Aristotle's Actual Infinities

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1 INTRODUCTION

According to a common telling, Aristotle suffered from a generalized horror of the infinite. He is supposed to have thought that the actual existence of any kind of infinite quantity, whether a magnitude or a plurality, is impossible or even unintelligible. At most, he is to have allowed some kinds of quantity to exemplify 'potential' infinity, in the sense that, for any (finite) quantity of the kind, it is possible for there to be another (finite) quantity of the same kind which exceeds it. He may also have countenanced a kind of infinitary process, in which finite quantity after finite quantity is generated, each exceeding the last. Still, when it comes to the straightforward, actual existence of something infinitely large, or of some things that are infinitely many, he is supposed to have held that this is not possible.[1]

1. Simplicius and H. Diels, *Simplicii in Aristotelis Physicorum libros quattuor priores commentaria* (Berlin, 1882), 493.18–19; G. Cantor, 'Über unendliche, lineare Punktmannigfaltigkeiten', *Mathematische Annalen*, 21 (1883), 545–91, 555; H. Scholz, 'Warum haben die Griechen die Irrationalzahlen nicht aufgebaut?' ['Irrationalzahlen'], in *Die grundlagenkrisis der griechischen mathematik* (Charlottenburg, 1928), 35–72, 53; E. Hussey, *Aristotle's Physics, books III and IV* [*Physics III–IV*] (Oxford, 1983), 82; R. Sorabji, *Time, creation, and the continuum: theories in antiquity and the early middle ages* [*Creation*] (Ithaca, N.Y., 1983), 211; W. Charlton, 'Aristotle's potential infinites', in Judson, L. (ed.), *Aristotle's Physics: a collection of essays* (Oxford: New York, 1991), 129–49, 144; U. Coope, *Time for Aristotle: Physics IV.10-14* [*Time*]

In this paper, I will argue that Aristotle was not so averse to infinity as all that. In particular, I claim, he did not deny that there can be an actual infinite *plurality*—that is, infinitely many things. I will focus on his treatise on the infinite in *Physics* 3. 4–8, and try to show that the negative statements about infinity in this treatise amount to no more than the following three claims:

1. It is not possible for anything to be infinitely large. (More strongly: there is a finite size S such that it is impossible for anything to be larger than S.)
2. It is not possible for anything to be infinitely small.
3. It is not possible for there to be an infinite number.

The first of these claims is a straightforward finitist claim about magnitude. It was highly controversial in Aristotle's time, and Aristotle argues for it at length in *Physics* 3. 5, making it the central claim of his treatise on the infinite. The second claim, denying the possibility of infinitesimal magnitudes, reflects a universal consensus of the day.[2] Aristotle can, and does, affirm it without argument, in *Physics* 3. 6. The third claim is widely thought to mean that there cannot be infinitely many things. But in fact it does not mean this. Rather, it reflects a then-standard understanding of what a number is: a

(Oxford, 2005), 10-1; J.M. Cooper, 'Aristotelian infinites', *Oxford Studies in Ancient Philosophy*, 51 (2016), 161–206, 187 n. 17

2. See note 73 below, and the paragraph to which it is attached.

number is a 'determined' or 'measurable' or 'finite' plurality. There may or may not be infinitely many things, but if there are, then there is no number of them. This claim required only minimal argument in Aristotle's intellectual context, and that is what it receives.

In short, I claim, Aristotle's discussion of infinity in *Physics* 3 does not contain any denial of the existence of an actually infinite plurality.

I am interested, more specifically, in infinite pluralities *of extended things*. These are important for understanding the metaphysics of continua and the parts of continua. I will argue that *Physics* 3 contains no denial of the existence of an actually infinite plurality of extended things, such as parts of a magnitude. Let me say a few words about why this matters.

1.1 Infinity and the metaphysics of parts

Aristotle holds that all extended things are without atomic parts, and are hence infinitely divisible. He also holds that, in general, the parts of a thing have being *in capacity* or *potentially* (δυνάμει), not *in activity* or *actually* (ἐνεργείᾳ).[3,4] His claims about the po-

3. See for example *De Anima* 3. 6, $430^b10–11$; *Metaph.* Δ 7, $1017^b6–8$; Z 13, 1039^a3-4, $^a6-7$, $^a13-14$; Θ 6, 1048^a32-3; *Phys.* 8. 8, $263^b3–6$.

4. The meaning, and the best translation, of the expressions δυνάμει and ἐνεργείᾳ is the subject of controversy. For convenience I will use the traditional translations, 'potentially' and 'actually', in the present

tentiality of a thing's parts give rise to difficult questions of interpretation, the answers to which are quite important for understanding Aristotle's overall metaphysical views. Perhaps the most fundamental question, and one on which commentators have reached no agreement, is the following: when Aristotle says that a thing's parts have being only potentially, does he mean (i) that there *are no parts* of the thing, that is, that aside from the whole thing there is nothing there to be identified or quantified over, even though the whole *could be divided* into fragments?[5] Or does he mean (ii) that there *are* parts of the thing, available to be identified and quantified over, but that these parts have a special mode of being, namely, potential being?[6] Let us say that according to interpretation (i), a whole has no 'identifiable' parts, and that according to interpretation (ii), a whole does have identifiable parts.

Coope has argued against interpretation (ii) along the following lines. If every possible part of a thing exists as something identifiable, then, because of infinite divisibility,

paper, while assuming as little as possible about the meanings of these terms.

5. Adherents of interpretation (i) include Sorabji, *Creation*, 211-2, Charlton, 'Aristotle's potential infinites', 132-4, Coope, *Time*, 10, and, seemingly, Cooper, 'Aristotelian infinites', 178-9.

6. Interpretation (ii) seems to be adopted by J. Bowin, 'Aristotelian Infinity', *Oxford Studies in Ancient Philosophy*, 32 (2007), 233–50, 245 and J. Beere, *Doing and being: an interpretation of Aristotle's Metaphysics* theta [*Doing and Being*] (Oxford; New York, 2009), and is energetically defended by C. Pfeiffer, 'Aristotle and the Thesis of Mereological Potentialism' ['Mereological Potentialism'], *Philosophical Inquiry*, 42 (2018), 28–66.

every extended thing contains an actually infinite plurality of parts. Even if these parts are mere *potential beings*, still the answer to the question 'how many are there?' is that there are actually infinitely many. However, says Coope, Aristotle denies the possibility of an actually infinite plurality. Therefore, he must think that there *are no parts* other than the ones that actually have being, or that are at least specially marked out in some way.[7]

As Coope's argument illustrates, there is a close connection between the 'why' and the 'what' of Aristotle's potentialism about parts. From the premise that the *reason why* Aristotle adopted his view was in order to avoid infinite pluralities, the argument concludes that we should interpret *what the view was* in a way that makes it avoid infinite pluralities.

Now, finitism is only one of several possible motivations for Aristotle's views about the potentiality of parts, and different motivations would be best served by significantly different theories. Here is an incomplete list of possible motivations.

FINITISM: Given the rejection of atomism, every extended thing has infinitely many parts. Aristotle adopted potentialism about parts in order to avoid actual infinite pluralities.

7. Coope's view contains a further subtlety (Coope, *Time*, 12): she holds that it is possible to create 'potential divisions' by marking them out in thought or in speech. Such potential divisions are identifiable and result in parts that are identifiable, even though they have merely potential being.

FEAR OF NIHILISM: If a magnitude is everywhere filled with divisions between its parts, then it is ultimately composed of nothing (or, almost as bad, it is composed of points).[8] Aristotle adopted potentialism about parts in order to maintain the reality of magnitudes, without being forced into the atomist doctrine that some magnitudes are indivisible.

UNITY: If something has parts that have being actually, then it *is* these parts, and consequently it is not a whole or a single thing. Aristotle adopted potentialism about parts in order to sustain the view that wholes exist and are one.

PRIORITY: If something has parts that have being actually, then its parts are prior to it. Aristotle adopted potentialism about parts in order to sustain the view that wholes are prior to their parts.

As we saw above, if Aristotle's motivation was finitism, then this will have called for a theory on which, in the absence of special 'marking out' or 'actualizing' circumstances, there simply are no parts of a whole. Other motivations, by contrast, seem to be served equally well by a theory on which there are parts, albeit ones that have being potentially (however this is to be cashed out), as by a theory on which there are no parts.

Aristotle's motivations could also make a second difference to the content of his view. If Aristotle was motivated by a concern for unity, then he ought to have thought

8. See *GC* 1. 2, esp. 316^{b}9–12, 317^{a}6–7; 1. 8, 325^{a}6–9

that the actual being of a whole precludes the actual being of *any parts at all*. Likewise if he was motivated by the priority of wholes over their parts. On the other hand, if he was motivated only by finitism, then he may have allowed some finite number of parts to actually have being. And if he was concerned only to avoid nihilism, then he may even have allowed infinitely many parts to actually have being, since a magnitude could contain infinitely many actual divisions without having actual divisions *everywhere*.[9]

A third difference follows from the second. It is sometimes held that, for Aristotle, parts can be brought into actual being by an act of thought or attention. On this view, when a geometer starts thinking about two parts of a line, those two parts acquire actual being. But if Aristotle's potentialism about parts was motivated by unity or priority, then, as we have seen, he should have held that the actual being of *any part* excludes the actual being of the whole. This, combined with the view that geometers bring parts into actual being by thinking about them, would yield the consequence that geometers destroy lines and figures by thinking about their parts. Aristotle presumably would not have accepted this consequence. Hence, the view that parts are brought into actual being

9. For example, a line could contain an actual division halfway along its length, a quarter of the way, an eighth of the way, and so forth. This would make for an actual infinite plurality, but it would not make for a line that is everywhere filled with divisions. It would not justify a nihilistic worry to the effect that the line is composed of nothing, or is composed of points.

by thought is compatible with finitist and fear-of-nihilism motivations for potentialism about parts, but does not pair well with motivations from unity or priority.

The present paper does not seek to settle the content of Aristotle's doctrine of the potential being of parts.[10] However, if my arguments succeed, they eliminate finitism as a plausible motivation for the doctrine, and thus have consequences for the doctrine's interpretation. They will thereby contribute, I hope, to our understanding of texts including *Physics* 6 and 8, which are more directly concerned with the parts of extended things, their quantity and metaphysical status.

1.2 Infinite pluralities in Aristotle? Initial evidence for and against

In the course of his treatise on the infinite in *Physics* 3. 4–8, Aristotle makes some remarks that have been thought to express an unrestricted denial of actual infinite quantities. The following two passages are often cited:

T1 It is apparent that the infinite cannot exist as something that exists actually. (*Phys.* 3. 4, 204^{a}20–1, as quoted by Scholz)[11]

10. A good recent discussion of the doctrine is Pfeiffer, 'Mereological Potentialism'. For discussion of a range of scholastic interpretations, see R. Pasnau, *Metaphysical themes, 1274-1671* (Oxford : New York, 2011), ch. 26.

11. Translations are my own unless otherwise noted. Quoted by Scholz, 'Irrationalzahlen', 54 without the immediately succeeding words, 'and as a substance and principle'.

T2 The infinite does not exist in any other way, but it does exist in this way: potentially and in the direction of reduction. (*Phys.* 3. 6, 206^{b}12–13)

One commentator has written of Aristotle's finitism that it 'must be defined as a *universal* renunciation of the infinite, thus as a renunciation of the infinite in *every* form'.[12] Another has written that Aristotle 'rejects the intelligibility of the actual existence of *any sort* of completed infinity'.[13]

On the other hand, a scattering of passages, outside Aristotle's treatise on the infinite in *Physics* 3, give reason for hesitation about such unrestricted interpretations. In these passages, Aristotle appears to say that there are infinitely many things, without adding any qualifications or reservations. I say 'appears to say', rather than 'says', partly because the word that I will translate as 'infinite' is sometimes translated instead—rightly or wrongly—as 'indefinite'. Not everyone will read these passages the same way I do. I do not claim that they are probative on their own, but I do think that they suffice to justify a review of the evidence, and that they will ultimately weigh on the same side of the scale as the arguments to be given in the course of this paper.

12. Scholz, 'Irrationalzahlen', 53, my translation, emphasis original. The sentence is most directly a description of the finitism at work in the scholarship of Spengler, but Scholz goes on to say that this finitism 'is indisputably a hallmark of man of classical antiquity since Plato and Aristotle' (p. 54).

13. Cooper, 'Aristotelian infinites', 187 n. 17, emphasis added. See also U. Coope, 'Aristotle on the Infinite', in Shields, C. (ed.), *The Oxford handbook of Aristotle* (Oxford, 2012), 267–86, 268.

To begin with, here is Aristotle's explanation as to why the phenomenon of homonymy is unavoidable:

T3 Names are finite, as is the plurality of phrases, but things are infinite in respect of number. It is necessary, then, that the same phrase and a single name signify several things. (*Soph. El.* 1, $165^{a}1$–13)

Next, here he is during a criticism of the view that souls are compounds of elements:

T4 There do not only exist these [*i.e.*, the elements], but also many other things; indeed, presumably the things composed of these are infinite in respect of number. (*De Anima* 1.5, $409^{b}28$–9)

In passage **T4**, Aristotle's remark about infinity does not seem to play an essential role in his argument, and it is perhaps not meant literally.[14] But in passage **T3**, the contrast between finite and infinite does play a role, and it is not obvious how any alternative contrast (such as definite versus indefinite) would yield Aristotle's conclusion.

