International Journal of Industrial Organization 63 (2019) 551–582

Contents lists available at ScienceDirect

International Journal of Industrial Organization

www.elsevier.com/locate/ijio

Prominence, complexity, and pricing☆

Ioana Chioveanu

Department of Economics and Finance, Brunel University London, Uxbridge UB8 3PH, UK

A R T I C L E I N F O

Article history:
Received 23 May 2018
Revised 30 October 2018
Accepted 29 December 2018
Available online 16 January 2019

JEL classification:
D03
D43
L13

Keywords:
Oligopoly markets
Consumer confusion
Prominence
Price complexity
Price dispersion

A B S T R A C T

This paper analyzes prominence in a homogeneous product market where two firms simultaneously choose both prices and price complexity levels. Market-wide complexity results in consumer confusion. Confused consumers are more likely to buy from the prominent firm. In equilibrium, there is dispersion in both prices and price complexity. The nature of equilibrium depends on prominence. Compared to its rival, the prominent firm makes higher profit, associates a smaller price range with lowest complexity, puts lower probability on lowest complexity, and sets a higher average price. However, higher prominence may benefit consumers and, conditional on choosing lowest complexity, the prominent firm's average price is lower, which is consistent with confused consumers' bias.

☆ Ioana Chioveanu thanks the Editor, Yongmin Chen, and two anonymous referees for their valuable comments, which significantly improved the paper. She is grateful to Uğur Akgün, Albert Banal-Estañol, John Bennett, Yiquan Gu, Paul Heidhues, Evagelos Pafilis, Alexei Parakhonyak, Pierre Régibeau, Rani Spiegler, Tobias Wenzel, Chris Wilson, Jidong Zhou, and audiences at the annual meetings of ASSET (2016), RES (2017), EEA (2017), CES (2017), at King's College London, and at Brunel University London for useful comments. The usual disclaimer applies.

E-mail address: ioana.chioveanu@brunel.ac.uk

https://doi.org/10.1016/j.ijindorg.2018.12.005

1. Introduction

Price complexity is a common feature of many markets, including those for retail financial and banking products, and retail supply of gas and electricity. It stems from the use of multi-part tariffs or partitioned prices, involved or technical language, or different price formats or information disclosure methods. A main concern is that complex pricing stifles competition by making it harder for consumers to understand firms' offers and identify the best deal.

The 2015 UK Competition and Market Authority investigation of the retail banking market found that "[t]here are barriers to accessing and assessing information on Personal Current Account charges" and "overdraft charges are particularly difficult to compare across banks, due to both the complexity and diversity of the banks' charging structures". The 2011 report by the UK Independent Commission on Banking mentions "evidence that complexity in pricing structures makes it difficult for consumers to receive good value". The 2007 EC study of EU mortgage credit markets and Woodward and Hall's 2012 study of US mortgage markets echo these concerns.[1]

Price complexity increases the time (or effort) consumers need to make a choice and the level of cognitive abilities and sophistication required to find the best offer. So, it may lead to consumer confusion and allow homogeneous product sellers to soften price competition and increase their profits.[2] Experimental research indicates that more fragmented multi-part tariffs can create confusion and lead to suboptimal consumer choices (see, for instance, Kalaycı and Potters, 2011; Kalaycı, 2015). These findings are consistent with evidence from the marketing literature that partitioned (or involved) pricing makes it difficult for consumers to compare competing offers (Greenleaf et al., 2016 reviews related work). Evidence of behavioral biases has also been found for US retail finance products (mortgage brokerage, loans, and credit card services) by Woodward and Hall (2012) and Stango and Zinman (2009a,b).[3]

In some markets where price complexity leads to consumer confusion, the choices of confused buyers are affected by firm prominence, which may be due to higher brand recognition (e.g., for a pioneer or incumbent product or an intensely advertised one), to product recommendations made by an expert, agent, or other consumers, to a more salient location (at eye-level, in a display, or at the top of an online search-outcome list), or to consumers' loyalty to an already familiar brand. See Armstrong et al. (2009) for a discussion of empirical evidence on prominence. For instance, consumers who shop for a mortgage or for insurance may be biased towards considering their current-account

[1] Carlin (2009) discusses empirical evidence of price complexity in financial markets and concludes that "many of the households who purchase retail financial products do not understand what they are buying and how much they are paying for these goods".

[2] When facing complex tariffs/markets, some consumers may rationally opt out of information processing due to its high cost. Or, they may be unable to deal with the complexity because they have poor numeracy skills and/or misjudge the information.

[3] See also Campbell (2016) for a thorough discussion of consumer ignorance in household finance.

bank. In retail energy markets that were previously monopolized, consumers may favor the 'familiar' regional incumbent over new entrants.[4]

This paper explores the relationship between price complexity as an obfuscation device and firm prominence and its implications in otherwise homogeneous product markets. We analyze the impact of prominence on firms' pricing and complexity choices and on market outcomes. In our model, a prominent seller and its rival compete for a unit mass of identical consumers with unit demands. Firms simultaneously and independently choose both their prices and price-complexity levels. The timing reflects the fact that in many environments, including banking and financial markets, firms can change relatively easily the price formats or the technical language employed in their price disclosures.

We formalize price complexity by allowing each firm to select a level from a closed interval. A firm's choice of complexity affects consumers' ability to understand its price offer. As a result, the firms' complexity choices affect market composition: some consumers are experts and purchase the lowest-price product, while others are confused.[5] A marginal increase in a firm's complexity level increases the share of confused consumers in the market. Confused consumers are unable to assess the firms' prices and make random choices, but are relatively more likely to select the prominent product as it enjoys higher recognition.[6]

We show that the nature of the equilibrium depends on the relative prominence of the two firms. Both firms have to balance conflicting incentives when setting their prices: to compete aggressively for the experts and to exploit the confused consumers. But the less prominent firm has stronger incentives to compete aggressively as it has a smaller base of confused consumers. In equilibrium, this friction rules out pure strategy pricing, so both firms randomize on prices. Moreover, the prominent firm also randomizes between the lowest and the highest price complexity levels and, for moderate levels of prominence, so does the less prominent seller. However, if the prominence level is high enough, the less prominent seller chooses the lowest complexity for sure as it benefits more from market transparency.

In equilibrium, whenever a firm randomizes on complexity, there is a positive relationship between prices and complexity levels.[7] When setting a relatively low price, a firm benefits from a lower complexity level as this is associated with a higher fraction of experts. In contrast, when a firm sets a relatively high price, it may benefit

[4] Hortaçsu et al. (2017) show that inattention and incumbent brands' advantages are sources of consumer inertia in the Texan residential electricity market. Analysing Mexico's private social security market, Hastings et al. (2017) show that firms' advertising and sales spending (which can be related to prominence) affects low-income or price-inelastic consumers' choices. See also Giulietti et al. (2014) for evidence from British electricity markets.

[5] This model is open to a default-bias interpretation whereby consumers are initially assigned to one firm, the prominent firm has an initial advantage (i.e., a larger base of consumers), and the extent of consumer inertia (i.e., the share of buyers who uphold their default option) is endogenously determined by firms' complexity choices.

[6] When confused, the consumers may use intermediaries who steer them towards the prominent product, may rely on persuasive advertisements, or may have stronger default biases.

[7] Armstrong and Chen (2009) and Chioveanu (2012) identify positive relationships between prices and product qualities in models where firms randomize on both dimensions.

from choosing a high complexity level, provided that it serves a large enough fraction of confused consumers.

The market outcomes reflect the differences in product salience. The prominent seller makes higher profits, chooses the highest complexity level with higher probability than the rival, sets a lower cut-off price below which prices are associated with the lowest complexity, and chooses the monopoly price with positive probability. As it sells to a larger share of confused consumers, the salient firm is more likely to choose high complexity and also, for a given complexity level, its incentive to set a high price is stronger. The less prominent seller's price is always below the monopoly level and its average price is lower than that of the rival.[8]

An increase in the level of prominence may lead to lower industry profit and higher consumer surplus. Such an increase affects firms' pricing directly, as it reallocates the confused consumers in favour of the salient firm. Moreover, it has an indirect effect on pricing as it affects firms' probabilities of choosing lowest complexity. As a result, our framework highlights a novel channel through which prominence affects market outcomes, related to the cut-off structure of firms' equilibrium strategies. An increase in prominence (weakly) increases the probability with which the less prominent firm chooses the lowest complexity, while it strictly decreases the corresponding probability of the salient firm. This tension underlies the non-monotonicity of consumer surplus in prominence. One implication is that in an environment where less prominent firms (e.g. new entrants) increase their relative salience (for instance, through advertising investments or sales efforts), this could be detrimental to consumers.

Conditional on choosing lowest complexity, the prominent firm's average price is lower. In this sense, confused consumers' bias for the prominent seller is consistent with the ranking of the average prices conditional on low complexity. In an extension, we show that a qualitatively robust cut-off mixed strategy equilibrium exists for more general confusion technologies if the marginal effect of a firm's price complexity increases in the rival's complexity choice.[9]

In spite of their prevalence, price complexity and firm prominence have only recently received attention in the economics literature. To analyze these phenomena, a recent stream of theoretical research develops the framework in Varian (1980), by endogenizing consumer heterogeneity. Carlin (2009) examines a homogeneous product market where identical firms compete in both prices and price complexity levels. Strategic price complexity leads to consumer confusion and softens price competition. Confused consumers make random choices, so each firm is equally likely to be selected. His findings are consistent with observed patterns in retail financial markets, such as price dispersion, positive mark-ups, and higher prices in more fragmented environments. Our analysis focuses on

[8] Gurun et al. (2016) show that lenders who advertise more sell more expensive mortgages and that the effect is stronger for less sophisticated consumers.

