Categoricity and Possibility. A Note on Williamson's Modal Monism

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Abstract: The paper sketches an argument against modal monism, more specifically against the reduction of physical possibility to metaphysical possibility. The argument is based on the non-categoricity of quantum logic.

Keywords: modal monism, quantum logic, non-categoricity

1 Modal monism and anti-exceptionalism

A widespread philosophical view about modality holds that there exists only one kind of necessity and possibility, to which all other kinds are to be reduced. The view is often called *modal monism*, and it has been expressed perhaps most famously by Wittgenstein in the *Tractatus*: "The only necessity that exists is logical necessity. (6.37) "Just as the only necessity that exists is logical necessity, so too the only impossibility that exists is logical impossibility." (6.375) According to this view, that is physically possible which is logically consistent with the laws of nature.

A similar view, according to which metaphysical and physical modality "stand or fall" together, has been more recently defended by Timothy Williamson: "In given circumstances, a proposition is *nomically possible* if and only if it is metaphysically compossible with what, in those circumstances, are the laws of nature (their conjunction is metaphysically possible)." (Williamson (2016), 455) Thus, nomical or physical possibility cannot be explained independently of metaphysical possibility.

It seems natural to take modal monism to provide support to Williamson's *anti-exceptionalism* about metaphysics, in general, and about the metaphysics of modality, in particular, which is expressed in the following way: "We should not treat the metaphysics and epistemology of metaphysical modality in isolation from the metaphysics and epistemology of the natural sciences." (*ibid*, 453) For if physical modality is explained in terms of metaphysical modality, then this anti-exceptionalist requirement is arguably satisfied.

In an earlier book, wherein Williamson mounts a defense of a metaphysical view he calls *necessitism*, i.e., the view that necessarily everything is such that necessarily something is identical with it, he similarly affirms anti-exceptionalism, which is here expressed somewhat differently: "We will be guided throughout this book by a conception of theories in logic and metaphysics as scientific theories, to be assessed by the same overall standards as theories in other branches of science." (Williamson (2013), 27)[1] These anti-exceptionalist declarations are meant to stand on their own, with no support from modal monism, which does not seem to be part of the picture yet: "Metaphysical possibility will not be assumed to be metaphysically basic, or fundamental, or irreducible, or perfectly natural, or anything like that." (*ibid*, 3, footnote 4)

Adopting scientific standards for the assessment of the metaphysics of metaphysical modality might be enough to ensure that this is not treated in isolation from the metaphysics of physical modality. The present paper argues, however, that modal monism should stay out of the picture, for it cannot add anything to support such treatment. The reason for this is that modal monism is false.

To be sure, modal monism has its own detractors within the field of analytic metaphysics (see, e.g., Fine (2005), for an argument in favor of modal pluralism). Here I will construct an argument against modal monism based on a technical result in the metatheory of quantum logic that has implications for our understanding of what quantum mechanics deems physically possible. To show that modal monism is false, I provide a counterexample to Williamson's reduction of physical possibility to metaphysical possibility. That is, I argue that there are propositions that quantum mechanics deems physically possible, but which are not metaphysically compossible with what in some circumstances are the laws of nature.

I start by providing some details about the quantum logical interpretation, or as I shall call it (following Stairs (2015)) the quantum logical *reconstruction* of quantum mechanics, enough to make intelligible the presentation of a technical result (due to Pavičić and Megill (1999)) that establishes the *non-categoricity* of quantum logic (where a logic will be called categorical with respect to an isomorphism class of structures if and only if all its models are in that class).

[1] The point is recalled towards the end of the book: "[T]he methodology of this book is akin to that of a natural science. [...] The theories are judged partly on their strength, simplicity, and elegance, partly on the fit between their consequences and what is independently known." (*ibid*, 423)

Afterwards I discuss the connection between this metatheoretical result and what quantum mechanics reveals about modality, suggesting an account of physical possibility that is independent of metaphysical possibility. Then I come back to modal monism to explain why I believe it is false, but also to reject an objection that might be raised against my argument, an objection that insists against taking the laws of quantum logic as laws of nature. An elaboration and defense of modal pluralism in the context of quantum mechanics will, however, be deferred to another paper.