Next, here is Aristotle explaining why there are no sciences of accidents:

T5 The producer of a house does not produce all the things that result simultaneously with the house's coming into being: for they are infinite. (*Metaph.* E 2,

14. Aristotle's point in the passage is that, under the assumption that 'like is known by like', the claim that a soul is composed of all four elements will not suffice to explain why the soul is capable of knowing all things. To make this point, all he needs is the fact that there are things other than the four elements.

1026^{b}6–7)

And here he is refuting an inadequate view about the unity of plot:

T6 A story is not one, as some suppose, if it is about one person. For infinitely many things befall the one person, and some of these do not constitute anything that is one. (*Poetics* 8, 1451^{a}16–19)

In passages **T5** and **T6**, Aristotle's reasoning would still go through if we take 'infinite' to stand for 'very many' or 'indefinitely many'. Still, we can easily make sense of the texts on the assumption that 'infinite' means infinite. For example, when you build a house, it is true that the thing you build is something taller than a meter, something taller than 1½ meters, something taller than 1¾ meters, and so on. Again, as Aristotle remarks in an allusion to Plato's *Sophist*, the house is something 'other than all the things that are':[15] for example, it is something other than me, something other than my father, something other than his father, and so on. Thus, Aristotle could sensibly say that infinitely many (types of) things result together with a house's coming into being.

Next, here is Aristotle drawing out consequences of the continuity of change:

T7 Everything that changes will have changed infinitely many times. (*Phys.* 6. 6,

15. *Metaph.* E 2, 1026^{b}9. The allusion is to Plato's *Sophist*, 256e5–6 where, after introducing a use of 'not' to mean 'other than', the Stranger says: 'concerning each of the forms, then, *what is* is many, but *what is not* is infinite in plurality' (περὶ ἕκαστον ἄρα τῶν εἰδῶν πολὺ μέν ἐστι τὸ ὄν, ἄπειρον δὲ πλήθει τὸ μὴ ὄν).

237a11, repeated at a16)

Finally, there are a handful of passages in which Aristotle refers to species as finite and to individuals as infinite.[16] This may suggest that he thinks there are infinitely many individual things.

It seems to me that passage **T7** is the most theoretically serious of the bunch. It is noteworthy that it appears within the *Physics*, the same work that contains the finitistic statements **T1** and **T2** quoted previously. More importantly, its infinitistic claim is, I think, unambiguous, and is embedded within a well-developed theory that justifies it. In the immediate context, Aristotle argues that whenever something *changes*, it *has changed* earlier, and also that whenever something *has changed*, it *changes* earlier.[17] To-

16. H. Bonitz, *Index Aristotelicus* (Berlin, 1870) refers us to *Topics* 2.2, 109b14, *Post. An.* 1.24, 86a4, *Rhet.* 1.2, 1356b31.

17. I here dogmatically assert what I think this means. In Aristotle's theory, a phrase of the form 'when *x* changes'—present tense—refers to an interval of time; a phrase of the form 'when *x* has changed'—perfect tense—refers primarily to an *instant* in time (see *Phys.* 6. 6, 236b19–23, 237a15; 6. 5, 235b32–3). (Aristotle allows talk of having changed—perfect—in an interval of time, albeit with special reference to the interval's end limit, at *Phys.* 6. 6, 237a3–5; he never allows talk of changing—present—in an instant.) In the context of *Phys.* 6. 6, it seems that an instant is 'earlier' than an interval if it is earlier than the *end limit* of the interval (compare: 'before you can say "Jack Robinson"'), and an interval is 'earlier' than an instant if the interval's end limit is earlier than or simultaneous with the instant. Thus, if we apply Aristotle's claims in alternation to an interval of time T_1 in which something changes, we first obtain an instant t_1 earlier

gether, these two claims entail that whatever changes has changed infinitely many times. A similar result follows also from an argument given earlier (*Physics* 6. 2, $232^{b}26$–$233^{a}8$), in which Aristotle showed that the time of any motion, as well as the distance that it traverses, has infinitely many parts. Whether or not these parts enjoy actual being, Aristotle writes as if they exist and are available to be referred to, to be named with letters, and to be ascribed determinate properties such as length and relative position.[18]

1.3 Infinite pluralities in Aristotle: outline of our argument

I do not claim that passages **T3–T7** conclusively establish that Aristotle accepted the existence of actually infinite pluralities. Some may be exaggerations, as when someone says of a play that it 'went on forever'. Some may be explained away in other ways. Still,

than the end limit of T_1, then an interval T_2 ending at or before t_1, then an instant t_2 earlier than the end limit of T_2, and so forth.

18. More strongly, certain aspects of Aristotle's phrasing, especially his use of perfect-tense verbs of motion, suggest, in my view, that he thinks of the parts discussed in *Physics* 6 as having actual being. The reason is that Aristotle indicates elsewhere, in *Physics* 8. 8, that there is an actual midpoint of motion wherever a moving thing *has come to be* (perfect) and *has come to be away from* (perfect). Despite some differences in vocabulary, it seems to me that the perfect tense claims of *Physics* 6 may fall under the criterion indicated in *Physics* 8. 8. The proposition that *Physics* 6 treats parts as having actual being is asserted by D. Bostock, 'Aristotle on continuity in *Physics* VI', in Judson, L. (ed.), *Aristotle's Physics: a collection of essays* (Oxford: New York, 1991), 179–212, 180.

on their face they suggest a fairly casual, unconcerned acceptance of the idea that there are infinitely many things. If Aristotle had wanted to avoid this impression, he could have chosen other words. If what he meant by 'infinite' in a given passage was 'very very many', then he could have said 'myriad' (μυρία, or even μυρία μυριάκις, 'myriad myriad', as at *GC* 1.2, 316ᵃ22). If what he meant was 'indefinitely many', he had words for that too.[19] Granted that Aristotle sometimes writes carelessly, still these passages should not be dismissed out of hand. Before we explain them away, we ought to check the strength of our reasons for wanting to explain them away.

Let us return, then, to *Physics* 3. How universally are his claims there meant to apply? Does he really renounce 'the (actual) infinite in *every* form'? It is true that he often speaks simply of 'the infinite', which may seem to suggest that he is talking without restriction of all sorts of infinity. It is also true that, in the introduction to his discussion of the infinite, he mentions a variety of types of quantity, including time, number, and magnitude (*Phys.* 3. 4, 203ᵇ16, 24, 25). Still, when he is ready to get down to business with his own battery of arguments, he makes it clear that he is mainly concerned with the question of infinite *magnitude*:[20]

> It is most proper to the physicist to examine whether there is an infinite percep-

19. Cf. 'the plurality is indefinite', τὸ πλῆθος ἀόριστον, *Phys.* 2. 6, 198ᵃ5. Aristotle speaks, by contrast, of a 'definite plurality' (πλῆθος ὡρισμένον) at *Phys.* 3. 7, 207ᵇ13; *Metaph.* Λ 8, 1073ᵇ13; *NE* 9. 10, 1171ᵃ1.

20. See also below, Section 2.2, 30–33

tible magnitude. (*Phys.* 3. 4, 204^{a}1–2)

Aristotle's focus on magnitude is appropriate. The topic is crucial to Aristotle's physics: for example, his theory of the four elements requires the universe to have an absolute center, whereas an infinitely large universe would have none (cf. *Phys.* 3. 5, 205^{b}31–5; *De caelo* 1. 7, 275^{b}13–15). In the other direction, the question of infinitely small magnitude is likewise important for physics: in Aristotle's view, the correct account of alteration, generation, and destruction depends above all on the fact that there are no minimal, indivisible magnitudes (*GC* 1. 2, 315^{b}24–8). When it comes to plurality, on the other hand, there are no obvious physical implications of the question whether there are, say, actually infinitely many numbers, or actually infinitely many points on a line.[21]

Furthermore, even when Aristotle uses the bare phrase 'the infinite', it is usually clear from context that he is talking about some particular kind of (putative) infinite quantity. I have not found any negative claims about the infinite in *Physics* 3. 4–8 that

21. Some commentators have thought to identify physical implications of infinite plurality, for example by way of Zeno's paradoxes of motion (D. Bostock, 'Aristotle, Zeno, and the Potential Infinite', *Proceedings of the Aristotelian Society*, 73 (1972), 37–51, 40), or by way of an inference from infinite plurality to infinite magnitude (Themistius, *In Phys.* 91.29–30; Simplicius, *In Phys.* 492.16–19; cf. D. Furley, 'Aristotle and the Atomists on Infinity', in Düring, I. (ed.), *Naturphilosophie bei Aristoteles und Theophrast* (Heidelberg, 1969), 85–96, 87–8). I do not agree with these commentators. For discussion, see XXX.

appear, in context, to be completely general and unrestricted. For an especially clear example, let us return to passage **T1**. Quoted in full, the sentence reads:

T1' It is apparent that the infinite cannot exist as something that exists actually *and as a substance and principle*. (*Phys.* 3. 5, 204^{a}20–1)

This passage comes from the middle of an argument designed to show that there is no substance whose very essence is to be infinite. It is clear that the quoted sentence is talking about this very special kind of being. It is not talking about infinite quantity in full generality.

When it comes to Aristotle's more specific claims about one sort of quantity or another, there are two kinds of claim in *Physics* 3 that have been read as denials of the existence of an actual infinite plurality.

The first kind of claim concerns *number*. Aristotle asserts that number is infinite potentially, but not actually (*Physics* 3. 7, 207^{b}11–12). I will discuss this in Section 2. The term 'number' (*arithmos*) in Aristotle, I will argue, signifies a specific kind of plurality. The special character of number, as against plurality more generally, guarantees that every number is finite. Hence, Aristotle's remarks about number in *Physics* 3. 4–8 are nearly tautologies, and Aristotle intends them as such. They are not substantive denials of the proposition that there are (actually) infinitely many things. Their main purpose is to serve as illustrations and analogues that help to clarify the substantive claims that Aristotle makes about magnitudes.

The second kind of claim concerns the *division of magnitudes*. Aristotle says that magnitudes are potentially but not actually 'infinite by division' (*Physics* 3. 6, 206^{a}16–18). This is commonly thought to mean that no magnitude actually has infinitely many parts: in other words, there is never an actually infinite plurality of parts of a magnitude. As I said above, I consider pluralities of parts to be the most important domain of application of the question of infinite pluralities. Therefore I devote a section to this case. I will argue in Section 3 that the phrase 'infinite by division' does not mean 'has (been divided into) infinitely many parts' but rather 'infinitely small'. Aristotle's claim about the infinite by division thus amounts to the proposition that, although there do exist smaller and smaller magnitudes without limit, there does not exist any infinitely small, that is, infinitesimal, magnitude.

There will remain one more issue to deal with, in Section 4. Aristotle ascribes a qualified kind of being to infinite number and to the infinite by division. He describes this qualified kind of being not only in terms of potentiality, but also in terms of coming to be: the infinite has being 'in virtue of always another and another thing coming into being'. This makes it sound as if the qualified being of infinite number consists in the occurrence of a particular kind of process or temporal sequence: presumably, one in which the total number of things continually increases over time. But if Aristotle did not deny the existence of infinite pluralities, it is hard to see why he would think this. If it is possible for there to be actually infinitely many things, then it is possible for every number actually to exist *simultaneously*, not merely one after another in a process. Thus, Aristotle's remarks about 'coming into being' may appear to be evidence that he rejected the possibility of actual infinite pluralities. I will attempt to explain away this appearance.

Before proceeding, I want to pause briefly to clarify the thesis of this paper. I am arguing that Aristotle in *Physics* 3 *does not deny* the existence of an actual infinite plurality. I do not claim to establish that he positively *accepted* the existence of such a plurality. My personal suspicion is that he did accept this, but I do not have an argument for the positive claim. Perhaps it is guaranteed that there are only finitely many entities at any given time, in the universe according to Aristotle. Still, if this is so, I say, then it will be the result of some confluence of physical and metaphysical commitments, and not the manifestation of any principled, universal rejection of infinite quantity. A consequence of my argument, if it is accepted, is that Aristotle's physics and metaphysics are not *driven by* any aversion to actual infinite plurality.

Here is an example of the sort of argument I am *not* concerned with here. Suppose Aristotle had some reason to think that, at any given time, there is a smallest actually existing whole body—that is, a body that is at least as small as every other whole body that actually exists at the time.[22] And suppose he denied, for some reason not having to

22. The germ of such a view may be found in *Physics* 1. 4, 187^{b}13–21. This passage inspired a doctrine of *minima naturalia* in later centuries, whose history is traced in P. Duhem, *Medieval cosmology: theories of*

do with infinity, that wholes have identifiable entities as parts. Then, given the finite size of the cosmos, along with one or two further assumptions, it would follow that there are only finitely many bodies at any given time. Suppose further that Aristotle thought that every non-bodily entity, apart from gods, depends on a body, and also that no single body has infinitely many non-bodily entities depending on it. From all this, it would follow that there are only finitely many entities at a time. The result of this argument would be a downstream kind of finitism about plurality, with which the present paper is not concerned.

2 NUMBER

My first task is to examine what Aristotle says about infinite number in *Physics* 3. 4–8.