[9] In the working paper, we also verify the robustness of our qualitative results in a modified model where expert consumers are biased towards the prominent firm's product (i.e. willing to pay a premium for it so long as the price is below their valuation), see Chioveanu (2017).

the interaction between price complexity and prominence, and shows that the latter has an impact on the equilibrium pattern. Specifically, we identify conditions where only the prominent firm randomizes in price complexity levels and show that consumer surplus may be non-monotonic in prominence.

Piccione and Spiegler (2012) study a duopoly market where consumers are initially assigned to one firm (their default option) and make price comparisons with a probability which depends on firms' chosen price formats. They consider a more general frame structure and identify a necessary and sufficient condition for firms to earn max–min profits in equilibrium. The analyses in Carlin (2009) and Piccione and Spiegler (2012, Section IV.B) focus on the polar case where both firms are equally prominent. Spiegler (2011, Chapter 10.4) provides a treatment for the other polar case where all consumers are initially assigned to the same firm, so there is extreme prominence. By allowing for arbitrary salience levels, our analysis fills the gap between these two polar cases, and shows that consumer surplus is not monotonic in prominence.

The symmetric oligopoly analysis in Chioveanu and Zhou (2013) shows that the equilibrium pattern depends on the relative effectiveness of frame differentiation and frame complexity as sources of consumer confusion. There, an increase in the number of firms induces firms to rely more on frame complexity and may harm consumers.

Gu and Wenzel (2014) propose a sequential model where a prominent seller and its rival compete in prices after committing to price complexity levels. They show that in equilibrium firms randomize in prices, but choose deterministic complexity levels. The salient firm chooses the highest complexity for sure, while the rival's choice depends on the market conditions. Consumer protection policies which reduce the share of confused consumers may backfire by making the less prominent firm increase its complexity. In contrast to our cut-off equilibrium model, in theirs, consumer surplus monotonically decreases in the level of prominence.

As they model complexity as a long-run decision, Gu and Wenzel's insights are relevant in markets where obfuscation relates to product design rather than price disclosure.[10] In our framework where both prices and complexity levels can be changed frequently, the prominent firm always randomizes on prices and price complexity levels, whereas for relatively high salience levels, the rival chooses the lowest complexity for sure. Moreover, a reduction in the share of confused always improves consumer surplus.

In a sequential search model where all consumers sample first one prominent firm, Armstrong et al. (2009) demonstrate that, with homogeneous products, the salient seller sets a lower price than its rivals, industry profits are higher, and consumer surplus and welfare lower than in a market where firms are equally prominent. They also show that prominence benefits both sellers and consumers when products are vertically differentiated (as the highest-quality producer has the strongest incentive to become salient). Armstrong and Zhou (2011) explore ways in which a firm can become prominent:

[10] See also Ellison and Wolitzky (2012), Wilson (2010), and Taylor (2017) for search-cost models of obfuscation.

intermediaries may steer consumers to one firm for a fee, price advertisements may affect the order in which firms' offers are sampled, or consumers' default biases may be a source of prominence. See also Rhodes (2011) for a related model and Armstrong (2017) for a recent review of the ordered search literature.

In our clearinghouse setting, the order of search is irrelevant but prominence affects the behavior of consumers who are confused by price complexity. We focus on environments where firms commonly employ complex prices, for example, consumer banking and energy retail markets. Prominence might be driven by default biases favouring the product under consideration or related ones or it may be due to persuasive advertising or marketing ploys which make a firm's product salient in a consumer's mind and so more likely to be considered.

By considering the interplay between complexity and prominence in a model with consumer confusion, this study contributes to an emerging literature that explores the interaction between boundedly rational consumers and strategic firms. See Ellison (2006), Spiegler (2011), Huck and Zhou (2011), Grubb (2015), and Spiegler (2016) for related discussions and surveys of recent work. Our model is also related to the literature on price dispersion (see Baye et al., 2006, for a review) and explores an asymmetric market where firms simultaneously choose prices and complexity, and randomize in both dimensions.

2. Model

Consider a market for a homogeneous product with two sellers, firms 1 and 2. The firms face zero marginal costs of production. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay up to $v = 1$. The firms compete by simultaneously and independently choosing prices (p_1 and p_2) and price complexity levels (k_1 and k_2). The timing reflects the fact that in many cases both complexity and prices can be changed relatively easily. The level of complexity k_i captures how difficult it is for consumers to assess the price of firm i. The firms set prices $p_i \in [0, 1]$ and can choose any complexity level $k_i \in [\underline{k}, \bar{k}] \subset R_+$ free of cost.

Depending on firms' complexity choices, some consumers may find it difficult to assess the price offers. For given k_1 and k_2, a fraction $\mu(k_1,\ k_2) \leq 1$ of the consumers are able to accurately compare the price offers and select the best deal (we refer to these as the 'experts' or 'informed'), but the remaining $1 - \mu(k_1, k_2)$ consumers are confused and make random choices, which may be biased due to firm prominence. Let $\mu(k_1, k_2) \in \mathcal{C}^2$.

If one firm unilaterally increases the complexity of its price, this lowers the fraction of expert consumers in the market ($\partial\mu/\partial k_i < 0$, for $i = 1, 2$), but does not affect the marginal impact of the rival's price complexity on consumers ($\partial^2\mu/\partial k_1 \partial k_2 = 0$). For simplicity, we assume that $\mu(k_1, k_2) = 1$ iff $k_1 = k_2 = \underline{k}$. That is, if both firms choose the lowest complexity level $\underline{k}$, all consumers are experts and buy the cheaper product.[11] In

[11] This is without loss of generality so long as the monotonicity assumptions in the text are satisfied.

Section 5, we explore the robustness of our results for alternative confusion technologies with $\partial^2\mu/\partial k_1\partial k_2 > 0$.

We focus on the interaction between price complexity and firm prominence. In our model, prominence is exogenous (it may be due, for instance, to higher firm recognition or perceived trustworthiness) and has an impact on product choice when consumers are confused by price complexity. It also affects the choice of informed consumers if the two firms offer the same price. Without loss of generality, firm 1 is a 'prominent' seller so that the consumers who are unable to assess the prices due to complexity are more likely to purchase its product. That is, a fraction $\sigma \in (1/2, 1)$ of the confused consumers buy from firm 1 and the remaining $1-\sigma$ buy from firm 2. Similarly, if both firms offer the same price, a fraction $\sigma \in (1/2, 1)$ of the experts buy from firm 1 and the remaining $1-\sigma$ buy from firm 2. As a result, the firms' profits are

$$\pi_i(p_i, p_j, k_i, k_j) = p_i \cdot [q_i(p_i, p_j)\mu(k_i, k_j) + s_i(1-\mu(k_i, k_j))]$$

where $q_i(p_i, p_j)$ is given by

$$q_i(p_i, p_j) = \begin{cases} 1, & \text{if } p_i < \min\{p_j, 1\} \\ s_i, & \text{if } p_i = p_j \le 1 \\ 0, & \text{if } p_i > \min\{p_j, 1\} \end{cases} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j$$

with $s_1 = \sigma > 1/2$ and $s_2 = 1 - \sigma$.

In line with closely related work, we assume that the confused consumers do not pay more than their reservation price ($v = 1$). This may be because they have a budget constraint and realize at checkout (or after purchase) if the price is higher than v and can decline to buy or return the product. Knowing this, firms do not have incentives to set prices above consumers' valuation.[12] Consumers' behavior is affected by market-wide complexity and prominence, but independent of how the firms' prices rank. This captures the idea that confusion due to complexity reduces consumers' price sensitivity and weakens price competition.

3. Preliminary analysis

We start by analyzing firms' price and complexity choices when market-wide complexity leads to consumer confusion and one firm is prominent. All proofs missing from the text are relegated to the appendix, unless specified otherwise. The following two results rule out the existence of pure strategy equilibria.

Lemma 1. *There is no equilibrium where both firms choose pure price-complexity strategies.*

[12] However, it can be shown that our results are qualitatively robust when confused consumers pay up to $1 + \varepsilon$ for $\varepsilon < \mu(\underline{k}, \bar{k})$.

Proof. Suppose firm i ($j \neq i$) chooses a deterministic complexity level k_i (k_j).

(i) If $k_i = k_j = \underline{k}$, all consumers are experts ($\mu(\underline{k}, \underline{k}) = 1$) and the firms make zero profits by competing à la Bertrand. But then firm i could profitably deviate to $k_i^d = k' > \underline{k}$ and a price $p_i = 1$ which would result in a non-trivial mass of confused consumers (i.e., $1 - \mu(k', \underline{k}) > 0$) and strictly positive profits. Hence, it must be that in any candidate equilibrium at least one firm (w.l.o.g. let it be i) chooses $k_i > \underline{k}$.

(ii) By (i) for any candidate equilibrium profile of price complexities (k_i, k_j), some consumers are confused, i.e., $1 - \mu(k_i, k_j) > 0$. But then for any such profile (k_i, k_j), there is a unique pricing equilibrium where firms randomize according to a c.d.f. on $[p_0, 1]$, with $p_0 = \sigma(1 - \mu(k_i, k_j))/\ [1 - (1 - \sigma)(1 - \mu(k_i, k_j))] > 0$ (see, for instance, Baye et al., 1992), and firm i makes profit $\pi_i = p_0[1 - s_j(1 - \mu(k_i, k_j))]$. But, as it must be that $k_i > \underline{k}$, firm i could profitably deviate to $p_i^d = p_0$ and $k_i^d = \underline{k}$ which would result in profit $\pi_i^d = p_0[1 - s_j(1 - \mu(\underline{k}, k_j))] > p_0[1 - s_j(1 - \mu(k_i, k_j))]$ as $\mu(\underline{k}, k_j)) > \mu(k_i, k_j)$. So, there can be no equilibrium where both firms choose pure price complexity strategies. □

This analysis focuses on $\sigma \in (1/2, 1)$, but the result in Lemma 1 carries over when $\sigma = 1/2$. When $\sigma = 1$, there is an asymmetric pure strategy equilibrium where firm 1 chooses $\bar{k}$ and firm 2 chooses $\underline{k}$. In that case, while there are both expert and confused consumers, but firm 2 does not serve latter and the deviation in part (ii) of the proof of Lemma 1 does not hold as $s_j = 0$ when $i = 1$.

