ} \gtrless 0$$

and

$$\frac{\partial x_0}{\partial Y} > 0$$

The first result is exactly the same as for the utilitarian welfare function and arises because the relative distribution of pollution over the space is important, as well as the absolute level. The second result indicates that as real income increases, the marginal opportunity cost of nearness to the centre rises in terms of the consumption good, for a given distance from the centre. Hence to obtain the maximum equal utility it is now relatively more desirable to locate everyone further out.

2.3.3 The Attainment of the Optimum by Decentralisation.

In this section we examine the conditions under which the optimum can be attained when the households and the producers make independent decisions. We consider initially the case where the optimal location of individuals is continuous (i.e. the set { P } can be represented by intervals of the real line), the levels of Y and Z are *fixed*, and the welfare criterion is the additive utility one. For this case we may obtain the optimum by defining a rental function p(x), over the range where we desire to locate the households, and a distribution of income such that households maximise their utilities at the given rental function to choose the optimum consumption function c(x):

$$\text{Max} \quad U(c, x, q(x, Z))$$
$$\text{s.t.} \quad c + p(x) = M \qquad (38)$$

The first order conditions for an optimum are[13]

$$\frac{dp}{dx} = \frac{U_x}{U_c} \qquad (39)$$

To obtain the optimal value of p(x),

$$\frac{dp}{dx} = \frac{U^*_x}{U^*_c} = \frac{U^*_x}{\lambda} \qquad (40)$$

Where the star indicates optimal values of the partial derivatives.

$$\text{Therefore} \quad \lambda \frac{dp}{dx} = U^*_x = U^{*t}_x - U^*_c \frac{dc}{dx} \qquad (41)$$

where 't' indicates the total derivative of U with respect to x.

Integrating (41) yields

$$p(x) = \frac{U(x)}{\lambda} - c(x) \qquad (42)$$

For the rental function to yield the optimum it has also to be true that no other feasible combination of c and x gives greater utility than the combination represented by the first order conditions (39). Such feasible combinations are given by

$$c + p(x) \leq c^*(x) + p(x^*) \qquad (43)$$

Where the x^* indicates the optimum choice. Substituting (42) gives:

$$c(x) + \frac{U(x)}{\lambda} - c(x) \leq c(x^*) + \frac{U(x^*)}{\lambda} - c(x^*)$$

$$U(x) \leq U(x^*) \qquad (44)$$

which establishes the desired proposition. A similar result can be obtained for the Rawlsian welfare criterion . We note that the decentralisation does not depend in either case on how U varies with x[14].

The above competitive realisation is, however, a very limited result, for it does not deal with the choice of Y and Z. When the production constraint is concave a local maximum can only be realised competitively, as long as care is taken to ensure that a condition similar to (38) is satisfied. For, if the government agency places a tax on the pollution Z of P_Z, and the wage per unit of labour force is so determined that the whole labour force is always employed, then the producer may be required to maximise profits at given prices:

$$\pi = f(n_1) - p_Z.(f(n_1) - g(n_2)) - W.(n_1 + n_2) \qquad (45)$$

The first order conditions yeild,

$$p_Z = \frac{f^1}{f^1 + g^1} = F_Z \qquad (46)$$

and it may be readily verified that the second order conditions will be satisfied if $F_{ZZ} < 0$ and W is chosen so that demand for labour is equal to N.

Setting $$P_Z = 2. \int_{x_0}^{x_0+n/2} - \frac{U_Z}{U_c} . \quad dx,$$ we obtain the necessary condition (9) for a local optimum. The sufficiency condition for local optimality, however, is that the sum of the marginal rates of substitution between Z and c be increasing with Z in the neighbourhood of the maximum. This has to be <u>independently</u> verified to ensure sufficiency. In the case of the additively separable utility function we have seen that it entails expression (38) being satisfied and we can interpret the terms involved. In diagram (7) below we illustrate such a local optimum. At price $p_Z^{(1)}$ the producer will choose α and at price $p_Z^{(2)}$ he will choose β. The latter is a local minimum and the former is a local maximum.

In the above we have assumed that the production constraint is concave, and as we have seen this will only be true in special circumstances. If the concavity of the production constraint is not satisfied then the price decentralisation will not guarantee even a local optimum. In diagram (8) below we illustrate such a case. α and β are the points chosen at prices $p_Z^{(1)}$ and $p_Z^{(2)}$ respectively. They are not local optima and the (global) optimum is attained by profit <u>minimisation</u> at γ with the price $p_Z^{(2)}$ for Z.

This section may be conluded by observing that there are substantial difficulties in the way of a price decentralisation of both producer and consumer decisions. A local optimum is possible if the production constraint is concave, providing that the concavity of the welfare criterion in the neighbourhood of the optimum, is independently verified. Without concavity in the production constraint a local optimum is not necessarily attainable by price decentralisation.

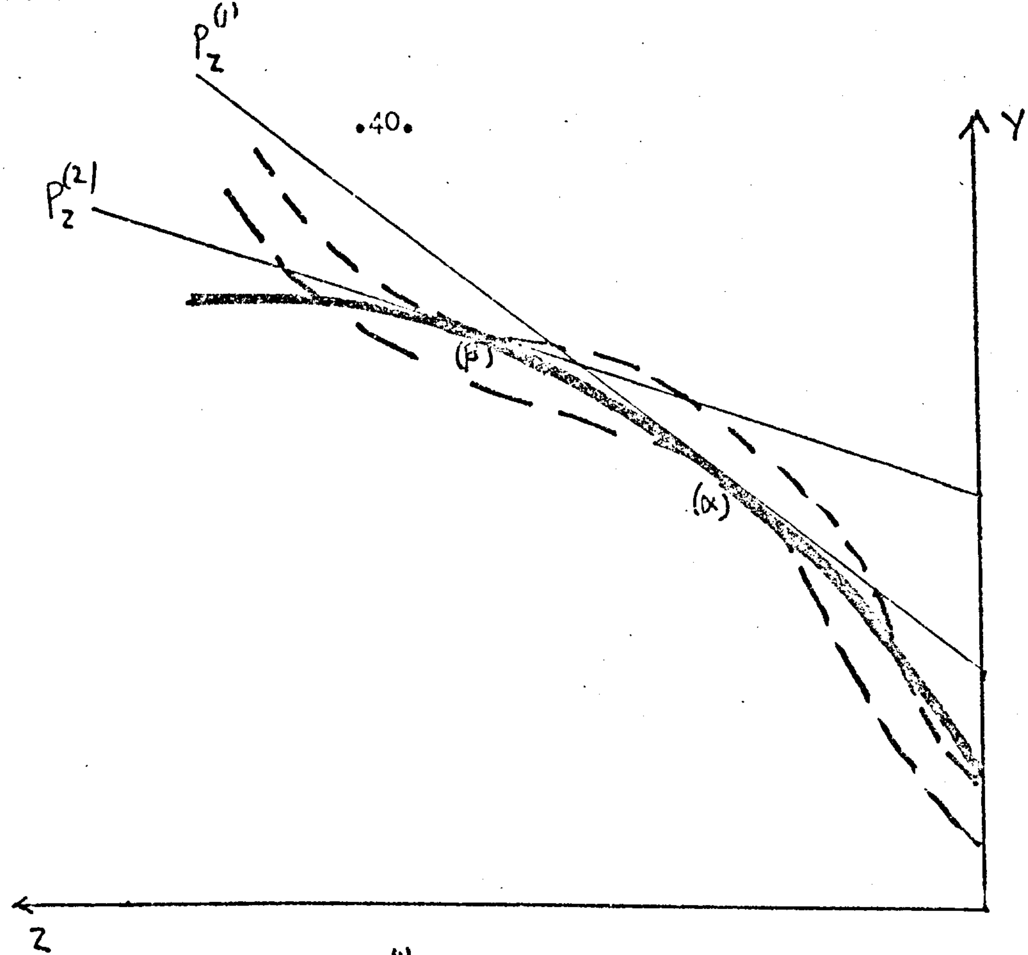


DIAGRAM 7

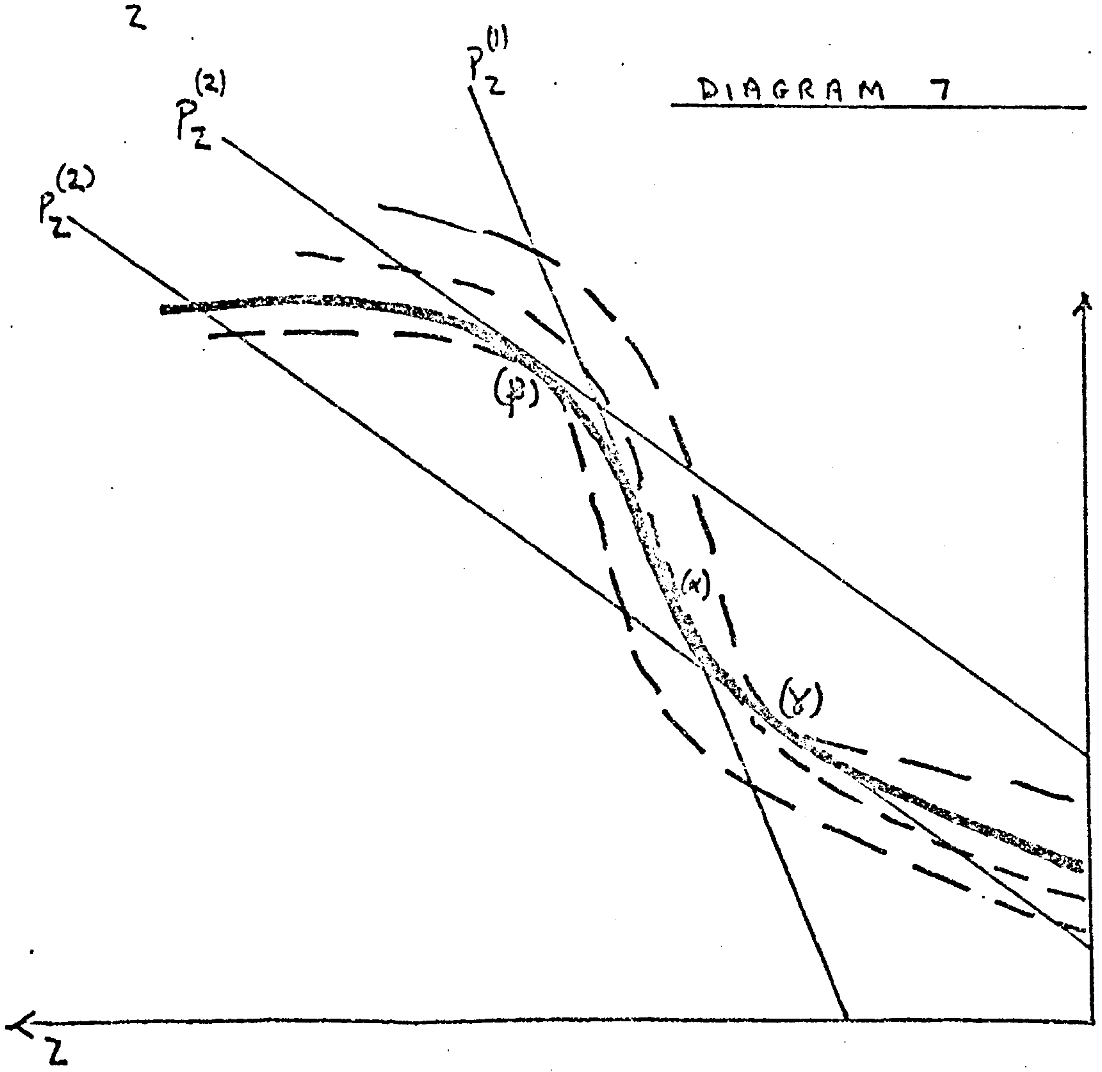


DIAGRAM 8

2.4. An Optimum Geography of Similar Towns.

2.4.1 Conditions defining the optimum size of town.

In this section we develop our analysis to take account of a variable town size. This is done by assuming that the population is to be divided into a number of identical towns so as to maximise some welfare criterion for the whole population. If the number of such towns is large, then we may conveniently represent the choice of this number by a continuous variable. We will assume that this is so.

The total population of the country is P, and it is to be located in t towns of size N each. If the welfare criterion for society is additive then we may represent it as

$$\Omega \quad (Y, Z, N) \quad = \quad 2. \frac{P}{N} \int_x \; U\{c(x), x, q(x,Z)\}.h(x) \; dx \tag{47}$$

$$s.t \quad 2.\frac{P}{N} \int_x \left\{ c(x) \; - \; \frac{F(N,Z)}{N} \right\} h(x) \; dx \quad = \quad 0 \tag{48}$$

We may write the Lagrangian as

$$L \quad = \quad 2. \frac{P}{N} \left\{ \int_x \left\{ U(c(x), \; x, \; q(x,Z) \; - \; \lambda \; (c(x) \; - \; \frac{F(N,Z)}{N})) \right\} h(x) \right\} dx$$

(49)

The necessary conditions for a local maximum are (8) and (9) above, and in addition,

$$\frac{\partial L}{\partial N} \quad = \quad 0 \tag{50}$$

We may obtain (50) explicitly and interpret it when the utility function is additively separable between x and c, and it is single peaked in x. As stated earlier these conditions imply that the optimal locations will be an interval of the real line. Then (49) may be rewritten as,

$$L = 2.\frac{P}{N} \int_{x_0}^{x_0 + N/2} \left\{ V(c(x)) + R\Big(x,\ q(x,Z)\Big) \right\} - \lambda \left\{ c(x) - \frac{F(N,Z)}{N} \right\} dx \tag{51}$$

Differentiating with respect to N gives,

$$\frac{\partial L}{\partial N} = -\frac{\Omega}{N} + 2.\frac{P}{N} \int_{x_0}^{x_0 + N/2} \frac{\partial}{\partial N} . \left\{ \lambda.\ \frac{F(N,Z)}{N} \right\} dx$$
$$+\ 2.\frac{P}{N} \left\{ \frac{1}{2} R\Big((x_0 + N/2), q\Big) - \frac{\lambda}{2} c(\ x_0 + N/2\ ) + \frac{\lambda}{2} \frac{F(N,Z)}{N} \right\} \tag{52}$$

$$= 2.\ \frac{P}{N} . \left\{ \frac{1}{2}\ R(x_0 + N/2, q) - \frac{\lambda}{2}\ c(\ x_0 + N/2\ ) + \frac{\lambda}{2}\ \frac{F(N,Z)}{N} \right\}$$
$$+\ 2\ \frac{P}{N} \left\{ \frac{1}{2}\ \frac{\lambda}{N} \Big\{ F_N.N\ -\ F(N,Z) \Big\} \right\}$$
$$-\ \frac{\Omega}{N} \tag{53}$$

From condition (8) we obtain,

$$c(x_0 + N/2) = \frac{F(N,Z)}{N}. \tag{54}$$ Hence $\frac{\partial L}{\partial N} = 0 \Rightarrow$,

$$2.\ \frac{P}{N}. \left\{ \frac{1}{2} R((x_0 + N/2), Z) + \frac{1}{2}.\ \frac{\lambda}{N} \Big\{ F_N.N - F(N,Z) \Big\} - \frac{N.\Omega}{P.2.N} \right\} = 0 \tag{55}$$

Rearranging, this gives

$$\left\{ \frac{N}{P} . \Omega \quad - \quad N R \left(x_0 + N/_2 , Z\right) \right\} = \lambda \left\{ NF_N - F(N,Z) \right\} \tag{56}$$

Condition (56) may be interpreted as follows,

If we take afew people from each town and set up a new town, then the gain of utility to the residents at a constant level of consumption is, $\frac{N}{P}.\Omega$. The loss of utility from existing towns is, $N.R((x_0 + N/_2), Z)$. Hence the net utility gain is the left hand side of (56). However such gain was calculated on the basis that no change in consumption per head occurs. The loss in output to all existing towns is NF_N, and the gain in output to the new town is $F(N,Z)$. The utility valuation of this change is the right hand side of (56). The marginal condition requires the gain in locational utility to be equal to the loss in consumption utility, so that no further marginal subdivision of the population is desirable.

Conditions (8), (9) and (56) are only necessary for a locally optimum geography. By an argument similar to that used in the previous section, these conditions, along with the concavity of Ω in Y, Z and N and the concavity of F(N, Z) in N and Z would be sufficient for a global maximum. However, as with the problem when N was held constant, there are no reasonable conditions to guarantee the concavity of Ω. The concavity of F(.) in N and Z requires that

$$F_{NN} < 0 \quad F_{ZZ} < 0 \quad F_{NN}F_{ZZ} - F_{NZ}^2 > 0 \tag{57}$$

Using the same notation as (19) and taking the implicit form of F(.) given in (18), the last condition reduces to,

$$\mathrm{Sgn} \quad (F_{NN}F_{ZZ} - F_{NZ}^2) = \mathrm{Sgn} \left\{ \Phi'' \psi'' (\Phi' + \psi')^2 \right\} < 0 \tag{58}$$

(58) is negative, since with decreasing returns to scale in pollution removal and increasing returns to scale in production Φ' and ψ'' are of opposite signs. It would appear therefore that even local concavity conditions cannot be satisfied for the function F(.). A local optimum would then be defined as a point when conditions (8), (9) and (56) are satisfied, and where L(.) is locally concave in Y, Z and N.

2.4.2. Some Comparative Static Observations.

The first point that we note here is that the necessary conditions for an optimum, and hence the optimum size of town, are independent of the size of the population P. This is of course because we have not considered any overall land area constraints, and any variation in the area occupied per individual. If such variations were considered, then the size of the population would be relevant.

If is of some interest also to compare the full optimum with the equilibrium which would prevail when there was no control on pollution and the town size was determined by competitive forces. Under these conditions an equilibrium town size will only exist, if the production process is such that the increasing returns postulated hitherto, only last up to a certain level of output, and beyond that level diminishing returns set in. Such a case is illustrated in diagram 8 below. There are increasing returns up to town size N^*, and then diminishing returns set in, the uncontrolled town size is then given by N^{**}. The whole production structure may then be represented as:

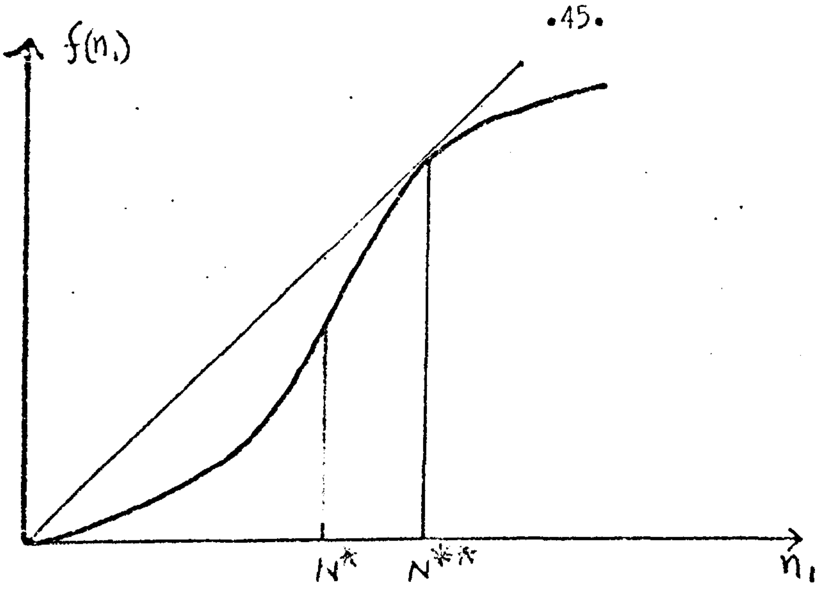


Diagram 8

$$y = f(n_1) \qquad f' > 0 \qquad f'' \geq 0 \text{ for } n_1 \leq N^* \qquad (59)$$

$$f'' < 0 \text{ for } n_1 \geq N^* \qquad (60)$$

$$Z = f(n_1) - g(n_2)$$

$$g' > 0 \qquad g'' < 0$$

$$n_1 + n_2 = N \qquad (61)$$

Now in the competitive situation in the long run the number of towns will expand to the point where zero excess profits are being made. This implies a competitive equilibrium, subscripted by c, at which we have

$$N_c = N^{**} \qquad Z_c = f(N^{**}) \qquad y_c = f(N^{**})$$

$$n_{2c} / n_{1c} = 0; \qquad F(N_c, Z_c) = N_c F_N(.) \qquad F_Z(.) = 0 \qquad (62)$$

At the full optimum it is clear that the proportion of the labour force employed in pollution removal cannot fall, but it is not clear whether the town size will rise or fall relative to the competitive equilibrium. There are two forces at work. On the one hand a smaller town means that there is a marginal consumption loss from further subdivision to cancel the marginal locational gain, but on the other hand a larger town may mean that more people can be allocated to pollution removal and hence the locational gains are smaller from setting up more towns. We may obtain the conditions under which the town size will <u>not</u> expand when we move from a competitive equilibrium to an optimum as follows. At the optimum the conditions defining N and Z are given by

$$N.F_N - F(N,Z) = A^* > 0 \qquad (63)$$

$$F_Z = B^* > 0 \qquad (64)$$

Where A and B are determined by N, Z and the optimal locations. Now if starting at N^{**} with $A = B = 0$, we find that $\frac{\partial N}{\partial A}$ and $\frac{\partial N}{\partial B}$ are negative, then the optimum cannot be attained at an $N > N^{**}$, when A^* and B^* 'are small numbers'.[16] To examine the conditions under which the two partial derivatives are negative, we obtain the following expressions by implicit differentiation of (63) and (64):

$$\frac{\partial N}{\partial A} = \frac{F_{ZZ}}{|D|}, \quad \frac{\partial N}{\partial B} = -\frac{N F_{NZ} + F_Z}{|D|} \qquad (65)$$

Where,

$$|D| = N(F_{NN}F_{ZZ} - F_{NZ}^2) + F_{ZN}F_Z \qquad (66)$$

From (65) and (66) we note that if,

(a) The production constraint is concave in Z and N in the neighbourhood of $N = N^{**}$

(b) $F_{ZN} \geq 0$

then ,

$$\frac{dN}{dA} < 0 \text{ and } \frac{dN}{dB} < 0.$$

Under these circumstances the full optimum is characterised by a no larger town size than the free competitive equilibrium with a no smaller proportion of the population employed in pollution removal. Hence the quantity of net pollution at such an optimum is not greater than that at a competitive equilibrium. When the town size under a competitive equilibrium is smaller than under the optimum, however, the net level of pollution must be lower in the latter case than in the former. For if this were not so, the competitive equilibrium would represent a greater level of welfare with less net pollution and greater output per head.

By introducing, in the above discussion, a range of the production function where there are decreasing returns to the production process, it would seem to be important to reconsider the sufficiency conditions discussed in the previous subsection. This is because, as we have seen above the full optimum may lie in the range where there are decreasing returns or in the range where there are increasing returns to $f(.)$. If the optimum lies in the former range then $F(.)$ is concave, and a local optimum is defined by the concavity of $F(.)$, and of Ω. It is of course still not possible to define a global optimum by conditions (8), (9) and (56), as there is no way of ensuring the concavity of Ω, and there is always the possibility of an equilibrium in the increasing returns range. Thus the problem of multiple equilibria remains as important as ever.

2.4.3

Some Comments on the attainment of an Optimum by Price Decentralisation

It would appear from the discussion so far, that the chances of attaining the optimum by price decentralisation are rather bleak. The difficulties that arise do so on account of the non concavity of both the F(.) function and of Ω . It is always possible to allocate the individuals optimally in the space around the town by competitive decentralisation, when Y, Z and N are fixed. When Y and Z are variable, but N is fixed, price decentralisation to a local optimum is possible as long as the production constraint is concave in Z, and as long as the local concavity of ω is independently verified. When N is also variable we have seen that in the increasing returns region for the production activity, the concavity of F(.) in N and Z will not be satisfied. In the region when $n_1 > N^*$ concavity will be satisfied. Since we wish to control the level of two variables N and Z to obtain the optimum, it would be natural to think in terms of two instruments; one, a pollution tax equal to the sum of the marginal rates of substitution between Z and C when the community is optimally located, and the other an employment tax, varying with the level of employment (and probably also with the level of pollution), and based on the difference between the average and marginal utilities in an optimally located town. However, formulating the problem in this manner it is easy to see that if firms act competitively to maximise profits, and if the equilibrium is one of zero profits, then the second order profit maximisation conditions will not be satisfied if the optimum value of n_1 is less than N^*. They will be satisfied under certain conditions if $n_1 > N^*$. Therefore if the optimum consists of a town size greater than that at which the increasing returns set in, then the production decentralisation may be feasible as suggested above[17]. It still remains true however that the concavity of Ω in Z and N will still have to be verified independently to ensure that the necessary conditions do not represent a constrained minimum, but rather a constrained <u>local</u> maximum.

Overall then it appears that the problems of optimal towns cannot in general be solved by mimicking the competitive model, when considerations of pollution are important and when there are increasing returns in the production process, for some range of output. It would be of some benefit to know just how much and with respect to which parameters the town size and the level of pollution are affected in order to attain the optimum, in comparison with the uncontrolled equilibrium. For if the differences are large, then the importance of direct controls on N and Z are to be emphasised more strongly. If, on the other hand the optimum town size is greater than N*, then some decentralisation may be feasible. It is to an examination of such differences, within a plausible parametric framework, that we now proceed.

2.5. A Parametric Formulation of Optimum Towns with Environmental Externalities.

2.5.1 Introduction

In this section we set out the problem examined in the rest of the chapter within a parametric framework and carry out some simulations to obtain the optimum town size and the optimum allocation of labour between the production and the cleaning sectors under varying assumptions regarding the social welfare criterion and the 'geography', and for various parameter values. The point of reference in setting up the framework and interpreting the results will be the "uncontrolled competitive town." This is the town with no control of pollution, and with the size determined by the level of employment at which the average product of labour in the production sector is maximised. If we fix the level of employment at which the decreasing returns to scale set in at 500,000, then the point at which the average product is maximised can be obtained from this figure and the parameters of the system. We may then compare the various optima with this by considering that if the uncontrolled competitive town has say 800,000 people and no pollution control, then the optimum town has 'x' people, of whom 'p' percent are employed in cleaning up pollution and the net level of pollution at the centre is 'y' percent lower.

It will be apparent that some of the parameter values considered here have little empirical foundation. While this is regrettable and implies that the model has little quantitative use it does not render the exercise valueless, for a range of values are considered and the model provides some interesting insight into the sensitivity of the results with respect to some of parameter values relative to others and indeed the direction in which the optimum moves as some parameters change is not intuitively obvious.

2.5.2 The Production Structure

We define the production of the consumption good by the following relations:

$$y = n_1^{k} \qquad k>1 \qquad 0 \leq n_1 \leq N^* \tag{67}$$

$$y = Bn_1^{\varepsilon} - c \qquad 1 \geq \varepsilon > 0 \qquad n_1 \geq N^* \tag{68}$$

B, C, positive constants.

By requiring continuity at $n_1 = N^*$ and differentiability at $n_1 = N^*$, we obtain the following expressions for C and B :

$$B = \frac{K}{\varepsilon}(N^*)^{k-\varepsilon} \qquad \text{(differentiability)} \tag{69}$$

$$C = \left(\frac{K}{\varepsilon} - 1\right) (N^*)^{k} \qquad \text{(continuity)} \tag{70}$$

Differentiability at $n_1 = N$ is assumed so as to ensure smoothness in the production function and prevent the results depending upon a 'kink' at that point.

We define the cleaning activity as producing an anti-pollutant, S, an amount given by the relation,

$$S = Dn_2^{\delta} \qquad 0<\delta \leq 1 \qquad n_2 \geq 0 \tag{71}$$

D, positive constant

and the net level of pollution, Z, is given by,

$$Z = y - S \qquad Z \geq 0 \tag{72}$$

In order to choose D, we require that when the consumption good process is employing N^* people, the pollution that they create can be cleared completely by $\xi . N^*$ people[18]. Hence,

$$D = \xi^{-\delta} . N^{*\,k-\delta}$$

Finally the labour employed in both the activities adds up to the total quantity of labour,

$$n_1 + n_2 = N \tag{74}$$

With such a production structure, the production constraint, $y = F(Z)$, for given $N \leq N^*$, may be concave. Concavity will be assured if,

$$k < 1 + \frac{n_1^*}{N - n_1^*} (1 - \delta) \tag{75}$$

where n_1^* is the solution to the equation,

$$n_1^k - (N^* - n_1)^\delta = 0 \tag{76}$$

However, as shown earlier, the production constraint $Y = F(Z, N)$ will not be concave. Furthermore the condition required to ensure that the optimum town be smaller than the competitive town, $(F_{NZ} \geq 0)$ cannot be guaranteed for any parameter values.

The production structure thus has the following undetermined parameter values,

k, ε, δ, ξ and N^*.

2.5.3 The Preference Structure and the Social Welfare Criterion.

The individual preferences are given by the utility function,

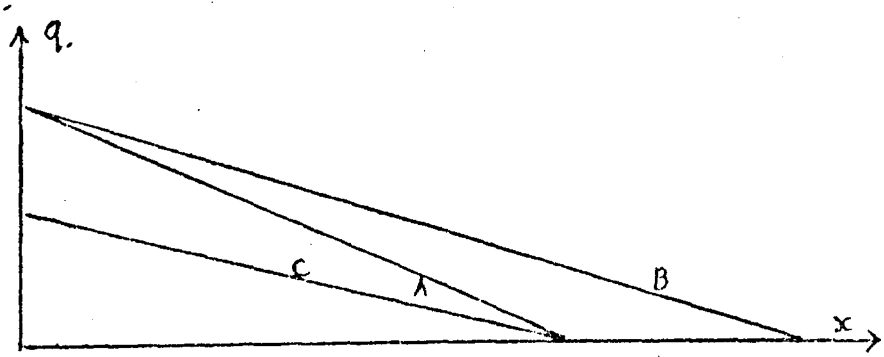
$$U = c^\alpha (T - x)^\beta (Z_m - q)^\gamma \qquad \begin{array}{l} \alpha > 0 \quad \beta > 0 \quad \gamma > 0 \\ \alpha + \beta + \gamma = 1 \\ T > x \geq 0, \; Z_m > q \geq 0 \\ c > 0 \end{array} \tag{77}$$

$$\log U = \alpha \log c \qquad q = Z_m, \quad x = T \tag{78}$$

$$q = \begin{cases} (Z(1 - r.x)) & x \leq \frac{1}{r} \quad r > 0 \\ 0 & x \geq \frac{1}{r} \end{cases} \tag{79}$$

Such a utility function expresses utility as an increasing function of the consumption good, nearness to the centre $(T-x)$ and the absence of pollution (eg. quietin the case of noise). The parameters T and Z_m may then be interpreted as follows; T is the maximum distance from the centre of the town that any household may be located, and Z_m is the maximum level of the pollution Z that may be tolerated. It is natural to think of a limit such as T - one cannot spend more than the whole working day commuting or the whole of ones wages on travel. The limit Z_m is a convenient translation of the measure of pollution from a 'bad' to a good. Its usefulness will appear again when we discuss the measurement of noise costs in Chapter 3. For the moment we may think of it as somehow related to the level of pollution produced at the centre in a competitive town. With noise pollution, $(Z_m - q)$ would be the level of quiet enjoyed by households at distance x from the centre. Z_m would be the maximum endurable noise at the centre. Normalising the coefficients α β and γ to add up to one, implies that the income elasticities of demand for the aggregate consumption basket, for 'nearness' to the centre, and for freedom from pollution are all unity. Furthermore the coefficients would then represent the proportion of ones income that is spent on each of these goods.

The distribution of pollution over space has a very simple assumed form. It is expected that the pollution will be the greatest at the centre and decline linearly until, at a distance of $\frac{1}{r}$ from the centre we arrive at a pollution-free zone. This may be a poor approximation for aircraft noise, which appears to decline more exponentially than linearly. It is not clear how good an approximation it is for urban noise or air pollution. It does however, have the advantage of allowing the social welfare criterion to be computed analytically for the chosen utility function. Furthermore the implication of a situation where pollution is more 'spread out' can be studied in this simple form by comparing situations such as A and B, or A and C in diagram 9. This can be done by varying r and Z



The social welfare criterion may be expressed as,

$$\Phi(.,\sigma) = 2 \int_{x_o}^{x_o+N/2} \frac{U(\cdot)}{1-\sigma}^{\frac{1}{1-\sigma}} dx \tag{80}$$

for a 'long and narrow' town. As $\sigma \to 1$ (80) is given by

$$\lim_{\sigma \to 1} \Phi(.,\sigma) = 2 \int_{x_o}^{x_o+N/2} \log U(.) \, dx \tag{81}$$

which is the additive social welfare criterion with a separable-utility function and an implied equal consumption per head used as the welfare criterion for much of the preceding analysis in this chapter. Henceforth when we consider the additive social welfare criterion we shall set $\sigma = 1$. For a circular town the same welfare criterion gives

$$\lim_{\sigma \to 1} \Phi(.,\sigma) = \int_{x_o}^{x_o+R} \log U(\cdot)x \, dx \tag{82}$$

where,

$$x_o+R = \sqrt{x_o^2 + \frac{N}{\pi \cdot \nu}} \tag{83}$$

ν = average density of the population per square mile.

With the Rawlsian social welfare criterion we have

$$\lim_{\sigma \to +\infty} \bar{\Phi}(.,\sigma) = U_0 \qquad (84)$$

Where U_0 is a given level of utility, independent of x. The consumption would then have to vary across x :

$$[C(x)]^{\alpha} \quad [T-x]^{\beta} \quad [Z_m-q]^{\gamma} = U_0 \qquad (85)$$

The preference structure and the social welfare criterion thus has the following undetermined parameters, α, β, T, Z_m, r, and ν.

2.5.4. Computing the optimum utility for a fixed population when

(1) There is an additive social welfare criterion and

(2) A Rawlsian social welfare criterion

(1) An additive social welfare criterion

(a) A long and narrow town

A necessary condition for optimality as obtained from (8) in the separable case is that

$$c = \frac{Y}{N} \qquad (86)$$

at all x where the residential density is positive. Since with the chosen utility function the residential area is an interval of the real line all we need to select is x_0, in order to obtain the set of points where the residential density is positive. In doing this we recall that all households must be located within a distance of $\bar{T}$ from the centre. Hence x_0 is given, for a long and narrow town as,

$$x_0 = \min [\ (T-N/_2),\ \max (o,x_0^*)\ ] \qquad (87)$$

where x_0^* is the solution to

$$\beta \log (T-x_0^*-N/_2) + \gamma \log (Z_m-Z(1-r(x_0+N/_2))) =$$
$$\beta \log (T-x_0^*) + \gamma \log (Z_m-Z(1-rx_0)) \qquad (88)$$

This is a polynomial in x_0^* which has to be solved numerically in general. When $\beta = \gamma$, however, the solution is given by

$$x_0^* = \frac{1}{2}\left[(T-N/2) - \frac{(Z_m-Z)}{Zr}\right] \qquad (89)$$

To compute the optimum utility for a given N (fixed population), we scan through the range of permissible values of n_2 and n_1 and compute utility for each set of values of n_2 and n_1, given the optimal location of households. Recall from (24) that $n_1 + n_2 = n$. The restrictions on the values of n_1 and n_2 are given by,

$$\max\left[o, \left(\frac{n_1^{k}-Z_m}{D}\right)^{1/\delta}\right] \leq n_2 \leq \left(\frac{n_1^{k}}{D}\right)^{1/\delta} \text{ for } n_1 \leq N^* \qquad (90)$$

$$\max\left[o, \left(\frac{Bn_1-c-Z_m}{B}\right)^{1/\delta}\right] \leq n_2 \leq \left(\frac{Bn_1-c}{B}\right)^{1/\delta} \text{ for } n_1 \geq N^* \qquad (91)$$

N^* is the point at which marginal increasing returns stop and decreasing marginal returns set in.

The right hand inequalities in (90) and (91) represent the restriction that the net level of pollution cannot be negative. The left hand inequalities represent the restriction that the allocation of labour is bound below by zero or that amount of labour that will just prevent the net pollution from rising above the maximum endurable - whichever is the greater. The value of the social welfare criterion for the optimal location of individuals is given by

$$\omega(y, Z) = 2.\left[\int_{x_0}^{x_0+N/2} \alpha \log\frac{y}{N}\,dx + \beta\int_{x_0}^{x_0+N/2} \log(T-x)\,dx + \gamma\int_{x_0}^{x_0+N/2} \log(Z_m-q)\,dx\right] \qquad (92)$$

Substituting in for q from (79),
this expression may be integrated, term by term, to give

$$W(y,Z) = \alpha\, N \log \frac{Y}{N} + 2\beta \Big[(T-x) - (T-x)\log (T-x) \Big]_{x_o}^{x_o + N/2}$$

$$+ \frac{2.\gamma}{r.Z} \Big[(Z_m - Z + rZx) \log (Z_m - Z + rZx) - (Z_m - Z + rZx) \Big]_{x_o}^{x_o + N/2} \qquad (93)$$

$$\text{if } Z \neq o, \quad x_o + N/2 < \frac{1}{r}$$

or,

$$+ \left\{ \frac{2.\gamma}{r.Z} \Big[(Z_m - Z + rZx) \log (Z_m - Z + rZx) - (Z_m - Z + rZx) \Big]_{x_o}^{\frac{1}{r}} + 2\gamma \Big[(x_o + N/2) - 1/r \Big] \log Z_m \right\}$$

$$\text{if } Z \neq o, \quad x_o + N/2 \geq \frac{1}{r} > x_o$$

or,

$$+ \quad \gamma . N \log Z_m \qquad \text{if } Z = o \text{ or } x_o \geq \frac{1}{r}$$

By evaluating the social welfare criterion for all different possible combinations of n_1 and n_2 that combination which maximises the criterion may be obtained, and from it the corresponding levels of **Z** and y.

(b) <u>A circular town</u>

With a circular town the additive social welfare criterion may be expressed as,

$$\int_{x_o}^{x_o + R} \left(\alpha \log \left(\frac{Y}{N}\right) + \beta \log (T - x) + \log (Z_m - q) \right) x \, dx \qquad (94)$$

where x_o is given by,

$$x_o = \min. \left\{ \left(T - \frac{N}{\pi . v} \right)^{\frac{1}{2}}, \ \max (o, x_o^*) \right\} \qquad (95)$$

and x_o^* is obtained by an expression analgous to (88) When $\beta = \gamma$, this expression becomes,

$$x_o^* = \hat{T}/2 - N/2\pi v \hat{T} \qquad (96)$$

Where $\hat{T} = T - \dfrac{(Z_m - Z)}{r.Z}$

Again to compute the optimum utility, one scans through the combinations of n_1 and n_2 satisfying (90) or (91) whichever is relevant, and integrating expression (94). This integration yields a series of forms somewhat more complex than (93), but similarly obtainable. Again the combination of n_1 and n_2 and the corresponding values of y and z, that maximise the social welfare criterion may be obtained.