In *Physics* 3. 4, Aristotle lists number alongside mathematical magnitude and 'that which is outside the heaven' as things that are sometimes held to be infinite because they do not 'give out in thought' (3. 4, 203^{b}23–5). He then raises the question of infinite plu-

infinity, place, time, void, and the plurality of worlds (Chicago, 1985), 35-45; A. Maier, 'Das Problem des Kontinuums in der Philosophie des 13. und 14. Jahrhunderts', *Antonianum*, 20 (1945), 331–68, 350-61; R. Glasner, 'Ibn Rushd's Theory of Minima Naturalia', *Arabic Sciences and Philosophy*, 11 (2001), 9–26; J. Murdoch, 'The medieval and renaissance tradition of Minima naturalia', in Lüthy, C., Murdoch, J.E., and Newman, W.R. (eds.), *Late medieval and early modern corpuscular matter theories* (Leiden; Boston, 2001), 91–131; J. Mcginnis, 'A Small Discovery: Avicenna's Theory of Minima Naturalia', *Journal of the History of Philosophy*, 53 (2015), 1–24.

rality, asking 'whether there is something infinite *or infinite* (plural) *in plurality*' (3. 4, 203^b34–204^a1, emphasis added). At the beginning of *Physics* 3. 6, he says that if there is no infinity whatsoever, then a consequence would be that 'number will not be infinite' (3. 6, 206^a9–12). Since he treats this consequence as unacceptable, and as a reason against completely eliminating the infinite, we may take him to affirm that number is in some way infinite.

On the other hand, there are two passages in which Aristotle denies the actual and unqualified existence of infinite number. In *Physics* 3. 5 he asserts:

> (i) there is no 'separate and infinite' number (3. 5, 204^b7–8).

And in *Physics* 3. 7, he asserts:

> (ii) number is 'potentially but not actually' infinite (3. 7, 207^b11–12);
>
> (iii) 'the infinity' (of number) 'does not persist but becomes' (207^b14).

These negative claims about the infinity of number are supported by a single brief argument. The argument reads as follows:

COUNTABILITY ARGUMENT: But indeed, neither is there an infinite number, in such a way as to be separate and infinite. For a number, or that which has number, is countable. If, then, it is possible to count that which is countable, then it would also be possible to go through the infinite. (*Phys.* 3. 5, 204^b7–10).

2.1 Number and plurality

Aristotle's rejections of actual infinite number, together with his countability argument, are sometimes read as a general denial of the existence of actually infinitely many things.[23] Such readings assume that Aristotle thinks himself entitled to an additional premise: namely, the premise that for any things, there is a number of all those things. If he adopted this premise, Aristotle could argue as follows:

1. For any things, there is a number of them all (premise)
2. Every actual number is finite (countability argument)
3. So, for any actually existing things, there is a finite number of them all (1,2)
4. So, there are not actually infinitely many things (3)

As I will now argue, however, there is good reason to doubt that Aristotle would adopt the additional premise. According to Aristotle, number is only a species of a more general kind of quantity, which he calls 'plurality' (πλῆθος). In *Metaphysics* I 6, for example, Aristotle writes:

'Plurality is, as it were, the genus of number; for a number is a plurality measur-

23. Aristotle's arguments about number are interpreted as applying to all plurality by E. Zeller, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung* (Tübingen, 1862), 294-5; Scholz, 'Irrationalzahlen', 55 n. 2; (seemingly) Hussey, *Physics III–IV*, 80, 82; Bowin, 'Aristotelian Infinity', 249.

able by one.' (*Metaph.* I 6, 1057^{a}2–4)

English translations of Aristotle do not typically show whether Aristotle is using the Greek word for 'number' or for 'plurality' in any given passage.[24] But at least in the present context, the difference between these terms is crucial. If number is only a species of plurality, then the claim that every number is finite does not entail the claim that every plurality is finite. Aristotle's countability argument leaves open the possibility that there are infinitely many things, things of which there is a plurality but no number.

In the remainder of the present section, I will review what Aristotle says about the relation between number and plurality, as well as noting some differences in his usage of the two terms. This will establish, I hope, that 'number' is officially a narrower term than 'plurality'. In the following section (2.2), I will look at the overall structure and content of *Physics* 3. 4–8, to see if there is any pressure from context in favor of reading Aristotle's argument and assertions about number more strongly than their literal reading. I

24. Translators often render both πλῆθος and ἀριθμός as 'number', including in the translation of *Physics* 3. 4–8. To give a few arbitrary examples, in Barnes, J., *The complete works of Aristotle: the revised Oxford translation* [*ROT*] (Princeton, N.J., 1984), πλῆθος is translated 'number' at *Metaph.* Α 3, 983^{b}19; Λ 8, 1073^{a}16; *Phys.* 2. 6, 198^{a}5, as well as at *Phys.* 3. 5, 204^{b}13. Additionally, 'number' is sometimes used in translations of passages that have no corresponding noun in the Greek: in the translation of *Physics* 3. 4–8, this occurs at 3. 4, 203^{a}19–20 ('limited in number', 'infinite in number'), 3. 5, 205^{a}29–30 ('infinite in number'), 3. 6, 206^{b}28 ('two in number'), 3. 7, 207^{b}10–11 ('the number of times a magnitude can be bisected is infinite') and 207^{b}12 ('the number of parts that can be taken always surpasses any definite amount').

will conclude that there is no such pressure, because Aristotle does not show a strong interest in the question of infinite plurality.

As mentioned above, Aristotle describes plurality as the genus of number in *Metaphysics* I 6, 1057^{a}2–4. In that passage, he describes the specific difference of number as 'measurable by one'. Aristotle does not explain what sorts of plurality *fail* to be measurable by one, but, given that a species always applies more narrowly than its genus,[25] he must think that such pluralities exist, or are at least conceivable. An infinite plurality is a plausible candidate for failure of measurability, since, as Aristotle notes in the *Physics*, something infinite is not measurable.[26] Though I am not in a position to provide a detailed reading of this very difficult chapter, *Metaphysics* I 6 appears to support the idea that (a) numbers are a special kind of plurality, and (b) there are properties special to number, as against plurality more generally, that guarantee finitude.

A second passage concerning the relation between number and plurality appears in *Metaphysics* Δ 13. Aristotle has divided the genus of quantity into plurality on the one hand, and magnitude on the other, and he has subdivided magnitude into length, width, and depth. He then states the following:

> Of these, plurality that is finite is number, length (that is finite(?)) is line, width is

25. *Topics* 4. 1, 121^{b}11–14; 4. 2, 123^{a}6–7; 4. 3, 123^{a}30; 4. 6, 128^{a}22–3; *Metaph.* Δ 3, 1014^{b}12–14

26. *Physics* 6. 7, 238^{a}12–15

> surface, and depth is body. (*Metaphysics* Δ 13, 1020ᵃ13–14)

Now, Aristotle does not explicitly state in this sentence that plurality is the genus of number, nor does he name a species of plurality other than number. Still, because of the sentence's phrasing and its position at the end of a series of divisions, the sentence strongly gives the impression that number stands to plurality as species to genus, with 'finite' as its differentia.[27] If this is correct, then *Metaphysics* Δ 13, like I 6, supports the claim that numbers are a special kind of plurality, and that finitude is special to number as against plurality more generally. (I will return below to a complication with *Metaphysics* Δ 13.)

A final passage worth noting appears in *Posterior Analytics* 1. 22, 84ᵃ11–17. In this passage, Aristotle reviews two ways in which a predicate may hold *per se* of a subject. One way is if the subject is mentioned in the definition of the predicate; the other way is if the predicate is mentioned in the definition of the subject. Aristotle gives the following example of each of the two ways:

> For example, odd of number—odd holds of number and number itself inheres in its account; and conversely, plurality, or the divisible, inheres in the account of number. (*Post. An.* 1. 22, 84ᵃ14–17, tr. J. Barnes (trans. and comm.), *Posterior ana-*

27. Similarly for body in relation to depth, as noted by H. Mendell, 'What's Location Got to Do with It? Place, Space, and the Infinite in Classical Greek Mathematics', in Risi, V. (ed.), *Mathematizing Space: The Objects of Geometry from Antiquity to the Early Modern Age* (Cham, 2015), 15–63, 47

lytics (Oxford, 1993))

Here, Aristotle says that plurality features in the definition of number. This suggests that plurality is either a genus or a differentia of number.[28] Since 'plurality' is a noun and not an adjective, it seems most likely that plurality is a genus. So this passage, again, confirms that numbers are a special kind of plurality.

In addition to these explicit statements about the relation between number and plurality, there is a noteable difference in Aristotle's concrete usage of the two terms. On the whole, the word 'number' is somewhat more common in the Aristotelian corpus than 'plurality', with 944 and 656 occurrences, respectively.[29] But in contexts where the notion of infinity is salient, 'plurality' is far more frequent than 'number'. Thus, we find the phrase 'infinite in respect of plurality' (ἄπειρα τὸ πλῆθος) five times (including a fragment), the phrase 'infinite in plurality' (πλήθει ἄπειρα) five times, and the phrase 'infinite according to plurality' (ἄπειρον κατὰ πλῆθος) twice.[30] Aristotle describes things as

28. Cf. *Post. An.* 1. 22, 83^{b}1; 2. 13, 97^{b}2–5

29. These numbers are from a search of the Aristotelian corpus in the TLG. (This includes dubious and spurious works.)

30. ἄπειρα τὸ πλῆθος: *GC* 1. 1, 314^{a}22; 1. 8, 325^{a}30; *Phys.* 1. 4, 187^{b}34; 2. 5, 197^{a}16; Fragment 5.32.208.17. πλήθει ἄπειρα: *De caelo* 3. 4, 303^{a}5–6; *Metaph.* α 2, 994^{b}28; I 6, 1056^{b}29; *Phys.* 3. 4, 203^{b}34–204^{a}1; *Pol.* 1. 8, 1256^{b}35. ἄπειρον κατὰ πλῆθος: *Phys.* 1. 4, 187^{b}8, 10

'finite in plurality' (πεπερασμένα πλήθει) four or five times.[31] By contrast, he uses the phrase 'infinite in respect of number' (ἄπειρα τὸν ἀριθμόν) only twice, and 'finite in respect of number' (πεπερασμένα τὸν ἀριθμόν) only once.[32] This fits with the evidence we saw above according to which there is a special association between number and finitude.

Aristotle's usage may be compared with that of Plato in his *Parmenides*, a text that appears to have influenced Aristotle's thinking about quantity.[33] On the whole, Plato uses the words 'number' and 'plurality' equally often in the *Parmenides* (25 and 22 occurrences, respectively). But the phrases 'infinite in respect of plurality' and 'infinite in

31. *Phys.* 3. 5, 204^{b}12; 6. 7, 238^{a}14; *De caelo* 1. 5, 271^{b}21, ^{b}22; *Metaph.* K 10, 1066^{b}28 (= *Phys.* 3. 5, 204^{b}12). This adds up to four or five depending on whether we count the *Metaphysics* passage separately from the *Physics* passage that it duplicates.

32. *DA* 1. 5, 409^{b}29; *SE* 1, 165^{a}12; *De Sensu* 6, 446^{a}19. Note: according to my argument it would be a contradiction in terms to say that there is *an infinite number* of things, but there is no contradiction in speaking of things as infinite *in respect of number*. In the latter phrase we use 'number' to specify the axis, as it were, along which the quantity in question is infinite. Such usage is, I think, not to be preferred, but neither is it strictly incorrect.

33. Waschkies, for example, traces parallels between Plato's *Parmenides* and Aristotle's *Physics* 6, in H.-J. WASCHKIES, *VON EUDOXOS ZU ARISTOTELES: DAS FORTWIRKEN DER EUDOXISCHEN PROPORTIONENTHEORIE IN DER ARISTOTELISCHEN LEHRE VOM KONTINUUM* (AMSTERDAM, 1977)

plurality' appear a total of nine times;[34] by contrast, the *Parmenides* does not contain any phrase such as 'infinite in number'. Thus, Plato is even more consistent than Aristotle on this point of usage.

Returning to Aristotle, there are two suggestive passages in which Aristotle says that *if* some principles or elements are finitely many, *then* we should ask what their number is.[35] Finally, at one point in *Physics* 3. 5, Aristotle speaks of one quantity 'having a number' in relation to another quantity in order to express the claim that the first is a finite multiple of the second.[36]

In sum, Aristotle's explicit statements about the relation between plurality and number, and also his actual usage of the two terms, support the view that numbers are a special kind of plurality, and that numbers are required to be finite in a way in which pluralities are not.

With this in view, let us return briefly to Aristotle's countability argument in *Physics* 3. 5, and ask whether it would make a convincing argument for the claim that every plurality is finite. The central premise of the argument is that every number is countable.

34. Part One: 132b2. Hypothesis II: 143a2, 144a6, 144e4–5, 145a3. Hypothesis III: 158b6, 158c6–7. Hypothesis VII: 164d1, 165c2.

35. *Phys.* 1. 2, 184^{b}19; *De caelo* 3. 4, 302^{b}10

36. '…if an equal amount of fire is so-and-so many times more powerful than an equal amount of air, provided only that it has some number…', *Phys.* 3. 5, 204^{b}17–18.