2 Quantum logic and orthomodular lattices

This section briefly introduces the standard approach to quantum logic and its semantics constructed in terms of orthomodular lattices. In doing so, it assumes the standard Hilbert space formalism of quantum mechanics. Thus, as is usual, the pure states of a quantum system are taken to be represented by unit vectors in an associated Hilbert space that represents the state space of the system, and quantum properties to be represented by linear closed subspaces of the Hilbert space.[2]

Let $\mathcal{QL}$ be a formal language that contains an infinite set of formulas, $p, q, ...$, and three symbols for logical connectives, $\sim, \wedge, \vee$ (that is, for negation, conjunction, and disjunction, respectively) such that if p and q are sentences in $\mathcal{QL}$, then $\sim p$, $p \wedge q$, and $p \vee q$ are also in $\mathcal{QL}$.[3] The semantics of this formal language is typically given by an ortholattice, which can be defined as an algebraic structure $\mathcal{LA}$, i.e., a set of elements, $a, b, ...$ (which are precisely the linear closed subspaces of a Hilbert space) together with the operations $', \cap, \cup$ (for orthocomplementation, join, and meet, respectively), such that any elements a, b, c in that set satisfy conditions like the following:

$$
\begin{gathered}
a \cap b = b \cap a \\
a \cap (b \cap c) = (a \cap b) \cap c \\
a \cap (a \cup b) = a \\
a = a'' \\
(a \cap a') \cap b = a \cap a' \\
a \cup b = (a' \cap b')'
\end{gathered}
$$

[2]For reference, see any standard introduction to quantum mechanics and its logical-algebraic formalism, e.g. Beltrametti and Cassinelli (1981).

[3]One can also define a variety of implication connectives. For a brief review, see Pavičić (2016). The first to define different quantum implications was Weyl (1940).

One further defines a supremum $1 := a \cup a'$, an infimum $0 := a \cap a'$, and a partial order $a \leq b := a \cap b = a$, $a \leq b := a \cup b = b$, as well as an equivalence relation $a \equiv b := (a \cap b) \cup (a' \cap b')$. Then, an ortholattice $\mathcal{LA}$ is said to be a model of $\mathcal{QL}$ if and only if for any sentences p, q in $\mathcal{QL}$ and any elements a, b of $\mathcal{LA}$ there is a map $h : \mathcal{QL} \longrightarrow \mathcal{LA}$, such that the following three conditions are satisfied:

$$h(\sim p) = h(p)' = a'$$
$$h(p \vee q) = h(p) \cup h(q) = a \cup b$$
$$h(p \wedge q) = h(p) \cap h(q) = a \cap b$$

The map h is a homomorphism, i.e., it preserves operational structure, by mapping negation to orthocomplementation, disjunction to join, and conjunction to meet. Thus, the algebraic relations between linear closed subspaces of the Hilbert space (i.e., the equations involving the ortholattice operations) are expressed by sentences of the formal language $\mathcal{QL}$, and are typically taken to represent compatibility relations between the properties of a quantum system (i.e., their co-measurability). But, of course, not all algebraic relations will do so, since in quantum mechanics not all properties of a system are co-measurable. In particular, distributivity, that is, the law according to which, for any elements $a, b, c \in \mathcal{LA}$, $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$, is usually taken to fail in quantum mechanics. As a result, weaker versions of distributivity have been considered, one of which is modularity.