2. A Rawlsian Social Welfare Criterion

From equation (85) we may express the Rawlsian Welfare Criterion as choosing a given level of utility U_0 for all individuals, by varying consumption $C(x)$ over x. It is easy to see, from the necessary condition that consumption at all extreme points of the set of household locations be equal,[19] that the choice of x_0 under such a welfare criterion will be the same as that under the additive criterion. Hence for a long and narrow town x_0 will be given by (87) and (88) and for a circular town it will be given by (95) and an expression analogous to (88), which yields (96) when $\beta = \gamma$. To compute the maximum value of U_0 available when n_1 and n_2 are given and the locations chosen we invert (85) to obtain:

$$C(x) = U_0^{\frac{1}{\alpha}} \cdot (T - x)^{-\beta/\alpha} \cdot (Z_m - q)^{-\gamma/\alpha} \tag{97}$$

Integrating with respect to x and multiplying by two gives,

$$2\int_{x_0}^{x_0+N/2} C(x)\,dx = Y = U_0^{\frac{1}{\alpha}} \int_{x_0}^{x_0+N/2} (T - x)^{-\beta/\alpha} (Z_m - q)^{-\gamma/\alpha}\,dx \tag{98}$$

This expression holds for a long and narrow town. A similar one may be obtained for a circular town. We define E_1 as,

$$E_1 \equiv 2.\int_{x_0}^{x_0+N/2} (T - x)^{-\beta/\alpha} (Z_m - q)^{-\gamma/\alpha}\,dx. \tag{99}$$

Then,

$$U_0 = \left(Y/E_1 \right)^{\alpha} \qquad (100)$$

The integral E_1 cannot be evaluated analytically but it may be computed by using numerical methods and hence U_0 may be obtained. To find the combinations of n_1 and n_2 that maximise U_0 for given n, we scan through the various combinations, taking account of (74), (90) and (91) and compute U_0 for each combination.

2.5.5 <u>Computing the optimum utility when the population is variable.</u>

To obtain the overall optimum, including the optimum town size, we have to scan through the range of town sizes. There is a natural maximum to the town sizes, called N_{max}, where,

$$N_{max} = 2T \qquad \text{for a long and narrow town} \qquad (101)$$

and $$N_{max} = \pi.T^2 \nu \qquad \text{for a circular town} \qquad (102)$$

This follows from the definition of T and the form of the utility function (77).

For the overall optimum, at the chosen N, n_1 and n_2 should be such that, Ω is maximised (as in equation (47)) where,

$$\Omega = \frac{P}{N}\,\omega(Y, Z) \qquad (103)$$

$\omega(Y, Z)$ is the maximum value of the social welfare criterion for given N. Since P is irrelevant to the optimisation, maximising Ω, amounts to scanning through N and, for each N computing $\omega(Y, Z)$ as indicated in the previous section. That value of N that maximises $\frac{W(Y, Z)}{N}$ is the desired overall optimum.

2.5.6. Investigating the Possibility of Multiple Equilibria for a Fixed Town Size.

In section 2.3.1 we discussed the possibility of multiple solutions to the necessary conditions that define the optimum to an additive social welfare criterion for a long and narrow town. It would be of some interest to investigate whether such multiple solutions were likely for plausible parametric values. Such solutions would represent multiple tangency points between the welfare indifference curves, and the production constraint, in diagram 5 and their existence and positions would help throw some light on how likely a competitive system is to go wrong in controlling pollution.

From equation (9) we know that a necessary condition for an interior optimum to the above problem is,

$$2\int_{x_0}^{x_0 + N/2} - \frac{U_Z}{U_C}\, dx = F_Z \qquad (104)$$

when $\sigma = 1$

Given (77) and (81) the L.H.S of (104) may be written as L_1 where

$$L_1 = 2.\ \frac{\gamma . Y}{\alpha . N} \int_{x_0}^{\min\left[x_0 + N/2,\ 1/r\right]} \frac{1 - rx}{Z_m - Z(1 - rx)}\, dx \qquad \text{for } x_0 \leq \frac{1}{r} \qquad (105)$$

$$0 \qquad \text{for } x_0 > \frac{1}{r}$$

This may be integrated to give,

$$L_1 = \frac{2.\gamma.Y}{\alpha..r.N.Z}\left[\left.\frac{Z_m}{Z}\left[\log\ (Z_m - ZV) - 1\right] + V\right|_{V(x_0)}^{\max\,\{V(x_0 + N/2),\ 0\}}\right] \text{for } Z \neq 0 \qquad (106)$$

$$\text{Where } V(x) = (1 - rx) \qquad (107)$$

From (67) to (72) the R.H.S. of (104) may be written as L_2, where,

$$L_2 = \frac{1}{1 + \dfrac{\delta D n_2^{\delta-1}}{K n_1^{K-1}}} \qquad \text{for } n_1 < N^*. \quad n_2 > 0 \qquad (108)$$

$$L_2 = \frac{1}{1 + \dfrac{\delta . D n_2^{\delta - 1}}{\varepsilon . \beta n_1^{\varepsilon - 1}}} \qquad \text{for } n_1 > N^*. \quad n_2 > 0 \qquad (109)$$

Thus by scanning through the permissible values of n_1 and n_2 all solutions of equation (104) can be computed, and furthermore the shape of the welfare indifference surfaces and the production constraint can be deduced.

2.5.7. Some Plausible Parameter Values for the System

Some Plausible Parameters of the Production Structure.

We consider the parameters in the following order,

Parameters of the production structure.

k, ε, N^*: The parameters of the production structure are chosen in such a way that the city size which maximises average product with no pollution control will be one with around a million inhabitants. This seems a typical city size to consider, and, as Clark (1968) points out, it appears to/be the emerging size in an industrial country. Consequently, in order to obtain an N^{**} of 1,000,000, N^* is fixed at 500,000. N^{**} may be expressed in terms of N^*, ε, and k by the following expression

$$N^{**} = \left[\frac{1}{k} \frac{(k - \varepsilon)}{(1 - \varepsilon)}\right]^{1/\varepsilon} . \; N^*$$

With the given value of N^*, we consider values of k ranging from 1.2 to 1.8 and values of ε ranging from 0.6 to 0.9. This gives an N^{**} ranging from about $3/_4$ million to $1^1/4$ million.

As a special case we consider the situation where $\varepsilon = 1.0$. In this event average product is always rising and the uncontrolled city size is defined as the maximum possible city size, for a monocentric city given the geographical and travelling limitations.

δ: The returns to scale in the cleaning activity are taken over the same range as the decreasing returns to scale for the production activity (0.6 to 0.9).

ξ: Unfortunately there are few estimates available of the resource costs of eliminating pollution in cities, and it is not clear that what estimates are available have a direct bearing on the value of ξ. As far as noise pollution is concerned no overall estimates are available. Urban noise has not been costed. Regarding aircraft noise, Walters (1974) has recently suggested that an expenditure of \$2 billion would be required to retrofit the existing fleet of U.S. airlines[20]. Unfortunately this tells us nothing of the effects on operating costs. (However this substantial reduction in aircraft noise would represent less than $^1/2$% of GNP in the U.S.) As far as air pollution is concerned, some better estimates are available. According to a report from the U.S. Environmental Protection Agency (1972), the costs of implementing the clean air acts amount to between 1 - 2% of G.N.P. These figures are reported by Beckerman (1974), who points out that the standards required by the new act represent very sharp reductions in air pollution. (e.g. an 86% reduction in the level of sulphur dioxide relative to its previous level). This would suggest that, the resource cost of bringing about a negligible level of air pollution would be somewhat above 2% of G.N.P. (with decreasing returns in the cleaning sectors the last few units of pollution are more costly in resources to clear).

Overall then the costs of reducing air and noise pollution in cities to negligible proportions could be anything from about 3% upwards. It is difficult to know what the upper bound is, and so values of ξ were tried from 0.05 up to a point where no abatement is undertaken at the optimum. This point appears to be around 0.20, indicating how high the cleaning costs would have to be for the maximum pollution to be optimal. Roughly speaking, a value of ξ of 0.05 represents a cost of $2^1/2$% of G.N.P. to get rid of all pollution and proportionately for higher values of ξ[21].

v: This variable represents the average density of the town. Most density studies of urban areas suggest a density function that declines exponentially with distance from the centre , although there is some evidence that some large cities in the U.S. are tending towards a uniform density pattern, with an average density of about 10,000 inhabitants per square mile. The average density for cities in the U.K. as given by the H.M.S.O. Handbook of Britain (1968) is about 18,000 per square mile. We take this density figure and recognise that the model will be biased in so far as a variation in density is an important adjustment factor in attaining the optimum. Thus for example the model will exaggerate the consequences for the optimum of a change in the rate of dispersion of pollution in the environment.

Parameters of the Social Welfare Criterion.

β: This parameter represents the preference for nearness to the city centre, and, with this form of utility function, a utility maximiser would be expected to spend the proportion β of his income on being near to the centre. While no direct estimate is available for this we take the expenditure on work transport as a proportion of total personal expenditure as a proxy for this. Regarding this we have some evidence from various surveys, including one by the U.S. Bureau of Labour, which is analysed by Oi and Shuldiner (1962). According to them households in cities of around a million spend 12.6 percent of their income on all travel. This will overestimate the expenditure on travel within the city. However, Oi and Shuldiner's Study indicates that a large proportion of travel expenditure is intra-city. Furthermore there is some suggestion that this number rises with real income and the U.S survey was conducted in 1950. Consequently a range of values of β, ranging from 0.1 to 0.15 were tried in the simulations, these representing estimates of 10 percent and 15 percent respectively.

γ: This represents the households distaste for pollution. In chapter 5 we discuss some evidence regarding household expenditure on quiet from house price differentials in noisy and quiet areas. These suggest an average expenditure of between 8 and 11 percent on quiet. Other studies of values of properties in zones affected by air pollution suggest values in the same range, but slightly smaller.[22] We have taken values of γ ranging from 0.08 to 0.16 in our simulations. Given β and γ, α is fixed as $1 - \beta - \gamma$, by the normalisation which allows us to interpret these coefficients as proportions of expenditure on the various goods.

T, Z_m: These two variables define the origins of the utility functions: T gives us the maximum distance fromthe town centre and Z_m the maximum level of endurable pollution. T was set at 10 miles - a distance which implies a width of 2.6 miles for a long and narrow city and which allows cities of up to six million people, at the densities chosen, with a circular land use pattern. This seems reasonable. Z_m was chosen as that level of pollution produced in a town of sixe N^{**} (maximising average product), with no resources devoted to abatement. From some sensitivity runs it appears that while optimum allocations of resource to abatement are affected by changes in the value of Z_m, the relative numbers are little changed and the optimum size of the town is quite insensitive to small variations in Z_m, and the choice of the origin of measurement does not appear to be crucial.

r: As far as noise nuisance is concerned, it appears from recent studies that traffic noise has an important but rather localised effect. Measures of such traffic noise are now available for some motorways and urban motor vehicle noise. In a recent book Anthorp (1973) reports on a study in Tokyo which calibrated median noise levels at different points in time against the traffic density. This shows " a high correlation between traffic density and measured noise levels"[23] In view of this evidence we shall take the traffic density as an approximate measure of noise levels. Some work has been done on this by the Traffic Studies group at University College/London and Vaughan et al, (1972) have shown that traffic density, as measured by total distance travelled per unit area on major roads, is a negatively exponential function of the distance from the town centre, reaching very low/levels within 2.5-3.75 miles for towns such as Reading and Luton. These towns have of course considerably less than a million inhabitants. In an unpublished paper Hutchinson has continued this work, and shown that parameters of the negative exponential depend on the city size, and with a city of about a million decline to very low levels with 5-10 miles of the city centre. Consequently we have taken/r as between 0.2 and 0.1. As far as air pollution is concerned, we are interested in the dispersal rate around a concentration of factories serving a population of about a million. (We ignore here the fact that such sources of air pollution are usually spread round the town rather than concentrated). From a brief examination of the National Survey of air pollution (1972) it would not seem unreasonable to take a dispersal rate leading to a 'base' level of air pollution within the same distances from the pollution centre. This is what we have done.

<u>2.5.8 Numerical Results</u>

<u>The effects on the optimum allocation of a fixed town size when,</u>

(a) <u>The social welfare criterion changes</u>

(b) <u>The geography of the town changes from long and narrow to circular</u>

(a) Whether the criterion of social welfare is an additive one, or a Rawlsian one makes little difference to the optimum proportion of the population to be devoted to pollution abatement and consequently of the optimum level of pollution. In all the cases considered the Rawlsian and additive criteria both gave optimum values for the above variables that were within eight percent of each other. The differences between the two criteria were greatest for low values of ξ with the Rawlsian criterion giving a lower allocation of resources to abatement and consequently a higher level of pollution. With

ξ at 0.2 the differences were less than two percent. In the case of the additive welfare function one chooses Y and Z to maximise the log sum of utilities over households. One might think of the Rawlsian function as choosing Y and Z to maximise the sum of utilities (with $\sigma = 0$) and then choosing the functional $c(x)$ to obtain equal utilities for all individuals. Thus while the additive welfare function does not do quite what the first part of the Rawlsian one would do, it does something similar. The numerical results suggest that the differences are rather small.

(b) In table 1 we present the optimum proportion of the labour resources that should be allocated to abatement in a long and narrow town and in a circular town, for three different values of ξ and two values of r and two combinations of β and γ. It appears that a circular town should go in for more clearance when the resource cost of such clearance is low, but that it should go in for less clearance when the cost of clearance is relatively high. These affects are accentuated when the pollution is more concentrated round the centre. There are two forces trading off against each other here. On the one hand a greater level of abatement allows more pleasant living, especially nearer the centre, but on the other hand it implies lower consumption. With a circular town the 'benefits' of clearance are greater in that more people can take advantage of the improved environment but on the other-hand the consequences of not clearing up are less bad than in a long and narrow town because even with very little abatement more people can live in pleasant areas. This leads to a more extreme behaviour in the circular optimum town: either you go in for a lot of clearance, or you go in for very little.

		$\alpha=0.7,\ \beta=0.15,\ \gamma=0.15.$		$\alpha=0.8,\ \beta=0.10,\ \gamma=0.10.$	
	ϵ \ r	0.15	0.20	0.15	0.20
Long and Narrow	0.05	4.15	2.70	1.83	1.13
Circular		6.58*	6.58*	4.42	0.05
Long and Narrow	0.10	2.91	1.78	1.13	0.65
Circular		1.12	0.05	0.16	0.00
Long and Narrow	0.20	1.83	1.02	0.54	0.27
Circular		0.21	0.00	0.05	0.00

Optimum percentage of labour force to be devoted to abatement

Table 1 $k = 1.3 \quad \delta = 0.7 \quad \epsilon = 0.7 \quad N^{**} = 925{,}000$

A star indicates that with that level of resources devoted to clearance all pollution is cleared.

This behaviour is repeated for the whole range of parameter values for which the optimum behaviour of long and narrow and round towns was computed. In diagram 10 we present a scale representation of the land areas available in the two kinds of towns.

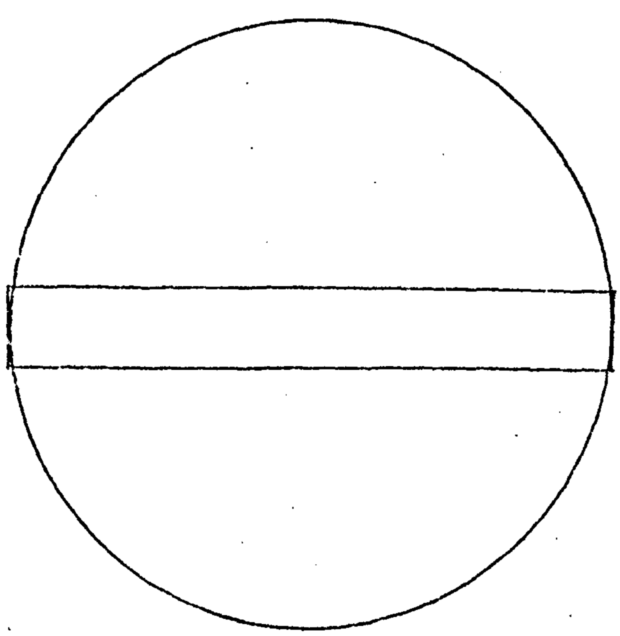


Diagram (10)

<u>Sensitivity of the allocation of labour to the two activities, when the town size is fixed, with respect to</u>

(a) <u>Changes in r and ξ</u>

(b) <u>Changes in k and ε</u>

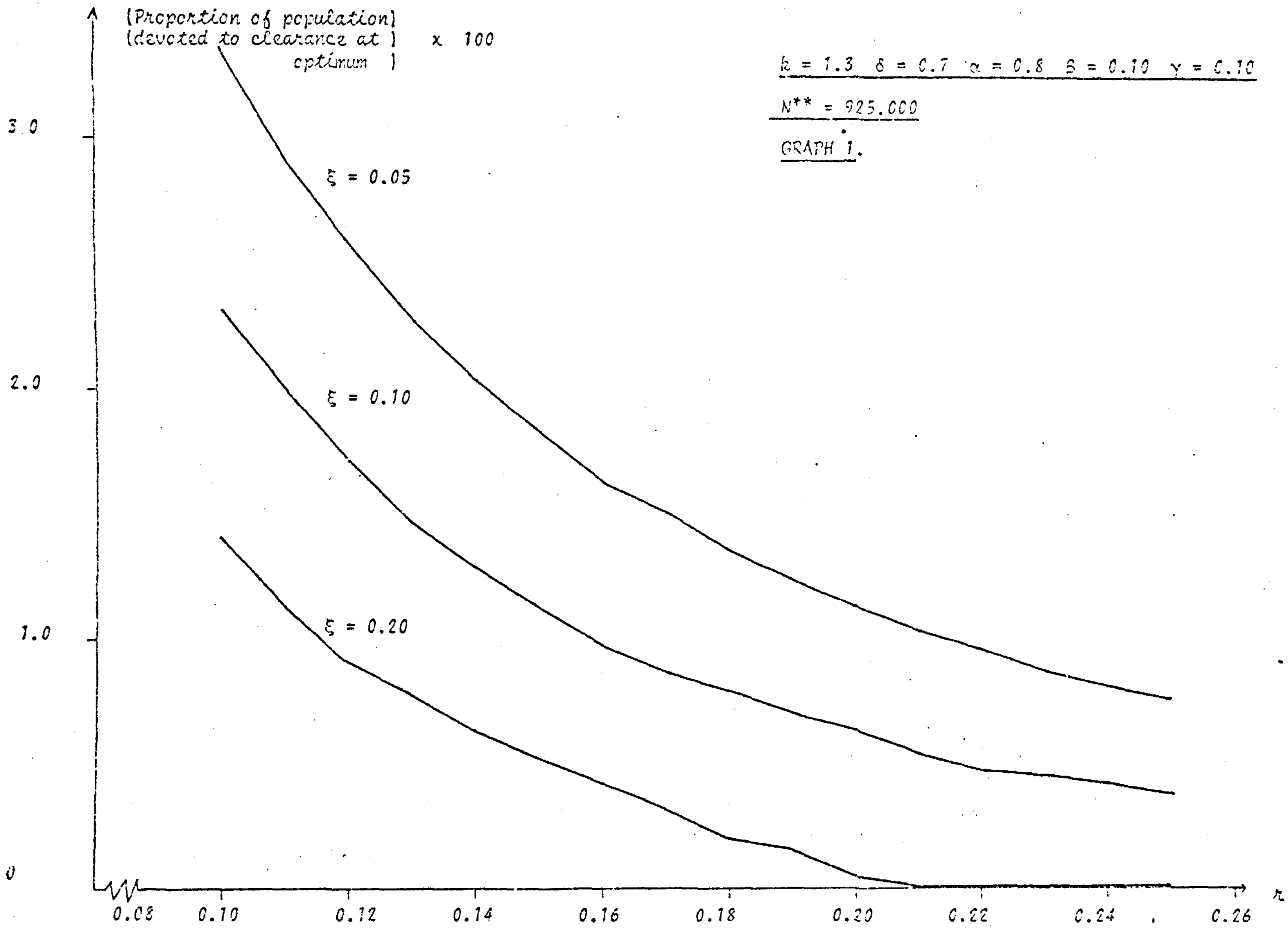
(c) <u>Changes in β and γ</u>

(a) <u>Changes in r and ξ</u>

In graph 1 we present the optimal percentage of the labour force that is allocated to abatement in a long and narrow town with an additive social welfare criterion, for different rates of pollution dispersal ranging from 0.10 to 0.25. This suggests that as the pollution gets more concentrated round the centre, the extent of abatement falls at a declining rate, with a higher cost of pollution abatement leading to a smaller allocation of resources to clearance and consequently to a higher level of optimal pollution. The rate of decline seems fairly constant for different values of ξ. These results hold quite widely across the parameter values, and are in accordance with what one would expect.

(b) <u>Changes in k and ε</u>

In table 2 we give the percentage of the labour force allocated to abatement for nine different combinations of k and ε and two combinations of utility parameter values. It seems clear that as the level of increasing returns for the intial part of the production function get greater the resources devoted to clearance fall. A higher value of k represents greater productivity in the consumption good sector. Consequently the opportunity cost of shifting resources to abatement is greater.


(Proportion of population)
(devoted to clearance at) x 100
optimum)
k = 1.3 δ = 0.7 α = 0.8 β = 0.10 γ = 0.10
N** = 925.000
ξ = 0.05
ξ = 0.10
ξ = 0.20
3.0
2.0
1.0
0
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
0.24
0.26
r

GRAPH 1.

ε \ k	$\alpha = 0.7$ $\beta = 0.15$ $\gamma = 0.15$			$\alpha = 0.8$ $\beta = 0.10$ $\gamma = 0.10$		
	1.2	1.3	1.5	1.2	1.2	1.5
0.7	5.57	4.16	2.70	2.69	1.84	1.00
0.8	5.12	4.20	3.20	2.32	1.75	1.15
1.0	5.20*	5.40*	4.45	2.75	2.50	1.45

TABLE 2 $\xi = 0.05$ $r = 0.15$

Optimum percentage of population to be allocated to abatement for a town of fixed size. (Long and Narrow with an additive social welfare criterion). A star indicates that with that level of resources devoted to clearance, all pollution is cleared.

With a log additive utility function, the marginal utility of abatement is independent of consumption and consequently not affected by changes in the value of k. Hence a fall in the allocation of labour resources to clearance with a rise in the value of k is what one would expect. With a non-separable utility function such a result need not hold. As ε increases for given k there is no clear indication as to how the optimal level of pollution goes.

(c) Changes in β and γ

It would seem that the optimal control of pollution is quite sensitive to the actual values of β and γ. The allocation of resources to abatement can vary by a factor of 3 to 4 between the extreme combinations of β and γ within the rather small range considered for these parameters. The differences are most accentuated when the value of ξ is high and when pollution is greatly dispersed over the environment. It would seem then that a satisfactory pollution control policy would have to pay great attention to the values of parameters that represented individual preferences concerning pollution and nearness to ones work centre.

Main Factors Affecting the Optimum Size of Towns

In table 3 we present some results regarding the optimum town when the town size is variable. A town constrained to a long and narrow geography and a town constrained to a circular geography are considered. We report, in each case the optimum town size, the optimum allocation of labour to abatement and the pollution level in the optimum town as a percentage of the pollution level in the uncontrolled town.

		Long and Narrow			Circular		
	ξ	0.05	0.10	0.20	0.05	0.10	0.20
$r = 0.15$	Optimum town size	700,000	700,000	700,000	800,000	750,000	750,000
	Optimum N_2 (%)	1.21	0.50	0.14	3.5	0.47	0.07
	(Z/Z_m) (%)	45.08	59.74	65.20	25.94	64.17	71.17
$r = 0.20$	Optimum town size	700,000	700,000	700,000	850,000	800,000	800,000
	Optimum N_2 (%)	0.64	0.29	0.07	0.0	0.0	0.0
	(Z/Z_m) (%)	53.07	62.16	65.94	82.47	82.47	82.47

TABLE 3 $\kappa = 1.3$ $\delta = 0.7$ $N^{**} = 925{,}000$ $\alpha = 0.8$ $\beta = 0.1$ $\gamma = 0.1$

For these parameter values the town size in the case of a long and narrow town is reduced by about 24 per cent and the circular town by between 8 per cent and 19 per cent, the lower reductions for the circular town being of course what one would expect. It is interesting to note that most of the fall in pollution at the optimum is achieved by choosing a smaller town size, and hence a lower level of pollution <u>created</u>, rather than a large devotion of resources to pollution <u>removal</u>. Comparing the pollution level in the circular and long and narrow towns one observes, again, a more extreme choice of pollution in the former than in the latter.

A number of combination β and γ were tried within the range selected and it was found that the optimum city size and the optimum pollution level went down as γ and β were raised. The maximum difference in optimum city size due to the values of these parameters was about 100,000.

It is interesting to compare the results obtained for values of ε which are less than one, and in the region 0.7 to 0.9, with those obtained when $\varepsilon = 1$. In the latter case average output in the production sector is always rising and the uncontrolled city size is defined by constraints on the city area imposed by the value of T. With a rectangular land area this implies, in our case, a maximum population of a million and in the circular case one of about $5^1/2$ million. We present, in graphs 2 and 3, the value of the social welfare criterion and the percentage of pollution cleared as a function of the city size for a long and narrow town, and in graphs 4 and 5 the same things for a circular town. Graphs 2 and 4 have $\delta = 0.7$ and graphs 3 and 5 have $\varepsilon = 1.0$. In all the cases where $\varepsilon = 1$ very little labour is allocated to abatement, and almost all the gains are obtained by reducing pollution through sacrificing consumption (reducing average output). In the circular city the fall in the population required to obtain the optimum size when $\varepsilon = 1$ is spectacular - the optimum size ranging from 1.9 million to 2.7 million depending on the parameter values.

Multiple Equilibria for a town of fixed size with increasing returns in Production.

In Section 2.5.6 we pointed out that it was possible to obtain some idea of shape of the production constraint and the welfare indifference curves drawn in diagram 5 by evaluating the left and right hand sides of equation (104), as given in equations (106) to (109). This was done for a range of values of κ, ε, γ, and with $\delta = 0.8$. This last value implied that the production constraint was concave. For 48 scans that were tried, 13 showed two interior solutions to equation 104. From the numerical values of the L.H.S. of 104 the welfare indifference curves seem to take the shape given in diagram 11 below. (The interior optimum giving the greater utility always represents a lower level of pollution). Thus a pollution tax policy equating the tax on pollution to ΣMRS could land up at either e_1 or e_2. In some cases the differences between e_1 and e_2 is quite large but it can be as little as e_1 representing 3.4 per cent of the labour force employed in abatement and e_2 representing 4.6 percent. Thus it is quite apparent that a market tax solution could easily lead to a local minimum, and, furthermore as the above figures show, one that is not patently absurd.

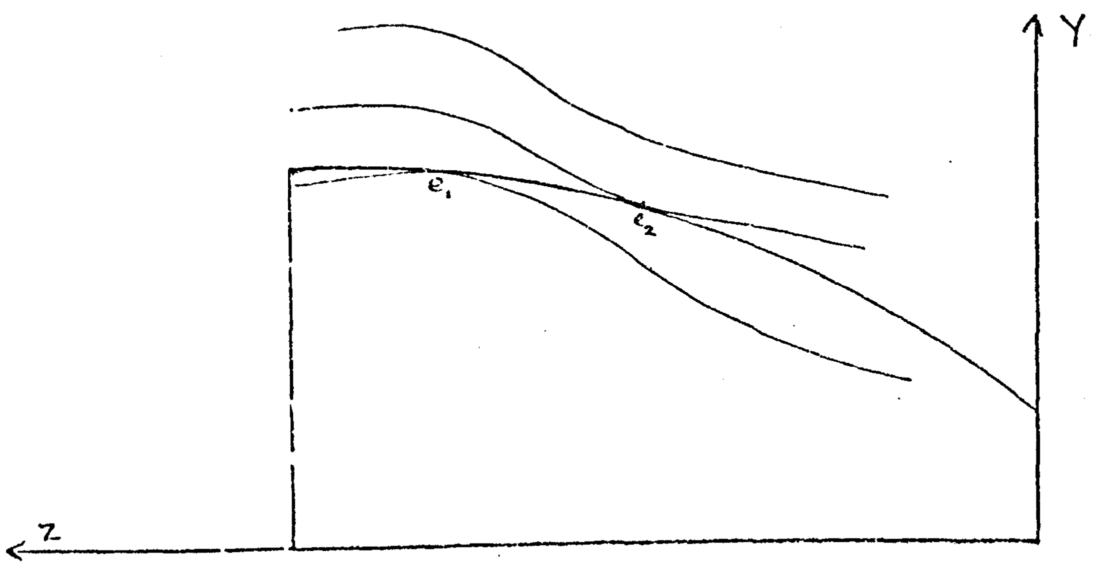
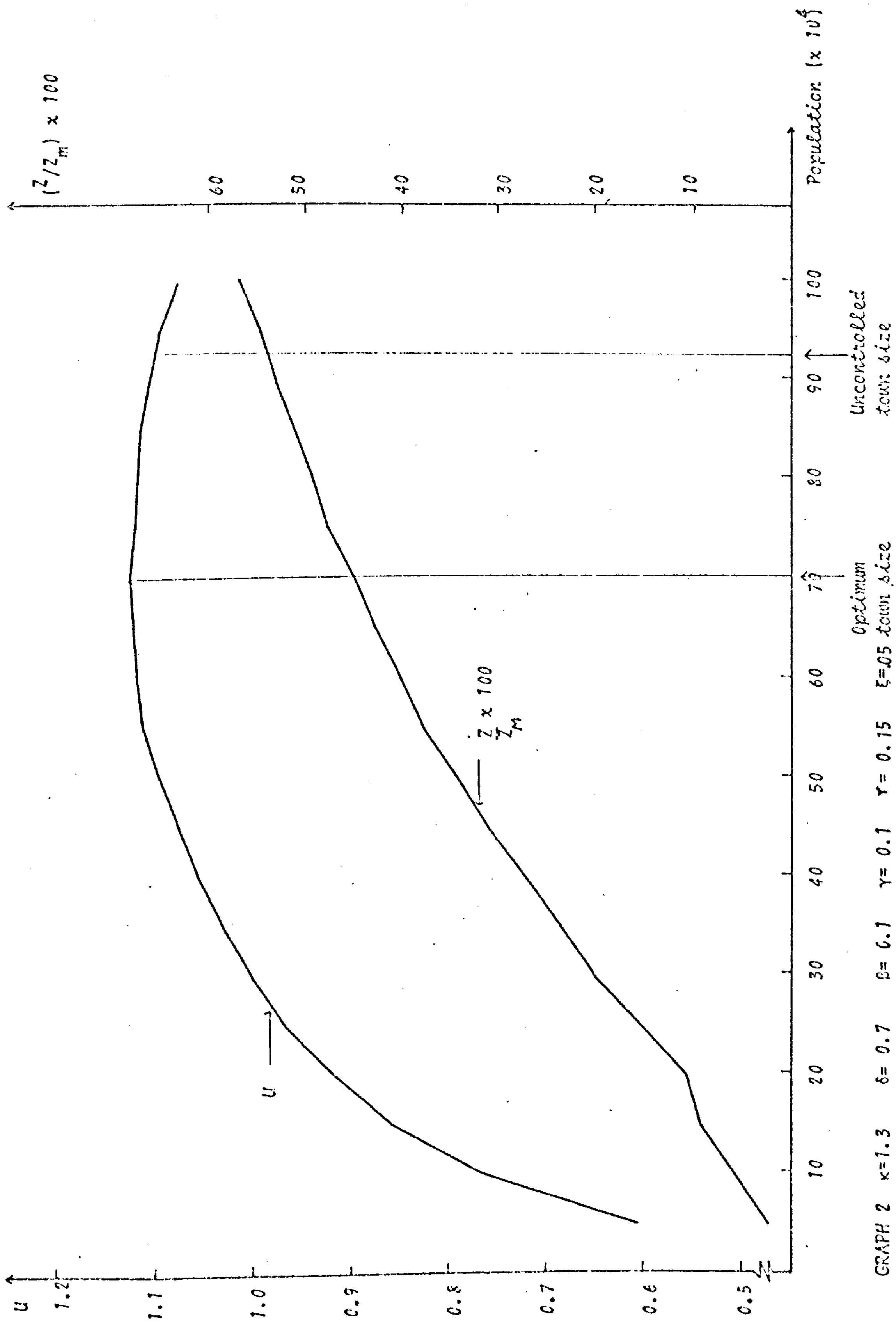
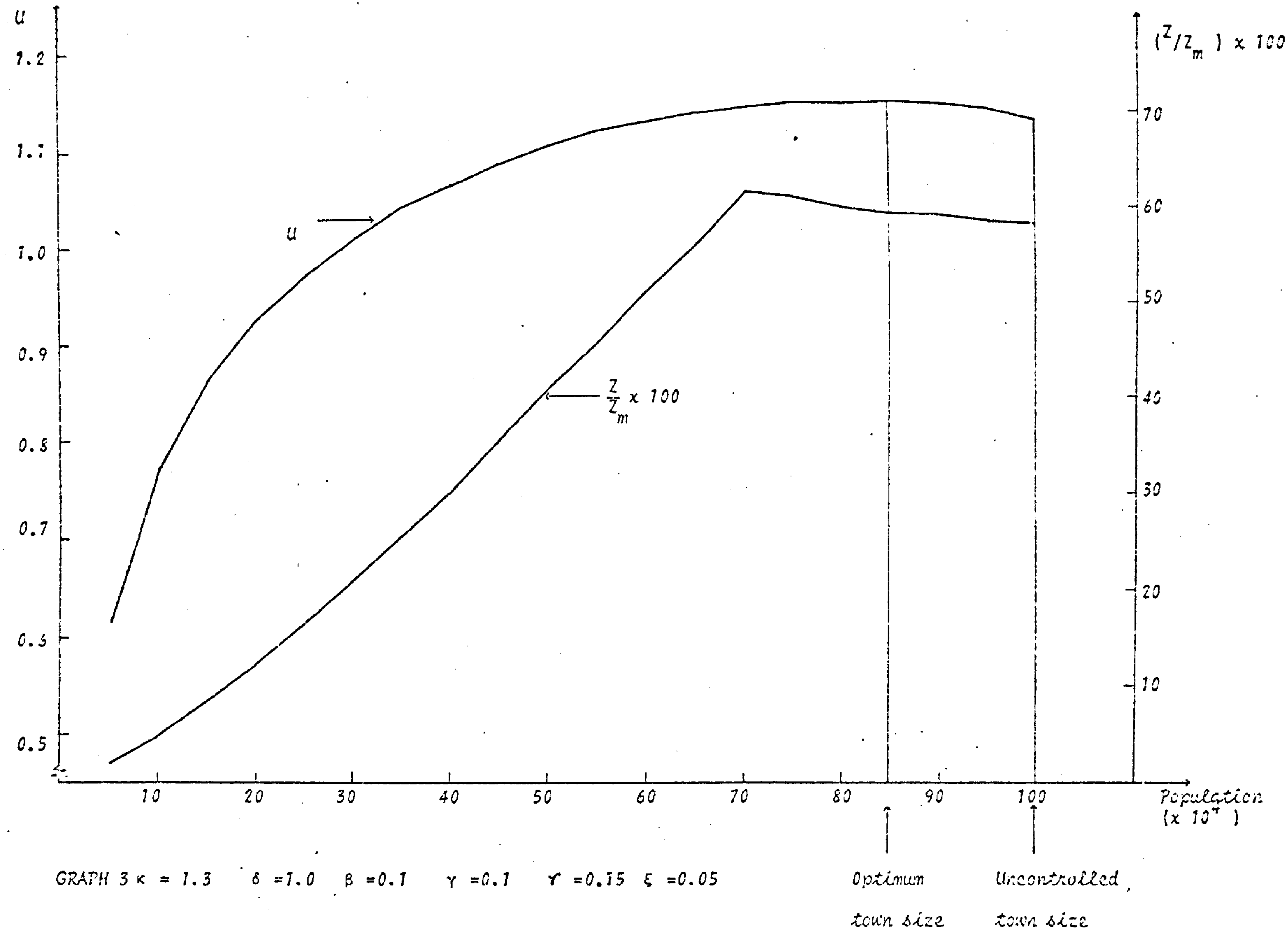