Lemma 1 implies that in any candidate equilibrium at least one firm randomizes on complexity levels. As a result, both firms face two types of consumers, confused and experts.[13] There is a conflict between the incentive to extract all surplus from confused consumers and the incentive to reduce price and compete for experts. This intuition underlies the following result, whose proof is standard and therefore omitted; see Varian (1980) and Rosenthal (1980).

Lemma 2. *There is no equilibrium where both firms use pure pricing strategies.*

Lemmas 1 and 2 show that in any duopoly equilibrium there must be dispersion in both prices and complexity levels. Firm i's strategy space is $[0, 1] \times [\underline{k}, \bar{k}]$. Denote by $\xi_i \equiv \xi_i(p_i, k_i)$ firm i's mixed strategy for $i = 1, 2$. ξ_i is a bivariate c.d.f. and can be written as $\xi_i = F_i(p_i)H_i(k_i \mid p_i)$, where $F_i(p_i)$ is the marginal c.d.f. of firm i's random price and $H_i(k_i|p_i)$ is the conditional c.d.f. of firm i's complexity level. (If the two random variables, p_i and k_i are independent, $H_i(k_i \mid p_i) = H_i(k_i)$.) For $F_i(p)$ and $H_i(k_i|p_i)$ to be well-defined

[13] We focus on a case where $\mu(\underline{k}, \underline{k}) = 1$. However, Lemma 1 is robust for $\mu(\underline{k}, \underline{k}) < 1$ so long as $\partial\mu/\partial k_i < 0$, for $i = 1, 2$. In that case, even for $k_i = k_j = \underline{k}$, firms face both experts and confused and so in the candidate price equilibrium, $\pi_1 = p_0[1 - (1 - \sigma)(1 - \mu(\underline{k}, \underline{k}))] = \sigma(1 - \mu(\underline{k}, \underline{k}))$. But, firm 1 can profitably deviate to $p_i^d = 1$ and $k_1^d = \bar{k}$ as $\pi_1^d = \sigma(1 - \mu(\bar{k}, \underline{k})) > \sigma(1 - \mu(\underline{k}, \underline{k}))$. As at least one of the firms chooses $k_i > \underline{k}$, part (ii) in the proof of Lemma 1 applies.

c.d.f.s they should be increasing on their supports. Using the approach in Narasimhan (1988) and Baye et al. (1992), we show that both firms choose prices according to c.d.f.s which are defined on a common interval $T = [p_0, 1]$ and are continuous everywhere except possibly at the upper bound $p = 1$; see Appendix A.1.

Suppose firm $i \neq j$ chooses a price p_i and complexity level k_i. Firm i's expected profit, which depends on firm i's choices and on the rival's mixed strategy ξ_j, can be written as

$$\pi_i(p_i, k_i, \xi_j) = p_i \left[\int_{p_i}^{1} \left(\int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j \mid p_j) \right) dF_j(p_j) \right] + p_i s_i \left[1 - \int_{p_0}^{1} \left(\int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j \mid p_j) \right) dF_j(p_j) \right].$$

The expected base of confused consumers is the term in the second square brackets in $\pi_i(p_i,\ k_i,\ \xi_j)$. The remaining consumers form the expected base of experts. But, the experts purchase from firm i only when it offers a lower price than its rival. The expected number of experts, conditional on firm i being the low price seller, is the term in the first square brackets. Firm i serves a share s_i of the expected base of confused consumers. Using Leibniz's Rule, the first derivative of $\pi_i(p_i,\ k_i,\ \xi_j)$ w.r.t. k_i is given by

$$p_i \int_{p_i}^{1} \left(\int_{\underline{k}}^{\bar{k}} \frac{\partial \mu}{\partial k_i} dH_j(k_j \mid p_j) \right) dF_j(p_j) - p_i s_i \int_{p_0}^{1} \left(\int_{\underline{k}}^{\bar{k}} \frac{\partial \mu}{\partial k_i} dH_j(k_j \mid p_j) \right) dF_j(p_j).$$

But $\partial^2 \mu / \partial k_i \partial k_j = 0$, so $\partial \mu(k_i,\ k_j)/\partial k_i$ is independent of k_j, and the first derivative becomes

$$p_i \frac{\partial \mu}{\partial k_i} [(1 - F_j(p_i)) - s_i].$$

Then, as $\partial \mu(k_i,\ k_j)/\partial k_i < 0$, to maximize its expected-profit firm i chooses

$$k_i(p_i) = \begin{cases} \underline{k} & \text{if } 1 - F_j(p_i) > s_i \Leftrightarrow p_i < \widehat{p}_i \\ \bar{k} & \text{if } 1 - F_j(p_i) < s_i \Leftrightarrow p_i > \widehat{p}_i \\ k, & \forall k \in [\underline{k}, \bar{k}] \text{ if } p_i = \widehat{p}_i \end{cases},$$

where the threshold price $\widehat{p}_i$ is implicitly defined by $F_j(\widehat{p}_i) = 1 - s_i$, whenever $\widehat{p}_i$ belongs to the support of F_j. Lemma 1 implies that at least one of the cut-off prices $\widehat{p}_i$ belongs to T, as at least one firm mixes on complexity levels. The next result follows.

Proposition 1. *In equilibrium, a firm's complexity choice depends only on its price. Firm i chooses its price according to a c.d.f. $F_i(p_i)$ with support $T = [p_0, 1]$. If $p_i < \widehat{p}_i$ ($p_i > \widehat{p}_i$) firm i chooses the lowest complexity $\underline{k}$ (highest complexity $\bar{k}$). If $p_i = \widehat{p}_i$, firm i is indifferent between any complexity level $k \in [\underline{k}, \bar{k}]$. If the cut-off price $\widehat{p}_i \in T$, then it is implicitly defined by $F_j(\widehat{p}_i) = s_j$. If $\widehat{p}_i \notin T$, firm i chooses a deterministic complexity level, but then it must be that firm j randomizes on complexity levels, i.e. $\widehat{p}_j \in (p_0, 1)$.*

When a firm mixes on complexity in equilibrium, there is a positive relationship between prices and complexity levels. If $\widehat{p}_i \in T$, at all prices below the cut-off level $\widehat{p}_i$, firm i chooses the lowest complexity and at all prices above $\widehat{p}_i$, it chooses the highest complexity level. Intuitively, when a firm chooses a relatively high price, its incentive to choose high complexity is stronger as it relies more on selling to confused consumers. In contrast, when setting a relatively low price, a firm has a stronger incentive to choose low complexity as this results in a larger base of experts.

We first analyze a situation where both firms randomize on complexity levels, and so the cut-off prices defined in Proposition 1 must satisfy $\widehat{p}_i \in T = (p_0, 1)$ for $i = 1, 2$. This implies that firm i chooses complexity level $\underline{k}$ with probability $F_i(\widehat{p}_i)$ and complexity level $\bar{k}$ with probability $1 - F_i(\widehat{p}_i)$. The threshold prices $\widehat{p}_i \in T$ are implicitly defined by $F_j(\widehat{p}_i) = s_j$ for $j = 1, 2,\ j \neq i$, where s_j is firm j's share of consumers confused by complexity (i.e., $s_1 = \sigma > 1/2$ and $s_2 = 1 - \sigma$). For expositional simplicity, denote:

$$\lambda_1 \equiv F_1(\widehat{p}_1) \text{ and } \lambda_2 \equiv F_2(\widehat{p}_2).$$

Consistency requires that $F_i(\widehat{p}_i) \in (0, 1)$ and $F_i(\widehat{p}_j) = s_i$. The following condition holds when both firms mix on both prices and complexity levels in equilibrium (see Appendix A.6).

Condition 1.

$$0 < p_0 < \widehat{p}_1 < \widehat{p}_2 < 1.$$

Below we illustrate the derivation of firm 1's expected profit. Consider a price $p \in [p_0, \widehat{p}_1)$. By Proposition 1, firm 1 associates prices in this range with complexity level $\underline{k}$. Then, its expected profit is

$$\pi_1(p, \underline{k}) = p\big[(F_2(\widehat{p}_2) - F_2(p)) + (1 - F_2(\widehat{p}_2))\mu(\underline{k}, \bar{k}) + \sigma(1 - F_2(\widehat{p}_2))(1 - \mu(\underline{k}, \bar{k}))\big]. \quad (1)$$

With probability $F_2(\widehat{p}_2)$, firm 2 chooses $\underline{k}$, so that all consumers are experts, i.e., $\mu(\underline{k}, \underline{k}) = 1$. The experts purchase from firm 1 if firm 2's price is higher, which happens with probability $F_2(\widehat{p}_2) - F_2(p)$. With probability $1 - F_2(\widehat{p}_2)$, firm 2 chooses $\bar{k}$ and there are $\mu(\underline{k}, \bar{k})$ informed and $1 - \mu(\underline{k}, \bar{k})$ confused consumers. All the informed purchase from firm 1 as it offers a lower price (firm 2 associates $\bar{k}$ with prices higher than $\widehat{p}_2$) and so does a share σ of the confused consumers. The first two terms in square brackets capture the expected number of experts, while the last term in square brackets gives the expected number of confused consumers.