A lattice is *modular* if and only if the law of modularity holds: if $a \leq c$, then $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$. As von Neumann first realized, however, modularity requires a finite dimensional Hilbert space, which he thought was improper for a truly general quantum logic.[4] A further weakening of distributivity leads to orthomodularity. A lattice is *orthomodular* if and only if the law of orthomodularity holds: if $a \leq b$, then $b = a \cup (a' \cap b)$. Orthomodular lattices give the standard algebraic semantics of the quantum logical calculus $\mathcal{QL}$. Thus, the homomorphism h, as defined above, maps sentences from $\mathcal{QL}$ to compatibility relations between the elements of an orthomodular structure of quantum properties.

[4] This is the reason von Neumann came to give up the Hilbert space formalism several years after he had introduced it. See the quotation at the end of the paper. For a detailed discussion, see e.g. Rédei (1996).

3 Non-categoricity and physical possibility

Having described these basic facts about quantum logic and its algebraic semantics, I turn now to a (so far unduly neglected) result in the metatheory of quantum logic, which shows that quantum logic is non-categorical (Pavičić and Megill (1999), Pavičić and Megill (2009)). More precisely, it is non-categorical with respect to the isomorphism class of orthomodular lattices, because one can show that not all its algebraic models are in this class: "one of its models is an orthomodular lattice, while others are nonorthomodular lattices." (Pavičić (2016), 2)

Nonorthomodular lattices are those in which the law of orthomodularity fails. Some of these nonorthomodular lattices, however, obey a weakened form of orthomodularity, appropriately called *weak orthomodularity*. To see what the law of weak orthomodularity states, consider an alternative definition of orthomodularity: a lattice is orthomodular if and only if for any elements $a, b \in \mathcal{LA}$, $a \equiv b = 1 \Rightarrow a = b$. That is, two elements of an orthomodular lattice are equivalent (in the sense of equivalence defined above) only if they are the same element. A weakly orthomodular, nonorthomodular lattice is then one in which for any $a, b, c \in \mathcal{LA}$, $a \equiv b = 1 \Rightarrow (a \cup c) \equiv (b \cup c) = 1$. In other words, such a lattice includes distinct elements that are indiscernible via orthomodularity.[5]

What Pavičić and Megill have proved, more precisely, is that there exists an orthomodular lattice, as well as a weakly orthomodular, non-orthomodular lattice, such that the quantum logical propositional calculus is sound and complete with respect to both.[6] One interpretation, proposed by Pavičić, is that quantum logic "can simultaneously describe distinct realities" (Pavičić (2016), 2). What this means, I take it, is that quantum logic can associate distinct compatibility structures of properties with one and the same quantum system: both an orthomodular structure, and a weakly orthomodular, nonorthomodular one.

[5]It might help to compare this to first-order Peano arithmetic: a nonstandard model includes distinct elements – nonstandard numbers – that are indiscernible by the application of the rules of the theory within its language.

[6]Related claims concerning the semantics of quantum logic were expressed first by Weyl, who pointed out that Birkhoff and von Neumann's quantum logic allows non-unique valuations of disjunctive and conjunctive formulas (Weyl (1940)). For discussion, see Toader (2020a). See Hellman (1980) for a proof that quantum logical connectives are not truth-functional. For an application of the non-truth-functional semantics developed by Arnon Avron and his collaborators (see, e.g., Avron and Zamansky (2011)) to the quantum logical formalism, see Jorge and Holik (2020).

This raises a couple of question, which I better attempt to answer now before I even formulate my argument against modal monism. First, one may point out that non-categoricity is an artefact of the quantum logical reconstruction of quantum mechanics, rather than a logical feature of the physical theory itself. Secondly, one may insist that one should reject a non-categorical theory as possessing an undesirable metatheoretical property.