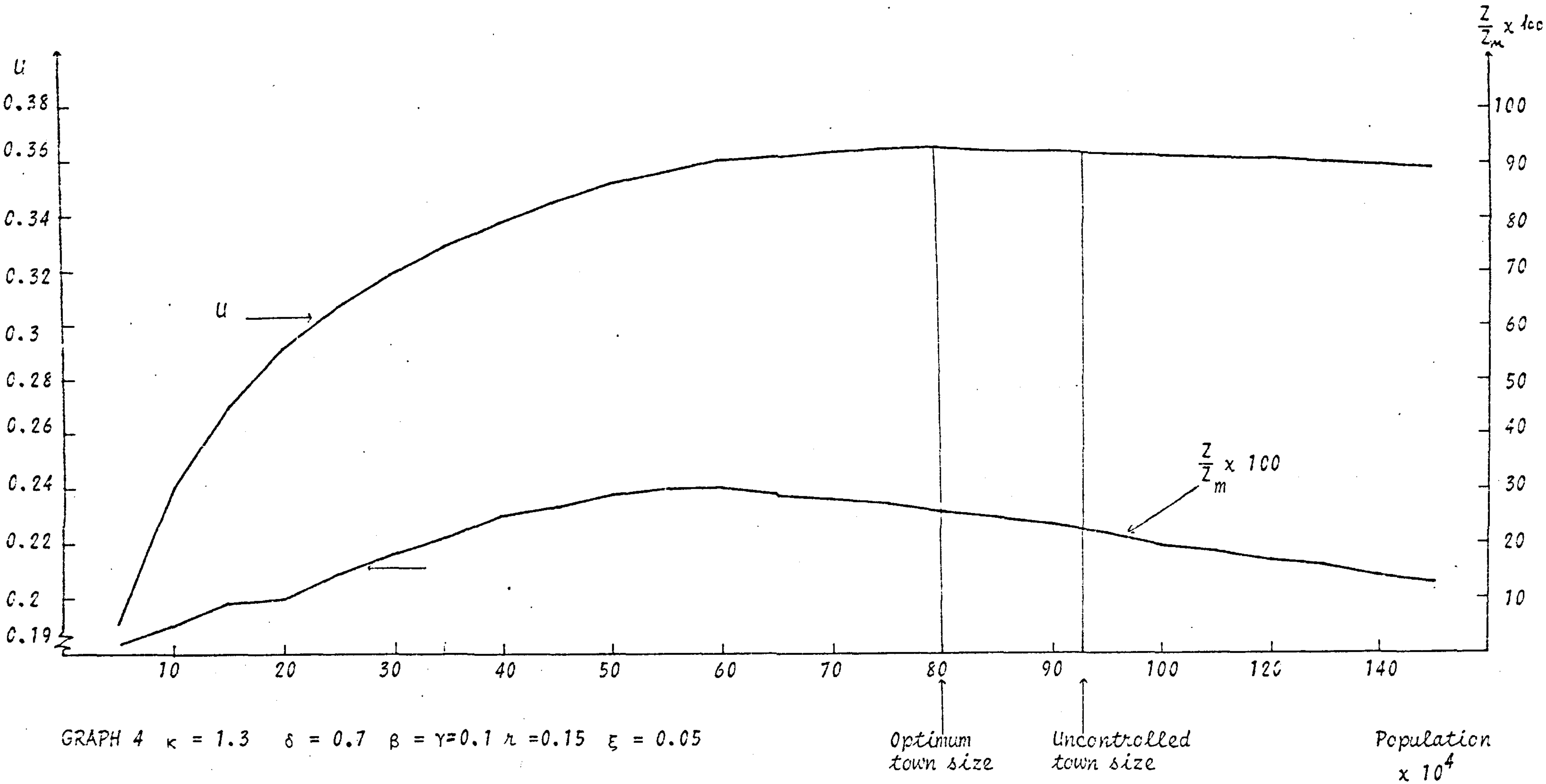
Diagram 11



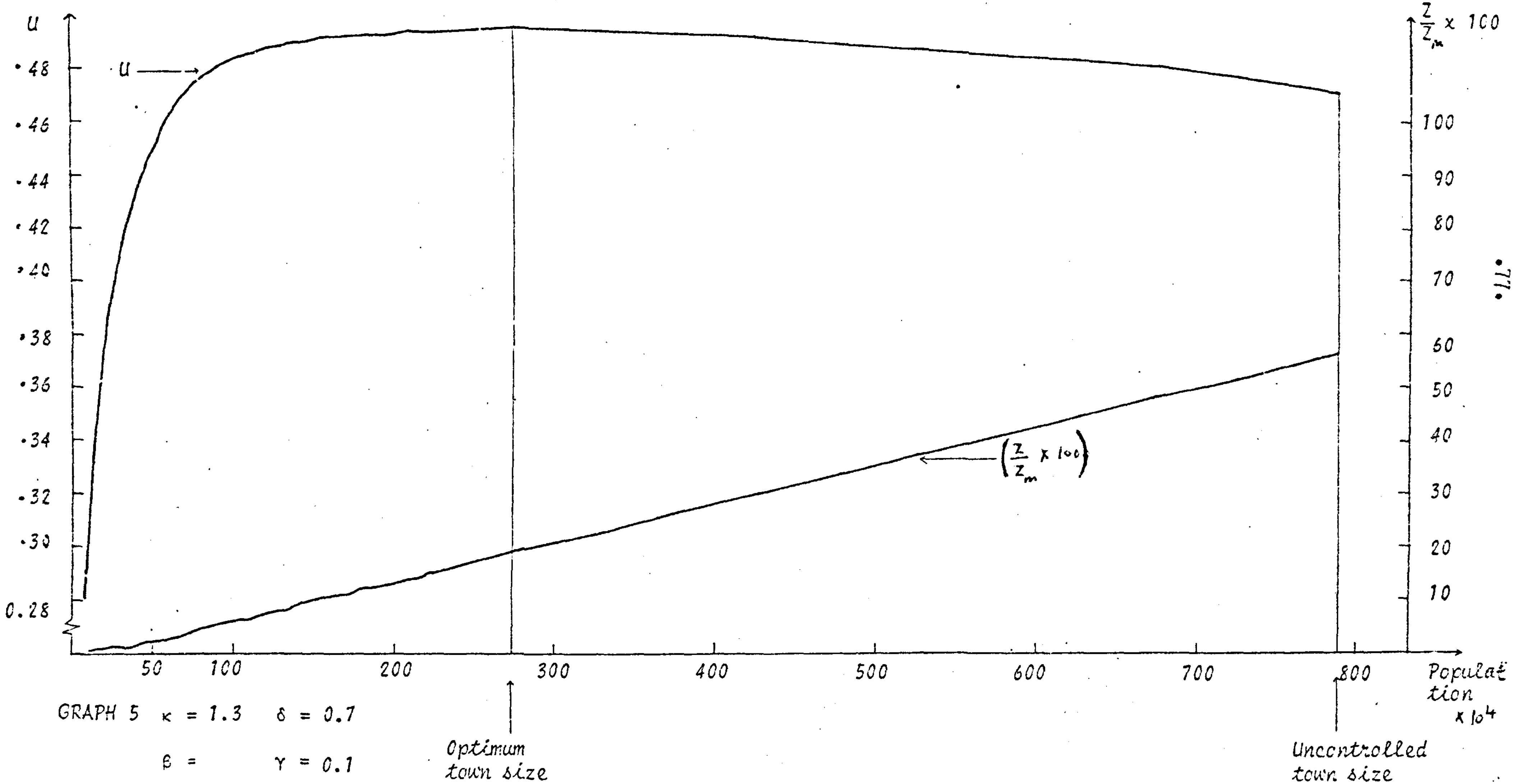
GRAPH 2 $\kappa=1.3$ $\delta= 0.7$ $\rho= 0.1$ $\gamma= 0.1$ $\tau= 0.15$ $\xi=.05$



GRAPH 3 $\kappa = 1.3$ $\delta = 1.0$ $\beta = 0.1$ $\gamma = 0.1$ $\Upsilon = 0.15$ $\xi = 0.05$



GRAPH 4 $\kappa = 1.3$ $\delta = 0.7$ $\beta = \gamma = 0.1$ $r = 0.15$ $\xi = 0.05$



GRAPH 5 $\kappa = 1.3$ $\delta = 0.7$

$\beta =$ $\gamma = 0.1$

$\xi = 0.05$

2.6 Conclusions

In this chapter we have considered the problem of optimum towns in the presence of environmental pollution. The structure of the town was taken simply as one with production concentrated at the centre and residential location organized in its vicinity. Initially we derived the necessary conditions defining the optimal locations and the optimal levels of pollution of a town of fixed size, when the social welfare criterion was an additive one and when it was a Rawlsian one. We observed that non-convexities had an important role to play, both as far as the production constraint was concerned and as far as the social welfare criterion was concerned. This meant that the problem could not be posed interestingly in a way which led to a unique interior optimum being defined by the first order conditions. Consequently a price decentralisation involved the possibility that equating the marginal product of pollution to the sum of the marginal rates of substitution between the public bad and consumption could lead to a constrained minimum position - a possibility that was lent some further credence when some numerical computations were done. In these circumstances it is important that not only should information regarding ΣMRS be obtained but also some information should be obtained regarding <u>its</u> rate of change.
We also examined certain features regarding the optimal location of households and the distribution of consumption across households. Here it turned out that little could be said regarding these factors when general utility functions were specified. With an additively separable utility function, however, consumption is independent of location and equal for all individuals, when an additive social welfare criterion is used. With a Rawlsian criterion consumption is equal at all extreme points of the set of locations (excluding the centre) and moves in an opposite direction to the locational utility as one traces its behaviour over the residential locations.

The locations are defined over an <u>interval</u> when the location utility along a ray from the centre is single peaked, for any given level of consumption, and the conditions for this to be so are stated. When the residential locations are defined over an interval, this interval does not necessarily move closer to the centre as the pollution level at the centre declines. Whether it does so or not depends on how the rate of dispersal is affected by changes in the level of pollution.

If the size of the town is not fixed then there are two choice variables, the size of the town and the level of pollution. The necessary conditions for optimality are given here, and a comparison is made with the uncontrolled competitive town. This town is defined as one which maximises average product, and pays no attention to the externality. For such a town size to exist, the increasing returns in production postulated hitherto have to be limited, and followed by some diminishing returns. If such diminishing returns do not exist then the uncontrolled city size is defined by some other constraint, such as land area available. The uncontrolled town cannot generally be shown to be larger than the optimum town. When the uncontrolled town size has some locally diminishing returns in its neighbourhood, and when the optimality conditions define an optimum 'close' to the uncontrolled town, then the conditions of the optimum town to be no larger than the uncontrolled town can be stated.

As with a fixed size town it is not generally possible to obtain the optimum by decentralisation. If the optimum is defined at a locally concave position in the production set, then, subject to the provisos made earlier, the optimum may be decentralised by using a pollution tax and an employment tax, and having producers who are price takers and profit maximisers.

A parametric representation of the model discussed above was made. Some numerical values were taken for the parameters, in order to obtain explicit solutions for the optimum level of pollution, the optimum allocation of labour to abatement, and the optimum size of town; and in order to examine the sensitivity of these variables to various assumptions. It turned out that whether the social welfare criterion is additive or Rawlsian made very little difference to the optimum values. The Rawlsian criterion implied slightly higher optimal values for pollution when the abatement costs were low. The geography of the town on the other hand matters rather a lot and a circular geography implied more extreme choices of pollution levels with different parameters. Also important in determining the optimal control of pollution were the values of the parameters representing the preferences. For quite small changes in these parameters, relatively large changes could be obtained in the optimum allocation of resources to abatement.

The optimum town size always emerged as smaller than the uncontrolled town size for the parameters chosen. The difference was less with a circular town than with a long and narrow town, except when average product was always rising, in which case the circular town is much larger in the uncontrolled state and reduces sharply in the controlled state. When comparing the optimum town with a town of a size fixed at the uncontrolled level but with optimal pollution control, it was observed that the former involved substantially less abatement than the latter. It appears therefore that where both town size and pollution levels are variable, there is a tendency for the optimum position to rely on reducing town size than on removing pollution.

The above conclusions are obtained in the context of a model that excludes several important aspects of an urban area. Other authors have examined some of the trade-offs represented by factors excluded here and their conclusions cannot be directly compared to ours. However, it is felt that while the actual 'numbers' reported here may be altered in a more general model, the qualitative conditions may still be of some importance.

FOOTNOTES TO CHAPTER 2

1. Setting A equal to 5 implies that the utility function is only defined for households living up to five miles from the centre of the town. We can think of this as implying that a monocentric town is bounded by natural considerations such as travel time.

2. Dixit: The Optimum Factory Town (1973).

3. Clearly we can always choose the units of measurement so that this is true. However, once we have done so the assumptions of concavity or convexity that are made with regard to 'natural' units may no longer hold with regard to these transformed units. While we recognise this possibility we do not deal with it.

4. This argument is taken from Mirrlees (1972).

5. The set is defined on E^1 with the centre of the town being represented by the origin. The argument in this and the succeeding section relies on the fact that we are considering a continuum of individuals. Thus is one individual occupies a unit length then a length dx will be occupied by 'dx' of one individual.

6. The indices of aircraft noise pollution discussed in chapter 5 certainly display this quality, and, from a cursory lock at the National Survey of Air Pollution (1972) it appears as if this is also the case for air pollution.

7. We assume sufficient continuity in the deriviatives for the inverses to be differentiable.

8. n_1* is the quality of labour devoted to pollution removal that gets rid of all pollution when the total labour force is N.

9. The consumption allocation rule then gives an equal allocation of consumption, and the location of individuals is over an interval. We drop the multiple 2 which indicated that the population is allocated on both sides of the town. In fact it is of no analytic importance and only has to be borne in mind when the computations are done.

10. This footnote has be excluded from the text.

11. If (a) the utility function is concave and completely and additively seperable in all its arguements, and, (b) the changes in the way that pollution spreads itself as the level of pollution rises are not such as to outweigh the tendancy to move the residential areas further out with the increase in pollution, then the first term will be positive. Again assuming concavity of the utility function, and that changes in pollution levels at distance X from the centre are in proportion to the changes at the centre, then the second term will be negative.

12. This occurs because of the constraint of equal utility. The indifference curves in the consumption - distance- from - the- centre space as shown in diagram 3 are not symmetric about the X axis and so higher levels of output require a different locational interval. This proposition is somewhat unintuitive.

13. The price function is being treated as differentiable, and again this assumes considerable smoothness in the relevant functions.

14. Assuming of course, that it is differentiable as stated earlier. This kind of decentralisation is not another example of the propositions found in general equilibrium theory, for we have a contin.um of consumers and non-convexities in the consumption set. Under these conditions an optimum cannot in general be decentralised.

16. We require that A^* and B^* be small numbers because the results require that the production function $F(\cdot)$ be locally concave at the A and B at which the partials are evaluated. The production process for the consumption good is locally concave from N^* onwards and in fact on all the computations done in optimum city size we never got anywhere as low as N^*. Thus as long as $F(\cdot)$ is concave in Z, the above results obtained are likely to hold.

17. THe firm is assumed to maximise profits subject to a given pollution tax P_Z, and a payroll tax dependant on the number of individuals employed, i.e. tte firm has the objective:

$$\text{MAX } \Pi = F(N,Z) - P(N) - P_Z.Z$$

The payroll tax $P(N)$ and the pollution tax P_Z have to be chosen so that the optimality conditions (63) and (64) are satisfied. From this it follows that the pollution tax will equal the sum of the individual marginal rates of substitution between Z and C when the population is optimally located. From the first order profit maximising conditions and the equilibrium condition that the number of towns be such that zero supernormal profits are made, we obtain :

$$P_N. N - P(N) \quad \frac{N\,U(\frac{Y}{N}) + 2\int_{x_0}^{x_0 + N/2} R(x,z)dx - NR(x_0 + N/2, Z)}{U_c\,(Y/N)} + P_Z\,Z$$

The term onthe right hand side is derived by replacing A^* in (63) with the value derived in (56). The RHS will be positive if the locational utility is a concave function of distance fromthe centre - the conditions for which we stated on page 12. If the RHS is positive, however, this implies that the function $P(N)$ is convex and the profit function is a concave function of N and Z. In that case the first order conditions do define the maximum profit position, when the function $F(\cdot)$ is concave in N and Z. Hence, given, the 'right' function $P(N)$ and the right value P_Z, profit maximisation subject to given prices will result in the necessary optimum conditions being satisfied when

(a) The optimum size of the labour force allocated to production is greater than N^*.

(b) The pollution removal process has decreasing returns to labour. ((a) and (b) imply that $F(\cdot)$ is locally concave in N and Z).

(c) The utility function is aditively seperable.

(d) Utility is a concave function of distance from the centre.

18. We choose an ξ by normalising around N^*. In retrospect this is not a good choice since the number can be more easily interpreted when it is defined with respect to N^{**}, the uncontrolled town size. Having chosen N^*, however, we estimated values of ξ, on the basis of the proportion of resources required to clear the pollution in am uncontrolled town and then worked back, via the production function to values of ξ with respect to N^*.

19. See pages 13 - 15 for details.

20. Walters (1974) page 152. The source of this estimate is not given in the monograph but for our purposes we are only interested in orders of magnitude.

21. This arises because of the choice of normalisation for ξ .

22. Ridker and Henning (1967) obtained some estimates of air pollution and its effects on residential house values. In a development and refinement of this study Anderson and Cocker (1969) estimate that at mean levels of air pollution, the marginal capitalised loss is about \$300 - \$700. "Marginal" here refers to an additional $10^{mg}/m^3$/ day of suspended particles plus an additonal 0.1 mg $So_3/100Cm^2$/ day of sulphation. Taking the capitalised loss at \$500 the total depreciation with taking the upper bound of air pollution (80 mg/m^3/ day and corresponding sulphation levels) as zero depreciation is \$4000. For a house valued a \$25,000 and an income/house price ratio of 3 this implies an expenditure of 4% of annual income to buy complete freedom from air pollution (we use here an annuitisation rate of 8%.) The house price of \$25,000 is used as relating to average U.S. income groups by Walters in compiling some illustrative figures for household expenditure on noise evasion. The income - house price ratio of 3 seems typical for U.S. (See appendix to chapter 3 for details). Clearly this figure of 4% must be regarded as an upper bound since most households buy less then total freedom from air pollution.

23. Anthrop (1973) page 60.

CHAPTER 3

The Location of Noisy Centres - The Measurement of Noise Costs to Households

Introduction

In the practical evaluation of the noise costs of establishing a noisy centre, the most significant component often turns out to be the costs to households.[1] In this chapter we are concerned with the measurement of such costs. The economic issues that are involved relate to the concept of a commodity called quiet and how an individuals demand for it can be measured from his behaviour. Section one considers a classically economic treatment of such a commodity. Given that quiet is a smooth, continuously adjustable, freely variable commodity, the demand for it can be identified and from this the costs of the imposition of a certain level of noise can be assessed. Two issues arise, however, that could lead to a modification of this treatment for quiet. The first is that quiet is often consumed as a joint good along with a number of other goods, notably residential ones, and is therefore not freely variable. The second is that the supply constraints on quiet are not only the overall constraint on the total amount of 'quiet' available, but also on the number of transactions of a given level of quiet that are possible. This leads to certain restrictions on the distributions of tastes and income under which a single equilibrium price for quiet will emerge.

Section two outlines the Roskill model of noise measurement. This contrasts with the more classical model in that the demand for quiet is treated as a discrete function and the concentration is on capturing the short run adjustment costs of the imposition of noise. A number of the underlying issues and assumptions of the model are investigated further. The first relates to the use of a noise annoyance scale to measure the noise annoyance disbenefits of various noise levels to individuals of differing perturbability. Some of the basic underlying difficulties in the use of such a scale are discussed. The second question that is considered is the treatment of the noise costs of inmovers in the Roskill Model. It is felt that this can be improved. The reasons for thinking so are given and a more consistent method of costing is suggested. The third factor to be considered concerns the welfare implications of the noise costs that are obtained by using such a model. It has been argued that a particular welfare implication can be made from these costs, and, depending on one's distributional judgements, these may or may not be the costs required in a cost-benefit analysis. Section three analyses the relationship between the noise costs obtained by a model such as the Roskill model and other desirable measures of the costs of noise, and obtains a specific relationship between them using Cobb-Douglas and Stone Geary utility functions.

The fourth assumption of the model that is investigated further is the relationship between the noise costs and other adjustment costs postulated in the model. In this sub-section, the so-called 'median' assumption is shown to be valid under very restrictive conditions and an alternative more general relationship is suggested. Some numerical examples of the application of the relevant algorithm are given.

The fifth assumption that requires further investigation is the one of discrete adjustment to noise. This requires that individuals who adjust to quiet do so by moving to completely quiet areas. A method of testing this assumption is suggested in section five. Finally, we consider some of the issues raised in the Roskill model regarding the treatment of costs over time and of uncertainty.

Section three concludes the chapter by discussing the relative merits of the improved Roskill model and the classical model of measuring noise costs and how both these models can be utilised in a framework that incorporates fully both the short run and long run costs of the imposition of noise. Apart from the occasional reference, when necessary to support a theoretical point, the empirical work in this field is not discussed or reported here. Such work which is related to the theoretical issues raised here is reported in chapter 5.

1. A classical model of the evaluation of noise costs

1.1. The four consumer surpluses and their use in the case of externalities

We begin this section by outlining the four interpretations that Hicks (1956) offered in his treatment of consumer surplus, and interpreting the various measures in the context of the imposition of an externality. Let there be a commodity q that is of interest to us and let p be its price initially. We consider an individual with income M who chooses q_0 units of commodity q and spends the remainder of his income Y_0 on other goods[2]. His position is represented by the point a. on diagram I. Now the imposition of an unfavourable externality would reduce the supply of q and consequently raise the price to p_1. If the externality were imposed the individual would move to b and we are interested in a money measure of the ensuing utility loss $U_a - U_b$. Hicks proposed the following four measures:

(a) If the individual could bribe the agency that was going to impose the externality and cause it to desist from so doing the maximum he would be willing to pay the authority would be M_0M_1. Hicks referred to this as the equivalent variation.

(b) If after the agency had imposed the externality, it was required to compensate the individual so that he could attain the utility level U_a that he enjoyed before, the minimum such compensation would be M_0M_2. This is known as the compensating variation.

(c) If the individual were to bribe the agency and if he knew that for some reason he could not change his consumption of q from q_0, then the maximum he would be willing to pay the agency would be $Y_0 - \hat{Y}_0$. This is known as the equivalent surplus

(d) Finally, if the agency had to compensate the individual after the externality had been imposed and he had moved to q_1, but could not now move from q_1, then the compensation required is $\hat{Y}_1 - Y_1$. This is referred to as the compensating surplus.

These concepts relate to the areas under the demand curves as follows: Diagram 2 represents the 'Marshallian' demand curves corresponding to diagram 1. D_a is the demand curve giving the relationship between price and quantity demanded when utility is held constant at level U_a. Similarly, D_b is the demand curve giving the same relationship when utility is held constant at level U_b. These are referred to as compensated demand curves. Finally, D_0 is the demand curve relating price to quantity demanded when money income is held constant at level M_0. The relative positions of D_a and D_b reflect a positive income elasticity for q.

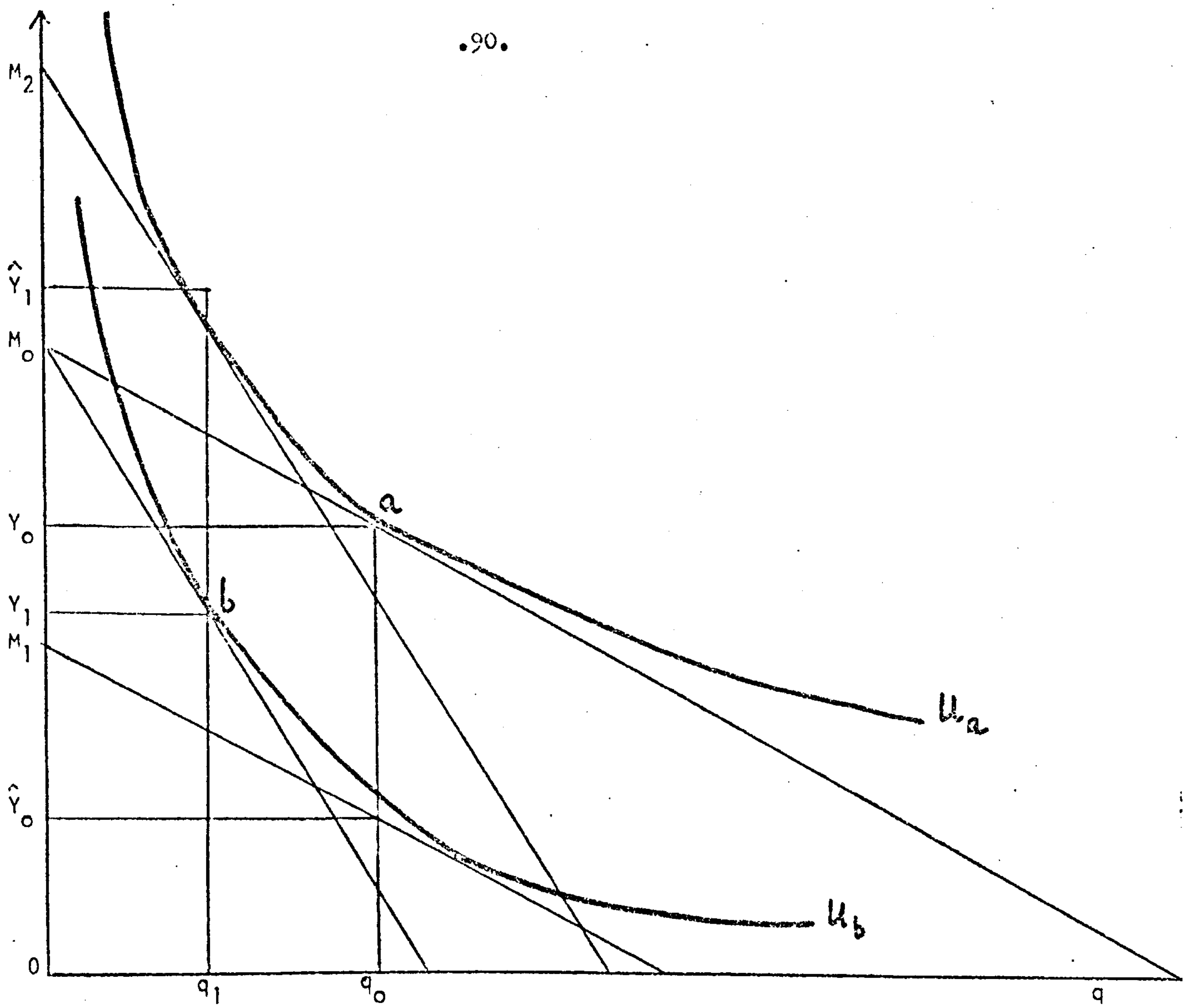


DIAGRAM I

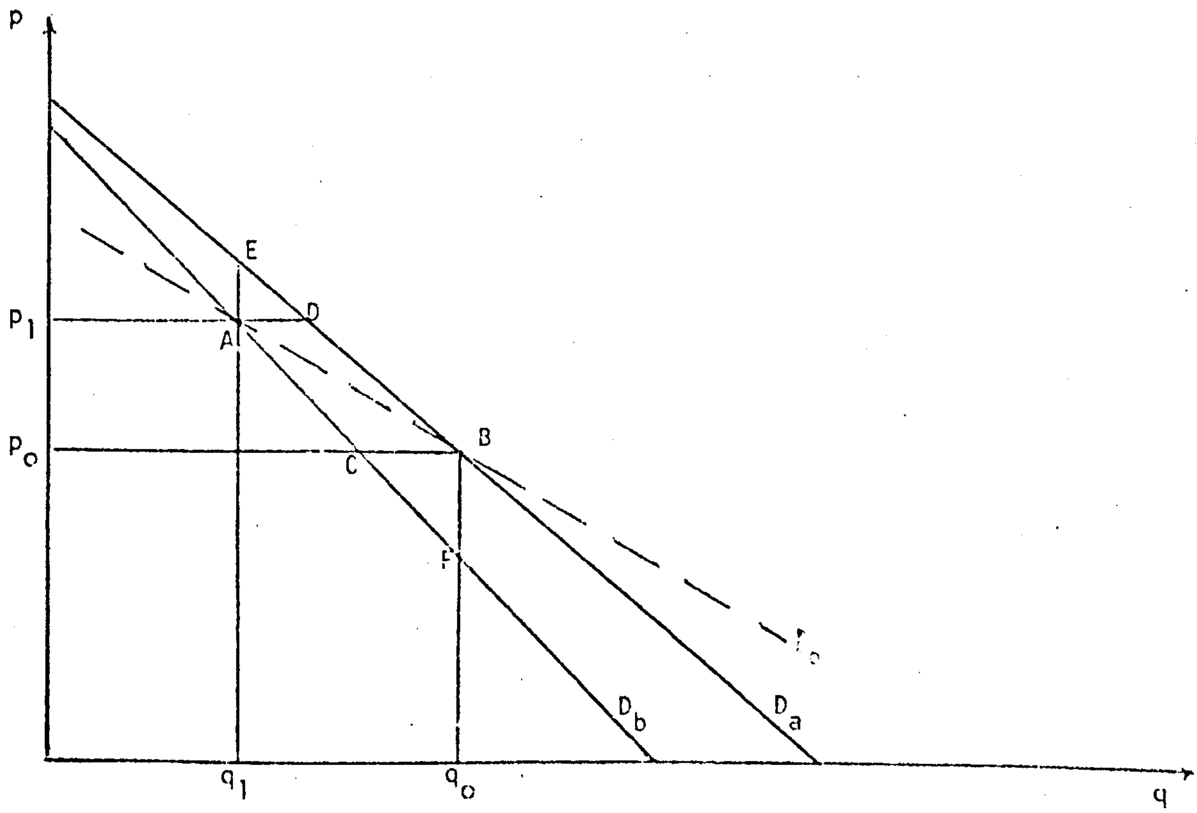


DIAGRAM 2

The compensating variation is given by the area P_0P_1DB , which is the "area under" the compensated demand curve D_a. The equivalent variation is given by the area P_0P_1AC , which is the area under the compensated demand curve D_b. Finally the compensating and equivalent surpluses are given by areas P_0P_1DB + AED and P_0P_1AC - CBF, respectively. It should be noted that normally the area under the uncompensated demand curve will not correspond to any of these concepts. It will correspond to all of them, however, if the income elasticity of demand for q is zero. In that case demand curves D_a and D_b coincide.

In the case of an externality the variational concepts of costs are more suitable when the individual can freely adjust the quantity of commodity q, i.e. when there are no significant adjustment costs and q is a continuous commodity. Whether we choose to use the equivalent variation or the compensating variation depends on our distributional value judgements, and to whom we accord the 'right' to impose the externality. If it is to the agency then the 'bribe' or equivalent variation seems appropriate. If it is to the individual then the compensating variation is more appropriate.

The concepts of compensating and equivalent surplus turn out, with some modifications, to be useful in measuring the costs of an externality when adjustment to the externality is lumpy and costly. Their use in the evaluation of noise costs is discussed further in section I.IV.

1.II The treatment of quiet

The above variational treatment of costs may be applied to the evaluation of noise costs through the demand for quiet. One important point needs to be made, however, regarding the commodity quiet. This is that the level of quiet can really only be interpreted as the lack of noise, and since there is a limit to the level of feasible noise, there must be a maximum level of quiet. Such a commodity can be treated in the standard economic framework by requiring that the marginal utility of further units of 'quiet', beyond the maximum always have a zero marginal utility. In that case we need never be concerned with such units.

In diagram 3 we consider an individual living in a previously quiet area with income M. As a result of the new noise centre somewhere in his vicinity he finds himself transferrred from a^* to b, but he also finds that a new price for quiet, p* , has emerged. The choice line now available to him is AF.[3]. There are two effects to separate out here. First there is the relative price effect, indicating that quiet is now relatively more expensive than other goods, and secondly there is an income effect, indicating that the individual has acquired an exchangeable asset - viz the number of units of quiet that he now lives under. To obtain the variational costs from the area under the compensated demand curve, given the equilibrium price p*, we proceed as follows: Construct a price line MD, parallel to AF. This gives the relative price effect when there is no income effect, and, under these circumstances the individual chooses the point c. The compensating variation now is MB. We now define the compensation cost as the minimum cost of restoring the individual to utility level U_a, given the new price for quiet, and we define the willingness cost as the maximum amount that the individual would be willing to pay to stay at a* and not have the price of quiet change from zero to p*.

To obtain the compensation cost we subtract from the compensating variation the income effect of the price change. The latter is given in diagram 3 as MA. Thus we have,

$$\text{Compensation cost} = MB - MA = AB$$

In terms of diagram 4, MB is given by

$$MB = p^*q_a + \text{area } Rq_aq_o$$

$$\text{and} \quad MA = p^*q_b$$

$$\therefore \quad AB = p^*(q_a - q_b) + \text{area } R\cdot q_a\cdot q_o$$

Similarly the willingness cost is the equivalent variation less the income effect of the price change. The equivalent variation is given by c*a* in diagram 3, and the income effect is again MA. In diagram 4 we have

$$c^*a^* = p^*q_c + \text{area } Q\cdot q_c\cdot q_o$$

$$\text{and} \quad MA = p^*q_b$$

$$\therefore \text{ willingness cost} = p^*(q_c - q_b) + \text{area } Q\cdot q_c\cdot q_o$$

The two demand curves in diagram 4 are the compensated demand curves, holding utility or 'real income' constant at U_a and U_c. In principle it is possible to obtain approximations for compensated demand functions, given the demand function for the product. For a discussion of such an approximation, see Hicks (1956).

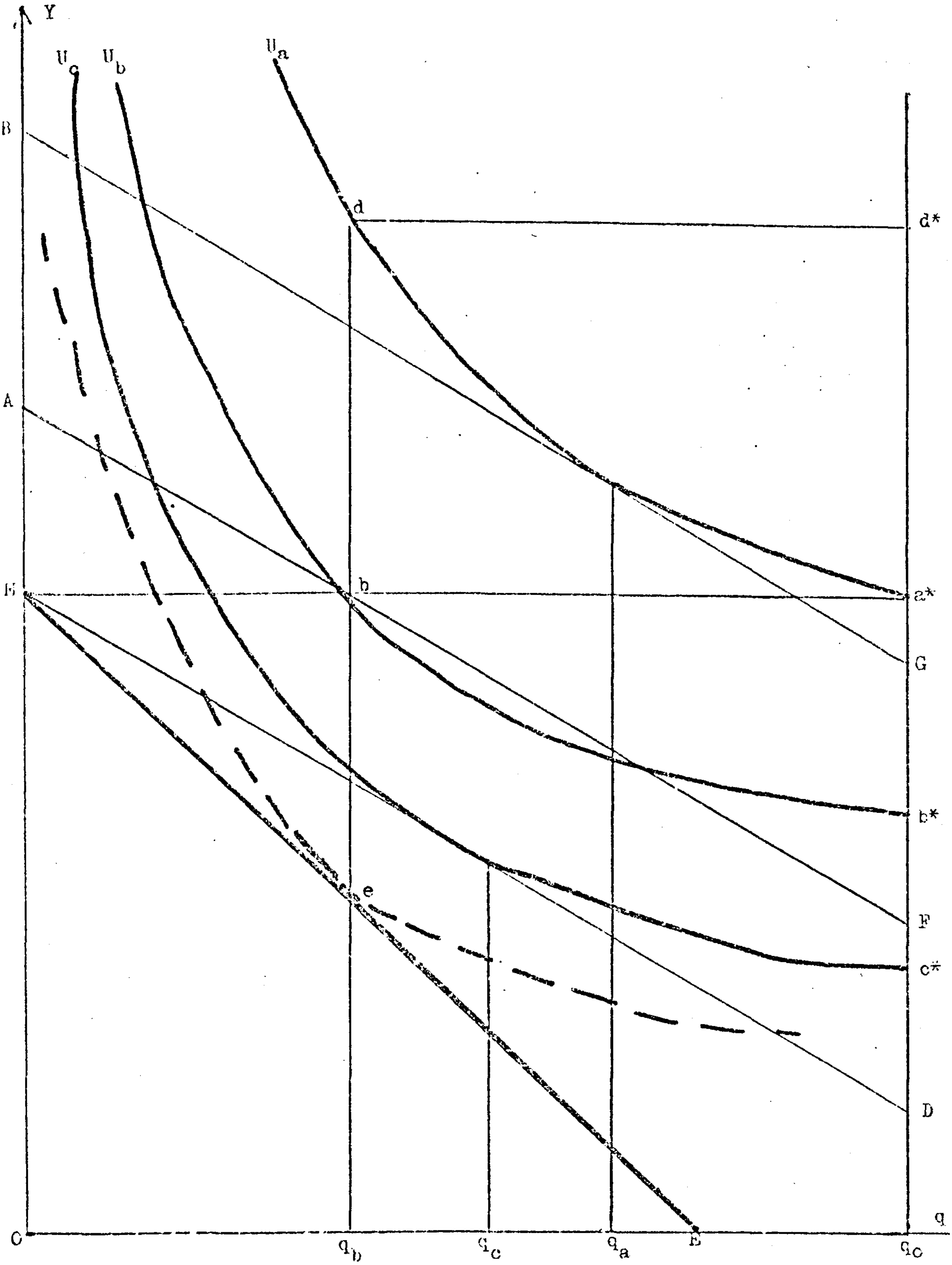


DIAGRAM 3

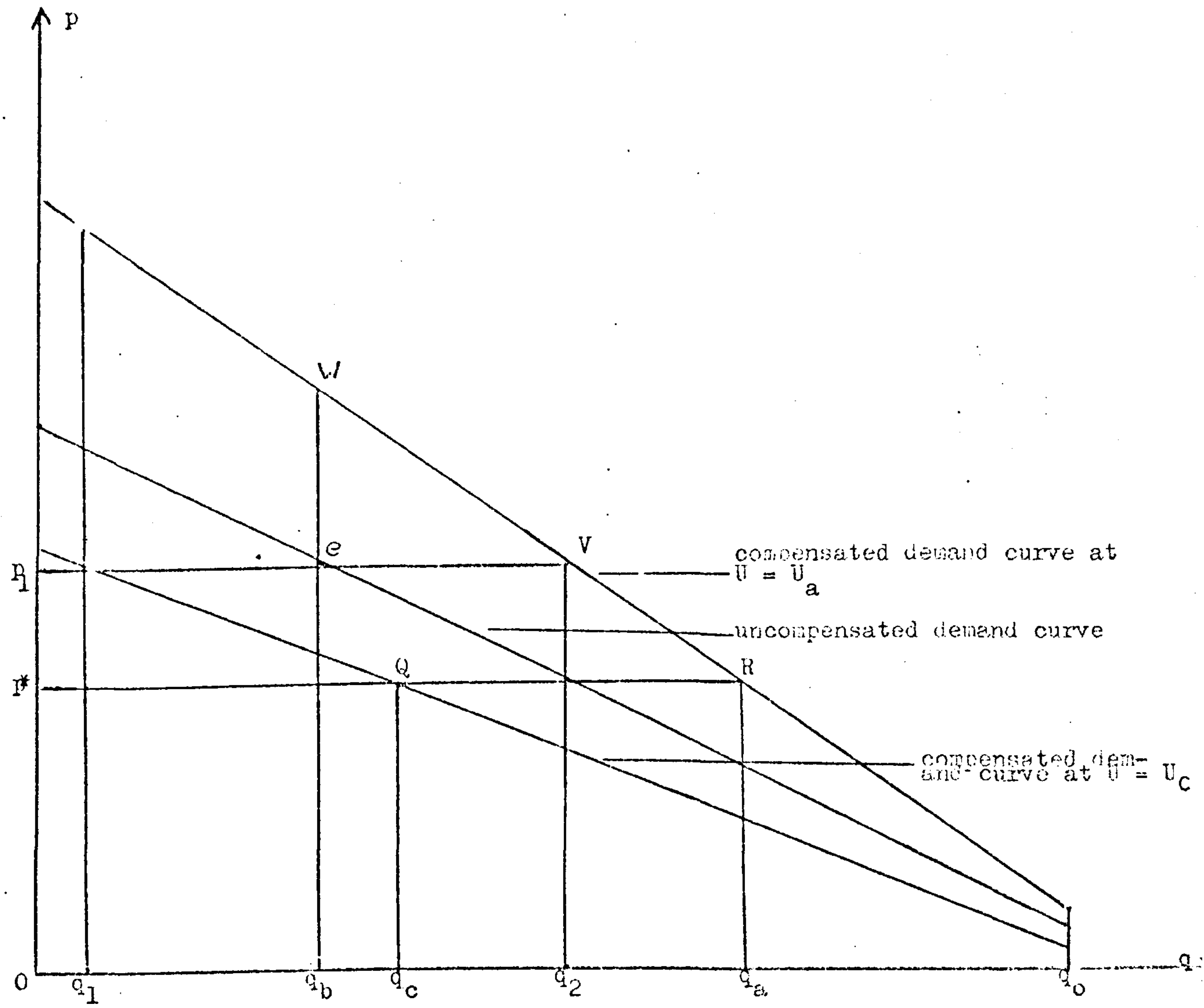

p
W
e
V
Q
R
p_1
p^*
0
q_1
q_b
q_c
q_2
q_a
q_o
q
compensated demand curve at $U = U_a$
uncompensated demand curve
compensated demand curve at $U = U_c$

DIAGRAM 4

To obtain an exact representation of compensated demand function, however, it is necessary to know the underlying utility function.

I.III The compensated demand functions in the case of a modified Cobb-Douglas utility function

The use of compensated demand functions for the measurement of noise costs has been stressed on account of the fact that the income elasticity for quiet is regarded to be substantially greater than zero and the size of 'income effect', consequent upon the price change is not negligible.[4] In this sub-section we are concerned with obtaining specific compensated demand functions for an individual given a specific utility function. The utility function chosen is the Cobb-Douglas or log linear utility function, modified to take account of the special nature of the commodity of quiet. Such a utility function implies unit elasticities of price and income for quiet in the relevant range of variations of the commodity and may be empirically unsatisfactory. In fact it is of interest for two reasons. First, in using these measures of consumer surplus we are assuming that cross price elasticities are zero or at least of negligible magnitude.[5] One utility function which generates demand functions with qualitatively plausible properties and has zero cross price elasticities is the Cobb-Douglas utility function. Second, it provides us with an exact representation of the compensated demand functions and an indication of the magnitude of error involved in the use of uncompensated demand functions. In section II we consider the implications of a Stone-Geary type utility function on the relation between the surpluses.