Would Aristotle have felt entitled to the premise that every *plurality*, and not just every number, is countable in the sense required by the argument? One sentence in *Metaphysics* Δ 13 might seem to suggest so (see Section 2.3 below). But Aristotle's argument exploits an etymological relation between the word 'number' and the word 'count', which is not available for the word 'plurality'. Further, we have seen that being 'measurable by one' is special to number, and this suggests that being countable may also be special to number. Finally, it is worth noting that the word 'uncountable' (ἀναρίθμητος) is applied fairly often in Greek literature to pluralities that are very large.[37] All in all then, the premise that every plurality is countable would be anything but secure. This is reason to think that Aristotle understood his argument as applying narrowly to number, not widely to plurality. (If you are doubting whether Aristotle would argue for a claim about number that is practically true by definition, note that the argument comes immediately after a similarly tautologous argument about magnitude, based on a definition of body as 'that which is limited by a surface': *Phys.* 3. 5, 204^{b}5–7.)

A similar point can be made about an argument that Aristotle gives in *Metaphysics* M 8, his only other argument in the corpus against the existence of an infinite number:

37. Sophocles, *Ajax*, 646; Herodotus, *Histories*, 1.126 l. 21, 2.134 l. 9, 7.190 l. 3, 7.211 l. 15, 9.79 l. 11; Isocrates, *Panegyricus*, 93.7, *De pace*, 118.3, *Antidosis*, 171.4, *Panathenaicus*, 98.6, *Ad Archidamum* 8.6; Aristophanes, *Vespae*, 1011; Xenophon, *Anabasis* 3.2.13, *Cyropaedia* 7.4.16, *De Vectigalibus* 4.25; Plato, *Theaetetus*, 175a3, *Laws* 804e6; Aristotle, *De caelo*, 292^{a}12.

> 'It is clear that number cannot be infinite; for an infinite number is neither odd nor even, but the generation of numbers is always the generation either of an odd or of an even number.' (*Metaph.* M 8, 1084ᵃ2–4)

The central premise of this argument is that every number is either even or odd. Now, Aristotle writes in the *Posterior Analytics* that odd and even should be defined as attributes of number.[38] From this it follows that nothing other than a number is even or odd.[39] Therefore, given that number is a species of plurality, he cannot adopt the premise that every plurality is either even or odd. The argument must be targeted narrowly at numbers. (Indeed, it seems to be targeted even more narrowly, against a specific philosophical conception of numbers. It is part of a polemic against Pythagorean and Platonist views of numbers as ideal, separate beings that are principles of things.)

2.2 Context: number means number

I have just advocated for the following claims. (1) Aristotle's explicit statements about the relation between number and plurality, in *Metaphysics* Δ 13, *Metaphysics* I 6, and *Posterior Analytics* 1.22, indicate that number is a species of plurality, and that there are properties special to number, as against plurality more generally, that guarantee finitude. (2) This indication is reinforced by the fact that Aristotle almost always, and Plato's Par-

38. *Post. An.* 1. 4, 73ᵃ37–40; 1. 22, 84ᵃ14–17; 2.13

39. 'Nothing outside number is odd', *Post. An.* 2. 13, 96ᵃ31–2

menides without exception, prefers phrases like 'infinite in *plurality*' over phrases like 'infinite in *number*'. (3) Aristotle's finitistic arguments in *Physics* 3. 5 and *Metaphysics* M 8 are convincing arguments about number, but would not be convincing arguments about plurality more generally.

Aristotle does not explicitly say that there are no actually infinite pluralities in *Physics* 3. 4–8. He only says this about numbers. I have been arguing that we should take him to mean what he says and no more. To this end, there is one more question to consider. Is there any reason deriving from context, or from the overall flow of argument in *Physics* 3. 4–8, that should make us think that Aristotle wants his claims to apply to plurality more generally than to number?

I believe the answer is 'no'. It is hard to prove this sort of negative claim, but one point at least should be made. Namely: Aristotle does nothing to suggest a strong interest in the question of infinite plurality one way or the other. The primary topic of the treatise is not plurality, nor quantity in general, but magnitude. Aristotle explains this in *Physics* 3. 4 and 3. 5:

> The main task of the physicist is to examine whether there is a perceptible *magnitude* that is infinite. (*Phys.* 3. 4, 204^{a}1–2, emphasis added)

> We are considering about perceptible things—the things our inquiry [*sc.* the *Physics*] is about—whether or not there is among them a *body* that is infinite in

> the direction of growth. (*Phys.* 3. 5, 204^{b}1–5, emphasis added)

These statements are borne out in Aristotle's practice. In his arguments against actual infinite quantities, contained in *Physics* 3. 5, Aristotle devotes 82 Bekker lines to magnitude, and especially body, whereas his argument about number (quoted in full above) barely stretches to four lines.[40] Again, in *Physics* 3. 6, when Aristotle discusses his distinctions between the potentially infinite and the actually infinite, and between the infinite by addition and the infinite by division, he indicates repeatedly that the main topic of his discussion is magnitude.[41] Apart from a premise introduced in service to an argument about magnitude (*viz.*, that it is impossible that there be infinitely many elements with infinitely many natural places, *Phys.* 3. 5, 205^{a}30–1), the topic of number or plurality does not recur until chapter 3. 7. And even in *Physics* 3. 7, the main role of number seems to be that of providing a contrast to the case of magnitudes: Aristotle makes the point that numbers collectively have a lower bound but no upper bound, whereas magnitudes collectively have an upper bound and no lower bound. Finally, in the brief concluding chapter, *Physics* 3. 8, in which Aristotle responds to three arguments in favor of an actual infinite, his responses mention magnitude but not plurality.[42]

40. On magnitude, I count 204^{b}5–6, 204^{b}11–205^{b}1, 205^{b}24–206^{a}8. On number, 204^{b}7–10.

41. *Phys.* 3. 6, 206^{a}16, 27, ^{b}1, 7–11, 19, 25

42. In *Phys.* 3. 8, Aristotle explains that (1) no 'actually infinite perceptible body' (208^{a}9) is needed to secure the inexhaustibility of becoming; (2) being finite does not entail being in contact with something; (3)

If Aristotle's treatise on the infinite had displayed a strong overall interest in settling questions about infinite plurality, this would be a reason to interpret him as settling such questions, even if he does not do so explicitly. However, we do not have a reason of this kind for interpreting his claims about number as applying to plurality in general.

Let us also look closely at the immediate context of Aristotle's argument about number, in *Physics* 3. 5. The structure of the surrounding text is as follows:

(i) Our question is whether there is a perceptible body that is infinitely large.

(ii) Considering the matter 'logically', there is no infinite body because….

(iii) And indeed, there is also no infinite number because…. — *four lines.*

(iv) Considering the matter 'physically', … — *scores of lines, all about body*.

Point (iii), Aristotle's argument about number, is not inferentially integrated into the surrounding argument: it does not support or get supported by any of the surrounding claims or arguments. Thus, its immediate context does not signal that we should interpret its terms in any special way. As best as I can tell, the main reason for the argument's position in the text is that it illustrates the 'logical' style of argument at work in point

thinking of something as bigger doesn't entail that it is or can be bigger. The third point could be relevant to number and plurality (cf. *Phys.* 3. 4, $203^{b}24$), but Aristotle does not mention number or plurality in *Physics* 3. 8.

(ii), Aristotle's argument about body.[43] Arguments (ii) and (iii) have in common a quasi-tautological flavor: first the finitude of every body is inferred from a definition of body as 'that which is limited by a surface'; then the finitude of every number is inferred from the etymological-*cum*-conceptual connection between number (*arithmos*) and counting (*arithmein*). Nothing here suggests that Aristotle intends to establish something stronger than what he literally says, namely that no individual number is infinite. If there are pluralities that are not numbers, I see no reason to think that he intended his conclusion to apply to them.

2.3 A Return to *Metaphysics* Δ 13

Above (Section 2.1), I cited *Metaphysics* Δ 13 in support of the claim that finitude is a distinctive property of numbers as against pluralities more generally. But at least one commentator has cited the same chapter of the *Metaphysics* in support of a conflicting claim, namely, the claim that 'quantity is, by nature, measurable or countable'.[44] From such a reading of the chapter one could arrive at the result that, since all pluralities are countable, all pluralities are finite. It is necessary to examine the relevant passage in

43. The difference between arguing 'logically' and 'physically' is discussed in M. Burnyeat, *A map of Metaphysics Zeta* (Pittsburgh, 2001), 19-25, 87, 124-125

44. Bowin, 'Aristotelian Infinity', 249

some detail, and consider which is the more satisfactory interpretation. This is the task of the present section.

In the opening lines of Metaphysics Δ 13, Aristotle offers the following definition and classification of quantities:

> [i] We call 'quantity' what is divisible into constituents of which both, or each, is such as to be a 'one' and a 'this'. [ii] Now a quantity is a plurality if it is countable, and a magnitude if it is measurable. [iii] We call 'plurality' what is potentially divisible into items that are not continuous, and 'magnitude' into items that are continuous. [iv] Of magnitude, what is continuous in one direction is length, in two directions width, and in three directions depth. [v] Of these, plurality that is finite is number, length (that is finite(?)) is line, width is surface, and depth is body. (*Metaphysics* Δ 13, 1020^{a}7–14)

Aristotle gives a general characterization of quantity in sentence [i], and then divides quantity into plurality and magnitude in sentences [ii] and [iii]. We will return below to the interpretation of [ii]. In a further step, in sentence [iv], he divides magnitude into length, width (or rather, area), and depth (or rather, volume).[45] Finally, in sentence [v], he states that a finite plurality is a number and that a finite length, width (area), or depth

45. For the view that 'width' and 'depth' mean area and volume here, see C. Kirwan (trans. and comm.), *Aristotle's 'Metaphysics', Books Γ, Δ, and E* (Oxford, 1971), 160-1

(volume) is a line, surface, or body, respectively.[46] By defining a number as a finite plurality, Aristotle implies—though he does not explicitly state—that pluralities in general need not be finite.

At this point, however, we appear to face a puzzle. In sentence [ii], Aristotle may be read as implying—though, again, he does not explicitly state—that a quantity is a plurality if and only if it is countable. (He explicitly states the 'if', not the 'only if'.) We know from *Physics* 3. 5 that countability implies finitude. Hence, sentence [ii] may seem to imply that every plurality must be finite. Sentence [v], on the other hand, implied that a plurality, unlike a number, need not be finite. How should we deal with the seeming tension between these two sentences?[47] Should we explain away the apparent implicature of sentence [ii], or the apparent implicature of sentence [v]? I will argue that we should honor the implicature of sentence [v], and that there are good reasons for choosing to explain away sentence [ii].

46. It is not certain whether 'finite' should be understood with 'length', 'width', and 'depth' as well as 'plurality' in the third step. In the view of Alexander (*in Metaph.* 396.32–3), it should not—Alexander says that Aristotle simply identifies line, surface, and body with length, width, and depth in the latter part of the sentence. The view that length is the genus of line is attested in *Topics* 6. 6, 143^{b}11–34, esp. ^{b}16. But in some other passages, Aristotle treats length and line as equivalent. This issue does not affect my purposes in the present paper.

47. For a recent discussion of the tension between the two sentences, see C. Pfeiffer, *Aristotle's theory of bodies* (Oxford, 2018), 204-5

First, the implicature generated by sentence [v] is quite strong. The context has the shape of a series of definitions by genus and differentia. Within this series, sentence [v] seems to express the view that *plurality* and *finite* are, respectively, the genus and differentia of number.[48] (Likewise, sentence [v] seems to say that *length*, *width*, and *depth* are the genera of line, surface, and body, respectively, while *finite* is the differentia of each.[49]) Now, Aristotle is committed to certain doctrines about genera and differentiae. Consider the following method for refuting claims to the effect that one thing is a genus of another, recommended by Aristotle in the *Topics*:

> See if the species and the (*sc.* purported) genus have an equal denotation … The elementary principle in regard to all such cases is that the genus has a wider denotation than the species and its differentia; for the differentia too has a narrower denotation than the genus. (*Topics* 4. 1, 121^{b}4–5, 11–14)[50]

48. So Pfeiffer, *Aristotle's theory of bodies*, 204

49. Aristotle twice denies that the same differentia can appear in two different genera when neither is subordinate to the other (*Categories* 3, 1^{b}16–24, *Topics* 1. 15, 107^{b}19–26). But in *Topics* 6. 6, 144^{b}20–30, Aristotle allows this, provided that both genera are subordinate to a single genus in common. Since plurality, length, width, and depth are all subordinate to the genus *quantity*, it is thus allowable for the same differentia, *finite*, to appear in each of them.

50. tr. *ROT.* 'has an equal denotation' and 'has a wider denotation' translate ἐπ' ἴσον λέγεται and ἐπὶ πλέον λέγεται, respectively.

In this passage, Aristotle states that every species and every differentia applies more narrowly than the genus to which it is subordinate. He repeats this claim elsewhere in the *Topics*.[51] Hence, if 'plurality that is finite' is a definition by genus and differentia of number, then *finite* and *number* must each apply more narrowly than *plurality*. In defining number as he apparently does in sentence [v], Aristotle commits himself to the view that not all plurality is number and not all plurality is finite.