Consider firm 1's expected profit for a price $p \in [\widehat{p}_1, \widehat{p}_2]$. By Proposition 1, firm 1 associates prices in this range with complexity level $\bar{k}$. Then, its expected profit is

$$\pi_1(p, \bar{k}) = p\big\{(F_2(\widehat{p}_2) - F_2(p))\mu(\underline{k}, \bar{k}) + (1 - F_2(\widehat{p}_2))\mu(\bar{k}, \bar{k}) + \sigma\big[F_2(\widehat{p}_2)(1 - \mu(\underline{k}, \bar{k})) + (1 - F_2(\widehat{p}_2))(1 - \mu(\bar{k}, \bar{k}))\big]\big\}.$$

The expected number of confused consumers is the term in square brackets. Firm 1 serves a fraction σ of this group. Firm 1 also serves the expert consumers if firm 2 chooses a higher price. With probability $F_2(\widehat{p}_2) - F_2(p)$, there are $\mu(\underline{k}, \bar{k})$ experts while, with probability $1 - F_2(\widehat{p}_2)$, there are $\mu(\bar{k}, \bar{k})$; this is reflected by the first two terms in curly brackets.

Consider firm 1's expected profit for $p \in (\widehat{p}_2, 1]$. By Proposition 1, firm 1 associates prices in this range with complexity level $\bar{k}$. Then, its expected profit is

$$\pi_1(p, \bar{k}) = p\big\{(1 - F_2(p))\mu(\bar{k}, \bar{k}) + \sigma\big[F_2(\widehat{p}_2)(1 - \mu(\underline{k}, \bar{k})) + (1 - F_2(\widehat{p}_2))(1 - \mu(\bar{k}, \bar{k}))\big]\big\}.$$

Echoing previous reasoning, with probability $F_2(\widehat{p}_2)$ firm 2 chooses $\underline{k}$, in which case there are $\mu(\underline{k}, \bar{k})$ informed and $1 - \mu(\underline{k}, \bar{k})$ confused consumers. A share σ of the confused consumers purchases from firm 1, the prominent seller. The experts do not purchase from firm 1 as firm 2's price is lower. With probability $1 - F_2(\widehat{p}_2)$, firm 2 chooses $\bar{k}$, so there are $\mu(\bar{k}, \bar{k})$ experts and $1 - \mu(\bar{k}, \bar{k})$ confused consumers. A share σ of confused consumers buy from firm 1. The experts purchase from firm 1 if it offers a lower price, which happens with probability $1 - F_2(p)$. The first term in curly brackets captures the expected number of experts, while the term in square brackets gives the expected number of confused consumers.

In Appendix A.2, we present firm 1's expected profits at p_0, $\hat{p}_1$and $\hat{p}_2$ and also when $p \to \widehat{p}_1$ and $p \to \widehat{p}_2$. There, we also derive firm 2's expected profit over the three price ranges, using the same approach as above. Next section combines these derivations to characterize the mixed strategy equilibrium and to identify a condition on the parameter values under which both firms randomize on both prices and complexity levels in equilibrium. When this condition does not hold – which happens when firm 1's level of prominence is relatively high – both firms mix on prices, but only the prominent firm randomizes on complexity levels.

4. Equilibrium analysis

In equilibrium, firm i's expected profit for any price-complexity combination $(p,\ k_i)$, which is assigned positive density in equilibrium, must be constant. Then, using expressions (A.1)–(A.3), (A.6), and (A.7) from Appendix A.2, we can write the price ratios $p_0/\ \widehat{p}_1$ and $p_0/\ \widehat{p}_2$ as functions of $\lambda_2 = F_2(\widehat{p}_2)$ and $\lambda_1 = F_1(\widehat{p}_1)$, firm 2's and firm 1's probabilities of choosing $\underline{k}$ in equilibrium, respectively. These ratios are presented in Appendix A.3. We then obtain the equilibrium values of λ_1 and λ_2,

$$\lambda_1 = \frac{(1-\sigma)[1 - \sigma(2-\sigma)(1 - \mu(\underline{k}, \bar{k}))]}{1 - (1-\sigma^2)(1 - \mu(\underline{k}, \bar{k}))} \quad \text{and} \quad \lambda_2 = \frac{\sigma[1 - (1-\sigma^2)(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(2-\sigma)(1 - \mu(\underline{k}, \bar{k}))}. \tag{2}$$

It can be checked that $\lambda_1 \in (0,\ 1)$ and $\lambda_2 > 0$. Furthermore, $\lambda_2 < 1$ holds iff the following condition is satisfied.

Condition 2.

$$(1-\sigma)/[\sigma(1-\sigma+\sigma^2)] > 1-\mu(\underline{k},\bar{k}).$$

As $\mu(\bar{k},\bar{k}) = 2\mu(\underline{k},\bar{k}) - 1$ and $0 \le \mu(\bar{k},\bar{k}) < \mu(\underline{k},\bar{k})$, it follows that $1-\mu(\underline{k},\bar{k}) \le 1/2$. For relatively low levels of prominence (that is, for $\sigma < 0.71$), this condition always holds and so firm 2 mixes between the highest and the lowest price complexity levels. More generally, for a given $\mu(\underline{k},\bar{k})$, the condition is satisfied when firm 1's level of prominence is not too high. However, Condition 2 gets more stringent as firm 1's prominence increases (the LHS of the inequality in the condition is decreasing in σ). When firm 1 is prominent enough, firm 2 benefits more from price transparency, as its share of confused consumers is relatively small.

In Appendix A.3, we show that when $\lambda_i \in (0,\,1)$, the consistency requirements also hold: $F_i(\widehat{p}_1) < F_i(\widehat{p}_2)$ for $i = 1, 2$, where $F_i(\widehat{p}_i) = \lambda_i$ and $F_i(\widehat{p}_j) = s_i$. Also there, we explore the firms' price c.d.f.s at the upper bound of the support. Using Lemma 4, we show that firm 2's price c.d.f. is continuous everywhere, while firm 1 has a mass point at the upper bound of the price c.d.f.'s support, $p = 1$. Then, we verify that p_0, $\widehat{p}_1$, and $\widehat{p}_2$ are well defined under Condition 2. Finally, we present the equilibrium cut-off prices in expressions (A.8) and (A.9), followed by the pricing c.d.f.s of the two firms. Using (A.1), (A.4) and (2), we obtain the equilibrium profit of firm 1 (π_1^*) and the lower bound of the price support (p_0).

$$\pi_1^* = \sigma(1-\mu(\underline{k},\bar{k}))\frac{2-\sigma-\sigma\big(\sigma^2-2\sigma+3\big)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))}; \tag{3}$$

$$p_0 = \sigma(1-\mu(\underline{k},\bar{k}))\frac{2-\sigma-\sigma\big(\sigma^2-2\sigma+3\big)(1-\mu(\underline{k},\bar{k}))}{\mu(\underline{k},\bar{k})+\sigma(1-\sigma)(\sigma^2-\sigma+1)(1-\mu(\underline{k},\bar{k}))^2}. \tag{4}$$

Using (4) and (A.6), we calculate firm 2's equilibrium profit,

$$\pi_2^* = \sigma(1-\mu(\underline{k},\bar{k})))\frac{2-\sigma-\sigma\big(\sigma^2-2\sigma+3\big)(1-\mu(\underline{k},\bar{k}))}{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))}. \tag{5}$$

Note that $\pi_1^*/\pi_2^* = \lambda_2/\sigma = (1-\sigma)/\lambda_1$.

Below we summarize our findings.

Proposition 2. *Under Condition 2, in the unique mixed strategy equilibrium firm i chooses the lowest complexity* $\underline{k}$ *with probability* $\lambda_i = F_i(\widehat{p}_i) \in (0, 1)$, *defined in (2) and highest complexity* $\bar{k}$ *with probability* $1-\lambda_i$. *Both firms randomize on prices in* $[p_0,\,1]$, *with* p_0 *given in (4). Firm 2's price c.d.f.* (F_2) *is continuous everywhere, while firm 1's price c.d.f.* (F_1) *is continuous on* $[p_0,\,1)$ *and has an atom at* $p = 1$. *Firm i uses* $\underline{k}$ ($\bar{k}$) *at prices below (above)* $\widehat{p}_i \in (p_0, 1)$. *The equilibrium profits* π_1^* *and* π_2^* *are given in (3) and (5).*

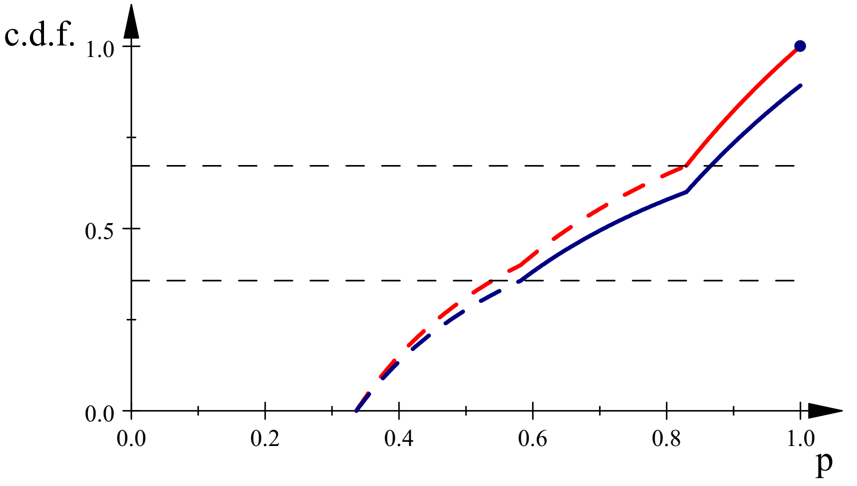

Fig. 1. The firms' price c.d.f.s for $\sigma = .6$ and $\mu(\underline{k}, \bar{k}) = .6$. $F_1(p)$ is the bottom line and $F_2(p)$ is the top line. The dashed lines correspond to prices associated with $\underline{k}$.

When firm 1's prominence is not too high in the sense that $\sigma > 1/2$, but Condition 2 is satisfied, both firms randomize on complexity levels and prices in equilibrium. In this case, the difference in the firms' shares of confused consumers is not too large. In the limit, when $\sigma \to 1/2$, $\lambda_1 = \lambda_2 = 1/2$, $\widehat{p}_1 = \widehat{p}_2$, and both firms' pricing c.d.f.s are continuous everywhere on their common support. This is consistent with the results in Carlin (2009). The following numerical example and Fig. 1 illustrate the result in Proposition 2.