To the first question: one should recall that the purpose of the quantum logical reconstruction is to reveal the logical structure of quantum mechanics – the very objective stated by Birkhoff and von Neumann already in 1936: "to discover what logical structure one may hope to find in physical theories which, like quantum mechanics, do not conform to classical logic."[7] Thus, to the extent that quantum logic succeeded in at least approximating the logical structure of quantum mechanics, and there is no serious reason to doubt that this is the case, its non-categoricity entails that quantum mechanics associates distinct compatibility structures of properties with a quantum system. In other words, quantum mechanics tells us that physical reality can be either an orthomodular ortholattice or a weakly orthomodular, nonorthomodular one. Despite the fact that quantum logic cannot itself be considered a physical theory, it is nevertheless physically salient because one of its metatheoretical properties has implications for our understanding of what quantum mechanics deems physically possible.[8]

To the second question: that one should characterize a non-categorical theory as semantically defective, as possessing an undesirable metatheoretical property, seems justified. Witness the many attempts to prove that theories like arithmetic and set theory are categorical, on the assumption that non-categoricity is a liability, one that implies for example the semantic indeterminacy of the languages of these theories, their inability to pick out their intended semantics.[9] However, from a metatheoretical perspective, there is arguably a significant difference between non-physical theories and physical ones like quantum mechanics. The categoricity of the latter has been long rejected as a kind of undesirable rigidity that unduly constrains its applicability.[10]

[7]Birkhoff and von Neumann (1936), 823. See also Suppes (1966).

[8]The categoricity problem of quantum mechanics is discussed at length in Toader (2020b)

[9]See, for discussion, Button and Walsh (2018).

[10]Consider, e.g., the following view: "A categorical theory is one such that any two models (true interpretations) of its underlying abstract formalism are isomorphic (structurally identical). Now a necessary condition for theory isomorphism is that the corresponding sets be similar, i.e. that there be a one-to-one correspondence between them. But we do not want such

More recently, non-categoricity appears to be considered as a theoretical advantage of a physical theory: "A theory that underdetermines its own interpretation is like a healthy breeding population: it has a shot at enough diversity to ... meet the variety of demands its scientific environment places on it." (Ruetsche (2011), 355) One sometimes speaks, very aptly, of "intended non-categoricity" (Rédei (2014), 80) to characterize the intention of the physicist to construct physical theories that allow for non-isomorphic models.

One fundamental idea underlying these evaluations of non-categoricity is that the non-isomorphic models of a physical theory represent distinct physical possibilities. This suggests a semantic account of physical modality, according to which a proposition is physically possible if and only if it is true in at least one model of a physical theory. This idea, I submit, should be understood as the basis for an independent account of physical possibility, i.e., one that does not reduce physical possibility to another kind of possibility, like metaphysical possibility. For on this account, a proposition may be considered physically possible independently of what is metaphysically compossible with what, in any circumstances, are the laws of nature. What is physically possible depends only on metatheory. But what holds at the metatheoretical level, i.e., that a proposition in the language of a physical theory is true in at least one model of that theory, need not be logically consistent with what the theory takes to be the laws of nature. This account represents a move towards modal pluralism.

4 Against Modal Monism

I think that everything is now in place for a presentation of my argument against Williamson's modal monism, an argument that draws, as already announced, on the non-categoricity of the quantum logical formalism. So let p be a sentence in $\mathcal{QL}$ that expresses a proposition $h(p)$ which holds in a weakly orthomodular, nonorthomodular structure of quantum properties, and let's assume that $h(p)$ entails weak orthomodularity, but not the negation of orthomodularity. Similarly, let q be a sentence in $\mathcal{QL}$ that expresses a proposition $h(q)$ which holds in an orthomodular structure of quantum properties, and let's assume that $h(q)$ entails orthomodularity.

a rigidity in physics for, even if two theories do have formally identical basic formulas (e.g. wave equations), they may refer to entirely different kinds of physical systems, these kinds being conceptualised as sets that need not be similar." (Bunge (1973), 166)

According to the independent account of physical modality, sketched above, each of $h(p)$ and $h(q)$ is physically possible, for each is true in at least one structure, i.e., $h(p)$ is true in a weakly orthomodular, nonorthomodular lattice, and $h(q)$ is true in an orthomodular lattice. This situation obtains because, as argued above, quantum logic is non-categorical, i.e., these two mutually non-isomorphic ortholattices are provably in the class of its models. From a quantum-mechanical point of view then, there is no reason to dismiss either $h(p)$ or $h(q)$ as a physical impossibility.