Christian Pfeiffer has suggested that the word πεπερασμένον, which I have been translating as 'finite', does not mean 'finite' but rather 'determinate' in the context of sentence [v]. This would mitigate the tension between sentences [ii] and [v], and Pfeiffer develops his proposal in a philosophically interesting way. However, I have not found any parallels for such a sense of πεπερασμένον in Aristotle's usage. It may be that the word means something other than 'finite' in Aristotle's reports of the views of Parmenides, Melissus, and the Pythagoreans.[52] But in a discussion of quantity, and in Aristotle's own voice, it seems very unlikely that the word means anything other than 'finite'.[53] (In addition, it is not clear to me that an indeterminate plurality would be

51. *Topics* 4. 2, 123^{a}6–7; 4. 3, 123^{a}30; 4. 6, 128^{a}22–3. See also *Metaph.* Δ 3, 1014^{b}12–14.

52. *Metaph.* A 5, 986^{a}18–19 (of the even and the odd, one is πεπερασμένον and the other is ἄπειρον), 986^{b}18–21 (the One is πεπερασμένον according to Parmenides, ἄπειρον according to Melissus); *NE* 2. 5, 1106^{b}29–30 (the bad belongs to the ἄπειρον, the good to the πεπερασμένον).

53. I base this claim on a survey of perfect participle forms of περαίνω in the Aristotelian corpus. The

countable. This means that the tension with sentence [ii] might still remain.) In sum, sentence [v] indicates quite strongly that 'infinite plurality' is a kind of quantity, with 'infinite' really meaning infinite.

Meanwhile, there is some independent reason to be cautious of sentence [ii]. First of all, in sentences [ii] and [iii], Aristotle appears to perform the same division of quantity—namely, into plurality and magnitude—twice over. This makes the passage read oddly, since Aristotle's phrasing does not suggest a do-over, but simply a movement from one point to the next.[54] Second, as Bonitz pointed out, dividing quantity by means of the predicates *countable* and *measurable* is not satisfactory, because quantities that are countable are also measurable.[55] Thus, for example, Euclid defines a prime number as 'that which is measured only by a unit', and a composite number as 'that which is measured by another number' (Euclid VII, defs 11, 13).[56] Sentence [ii] thus seems to yield the result that numbers are both pluralities and magnitudes, which is unacceptable.

TLG returned 290 hits; apart from the three passages cited in n. 52, I did not notice anything that might supply a parallel supporting Pfeiffer's proposal.

54. The two sentences lead with the particles μὲν οὖν and δέ, respectively. It would be easier to read [iii] as going over the same ground as [ii] if it led with πάλιν or ἔτι or καί ('again', 'furthermore', 'also').

55. H. Bonitz and Aristotle, *Aristotelis Metaphysica* (Bonnae, 1848), 257

56. For talk of numbers as measured or measurable in Aristotle, see for example *Post. An.* 2. 13, 96^{a}36; *Metaph.* Δ 15, 1021^{a}12–13; I 1, 1052^{b}20–22; I 6, 1056^{b}23–4; N 1, 1088^{a}4–6

Studtmann proposes that numbers are measured 'in a different sense' from magnitudes,[57] but the standard ancient technical notion of measurement applies to both, without significant difference in sense.[58]

Let me return to the first of these two points, about the redundancy between sentences [ii] and [iii]. Labeling them according to what they say about plurality, these sentences state the following:

COUNTABLE (SENTENCE [ii]): a quantity is a plurality if it is countable; a quantity is a magnitude if it is measurable.

57. P. Studtmann, 'Aristotle's Category of Quantity: A Unified Interpretation', *Apeiron*, 37 (2011), 69–91, 82

58. The notion of measurement is not defined in Euclid nor, as far as I know, in any ancient text. Mueller explains the notion as follows: 'one positive integer measures a second when it divides the second evenly, i.e., when the second positive integer can be segregated into some number of parts each equal to the first. The notion of measurement can, of course, be applied geometrically as well, and Euclid does do so' (I. Mueller, *Philosophy of mathematics and deductive structure in Euclid's Elements* (Cambridge, Mass., 1981), 61). Menn gives a similar explanation: 'X is measured by Y iff X is the sum of finitely many equal constituents each equal to Y' (S. Menn, 'Ig2a: Iota on unity, and consequences for the ἀρχαί', in *The Aim and Argument of Aristotle's Metaphysics* (2018), 7). If notions such as 'segregated', 'sum', and 'equal' have different senses in application to numbers and to magnitudes, then 'measurement' will have correspondingly different senses. But it is not clear to me that (Aristotle would have thought that) the antecedent of the foregoing conditional is true.

DISCRETE (SENTENCE [iii]): a plurality is a quantity that is divisible into non-continuous items; a magnitude is a quantity that is divisible into continuous items.

Both of these statements may strike the eye as exhaustive divisions of quantity and as definitions of the two species of quantity. On the other hand, as noted above, Aristotle's phrasing does not suggest that he intends to be dividing the same thing into the same pair of species twice over. Of the two statements, it seems to me that DISCRETE deserves to be taken more seriously.[59] That is, DISCRETE has better claim than COUNTABLE to be read as part of Aristotle's official sequence of definitions in *Metaphysics* Δ 13. And DISCRETE, more than COUNTABLE, is formulated on its face as an exhaustive division of quantity. But if COUNTABLE is not a pair of definitions, and is not clearly intended as an exhaustive division, then there is little pressure to interpret its 'if's as 'if and only if's. And if its 'if's are not 'if and only if's, then the sentence does not imply that every plurality is countable.

Here is a reason why DISCRETE looks more like part of Aristotle's official sequence of definitions. Key terms in DISCRETE, namely 'divisible' and 'continuous', appear in the definitions that precede and follow it in sentences [i] and [iv]. 'Divisible' was a key term in

59. In addition to the following arguments in the main text, we may note that sentence [iii] contains the word λέγεται ('is called') whereas sentence [ii] does not. Throughout *Metaphysics* Δ, Aristotle uses the word λέγεται in his leading characterizations of key terms. This is evidence that sentence [iii] belongs more centrally to the project of Δ than sentence [ii]. I owe this observation to Marko Malink.

sentence [i], and 'continuous' is a key term in sentence [iv]. It thus results that in the sequence of sentences [i], [iii], [iv], each subsequent definition subdivides a differentia from its predecessor. First, a quantity was defined as something *divisible* into such-and-such constituents; then, *divisible* is split into what is divisible into *non-continuous* or *continuous* constituents; finally, *continuous* is split into what is continuous *in one*, *two*, or *three directions*. This structure is a desirable, even if not indispensable, feature in definitions.[60] With COUNTABLE, by contrast, the terms 'countable' and 'measurable' neither look back in any clear way to the initial definition of quantity, nor, going forward, is there mention of counting or measurement in the remainder of Δ 13.

Furthermore, the wording of DISCRETE, more than that of COUNTABLE, makes it appear on its face as an exhaustive division of quantity. This is because DISCRETE employs a pair of contradictory predicates, 'not continuous' and 'continuous'. Since quantity has just been defined as divisible, it suggests itself that every quantity must be divisible *either* into items that are not continuous, *or* into items that are continuous. By contrast, there is nothing obvious in the text to suggest that every quantity—that is, everything that is divisible into items that are 'ones' and 'thises'—must either be countable or

60. See *Parts of Animals* 1. 3, together with the comments on this chapter in D.M. Balme (trans. and comm.), *Aristotle's De partibus animalium I and De generatione animalium I (with passages from II. 1-3)* (Oxford : New York, 1992), 101, 103 and J.G. Lennox (trans. and comm.), *On the parts of animals* (Oxford, 2001), 165-6; *Metaphysics* Z 8, 1038^{a}9–15.

measurable. So, again, there is reason to treat DISCRETE as a pair of definitions that exhaustively divides quantity, and to interpret COUNTABLE in some other way.

I offer, somewhat tentatively, a final consideration against reading sentence [ii] (COUNTABLE) in such a way that it entails that all quantities are finite. It seems to me that the purpose of Aristotle's discussion of philosophical terms in *Metaphysics* Δ is not only to help Aristotle himself discover and state the truth more clearly, but also to help him state and explain the views of other philosophers with whom he is in dialogue.[61] Thus, I expect his definitions in Δ to guide how he describes, in his own words, the views of other philosophers. Given this, if COUNTABLE implied that all quantities are finite, this would have somewhat surprising terminological and classificatory implications. Abstracting from the question whether there are any things that are infinite (as a number of philosophers other than Aristotle believed there were[62]), COUNTABLE would tell us something about how to *classify* the predicate 'infinite'. It would tell us that *infinite* does not belong under the heading *quantity*. (I believe I am in agreement with Alexan-

61. Towards this second purpose, Aristotle sometimes comments on how other philosophers use the terms that he discusses in *Metaphysics* Δ. For example, in his discussion of the term *element* in Δ 3, he explains why some philosophers say (as Aristotle himself does not) that genera are elements (1014^{b}9–11). Again, the distinction between *monad* and *point* in Δ 6, 1016^{b}25–6, 30–1, seems to apply to Pythagorean and other views more than to Aristotle's own: it is not clear that Aristotle believes that there are indivisible objects without position (the definition of *monad*).

62. Democritus, Leucippus, Anaxagoras, Plato (REFS)

der on this point.[63]) Hence, for example, if we wanted to say how many atoms Democritus thought there are, COUNTABLE would tell us to say that, according to Democritus, *there is no quantity* of all the atoms in the universe.

This is not how Aristotle talks, however. In *Physics* 1. 2 and 3. 5, Aristotle unambiguously classifies the infinite as belonging to the category of quantity.[64] When he describes the views of Democritus, he says that, according to him, atoms are 'infinite in plurality'.[65] Thus, Aristotle includes the supposed infinity of atoms under the category of quantity, and, in particular, under plurality. So, then, it seems he ought not to define infinity and quantity in such a way that they are mutually exclusive. Given that we are not forced to read COUNTABLE as doing this, I think it best not read it so.

If COUNTABLE is not a definition of plurality and magnitude, nor an exhaustive division of quantities into countable and measurable, we might wonder what it is doing in

63. In Alexander's commentary (*in Metaph.* 396.14–18), COUNTABLE raises the question whether the infinite is 'under quantity' (ὑπὸ τὸ ποσόν, 17), and Alexander proposes a special meaning for 'measurable' in order to reach an affirmative answer. This is desirable, he thinks, because the infinite is defined as 'untraversable quantity' (ποσὸν ἀδιεξίτητον, 17–18). Thus, Alexander seems to be concerned about definition and classification, not about the substantive question whether anything is in fact infinite.

64. *Physics* 1. 2, 185^{a}32–4 ('Melissus says that what is is infinite. Hence, what is is a quantity, since the infinite is in quantity'), 185^{b}2–3 ('the definition of infinite makes use of quantity'); 3. 5, 204^{a}28–9 ('it is impossible for the actually infinite to be indivisible; for it is necessarily a quantity').

65. ἄπειρα τὸ πλῆθος, *GC* 1. 1, 314^{a}22; πλήθει ἄπειρα, *De caelo* 3. 4, 303^{a}5

the text at all. I do not have a definitive answer to offer, but I have a suggestion. Aristotle's account of quantity in the first sentence of *Metaphysics* Δ 13 is highly abstract. Its connection to intuitive notions of quantity is not obvious. Perhaps the purpose of Aristotle's statement about countable and measurable quantities is to help the reader connect the technical characterization of quantities, as things that are divisible in a certain way, with a more familiar and traditional conception of quantities, as the sorts of things that are counted and measured. Aristotle lets the reader know that, when something is divisible in the way he has just specified, it may be countable, in which case it falls under the genus of plurality, and it may be (not countable but) measurable, in which case it falls under the genus of magnitude.[66] This helps the reader get a sense of what is going on, before being confronted with DISCRETE, which is a more formal and accurate division of quantity into plurality and magnitude.

According to the interpretations I have given in this section, *Metaphysics* Δ 13 and I 6, as well as *Posterior Analytics* 1. 22, all describe number as a species of plurality. Thus, they all imply, even if they do not outright state, that not every plurality is a number. According to I 6, the distinguishing feature of number is being measurable by a one.

66. It may also, according to the interpretation I am putting forward, be neither countable nor measurable. The statement can serve the purpose just suggested without implying or presupposing that every quantity is either countable or measurable.

This is plausibly identified with being countable, assuming that to measure a plurality is to count it ('by a one' doesn't seem to add anything, since a measure is always a one, according to I 1, 1052^b25–6). According to Δ 13, the distinguishing feature of number is being finite. In the light of Aristotle's argument in *Physics* 3. 5, every countable plurality will be a finite plurality. Aristotle does not provide us with the converse, that every finite plurality is a countable plurality, but this is at least not implausible. If Aristotle accepted the converse, then his accounts of number in Δ 13 and I 6 would be in agreement.

The conclusion of this section is that when Aristotle says that there cannot be an actual infinite number, he is not saying, and does not think he is saying, that there cannot be an actual infinite plurality.

3 INFINITE BY DIVISION

I now turn to Aristotle's claims about the infinity by division of magnitudes.