Let the tastes be represented by:

$$U = q^{\alpha} \cdot Y^{I-\alpha} \qquad Y \geq 0,\ 0 \leq q \leq q_0 \qquad (I)$$

$$U = q_0^{\alpha} \cdot Y^{I-\alpha} \qquad q \geq q_0 \quad Y \geq 0 \qquad (2)$$

(2) indicates that q_0 is the maximum level of quiet that is of interest. The uncompensated demand functions are given by:

$$q = \frac{\alpha \cdot M}{p} \qquad \text{for} \qquad p \geq \frac{\alpha M}{q_0} \qquad (3)$$

$$q = q_0 \qquad \text{for} \qquad p \leq \frac{\alpha M}{q_0} \qquad (4)$$

After the introduction of the noise the equilibrium price is p^* .
If $p^* \leq \frac{\alpha M}{q_o}$ then the cost associated to the individual, with the advent of noise is given by the distance a*F in diagram 3:

$$a^*F = (q_o - q_1)\, p^* \qquad (5)$$

which is the market valuation of the noise imposed. This cost will measure both the compensation and willingness costs, as given in the previous section[6].

If $p^* > \frac{\alpha \cdot M}{q_o}$ then we may use the expression obtained in the previous section to derive the compensation and willingness costs. First it is necessary to define the compensated demand curve corresponding to utility function (1). To obtain such a demand curve we consider the expenditure function corresponding to (1). This expenditure function, which gives the minimum expenditure required to attain a given level of utility, at given prices, is expressed as:

$$M = \alpha^{-\alpha}\,(1-\alpha)^{\alpha-1}\;p^{\alpha}\cdot U \qquad (6)$$

Differentiating this with respect to p , we obtain the compensated demand function, with q as a function of p, and U:

$$\frac{\partial M}{\partial P} = q = \left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)} \cdot p^{\alpha-1} \cdot U. \qquad (7)$$

Inverting this function we obtain:

$$p = \frac{\alpha}{1-\alpha} \cdot \left(\frac{U}{q}\right)^{\frac{1}{1-\alpha}} \qquad (8)$$

For the compensating variation we wish to measure the area given in diagram 4 as $R \cdot q_a \cdot q_o$. This may now be expressed as:

$$Rq_a q_o = \int_{q_a}^{q_o} \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{U_a}{q}\right)^{\frac{1}{1-\alpha}} dq \qquad (9)$$

Where $U_a = q_o^{\alpha} \cdot M^{1-\alpha}$ Substituting this in (9) and rearranging gives:

$$Rq_a q_o = \left(\left(\frac{\alpha}{1-\alpha}\right) \cdot M \cdot q_o^{\frac{\alpha}{1-\alpha}} \right) \int_{q_a}^{q_o} q^{-\frac{1}{1-\alpha}} \, dg \qquad (10)$$

which, integrated, yields:

$$Rq_a q_o = M \cdot q_o^{\frac{\alpha}{1-\alpha}} \left(q_a^{-\frac{\alpha}{1-\alpha}} - q_o^{-\frac{\alpha}{1-\alpha}} \right) \qquad (11)$$

q_a may be obtained directly, from the compensated demand function (7). When we take into account that it may not exceed q_o, it may be written as,

$$q_a = \min \left[q_o ; \left(\left(\frac{\alpha}{1-\alpha}\right)^{\cdot(1-\alpha)} (p^*)^{(\alpha-1)} \cdot q_o^{\alpha} \cdot M^{1-\alpha} \right) \right] \qquad (12)$$

Similarly one may obtain the equivalent variation, which holds utility constant at U_c. This is given by the area $Q.q_c.q_o$ in diagram 4. If we recall that $q_c = \min \left(\frac{\alpha M}{p^*}, q_o\right)$, we may proceed as above, to obtain the expression:

$$Q \cdot q_c \cdot q_o = M \cdot (1-\alpha)^{\left(\frac{1}{1-\alpha}\right)} \cdot \left(q_c^{-\frac{\alpha}{1-\alpha}} - q_o^{-\frac{\alpha}{1-\alpha}} \right) \cdot q_c^{-\frac{\alpha}{1-\alpha}} \qquad (13)$$

Given the areas $Rq_a q_o$, and $Q \cdot q_c \cdot q_o$ the compensation and willingness costs can be derived straightforwardly, as indicated in the previous section.

It is of interest to compare the compensation and willingness costs in the case of a typical kind of figure that is obtained in empirical work for p^*, along with values of α that would appear to be plausible, given households calculated willingess to pay for quiet. The empirical data is discussed in greater detail in chapter 5, and the representative values used in the calculations that follow, are given in an appendix to this chapter. In table 1 below, we list the compensation and willingness costs for a household with an annual income of £2,000, with a given preference for quiet indicated by the value of α. The level of quiet is measured in units for zero to 25, where 25 indicates the maximum quiet (no noise) and zero indicates the maximum level of noise. (For details of the scale see the appendix). The different values of q_b that are considered for each α , indicate the different levels of noise imposed. Thus a value of q_b of 8 indicates that when the noise is introduced the household's level of quiet is reduced from 25 to 8.

We note that the divergence between the compensation costs and willingness costs is greater when α is below the level at which the individual continues to consume q_0 units of quiet. Once $\alpha \geqslant \frac{p^* q_0}{M}$; then, of course the compensation and willingness costs are the same, and are given by equation (5). The percentage divergence is also greater between the two costs, when the level of noise imposed is small (ie q_b is large). Thus it would appear that differences between the two kinds of costs could be quite large, when the values of α are relatively <u>low</u> - ie when we are concerned with individuals who do not express a relatively strong distaste for noise.

TABLE I

	q_b	5	10	15	20
α = 0.08	Compensation Cost	175	130	85	40
	Willingness Cost	153	108	63	19
	(Compensation-Willingness) / Willingness X 100	14.4	20.4	34.9	110.5
α = 0.11	Compensation Cost	180	135	91	46
	Willingness Cost	179	134	90	45
	(Compensation-Willingness) / Willingness X 100	0.6	0.7	1.1	2.2

Compensation and Willingness costs in £s per annum for a household an annual income of £2,000. For value of α in excess of 0.1120 both costs are equal

1 . IV Some extensions to the application of the classical model.

Adjustment costs

In the above analysis it has been assumed that there are no adjustment costs, there is perfect information, and the purchase of quiet is independent of other goods. Adjustment costs arise when the consumption of different levels of noise require some lump sum expenditures such as moving locations. In this case some individuals who would have adjusted their consumption level will be deterred from so doing, while others, taking account of the adjustment costs, will change their level of q. The costs of those who do adjust their consumption level are given by the previous section, with the additional requirement that the adjustment costs be added to the costs already calculated. For those who do not adjust their consumption level, we need to invoke the Hicksian surplus concepts outlined in section 1.I. Consider diagram 3. As a result of the noise the individual is placed at b. If he is not going to move as a result of the noise then the maximum bribe he would pay to the agency imposing the noise is a*b*, while the minimum compensation required to restore him to his original utility level is a*d*. These measures can be related to Hicks' compensating and equivalent surpluses as follows:

In diagram 4 the middle demand curve is the uncompensated demand curve, and the point p_1q_b on it represents the point e in diagram 3, lying on budget line ME. Now the compensating surplus, which is the distance de in diagram 3, is represented in diagram 4 by the expressions

$$p_1 q_b + \text{area } e\, v\, q_o q_b + \text{area } wev$$

From this must be subtracted the income effect corresponding to b.e, which is given in money terms by $p_1 \cdot q_1$

Hence the compensation cost for the non mover is given by

$$\text{Compensation Cost} = \text{area } (Veq_o q_b) + \text{area } (wev) + p_1(q_b - q_1)$$

The points q_b and q_1 are of course the points of tangency of a budget line with slope $-p_1$ with the indifference curves representing utility levels U_a and U_c respectively. (These lines are not drawn into diagram 3, so as to keep the diagram simple). Given the compensated and uncompensated demand curves, all these areas may be calculated straightforwardly.

The willingness cost for the non-mover, is given by c*a* – c*b*. C* a* is of course the equivalent surplus and the equivalent variation, and can be measured in terms of the demand curves by the formula given in section 1.II. c*b*, however, cannot be represented in terms of a straightforward income effect. If the utility function underlying the demand function is known then an exact value for it can be obtained. The distance DF used in obtaining the willingness cost for movers will in general not equal b^*c^*.[7]. It may be possible to obtain an estimate of this measure, in terms of measureable variables, but we have not succeeded in doing so.

Joint Goods

Apart from the adjustment costs discussed above, a further related problem arises in this treatment of noise costs. It has been assumed so far that the purchase of quiet is independent of all other goods, and can be combined with them in any desired mixture. However, in many cases the purchases of quiet will necessitate the physical movement from one location to another and a change of residence.[8] A residential location is a collection of goods, the individual components of which may not be free to vary independently of each other. Thus ideally one should consider the joint demand for all these commodities. This is, however, impossible in practice. One has therefore to work on the assumption that in considering what level of quiet to enjoy, the individual will take account of the difference in value to him between his present location and the 'best' alternative location at each level of quiet. For each level of quiet this figure may be referred to as the householder's surplus. In our measurement of noise costs we may treat such a householder's surplus as an additional adjustment cost and again divide consumers into those who would move and those who would not. Those who move would now be accorded the compensation costs plus the adjustment costs plus the householder surplus. Those who did not move would have their compensation costs assessed exactly as before.

I.V Dynamic Considerations

There is perhaps a distinction to be drawn here between those who adjust to an increase in the noise level in the short run and those who do so in the long run; or more generally it is important to consider the time period of adjustment to the imposition of the noise. Those households that adjust in the short run immediately after the noise arrives are very likely to incur some householder surplus loss and may be regarded as having incurred the movement costs specifically with respect to the noise externality.

Hence the treatment of their costs outlined above seems appropriate. However, in the long run it may be that more combinations of goods that individuals wish to consume will make themselves available through the residential and commercial markets and through the provision of the relevant public services. In that case, the long run mover may be regarded as losing less householders surplus. Furthermore, it is statistically observed that most of the population move locations in the long run anyway, and so the movement costs incurred by them do not apply to the noise externality[9]. Thus the long run treatment of noise costs approaches more closely to the classical model outlined in Section I.II.

This distinction indicates that in a simple model the overall noise costs, N, would be assessed by taking a discounted sum of the annual costs as follows:

For short run movers

$$N = \sum_{i=1}^{T_s} N_{vi} \left(\frac{1}{(1+r)}\right)^i + (M + S) \left(\frac{1}{(1+r)}\right)^{T_s} \tag{14}$$

where $T_s \leqslant \bar{T}$, and for long run moves,

$$N = \sum_{i=1}^{\bar{T}} N_{si} \left(\frac{1}{(1+r)}\right)^i + \sum_{i=\bar{T}+1}^{T_L} N_{vi} \left(\frac{1}{(1+r)}\right)^i \tag{15}$$

Where T_s is the number of years after which the short run mover moves out, N_{vi} is the annual variational cost (compensation or willingness) and N_{si} is the annual surplus cost (compensation or willingness)[10]; M is the movement cost, S the surplus, $\bar{T}$ the maximum length of time within which the short run mover moves out of the noisy location, and T_L the length of time after which a long run mover leaves the location.

This issue is important only if $\bar{T}$ is small relative to T_L. If T is a long period then the distinction between the short and long run movers is blurred.

I.VI Supply Constraints

The final problem discussed regarding this treatment of quiet is the one relating to supply constraints. With a normal good, there are no constraints regarding the number of transactions of any given size made and the equilibrium price is determined by equating agregate supply and

and aggregate demand for the commodity. If, however, the purchase of a given amount of quiet is related to the use of a specific location and there are a fixed number of such locations then the number of household transactions of a given level of quiet are also restricted.

In general this implies that there will not be a unique equilibrium price per unit of quiet, as there are as many constraints to be satisfied as there are levels of quiet. Under some restrictive conditions on the distributions of tastes, incomes and surpluses, there will be a unique price for quiet. What these conditions amount to, is requiring that in addition to the aggregate demand and supply being equal at a given price p, it should 'work out' that the number of transactions demanded for any given level of quiet should not exceed the number of transactions possible at that level of quiet. Whether or not this is the case is ultimately an empirical question. However, it should be noted that this problem is not one of the choice of units of measurement of quiet. Indeed, in the absence of any restriction on transactions there will be <u>a</u> measure of quiet, such that, given the definition of an origin, only measures that are proportional to that will yield an equilibrium constant price per unit of quiet. If such a measure is then transformed so as to yield a constant price per unit of quiet in the presence of restriction on transactions, then the transformed units of quiet will not normally be admissible in the calculation of individual welfare costs.

I.VII The Aggregation of Individual Noise Costs

So far in this section we have considered how one might calculate the noise costs of an individual, given some information regarding his tastes for quiet through a utility function in quiet and an aggregate of other goods or directly through a demand function for quiet. However, in society there is a variation of incomes, tastes for quiet and householder surpluses, and it remains to consider the issues raised in the aggregation of their noise costs. Once the joint distribution of incomes, householder surplus, and a taste parameter is available, there is no real difficulty. We calculate the noise cost for a mover and for a stayer, given specific values of these variables and given a certain level of imposed noise. We then take the <u>lower</u> of the two costs and weight it by the proportion of the population in that noise zone that has these values of the variables.

The weighted sum of the noise costs of all noise zones is the total noise cost.

To obtain such a distribution, we may proceed by survey methods to collect information regarding income and surplus (although there are some difficulties regarding the latter that are discussed in section II.III), but we cannot obtain information regarding tastes in this manner. The parametric representation of tastes could be with respect to the price and income elasticities of the demand function or it could be with respect to a parameter of the utility function (such as in the case of the Cobb-Douglas utility function used in section I.III). In either case the estimation of such a distribution could proceed in one of two ways. First it could be done as an integral part of a study of the implicit market for quiet, which would have to be carried out to obtain an estimate of the demand function for quiet, and second it could be done by constructing an ordinal index of the strength of preferences regarding noise from a questionnaire, and using this in conjunction with the information on the price of quiet, and the other costs of adjustment, to infer a distribution of tastes for quiet. Neither approach has been fully attempted so far, although the second approach is very much in the spirit of the work of the Roskill Commission that is discussed in the next section. It seems appropriate, therefore, to consider that work, and to discuss the question of the appropriate method of measurement of the distribution of tastes when evaluating the relative merits of the two approaches. It is to the Roskill approach to the measurement of noise costs that we now turn.

I.VIII The Welfare measurement of costs, when there is a change in the price of more than one related good.

In the preceding analysis we have been concerned with a change in the price of quiet, as a result of some government action, and we have analysed the consequences of such a change, independently of movements in any of the other prices. If, however, the same government action significantly effects the prices of any of the other commodities in that area, then the costs or benefits of such effects have to be considered in establishing a ranking alternative government actions.

Initially it might seem desirable to take the sum of the relevant variational or surplus costs to obtain the total cost of the project. Such variational or surplus costs are measured as the areas related to the compensated demand curves and indicated on diagram 2. However, when there are changes in relative prices the compensation measures (variational and surplus) may not rank different alternatives according to their relative affects on utility. This point has been made by Foster and Neuberger (1974) and can be demonstrated as shown in diagram 5. Originally the individual is at α. In the case of project I, relative prices are so changed that his budget line moves from AB to CD, and in the case of project 2, relative prices are so changed that his budget line moves from AB to EF. The gain in utility measured by a compensating variation CV_1 in case 1 is given by

$$CV_1 = \Delta_1 \; P_{x_2}^{(I)}$$

where $P_{x_2}^{(I)}$ is the price of x_2 in case I. Similarly in case 2

$$CV_2 = \Delta_2 \; P_{x_2}^{(2)}$$

But $$P_{x_2}^{(2)} > P_{x_2}^{(0)} > P_{x_2}^{(I)}$$

and, as drawn, $$\Delta_1 > \Delta_2$$

therefore $CV_1 > CV_2$

However, the respective utility positions obtained indicate that

$$U_2 > U_1$$

If the line EF were tilted towards B we could have the same relative values for the CVs but the $U_2 < U_1$. Thus it follows that the compensating measure need not be monotonically related to changes in utility when there is more than one price change. However, the same is not true of the equivalent measure if the individual is unsatiated. This can be seen simply as follows:

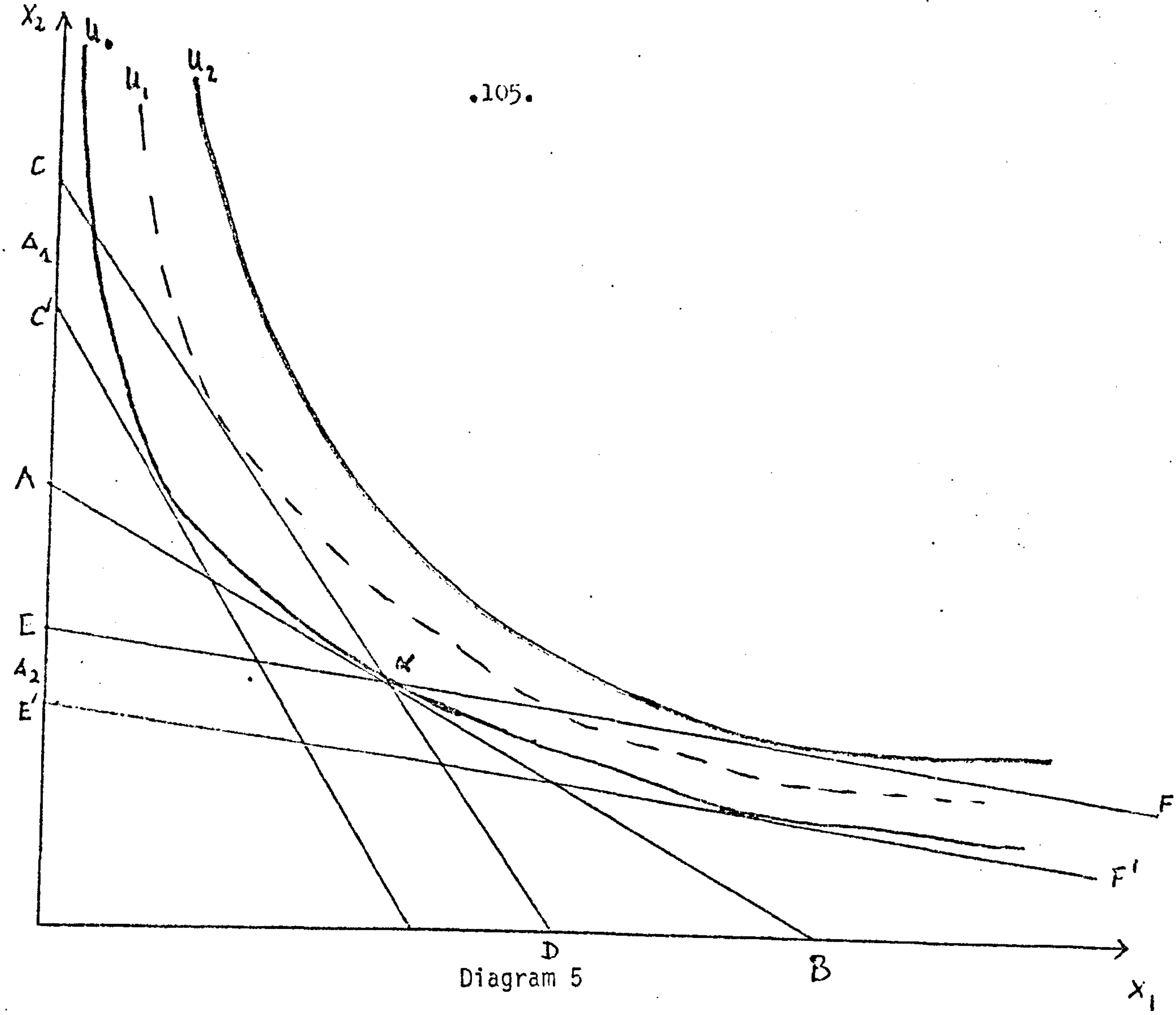


Diagram 5

Originally the price vector for commodities is P and income is M. In case I the price vector is changed to P* and in case 2 the price vector is changed to P**

To find the equivalent variation EV, we wish to find an M* and an M**:

$$U\ (P^*, M) = U\ (P, M^*), \text{ and}$$
$$U\ (P^{**}, M) = U\ (P, M^{**})$$

if $M^{**} - M > M^* - M$, then $EV_1 > EV_2$
and $U\ (P, M^{**}) > U\ (P, M^*)$
therefore $U\ (P^*, M) > U\ (P^{**}, M)$.

Thus it would appear that whereas EV is monotonically related to utility, CV need not be. While this point is important, it is also important to bear in mind that in ranking alternatives, it is not sufficient to rank the utilities of individuals. This is because almost all problems involve the effects on the utilities of several people, as well as some direct production costs. Thus the particular cardinalisation chosen to represent utilities will be of great importance. In section I. I we have given some interpretation to the various measures of a utility change brought about by a disbenefit causing the price of one good to rise. These interpretations relate to the accordance of pollution rights and the distributional judgements inherent therein. If therefore we can obtain the same ranking of alternatives using only the equivalent measures as we can using the combination of compensating and equivalent measures that our distributional judgements would like, then the cardinalisation represented by the latter is surely preferable and these costs should be used. In the probably unlikely event of the equivalent measures producing a different ranking from the desired measures, we are compelled to use the equivalent measures.

II.I The Roskill Approach to the Measurement of Noise Costs

In this section we consider the measurement of noise costs pioneered in the Commission on the Third London Airport (1970), volume 7, parts 1 and 2. The method of measuring noise costs used by the Commission differs from the classical treatment outlined in the previous section because it does not use the concept of the demand for quiet as such. When noise is imposed on individuals they either adjust to it by moving out of the noisy area and into a completely quiet area, or they suffer the consequences of a higher noise level. The action that they choose depends on which is the less costly of the two alternatives. Thus in the individual's decision making, noise is treated as an attribute - either you suffer it or you do not.

The Roskill model started by using a measure of noise called the noise and number index (NNI). When aircraft noise was imposed on a previously quiet area, this lead to a depreciation in the value of household property, that was related to the NNI level imposed in that area. This depreciation occurred as some residents moved out of the area on account of the noise, and others moved in. The noise costs to households were then assigned as follows:

(i) Households that moved out on account of the noise were assigned a cost equal to R + S + D where R was the movement cost, S the householder's surplus and D the depreciation in the property on account of the noise.

(ii) Households tnat stayed in the area were allocated a noise cost N, where N was a measure of the "noise annoyance disbenefit".

(iii) Households that moved into the noisy area were assigned a noise cost of N which was exactly equal to their gain in depreciation D. Thus they were assumed to have no noise cost.

(iv) Households that moved out of the area for reasons other than noise were assigned a cost of D.

The model was applied to noise zones of 35 NNI and above only. The values of R and D were obtained empirically, as was the distribution of S in the population involved. The distribution of N in each noise zone was obtained as follows: households in each noise zone were asked a number of objective and subjective questions regarding various aspects of aircraft noise and according to their responses, they were rated on a five point noise annoyance scale, with a higher rating representing greater annoyance. The median value, on the noise annoyance distribution thus constructed, was then accorded a noise annoyance disbenefit equal to the depreciation in that noise zone. This was done for each noise zone and as the distributions of noise annoyance and the levels of depreciation were different in the different noise zones this gave a number of different money values for the different points on the noise annoyance scale. By interpolating linearly in between points the noise annoyance scale was converted into a money measure and the distribution of N obtained for each noise zone. The justification given for equating the median of the noise annoyance scale with the house price depreciation was that the equilibrium fall in house prices is determined by households whose noise annoyance is greater than the depreciation moving out of the area, and households whose noise annoyance is less than the depreciation moving into the area. At the margin the inmovers gain, D, is equal to his noise annoyance cost, N, while the outmovers loss, also D, is equal to his 'saving' in noise annoyance cost, N. As there are as many outmovers as there are inmovers, it is alleged that this implies that the cost associated with the median noise annoyance is equal to the depreciation. All the costs, S, D,R, and N were calculated on an annual basis and S, D, and R were assumed to grow over time, to reflect an increasing valuation of quiet in the case of D and an increasing valuation of commodities with a high service content in the case of R. Heuristically the logic of the model can best be seen by considering diagram 5 below which represents the density distribution of N in the population. For a given householder surplus $\bar{S}$ [11], the right hand tail will represent the initial outmovers ($N > R + \bar{S} + D$), while the left hand tail will represent the initial inmovers ($N > D-R$)[12]. The central section represents those who stay. Over time the distribution of N will tend to shift to the left as the right hand tail disappears and to the right as the median value of N, which equals D, grows as D grows. Thus a new group of outmovers and inmovers will appear annually, and their costs will be assessed in the same way as the initial years costs were assessed. Those who do not move out or in on account of the noise, suffer a noise annoyance disbenefit N according to their position in the distribution for some period of time and then they are assumed to move out for reasons unconnected with noise, and suffer a loss of depreciation D. Thus we have the basis of dynamic model for costing noise in a situation where adjustment costs are important. The sum of the annual noise costs, appropriately discounted, is equal to the total noise cost of the imposition of noise.

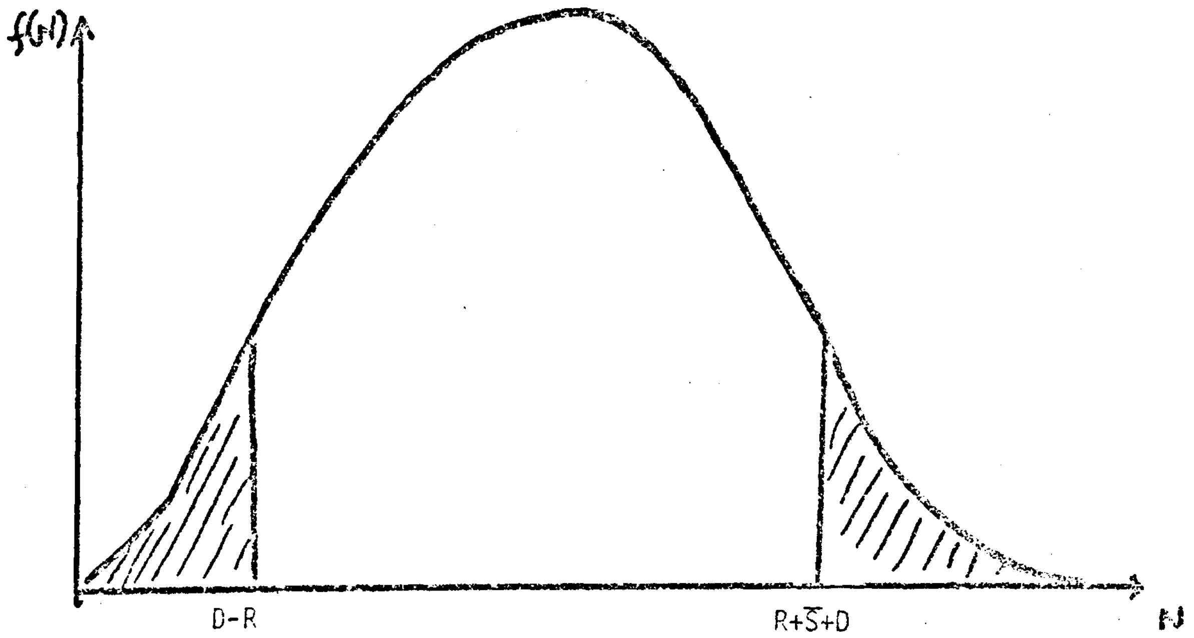


DIAGRAM 6

II.II Criticisms and Comments on the Roskill Model

The method described in the previous section played a central role in the evaluation of noise costs by the Roskill Commission. The work of the Commission in general, and the costing of noise in particular has drawn a range of comments and criticisms. A summary of the various positions taken is given in a paper by John Adams (1971). At the extreme there are the comments by P. Self (1970) in New Society where he describes the work of the Commission as, "nonsense on stilts", "lunatic logic" and "a porridge of bogus accountancy". Mr Adams himself takes a rather similar line. Thus elsewhere he states, "(the measurement of noise costs) is clearly not a question amenable to quantification. (Roskill) created a cost-benefit fantasy world in which things ...... of real importance, such as friends, neighbours and human lives were treated in a derisory manner". Adams (1972). The comments are given, not so that they may be argued here, but because they do indicate that individuals who are concerned about these things do believe that the measurement of noise costs by <u>any</u> means whatsoever, is not a feasible exercise. Unless there are some metaphysical reasons for rejecting measurement however, the rational procedure seems to be to judge the measurement of noise costs on the theoretical and empirical grounds by which they were derived. This indeed is the position adopted by most of the people that have considered the question. After having examined the noise estimates in more detail, suggested improvements where possible, and considered alternative methods of measurement , we may evaluate whether the problem has been satisfactorily tackled. This is what we hope to do at the end of this chapter.

The following is a list of the important aspects of the Roskill noise methodology that have drawn criticism and further comment:

(a) The measurement of noise by the noise and number index.
(b) The data on which the house price depreciation estimates are based.
(c) The use of a noise annoyance scale.
(d) The treatment of inmovers.
(e) The Welfare implications of the noise costs measured by N.
(f) The use of the median assumption in calculating N.
(g) The use of noise as an attribute in the decision-making process.
(h) The treatment of time and uncertainty.

Items (a) and (b) are essentially empirical questions and these are discussed along with the other empirical issues in chapter 5.. In the remainder of section II, we consider in detail items (c)-(h).

II.III <u>The Use of a Noise Annoyance Scale</u>

The noise annoyance scale was used initially in the construction of the NNI (McKennell 1963). Households were asked a number of questions. They were first asked, 'Does the noise of aircraft bother you, very much, moderately, a little, or not at all?' If they were at least a little annoyed they were then asked, 'Does the noise of aircraft, (a) ever wake you up, (b) interfere with listening to TV or radio, (c) make the house vibrate or shake, (d) interfere with conversation (e) interfere with or disturb any other activity, or bother,annoy or disturb you in any other way?' From their responses, households were rated on a five point scale where for example a rating of 2 corresponds approximately to being a little annoyed, a rating of 3 to being moderately annoyed, and so on.

Thus the noise annoyance scale provides an ordinal rating of the strength of feeling about noise. It was first used to construct a noise index. Given the noise index, this tells us the distribution of noise annoyance in a given noise zone. There are three economic points to be made regarding such a scale. First, the population in a given noise zone may already be self selected. If the noise has persisted for some time then the distribution of noise annoyance will be biased in favour of the less annoyed and this will underestimate the noise costs of introducing noise in a new area. McKenell's survey of noise was conducted in 1961, about two years after the introduction of jet noise around Heathrow Airport, London, and so one may assume some adjustment. This was not allowed for, and it is not clear how it can be allowed for. It appears, in general, not possible to obtain satisfactory answers to 'as if' questions in areas where there is no noise, thus excluding the possibility of surveying the distribution of annoyance in an area where there is no noise[13]. Second, there is the well known Public Good problem regarding the revalation of preferences in situations where individuals may believe that they have something to gain or lose by giving a particular answer. It is plausible, for example, that an airport worker will understate his noise annoyance in the belief that if too many people 'complain', the airport may be moved, while someone else who has nothing to lose by the airport being moved may will exaggerate his annoyance (but perhaps not overdo it, lest the questionnaire is thrown out!)[14]. The resultant distribution from the interaction of these forces may be far removed from the object of initial interest in conducting the survey. To be sure, a well designed survey does not make its object transparent, and has questions that check against each other. However, it is difficult to believe that the object does not become clear to the respondent, and that his response is determined by his self interest. Third, it is important to remember that the noise annoyance scale is only an ordinal scale of annoyance. In converting it into a money measure of the noise annoyance disbenefit, the median assumption is used to obtain the money value of levels of annoyance that lie, in general, between the integer ratings 0-4. Thus to obtain the noise annoyance disbenefit of a rating of 1, 2, 3, or 4, it will be necessary to interpolate between the values obtained. In the absence of any other basis for interpolating, this is done linearly. Thus there is a strong presumption towards cardinality in associating money values with points on the noise annoyance index. Given the strongly ordinal assumptions behind its construction this is a matter of some concern.

While these issues have been raised, it remains unfortunately true that they cannot be satisfactorily resolved, and furthermore, the magnitude of the erros caused by the presence of these problems remains undetermined. Overall, the use of a noise annoyance scale to establish the noise annoyance disbenefit remains, in our view the ~~most susceptible~~ part of the Roskill methodology most susceptible to criticism.

II.IV The Treatment of Inmovers

The Roskill model assigns no noise costs to inmovers. Their noise annoyance disbenefit N, is assumed to equal the depreciation in property values that they obtain. This is difficult to justify. If a household moves into a noisy area, on account of the noise it's net gain is D - R - N where R is the annuitised movement cost.[15] These gains can be straightforwardly calculated, given the distribution of N, as shown in diagram 5, by reading off the left hand tail. The reason why the commission's study ignores these gains is because a large number of inmovers are regarded as ignorant of the true noise annoyance, and only discover it after having moved in. The evidence given for this is that recent inmovers in noisy areas often have high noise annoyance ratings. However, if this is the case, then their net costs should be taken into account and there are no reasons for believing that the costs of this group exactly or even approximately outweigh the gains of the informed inmovers. In section II.VIII we consider some subsequent work that has been done to treat the costs of uninformed inmovers systematically. If such a procedure is adopted then the gains of the informed inmovers should be taken into account and this can be done, as indicated, quite straightforwardly.

II. V The Welfare Implications of the Noise Costs Measured by N

In section I of this chapter, we observed that there are four concepts of the costs of externalities: two variational concepts and two surplus concepts. It turned out that for the noise problem the variational concepts were relevant when we are concerned with movers and the surplus concepts are relevant when we are concerned with non-movers. There are two variational and two surplus concepts because in each case we can consider either the maximum amount that the individual would be willing to pay so that he is no worse off than he would be if the noise were imposed, or the minimum compensation that he would require to restore him to his original level of satisfaction once the noise is imposed. Which of these we use depends on our allocation of pollution rights and therefore on distributional judgements. If we grant the pollution rights to the households in the area where the noise is imposed, then the compensating measure is appropriate. If we grant the pollution rights to the government agency that decides on the flying regulations and plans the airports, then the willingness-to-pay measure is more appropriate. Thus it appears that there are two questions to ask regarding the noise costs measured by N. First, does it measure the variational costs for movers and surplus costs for non-movers; and second does it measure the willingness-to-pay or compensation cost in each case? Having considered these questions we then examine whether within the framework of the Roskill model it is possible to derive information on costs other than those obtained, by the use of suitable utility functions.

The distribution of noise costs N is obtained by equating the depreciation in house prices in a given noise zone to the median noise rating in that zone, and by assuming that the movers out of noise zones move into completely quiet ones. The heuristic explanation that was given for the median assumption, was that the equilibrium depreciation was a market phenomenon, in which the marginal buyer and seller have no net gains. We consider the position of a marginal seller in diagram **7**, which considers the same variables as diagram 3. Originally he is at a* enjoying the maximum quiet. When noise is introduced he finds himself at b. Should he wish to move, the choices available to him are given by AF, which does not go through b, because there is a lump sum moving cost. If he is a marginal seller, he will be indifferent between staying at b and moving to F, and this is shown in the diagram. The quantity a*F is equal to D + R. If we equate this to N, for the appropriate noise annoyance rating, then we are measuring the willingness-to-pay cost, which happens to be both the variational and surplus concept, as the individual is moving to complete quiet.[16] Now the Roskill model equates D to the median of the noise annoyance scale, and the most favourable interpretation that we can place on this act is to say that, D + R equalling some point on the noise annoyance scale implies D equalling the median point on that scale. We will return to the median assumption in the next section. At this stage the best that can be done is to suggest that if individuals do adjust completely to noise, then the noise annoyance disbenefit obtained by the Roskill model is the willingness-to-pay cost, which is both the variational and surplus concept. Thus the willingness to pay costs for movers and non-movers are the same. The compensation costs, however, will not be the same. Again if we can assume complete adjustment, then the compensation cost for the mover is equal to R + S + D, where this is the sum of costs incurred in moving to complete quiet. For the non-mover

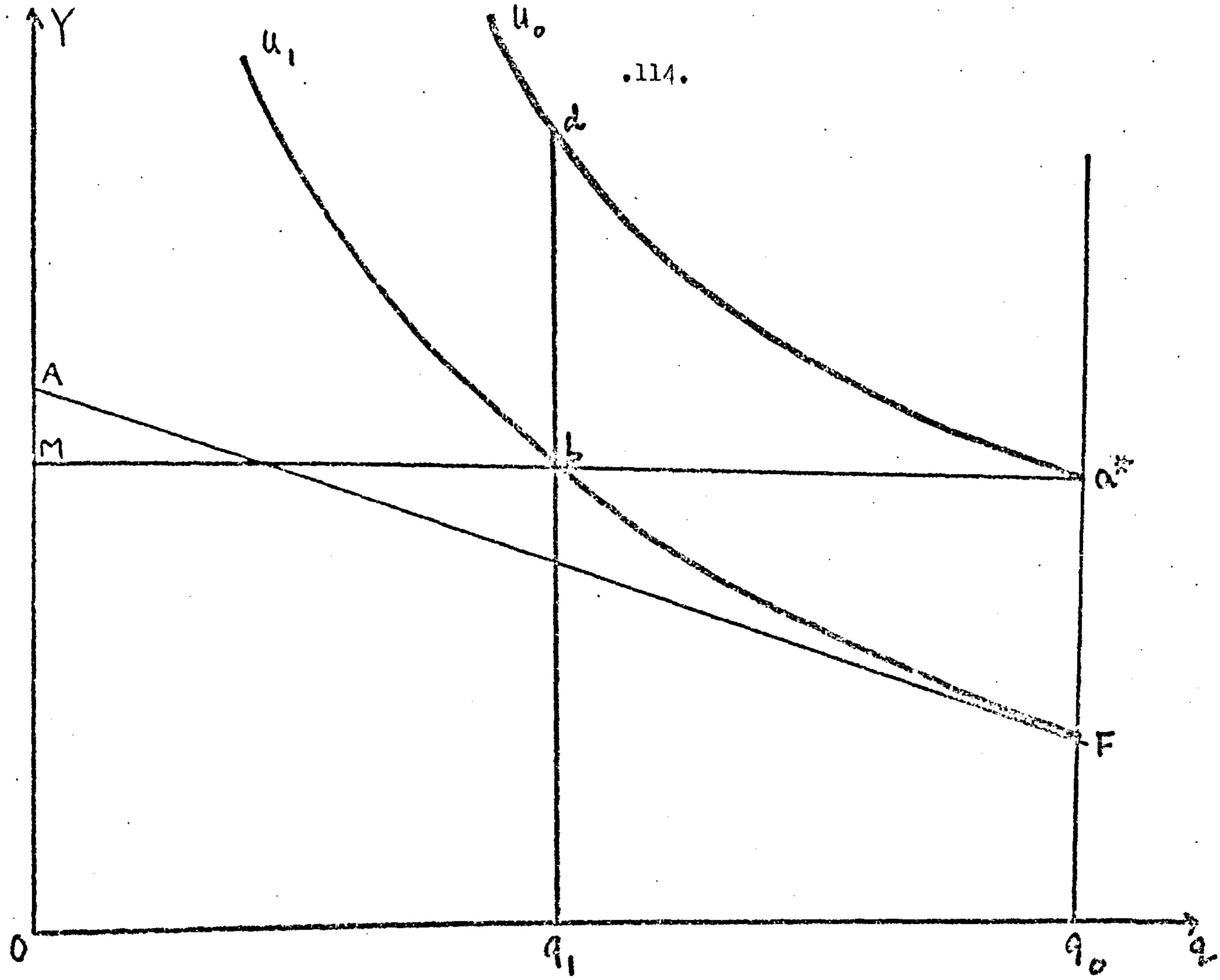


DIAGRAM 7

the compensating surplus, which is the relevant cost is given by bd in the above diagram.