But recall that, according to Williamson's modal monism, that is physically possible which is metaphysically compossible with what in some circumstances are the laws of nature. So take the following circumstances: an orthomodular structure of quantum properties – an orthomodular world, as it were. On Williamson's view, then, $h(q)$ would be physically possible, since it is logically consistent, and thus metaphysically compossible, with the laws of an orthomodular world: by assumption, $h(q)$ entails orthomodularity. Furthermore, $h(p)$ would be physically possible as well, since it is logically consistent, and thus metaphysically compossible, with the laws of an orthomodular world: by assumption, $h(p)$ does not entail the negation of orthomodularity.

So far, so good. Nevertheless, consider now the following, different circumstances: a weakly orthomodular, nonorthomodular structure of quantum properties – a weakly orthomodular, nonorthomodular world, that is. On Williamson's view, $h(p)$ would be physically possible, since it is logically consistent, and thus metaphysically compossible, with the laws of a weakly orthomodular, nonorthomodular world: by assumption, $h(p)$ entails weak orthomodularity. However, on the same view, $h(q)$ would turn out to be physically impossible, because $h(q)$ is not metaphysically compossible with the laws of a weakly orthomodular, nonorthomodular world, since their conjunction is logically inconsistent: by assumption, $h(q)$ entails orthomodularity.

It follows from all this that there are propositions which quantum mechanics deems physically possible, but which according to Williamson's modal monism are not metaphysically compossible with what, in some circumstances, are the laws of nature. This, I think, constitutes a sufficient reason for rejecting the reduction of physical possibility to metaphysical possibility, which characterizes modal monism, at least in the version that appears to be advocated by Williamson. Furthermore, insofar as it is actually based on modal monism, his anti-exceptionalism about the metaphysics of metaphysical modality turns out to be improperly justified.

One may object that orthomodularity and weak orthomodularity cannot be considered laws of nature (Mittelstaedt (2012)). If so, the above would not really provide a counterexample to Williamson's reduction of physical possibility to metaphysical possibility. This objection raises, of course, a more general question: what are the laws of nature according to quantum mechanics? Typically, laws of nature are supposed to express relations between the states of a physical system, relations that allow us to make testable predictions about the states of a system. So the laws of nature are typically dynamical laws, as they describe trajectories through the state space associated with a system. But in quantum logic, of course, there are no dynamical laws, there is no expression of Schrödinger's equation.

This may be considered as a shortcoming of the quantum logical reconstruction of quantum mechanics, but only if one assumes that any reconstruction must describe the dynamics. This, however, has never been the purpose of quantum logic. As mentioned above, that purpose was to uncover the logical structure of quantum mechanics, the compatibility structure of properties possessed by a quantum system. But a dynamical law does not tell us anything about exactly what quantum properties are compatible (i.e., co-measurable). In the quantum logical reconstruction, as von Neumann emphasized in a famous letter, the focus is not on the states of a system any more, but on its properties: "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space any more. ... Now we begin to believe, that ... it is not the *vectors* which matter but the *lattice of all linear (closed) subspaces*. Because: ... the *states* are merely a derived notion, the primitive (phenomenologically given) notion being the *qualities*, which correspond to the *linear closed subspaces*." (von Neumann to Birkhoff, November 6, 1935)

Orthomodularity and weak orthomodularity are laws of nature in the sense that they describe compatibility relations between quantum properties. These are not dynamical laws, since they do not describe trajectories through state space, but they allow us to predict what properties of a system are compatible in certain circumstances. The law of orthomodularity, for example, tells us that no measurement undertaken in an orthomodular world can reveal an instance of an nonorthomodular compatibility structure of properties. It can furthermore be easily seen that such laws support counterfactuals. Thus, the difference between orthomodular and weakly orthomodular, nonorthomodular worlds is not merely factual, but nomological. So the objection fails.

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