Example 1. When $\sigma = .6$ and $\mu(\underline{k}, \bar{k}) = .6$, in equilibrium firm 1 and 2 choose $\underline{k}$ with probabilities $\lambda_1 = .357$ and $\lambda_2 = .672$, respectively. The two firms randomize on prices according to the following c.d.f.s, which are illustrated in Fig. 1,

$$F_1(p) = \begin{cases} .846 - .284/p & for\ p \in [p_0, \widehat{p}_2) \\ 1.171 - .474/p & for\ p \in [\widehat{p}_2, \widehat{p}_1] \\ 2.131 - 1.422/p & for\ p \in (\widehat{p}_1, 1] \end{cases} \text{ and}$$

$$F_2(p) = \begin{cases} .948 - .319/p & for\ p \in [p_0, \widehat{p}_2) \\ 1.313 - .531/p & for\ p \in [\widehat{p}_2, \widehat{p}_1] \\ 2.593 - 1.593/p & for\ p \in (\widehat{p}_1, 1) \end{cases},$$

where $p_0 = .336$, $\widehat{p}_1 = .582$, and $\widehat{p}_2 = .829$. Firm 1 and firm 2 make profits $\pi_1^* = .319$ and $\pi_2^* = .284$, respectively. Firm 1's atom at $p = 1$ is $\phi = .108$.

When Condition 2 does not hold, the results in Proposition 2 no longer apply as the derived λ_2 is weakly larger than 1 (and to be a well-defined probability and for both firms to randomize on complexity, λ_2 should be strictly smaller than 1). In this case, because firm 1's prominence advantage is large enough, firm 2 serves a relatively small share of confused consumers. Then, firm 2 relies more on expert consumers and so benefits more from market transparency than from confusion. We prove the following result in Appendix A.4.

Proposition 3. *When Condition 2 does not hold, in the unique mixed strategy equilibrium firm 2 chooses* $\underline{k}$ *for sure and firm 1 chooses the lowest complexity* $\underline{k}$ *with probability* $\lambda_1^h = F_1^h(\widehat{p}_1^h)$ *and the highest complexity* $\bar{k}$ *with probability* $1-\lambda_1^h$*, where*

$$\lambda_1^h = \frac{(1-\sigma)[1-\sigma(1-\mu(\underline{k},\bar{k}))]}{1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))}.$$

Both firms randomize on prices in $[p_0^h, 1]$*, with* $p_0^h = \sigma(1-\mu(\underline{k},\bar{k}))$*. Firm 2's price c.d.f.* F_2^h *is continuous everywhere, while firm 1's price c.d.f.* F_1^h *is continuous on* $[p_0^h, 1)$ *and has an atom at* $p = 1$*. Firm 1 uses* $\underline{k}$ $(\bar{k})$ *at prices below (above)* $\widehat{p}_1^h = (1-\mu(\underline{k},\bar{k})) \in (p_0^h, 1)$*. The equilibrium profits are given by*

$$\pi_{h1}^* = \sigma(1-\mu(\underline{k},\bar{k})) \text{ and } \pi_{h2}^* = \sigma(1-\mu(\underline{k},\bar{k}))\frac{1-\sigma(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))}. \quad (6)$$

When prominence is large enough, firm 2 chooses the lowest complexity for sure to minimize the number of confused buyers and reduce its disadvantage. The prominent firm, as before, associates lower prices with the lowest complexity (at those prices it benefits from more transparency) and higher prices with highest complexity (at those prices it relies more on confused consumers). Specifically, firm 1 chooses complexity $\underline{k}$ for all prices $p < \widehat{p}_1^h \in (p_0^h, 1)$ and $\bar{k}$ for all prices $p \geq \widehat{p}_1^h$. Proposition 1 then requires that firms' pricing c.d.f.s satisfy $F_2^h(\widehat{p}_1^h) = 1-\sigma$ and $F_1^h(1) \leq \sigma$ (that is, $\widehat{p}_2^h \geq 1$).[14] The following example and Fig. 2 illustrate the results for relatively high prominence.

Example 2. When $\sigma = .8$ and $\mu(\underline{k},\bar{k}) = .6$, in equilibrium firm 1 chooses $\underline{k}$ with probability $\lambda_1^h = .145$, while firm 2 chooses $\underline{k}$ for sure. The two firms randomize on prices according to the following c.d.f.s, which are illustrated in Fig. 2,

$$F_1^h(p) = \begin{cases} .726 - .232/p & for\ p \in [p_0^h, \widehat{p}_1^h) \\ 1.113 - .387/p & for\ p \in [\widehat{p}_1^h, 1] \end{cases} \text{ and}$$

$$F_2^h(p) = \begin{cases} 1 - .32/p & for\ p \in [p_0^h, \widehat{p}_1^h) \\ 1.533 - .533/p & for\ p \in [\widehat{p}_1^h, 1) \end{cases},$$

where $p_0^h = .32$ and $\widehat{p}_1^h = .4$. Firm 1 and firm 2 make profits $\pi_{h1}^* = .32$ and $\pi_{h2}^* = .232$, respectively. Firm 1's atom at $p = 1$ is $\phi^h = .274$.

[14] As by Lemma 1 $F_1^h(\widehat{p}_2^h) = \sigma$, if $F_1^h(1) > \sigma$ then $\widehat{p}_2^h < 1$ and the candidate $\lambda_2^h = F_2(\widehat{p}_2^h) < 1$. But this is inconsistent with an equilibrium where firm 2 chooses $\underline{k}$ for sure.

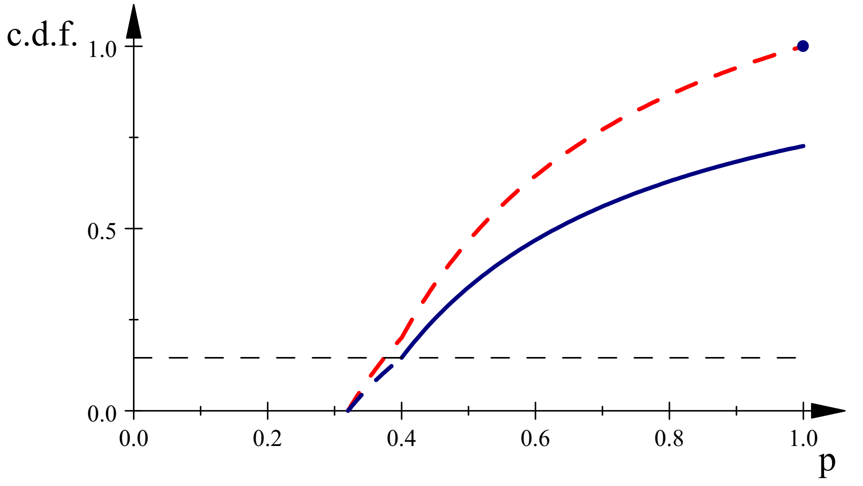

Fig. 2. The firms' price c.d.f.s for $\sigma = .8$ and $\mu(\underline{k}, \bar{k}) = .6$. $F_1^h(p)$ is the bottom line and $F_2^h(p)$ is the top line. The dashed lines correspond to prices associated with $\underline{k}$.

The remainder of this section uses Propositions 2 and 3 to explore the role of prominence on market outcomes.

In Examples 1 and 2 where $\mu(\underline{k}, \bar{k}) = 0.6$, an increase in σ from.6 to.8 results in a decrease in industry profit from.603 to.552. This shows that an increase in the prominence level might harm industry profit, in which case it benefits the consumers, as total surplus is normalized to one. It also indicates that markets where an entrant competes with a prominent enough incumbent may be more competitive than markets where the differences in prominence between suppliers are relatively smaller, which may be of relevance in retail electricity markets where new entrants facing incumbent suppliers could become more prominent over time. Holding $\mu(\underline{k}, \bar{k}) = 0.6$, Fig. 3 illustrates individual and aggregate profits as functions of the level of prominence. In this case, total industry profit is lowest and consumers surplus highest at $\sigma = 0.754$, which is the cut-off prominence level for the two types of equilibria presented in Propositions 2 and 3.

Example 3. Suppose $\mu(\underline{k}, \bar{k}) = 0.6$. Then, Condition 2 holds iff $\sigma <$ 0.754.

In our framework, an increase in the level of prominence affects pricing, and ultimately profits, in two ways. First, prominence has a direct effect on prices as it reallocates the confused consumers in favour of the salient firm. Second, prominence affects the firms' probabilities of choosing the lowest price complexity, i.e., λ_1 and λ_2 – which endogenize the expected share of confused consumers – and, therefore, it also has an indirect effect on prices through this channel.

In settings where the total share of confused consumers is exogenous, only the direct effect plays a role. In that case, an increase in the level of prominence boosts industry profit and so harms consumer welfare; this can be easily checked, for instance,

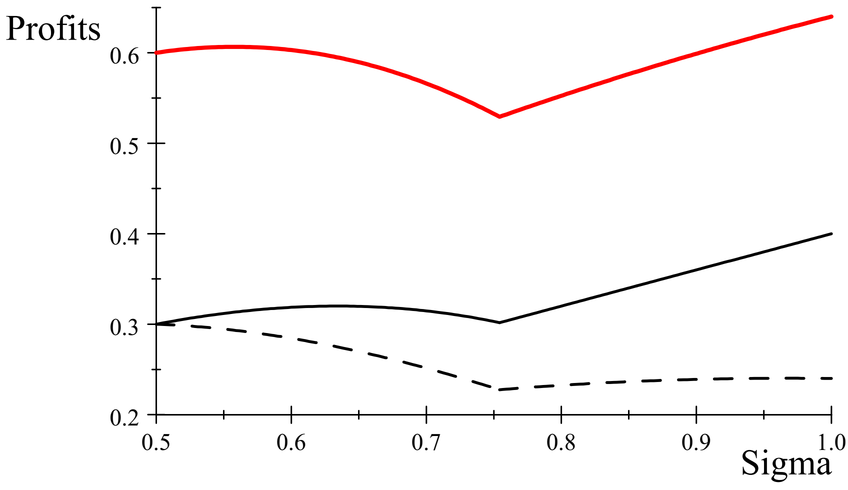

Fig. 3. The profits of firm 1 (medium solid) and firm 2 (dashed), and total profit (thick solid) for $\mu(\underline{k}, \bar{k}) = .6$.

in Narasimhan (1988). Gu and Wenzel (2014) show that this result is qualitatively robust in a sequential set-up where a salient firm and its rival first commit to complexity levels and then compete in prices. In their analysis, although the share of confused is determined endogenously, firms choose deterministic complexity levels and the pricing stage is similar to Narasimhan (1988).