> Everything is infinite either according to addition or according to division, or in both ways. (*Physics* 3. 4, 204^a6–7)

> We have said that magnitude is not actually infinite. But it is infinite by division: for it is not difficult to do away with indivisible lines. It remains, then, that the infinite exists potentially. (*Physics* 3. 6, 206^a16–18)

What sort of infinity is it, precisely, that exists potentially but not actually according to the above passage? According to most commentators, Aristotle's claim is that no magnitude is actually divided into infinitely many parts.[67] But an alternative interpretation is suggested by a remark of Zeller's, who notes that according to Aristotle a magnitude has no infinitely *small* part.[68]

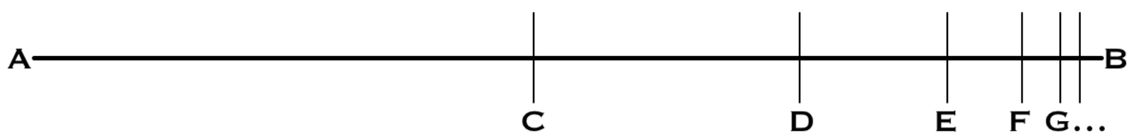
I will argue that Zeller's interpretation is correct: as Aristotle conceives of it, for a magnitude to actually be infinite by division is for the magnitude to actually be infinitely small. Correspondingly, for a time to actually be infinite by division is for the time to actually be infinitely brief; and so on for other kinds of quantity.

There is, of course, a very close correlation between how many parts a thing has and how small the thing's parts are. It may be impossible to determine conclusively which of these two things—smallness of parts or plurality of parts—Aristotle has in mind in every single passage: much of what he says can be made sense of on either interpreta-

67. Themistius and H. Schenkl, *Themistii in Aristotelis Physica paraphrasis* (Berlin, 1900) 91.27–8; Simplicius and Diels, *Simplicii in Aristotelis Physicorum libros quattuor priores commentaria* 492.16-17; Furley, 'Aristotle and the Atomists on Infinity', 87; Hussey, *Physics III–IV*, 82, Sorabji, *Creation*, 211; Coope, 'Aristotle on the Infinite', 268-9

68. Zeller, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*, 297. Charlton, 'Aristotle's potential infinites', 142, also describes the infinite by division in terms of smallness.

tion. Even so, the choice of interpretation is significant. Consider the division of a line in the manner of Zeno's Dichotomy, as follows:



If we suppose that line AB has infinitely many parts AC, CD, DE…, each part being half the length of the part to its left, then it does not follow that there exists a part of AB that is infinitely small. To the contrary, each of the parts just described has a finite length, namely the length of AB divided by some power of 2. Hence, if I am right about the meaning of 'infinite by division', then Aristotle's assertion that no magnitude is actually infinite by division is perfectly consistent with the proposition that line AB actually has infinitely many parts. I am not arguing that Aristotle positively *endorsed* the latter proposition, but I am arguing that he does not deny it in *Physics* 3. 4–8.

3.1 Considerations in favor of the smallness reading

There are three main considerations which, I believe, tell in favor of interpreting 'actually infinite by division' to mean actually infinitely small, rather than meaning actually divided into infinitely many parts.

(1) Infinite smallness was a recognized topic, and we might expect Aristotle to address it in his *Physics*. Aristotle states twice in *Physics* 3 that Plato posited two infinites, which he describes once as 'the big' and 'the small' (3. 4, 203^{a}15–16) and once as the infinite 'in the direction of growth' and 'in the direction of reduction' (3. 6, 206^{b}27–9). Earlier, in *Physics* 1. 4, Aristotle discussed the views of Anaxagoras, who pronounced: 'All things were together, infinite both in plurality and in smallness' (Fragment 1). The case of Anaxagoras shows that infinite smallness was a topic of relevance to physics, and not only to metaphysics in the Platonic style. If 'actually infinite by division' means 'actually infinitely small', then Aristotle addresses this recognized topic.

It might be objected that, if Aristotle were taking up the topic of infinite smallness from Anaxagoras and Plato, he would have indicated this more clearly. Why, for example, does he not repeat the word 'small' or 'smallness', which he used when reporting their views? The answer to this has two parts. First, Aristotle has a reason not to frame his discussion in terms of the small (μικρόν). For when he writes that 'everything is infinite either according to addition or according to division' (*Phys.* 3. 4, 204^{a}6–7), he is making a claim that applies equally to every kind of quantity—be it magnitude, time, plurality, or something else. The term 'small', however, applies primarily within the genus of magnitude, and is not properly used of numbers or pluralities, according to Aristotle.[69] Hence, this term would not suit his purposes.

69. At *Metaph.* N 1, 1088^{a}17–21, Aristotle appears to indicate that big and small (μέγα, μικρόν) are *per se*

Second, Aristotle does give positive indication that his vocabulary of 'addition' and 'division' corresponds to Plato's opposition between 'large' and 'small'. He does this in *Physics* 3. 6, 206^{b}27–9. Aristotle has previously admitted a qualified kind of infinite by division, and an even more qualified kind of infinite by addition. He has just pointed out an important difference between these two qualified infinites, which is as follows. When magnitude is qualifiedly infinite according to addition, it is *not* the case that every definite magnitude will be surpassed—rather, the magnitudes that can be taken asymptotically approach a finite upper bound. But when magnitude is qualifiedly infinite according to division, it *is* the case that every definite magnitude will be surpassed—that is, for every definite magnitude, a smaller magnitude can be taken (*Phys.* 3. 6, 206^{b}17–20). Aristotle highlights the importance of this difference by means of a reference to Plato:

> The very reason why Plato made the infinites two was because it seems to surpass and go to infinity both in the direction of growth and in the direction of reduction. (*Phys.* 3. 6, 206^{b}27–9).

attributes of magnitude, while many and few (πολύ, ὀλίγον) are *per se* attributes of number. Cf. *Phys.* 4. 12, 220^{a}32–^{b}3; *Metaph.* I 6, 1056^{b}10–11. He does occasionally speak of a 'small time' (μικρὸς χρόνος), at *PA* 2. 17, 660^{b}17; *GA* 1. 18, 725^{b}9; *Metaph.* Λ 7, 1072^{b}15. It is also true that he once speaks of a 'small plurality of earth' (μικρὸν πλῆθος γῆς), *De caelo* 4. 2, 309^{a}32. But this is the exception that proves the rule. Aristotle never speaks of a large or small number, nor of a large plurality.

Aristotle's point seems to be that in one direction, the direction of 'reduction', Plato's reason for positing an infinite is satisfied: magnitudes do indeed 'surpass and go to infinity' in the direction of reduction. But in the other direction, the direction of 'growth', they do not. This means that, judging by Plato's standards, magnitudes are not even *potentially* infinite in the direction of growth (cf. 206^{b}25–6).

Now, in his reference to Plato, Aristotle repeats the word 'surpass' from earlier in the passage, while substituting the phrases 'in the direction of growth' and 'in the direction of reduction' (ἐπὶ τὴν αὔξην, ἐπὶ τὴν καθαίρεσιν) for the phrases 'according to addition' and 'according to division' (κατὰ πρόσθεσιν, κατὰ διαίρεσιν), respectively. This indicates that he understands the former pair of terms to be the Platonic equivalent of the latter pair. Moreover, a comparison of Aristotle's two descriptions of Plato's infinites in *Physics* 3. 4, 203^{a}15–16 and 3. 6, 206^{b}27–9 suggests that Aristotle identifies 'infinitely large' with 'infinite in the direction of growth' and 'infinitely small' with 'infinite in the direction of reduction'. Thus, Aristotle may be taken to indicate, if not clearly, then at least as clearly as can be expected of Aristotle, that his 'by addition' and 'by division' correspond to Plato's 'large' and 'small'.

(2) We should expect the pair 'infinite by division' and 'infinite by addition' to refer to a proper pair of opposites. Given this, a magnitude's infinity by division and a magnitude's infinity by addition should both belong to the same genus of quantity.[70]

70. Opposites are members of the same genus (or perhaps of opposite genera—but the genera of quanti-

Let us consider, then, what the phrase 'actually infinite by addition' means, as applied to magnitude. What is Aristotle saying when he says that no magnitude is actually infinite by addition? It appears that the phrase means actually infinitely large. All of Aristotle's negative arguments concerning infinite body in *Physics* 3. 5 have the conclusion that there is no infinitely large body. Admittedly, the phrase 'by addition' does not appear in these arguments—Aristotle instead uses the phrase 'infinite in the direction of growth' (ἐπὶ τὴν αὔξησιν) (*Phys.* 3. 5, 204^{b}4). Still, as noted above, Aristotle appears to identify 'infinite in the direction of growth' with 'infinite by addition' at *Physics* 3. 6, 206^{b}24–9. The beginning of *Physics* 3. 6 confirms this: coming immediately after the arguments of *Physics* 3. 5, we find a statement of the distinction between infinite by addition and infinite by division (206^{a}15–16), and a claim that magnitude is infinite *potentially* and *by division* (206^{a}17–18). The context suggests that this claim is meant to be understood by contrast with the result just established in 3. 5, which Aristotle would therefore be construing as the result that magnitude is not *actually* infinite *by addition*.

If for a magnitude to actually be infinite by addition is for the magnitude to actually be infinitely large, and if 'infinite by division' is the opposite of 'infinite by addition', then for a magnitude to actually be infinite by division is for the magnitude to actually be infinitely small.

ty do not seem to have opposites), and belong to the same genus of subject: *Cat. 6,* 6^{a}17–18, 14^{a}15–16; *Post. An.* 1. 4, 73^{b}21–2; *Topics* 4. 3, 123^{b}3–4; *GC* 1. 7, 324^{a}2–3; *Metaph.* I 8, 1058^{a}10–11.

On the more usual interpretation, for a magnitude to actually be infinite by division is for the magnitude to actually have infinitely many parts. On this more usual interpretation, we must either deny that 'infinite by addition' and 'infinite by division' are properly opposites (since the former is a matter of how large, while the latter is a matter of how many), or we must give a different sense to 'actually infinite by addition'. Either of these moves may be defensible, but I believe that the 'smallness' reading of 'infinite by division' has the advantage here.

(3) Aristotle does not argue in *Physics* 3 for the non-actuality of the infinite by division, but simply asserts it. The lack of argument has troubled commentators.[71] But if Aristotle is denying that any magnitude is actually infinitely small, then it is understandable why he doesn't argue his point. Aristotle is saying something universally accepted in the mathematics of his time. For example, in Euclid X.1 it is assumed that, given two unequal magnitudes, some finite multiple of the lesser magnitude will exceed the greater magnitude (Euclid X.1, 11–12). This assumption would not be justified if it were ever the case that one of two magnitudes is infinitely small. Scholars have occasionally used the language of infinitesimals in discussing the Pythagoreans and other Presocratic figures such as Anaxagoras,[72] but I think it can be said with Schofield that the infinitesi-

71. Bostock, 'Aristotle, Zeno, and the Potential Infinite', 38; Hussey, *Physics III–IV*, 82, Coope, 'Aristotle on the Infinite', 268

72. E. Frank, *Plato und die sogennanten Pythagoreer: ein Kapitel aus der Geschichte des griechischen*

mal is 'an un-Greek notion'.[73] By contrast, the idea that one can divide a finite magnitude into infinitely many parts, as in Zeno's Dichotomy, was articulated and discussed in Aristotle's time. Thus, it is easy to see why Aristotle would take a stand on infinite smallness without argument, but harder to see why he would reject infinite pluralities of parts without argument.

In addition to these three main considerations, there are a few smaller textual points that may lend support to the 'smallness' reading of the phrase 'infinite by division'. The first is that Aristotle occasionally substitutes a word meaning 'reduction' or 'diminution' (καθαίρεσις) in place of the word 'division' (διαίρεσις) in his discussion of the infinite (*Phys.* 3. 6, 206^{b}13, 207^{a}22–3; 3. 8, 208^{a}21–2). It is easy to assimilate division and reduction to each other as both producing *smaller* entities, but more difficult to interpret them both as producing *more numerous* entities.

Secondly, Aristotle associates division with reduction in size when he relates it to 'surpassing every magnitude' (*sc.* in smallness): 'It is reasonable that there should not be

Geistes (Halle (Saale), 1923), 47, 62; H. Hasse and H. Scholz, 'Die Grundlagenkrisis der Griechischen Mathematik', 9; D. Sider, *The fragments of Anaxagoras* (Meisenheim am Glan, 1981), 88, 135

73. M. Schofield, 'Review of 'The Fragments of Anaxagoras', David Sider', *The Classical Review*, 32 (1982), 189–91, 190. B.L. van der Waerden, 'Zenon und die Grundlagenkrise der griechischen Mathematik', *Mathematische Annalen*, (1940), effectively counters earlier arguments for attributing a theory of infinitesimals to some figure earlier than Zeno of Elea.

held to be an infinite in respect of addition such as to surpass every magnitude, but that there should be thought to be such an infinite in the direction of division' (*Phys.* 3. 7, 207a33–5, tr. Barnes, *ROT*).

I will mention one last point, although I am not sure that it tells one way or another. When Aristotle says for the first time that the division of magnitudes is a source of infinity, he appeals to mathematics: 'for mathematicians too employ the infinite' (3. 4, 203b17–18). It seems likely that Aristotle is thinking of mathematical proofs in which, after a certain magnitude has been given, it is shown or assumed that a magnitude smaller than the given magnitude can be constructed (see for example Euclid X, prop. 1; Euclid XII, prop. 2). These proofs rely on the assumption that there is no finite lower bound on the sizes of magnitudes, and in that respect they employ the—potential—infinite smallness of magnitude.[74]

74. At the same time, the proofs require constructing smaller and smaller magnitudes *by means of repeated divisions*, and hence they also rely on the assumption that there is no finite upper bound on the *number of times a magnitude can be divided.* This is why I am not sure that Aristotle's appeal to mathematics tells one way or the other. I am inclined to think that *smallness* of parts figures more prominently than *number* of parts in the ways that these proofs were presented and understood, but this would require lengthy discussion.