It would appear, therefore, that the Roskill assumption of all households moving out of noisy areas, moving into completely quiet ones does considerably simplify the problem of identifying the noise costs. We will consider some ways of testing this assumption in section II.VI.

There has been some criticism of the Roskill noise costs on the grounds that they claim to measure the compensating costs, whereas in fact they do not. From our analysis this criticism appears to be valid as far as the costs of non-movers are concerned. In this case it seems reasonable to ask whether some information on the compensating surplus for non-movers can be obtained, given the equivalent surplus represented by N. This can be done in specificcases when preferences of the household can be represented by analytical utility functions. For illustrative purposes we consider the Cobb-Douglas utility function that was described fully in section I.III.

$$U = K \cdot q^{\alpha} \cdot Y^{1-\alpha} \tag{16}$$

where

$K > 0 \qquad 0 < \alpha < 1$

q = amount of quiet

Y = expenditure on other goods (annual)

If we consider the original income as M, and define the original level of quiet as q_o, and the new level of noise suffered as q_1, then we are interested in the value of bd, given the value a*F. We observe the following relationship:

$$\frac{bd}{a^*F} = \frac{M}{M - a^*F} \tag{17}$$

This can be seen as follows:

$$bd = \left(\frac{U_o}{Kq_1^{\alpha}}\right)^{\frac{1}{1-\alpha}} - \left(\frac{U_1}{Kq_1^{\alpha}}\right)^{\frac{1}{1-\alpha}} = \frac{U_o^{\frac{1}{1-\alpha}} - U_1^{\frac{1}{1-\alpha}}}{(Kq_1^{\alpha})^{\frac{1}{1-\alpha}}} \tag{18}$$

$$a^*F = \left(\frac{U_o}{Kq_o^{\alpha}}\right)^{\frac{1}{1-\alpha}} - \left(\frac{U_1}{((Kq_o)^{\alpha})}\right)^{\frac{1}{1-\alpha}} = \frac{U_o^{\frac{1}{1-\alpha}} - U_1^{\frac{1}{1-\alpha}}}{(Kq_o^{\alpha})^{\frac{1}{1-\alpha}}} \tag{19}$$

$$\frac{bd}{a^*F} = \left(\frac{q_o}{q_1}\right)^{\frac{\alpha}{1-\alpha}} \tag{20}$$

and since $q_1^{\alpha} M^{1-\alpha} = q_o^{\alpha} (M - a^*F)^{1-\alpha}$ (21)

$$\left(\frac{q_o}{q_1}\right)^{\frac{\alpha}{1-\alpha}} = \frac{M}{M - a^*F} = \frac{bd}{a^*F} \tag{22}$$

The relationship between bd. and a*F would suggest the following things:

(i) The compensation cost is never less than the willingness to pay cost. The two, however, only differ substantially if the willingness to pay cost is a substantial part of household income. A willingness to pay of about 8% of income is about average which would suggest that the compensation cost be about 8.7% greater than the willinness to pay cost.[17]

(ii) In the utility function tastes are represented by the parameter α. For a given value of α, and a given level of quiet q_1, the value of N_w is proportional to income. Thus for given tastes, and a given level of noise imposed, the percentage difference between the compensation and willingness costs is independent of income.

The Cobb-Douglas utility function is used here to illustrate the relationship between the two kinds of costs. It is valid only if the price and income elasticities for quiet are unity. Empirically this is known to be unsatisfactory, especially as regards the income elasticity. A more realistic relationship between the costs can be obtained if we consider that there is a minimum level of expenditure on other commodities than quiet, such that for any household at a level of income equal to or below that, the household would purchase no quiet at all, in the event of an airport being placed near him. This has the advantage of not assuming that the demand for quiet falls only proportionately with income. We may modify the Cobb-Douglas utility function to take account of such a factor as follows:

$$U = Kq^{\alpha} (Y-E)^{1-\alpha} \tag{23}$$

where E is the minimum level of expenditure on other goods.[18] The demand function for quiet implied by this is

$$q = (1-\alpha) \frac{(Y-E)}{p_q} \qquad q<q_0 \qquad p> \alpha\frac{(M-E)}{q_0} \tag{24}$$

where p_q is the price of quiet.

This given an income elasticity for quiet that is always greater than one, and decreasing as income decreases. This implies the following relationship between a*F and bd.

$$\frac{bd}{a^*F} = \frac{M-E}{M-E\cdot a^*F} \tag{25}$$

In this case we note the following:

(i) For a given money value of expressed willingness to pay the compensation required will be higher for a person with low income than if E were set equal to zero.

(ii) For an expressed willingness to pay that is a given proportion of an individual's income the percentage difference between the compensation cost and the willingness to pay cost will now increase as M gets smaller relative to E. As a comparison with the figures obtained in the Cobb-Douglas case we note the following

If N_W = 0.084 then

Compensation cost = 1.087 willingness cost if no account is taken of E

Compensation cost = 1.667 willingness cost if M = 1.25 E

Compensation cost = 1.190 willingness cost if M = 2 E

Compensation cost = 1.119 willingness cost if M = 4 E

(iii) For a given set of preferences for quiet, (i.e. a given α) the percentage divergence between the compensation cost and willingness cost is independent of income for a given level of imposed noise, for all levels of income greater than the minimum level of consumption. This result holds in the case of the straightforward Cobb-Douglas utility function as well. (when E may be regarded as zero).

Overall, these comparisons serve to show that it is possible to obtain information on compensation costs regarding non-movers within the Roskill methodology, if it is possible to represent the tastes by a satisfactory set of preferences. If this cannot be done, but some knowledge is available about the price and income elasticities for quiet, then we may proceed to obtain direct estimates of the compensation costs by using the demands methods given in section I of this chapter.

II.V The Use of the Median Assumption in Calculating N

In section II.I we explained the rationale given for equating the house price depreciation in a certain noise zone with the median noise annoyance level in the noise annoyance distribution for that zone. The three basic assumptions underlying this view are that when the residential market adjusts as a result of the noise,

(a) The number of inmovers is equal to the number of outmovers.

(b) The depreciation is so determined that the marginal inmover and the marginal outmover gain nothing by moving.

(c) The inmovers' net gain is D-N and the outmovers' net gain is N-D.

Given these assumptions it is clear that D will equal the median of the distribution of noise annoyance <u>among the inmovers and outmovers</u>. This may or may not imply that D is equal to the median of the noise annoyance distribution for the population. Furthermore, whereas assumptions (a) and (b) are acceptable, it is not possible to accept assumption (c). The net gains of the inmovers and those of outmovers must include losses of movement costs and of householder surplus. When both these factors are brought into consideration, the validity of the original argument for the median assumption breaks down, and one is left with an assumption which is very important and which cannot be satisfactorily justified.

What is really involved here is an attempt to derive a noise annoyance cost function which associates with each noise annoyance level x, a cost c(x).[19] We first consider cases where the cost function obtained by the use of the median assumption is valid. The conditions required for its validity turn out to be rather restrictive. We then go on to consider an alternative approach for obtaining c(x) which involved the use of data on inmovers and outmovers as a proportion of the population in an area, and an algorithm for dealing with the difficulties raised by the existence of a distribution of householder surpluses. Finally we conclude this subsection by making some general observations on the applicability of such an alternaitve procedure.

Let the noise annoyance scale x be defined over a range $[0,T]$ and let there exist a cost function c(x), defined over this noise annoyance scale, such that c(o) = 0. Then we may state the following lemmas:

<u>Lemma 1</u>

If (a) The depreciation is so determined that the marginal inmover and outmover have zero net gains;

(b) the density distribution of the population over the noise annoyance scale x is symmetrical and unimodal;

(c) c(x) is a linear function;

(d) all inmovers and outmovers have the same surplus $\bar{S}$;

then the mean and median of the distribution of noise annoyance $T/2$, has a cost D associated with it.

Proof

Since the number of inmovers is equal to the number of outmovers as a result of the noise, and the marginal inmover and outmover gain nothing, there exists a λ on the noise annoyance scale:

$$c(\lambda) = D-R-\ddot{S} \qquad (26)$$

$$c(T-\lambda) = D+R+\ddot{S} \qquad (27)$$

By linearity of the noise scale, and since $C(o) = o$

$$c\left(\frac{T}{2}\right) = D \qquad (28)$$

With a linear cost function we may write $c(x) = \alpha.x \quad , \quad \alpha > o$

Lemma 2

If assumptions (a) to (c) of lemma 1 hold and there is a distribution

(d) of householder surplus S over the population such that the distribution is defined over a range $[0, S_{max}]$, where

$$S_{max} \leq \min \left[D-R \ , \quad \alpha T-D-R \right] \qquad (29)$$

(e) The distributions of householder surplus and noise annoyance are independent of each other, then the mean and median of the noise annoyance distribution, $\frac{T}{2}$ has a cost D associated with it.

Proof

Let F(x) be the cumulative distribution corresponding to the density distribution f(x). Then for each surplus level S, the proportion who move in are given by F(D-R-S), and the proportion who move out are given by 1-F(D+R+S). Let g(s) be the density distribution of the population. Since g(s) and f(x) are independent, the equality of the number of inmovers to the number of outmovers gives:

$$\int_{S=o}^{S=\min(S_{max},\ D-R)} F(D-R-S)\ g(S)dS \qquad \int_{S=o}^{S=\min(\alpha T-D-R\ ,\ S_{max})} \left[1-F(D+R+S)\right] g(S)dS \qquad (30)$$

Given assumption(d), and rearranging terms,

$$\int_{S=o}^{S_{max}} \left[F(D-R-S) + F(D+R+S)\right] g(S)dS = 1 \qquad (31)$$

Now by assumption (b) we know that if $F(D) < 0.5$, then $F(D-R-S) + F(D+R+S) < 1$ and the above equation cannot be satisfied. (Recall that $g(S) = 1$ and $\int g(S)dS = 1$). If, however, $F(D) > 0.5$, then $F(D-R-S) + F(D+R+S) > 1$, and again the above equation cannot be satisfied. Therefore it can only be satisfied when $F(D+R+S) + F(D-R-S) = 1$ and that occurs when $F(D) = 0.5$, indicating that $c\left(\frac{T}{2}\right) = D$.

These two lemmas indicate that only under restrictive conditions can the median assumption be shown to be valid.[20] From lemma 1 it follows that if all the inmovers and outmovers have a fixed surplus then linearity of the cost function and unimodal symmetry of the annoyance distribution will do. One naturally thinks of noise movers as having a small or even zero surplus although this need not be the case. It is more difficult, however, to accept the linearity assumption which implies a cardinal interpretation to what is essentially an ordinal scale. If, the movers have considerably variable surpluses, then from lemma 2, the further requirement (d′) is that the range of such a surplus be small - so small that no inmover is excluded from moving in on account of his surplus, and no outmover is excluded from moving out on account of his surplus, given the value of D and R. Again this condition seems unlikely to be satisfied.

An alternative procedure for obtaining the cost function $c(x)$ is one that attempts to infer such a function from information regarding turnover rates in residential property in various noisy zones. For each noise zone z, we have a noise annoyance density distribution $f(x,z)$[22]. From these distributions we wish to obtain a cost function $c(x)$. If this noise cost function is strictly monotonic and differentiable then there exists a derived distribution $g(c,z)$ where

$$g(c,z) = f((x(c),x) \cdot \left|\frac{dx}{dc}\right| . \tag{31}$$

and $x(c)$ is the inverted cost function.

$g(c,z)$ is the density distribution, giving the proportion of the population that has a noise nuisance cost of between c and c+dc in zone z as $g(c,z)dc$. Let us assume initially that there is no distribution of surplus among the population and that everyone has a surplus S. We will relax this assumption later. Then, if noise is introduced into a previously quiet area, making it into a noise zone of level z, all outmovers will suffer a loss of D+R+S. Since at the margin there is no net gain for the outmover of the inmover, we have that:

$$G(c^*,z) = 1-t(z) \tag{32}$$

$$G(c^{**},z) = t(z) \tag{33}$$

$c^* = D+R+S$, $c^{**} = D-R-S$, and t is proportion of the residents that move out of the area as a consequence of the noise zone being created. $G(c,z)$ is the cumulative distribution corresponding to the derived distribution $g(c,z)$.

Now for each noise zone z there will be values of $t(z)$ from which the function $c(x)$ can be constructed, as shown below in diagram 7. We plot the cumulative distribution $F(x,z)$ in the southwest quadrant. In the north east quadrant we plot the combinations of c^* and $G(c^*)$, and c^{**} and $G(c^{**})$ as given in the expressions above. By tracing through diagram we can thus obtain two points relating c to x. Repeating the exercise for different noise zones, we get further pairs of points in the c-x space. By interpolating between these points we have an approximation to the $c(x)$ function.

Thus for the case where there is no distribution of surplus and where the function $c(x)$ is strictly monotonic, there is a procedure for deriving this function by a consideration of the turnover rates in the various noise zones. In most applications, however, the distribution of the surplus S is an important factor, and it is to the implications of such a distribution on the derivation of the function $c(x)$ that we now turn.

Let us assume that there is a density distribution of surplus, given by $h(s)$, and that the distrubiton of surplus is independent of the distributions $f(x,z)$, for all z. We proceed by ignoring the distribution $h(s)$, and by choosing, for each z, two values of S, viz., $S_L(z)$ and $S_u(z)$. Given these values the cost function $c(x)$ is constructed, as outlined in the previous section with the knowledge of the turnover rates in each of the noise zones. Now given the cost function, the cumulative distribution of noise annoyance costs, $G(c,z)$ can be obtained by a summation over c of the terms given in equation (32). With this cumulative distribution, the predicted turnover rate in zone z is expressed as $E[t(z)]$, where[23]

$$E\left[t(z)\right] \quad : \quad 1 - \int_S G(A+S;z)h(S) \tag{35}$$

$$A \quad = \quad D(z)+R \tag{36}$$

We are concerned with the choise of the values $S_L(z)$ and $S_u(z)$, such that the cost function obtained predicts, as 'well' as possible, the observed turnover rates. Thus one possible way would be to try all possible combinations of $S_L(z)$ and $S_u(Z)$ but that would be extremely

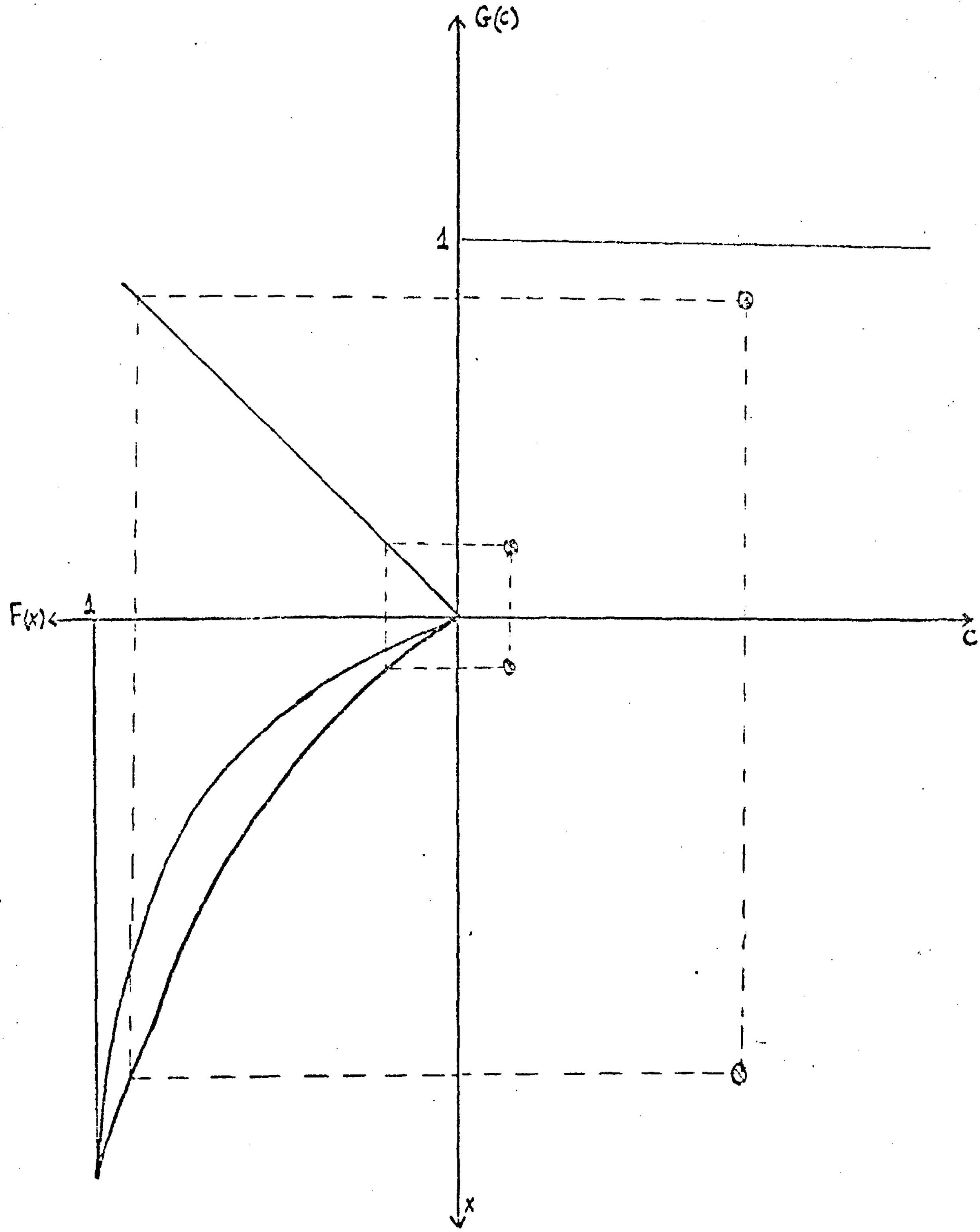


DIAGRAM 8

laborious. The following algorithm seems to suggest itself:

If $E[t(z)] > t^*(z)$

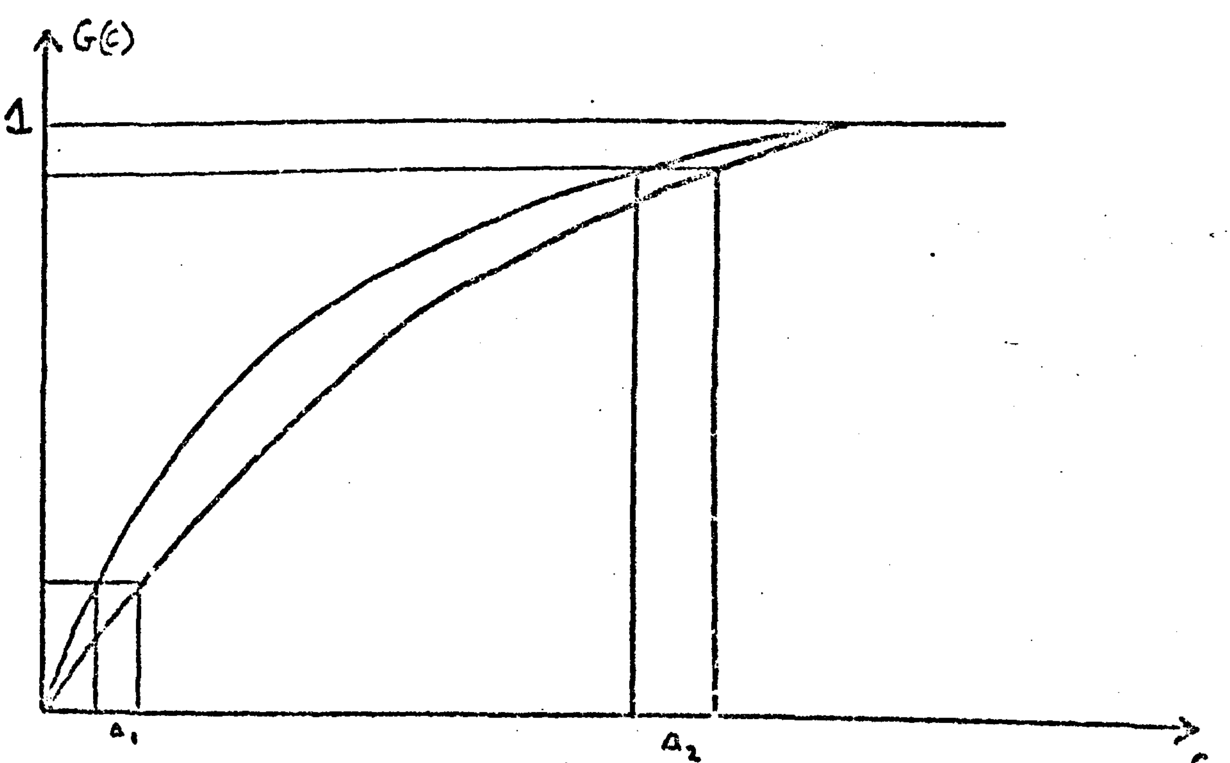
where $t^*(z)$ is the observed turnover rate in zone z, then we wish to lower the predicted turnover rate. From equation (35) it is clear that the cumulative function $G(c,z)$ must shift upwards as shown in diagram 8. For this to happen, the cost associated with a given turnover rate must fall. For a turnover rate of $t(z)$ we associate a cost of $D-R-S_L(z)$ with $t^*(z)$ and a cost of $D+M+S_u(z)$ with $1-t^*(z)$. Thus, if these costs are to fall $S_L(z)$ must be raised and $S_u(z)$ lowered.

Therefore, if we observe that the predicted turnover rate exceeds the observed turnover rate we should lower $S_u(z)$, and raise $S_L(z)$, and vice versa. There are, however, some natural restrictions on the value that $S_u(z)$ and $S_L(z)$ can take:

(a) $D(z)-R-S_L(z) \geqslant 0$ for all z

(b) The choice of the relative values of $S_L(z)$ and $S_u(z)$ for all z is such that the cost function obtained is non decreasing in x.

Condition (a) is necessary to ensure that the costs associated with given values of x are not negative. Condition (b) seems to be a natural restriction to place on a noise annoyance scale that measures increasing levels of annoyance



Δ_1 and Δ_2 are the required reductions in c

DIAGRAM 9

In any practical application of a procedure such as that outlined above, it will not be possible to obtain predicted movement rates that are exactly equal to the observed movement rates. This is because the data on the functions $f(x,z)$ and $h(S)$, and on the turnover rates is obtained by statistical methods that are liable to sampling error, and because the construction of the $c(x)$ function will involve a considerable amount of interpolation. It will be necessary therefore to have some criteron of 'goodness - of fit'. One such criterion could be, for example, to minimise:

$$\sum_z \left[\bar{E}(t(z)) - t^*(z)\right]^2$$

This penalises one large deviation more than two small ones, and therefore emphasises a uniform closeness to the observed rates.

In chapter 5 we discuss some of the evidence on turnover rates in areas inflicted by noise. There are several difficulties of measurement and interpretation, but one of the tentative conclusions is that about six to nine per cent of the residents of a region inflicted with noise might move, in regions where the noise level is 27.5n.e.f. and over (This is approximately equivalent to noise zones of 40 n.n.i. and over). Taking three noise zones for which the Roskill Commission had collected data on the $f(x,z)$ functions, we proceeded to apply the above algorithm, when the observed turnover rates in each of the zones was eight per cent. The details of the data used, and the assumptions regarding movment costs and depreciation levels is given in an appendix to this chapter. We summarise here the iterations in Table 2 and graphs A and B. In Table 2 we observe that the sum of squares of deviations falls between each step. In graph 1 we show the changes the function $c(x)$ as the iterations are carried out. Graph 1 corresponds to step I in the table, graph 2 corresponds to step II in the table and graph 6 corresponds to step III in the table. Graphs 3 to 5 are not tabulated, but all show a uniform fall in the sum of squares of deviations.[24] In graph B we plot the best estimated cost functions using turnover rates of eight per cent, and the cost function obtained by the use of the median assumption.* The main points to note in graph B are:

(i) The median assumption probably seriously underestimates the noise costs at the top end of the scale. This will imply that the Roskill

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* The costs presented here, as well as the value of $S_U(z)$ and $S_L(z)$, are all annuitised costs in income units. Thus for example a value of c of 0.10 implies that the annual noise nuisance cost is 10% of annual income.

TABLE 2

	z(NEF)	$S_L(z)$	$S_V(z)$	$E\ t(z)$	$t^*(z)$	$E\ T(z) - t(z)\ ^2$	
I	37.5	0.0032	0.08	0.123	0.080	0.0018	
	33.5	0.0032	0.08	0.131	0.080	0.0026	
	27.5	0.0032	0.08	0.204	0.080	0.0154	
							0.0198
II	37.5	0.0032	0.07	0.061	0.080	0.0004	
	33.5	0.0032	0.06	0.077	0.080	0.0000	
	27.5	0.0032	0.05	0.163	0.080	0.0069	
							0.0073
III	37.5	0.0496	0.07	0.55	0.080	0.0006	
	33.5	0.0317	0.06	0.066	0.080	0.0002	
	27.5	0.0048	0.05	0.100	0.080	0.0004	
							0.0012

Restrictions on values of $S_L(z)$ and $S_V(z)$:

Condition (a) implies

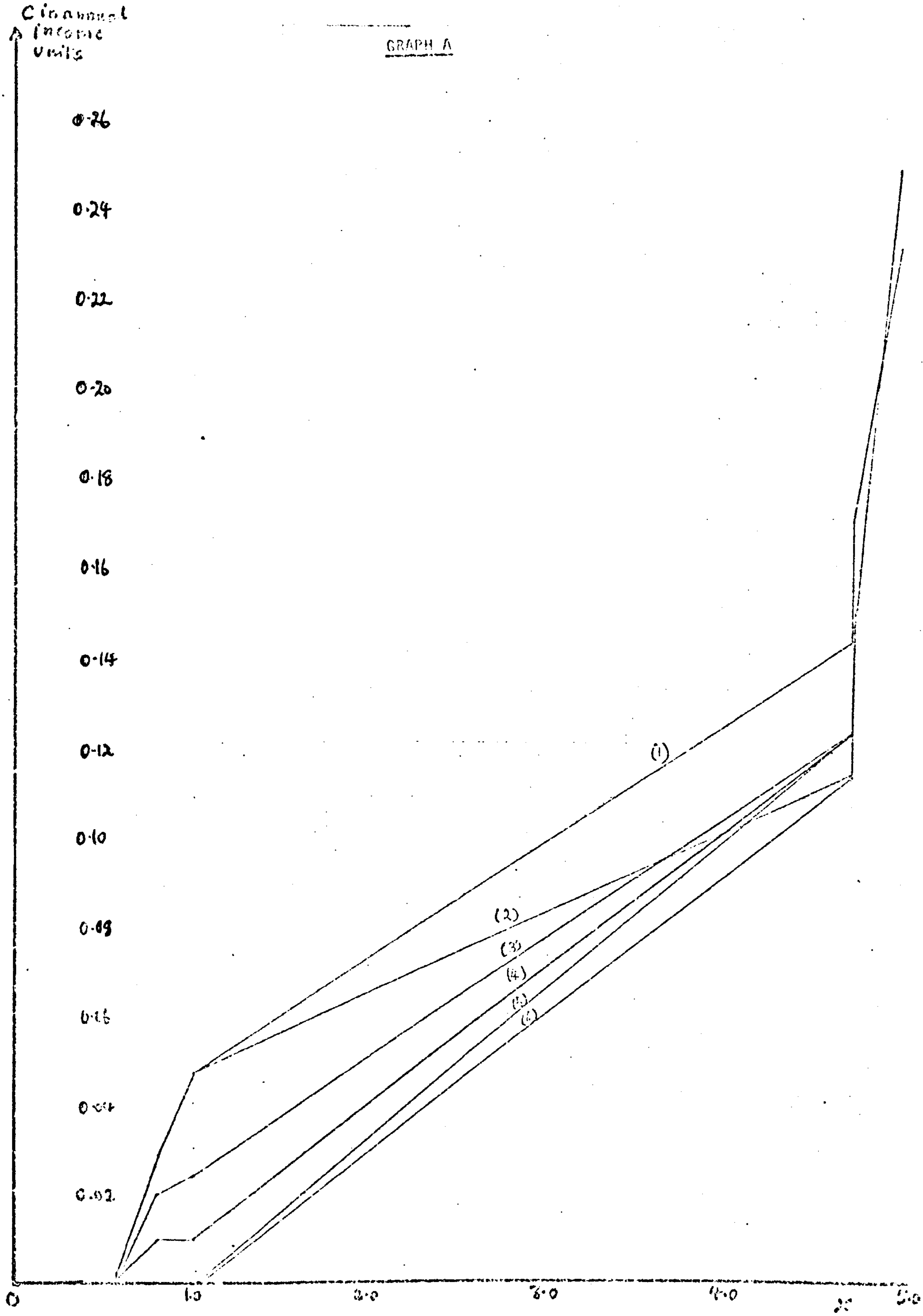
$$S_L(27.5) \leqslant 0.0048$$
$$S_L(33.5) \leqslant 0.0317$$
$$S_L(37.5) \leqslant 0.0496$$

Condition (b) implies that

$$S_L(37.5) \geqslant S_L(33.5) \geqslant S_L(27.5)$$

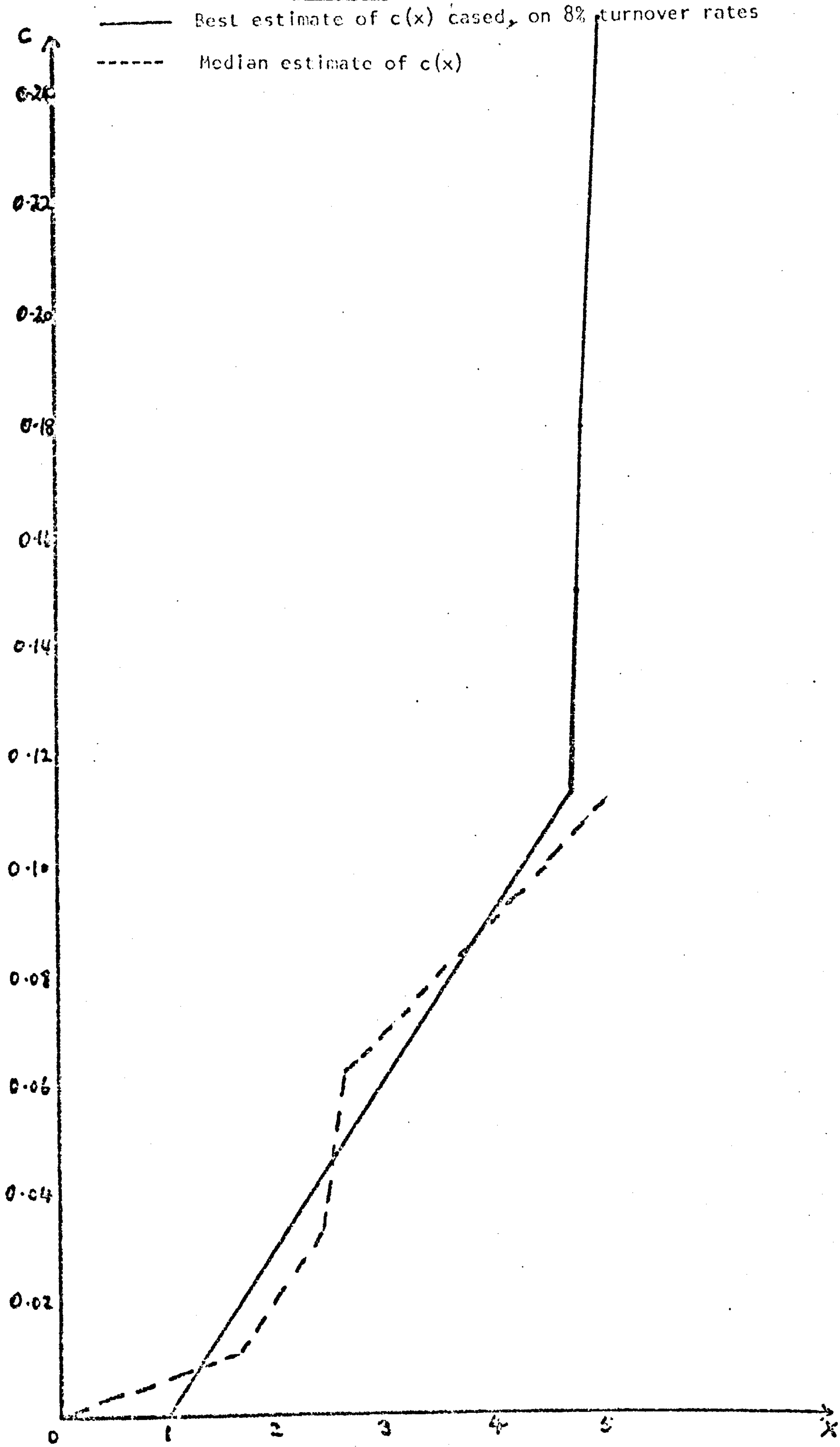
All values of S are expressed in annual income units, e.g. S = 0.08 implies that the annuitised value of the householder surplus is 8% of annual income.

GRAPH A



C in annual income units
0·26
0·24
0·22
0·20
0·18
0·16
0·14
0·12
0·10
0·08
0·06
0·04
0·02
0
1·0
2·0
3·0
4·0
5·0
(1)
(2)
(3)
(4)
(5)
(6)

GRAPH B



Best estimate of c(x) cased, on 8% turnover rates
Median estimate of c(x)
c
0·24
0·22
0·20
0·18
0·16
0·14
0·12
0·10
0·08
0·06
0·04
0·02
0
1
2
3
4
5
x

noise model will generate much lower movement rates and lower noise costs for the higher noise nuisance ratings, especially in very noisy zones. In fact if we consider the movement rates predicted by noise cost function as estimated by the median assumption, we obtain the following movement rates[25]:

TABLE 3

NEF	Movement rates predicted by noise model using median assumption %
20-25	I4.9
25-30	I2.I
30-35	5.9
35-40	0.0

The figures for the 30-35 nef and 35-40 nef appear to be at variance with what statistical data is available on movement rates*

(ii) If the observed movement rate is around 8%, then the middle section of the noise cost function (x=2 to x=4) is probably not too bad as estimated by the median assumption.

(iii) Again with observed movement rate of around 8%, it is probably safe to ignore the gains to inmovers, as the best estimate of the cost function appears to be c(x)=0 for x $0 \leq x \leq 1$.

However, it is by no means established that the movement rate is 8%. A closer examination of the statistical evidence on these is necessary, and such evidence could lead to the costs established by the median assumption as being even more unsatisfactory.

II.VI The Use of Noise as an Attribute in the Decision-Making Process

In the noise model used by the Roskill Commission households were assumed to choose between the option of staying and suffering the noise, or moving to a completely quiet zone. In reality of course, households have a whole range of intermediate options involving moving to different noise zones.

* The very low turnover rates in the 30 nef and above zones are predicted because the distributions of noise nuisance are rather closely bunched together at the top end for all noise zones and the higher depreciation levels in the very noisy zones lead to median calculated values of N to move more slowly than depreciation rates.

The choices available are represented in diagram 9. Originally the household is at F, enjoying the maximum quiet. He then finds himself at N, as a result of the imposed noise in the environment. The choices available to him are to stay at N, or to choose a point on the line QE. This line is below N, because there are movement cost losses of surplus involved in any move. Such fixed costs of moving are represented by NK. The household will evaluate the maximum utility attainable by moving (call it U_m) and compare it with the utility of staying (call it U_s) If the former is greater than the latter he will move, otherwise he will stay.

The question involved here is whether, for an outmover, it is valid to assume that U_m passes through E. If it does then the relevant comparison is between FE and FO, where O is the point of intersection of the indifference curve through N, representing utility U_s, with the maximum quiet line. If FO is greater than FE the householder moves; otherwise he stays. Intuitively one might regard the fixed cost of moving as implying a declining unit cost to the purchase of quiet, and the greater the fixed cost relative to the price of quiet the more likely it is that a household that finds it beneficial to move will only find it so when purchasing a lot of quiet. In practice the fixed costs are always quite substantial. Even in the absence of any householder surplus, the movement costs are about nine per cent of the houseprice.[26] These compare with a depreciation of about fourteen per cent of the house price in moving from a 30 n.e.f. noise level (quite noisy) to a quiet zone. Thus one might expect the assumption that noise can be treated as an attribute to be reasonably satisfactory in the context in which it is used.