The comparative statics of consumers surplus with respect to the prominence level in our model is different from the corresponding result in the sequential move setting of Gu and Wenzel (2014). However, our non-monotonicity result is not due to the timing of the game per se but to the cut-off structure of the equilibrium – that is, to the statistical dependence between firms' pricing and price complexity equilibrium strategies.[15] Therefore, the indirect effect on firms' probabilities of choosing the lowest price complexity identifies a novel channel through which prominence affects industry profit and consumer surplus. This novel effect is related to the conflicting incentives of the two firms identified in our next result.

Corollary 1. *In the mixed strategy equilibrium, firm 2's probability of using the lowest complexity (λ_2) weakly increases in the level of prominence (σ), while firm 1's corresponding probability (λ_1) decreases in σ.*

An increase in the prominence level increases the salient firm's share of confused consumers and so it lowers its incentive to choose the lowest price complexity level ($\underline{k}$). The larger is firm 1's share of confused consumers, the more the firm benefits from

[15] For instance, in a simultaneous move setting where price format differentiation is the main source of confusion (rather than complexity), price and price format decisions are independent in equilibrium and consumer surplus decreases in the degree of asymmetry/prominence; see Chioveanu (2019).

confusion. In contrast, an increase in prominence decreases the less salient firm's share of confused consumers and so it boosts this firm's incentive to choose $\underline{k}$. The lower is firm 2's share of confused consumers, the more it benefits from transparency.

For relatively high levels of prominence – i.e., when Condition 2 does not hold – firm 2 chooses $\underline{k}$ for sure (i.e. $\lambda_2 = 1$) and, unlike its rival, it cannot directly affect market transparency by adjusting its probability of choosing $\underline{k}$ in response to an increase in σ. In this range, industry profit strictly increases and consumer surplus strictly decreases in σ (see Appendix A.5 for the details).

For relatively low levels of prominence – i.e., when Condition 2 holds – both firms can adjust their probabilities of choosing $\underline{k}$ in response to an increase in σ and they have conflicting incentives. In this range, industry profit may decrease and consumer surplus may increase in σ. Numerical examples suggest that this is the case in the range of σ's where $\lambda_2 < 1$ is close to 1 and where an increase in prominence makes the less prominent firm a more aggressive competitor.

In our simultaneous move setting, the direct and indirect effects of prominence on pricing cannot be clearly separated and general comparative statics analysis is intractable. However, a combination of numerical simulations and analytical results provide further insights. Over the range of σ's where Condition 2 holds, industry profit has an inverted-U shape. Outside this range, it is strictly increasing. As a result, consumer surplus is maximized either at the cut-off prominence level for the two types of equilibria presented in Propositions 2 and 3 (this happens for $\mu(\underline{k}, \bar{k}) \lesssim 0.8$, see Example 3 for an illustration) or as $\sigma \to 0.5$ (which happens for $\mu(\underline{k}, \bar{k}) \gtrsim 0.8$). The cut-off prices of firms 1 and 2, are weakly decreasing and, respectively, increasing in σ: $\widehat{p}_1$ ($\widehat{p}_2$) is strictly decreasing (increasing) in σ, while $\widehat{p}_1^h = 1 - \mu(\underline{k}, \bar{k})$ ($\widehat{p}_2^h = 1$) are constant and so independent of σ. The lower bound of the firms' price support (p_0) has an inverted-U shape over the range of σ 's where Condition 2 holds and it is strictly increasing outside this range ($p_0^h = \sigma\widehat{p}_1^h$). The likelihood that the prominent firm chooses the monopoly price strictly increases in σ.

Corollary 2. *In the mixed strategy equilibrium, (i) the more prominent firm makes higher profits than the rival; (ii) the price distribution of the prominent firm first order stochastically dominates the one of the less prominent firm; (iii) the more prominent firm's average price is higher than that of the less prominent firm, and (iv) the less prominent firm chooses the lowest complexity ($\underline{k}$) with higher probability than the rival.*

The prominent firm attracts a larger share of confused consumers, and so it benefits more from market-wide confusion. For this reason, it chooses the highest level of complexity with higher probability than its rival, has lower incentives to compete for the expert consumers, and therefore it chooses a higher average price. The combined effect of charging higher prices (in the first order stochastic dominance sense) and attracting a higher share of the confused consumers allows the prominent firm to make higher profits in equilibrium. Confused consumers' bias in favor of the prominent firm appears to be inconsistent with the ranking of the average prices. Our next result focuses on the ranking of the average prices, conditional on these being associated with the lowest complexity ($\underline{k}$).

Corollary 3. *In the mixed strategy equilibrium, the more prominent firm chooses a lower cut-off price – below which it uses the lowest level of price complexity* $\underline{k}$ *– than its rival* ($\widehat{p}_1 < \widehat{p}_2$ *when Condition 2 holds and* $\widehat{p}_1^h < \widehat{p}_2^h = 1$ *when it does not). Furthermore, conditional on choosing the lowest complexity, the more prominent firm offers a lower average price than its rival:* $E(p_1 \mid p_1 < \widehat{p}_1) < E(p_2 \mid p_2 < \widehat{p}_2)$ *when Condition 2 holds, and* $E(p_1 \mid p_1 < \widehat{p}_1^h) < E(p_2 \mid p_2 < \widehat{p}_2^h)$ *when it does not.*

We prove this corollary in Appendix A.5 and sketch here the intuition. The price c.d.f.s of the two firms, conditional on price being strictly less than $p<1$, are identical. This is because unconditional price densities are proportional everywhere below $p = 1$, which can be easily seen in Examples 1 and 2. Combined with the fact that in equilibrium the cut-off price below which firm 1 chooses $\underline{k}$ is lower than the cut-off price of firm 2 (that is, $\widehat{p}_1 < \widehat{p}_2$, if Condition 2 holds, and $\widehat{p}_1^h < \widehat{p}_2^h$, if it does not), this proves the corollary.

Hence, in our model, consumers' bias for the prominent firm is consistent with the ranking of the average prices conditional on the lowest complexity. For example, if information on prices associated with the lowest complexity gets aggregated through interactions between confused consumers (e.g. on social media), then the raking of these conditional prices would confirm the consumer bias for the prominent firm ex-post.

Another interpretation, suggested by a referee, could make confused consumers' bias consistent with market outcomes ex-post. Suppose that confused consumers are more likely to buy from the firm with the largest market share. Then, in markets where the share of experts is small enough, the prominent firm's market share will be larger than the rival's and so confused consumers' bias for this firm would be confirmed by the market shares.

5. Alternative confusion technologies

The main analysis assumes that a marginal increase in firm i's complexity reduces the fraction of experts in the market but does not alter the effectiveness of the rival's marginal increase in price complexity on consumers, that is, $\partial^2\mu/\partial k_1 \partial k_2 = 0$. Below we prove that there exists an equilibrium which is qualitatively consistent with the one in the main analysis whenever $\partial^2\mu/\partial k_1 \partial k_2 > 0$. As $\partial\mu/\partial k_i = \mu_i < 0$, this condition requires that the magnitude of the marginal impact of firm i's complexity be decreasing in firm j's complexity ($\partial|\mu_i|/\partial k_j < 0$).[16] More specifically, we show that if the rival uses a mixed strategy with a positive relationship between price and price complexity, it is a best

[16] An example of confusion technology which satisfies this assumption is $\mu(k_1, k_2) = (\underline{k})^2/(k_1 k_2)$.

response for a firm to associate prices below a threshold with the lowest complexity and prices above it with the highest complexity.

Suppose that firm j uses a mixed strategy ξ_j so that $dk_j(p_j)/dp_j \geq 0$. Consider the expected profits of firm i presented in Section 3:

$$\pi_i(p_i, k_i, \xi_j) = p_i \int_{p_i}^{1} \left(\int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j \mid p_j) \right) dF_j(p_j) \\ + \; p_i s_i \left[1 - \int_{p_0}^{1} \left(\int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j \mid p_j) \right) dF_j(p_j) \right].$$