4 POTENTIALITY AND BECOMING

So far I have asked what it is that Aristotle *denies* when he denies that magnitudes and numbers are *actually* infinite. A complementary question is, what does Aristotle *affirm* when he affirms that magnitudes and numbers are *potentially* infinite?

This related question may seem to cause problems for the interpretation advocated in the present paper. This is because Aristotle appears to appeal to *processes* of a certain kind when he discusses the potential being of the infinite, and especially when he explains the (qualified) sense in which this potential being can be actualized. Aristotle says, namely, that magnitude is sometimes actually infinite by division 'in virtue of always another and another thing coming into being' (*Phys*. 3. 6, 206^{a}22–4, cf. ^{b}13–14). This is widely taken to mean that the actual, or quasi-actual, existence of the infinite by division consists in the occurrence of a process resulting in ever smaller and smaller (or more and more numerous) magnitudes, such that the process can or does go on without limit. Analogously, the existence of infinite number would consist in the occurrence of a process resulting in ever greater and greater numbers of things, such that the process can or does go on without limit. In short, the story goes, the infinite exists insofar as processes to infinity occur.

Now, in an infinite process of numerical increase there is, at each time, a finite total number of things. And in an infinite process of decrease in magnitude there is, at each time, a finite smallest magnitude. Under the interpretation I have been advocating, there seems little reason for Aristotle to appeal to such processes. I have argued that, for all Aristotle says, it is possible for there to be an infinite plurality of things, including an infinite plurality of parts of a magnitude, all at once. If such pluralities sometimes exist, then there sometimes are—simultaneously—smaller and smaller magnitudes without limit, and there sometimes are—simultaneously—greater and greater numbers without limit. (Of course, it remains true that each individual magnitude has finite size, and that each individual number is finite.) There is no need to introduce processes of numerical increase or of decrease in magnitude, according to my interpretation. If Aristotle thought that processes are essential to the existence of the infinite, then this would suggest a commitment to the view that there is never an infinite plurality.

In this section I will discuss the principal relevant passages, and attempt to show that they do not, on balance, constitute evidence against my interpretation.

As we have already seen, Aristotle claims that magnitude is only potentially, never actually, infinite by division. He expresses this claim as follows:

> As for magnitude, we have said that it is not infinite in actuality, but it is infinite by division: for it is not difficult to do away with indivisible lines. It remains, then, that the infinite exists potentially. (*Phys.* 3. 6, 206^{a}16–18)

In these lines Aristotle, somewhat confusingly, pairs two orthogonal distinctions. On the one hand, he says that magnitude is potentially but not actually infinite. On the other hand, he says that magnitude is infinite by division but not by addition. It follows that the only sense in which magnitude is infinite is that it is potentially infinite by division.

In the immediately following lines, Aristotle explains and qualifies this claim. He notes that he has appealed to a rather unusual variety of potential being, namely one such that a corresponding *actual* being is not possible. Then, he introduces a second way of being, and suggests that magnitudes can actually be infinite by division in this second way. (Presumably, then, when something actually is in the second way, it only potentially is in the first way.[75]) The lines in question read as follows:

> We should not understand being potentially in the following way: just as, if this is capable of being a statue then this will also be a statue, so also, what is potentially infinite is that which will actually be infinite.[76] But—since being is multiple—just as a day and a contest exist in virtue of always another and another thing coming into being, so also does the infinite exist. For those things too ad-

75. Bowin, 'Aristotelian Infinity', 234-5

76. Like other commentators, including W.D. Ross (ed. and comm.), *Aristotle's Physics, a Revised Text With Introduction and Commentary* (Oxford, 1936), 555, Coope, 'Aristotle on the Infinite', 271, and, tentatively, Bostock, 'Aristotle, Zeno, and the Potential Infinite', 37, I take the phrases 'will be a statue' and 'will actually be infinite' to be statements of possibility, despite their plain future indicative form. By contrast, J. Hintikka, 'Aristotelian Infinity', *The Philosophical Review*, 75 (1966), 197–218, 206 reads them as statements of actual fact.

> mit both of existing potentially and of existing actually. (*Phys.* 3. 6, 206^{a}18–24)

In light of the immediately preceding lines, the phrase 'the infinite' in this passage appears to refer to a magnitude being infinite by division. Thus, the passage states that, in one way of being, magnitude is potentially infinite by division, but never will or can be actually infinite by division. In another way of being, on the other hand, magnitude is both potentially and (sometimes) actually infinite by division. This other way of being, Aristotle says, is enjoyed 'in virtue of always another and another thing coming into being'.[77] A few lines later, Aristotle will vary his formulation so as to read, 'in virtue of always another and another thing *being taken*'.[78] Aristotle reaffirms later in the chapter that magnitude sometimes is actually infinite by division in this second way of being:

> The infinite does not otherwise exist, but it does exist in this way: potentially and in the direction of reduction. And it also exists actually, in the way that the day exists and the contest exists. (*Phys.* 3. 6, 206^{b}12–14)

For convenience, let us express the first way of being by saying that something 'is-1' or 'exists-1', and to the second way of being by saying that something 'is-2' or 'exists-2'. Aristotle says that magnitude potentially but never actually is-1 infinite by division, while magnitude both potentially and sometimes actually is-2 infinite by division.

77. τῷ ἀεὶ ἄλλο καὶ ἄλλο γίγνεσθαι, *Phys.* 3. 6, 206^{a}22

78. τῷ ἀεὶ ἄλλο καὶ ἄλλο λαμβάνεσθαι, *Phys.* 3. 6, 206^{a}27–8

Aristotle speaks about the infinity of number in similar terms to the infinity by division of magnitude:

> Thus [number] is potentially, not actually [infinite], but what is taken continually exceeds every defined plurality. But this number is not separable from the bisection, nor does the infinity persist but comes into being, just like time and the number of time. (*Phys.* 3. 7, $207^{b}11$–15)

In this passage, Aristotle tells us that number is potentially but not actually infinite, and appears to associate the potential infinity of number with the fact that, for any definite (that is, finite) plurality, a number can or will be taken that exceeds the plurality. After a somewhat mysterious remark about separability, which I will not attempt to interpret, Aristotle then states that the infinity of number 'does not persist, but comes into being'. This appears to be a back-reference to Aristotle's description of how the infinite by division exists 'in virtue of always another and another coming into being'. I will suppose, then, that the infinite in the case of number and the infinite by division in the case of magnitude exist in the same three ways: they potentially exist-1, they potentially exist-2, and they sometimes actually exist-2.

What does this all mean? Aristotle's statements here are difficult to understand fully. For one thing, it is hard to know what sense to give the claim that certain things *potentially* are-1 a certain way, combined with the claim that it is impossible for the things *actually* to be-1 that way. In other texts, Aristotle appears to draw inferences from 'potentially is …' to 'is capable of being …' (that is, from δυνάμει to δυνατόν).[79] And Aristotle asserts that if something is capable of being, then it is not impossible for it to actually be.[80] It is true that Aristotle explicitly tells us not to understand the sentence 'magnitude is potentially infinite by division' in the same way as we would understand the sentence 'this is capable of being a statue' (*Phys.* 3. 6, 206ᵃ18–21, quoted above; cf. *Metaph.* Θ 6, 1048ᵇ9–17). So we are meant either to resist the inference from 'potentially' to 'capable', or the inference from 'capable' to 'possibly actual' (I am not sure which). But when we follow Aristotle's command, it is unclear what we are taking 'potentially' to mean.

Furthermore, as we will see, there are difficulties in understanding Aristotle's explanation of the being-2 of the infinite. What mode of being, exactly, does he attribute to days and to contests? And how, exactly, are we meant to interpret the analogy between days and contests, on the one hand, and infinite number and infinitely small magnitude, on the other?

I cannot give a fully worked out and defended resolution of all the difficulties posed by these passages, but I will do my best to make sense of them. And I will try to make it plausible that these passages do not posit an essential connection between the being of

79. *GC* 1. 2, 316ᵇ22 (δυνάμει) to 23 (δυνατόν); *Metaph.* N 2, 1088ᵇ18 (δυνάμει) to 19 (δυνατόν). See also *Phys.* 8. 4, 255ᵃ35–ᵇ1; *Metaph.* Θ 7, 1049ᵃ4–5.

80. *Phys.* 7. 1, 242ᵇ72–243ᵃ31; 8. 5, 256b10–12; *Metaph.* Θ 3, 1047ᵃ24–6; Θ 4, 1047ᵇ10–11. See also *Pr. An.* 1. 15, 34ᵃ25–9

the infinite and the occurrence of a process, in a way that would tell against the simultaneous existence of infinitely many magnitudes or other things.

4.1 Process-based interpretations and some objections to them

According to Bowin, when Aristotle likens the existence of the infinite to the existence of a day or a contest, he is telling us 'that infinity exists as a process exists'.[81] Bowin claims that, on the one hand, a process can either potentially be occurring or actually be occurring, while on the other hand it is true to say even of an actually occurring process that it exists *potentially*. He explains as follows:

> Aristotle tells us at the beginning of book 3 of the *Physics* that a motion exists only in so far as, and as long as, it has the unfulfilled potentiality of being completed by the arrival of the moving thing at a goal state that is intrinsic to the motion. … [A]s long as the moving thing is en route, the motion is an actuality *qua* existing potentially since it is potentially, but not actually, completed. Once this potentiality is realized, the motion no longer exists.[82]

In short, Bowin's view is that infinity exists in the form of a process; that an infinite process can either potentially or actually occur; and that when an infinite process is actually occurring, it still exists merely potentially because all processes exist merely po-

81. Bowin, 'Aristotelian Infinity', 239; cf. p. 234.

82. Bowin, 'Aristotelian Infinity', 240

tentially. Thus, he explains Aristotle's comparison to days and contests, and also the sense in which the infinite potentially exists-1, by identifying the infinite with processes.

Bowin's interpretation is subject to a number of objections. First, it is worth noting that days and contests are not standard examples of a motion or process (κίνησις) in Aristotle. When Aristotle wants to invoke the notion of a process, he normally draws on a small group of examples including walking, healing, and house-building. Aristotle never says that a day or a contest is a process or motion, and there is some evidence that he would deny that this is what they are.[83]

Second, there is reason to resist Bowin's claim that a motion exists potentially while it occurs. Bowin explains this claim by saying that a motion has, for as long as it exists, an unfulfilled potentiality to be completed. However, Aristotle does not say that motions have potentialities, unfulfilled or not. As far as I have seen, substances are the only things to which Aristotle attributes potentialities. On another point, I am not sure it is correct to say that a motion no longer exists as soon as it is completed. Aristotle says

83. A day is treated as a hylomorphic compound of air and light in *Metaphysics* Λ 4, 1070^{b}22, a conception that seems to cohere with remarks made at Δ 24, 1023^{b}6–8 and at H 5, 1045^{a}2–3. It might also be thought of as an interval of time. As for 'contest' (ἀγών), this word can denote a multi-day festival such as the Dionysia (*Ath. Pol.* 56.5.2–3, 57.1.6–7) or the Olympics. Perhaps a festival of several days is *composed* of κινήσεις, but it is surely not itself a κίνησις. Even a single competitive event such as a wrestling match, a race, or a comedic performance is not plausibly regarded as a single κίνησις, even if it is composed of κινήσεις. (For the conditions on being one κίνησις, see *Physics* 5. 4.)

that a motion is completed in the whole time in which it occurs, as well as at the end limit of this time (*NE* 10. 4, 1174^{a}21). The motion exists in the whole time of its occurrence, so, in this sense at least, it exists when it is completed. As for the end limit, to say that a motion no longer exists in that moment is strictly analogous to saying that a body is not present at its surface. These two analogous claims both seem dubious.[84] Finally, even if we grant that every motion is 'potentially but not actually *completed*' throughout its existence, the inference from there to 'potentially but not actually *existing*' is not secure. As a textual matter, I have not found any passages where Aristotle says that motions exist potentially while they occur. Aristotle seems to describe motions uniformly as activities or actualities, even if they are 'imperfect' or 'incomplete' (ἀτελής) ones.