Whether this is the case or not, however, can only be established by further specifying the underlying utility function for the household. For example, if we take the Cobb-Douglas utility function, modified to take account of a maximum level of quiet, as presented in section I.III, then we may present the choice as follows:

If the household was enjoying q_o units of quiet, where q_o is the maximum amount of quiet possible, and now finds itself at x units of quiet then it will move if

$$q_1^{\alpha} Y_1^{1-\alpha} - x^{\alpha} M^{1-\alpha} > o$$

where q_1 and Y_1 represent the utility maximising choices along the budget line

$$M + p \cdot x - p.q - L = o$$

where p is the price of a unit of quiet and L is the fixed cost (movement cost plus loss of household surplus) of moving locations. The utility

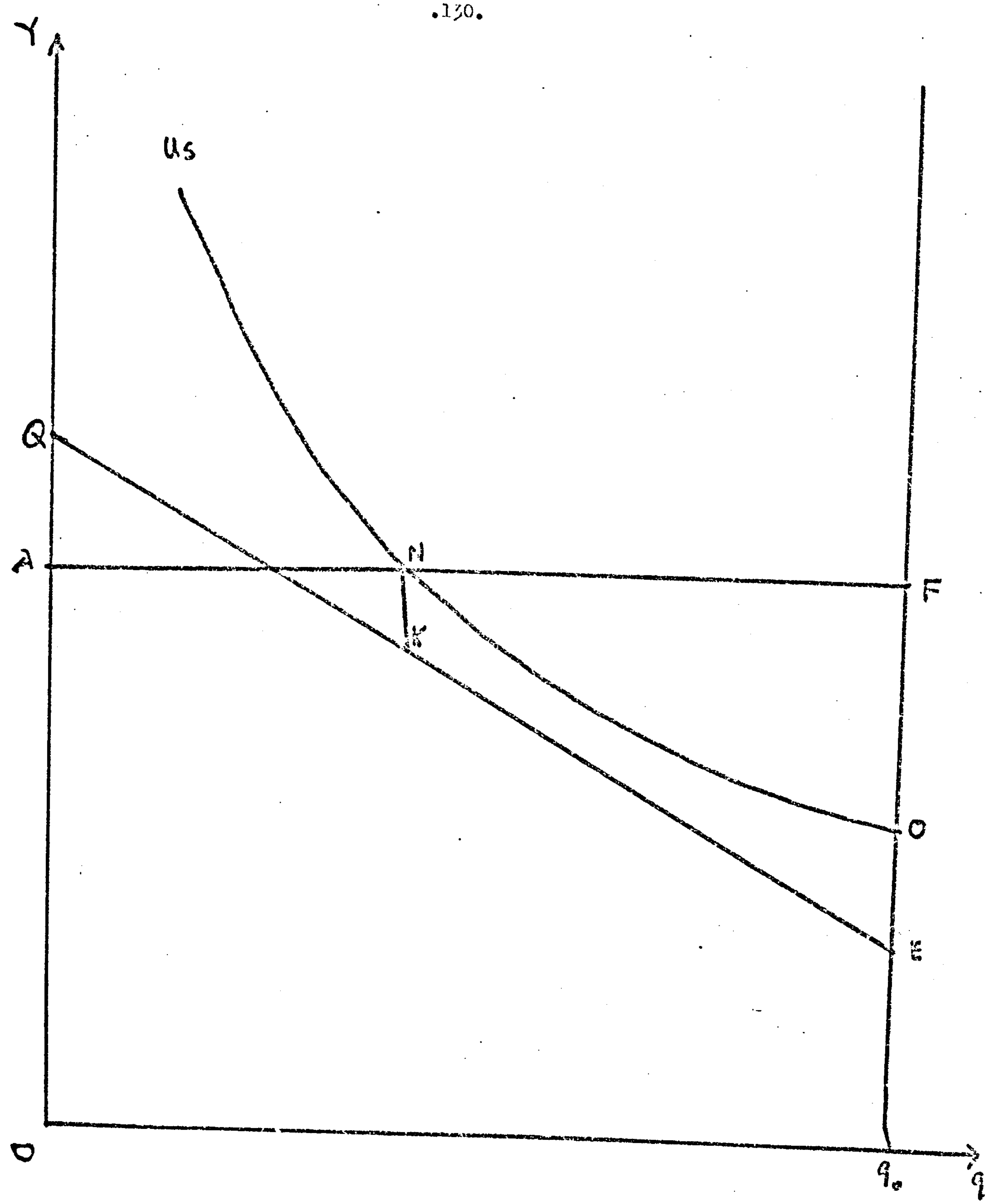


DIAGRAM 10

maximisation gives:

$$q_1 = \min\left[\alpha\ \frac{(M+px-L)}{p},\ q_o\right]$$

$$Y_1 = \max\left[(1-\alpha)(M+px-L),\ M-p(q_o-x)-L\right]$$

The min andmax terms indicate that no individual can consume more than q_o units of quiet, no matter what his α.

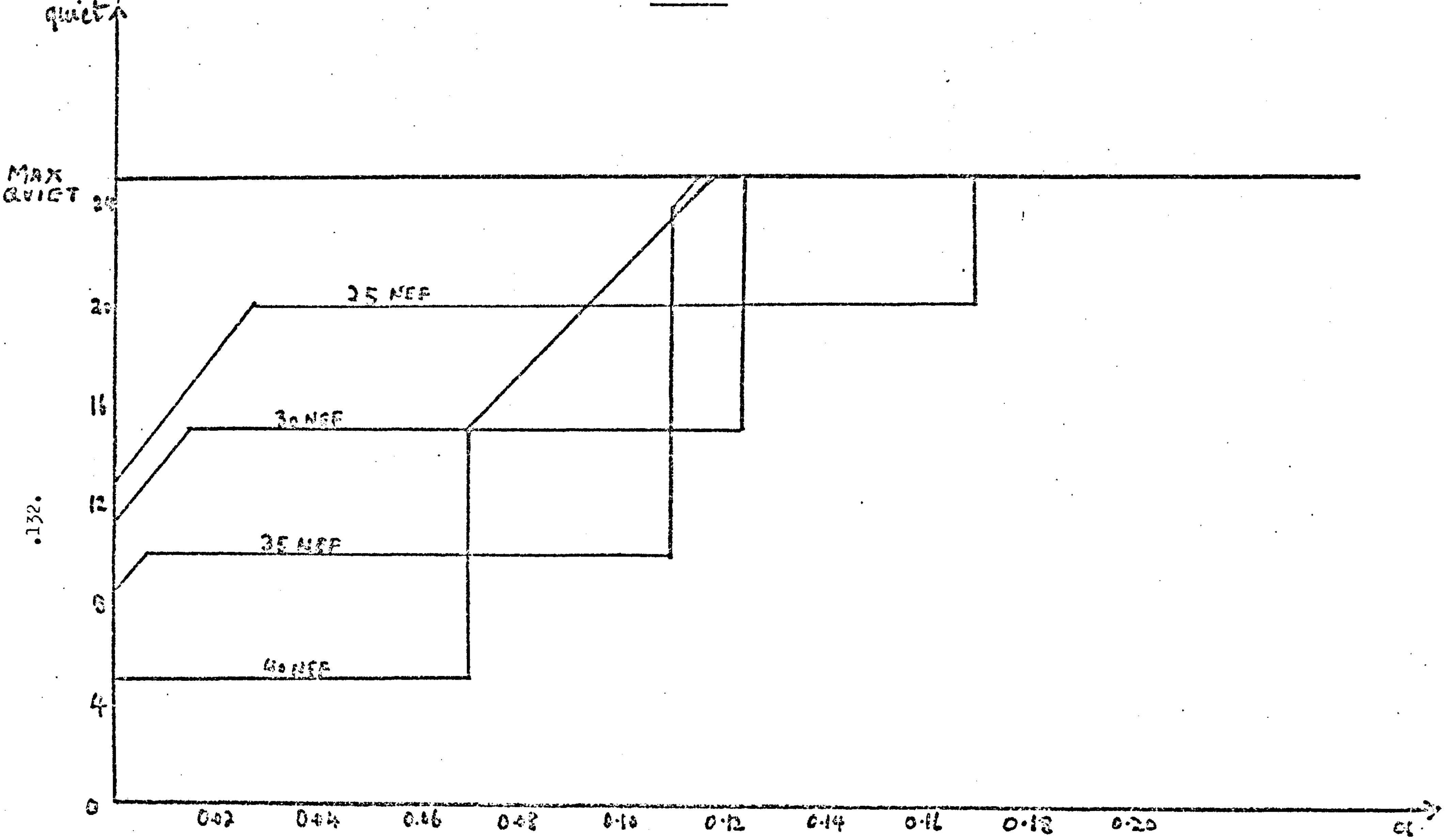
For illustrative purposes we consider a household with an annual income of £3000. L is set equal to R, the movement costs alone, and the data on R and p are taken as given in the appendix.

In graph C we plot α, the parameter indicating the preference for quiet on the horizontal axis and the level of quiet that the household chooses to live under as the vertical axis. The maximum level of noise for residential purposes is taken as 45 n.e.f. This implies that all households with $\alpha > o$ will move from a 45 n.e.f. noise level. 20 n.e.f. is taken as the no noise level. This means that there are 25 units of quiet. Households are considered at the 25, 30,35 and 40 n.e.f. zones, and the stayers are represented by the horizontal part of the graph for each zone. Those households with very low alphas move into even more noisy areas, whereas those with high alphas (a strong liking for quiet) move into quieter areas. It will be observed that, for the 25-30 n.e.f. zones all households that move into quieter areas, move to the maximum quiet, whereas in the 40 and 35 n.e.f. zone there are some outmovers that move into less than the completely quiet zone. Such households could be substantial, depending on the distribution of α in the population especially in the very noisy zone (40 n.e.f.) Overall, however, it does appear that, except for the noisest zone the assumption that the decision to move to quiet areas can be made with noise regarded as an attribute is reasonably valid.

Although these results have been presented for a given income level, they are in fact very insensitive to changes in the level of income. Also, increasing the level of L to include householder surplus only acts to reinforce the conclusions reached here.

While the choice of utility function used in the above illustration has its already spelt out limitations, it is difficult to believe that an alternative utility function would radically alter the conclusion that, given the kinds of values taken by the relevant variables, the treatment of noise as an attibute in this context is satisfactory all except the noisiest zones.

GRAPH C


quiet
MAX QUIET
24
20
16
12
8
4
0
25 NEF
30 NEF
35 NEF
40 NEF
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20

II.VII The Problems of Time and Uncertainty

Uncertainty

The model outlined so far, and commented upon, attempts to measure the costs of noise from a household decision making model which is based on the assumptions that the household chooses that course of action which is most beneficial to itself, and it does so with complete information regarding the relevant costs and prices, and regarding its own tastes. The assumption of complete information, however, is rather unsatisfactory in this context. The development of serious aircraft noise is a comparatively new phenomenon, and careful marginal judgements with resepct to it may be difficult, especially for those with little or no experience of noise. It is for this reason that the Roskill Commission did not measure the noise costs or benefits of inmovers. However, it remains true that if there is some depreciation in house price due to noise, then there must be some households that choose to move in on account of their subjective evaluation of the noise costs. Furthermore the social costs of noise will depend on the relationship between the subjective evaluation and the realised noise costs. Such a relationship may be a completely 'random' one, or it may be that the 'subjective' evaluation of noise costs, consists of a probability distribution of such costs, and the probabilities reflect the long run frequencies of the realised noise costs. In the case of the subjective noise costs and the realised noise costs being randomly related, we may proceed as before, including in the calculation of the function $c(x)$. When the noise costs of inmovers are to be assessed, however, the treatment is as follows:

A uniform random relationship implies that, irrespective of one's subjective noise cost, the actual noise cost, N could take any of the values in the range of N. That is to say, each value of N has the same probability of occurring. Let the range of values of N be from zero to T. Then the expected noise costs are given by:

$$\text{Expected Noise Costs} = -\sum_{N<\min(T,D+R)}(D-R-N)\frac{1}{T} + \sum_{N>\min(T,D+R)}(D+2P)\cdot\frac{1}{T}$$

The first term indicates the expected gain, when the individual noise cost is less than D+R, and he stays in the noisy zone. The second term is the expected loss if N exceeds D+R and the individual leaves the noisy zone, to return to the quiet zone. (This being his least costly course of action).

Assuming no risk aversion, the individual will move into the noisy area only if his expected noise cost is non positive. Thus if we add all the negative expected noise costs we obtain the long run or expected noise costs of inmovers.

Such a procedure is based on the assumption of a uniform random relationship between the subjective noise cost and the realised noise cost. If, however, the 'subjective' noise cost does bear some other relation to the possible outcomes, then the problem is more complex for the probability distribution of possible noise costs has to be ascertained. In our present state of knowledge it would not appear to be possible to construct the subjective probability distributions of individuals views' of their noise costs. However, taking a uniform random relationship, which may be interpreted as a case of equal ignorance of all possible states, gives some idea of the expected noise costs in a situation of considerable uncertainty.

The presence of risk aversion in individual behaviour would further complicate the problem, and would require the specification of a Von-Neumann-Morgernstern type utility function with a risk parameter, for the solution of the problem. However, in the absence of any reasonable basis for making such a specification, it should be noted that the actual noise costs incurred by a risk averse population will be lower than those incurred by a risk neutral population. This is because, being risk averse they will tend to stay where they are more frequently, and this action has a zero cost or benefit associated with it. Similarly the gains obtained will be smaller too, thus giving an overall distribution of actual gains and losses that is more closely bunched around zero than with risk neutral behaviour.

Time

The annual noise costs are discounted to a given year and added together to obtain the total noise costs. The Roskill Commission truncated the noise costs in the year 2005. Also, to allow for growth of real incomes, and a change in relative prices they allowed for a growth in the values D, R and S overtime. The choice of the year truncation raises an interesting economic problem. It would be a desirable property of the truncation point if the discounted sum of net benefits of the whole project for any discount rate should not change sign for truncation points greater than the one chosen. If it did change sign then it would imply that the costs and benefits that are ignored could have a decisive effect on the project depending on the discount rate. In this connection Sen (1973) has shown, in an unpublished paper, that if certain regularity conditions are satisfied and in addition the discounted sum of the net benefits <u>after</u> the point of truncation are <u>positive</u> then that point satisfies the above property. In practice, however, it is extremely difficult to establish the last condition, and a point of truncation is chosen when it is felt that values of the relevant variables beyond that point are too uncertain to be held with any credibility. This does mean of course that different views of the future beyond the point of truncation could reasonably be the source of differences regarding the desirability of the whole project.

It is certainly important to take account of changes in real income and relative prices over time. As real income grows over time, the positive income elasticity for quiet implies an increase in the demand for quiet. Given limitations in the supply side this would lead to an increase in the price of quiet, although in measuring this it is important to take into consideration changes in the levels of noise produced by aircraft of different design and different volume in the future. Similarly, an increase in the relative price of services would lead to an increase in the costs of moving. Householder surplus is usually related to the expenditure on housing. Whether this increases over time or not, depends on the relative values of the income and price elasticities of housing. If we define Φ as the percentage change in real expenditure on housing,

then $\Phi = \eta . g + (1+\varepsilon)\psi$

where ε = price elasticity of demand for housing

ψ = rate of increase of house prices in real terms

η = income elasticity of demand for housing

g = rate of growth of real income

Since ψ is often considered to be positive, whether Φ is positive or not depends on the relative values of the variables. It is by no means clear that it should equal g, as assumed by the Roskill Commission.

III. Conclusions

In this chapter we have considered, in some detail, two alternative methods of measuring noise costs to households. The first method begins by treating quiet as a typically economic good, and then makes allowances for factors which are special to quiet, such as adjustment costs. The second method starts out by taking account of the special features of noise in constructing a noise model. When considered more closely, this model can be regarded as a special case of the first method - a case where noise can be treated as an attribute rather than as a continuously variable commodity in the households decision as to whether it should move or not. We have considered the validity of this simplifying assumption, and conclude that it is probably valid for low and medium levels of imposed noise, but may not be valid for high levels of imposed noise.

If noise is treated as an attribute, the measurement of noise costs can proceed without any specific demand function for quiet. The approach that is taken consists of deriving a noise annoyance disbenefit distribution from an ordinal scale of noise annoyance, along with data on house price depreciation, movement costs, and householders surplus. We have discussed this approach and considered a number of problems. By far the most awkward problem is the one of constructing a suitable noise annoyance scale from questionnaires on issues related to noise, carried out in noisy zones. It is difficult enough to state the direction of any biases caused by using such a scale, let alone quantify them. Yet there are reasons for thinking that such biases might not be insubstantial. However, if the noise annoyance scale can be relied upon, then it should be possible to obtain reasonable estimates for the noise costs, providing that some information can be obtained on turnover rates in zones where the noise is introduced, and providing that the data on depreciation, movement cost and surplus is satisfactory. Such estimated costs will consist of movers costs and stayers costs. The movers costs are both the compensation and willingness costs as defined earlier, but the stayers costs are the willingness costs only. From these, some idea of the compensation costs can be obtained, given an underlying utility function between quiet and other commodities. Such a function would imply, however, a demand function for quiet, and some values for the price and income elasticities for quiet, which this approach aimed to do without.

The more classically economic method of costing noise does away with a noise annoyance scale, and the consequent estimation of the noise annoyance disbenefit, but requires an estimated demand function (or functions) for quiet. This estimation is not an easy task, but the data is available, and it should be possible. Indeed in some hitherto unpublished work, Walters (1973) has already derived some preliminary estimates for the price and income elasticities for quiet. The measurement of noise costs based on these has a number of advantages:

(a) An estimated demand function for quiet would give some confidence interval for the estimated parameters. With these it should be possible to obtain confidence intervals on the derived noise costs. This is not possible with the Roskill or modified Roskill models.

(b) There will be no need to assume that noise is an attribute - an assumption that could be misleading in some cases.

(c) It will be possible to provide a fuller treatment of the compensation and willingness costs of noise. Such a treatment is not possible with the Roskill model, without specifying the utility function, which amounts to having some idea of the demand function.

(d) The step by which the noise annoyance scale is converted into money values will be avoided. This means that the dubious median assumption, or the difficult alternative of measuring turnover rates will not be necessary.

APPENDIX TO CHAPTER 3

In calculating the cost functions the following data was used:

(1) The functions f(x,z) were taken from the McKennel Survey (McKennel 1963). This gave the distribution annoyance at various nni levels. The nni values were converted to nef values by using conversion factors constructed by Abrahams. (Abrahams, 1973).

(11) The distribution of surplus h(s) was taken from the Roskill Report, Vol.7, Table 20.1. This gave the consumer surplus as a percentage excess of the market price of a house and listed the percentage of a sample of householders who stated that surplus.

(111) House price depreciation was set at 1.4 per cent of the house price with the noise, per nef. Details of this estimate are given in chapter 5.

(1V) Movement costs were fixed at 9 per cent of the house price. Again this estimate is explained in more detail in chapter 5.

(V) To measure everything in income units a ratio of 4 was assumed between annual income and house value. The constancy, or other wise, of the proportion of income spent on housing has long been a matter of debate. The current evidence on this question is summarised in a survey by de Leeuw (1971), where he argues that the income elasticity of demand for housing is about unity with respect to permenant income, implying a constant proportion of income being spent on housing services. The ratio of expenditure on housing services to house value however, appears to be a marginally declining one as house value increases. With the fitted equations relating the log of housing expenditure to the log of permenant income, it is possible to estimate the proportion of income spent on housing from the constant term of the equations when the income elasticity is about unity. Unfortunately this does not yield sensible values when applied to those results quoted by de Leeuw. (In the case of equations relating house value to income they imply the ratio of income to house value as being greater considerably than one!) In a footnote in the same paper, de Leeuw states that this ratio is probably between 2 and 3 in the United States. This range seems rather low to us for British data, where the ratio is probably closer to 4 - a value that was considered to be about right for Australian data in the course of the Sydney Airport Study.

(VI) To annuitise the costs a discount rate of eight per cent was used.

Finally, the functions $f(x,z)$ were only used for those zones where $D(z)-R>o$, since if $D(z)-R<o$ then there will be no inmovers and hence the level of depreciation must be a hypothetical figure. This excludes the possibility of uninformed behaviour, but this analysis only makes sense if such behaviour is excluded.

In calculating the willingness and compensation costs in section 1, the price of a unit of quiet, p^*, was set at 1.4% of the house price per unit of quiet. To relate this to income, assumption (v) above was made. All costs were annuitised using a discount rate of 8%. The units of quiet are measured from 0 to 25. 25 units of quiet may be taken to correspond to 20 n.e.f., the cut off point in the noise scale used by the Roskill Commission. 0 units of quiet would then correspond to 45 nef, and, given the utility function assumed, this would imply that at that noise level all households with any positive degree of perturbility would move out.

FOOTNOTES TO CHAPTER 3

1. In an evaluation of the costs of aircraft noise the Roskill Commission calculated that residential noise costs were the easily biggest single component of the total noise costs and ranged, for a prospective third London airport, from about £10 million for an airport at Foulness to about £72 million for an airport at Nuthampstead. See Roskill (1970) for details.

2. Expressing an indifference curve between quiet and expenditure in other goods, the indifference map will depend on the prices of all other goods. We assume that such prices are held constant in this analysis.

3. It is entirely possible of course, that as a result of the noise the household would be better off than with no noise. For if his indifference curves were very 'flat', then some points on the new price line could lie above U_A and moving to a lower level of quiet represented by one of these the household would be at a higher level of welfare than that represented by U_A.

4. In chapter 5 we cite some of the evidence suggesting that the income elasticity of demand for quiet is about 2. This kind of elasticity, along with the 'large' price change being considered are the classic ingredients of important income effects.

5. Where cross price elasticities are not negligible it would be necessary to take account of the effects on other prices, and the shifts they would cause to the demand curves for other commodities. Since such effects are rarely considered it is implicitly being assumed that cross price elasticities are negligible.

6. If households are sufficiently sensitive to move to a completely quiet area then their willingness and compensation costs are indistinguishable. Later when adjustment factors are considered this statement will have to be qualified.

7. With Cobb-Douglas or Stone-Geary type utility functions, implying a constant unit income elasticity and a greater than unity income elasticity, declining with income, respectively, it can be shown that DF being greater than the distance between U_c and U_b measured in expenditure terms at q_b, is consequently larger than c* b* (see section ~~II~~ V for details). For other forms of utility function we cannot say how DF and c* b* will be related.

8. In considering any serious adjustment to noise we assume that moving residence is necessary. This is in accordance with the general belief that adjustments to noise through sound proofing fail to offer extensive protection against noise. The adjustment costs associated with moving

locations that is referred to as householders surplus may itself be measured in compensation terms - the extra money above the sale price of the house required to compensate you for moving locations, or in willingness to pay terms - the amount you would be willing to pay above the purchase price to buy the particular house you live in. We discuss these differences more fully in Chapter 5, section four. In summary, the willingness concept is the relevant one for decisions regarding moving or staying; once a householder decides not to move the concept is irrelevant, and if the householder moves his costs will be measured as before except that to obtain an overall compensation cost a compensation householder surplus is added and to obtain an overall willingness cost a willingness-to-pay householder surplus is added. For some interesting theoretical and empirical observations on this question the reader is referred to Starkie and Johnson (1973).

9. From data given by the General Householder Survey (1973) we observe that about 40 percent of the population move in a period of five years anyway. See General Household Survey Table 5.5.2.

10. The variational cost is used for short run movers because we consider them as adjusting their level of quiet in the short run in response to the externality. We use the surplus cost for the long run movers while they can be regarded as non-adjusters - i.e. until that period of time is past after which we may regarded all householders as adjusting. Then we regard them as free to adjust and use a variational cost.

11. We consider a fixed householder surplus for diagramatic simplicity. Otherwise we would have to do the analysis for every level of householder surplus.

12. We ignore here the surplus for inmovers. In practice it turns out to be unimportant since only households with no noise annoyance and no surplus would move into noisy zones in the short run.

13. This was considered in conjunction with the Sydney Airport Study and after further enquiry it was decided that surveying annoyance even in areas where there had been noise some years back was not feasible.

14. The author actually experienced an interview during which the husband refused to let his wife talk, and claimed that he did not suffer any noise annoyance, although one could not hear him when a plane flew over head and the house visibly shook. It turned out that he was employed at the airport and there was some possibility of his airline cutting back staff!

15. Again for the same reason, we ignore householder surplus among inmovers.

16. This marginal condition of N=D+R will only measure a*F if of course the household does move to complete quiet. We return to this point later.

17. We remark here that these differences are quite small, and non movers costs being the major component of the noise, compensation and willingness costs, at least as measured by unit elasticities are well within the margins of error of such an analysis.

18. Such a concept as E should be quite measureable. One interpretation might be the poverty line as defined in Social Welfare Programmes. This allows for local factors and brings in the size of the family which would appear a desirable thing to do.

19. In constructing such a function a noise cost of c for a noise annoyance rating of x is assumed to hold in all noise zones.

20. It should be stressed here that the above conditions for the median assumption to be valid are sufficient but not necessary. Thus it may be possible for the assumption to be valid when these conditions are not satisfied. However, what we have shown is that, in examining the conditions under which the median assumption is valid, we have not been able to provide a theoretical rationalisation for its use. In the absence of any such rationalisation, therefore, we must remain sceptical of the validity of this assumption.

21. This footnote has been excluded from the text.

22. We use this notation to refer to the proportion of households with a noise annoyance rating of X in a noise zone Z. It is obtained by sample survey methods.

23. The predicted turnover rate is constructed by assuming that all households with noise costs greater than depreciation plus adjustment costs move out of the noise zone.

24. From this analysis it does appear that the algorithm would work better if the noise annoyance rating scale was continuised in some appropriate fashion. Failing that a closer approximation to the observed rates would be obtained if the interpolation betweeen points on Graph A was done convexly rather than by straight lines.

25. Details of these estimates are given in Chapter 5 section 5.5 Effectively thay are derived from a Roskill type noise cost model with a constant level of noise imposed and with the price of quiet rising relative to all other variables. These movements are the ones that occur in the short term (first eight years). After that, since gradually but inexorably the valuation of quiet is assumed to rise there will be further adjustments.

However, for the purposes of estimating <u>initial</u> noise costs it is the short term movements that we are interested in.

26. This estimate is taken from the Sydney Airport Study. The Roskill Commission used similar figures.

Chapter 4.

Control of the Level of Noise

4.1. INTRODUCTION

In this chapter we discuss the economic aspects of the control of the level of noise. In doing this we draw from the large and growing literature on the control of externalities and Public Bads, and, in the light of the specific nature of the noise problem, evaluate the various forms of control which are used and which have been proposed to deal with this problem. This is done in this section. In the next section the most suitable method of charging for noise is examined more fully within the context of a formal general equilibrium model incorporating the relevant variables, and some of the problems and difficulties that may be encountered in its implementation are discussed. Such a scheme would require considerable further work and may or may not be operational. In the meantime the urgency of the noise problem has resulted in a series of measures to control noise. These have a varying degree of appeal to social or economic criteria and in section three we evaluate such direct controls as already exist. Most of the discussion in this chapter is in the context of aircraft noise, and certainly sections two and three deal with aspects of the noise problem that have little relevance to the questions raised by the existence of urban noise. Section four says what little can be said about the economic aspects of the control of urban noise.

4.1.1. A general discussion of noise as an externality and as a Public Bad

An externality is said to exist when an activity pursued by one agent in the economy has a direct effect on the utility or the profits of some other agent in the economy, and when no market exists which directly takes account of such an effect[1]. It is clear how noise complies with this definition. The noise generated by motor vehicles, aeroplanes, and individuals affects the utility of other individuals, and although the markets for related commodities may reflect such influences, there is no direct market for quiet, or freedom from noise. When the activity of some agent has a direct and deleterious effect on the utility of all households, and the above conditions apply, then we may refer to that aspect of the activity in question that produces this undesirable effect as a Public Bad. Whether or not noise is a Public Bad is not so clear. Many kinds of noise, such as motorway noise have a rather localised effect and cannot be said to directly influence the utility of many households. Other kinds of noise are rather broader in their impact, and can be said to influence a large part of the population. A household may be influenced by the existence of noise, even when it does not appear to suffer the consequences of this noise. This is because it may have taken some approriate action, at some cost to itself, to avoid this noise. On the other hand some household may be seen to be suffering some noise, but might be better off than if the noise did not exist. This is because its preferences for quiet may be small, relative to the implicit price for quiet which emerges through the markets of complementary commodities. As we saw in chapter 3 this can easily lead to a higher level of welfare for some household, relative to the no noise position, in which case it is perhaps inappropriate to refer to noise as a Public Bad. It is widely recognised, however, that a pure public good, or bad, hardly ever exists. The concept, nevertheless, is useful in deriving the conditions defining the optimum level of some commodities, and on considering the attainment of such optima. The validity of the concept depends on whether a large, and approximately identifiable, group of households is influenced by a particular activity[2]. This would appear to be the case with aircraft noise, but not with some other kinds of noise. Hence their analysis and treatment would differ.

4.1.2. Various methods of controlling externalities and Public Bads.

In order to keep ideas concrete, and to concentrate on the matter at hand, we will discuss the control of externalities in the context of aircraft noise. We have in mind an airport surrounded by residential dwellings. Households can select the level of noise they suffer, but at a cost. For simplicity we may think of any changes in the noise level at the centre as giving an equal change in the noise level at any point[3]. How can we obtain the optimal level of noise pollution?

(a) A Pseudo-Market

The public bad exists because, for legal and/or technical reasons there is no market in noise pollution. Given the possiblity of the measurement of noise, the Government may set up a framework within which the rights to pollute may be bought and sold between the airport authority and the residents in the neighbourhood. If the environmental rights are accorded to the residents then they could sell them to the airport authority either individually or as a group, with a marginal increase in noise being sanctioned if all households receive their marginal costs of evading the extra noise. The airport authority[4] would be required by the Government to act as a marginal cost pricing industry - i.e. not to exploit its monopoly power as a buyer of pollution rights - in deciding how much to buy of them. It would then choose to buy that quantity where the marginal gain from creating more noise just equalled the price. The equilibrium level of production of the externality is given where the sum of the marginal costs of residential noise evasion equal the marginal gains to the airport authority of noise creation.

If the environmental rights were given to the airport authority, the households would each pay a charge equal to the marginal noise evasion cost to the airport, and the equilibrium would be defined by the equating of the marginal noise evasion cost to the sum of the marginal benefits to all households of a reduction in the noise level. In this case the measure of noise evasion has to be <u>relative</u> to some level of noise. One naturally thinks of this as being that level which would prevail in the absence of any controls on noise, although other 'origins' of measurement may be used. When the marginal conditions determine a unique equilibrium the outcome is the same either way, whoever has the rights, the only difference being that in the former case the households are better off than in the latter case, with the opposite being true for the airport authority. The usual caveat to the above statement is that neither of the parties 'shuts down' operations, as the rights are transferred from one group to another. Households cannot 'shut down' in any real sense. They may vary the level of quiet that they choose, according to whether they have the rights or not, and hence the 'marginal benefits of noise evasion schedule' may differ from the 'marginal costs of noise evasion schedule'. Such a difference will be closely related to the difference between the equivalent and compensating costs discussed in the previous chapter, with the marginal benefits being measured by the willingness to pay for extra units of quiet and the marginal costs being measured by the compensation required for taking away an extra unit of quiet.

Given that the income effects of the onwership of pollution rights are small, such cost differences will also be small, and consequently the levels of quiet chosen under the two alternative schemes will not be far apart. The airport authority, however, can shut down if the noise charges are too high. In a recent and as yet unpublished paper Starrett and Zeckhauser (1971) have shown that such a shut down is only possible when the production possibilities for the airport authority are non-convex. However, such non-convexity is quite plausible and is illustrated in diagram 1 below.

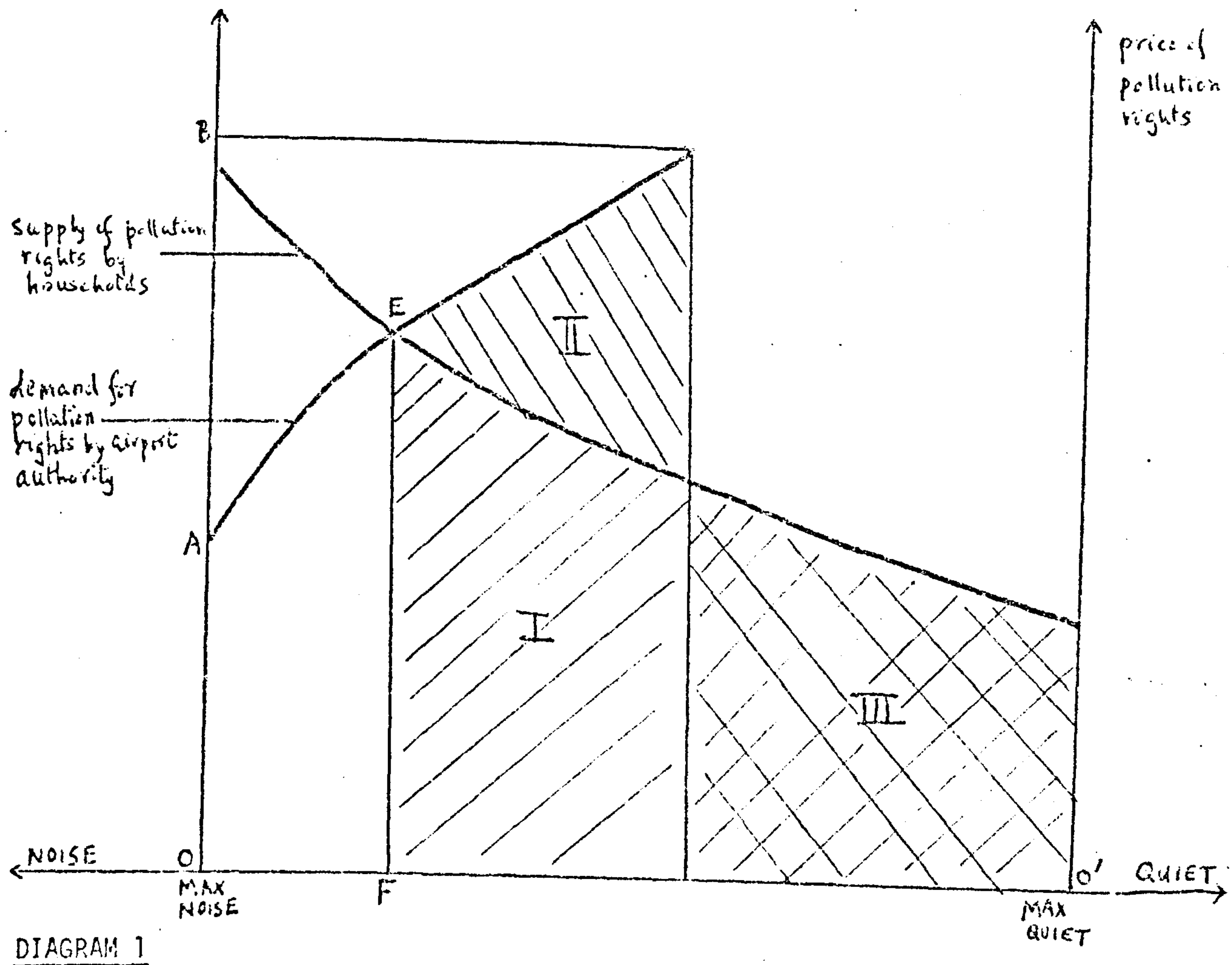


DIAGRAM 1

The horizontal axis measures quiet from left to right and noise from right to left. The airport authorities demand schedule shows that they will purchase all pollution rights until the price rises above OA and will then purchase progressively fewer rights until the price reaches OB. At that point it is no longer possible for them to operate an airport in their regulated capacity, and so they shut down operations[5]. Hence there are two equilibria, one at E, and one at $0'$. Which one is superior depends on the relative sizes of the areas (I + II) (the benefits of the airport staying open), and areas (I +III) (the benefit of the airport closing down). The kind of schedules indicated below are by no means implausible. What their presence points to is that while a pseudo-market determined charge for pollution rights is possible in principle, such a charge, and the equilibrium associated with it, has to be compared with the other possibility of shutting down the airport. Conversely the decision to locate an airport cannot be solved by discovering whether an equilibrium tax rate exists. Such a solution may be dominated by one where there is no airport.

The kind of comparison of equilibria that is suggested here involves the measurement of surpluses and consequently would be conducted within the framework of a benefit cost analysis.

Apart from this consideration, there are several problems with the implementation of an pseudo-market approach to controlling noise. The airport authority is a single buyer or seller of pollution rights and consequently government regulation would be required to avoid the use of monopoly power, where the authority is a private concern. Where the authority is a public concern the demand for pollution rights or the supply of an improved environment will be influenced by any financial control on the authorities operations and any pricing controls on the authorities other activities. The households on the other hand are in a classical public good type situation. When they have the pollution rights they will be inclined to overstate their marginal noise evasion costs and when the authority has the rights the reverse will be the case[6]. The difficulties raised by such issues are very serious. We can, under certain conditions, get information regarding household costs schedules from their market behaviour and do not have to resort to questionnaires to obtain subjective information. The circumstances under which this can be done are analysed in the next section. Where such information is gathered by a third party, however, the principle of a market between two groups of agents no longer exists and the information efficiencies that are one of the important advantages of the market type approach disappear.

(b) Taxes

With a government agency being able to assess the noise costs of households with reference to market data and being frequently involved in the operations of the airport authority, it would seem easier for a taxation scheme to be used to correct the externality. In such a scheme the government obtains information regarding the supply price of pollution rights at the given level of noise and places a tax on the airport authority. This leads to a reduction in the noise level and to a recalculation of the supply price, the procedure resulting in an equilibrium position if it is a stable procedure. Given that the demand and supply schedules for pollution rights are as shown in diagram 1 (with a downward sloping demand curve for pollution rights and an unward sloping supply curve for them), then we will end up by such a procedure at either equilibrium E or 0^1. The difficulty is that the one that we end up at need not be the optimal one. However, given that the system is stable around equilibrium E, and we have independently decided which of the equilibria E or 0^1 we prefer, a taxation procedure would overcome the difficulties associated with direct public involvement in the pseudo-market approach. At the equilibrium the government imposes a tax of OD per unit of quiet that the airport authorities take away, leading them to reduce quiet by $0^{/}F$ units. Alternatively the government could pay the authority an amount equal to OD per unit of noise that it reduced below the maximum. The authority would then be willing to reduce noise by OF units again leading to the optimum position. The tax revenue raised, or the bribe spent would be assumed to come from the overall government budget and the usual rules relating to government taxation (in the case of a bribe) or expenditure (in the case of a noise charge) would apply[7].
Such a scheme does not involve either raising money from households or paying money out to households, although there is no reason why this should not be done, if it were thought desirable on political grounds. In the light of the political lobbying that results whenever decisions regarding aircraft noise are concerned, it is important to consider the possibility that households be compensated for noise nuisance, and the role that such compensation might play in enabling an optimum level of pollution to be achieved. Such difficulties as may arise in providing a compensation scheme are discussed when we consider below the direct controls on aircraft noise.

At this stage we may note that such compensation is not an integral part of a tax scheme for attaining the optimum level of noise control.
In practical terms a taxation scheme has a number of advantages over a market scheme as far as the noise problem is concerned. Administratively it would be much easier to operate. The problem of stability, while theoretically relevant is unlikely to crop up for the demand schedule for pollution rights is probably inelastic enough to ensure stability.
The main difficulties that a tax scheme would encounter would be with regard to the airport authorities' ability to calculate its demand curve for pollution rights and for the government to calculate the marginal noise evasion costs to households. Regarding the former as we mentioned earlier that an airport authority is being considered as a government regulated industry applying the appropriate welfare pricing policy to its services and calculating its demands for factors and pollution rights on that basis.
At present we are still a long way from applying fully the principles of regulated industries to the business of airports. In fact many aspects of airport pricing policy show so little regard for considerations of cost and allocative efficiency[9] that it sometimes seems heroic to assume that the kind of calculations underlying the demand schedule for pollution rights indicated in diagram 1 will ever be undertaken, and the regulatory procedures applied to the airport authorities so that they act as price takers and cost minimisers and pursue the desired pricing policies[10]. This does not, however, invalidate the consideration of charge schemes at a theoretical level, for such consideration must inevitably precede, and indeed has often preceded the implementation of new pricing policies in other spheres[11]. In the meantime, however, it is necessary to recognise that such schemes are a long way off and that something has to be done to control noise levels.
Regarding the latter, there are also substantial difficulties in obtaining the relevant information - difficulties that may be insurmountable. We consider these more fully in the next section.

(c) The use of direct controls

Over the last decade and a half a number of controls over aircraft noise have emerged. These constitute :-

(i) Controls on the location of airports
(ii) Controls on the noise levels of aircraft
(iii) Controls on flying hours
(iv) Controls on flight paths
(v) Zoning

As far as item (i) is concerned, we have observed above that the controls on airport location may be necessary even when a market or tax scheme is in operation: Decisions regarding airport location have been formalised so that the economic costs and benefits can be enumerated within one framework, and the costs of noise within this framework have been examined in chapter 3. As far as controls on items (ii) to (v) are concerned, we must recognise that, while they are in part ad hoc, and the levels of control not fully justifiable, it may be essential to retain some of them along with a charges scheme, albeit in a form modified from the present one. This is because certain aspects of noise nuisance cannot be properly measured or may be too costly to measure and hence cannot be part of a tax adjustment procedure. Nevertheless one can appeal to economic principles to assess these controls in a qualitative fashion. We attempt to do this in section three of this chapter, recognising that while these controls were conceived as measures arising from a growing but unsystematic concern for the nuisance of noisy aircraft, they may still have a role in an optimal control of such noise.