The f.o.c. of firm i's expected profit maximization w.r.t. k_i requires that

$$p_i \left(\int_{p_i}^{1} \mathbb{E}(\mu_i(p_j) \mid p_j) dF_j(p_j) - s_i \int_{p_0}^{1} \mathbb{E}(\mu_i(p_j)) dF_j(p_j) \right) = 0 \;, \tag{7}$$

where $\partial\mu(k_i, k_j(p_j))/\partial k_i \equiv \mu_i(k_i, k_j(p_j))$ gives the marginal impact of k_i on μ and $\mathbb{E}(\mu_i(p_j) \mid p_j) = \int_{\underline{k}}^{\bar{k}} \mu_i(k_i, k_j(p_j)) dH_j(k_j \mid p_j)$ is the expected marginal impact of an increase in k_i on the fraction of experts conditional on firm j's price. For given ξ_j, $\int_{p_0}^{1} \mathbb{E}(\mu_i(p_j)) dF_j(p_j)$ – the overall expected marginal impact of an increase in k_i on the fraction of experts – is a constant. At $p_i = p_0$, the term in brackets becomes $(1 - s_i) \int_{p_0}^{1} \mathbb{E}(\mu_i(p_j)) dF_j(p_j) < 0$ and when $p_i \to 1$, it converges to $-s_i \int_{p_0}^{1} \mathbb{E}(\mu_i(p_j)) dF_j(p_j) > 0$. So, there is at least one price $\widehat{p}_i \in (p_0, 1)$ which satisfies (7). Moreover, $\widehat{p}_i$ is unique if

$$d\left(\int_{p_i}^{1} \mathbb{E}(\mu_i(p_j) \mid p_j) dF_j(p_j) \right) / dp_i = \int_{p_i}^{1} \left[\frac{d(\mathbb{E}(\mu_i(p_j) \mid p_j))}{dp_i} \right] dF_j(p_j) - \mu_i^e(p_i) F_j'(p_i) > 0 \;,$$

where the equality follows from Leibniz's Rule. As $-\mu_i^e(p_i) > 0$ and $F_j'(p_i) > 0$, this condition holds if $d\mathbb{E}(\mu_i(p_j) \mid p_j)/dp_i > 0$. But, as $dk_j(p_j)/dp_j > 0$, a sufficient condition is then $\partial\mu_i(k_i, k_j)/\partial k_j = \partial^2\mu(k_i, k_j)/\partial k_i \partial k_j > 0$. Hence, whenever $\partial^2\mu/\partial k_i \partial k_j > 0$ there exists a unique $\widehat{p}_i \in (p_0, 1)$ which satisfies (7) and it follows that firm i's complexity level choice is

$$k_i(p) = \begin{cases} \underline{k} & \text{if } p < \widehat{p}_i \\ \bar{k} & \text{if } p > \widehat{p}_i \\ k, & \forall k \in [\underline{k}, \bar{k}] \text{ if } p = \widehat{p}_i \end{cases} ,$$

whenever $\widehat{p}_i$ belongs to T_j the support of F_j. Lemma 1 implies that at least one of the cut-off prices $\widehat{p}_i$ belongs to T_j. This shows that a mixed strategy equilibrium like the one analyzed in our benchmark model exists for a more general confusion technology.

6. Conclusions

We analyze the interplay between consumer confusion due to price complexity and firm prominence in a model where two firms compete by simultaneously choosing prices and the complexity of their price offers. One of the firms enjoys a higher level of prominence, which may be due to higher brand recognition, industry dynamics, or advertising effort/spending. Price complexity leads to consumer confusion so that some buyers are able to identify the best offer, while others may get confused. The share of confused consumers is determined by market-wide complexity. The confused consumers shop at random and favor the more prominent firm, in the sense that they are more likely to buy from it.

In equilibrium there is dispersion in both prices and complexity levels. The nature of the equilibrium depends on the level of prominence. For moderate levels of prominence, both firms mix on price complexity levels, while for high levels of prominence, the less prominent firm chooses the lowest price complexity for sure. The prominent firm makes higher profits, chooses higher prices on average and the lowest complexity level with lower probability, and sets the monopoly price with positive probability.

In our model, a decrease in prominence may increase industry profits and harm consumers. In addition, conditional on choosing the lowest complexity, the prominent firm sets a lower price, on average, which is consistent with confused consumers' behavior. This suggests that it may be useful to investigate in the future an alternative model where confused consumers' beliefs about the price ranking is based on the average prices conditional on lowest complexity and where, as a result, prominence is endogenous. Finally, our analysis shows that a qualitatively similar equilibrium exists with alternative confusion technologies if the marginal impact of an increase in one firm's complexity increases in the rival's complexity level.

Appendix A

A.1. Properties of the pricing distribution functions

The proofs of the lemmata below are standard and presented in the working paper version (Chioveanu, 2017).

Lemma 3. *The supports of the pricing c.d.f.s,* T_1 *and* T_2 *are both connected intervals (i.e., there are no gaps in either of them).*

Lemma 4. *Neither firm can have a mass point in the interior or at the lower bound of the other firm's price c.d.f. support. Moreover, firm* i *cannot have a mass point at the upper bound of* T_j *if firm* j *has a mass point there.*

Lemma 5. *In equilibrium, it must hold that* $T_1 = T_2 = [p_0, p^h]$ *for* $p_0 < p^h \leq 1$.

Lemma 6. *In equilibrium,* $\sup T_1 = \sup T_2 = 1$.

A.2. Expected profits

Derivation of firm 1's expected profit

- Suppose firm 1 chooses a price $p \in [p_0, \widehat{p}_1)$.

Using (1), as $F_2(\widehat{p}_2) = \lambda_2$, firm 1's expected profits at $p = p_0$ and when $p \to \widehat{p}_1$ are

$$\pi_1(p_0, \underline{k}) = p_0\big[1 - (1-\sigma)(1-\lambda_2)(1-\mu(\underline{k}, \bar{k}))\big]; \tag{A.1}$$

$$\lim_{p \nearrow \widehat{p}_1} \pi_1(p, \underline{k}) = \widehat{p}_1\big[\sigma - (1-\sigma)(1-\lambda_2)(1-\mu(\underline{k}, \bar{k}))\big]. \tag{A.2}$$

- Suppose firm 1 chooses a price $p \in [\widehat{p}_1, \widehat{p}_2]$.

As $1 - \mu(\underline{k}, \bar{k}) = \mu(\underline{k}, \bar{k}) - \mu(\bar{k}, \bar{k})$, using $F_2(\widehat{p}_2) = \lambda_2$, $\pi_1(\widehat{p}_1, \bar{k}) = \lim_{p \nearrow \widehat{p}_1} \pi_1(p, \underline{k})$, as given in (A.2). By Proposition 1, $F_2(\widehat{p}_1) = 1 - \sigma$ and the expected profit at $p = \widehat{p}_2$ is

$$\pi_1(\widehat{p}_2, \bar{k}) = \widehat{p}_2\{(1-\lambda_2) - (1-\mu(\underline{k}, \bar{k}))[2(1-\sigma-\lambda_2) + \sigma\lambda_2]\}. \tag{A.3}$$

- Suppose firm 1 chooses a price $p \in (\widehat{p}_2, 1]$.

As $1 - \mu(\underline{k}, \bar{k}) = \mu(\underline{k}, \bar{k}) - \mu(\bar{k}, \bar{k})$ and $F_2(\widehat{p}_2) = \lambda_2$, firm 1's expected profit becomes

$$\pi_1(p, \bar{k}) = p\{(1-F_2(p))(2\mu(\underline{k}, \bar{k}) - 1) + \sigma(1-\mu(\underline{k}, \bar{k}))(2-\lambda_2)\}. \tag{A.4}$$

It can be checked that $\lim_{p \searrow \widehat{p}_2} \pi_1(p, \bar{k}) = \pi_1(\widehat{p}_2, \bar{k})$ as presented in (A.3).

Derivation of firm 2's expected profit

- Suppose firm 2 chooses a price $p \in [p_0, \widehat{p}_1)$.

By Proposition 1, this price is associated with complexity $\underline{k}$, so firm 2's expected profit is

$$\begin{aligned}\pi_2(p, \underline{k}) = p\big\{&(F_1(\widehat{p}_1) - F_1(p))\mu(\underline{k}, \underline{k}) + (1-F_1(\widehat{p}_1))\mu(\underline{k}, \bar{k}) \\ &+ (1-\sigma)\big[F_1(\widehat{p}_1)(1-\mu(\underline{k}, \underline{k})) + (1-F_1(\widehat{p}_1))(1-\mu(\underline{k}, \bar{k}))\big]\big\}.\end{aligned} \tag{A.5}$$

With probability $F_1(\widehat{p}_1)$, firm 1 chooses $\underline{k}$, so that there are $\mu(\underline{k}, \underline{k})$ informed and $1 - \mu(\underline{k}, \underline{k})$ confused consumers. A share $1 - \sigma$ $(<\sigma)$ of the confused purchases from firm 2, the less prominent seller. The experts purchase from firm 2 if firm 1's price is higher, which happens with probability $F_1(\widehat{p}_1) - F_1(p)$. With probability $1 - F_1(\widehat{p}_1)$, firm 1 chooses $\bar{k}$, so there are $\mu(\underline{k}, \bar{k})$ informed and $1 - \mu(\underline{k}, \bar{k})$ confused consumers. All experts purchase from firm 2 as it offers a lower price (firm 1 associates $\bar{k}$ with prices higher than $\widehat{p}_1$) and so does a share $1 - \sigma$ of the confused consumers. The first two terms in the curly brackets capture the expected number of experts, whereas the term in square brackets gives the expected number of confused consumers. Using $\mu(\underline{k}, \underline{k}) = 1$ and $F_1(\widehat{p}_1) = \lambda_1$, it follows that,

$$\pi_2(p_0, \underline{k}) = p_0[1 - \sigma(1-\lambda_1)(1-\mu(\underline{k}, \bar{k}))] \quad \text{and}$$

$$\lim_{p \nearrow \widehat{p}_1} \pi_2(p, \underline{k}) = \widehat{p}_1(1-\lambda_1)[1-\sigma(1-\mu(\underline{k},\bar{k}))]. \tag{A.6}$$

- Suppose firm 2 chooses a price $p \in [\widehat{p}_1, \widehat{p}_2]$.