Instead of identifying the infinite with a motion, an alternative strategy is to identify it with an *endpoint* of motion. It is uncontroversial that, for Aristotle, an endpoint of motion exists merely potentially while a motion occurs (see for example *Phys.* 3. 1, 201^{b}11–15). While a body becomes hot, it is potentially hot, and while a man walks to Corinth, he is potentially in Corinth. Perhaps, then, the infinite in number or magni-

84. We ascribe properties to bodies at their surfaces, such as colors, or the property of being in contact with something, or the property of being limited. The same should go, *mutatis mutandis*, for motions and the instants that limit their times. —I say this with a view to the Aristotelian continuum, in which lower-dimensional items are not parts of higher-dimensional items, and there is no distinction to be made between open and closed intervals.

tude sometimes exists as a—never attained—endpoint of motion. We could say that the infinite actually exists-2 when there occurs a motion towards infinite number or towards infinitesimal magnitude, and that it potentially exists-1 whenever such a process can or does occur, but that it never actually exists-1 because such a process never attains its endpoint.[85]

The problem for this line of interpretation is that Aristotle cannot admit the possibility of the required sort of process to infinity. To begin with, there are practical problems involved in bringing about such a process: where are the suitable patients, things that will stick around forever while they are made ever smaller and/or more numerous? Where is a suitable agent, someone or something that will do the dividing or reducing or multiplying?[86] Aristotle's eternal beings, such as God and the heavenly spheres, do not spend their time on such tasks as the division of magnitudes. Mortal beings, such as ourselves, necessarily die after some finite lifespan. Perhaps a task could be passed on through the generations, but this is not something that people in fact do, and Aristotle seems to say that magnitudes really are, on occasion, actually infinite by division (e.g., at $206^{b}13$–14).

85. Interpreters who appeal to some sort of process to infinity include Bostock, 'Aristotle, Zeno, and the Potential Infinite'; Coope, 'Aristotle on the Infinite'; Hintikka, 'Aristotelian Infinity'(?); more refs.

86. Simplicius *In Phys.* 493.33–494.4, J. Lear, 'Aristotelian Infinity', *Proceedings of the Aristotelian Society*, 80 (1979), 187–210

More importantly, there is a problem of principle. In Aristotle's view, it is a necessary truth that all eternal processes or sequences are cyclical.[87] But the interpretation now under consideration requires an eternal *linear* process of division, reduction, or multiplication. Such a process is impossible in Aristotle's view.

Coope has proposed the following response to this problem. She grants that an infinite process cannot *occur* (aorist or perfective aspect). However, Coope suggests, such a process can nevertheless *be occurring* (progressive aspect).[88] For comparision, it sometimes happens that a person *is knitting* a sweater, but then stops and never *knits* the sweater. On Coope's proposal, this is what always and necessarily happens with processes to infinity. The idea is that a magnitude can be undergoing a process of division that has no intrinsic tendency to stop, even if it is necessary that some extrinsic factor, such as the death of the divider, will halt the process after a finite amount of time and yield a finite total number of divisions. Coope proposes that a magnitude qualifies as undergoing division to infinity, and is (that is, is-2) actually infinite by division, when it is undergoing such a process.

87. *GC* 2. 11, $338^{a}2$–5; *Phys*. 6. 10, $241^{b}18$–20. Cf. *Post. An.* 2. 12, $95^{b}38$–$96^{a}7$

88. Coope, 'Aristotle on the Infinite'. So also Bostock, 'Aristotle, Zeno, and the Potential Infinite', 39; H. Weidemann, 'Potentiality and Actuality of the Infinite: A Misunderstood Passage in Aristotle's Metaphysics (Θ.6, 1048b14-17)', *Phronesis*, 62 (2017), 210–25, 219

Coope's proposal relies on the claim that in some cases, it is possible for a kind of process to be occurring although it is impossible for that kind of process to occur. However, Aristotle denies this. He writes that 'it is impossible for that which does not admit of having-come-to-be (γενέσθαι, aorist) to come to be (γίγνεσθαι, present)', in *De caelo* 1. 7 (274^{b}13–14). He says the same in *Physics* 6. 10 (241^{b}3–11). On a closely related point, he says that if it is impossible for something to *be*, then it does not *come to be* (γίγνεσθαι, present: *De caelo* 4. 4, 311^{b}32–3; cf. *Physics* 8. 9, 265^{a}19). In all these passages, Aristotle uses this point to argue that there is no such thing as locomotion to infinity, on the grounds that it is impossible to *have traversed*, or to *be at*, an infinite distance. The application to the present case is unmistakeable. It is not only impossible for an infinite (linear) process to *occur*, it is impossible for such a process to *be occurring.*

Bowin has offered a different response to the problem. While granting that 'all genuine changes are, in fact, finite,' Bowin proposes that some changes 'could, counterfactually, have gone on indefinitely.'[89] He suggests that this might, in some way, suffice to allow us to speak of the existence of the infinite.

There are two ways of reading the claim that a given change could have gone on indefinitely. Under one reading, the claim means that the change in question could have lacked any definite goal or endpoint.[90] As Bowin sees, Aristotle does not think that any

89. Bowin, 'Aristotelian Infinity', 248

90. By 'endpoint', I mean whatever Aristotle means by 'that to which' (εἰς ὅ) in *Physics* 6; I take this to be

change could have gone on indefinitely in this sense: Aristotle treats it as a necessary truth that every change has a definite goal and endpoint (see for example *Phys.* 5. 1, 224^b1, 225^a1; 6. 10, 241^a26 ff.). So presumably Bowin intends his proposal to be read in another way. Under a second reading, the claim means that a change could have gone on longer than it in fact did, and that there is an indefinite range of distances by which it could have gone on longer. (The walk could have proceeded to a place a mile away, or two miles away, or three miles away, or….) It is possible that Aristotle believed this, but I do not know of any passage where he says so. Pointing in the opposite direction, Aristotle includes sameness of starting point and endpoint as necessary conditions on oneness of change.[91] The exact meaning of this is open to interpretation, but it suggests that if something had moved farther than it actually did, then it would have undergone a different motion, not one and the same motion for a longer distance. I do not see good reason to think that, for Aristotle, some changes could have gone on indefinitely.

In sum, attempting to understand the existence of the infinite for Aristotle in terms of processes leads to problems. The problems may be surmountable, but they are serious. It is worth considering a different approach.

where the change actually ends. By 'goal', I mean whatever Aristotle means by τέλος; I take this to be where the change would end if nothing interfered (*PA* 1. 1, 641^b24–5).

91. At Phys. 5. 4, 227^b14–20, Aristotle indicates that sameness of starting point and endpoint is necessary but not sufficient for oneness in kind; more strongly, sameness of *path* is necessary.

4.2 A different approach

Aristotle sometimes uses process-verbs, such as 'go' or 'become', in ways that do not implicate a genuine process or occurrence in time. For example, in a discussion of chains of predication, Aristotle asks whether it is possible for a chain to 'go' (ἰέναι) to infinity (*Post. An.* 1. 19). For another example, the verb 'come to be' is used to express logical consequence when Aristotle writes, 'it is proven that the diagonal is incommensurable through the fact that odd numbers *come to be* equal to even numbers when the diagonal is posited as commensurable' (*Pr. An.* 1. 23, 41^{a}26–7, emphasis added).

I propose that something similar is happening in *Physics* 3. 6–7. Aristotle occasionally moves away from the language of 'coming into being' in *Physics* 3. 6, and speaks intead of 'being taken' at 206^{a}22 and ^{a}28.[92] Similar to his use of 'go' in *Post. An.* 1. 19, 'come to be' might serve as a device for talking about the successive members of a sequence, regardless of whether or not the sequence is one that unfolds in time.

92. It is true that 'taking' (λαμβάνειν) typically signifies an act of the soul, which occurs at a time. But it seems that a series of takings can also be accomplished by stipulation, as in Euclid VIII.3.7–15, 'Let two numbers be taken […] three […] and successively one more, until what is taken becomes equal in plurality to A, B, C, D'. The force of the proof does not seem to depend on anyone's actually taking particular collections of numbers in sequence.

One can see how a non-temporal sequence of magnitudes might be thought to ground a qualified kind of actual existence for the infinitely small. Consider the sequence of parts of a line that are assumed to exist in Zeno's Dichotomy: half the line, then a quarter, an eighth, a sixteenth, and so forth. Suppose that all these parts exist, actually and simultaneously. No single one of these parts is infinitely small: the sequence does not contain any infinitesimal lines. Hence the infinitely small does not actually exist-1. On the other hand, there actually exists a sequence that approaches arbitrarily close to zero, with every line succeeded by a smaller line. Such a sequence might be described as a qualified manifestation or actualization of infinitely small magnitude. It might be said that infinitely small line actually exists-2, that is, actually exists in a qualified way, in virtue of the way that line follows upon line in this sequence. I propose that this is a case in which the infinite by division exists 'in virtue of always another and another coming into being' or 'being taken'.

A parallel account can be given for Aristotle's claim that the infinity of number comes into being rather than persisting. Suppose that there are actually infinitely many things, and hence that every number of things exists actually and simultaneously. Consider the sequence of numbers two, three, four, and so forth. No single number in the sequence is infinitely great. Hence infinite number does not actually exist-1. On the other hand, there actually exists a sequence that proceeds beyond every numerical bound, with every number succeeded by a greater number. Such a sequence might be described as a manifestation or actualization of infinitely great number. It might be said that infinitely great number actually exists-2, that is, actually exists in a qualified sense, in virtue of the way that number follows upon number in the sequence. I propose that this is a case in which the infinity of number 'does not persist but comes into being'.

If this is correct, then Aristotle's comparison to days and contests does not rest on ideas about time or unfolding in time. Nor does it involve the metaphysics of motion or process. Rather, the point of the comparison is to elicit the more abstract idea of something's supervening on a sequence. The parallelism between infinite magnitude or number, on the one side, and days or contests, on the other, is somewhat loose on this interpretation. This may seem to be a disadvantage. But the interpretation has an advantage in explaining why Aristotle reached for an analogy at all, instead of saying directly what he meant. Aristotle knows how to say explicitly that a process proceeds to infinity, if that were what he was talking about (see for example *Physics* 6. 10, 241^b11–12). According to my proposal, Aristotle is trying to express a more difficult idea, which he may well have been unable to state directly and precisely.

My proposal is still rather vague in its details. But I hope I have said enough to make it plausible that one can make decent sense of Aristotle's talk about potentiality and becoming, without imputing to him a rejection of actual infinite pluralities.

5 CONCLUSION

The picture of Aristotle as a forefather and paragon of finitism is deeply entrenched in our tradition. I have been trying to shake loose our conviction in the accuracy of this picture, to make a case that the texts of Aristotle—at least the ones that we have today—do not establish him as a finitist about plurality. Aristotle, I believe, says nothing to rule out the existence of an actual infinite plurality.

Perhaps the reader has not (yet?) come around to my view. Perhaps you are thinking that, if Aristotle's strictures on the infinite did not apply to pluralities, then he would have said so explicitly. I have tried to somewhat disarm this thought above (Section 2.2, pp. 30–33), by pointing out how clearly Aristotle marks the fact that he is focused on magnitude, and not on plurality or quantity more generally. But one might still be troubled by Aristotle's silence about plurality, especially when combined with his universal-sounding claims about 'the infinite'. If so, then let me call attention to another bit of silence on Aristotle's part, which may provide a counterweight.

Prominent philosophers before Aristotle, including Leucippus, Democritus, and Anaxagoras, believed in an infinite plurality of principles: infinitely many atoms or infinitely many homoiomerous stuffs. Aristotle engages fairly extensively with their views. If he had known of some general reason for denying the possibility of actually infinite pluralities, then this would have been highly relevant to his discussions of these thinkers. Aristotle does not offer any such general reason. Instead, he makes points such as the following:

1. If the same results can be derived from finitely or infinitely many principles, then it is better to posit finitely many principles, and the fewer the better. (*De caelo* 3. 4, 302^b26–30, 303^a17–18; *Phys.* 1. 6, 189^a14–16; 8. 6, 259^a8–12)
2. There are finitely many differences between kinds of body. (*De caelo* 3. 4, 302^b30–303^a3, 303^a19–20)
3. There are finitely many kinds of simple locomotion, corresponding to finitely many places that can serve as the goals of such locomotion. (*De caelo* 3. 4, 303^b5–8; *Phys.* 3. 5, 205^a29–31)
4. The infinite *qua* infinite is unknowable. (*Phys.* 1. 4, 187^b7; 1. 6, 189^a12–13)

None of these points is a reason to deny the existence of an actually infinite plurality of bodies or other particulars. Particulars are not, in general, principles (1). Infinitely many particulars could belong to a finite number of kinds, and have finitely many kinds of simple locomotion (2, 3). Particulars are not objects of scientific knowledge according to Aristotle, so it is not a problem if they are unknowable *qua* particulars (4).[93]

93. Universals are finite and knowable, while particulars are infinite and unknowable: *Post. An.* 1. 24, 86^a3–10. Hence I do not think Lear was right to say that for Aristotle 'the possibility of philosophy—of man's ability to comprehend the world—depends on the fact that the world is a finite place containing objects that are themselves finite' (Lear, 'Aristotelian Infinity', 202, quoted approvingly by Bowin, 'Aristotelian Infinity', 233).

Now, just because Aristotle doesn't say something in his surviving texts, it doesn't mean we can be sure he didn't think it. But if we are in the mood of being troubled by Aristotle's silences, then we should be struck by Aristotle's refusal to criticize his predecessors for the bare fact that they posit infinite pluralities. I offer this argument from silence as a counterweight to arguments from silence on the finitist side.

If Aristotle did not deny the existence of actual infinite pluralities, then this opens up some interesting philosophical and historical questions. It affects how we should think about the metaphysical status of parts in Aristotle's thinking. It affects how we should understand certain strands of thought that run from Zeno of Elea through the atomists to Aristotle. And lastly, it invites us to speculate about who else in antiquity, if not Aristotle, may have originated the finitist views about plurality that have at times so dominated the philosophical landscape.

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