(d) Internalising the externality

One solution that has been posed to deal with the control of aircraft noise has been the idea of the 'expanded firm'. In an article investigating the legal aspects of noise, Baxter and Altree (1972) consider the possiblity that the airport authority should own all the land in the vicinity of the airport that is affected by noise. Given the object of maximising profits subject to given prices, such an authority would have to trade-off the profitability of further flying activities against the reduced rental income from the residential dwellings that are now inflicted by further noise. Such a procedure could, in principle, emerge at the equilibrium E. (Again the shut down or set-up decision cannot be solved by this procedure). However, as Baxter and Altree recognise, and as Walters has pointed out, the size of such an expanded firm would be gigantic, incorporating in the case of

London's Heathrow airport for example, the ownership of about 80,000 households[8] by the airport authority! The efficient management of regulated industries of the size envisaged raises many problems, more perhaps than such an expanded firm solves. When the airport is not located close to large populations, then the possibility of an expanded firm may be more realistic. In such circumstances the purchase of the noise affected land may be considered. The appropriate price for such a transfer would be the price that prevailed before the airport was considered, thus excluding all externalities associated with the airport.

(e) Compensation

In the discussion of noise nuisance the idea has been considered that parties affected by a sudden increase in aircraft noise should be compensated for such an increase. In fact there is a legal history of such compensation being claimed in the U.S. There are recorded cases of the claims being denied and being upheld depending on individual circumstances, and a good summary of this development is contained in an article by Baxter and Altree. The current situation seems to be that increased aircraft noise nuisance is compensable, providing that some depreciation to the property affected can be shown to have occurred on account of the noise nuisance. The very presence of aircraft noise as measured by one of the recognised noise measures, does not however, constitute evidence of such depreciation, and indeed there does appear to be scepticism of the validity of these measures. The compensation where it is provided, is related to the extent of the depreciation established.

Such compensation as we have discussed above does not correspond to an economic concept of compensation. In chapter 3, where we considered the costs of noise nuisance in some detail, we distinguished the compensation costs for movers and the compensation costs for stayers, and that there is perhaps a useful distinction to be drawn between the long term movers' costs and the short term movers' costs. For movers such costs consist of the depreciation in property values, relative to the noise level in their new location, movement costs and some element of lost householder surplus. If we wish to consider the refinement of distinguishing between short and long term movers, then movement costs are to be ignored in the long term, and the surplus element considerably diminished. For stayers the compensation costs are related to the households preference for quiet. Such a model of compensation costs, is, however, of little use in practice. This is because compensation is paid to each household and, while a model can calculate total noise nuisance costs on the basis of the model outlined above, by deriving some knowledge of the distribution of preferences from market data and econometric estimation, such a model cannot then assign individual costs to individual households.

Thus the construction of a suitable scheme for compensation based on market data is not possible. This brings us to the question of developing such a compensation scheme by obtaining individual household information, and as we have pointed out this line of approach is fraught with the difficulties associated with asking people questions when they know they stand to benefit according to the answer they give. It would appear then that a satisfactory compensation scheme is not feasible. Furthermore it is not sufficient in our view to say that a scheme based on the payment of the estimated depreciation to all residents is an adequate approximation to an 'ideal scheme'. The notion of economic compensation is a sum of money required to restore the individual to his original level of satisfaction and the difference between such a sum and the estimated depreciation could be arbitrarily large.

This does not mean, however, that some form of compensation cannot be justified on political grounds, and a rule of thumb based on depreciation, would, if acceptable, be as good as any. The argument one might suggest here is that there is a strong opposition to decisions regarding airport noise in general and airport location in particular, because a large identifiable group of people are adversely affected. Making the optimal economic decision may require for example maximising the discounted sum of benefits less costs. This, however, cuts little ice with the action groups formed to preserve their share of the national distribution of welfare. If some payments were made to this group of people, in such a way that some gained a little relative to the no noise situation and some lost, then the solidarity would be fragmented. We may then consider the payment of compensation as an instrument for permitting decisions based on a benefit cost principle to operate. Alternatively we may regard compensation as the best available means of redressing any undesirable changes in the distribution of welfare caused by the imposition of noise in a particular area. Although, as we point out, compensation would not provide an accurate guide to correcting such changes, it would work in the right direction and might be the best guideline available.

4.2. Measures Relating to the Use of Charges in Controlling Aircraft Noise

In defining the optimum charges in the previous section we relied on a marginal argument and a partial equilibrium approach. This is a useful tool of analysis, based essentially on a consideration of consumer and producer surpluses, but it masks a number of difficulties in defining the equilibrium and discovering the conditions under which a multiplicity of equilibria can be ruled out. As we have already seen in a rather restricted context, if there is more than one equilibrium a charge scheme may choose the wrong one from an optimality viewpoint. In the context that we discussed, we could rule out either one of the equilibria by a benefit-cost type of analysis. However, it may be that the demand schedule for pollution rights and the supply schedule are not 'nicely' sloped as shown but cross over at a number of points. In that case a charge system may end up at either a local or a global maximum or minimum, and knowing that is not very helpful. The questions we are concerned with asking in this section are, what assumptions have to be made about the technology and about the factors determining household utilities to ensure that a unique interior optimum exists, and are these assumptions reasonable ones to make? These questions are better considered within the context of a simple general equilibrium model.

4.2.1. The Scarcity of Quiet

A supply curve of pollution rights by households will only exist if there is a scarcity of the supply of quiet. But what is the cause of this scarcity? After all, there is plenty of land which is quiet and where one can live without suffering any aircraft noise. Indeed if one could combine quiet, and all the other goods that one wished to consume in any manner that pleased one, then there would be a negligible demand for noise affected land and hence quiet could not be a scarce commodity[8]. The scarcity of quiet arises because the choice of a certain level of quiet generally entails some sacrifice - the convenience of being near to ones place of work for example, or near ones friends. When one considers the purchase of quiet immediately after the arrival of some aircraft noise, such factors will weigh more heavily than if one has to make these decisions over a long period, where

the set of choices is more flexible. In the assessment of the costs of noise to households we captured such considerations by using the concept of householders' surplus. Here, however, we argue that the existence of some such surplus is essential to the existence of a scarcity of quiet. Perhaps the best way to capture this aspect of quiet is to use the concept of locational inconvenience. This is the minimum inconvenience, suitably measured, that is involved in consuming any given level of quiet. There may be several dimensions to this and from the analytical point of view there is no reason why there should not be. However, to keep the exposition simple, it will be assumed that such inconvenience is uni-dimensional. (It is easy to think of many aspects to such inconvenience. However many inconveniences of consuming a different level of quiet are purely short term and disappear once the move is made. In discussing a concept of inconveniences that is relevant for a scheme for noise charges the relevant inconveniences, however, are only the long term ones. One example of these is the distance involved in travelling to work.)
We will not assume that locational inconvenience either increases or decreases with the amount of quiet chosen. There is no a priori reason to assume that either is the case, and even when locational inconvenience is measured in terms of the distance from one's work place to one's residence, the above relationship will not be monotonic unless the noise centre and the work centre are coincident for all individuals - an unlikely state of affairs in practice. An important assumption regarding the relationship between locational inconvenience and the level of quiet chosen that is made, however, is that locational inconvenience is a convex function of the level of quiet. This assumption is necessary to ensure a unique equilibrium for a given distribution of income. It implies that as you move away from some 'desired' location which exists at some level of quiet, your chosen location involves you in progressively more inconvenience as you choose increasingly higher or lower levels of quiet. An illustration is given in diagram 2 below. This household's desired location is at a level of quiet $\bar{q}$. If it wishes to consume any other level of quiet, it is forced to suffer some locational inconvenience, and this inconvenience increases marginally as the level of quiet differs from q.
On a priori grounds one can argue for and against this assumption.

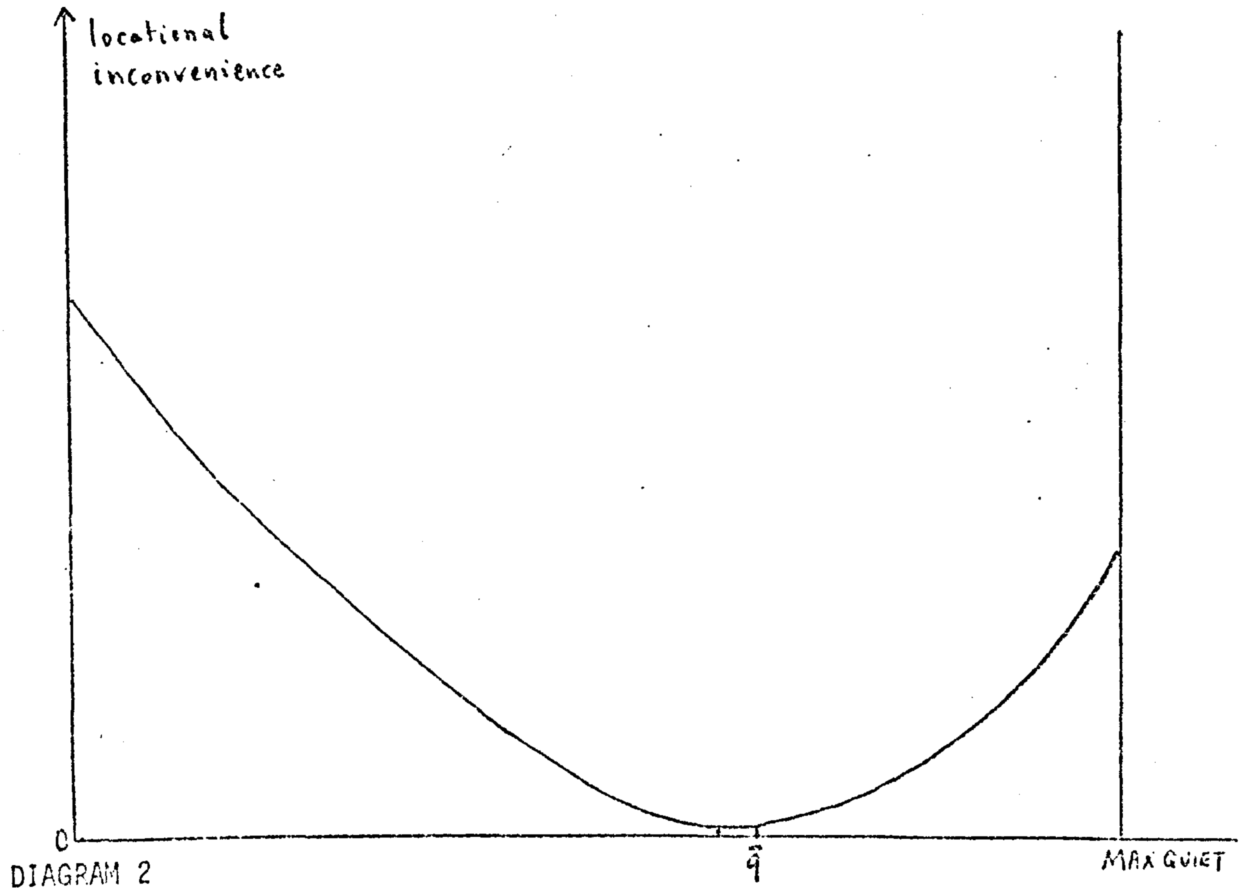


DIAGRAM 2

The noise level falls at a declining rate as the distance from the noise source increases. Given an ideal location at some position relative to the noise source, the distance that one would have to move from this location to acquire a lower level of noise will increase at an increasing rate, whereas the distance moved to acquire higher levels of noise will decrease at an increasing rate. Thus if locational inconvenience were defined only with respect to the distance from some ideal location, it would be convex for lower levels of noise and concave for higher levels of noise than that at the ideal position. However all choices regarding housing and social amenities at higher levels of noise tend to get progressively more limited, and the inconveniences represented by these factors could counterbalance the concavity obtained by distance considerations. Conversely, however, the expanded choice at lower levels of noise should ease the locational inconvenience and weaken the convexity argument there. Hence it is not clear whether convexity is a justified assumption and some empirical evidence on this question would be welcome. If, however, this assumption seriously breaks down then there may well be multiple equilibria to any charge scheme such as that discussed above.

In diagram 2 we implicitly assume that the level of noise at the source and its distribution is constant. This implies that the noise contours are fixed, which in turn implies that the numbers of each type of aircraft taking off and landing and their timing and flight paths are fixed. In this analysis we shall assume that the timing of aircraft movements and their flight paths are fixed in advance. Later we shall discuss why the former are probably best determined by direct control, while the latter are chosen so as to minimise the noise costs, given the flight routes. This leaves us with the effect of changes in the noise at the source on the relationship between quiet consumed and locational inconvenience. If any increase in noise at the centre implies an equal increase in the noise at any point, then it is clear that locational inconvenience may be defined as a function of the difference between the level of noise at the source at the level of noise at any point. If we consider the technical factors regarding the measurement of noise it appears that such an assumption is justifiable, as long as any adjustments to reduce the level of noise by limiting flights are such as to limit movements on all paths approximately equally, and as long as changes in the noise levels of aircraft are obtained primarily by measures such as retrofitting the engines and not by developments in the rate of climb or descent of aircraft . In our analysis we shall make these simplifying assumptions. A more complex approach to the problem is possible but it results in expressions which can only be interpreted with a great deal more technical information.

Having considered the nature of the commodity quiet in some detail, we may proceed to construct a simple general equilibrium model incorporating these features, along with some overall production constraints.

4.2.2. A Formal Model to Derive the Optimum Conditions

A.1. The community has a number of households, H, and an overall social welfare function

$$W = W(U^1, U^2, \dots U^h, \dots U^H) \qquad (1)$$

This function is defined over the whole utility space, is twice differentiable, strictly quasi-concave, and of the Pareto family (i.e. monotonically increasing with respect to all its arguments).

A.2. The utility function of any one household, h, is given by,

$$U^h = U^h(c^h, l^h, n^h) \tag{2}$$

This function is defined over the household's consumption set, is twice differentiable, and strictly quasi-concave in its arguments. The variables are defined as follows,

c^h: This stands for the quantity of commodity c that is consumed by household h. It represents those goods not dealt with directly in the model. $c^h \geq o, U^h_c \geq o$

l^h: This is the long run locational inconvenience suffered by household h. In accordance with the discussion in previous section we may write this as,

$$l^h = l^h(Z-n^h) \qquad l^h \geq o. U^h_l \leq o \tag{3}$$

This function is strictly convex.

We define the noise level at the noise centre as Z, and the noise level at the position occupied by the household h as n^h.

$$Z \geq o, \; n^h \geq o. \; U^h_n \leq o$$

A.3 Given the other factors of production, which are assumed to be fixed in supply and fully employed, the efficient production possibilities in the economy are defined by the values of Y,Z and L, where

$$T(Y,Z,L) = o \tag{4}$$

$$Y = \sum_h c^h \qquad L = \sum_h l^h \tag{5}$$

T(.) is assumed to be a strictly concave function. This assumption of concavity implies that the production set defined by $T(Y,Z,L) \geq o$ is convex. This limits the extent to which there may be increasing returns in any of these activities. The way in which Y and Z would be traded off, for given L, and the conditions under which the frontier in Y and Z would be concave were discussed in some detail in Chapter 2. Similar considerations would apply to the assumption of concavity with respect to L.

The optimum is defined as the maximum welfare attainable, subject to the production constraint and the non-negativity constraints. The problem is one in quasi-concave programming, in which, subject to some regularity conditions, we may define an interior optimum (i.e. one where $c^h > o$, $l^h > o$ for all h and $Z > o$), as a point where[12]:

$$\sum_h \frac{U_l^h}{U_c^h} \, l^h_{(Z-n)} = \frac{T_Z}{T_Y} + \frac{T_L}{T_Y} \sum_h l^h_{(Z-n)} \qquad (6)$$

$$-\frac{U_n^h}{U_c^h} + \frac{U_l^h}{U_c^n} \, l^h_{(Z-n)} = \frac{T_L}{T_Y} \, l^h_{(Z-n)} \qquad (7)$$

Condition (6) is a marginal condition which may be reasoned as follows: if we increase the level of noise at the centre by one unit, then all households need to buy another unit of quiet to keep them at the same net level of noise. This involves them in some locational inconvenience, the marginal opportunity cost of which, in terms of the consumption good, is the LHS of expression (6).
The increase in the level of noise, however, permits some further output of the consumption good (as less resources are spent on abatement). Part of this further output has to be sacrificed in order to be able to produce the extra goods and services required to make the locational adjustment. The condition requires that the net increase in the output of the consumption good be equal to the sum of the marginal opportunity costs to all households. Condition (7) states that if any household decides to enjoy an extra unit of quiet then the sacrifice of its consumption good that it is willing to make in order to do this should equal the marginal fall in the production of Y required to make the extra goods and services that the locational adjustment involves, available. From this it is clear that at the optimum, the marginal locational inconvenience cannot be negative; for if it were then a further reduction in the level of noise under which a household lives would be desirable - as it would provide a utility gain and no resource cost. Hence at the optimum, $l^h_{(Z-n)} \geq o$. Given our assumptions of the quasi-concavity of (1) and (2), the convexity of 3 and the concavity of (4), conditions (6) and (7)

are also sufficient for an optimum

4.2.3. On Attaining the Optimum - Prices, Charges and the Government's Role

In considering the attainment of the optimum we will assume that the consumption good is competitively produced and that the householders are utility maximisers and price takers. The production of the goods associated with locational adjustment (here one thinks mainly of transport services) and of the activities that produce the noise are subject to government regulation.

Given the set-up outlined above, it is fairly clear that in a society which allocates scarce resources by market forces, a premium will emerge for quiet areas that have a locational advantage for a lot of people. For while there is no scarcity of quiet as such, prime locations that are quiet will earn an economic rent by virtue of their position. The extent to which this will happen will depend on the relative distributions of locational inconvenience for various households and the distribution of quiet.[13]

We will assume that this premium takes the form of a price for quiet. As mentioned earlier, there is some empirical evidence for this. Given such a price for quiet, households will face the following maximisation:

$$\max \quad U^h(c^h, l^h(Z-n^h), n^h) \tag{8}$$

$$\text{s.t.} \quad c^h + P_L l^h + P_Q(Z-n^h) - m^h = 0 \tag{9}$$

Where P_L and P_Q are the consumer prices of goods L and Q respectively, expressed in units of the price of the consumption good, and m^h is the income level of household h. The necessary condition for a maximum is

$$-\frac{U^h_l}{U^h_c} \cdot l^h(Z-n) + \frac{U^h_n}{U^h_c} = -P_L l^h(Z-n) - P_Q \tag{10}$$

equating the marginal rate of substitution between noise and consumption (including the effect via l) to the marginal cost to the consumer.

If the regulated agencies that are involved in the production of L and Z are obliged to follow a marginal cost pricing policy, then we can see from condition (6) that the charge on the production of noise, Z, at the source should be P_Z, where,

$$P_Z = -P_L^* \sum_h l^h_{(Z-n)} \quad + \sum_h \frac{U^h_l}{U^h_c} \cdot l^h_{(Z-n)} \tag{11}$$

P_L^* is the marginal cost price for the producer which, as we shall see will differ from the consumer price. We may substitute for the second expression on the right hand side of (11) by summing (10) over h and rearranging to give,

$$P_Z = (P_L - P_L^*) \sum_h l^h_{(Z-n)} \quad + \quad \sum_h \left(P_Q + \frac{U^h_n}{U^h_c}\right) \tag{12}$$

(12) equates the optimum charge on Z to the sum of the marginal locational inconveniences, valued at the difference between the consumer price of L, (P_L) and the marginal opportunity cost of L in production (P_L^*) plus the sum of the differences between the price of quiet and the marginal rate of substitution between quiet and the consumption good[14].
We cannot evaluate such a charge on noise in terms of market data alone. We require to know the marginal long run locational inconveiences of choosing a different noise level. While it may not be possible to capture these fully, some of the main components may be obtained by considering, for example, the total amount of extra travel involved to the household in living under a marginally different noise level. We also need to know, however, the marginal rate of substitution between quiet and consumption and such information can only be obtained by subjective questionnaires, which have, as we already indicated, a large number of difficulties associated with them.
There is a possibility, however, that the noise charge may be expressed independently of this marginal rate of substitution. The reason for this is that locational inconvenience, certainly as measured by the extra travel incurred over that at some ideal location, may well be dependent on a number of other factors, such as the area required, the levels of other forms of environmental pollution and so on. If some of these factors are not directly priced and the household chooses the level of n to maximise utility given the overall budget constraint, then it will equate the marginal rate of substitution between n and c to the price of quiet. In that case it follows from (11) that the charge P_Z is given by

$$P_Z = (P_L - P_L^*) \sum_h l^h_{(Z-n)} \tag{13}$$

relating P_Z to the sum of the marginal locational costs valued at the difference between the producer and consumer prices. It is the relationship between these, therefore, that we have to consider.

When the producer price for L is a marginal cost price, then it follows from condition (7) that the marginal resource cost to household h of choosing a marginally different level of quiet is, $P_L^* l_Z^h$. However, the actual marginal cost to household h is, by equation (10), $P_L l_Z^h + P_Q$ From this we can see that these two marginal costs cannot generally be the same for <u>all</u> households. Since, in general the marginal inconveniences will differ for different households and since there is only one value for P_L [15], this value of P_L will have to be chosen as a compromise over various households. We consider one criteria for choosing such a price below.

One case, however, where no compromise is required in choosing P_L is where all households have equal marginal inconveniences of adjusting their level of quiet. In that case, P_L is given by,

$$P_L = P_L^* - \frac{P_Q}{l_Z^h} = P_L^* - \frac{P_Q . H}{\sum_h l^h_{(Z-n)}} \tag{14}$$

where $H / l^h_{(Z-n)}$ is the reciprocal of the average of the marginal inconveniences (being equal to marginal in this case for all households). From (14) it follows straightforwardly that the RHS of (13) may be rewritten as, $-H.P_Q$. Hence we have the simple result,

$$P_Z = -HP_Q \tag{15}$$

which states the noise charge merely as the sum of the marginal noise evasion costs where such costs are measured only by the price of quiet. From (14) it may be reasonable to use a noise charge given by (15) when marginal locational inconveniences have a very small variance around a given mean.

If the distribution of these inconveniences cannot be satisfactorily approximated by a mean then we have to choose P_L by some suitable criterion. One such criterion which has some appeal is to minimise the sum of the deadweight losses of surplus for all households. Consider diagram 3. We measure the level of quiet from left to right, and we plot the marginal benefit of noise reduction which in the neighbourhood of the equilibrium we know will be sloped as shown, as well as the

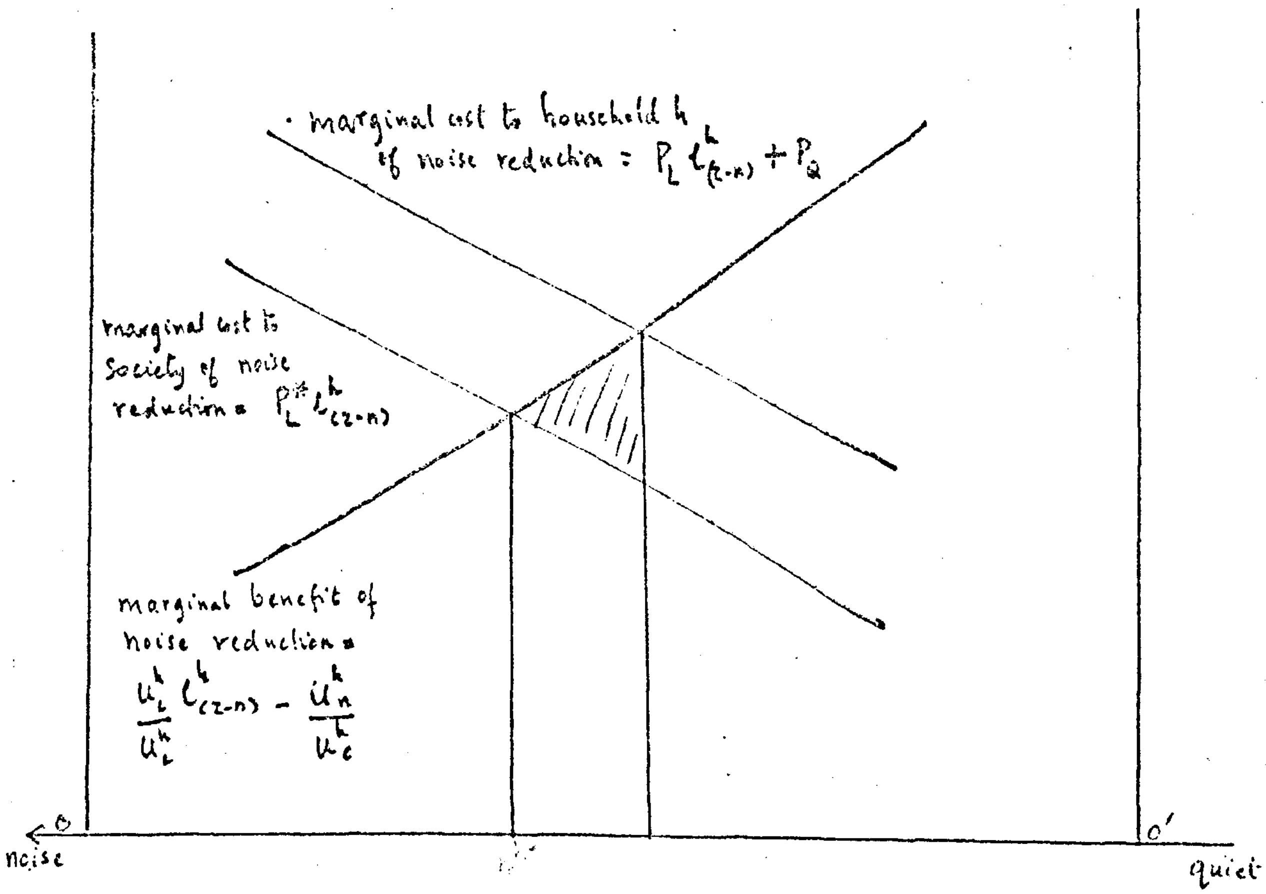


DIAGRAM 3

private and social marginal costs of noise reduction, which in the neighbourhood of the equilibrium will be upward sloping. For the household shown the deadweight loss of the divergence between the two costs is the shaded area. Given the choice of P_L a number of such areas will appear for different households. If we minimise the sum of such areas we are choosing a surplus- loss minimising criterion, which was made famous in another context by Hotelling (1938) and has been used recently in the welfare pricing literature.

For small areas such as the one shaded, we may approximate it by the area of a triangle. Simple manipulations yield the deadweight loss for household h as d^h, where

$$d^h = \frac{1}{2} \quad \frac{\{(P_L - P_L^*)1^h_{(Z-n)} + P_Q)^2\} \, \iota . n^{h*}}{\{\frac{\iota}{\nu} P_L^* \, 1^h_{(Z-n)} + P_Q + P_L \, 1^h_{(Z-n)}\}} \tag{15}$$

Where ι is the price elasticity of demand for quiet, (measured positively), ν the elasticity of the marginal inconvenience with respect to quiet, and n^{h*} the chosen level of noise suffered by the household. While the expression looks somewhat complex, there is nothing in it that is not in principle 'knowable'. P_L is what we wish to choose given P_L^*. To do this we minimise $\sum_h d^h$, with respect to P_L, recalling that n^{h*} will be a function of P_L. The expression derived to define the minimum is somewhat cumbersome but involves the expressions already listed, as well as the price elasticity of demand for quiet with respect to the price of L. Again, given the information for suitably defined groups of households, it should be possible, by numerical methods to calculate the optimal consumer price for L.

In concluding this section we note that once we start to examine the marginal analysis of diagram 1 more closely we discover a structure that is not easy to analyse and that does not readily make available a method of calculating the supply price of pollution rights. If the convexity assumptions of economic analysis are to carry over to this problem, then we have to assume that $1^h(.)$ is convex with respect to its argument. We do not know if this is a valid assumption. Once we devise the optimum conditions these can be easily interpreted. However, to attain them by a system of prices and charges would most probably require the validity of the assumption that allows us to express the charge in terms of equation (13) rather than equation (12). When this assumption is valid, a charge system may be feasible - if the required information regarding marginal inconvenience is available. Furthermore, from such information we will be able to see how widely distributed the marginal values are over households, and whether the simple formula of (15) is acceptable or not. If it is not acceptable, a more complex approach, along the lines suggested will be required. Overall this discussion places a considerable emphasis on direct controls until further empirical work establishes the possibility or otherwise of noise charges.

4.3 Measures Relating to the Direct Control of Aircraft Noise

In section one we listed the items by which direct control of aircraft noise nuisance was achieved at an operating airport, as controls on the noise levels of aircraft, on the flight paths of aircraft, on flying hours, and on the zoning of land for non residential use.

(a) Aircraft Noise Levels

The systematic control on aircraft noise levels started with a system of noise certification for new types of sub-sonic jet aircraft. This was agreed to by the International Civil Aviation Organisation in 1969, and, according to the Noise Advisory Council, will ensure that new aircraft will be half as 'noisy', weight for weight, as current types. In choosing these levels of cetification no analysis was done of the costs of implementing these proposals, relative to the benefits of noise reduction. Furthermore these noise levels apply to new aircraft, and not to existing aircraft, although the matter of adapting existing engines to reduce their noise levels is being given some consideration. Given any permissible noise level for existing aircraft, however, economic factors regarding the relative cost of quietening existing aircraft and buying quieter new ones will determine the chosen rate of obsolescence of the existing aircraft fleet by the airline operators. Even ignoring the external benefits of quieter aircraft, it is difficult to see how the levels of controls on new and old aircraft can be decided upon without some idea of overall desired noise levels in the future and some knowledge of the sensitivity of the rate of obsolescence to the relative costs mentioned. So far as one can gather such factors have not been systematically considered.

(b) Aircraft Flight Paths

The matter of the choice of flight paths has received more attention with regard to the economics of the question. The issue here is, what paths should departing aircraft take to their respective air corridors, and what approaches should arriving aircraft use? In answering this question there are a number of costs to consider. The noise costs imposed on households, the airline operating costs, and the costs to the control authorities. All these will vary with the flight paths that are chosen. The chosen strategy should be such that, for a given scale of operations, the overall costs of all groups are minimised subject to whatever technical constraints exist. The component of these costs that represent the noise nuisance should value the falling away of a unit of quiet at the long run marginal noise nuisance costs of all the households, and, as we saw in the previous section, these will depend in general upon both the price of quiet to households <u>and</u> the long run locational inconveniences of adjusting to a marginally different noise level.

In the simple case where the value of a unit of quiet is just the price of quiet times the number of households affected (see equation 15) there is a case for concentrating aircraft flight paths over a few households rather than dispersing them. The argument for this is that most noise measures appear to have the cardinal property that households are willing to pay the same amount for each unit of quiet as measured by them. However, the marginal increase in the noise level at any point is a declining function of the number of aircraft along a particular flight path[16]. Thus if there were two flight paths over identical concentrations of population, it would be better, *ceteris paribus*, to concentrate all flights on one flight path, for this would minimise the noise nuisance costs given the aircraft and their operations. Indeed a similar argument may be applied to the choice of the number of airports - fewer airports over a uniform population density represent lower noise nuisance costs. However it is difficult to see how this conclusion can be carried over to the case where the marginal valuation of a unit of quiet cannot be adequately represented in terms of equation (15). In that event the long run marginal locational inconveniences will have an important role to play and these might have different implications for the pattern of flight paths.

(c) Flying Hours

Most airports regulate the arrival and departure of aircraft during the night hours. This seems an appropriate matter for regulation, the argument being that if a charge was imposed for night flying it would be so high as to eliminate all use of airport facilities during the night. In these circumstances it seems administratively more convenient to ban night flying altogether.

(d) Zoning

Some municipal authorities ban, or severely restrict the building of residential dwellings in the noisiest zones. There is considerable scope for zoning in optimal land use patterns, and indeed with long term considerations of the optimal location of industry it may be best to concentrate industrial activity near the airport. However, one argument commonly used to restrict residential building near the airport in an otherwise residential area is unpersuasive. This is that uninformed households will suffer as a consequence of buying houses in very noisy areas. If lack of information is a serious problem then of course the answer is better information and not zoning.

Whether zoning land near an airport for industrial purposes is desirable from the viewpoint of the optimal location of industry is a question that involves a whole host of issues that are beyond the scope of this study. It is clear, however, that any policy dealing with long run noise control, would be incomplete without a consideration of optimal land use patterns.

4.4. Urban Noise

The discussion in this chapter has been concerned with the control of aircraft noise. This is not, however, the only important source of noise nuisance. To many people traffic noise and neighbourhood noise are a more immediate source of annoyance than aircraft noise. A recent report of the Noise Advisory council[17], stated that at least a fifth of the urban population are exposed in their homes to 'undesirable' levels of traffic noise and that "traffic noise is by far the most widespread source of noise nuisance and the most urgent target for abatement action"[18]. This finding is in keeping with the surveys done in other countries. The O.E.C.D. (1971) report on urban noise lists surveys in many urban areas, even with large airports, finding that surface traffic noise is the most predominant and widespread source of noise nuisance.

While many of the analytical tools developed to deal with costing aircraft noise can be extended to traffic noise, and indeed some urban motorway studies have evaluated the environmental costs of such schemes,[19] the same is not true of the control of urban noise levels. In any real urban environment it is no longer possible to think of the noise as emanating from one source and spreading out in the vicinity of that source. Thus it is not appropriate to treat traffic noise as a Public Bad in the way that we did in section 2 above. In these circumstances it is natural to resort to a system of standards for the engine noise for vehicles, employed in conjunction with a suitable policing system. The levels at which these standards should be set, is not however, an easy question to answer. In chapter 2 we explored some of the broad conceptual problems that arose with the existence of environmental externalities such as noise, in a highly simplified urban setting. The general picture that emerged there, was that optimal controls are extremely difficult to implement even in a very simple setting, but given some information regarding tastes and technology, it should be possible to obtain rough orders of magnitude of the proportion of resources that should be devoted to such things as noise abatement, and to obtain some idea of how much control should be exercised on city size, in varying circumstances.

The policy thinking on the issue of urban noise has followed very much a quantity control approach. In its recommendations on urban noise the O.E.C.D. urged that governments should adopt more effective enforcement procedures for the maximum permissible noise emissions, they should control heavy night traffic in some residential zones, improve methods of traffic flow control to avoid disturbance from noisy acceleration, and encourage the use of noise screens and other artificial noise attenuating barriers[20]. They also recommended that the government should support detailed studies on the costs of noise abatement by these methods. Such studies have still to be carried out with regard to many aspects of urban noise control.

FOOTNOTES TO CHAPTER 4

1. Every economist has his pet definition but this broadly includes the main features-no explicit market and the influence of activities other than those under ones control on utilities and profits.

2. We make the point that to be influenced by a particular activity, one does not have to be observed to suffer it, for evasive action may be the form that the influence takes.

3. This approximation is effectively assuming that the noise function, which may be written in general as $N = N(Z, [x])$ (where Z is the noise level at the centre, $[x]$ is the coordinate position of a given location relative to the noise source and N is the noise level at this location) may be expressed in a seperable form as $N = A(Z) + B([x])$. This greatly simplifies the analysis, and as an approximation for a range of noise levels generated along given flight paths by conventional aircraft, it proves to be reasonably satisfactory.

4. We assume that the airport authority is the agency that is taxed or charged, rather than the individual airlines whose behaviour the noise controls will finally influence. This would probably be necessary for administrative convenience, as well as for the resolution of conflicts of interest that would arise between airlines when noise charges are imposed. On this latter point see, for example, Section 4.3.(b).

5. In practice of course this would not often happen. In order to keep the airport open the government would allow the authority to deviate from its regulating behaviour and subsidise it to keep it from passing on the noise charges to the airlines. However the diagram indicates the demand for pollution rights schedule that is relevant for a calculation of surpluses.

6. When households have the pollution rights any individual household will recognise that the demand for these rights will be negligibly affected by his supply price whereas if the authority has the rights we have the classic free-rider case. The effects of under and over-reporting on the equilibrium achieved is more fully discussed in Malinvaud (1971).

7. i.e. government taxes and spending does not influence marginal conditions elsewhere in the economy. The issues raised by second best considerations when there are commodity and profits taxes do not illuminate this type of partial analysis.

8. The creation of noise reduces the total amount of quiet land available but this effect is very small indeed. The main effect of noise is to reduce the amount of quiet land available in locations that have great desirability because of the convenience of living there.

9. For a discussion of some of the more obvious misallocative aspects of airport charges the reader is referred to Walters (1973)

10. The broader questions of airport pricing policies are being investigated by Lenhoff and some preliminary results are given in Lenhoff (1973). Clearly much more is involved than airline charges, with the airport providing extensive freight and passenger services, and the pricing policies of all these items have to be considered together.

11. The story of how welfare principles were applied to the public utilities, especially electricity, in France is fascinatingly told by Allais, Boiteux and Massé in Nelson (ed.) (1964).

12. We may safely consider an interior optimum here. It is inconceivable that at the optimum there will be <u>no</u> noise pollution or that any household will consume nothing, and it is very likely that all households will suffer <u>some</u> locational inconvenience.

13. For example we may obtain the price for quiet explicitly in terms of distributions, when all locational inconvenience is measured as the shortest distance to the centre of the town, and the proportion of a 'ring' of land at distance r from the centre that is affected by a noise level N is known. The mathematical derivation, however, is of little economic value.

14. $\frac{U^h_n}{U^h_c}$ is the marginal rate of substitution between noise and the consumption good and is therefore equal to minus the MRS between quiet and the consumption good.

15. There may of course be more than one value of P_L - an example would be when there are different commuter subsidies to different parts of the town. However, we may assume that there could not in general be as many consumer prices for L as there are different marginal inconveniences, hence the need for some principle to choose the values of P_L.

16. The noise level at any point is, broadly speaking, linearly related to the log of the number of aircraft going over that point. Hence the marginally declining rate of increase.

17. Noise Advisory Council 1972 (b)

18. Unfortunately the Noise Advisory Council nowhere state what they mean by undesirable. We presume that this figure is derived from the Wilson Reports which cites, that for exposure levels higher than 55dB(A) mean energy value the number of individuals considerably disturbed often exceeds 20%.

19. For a review of some of these studies the reader is referred to Urban Motorways Study (1973).

20. O.E.C.D. (1971)

Chapter 5

Empirical Evidence

5.1. INTRODUCTION

So far in this thesis, we have considered various aspects of the economics of noise, assuming that noise could be satisfactorily measured, that an implicit price for quiet could be ascertained, and that as far as the measurement of noise costs was concerned, reasonably adequate measurement could be made with regard to the household surplus and turnover rates. Being able to make these assumptions, and being able to take some rough orders of magnitude for these variables, made it possible to analyse the question of noise pollution in an economic framework, and to give some idea of the relative importance for some issues relative to others. Thus it is clear that empirical work in this field has been of paramount importance in development of noise economics. In this chapter we present a brief review and assessment of some of the current empirical evidence on the issues listed above. In section two we discuss the various measures of noise nuisance. Their 'validity' must be intrinsically tied up with the implicit market valuation of units of this measure, and we consider the evidence on the price of quiet that has so far been obtained by studies of house price depreciation in noisy areas around airports, in section three. Section four reviews some of the evidence collected regarding householder surplus and section five examines the evidence on turnover rates in noisy zones, which may be of some importance in evaluating noise costs using a Roskill type method. The evidence regarding the measurement of noise and the price of quiet has been well summarised by Walters in his recent but as yet unpublished monograph on the economics of noise. We add little to this, except some differences in interpretation, and some further discussion on turnover rates.

5.2. The Measurement of Noise

There are now several measures of aircraft noise and at least two measures of traffic noise that have been designed to capture the noise annoyance caused by these sources.[1] The construction of these measures has proceeded in much the same way, and the principles involved can be indicated by considering the Noise and Number Index that was developed by McKennell (1963) as part of a study of aircraft noise annoyance around London (Heathrow) Airport. From a social survey of households in noise affected areas around the airport, every household was given a noise annoyance rating on a noise annoyance scale, as outlined in chapter 3, section II,III. At the same time measures of the noise level around the household, as given by the average loudness of aircraft flying overhead, the median loudness, the duration of aircraft noise, the number of aircraft, and other indicators, were tabulated. The noise annoyance ratings were then regressed against combinations of the above variables, and the combination giving the best fit in terms of minimising the unexplained sums of squares, was then selected as a measure of the noise nuisance. For the noise and number index the best fit was obtained by:

$$NNI = PNdB + 15 \text{ Log } N - 80 \qquad (1)$$

NNI = noise and number index,
Where PNdB is the average peak loudness of aircraft, and N is the number of aircraft heard on a summer's day around Heathrow.
Regressing the noise annoyance rating against NNI gave an R^2 of 0.46 which is significant at the 10% level of confidence.
There are a number of difficulties with the noise and number index. The predetermined variables are strongly correlated and this leads to large standard errors on the coefficients when estimated by ordinary least squares. Furthermore there was hardly any difference in terms of R^2 between choosing N and Log N in equation (1), although the implications for policy of the two may be substantially different.[2] This suggests that the sensitivity of any results to minor changes in the estimation of the noise index is quite large, Finally, the NNI

does not differentiate between night and day flights, and so relates to the actual division between night and day flights at Heathrow, when the index was constructed. At that time approximately a quarter of the flights were night flights.[3] This means that if a different combination of night and day flights were to exist then the NNI would be invalid if night flights had a different noise annoyance effect from day flights - which is very likely to be the case. A more sophisticated and more recent measure of noise nuisance is the noise exposure forecast (NEF). This replaces the PNdB measure of the noise loudness of aircraft by a means called the effective perceived noise level, EPNdB, where the latter takes account of the duration of loudness, as well as a technical correction for the presence of pure tones in the noise. The total noise exposure at a given point is viewed as being composed of noise produced by different aircraft flight paths. For a specific class of aircraft, i, on flight path j, the NEF_{ij} is given by:

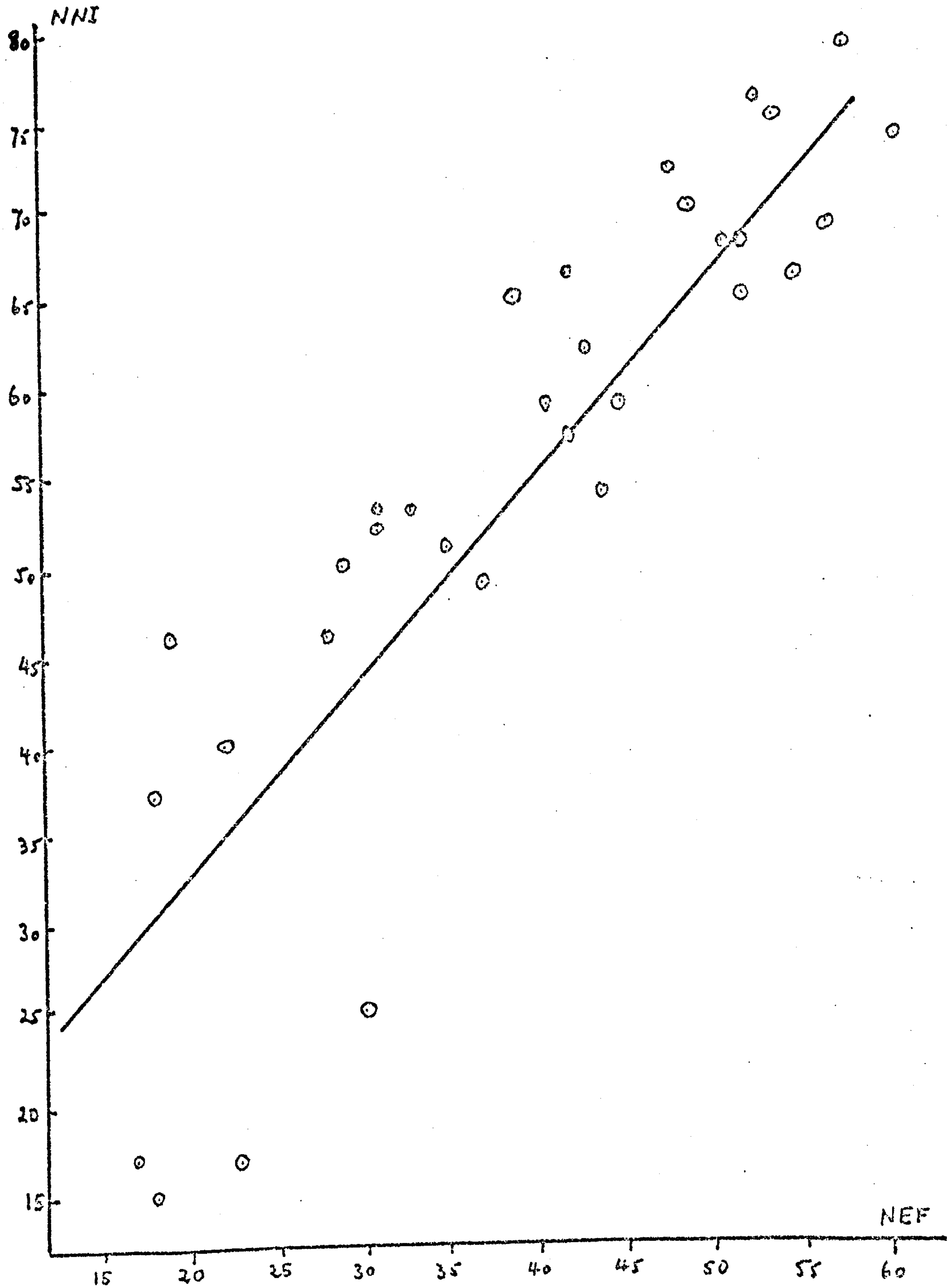
$$NEF_{(ij)} = EPNdB_{(ij)} + 10 \text{ Log} \left[\frac{N_{(ij)}(\text{Day})}{K_D} + \frac{N_{(ij)}(\text{Night})}{K_N} \right] - C \qquad (2)$$

Where $N_{(ij)}$(day) and $N_{(ij)}$(night) are the numbers of the aircraft of type i, on flight path j, during the day (07. 00 - 22.00 hours) and during the night (22.00 - 07.00 hours) respectively. The choice of K_D and K_N, both constants, is so made as to imply that a single night time flight contributes as much to NEF as approximately 17 day time flights. The constant C is chosen so that the NEF numbers lie in a range where they are not likely to be confused with other noise ratings. The total NEF at a given ground position is determined by the summation of all individual $NEF_{(ij)}$ values on an energy basis:

$$NEF = 10 \text{ Log} \sum_i \sum_j \text{antilog} \frac{NEF_{(ij)}}{10} \qquad (3)$$

This is a much more sophisticated measure than NNI. However its relation to annoyance ratings has not been fully investigated, although it has been succesfully used to analyse complaints about noise around some airports and to predict their patterns around others.[4]

Other noise indices constructed in much the same spirit but differing in detail are the Indice Isopsophique in France and the Störindex in Germany. Regarding traffic noise, both the Traffic Noise Index and the Mean energy level have been shown separately to correlate well with nuisance, and it is understood that the relative merits of these have now been tested in France in a separate Social Survey.[5] Various measures of noise, whether traffic or airport are evidently closely related. For a given type of noise, it is not possible to obtain a unique relationship between them. For the kind of variations in aircraft numbers and their breakdown between day and night, in duration and loudness of aircraft and other factors, approximate relationships can be derived. The Roskill Report showed this relationship between the NNI, the Isopsophique and/the Störindex. In graph 1 we present a similar relationship between the NNI and the NEF, as calculated by S. Abhrahams of the Civil Aviation Authority. The line fitted by the least squares suggests that approximately 1 NEF = 1.35 NNI. As they stand, all these measures of annoyance have no obvious and relevant cardinalisation. That is to say the measures do not have the property of two units representing twice as much 'annoyance' as one, or twice as much 'damage' to an individuals psyche as one. The coordination that an economist would naturally choose for measuring a commodity is that each unit should have the same market price. Therefore the line of investigation that one should pursue in assessing the suitability of these measures, is to see whether they, or any monotonic transform of them, has the same unit value in an implicit market where quiet is traded. We turn to this in the next section. Before we do so, however, it does seem worth considering a criticism of the use of noise and number index. Mrs. Paul (1971) states that the index was derived in a situation where there were few cases of moderate loudness and many flights or of few flights and extremely loud noise, and that it was unjustifiably used to predict noise levels and costs in circumstances where both were true. In other words, the index does not hold outside the narrow range within which it was constructed. The validity of such an index cannot depend, however, on the correlation coefficient quoted earlier between noise annoyance ratings and the noise index. It is quite conceivable that a smaller coefficient could


NNI
80
75
70
65
60
55
50
45
40
35
30
25
20
15
NEF
15
20
25
30
35
40
45
50
55
60

GRAPH II

result in a perfectly satisfactory index. The test of this must rest on whether a normalisation can be found of the given measure such that it produces a given unit price for quiet, under varying noise and number combinations .

5.3. The Price of Quiet

5.3.I. Techniques of Investigation in Statistics of the Price of Quiet

Empirical studies on the price for quiet are concerned with establishing a house price differential for similar houses in different noise zones. By similar, we mean houses with a given level of accommodation, access, and amenities. Matched samples are constructed, at different noise levels and the average prices compared in a cross-section study, which is the main form of investigation undertaken, although there are some studies that compare the average rates of appreciation in matched samples at different noise levels over time. The construction of matched samples is not an easy task, as the number of features that define a residential dwelling are many in number, and in some cases it is not possible to define them in comparable terms. This was found to be partly the case in the Roskill study, and led there to an inquiry of estate agents in the vicinity of London's two airports. This was designed to get their assessment of house price differentials due to noise, for subjectively matched houses in different noise zones. Although this work was conducted with some care, it is undoubtedly preferable to have a statistical study based on actual data for house prices. This involves less subjective judgement and permits a calculation of standard errors of house price differentials. In addition to the above problem of matching samples, a further difficulty arises in these investigations that attempts to calibrate actual house price differentials. This is that data on the prices at which houses are sold is not always obtainable. We find this problem especially in the U.K., and it implies that we have to rely on self-assessments of house values, which are not really satisfactory, or on valuations for tax purposes, which tend to underestimate house values. Any interpretation of the data has therefore, to bear in mind the data source used for house prices.

5.3.2. Main Results of Studies on House Price Differentials

About a dozen studies have now been done regarding the effects of aircraft noise on house prices. Most of these have been undertaken in America (U.S. and Canada) although some of the pioneering work was done in Britain as part of the Roskill Report, and a study was recently done in Australia as part of the Sydney Airport study scheme. Walters has recently attempted to bring together the conclusions of the British and the American Studies and present them in a comparable fashion. His basic conclusions are that aircraft noise, nuisance, as measured by any of the indices used in these countries, does have some effect on house prices, and as a rough guide one may say that a house valued at around £10,000 $ (U.S.) 25,000 in the late sixties would depreciate between 0.4 and 0.7 percent per unit increase in the NNI in the U.S. and between 1.0 and 1.4 percent per unit increased in the NNI in the U.K. These results are obtained by measurements in noise zones of between 40 and 55 NNI, and if we may linearly extrapolate the actual depreciation levels per NNI in this range to lower NNI levels, we find a zero differential at between 28 and 30 NNI. The Australian study conducted in the Sydney region suggests a value of about 1.0 per cent per NNI. . While each of the studies does not by itself provide overwhelming evidence on the quantitative effects of noise on house prices, together they certainly do offer a range within which the price of quiet probably lies. Given the approximate translations between the noise indices, these results also give some idea of the price of quiet as measured by these indices.

5.3.3. Comment on the Cardinality of the Noise Measure.

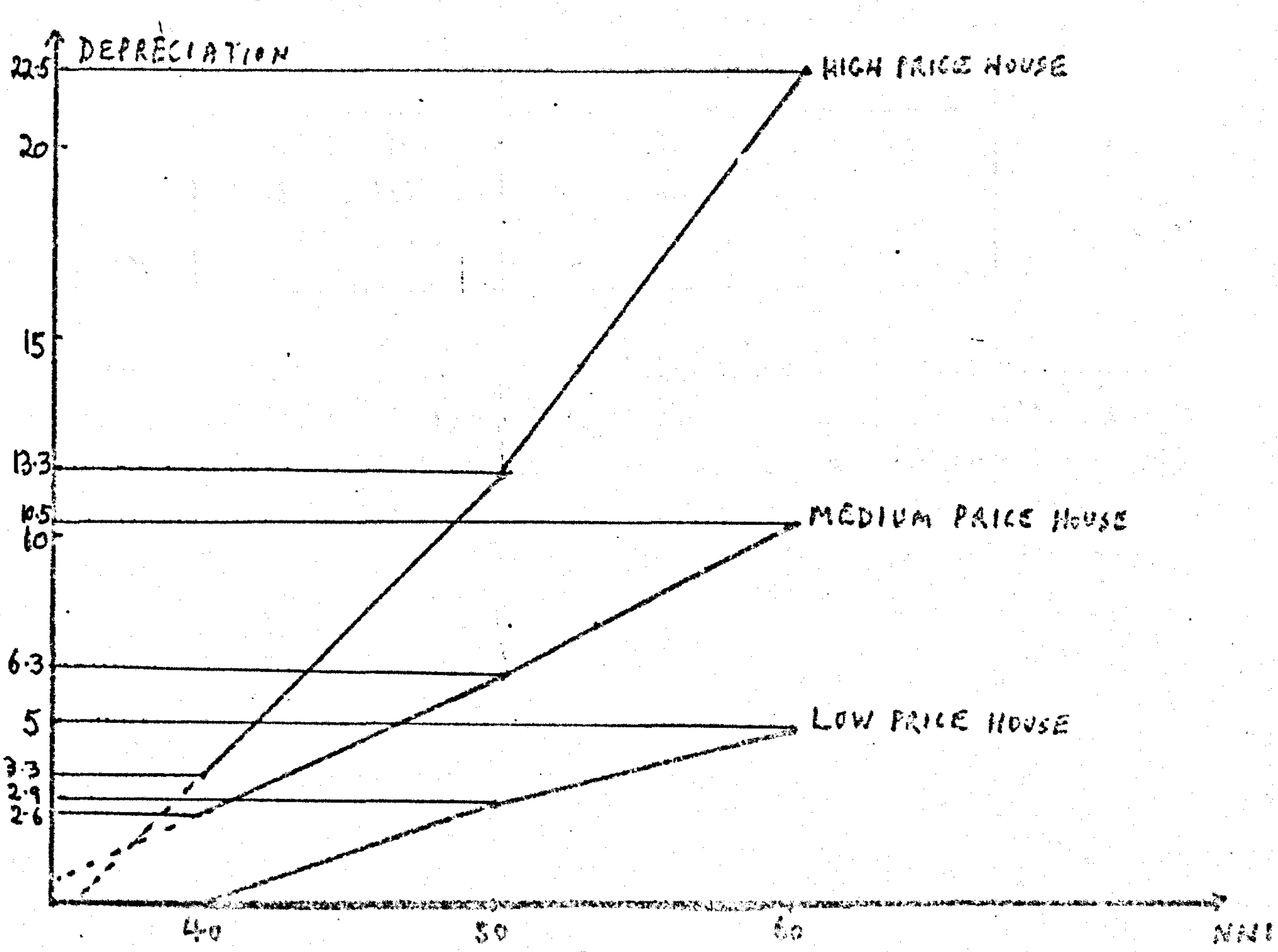
An issue of some importance to the measurement of noise and to the price of quiet is how the depreciation per unit change in the noise index varies with the level of noise. As we mentioned earlier, if this depreciation is constant over a wide range of noise levels then that measure of noise can be treated as an economic measure as it stands. If the depreciation is not constant over the experienced noise range, then it would be necessary to look for a transform of the noise measure that would produce a constant depreciation, and this transformed measure would then be an economic measure of noise.

The evidence on the constancy of this depreciation over the noise range is somewhat ambiguous. Only two studies have any real bearing on this question. They are the Roskill Study, and a study by Emerson(1969) on aircraft noise levels and depreciation in the Minneapolis area.

We briefly summarise the relevant sections of these investigations below:

The Roskill Study

The date collected around Heathrow airport during the Roskill Study provides us with average rates of depreciation over 10 NNI range zones for 3 house price groups. Graph two below plots the percentage depreciation compared with houses below 35 NNI, against the mid point of the NNI range.[6]



GRAPH III

Although the evidence is scant, it does suggest an approximately linear relationship between noise levels and depreciations, at least for the medium and low price houses, and this would appear to be reasonably consistent with the assumption of a constant depreciation rate per NNI for a given house price group. For different house price groups the price of quiet and the zero point of the noise scale are predicted to be different, however, and this would make a unique index of noise that applied to all income groups impossible.

Emerson

Emerson's study, a Ph.D. dissertation at the University of Minnesota, is extensively reported in Walters' monograph, where it is described as the most sophisticated and comprehensive study so far carried out in the United States. It analysed a cross-section of 222 sales of single family houses in 1967, using 26 independent variables including the level of aircraft noise as measured in steps of 5 CNR noise units (a unit change in CNR being approximately 1½ NNI units).

A summary of the noise depreciation results of this study were:-

CNR	Percentage depreciation due to noise
95 to 119	2.7
120 to 124	4.6
125 and over	9.6

This suggests that there is a substantially increasing rate of depreciation per unit of CNR as the level of CNR increases. However, as Walters points out, the method of estimation was constrained to a non-linear relationship between the depreciation rate and the level of noise. Thus we do not know how well a linear relationship would have fitted, had it been tried.

In conclusion on this question one must say that the issue is not at all clearly resolved. There is some evidence from the Roskill data that is not inconsistent with the linearity hypothesis, and some evidence from Emerson's study that is suggestive of a marginally increasing depreciation rate. However, we must await further evidence on this question. There are a number of difficulties in the comparative analysis that we have done so far which it would be desirable to tackle, and until more firm evidence is forthcoming an hypothesis of a constant depreciation cannot be said to have been disproved.

5.3.4. The Price and Income Elasticities of the Demand for Quiet

From the Roskill data we obtain a relationship between the house price and the percentage depreciation that the house suffers. If one may assume that the price of the house is proportional to the permanent income of the household, and there is some evidence for this, then the implied income elasticity of demand for quiet is about 2. This conclusion is derived from cross section data and to date there is no time series evidence on the demand for quiet. Furthermore, there is no real evidence on the price of elasticity of demand for quiet. Walters states that "the data are broadly consistent with a unit elasticity of demand - but converseley one cannot claim that such a hypothesis has been critically tested with such figures." [10]

5.4. Householder Surplus and Movement Costs

Householder Surplus

Householder surplus is intended to capture the locational advantages of a particular house, over the best available alternative. One can think of this household consuming many goods, the unit price of which to him depends on his location. If he moves to a different area he has to pay a higher unit price for these goods and services and the householder surplus is a measure of the utility difference implied by the two price confugurations[11]. As we discussed in Chapter three, such a utility difference can be measured in money terms, in four different ways: two relating to the money cost of ensuring the pre-moving utility level at the new prices and two relating to the reduction in money income that would ensure post moving utility at the old price.
The relevance of this distinction for the calculation of noise costs as outlined in Chapter three is clear. In order to decide whether a household would move or stay, the concept that is relevant is the willingness-to-pay one (relating the pre-moving utility level to the new prices). Once this decision is established the non-movers costs are assesed as outlined before - the matter of surplus no longer being relevant - and the movers costs are now assesed as either willingness costs or compensation costs. In order to do this we obtain one component from the demand curve for quiet, and add to it the adjustment costs.[12] If we now want the overall movers costs as willingness costs then we must add a willingness-to-pay hoseholder surplus and if we want the movers costs as compensation costs then we must add a compensation householder surplus.

While these distinctions may be important they have not been examined comprehensively and the empirical work on this question has mainly taken the form of questionnaires on a sample of houseowners and renters in order to ascertain what price they would take to move, net of any taxes or removal costs. The surveys of this type that have been conducted by the Roskill Commission[13] and by the Urban Motorway[14] Study group have proved to be fairly satisfactory as far as the consistency of the sample results was concerned, and broadly in keeping with some other knowledge in the distribution of these surpluses . Given the general success of these surveys it should prove worthwhile to attempt to derive willingness-to-pay and compensation surplus distributions, separately.

Movement Costs

The costs of moving house include the removal expenses, conveyancing charges and similar items. It has been estimated that these amount to anything between 8 - 16 per cent of sale price, depending on the country considered. The lower figure relates to Australian data, while the Roskill Commission estimated the costs around Heathrow to be 16 per cent.

5.5. Turnover Rates

Most people doing empirical work in the field of noise economics have recognised that in a noisy area, the proportion of the population moving out will be larger than in a quiet area for some time after the noise has been imposed. These extra movements are generated by a dynamic adjustment process in which households decide to move or to stay as they evaluate their dislike for noise relative to the costs of moving out. Inevitably such a process is spread out over time as it takes a while to appreciate the consequences of noise, and there is a natural inertia in making any adjustments.

Some estimates have been made of the relative movement rates at a point in time, in noisy and quiet area that have similar residential dwellings with similar facilities. For details of these estimates, the reader is referred to Walters, who concludes by saying that "when properly interpreted ... the orders of magnitude of the movements that are attributable to noise are approximately the same in both studies - say between 20 and 30 per cent more movers than there would be under normal non-noisy conditions. There is no evidence of dramatic sustained increases in movement rates - although in certain shock years the movement rates may be as much as 50 per cent above normal".[15]

While these estimates are of some interest, they are noticeably incomplete for our purposes. First, they do not distinguish between movement rates at different noise levels, and secondly they do not tell us anything of the total inital population that would move out following the introduction of noise in a given area. Both these issues are of some importance in assessing the validity of a Roskill type noise model, as well as in constructing a noise model not dependent on the unsatisfactory median assumption.

As far as we are aware there are only two sources of evidence on these questions. The first arises from the McKennell Study[16] in which an assessment of a household's desire to move was made from a series of questions asked of it. These yielded the following result in columns I and II :-

I	II
NNI	Percentage of people who feel like moving because of the noise
35-45 (30 NEF)	3.1
45-55 (37 NEF)	8.0
55+	11.0

Source Walters: page 125 Table 6.11

It is not clear, however, how this data is related to actual movement rates. There is no known way of doing this and so we can only take these as some indicators of moving intentions in different noise zones. The second source of evidence on this question was collected and analyzed during the Sydney Airport study. Unfortunately, we are not able to quote the data collected there, but the method is of some interest. All house sales by homeowners and all moves by renters were recorded on an annual basis over a period which began before aircraft noise was a serious problem and continued until well after the initial shock of the noise was felt. For homeowners and renters, the houses vacated as a percentage of the stock of such houses were listed for a quiet zone and for zones, at various noise levels, which were otherwise comparable with the quiet zone. Thus the percentage turnover rates for zones of various noise levels can be obtained and compared with turnover rates in the quiet zone. Now these figures will include houses which have been sold once, houses which have been sold twice, three times and so on. We wish to isolate from these, the percentage of houses in each zone that have been sold <u>once</u>, as that figure would be an estimate for the proportion of the original population that moved out on account of the noise. While we cannot do this precisely it does appear that a <u>lower</u> estimate of once vacated houses is given by assuming that the distribution of changes in occupancy follows a Poisson distribution[17]. On the other hand an upper bound is obtained by taking all movements as being single movements. If the number of movements follows a Poisson distribution, then, since we know the mean number of movements in zone Z from our statistics as m(Z), we may calculate the number of single movements as

$$e^{-m} . m.$$

Proceeding on this basis it appears that about five to nine percent of the population move out of the noisy zones on account of the noise, within a decade of the arrival of the noise. The somewhat higher turnover figures are obtained in the zones with a history of higher noise levels.

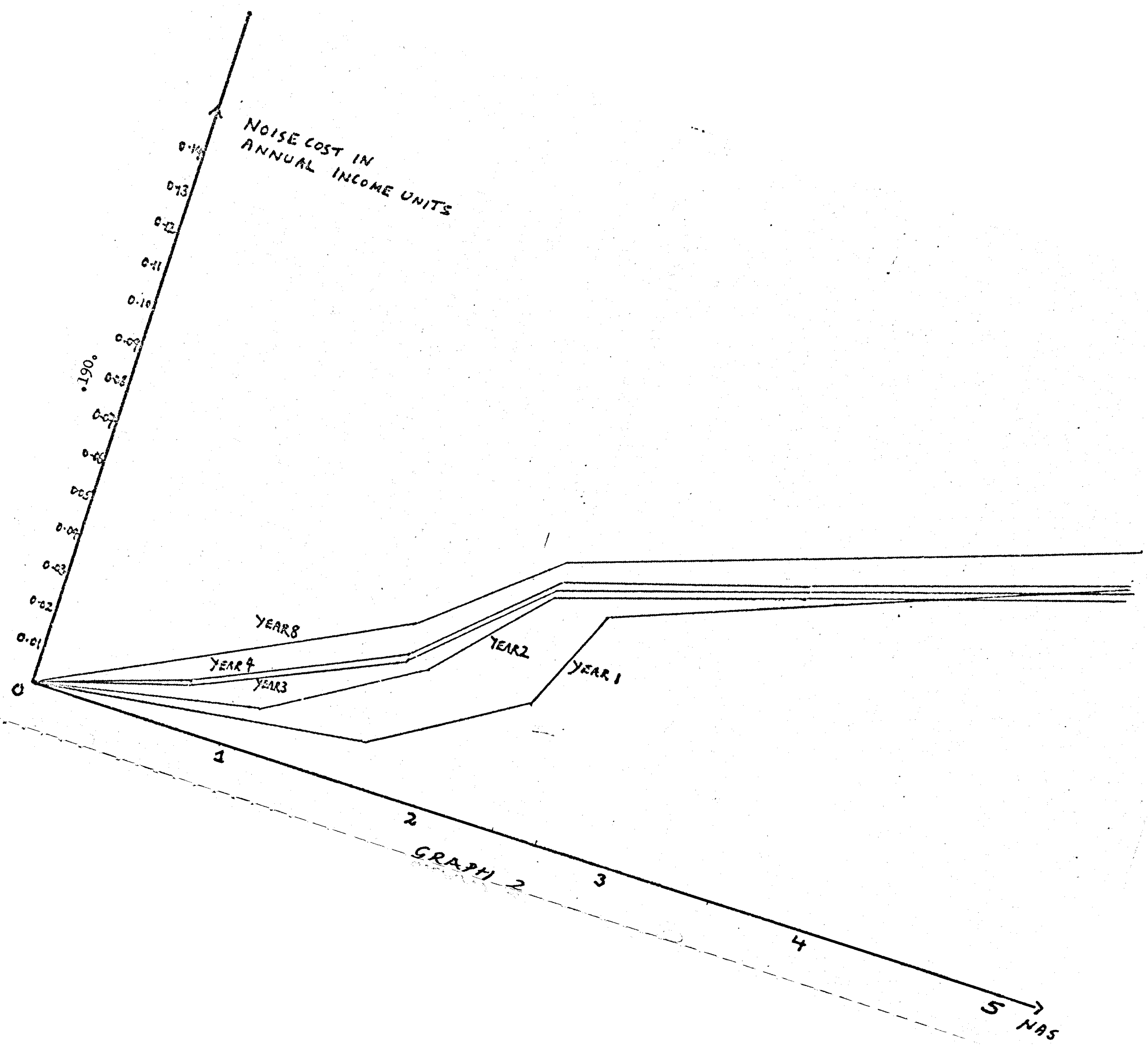
It is interesting to compare the movement rates suggested by the above analysis and their time profile, with that predicted by the use of the Roskill model. To do this we need to calculate the noise cost function for each year after the noise has been imposed. The assumption on which this was done by the Roskill

Model is discussed in some detail in chapter 3 but briefly it involves taking the distributions of the noise annoyance scale (NAS) on each noise zone, and associating with the median point of that scale the depreciation that would be observed in that noise zone. After this is done for the first year the noise function is constructed by joining up the points obtained from the medians in each distribution of NAS. Then, using the rule that if movement costs plus depreciation plus surplus is less than the noise costs the households for whom this is true move out, we obtain a new distribution of the NAS for each noise zone[18]. The noise function may then be recalculated for the following year with these new distributions and with the further assumption that owing to the income elasticity for quiet being greater than one, household depreciation is growing 2% per annum faster that movement costs, surplus, income, and expenditure on housing, all of which are growing at the same rate. This latter assumption is in fact taken from the Roskill Report. The details of the initial data for the first round calculations are given in the Appendix to Chapter 3. In graph 2 we plot the calculated noise cost function for years one to four and year eight as obtained by the above procedure. In the table below we give the implied movement rates due to noise, year by year, for the first ten years.

NEF \ YEAR	1	2	3	4	5	6	7	8	9	10	Total
22.5	9.1	5.8	0	0	0	0	0	0	0	0	14.9
27.5	10.1	2.0	0	0	0	0	0	0	0	0	12.1
33.5	4.1	0	0	0	0	0	0	1.8	0	0	5.9
37.5	0	0	0	0	0	0	0	0	0	0	0

percentage movement rates per annum implied by Roskill Model

These movement rates are not complete, as over time the noise costs will tend to rise faster than movements costs plus depreciation[18] and consequently over a very long period (100 years?) the whole noise zone must become populated by imperturbables[19]. However these figures offer an interesting comparison with observed rates. In doing this it must be remembered that


NOISE COST IN ANNUAL INCOME UNITS
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
0
1
2
3
4
5
NAS
YEAR 8
YEAR 4
YEAR 3
YEAR 2
YEAR 1

GRAPH 2

the observed rates are obtained for a different noise profile over time, whereas the above calculations are based on an assumed constant noise level. From the way the Roskill model works however it appears that if noise were growing as suggested say by the Sydney experience in the sixties, then the Roskill model would generate rather lower movement rates than we predict it to do. Furthermore we have assumed an initial depreciation level of 1.4 per cent of the house price per NEF. This is probably about right for the late sixties/early seventies and too high for the early sixties. So again using the Roskill model to replicate the early sixties behaviour we would find it generating rather lower movement rates. (Regarding the choice of this initial depreciation rate it is important to remember that the annual movement value in the Roskill model are rather sensitive to this number.)

With all these qualifications, however, it does seem that the Roskill model does generate what is possibly a little too high a movement rate in the lower noise zones, and very likely too low a movement rate in the higher noise zones.

FOOTNOTES TO CHAPTER 5

1. For a discussion on measures of traffic noise see Langdon and Scholes (1973), and Scholes (1970).
2. If the noise index is constructed using N rather than log N then the marginal increase in the noise level would be a constant, with respect to the number of aircraft and the desirability of concentrating aircraft activities as discussed in the previous chapter would no longer hold, _if_ the price per unit of quiet were still estimated to be constant. Whether or not this would be the case, would depend on how sensitive the estimation techniques for the price of quiet were to such a change in assumption.
3. This figure is quoted by Mrs. Paul (1971), and taken from the Roskill Report.
4. For an account of the NEF index and its validation see Tracor (1970).
5. Unfortunately it has not been possible to obtain a copy of this survey.
6. This graph is drawn from data taken from Walters (1974) Table 63. The broken lines indicate extra extrapolation to lower values.
7. This footnote has been excluded from the text.
8. This footnote has been excluded from the text.

10. Walters (1974) page 111.

11. In our view this is the best way to interpret householder surplus. It may seem somewhat restrictive in that certain features that make one dwelling desirable relative to another are not priced (e.g. friends, local amenities etc.). However, if an individual can trade these off against some money value then there must be some underlying way of valuing these things, such that different locations involve different costs of conducting certain activities. For convenience we refer to their activities as measurable with a unit implicit price.

12. For details of this calculation the reader is referred to Chapter 3 pages 13 and 14.

13. The survey methods used are discussed in Roskill (1970) and the distribution of surplus obtained is given in their report, "Commission on the third London airport", Vol. VII Table 20.1

14. The Urban Motorway Project Team's survey methods and results are reported in Urban Motorways Study Group (1973).

15. Walters (1974) page 127.

16. McKennell (1973), Appendix to the Report on Aircraft Noise Annoyance around London Heathrow Airport.

17. A Poisson distribution for the number of changes of occupancy was suggested by Walters. We have been able to find no published evidence for this although casual observation on turnover data suggests a J shaped distribution. We attempted to derive some direct evidence bearing on this question by considering some British data from the General Household Survey (1973), in table 5.52, this gives the distribution of the number of moves undertaken in the past five years by a sample of households. If we assume that a household moving m times in 5 years moves every $\frac{5}{m}$ years then we may construct a distribution in the number of changes of occupancy as follows:
Let the proportion of households moving every m years be P(m), and let the percentage of an initial population in year o that moves in year t be q_t. Then

$$q_t = \sum_{m=t}^{5} \frac{1}{m} \quad P(m)$$

The houses vacated for an j th time in year k ($k \geq j$) are given by V_{jk} where

$$V_{jk} = \sum_{m=1}^{k-j+1} P(m)V_{j-1,k-m}$$

The upper limit arises because we do not allow for more than one move a year. This is a recursive expression, where $V_{1t} = q_t$. Finally the proportion of households that are vacated n times altogether in five years is given by h(n) where

$$h(n) = \sum_{k=n}^{5} \quad V_{nk}. \quad (1- \sum_{m=1}^{k+1} P(m))$$

Admittedley this is a crude method of calculating h(n), as it does not allow for multiple moves within one year, and assumes an even spread of moves for those who say they move m times in five years. Nevertheless it should give us some idea of the distribution of occupancies of a given house. In the table below we give the distribution of the number of moves in the past five years, the proportion of houses changing hands n times in the past five years, and the Poisson distribution of change of occupancy based on the mean of this distribution being calculated by taking the estimated percentage of houses changing hands as given by the sum of column 2.

I	II	III	IV
n	% of houses moving n times	% of houses changing hands n times	Poisson distribution of III
0	64.6	64.30	69.97
1	23.4	33.97	25.00
2	6.7	1.72	4.45
3	3.2	0.01	0.53
4	1.1	0.00	0.05
5	1.0	0.00	0.00

Source: General Household Survey (1973) Table 5.52. Sample Size 11,899.

Clearly the number of changes given by the Poisson distribution falls more slowly than the sample, and therefore it underestimates the percentage of the original population that move out due to noise.

However, it can only be stressed that these numbers are suggestive - they do indicate a sharply declining distribution of changes in occupancy, with the Poisson distribution underestimating the rate of decline.

18. In constructing the new distribution we assume that all inmovers due to noise are imperturbables. This assumption is probably valid, given the small movement rates and the numbers of households with a zero noise annoyance score. In fact even if some of the movers had annoyance scores of 1 or 2 this would make no difference to the movement rates.

19. The movements caused by the rising valuation of quiet are the long term movements. These are distinguished from the short term movements, which appear to be concentrated in the first two years, with movements in the eighth year being somewhat ambiguous with respect to the divison.

Chapter 6
Conclusion

In this thesis various aspects of noise pollution have been surveyed. We started in Chapter 1 by stating the limitations of this survey and pointed out some important aspects that do not receive much attention here. Undoubtedly there is a lot to be done on ascertaining more fully the physical and psychological consequences of living in a very noisy environment for prolonged periods. However, such factors cannot be adequately discussed in an economic framework and only when the 'facts' are available will we be able to take account of them in our economic models. The complementary roles of the engineer and economist are apparent at a very early stage to anyone working in this field, and indeed the economist would have very little to say, had the engineer not developed suitable calibrations of various types of noise.

In Chapter 2 we use a simplified theoretical model to examine the consequences of a spatially distributed type of pollution on the optimum structure of towns. Although the models used in this knid of analysis are highly unrealistic they do, nevertheless, provide us with some insights into the relavant issues. In this case we observe that the spatial considerations of the pollution <u>alone</u> cause great difficulties in obtaining an optimum by price methods, and even if the overall convexity conditions are satisfied in production, a government agency would have to be concerned both with the sum of the marginal rates of substitution between the private good and the public bad, as well as the direction of the rate of change of the sum. Multiple equilibria are very likely to be present and a price decentralised economy could easily settle at a sub-optimum position. In addition to the problems raised by spatial factors we have to take account of the technology of production which must have increasing returns for some range at least, if the existence of towns is to be justified. Constructing a plausible technology and taking reasonable parameter values we examined the sensitivity of the optimum solutions to various parametric and geographic assumptions. Here we found that the optimum solutions were rather sensitive to small changes in the parameters representing preferences and to restrictions on the use of land area available.

Also it turned out that the policy prescriptions for optimal pollution control could differ greatly if the town size could not be controlled from those which would exist if it did.

From this rather general analysis we moved to considering the problem of the measurement of noise costs. The analytical framework used here is a partial equilibrium one with all its attendant shortcomings. However, where it is possible to treat the movement in the price of quiet in relative isolation from other prices it provides a most useful basis for comparative cost studies as well as calculations of compensation for noise imposed. We considered basically two different approaches - one starting with orthodox economic theory and adding various bits that were special to the noise problem, and the other taking the Roskill noise model and strengthening those parts of it that are somewhat suspect. In the end one can conclude that the Roskill model was a very reasonable first attempt at this kind of thing, although, if done again it would be better to use the more orthodox approach outlined in the first part of this Chapter.

In Chapter 4 we considered various problems relevant to the control of noise. There is clearly much scope for the application of economic principles to this area and in many instances all we could do was to indicate the shortcomings of existing practices. The standard theory of externalities and Public Bads, while it applies broadly to the matter of noise pollution, cannot be straightforwardly translated into operational terms. The difficulties inherent in a pollution tax are discussed at some length and it would appear that in general any noise tax would be related to the price of quiet as well as to the relationship between the consumer and producer prices of the services that one has to buy in order to be quiet. What we refer to as locational inconvenience. Such a relationship is not straightforwardly defined and while in some cases a simplification may be obtainable, there are considerable data problems involved in constructing a satisfactory noise tax. The reliance therefore on direct controls places all the more importance on having a good method of measuring the costs of implementing noise (or equally the benefits of reducing it). It is only by this means that a reasonable decision can be made regarding the choice between various noise abatement procedures.

In Chapter 5 we discussed various bits of empirical evidence. Here, too, there is great need for good statistical and econometric work, especially with regarding the measurement of house price depreciation due to noise. The price and income demand elasticities for quiet, constructed from the available data are at best only indicative and these estimates could be improved. Nevertheless some results have been achieved, contrary to the scepticisim of many people, and those results are, broadly speaking, consistent with each other. Such work, and the analysis in this dissertation, shows that economic theory and econometrics have a great deal to contribute in the analysis of pollution problems.

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