  By Proposition 1, it associates this price with $\underline{k}$. Then, firm 2's expected profit is

$$\begin{aligned}\pi_2(p,\underline{k}) &= p\{(1-F_1(p))\mu(\underline{k},\bar{k}) + (1-\sigma)[F_1(\widehat{p}_1)(1-\mu(\underline{k},\underline{k})) + (1-F_1(\widehat{p}_1))(1-\mu(\underline{k},\bar{k}))]\}\\ &= p\big[(1-F_1(p))\mu(\underline{k},\bar{k}) + (1-\sigma)(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))\big].\end{aligned}$$

The logic behind the expression above is similar to the one for (A.5). But when firm 1 uses $\underline{k}$, it attracts all the experts, as it offers a lower price. It is easy to check that $\pi_2(\widehat{p}_1, \bar{k}) = \lim_{p \nearrow \widehat{p}_1} \pi_2(p, \underline{k})$ as given by (A.6), and that the expected profit at $\widehat{p}_2$ is

$$\pi_2(\widehat{p}_2, \underline{k}) = \widehat{p}_2(1-\sigma)\big[1-\lambda_1(1-\mu(\underline{k},\bar{k}))\big]. \tag{A.7}$$

- Suppose firm 2 chooses a price $p \in (\widehat{p}_2, 1]$.

  By Proposition 1, it associates this price with complexity level $\bar{k}$. Its expected profit is

$$\begin{aligned}\pi_2(p,\bar{k}) &= p\big\{(1-F_1(p))\mu(\bar{k},\bar{k}) + (1-\sigma)[F_1(\widehat{p}_1)(1-\mu(\underline{k},\bar{k})) + (1-F_1(\widehat{p}_1))(1-\mu(\bar{k},\bar{k}))]\big\}\\ &= p\big\{(1-F_1(p))(2\mu(\underline{k},\bar{k})-1) + (1-\sigma)(2-\lambda_1)(1-\mu(\underline{k},\bar{k}))\big\}.\end{aligned}$$

A.3. Equilibrium analysis

Price ratios using the firms' constant profit conditions

In equilibrium, firm *i*'s expected profit for any price-complexity combination $(p,\ k_i)$, which is assigned positive density in equilibrium, must be constant.

Using (A.1)–(A.3), the constant profit conditions for firm 1 give the following price ratios expressed as functions of λ_2:

$$\frac{p_0}{\widehat{p}_1} = \frac{\sigma-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))}{1-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))} \quad \text{and}$$
$$\frac{p_0}{\widehat{p}_2} = \frac{1-\lambda_2-[2(1-\sigma)(1-\lambda_2)-\sigma\lambda_2](1-\mu(\underline{k},\bar{k}))}{1-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))}.$$

Using (A.6) and (A.7), the constant profit conditions of firm 2 lead to the following price ratios expressed as functions of λ_1

$$\frac{p_0}{\widehat{p}_1} = \frac{(1-\lambda_1)[1-\sigma(1-\mu(\underline{k},\bar{k}))]}{1-\sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))} \quad \text{and} \quad \frac{p_0}{\widehat{p}_2} = \frac{(1-\sigma)\big[1-\lambda_1(1-\mu(\underline{k},\bar{k}))\big]}{1-\sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))}.$$

Equilibrium λ values

We show below that equilibrium λ_1 is always well defined and that λ_2 is well defined when Condition 2 holds. The expression for the λ's is given in (2).

(i) It is easy to see that $\lambda_1 < \sigma$ and that $\lambda_2 > 1-\sigma$ as $1 > \sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))$.

(ii) We now check that $\lambda_i \in (0,\ 1)$.

- As $\mu(\underline{k},\bar{k})+\sigma^2(1-\mu(\underline{k},\bar{k}))>0$, $\lambda_1>0 \Leftrightarrow 1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))>0 \Leftrightarrow 1/(1-\mu(\underline{k},\bar{k}))>\sigma(2-\sigma)$. This always holds as the RHS is lower than 1 and the LHS larger than 1.
- $\lambda_1<1 \Leftrightarrow (1-\sigma)[1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))]<\mu(\underline{k},\bar{k})+\sigma^2(1-\mu(\underline{k},\bar{k})) \Leftrightarrow \sigma/(1-\sigma)\left(1-\sigma+\sigma^2\right)>(1-\mu(\underline{k},\bar{k}))$, which holds as the LHS is always larger than 1.
- $\lambda_2>0$, by the same argument used to show that $\lambda_1>0$.
- $\lambda_2<1 \Leftrightarrow (1-\sigma)/[\sigma\left(1-\sigma+\sigma^2\right)]>(1-\mu(\underline{k},\bar{k}))$, which gives Condition 2.

Mass point at the upper bound

If both firms' price c.d.f.s were continuous everywhere (that is, if $F_1(1)=F_2(1)=1$), then using (A.4) and (A.8), it should be that $\pi_1^*=\sigma(2-\lambda_2)(1-\mu(\underline{k},\bar{k}))$ and $\pi_2^*=(1-\sigma)(2-\lambda_1)(1-\mu(\underline{k},\bar{k}))$. Then, the lower bounds of the supports should be

$$p_0^1=\frac{\sigma(2-\lambda_2)(1-\mu(\underline{k},\bar{k}))}{1-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))}>p_0^2=\frac{(1-\sigma)(2-\lambda_1)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))}.$$

The inequality uses the fact that $\lambda_1/(1-\sigma)=\lambda_2/\sigma$.[17] This contradicts Lemma 5. Suppose now that firm 2 had a mass point, so that $F_2(1)<1$. By Lemma 4, it must be that $F_1(1)=1$ and firm 2's profit is $\pi_2^*=(1-\sigma)(2-\lambda_1)(1-\mu(\underline{k},\bar{k}))$. But then if firm 2 deviates to p_0^1, it makes profits $[1-\sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))]p_0^1>(1-\sigma)(2-\lambda_1)(1-\mu(\underline{k},\bar{k}))$. A contradiction.

So, it must be that firm 2's price c.d.f. is continuous everywhere, while firm 1 has a mass point at $p=1$. Then, at $p=1$, firm 1's expected profit is

$$\pi_1(1,\bar{k})=\sigma\left[1-\lambda_2\mu(\underline{k},\bar{k}))-(1-\lambda_2)\mu(\bar{k},\bar{k})\right]=\sigma(2-\lambda_2)(1-\mu(\underline{k},\bar{k})).$$

Equilibrium profits and boundary prices

We present the boundary price p_0 and the cut-off prices $\widehat{p}_1$ and $\widehat{p}_2$ as functions of λ_2 and check that they are consistent with Condition 1.

$$p_0=\frac{\sigma(2-\lambda_2)(1-\mu(\underline{k},\bar{k}))}{1-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))};$$
$$\widehat{p}_1=\frac{\sigma(2-\lambda_2)(1-\mu(\underline{k},\bar{k}))}{\sigma-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))};$$
$$\widehat{p}_2=\frac{\sigma(2-\lambda_2)(1-\mu(\underline{k},\bar{k}))}{1-\lambda_2-[2(1-\sigma-\lambda_2)+\sigma\lambda_2](1-\mu(\underline{k},\bar{k}))}.$$

Consider a situation where both firms randomize on prices and complexity, so $\lambda_2\in(0,1)$. Also, by Proposition 1, $F_2(\widehat{p}_1)=1-\sigma$. As $\widehat{p}_1<\widehat{p}_2$, it must be that that $F_2(\widehat{p}_1)=1-\sigma<\lambda_2=F_2(\widehat{p}_2)$ (see Lemmas 3 and 4).

- $\widehat{p}_1>p_0 \Leftrightarrow 1-\sigma>0$, so it clearly holds.
- $\widehat{p}_2>p_0 \Leftrightarrow -\lambda_2-(1-\lambda_2)(1-\sigma)(1-\mu(\underline{k},\bar{k}))<0$ which holds for $\lambda_2\in(0,1)$.
- $\widehat{p}_1<1 \Leftrightarrow -\sigma\lambda_2\mu(\underline{k},\bar{k}))<(2\sigma-1)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))$ which holds for $\lambda_2\in(0,1)$.

[17] It can then be reduced to $1-\mu(\underline{k},\bar{k})<(2\sigma-\lambda_2)/\ [2\sigma-\lambda_2+\sigma(1-\sigma)(\sigma-\lambda_2)]$. But as $\lambda_2>\sigma$ for $\sigma\geq 1/2$ the RHS is larger than 1, while the LHS is smaller than 1.

- $\widehat{p}_2 < 1 \Leftrightarrow \big(2\mu(\underline{k},\bar{k})-1\big)(\lambda_2-1)<0$.
- $\widehat{p}_2 > \widehat{p}_1 \Leftrightarrow \sigma-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k})) > (1-\lambda_2)-[2(1-\sigma-\lambda_2)+\sigma\lambda_2](1-\mu(\underline{k},\bar{k})) \Leftrightarrow 1-\sigma-\lambda_2<0$.

Below we check that the equilibrium profits are well defined and present the equilibrium values of the cut-off prices.

▶ π_1^* given in (3) is well defined. Clearly, $1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))>0$. Also, under Condition 2, $\big[2-\sigma-\sigma\big(3-2\sigma+\sigma^2\big)(1-\mu(\underline{k},\bar{k}))\big]>0$. It follows that $\pi_1^*>0$. Then, $\pi_1^*<1$ as

$$\frac{2-\sigma-\sigma\big(3-2\sigma+\sigma^2\big)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))}<\frac{1}{\sigma(1-\mu(\underline{k},\bar{k}))}$$
$$\Leftrightarrow [1-(2-\sigma)\sigma(1-\mu(\underline{k},\bar{k}))]^2+(2\sigma-1)\sigma^2(1-\mu(\underline{k},\bar{k}))^2>0.$$

▶ π_2^* given in (5) is well defined. Under Condition 2, as $\sigma>1/2$, it follows that $2-\sigma-\sigma\big(\sigma^2-2\sigma+3\big)(1-\mu(\underline{k},\bar{k}))>0$. The, clearly $\pi_2^*>0$. Noting that $\pi_2^*<\pi_1^*$ as $1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))>1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))\Leftrightarrow\sigma>1/2$, it follows that $\pi_2^*<1$.

▶ Below are $\widehat{p}_1$ and $\widehat{p}_2$.

$$\widehat{p}_1=\frac{\sigma(1-\mu(\underline{k},\bar{k}))[2-\sigma-\sigma(3-2\sigma+\sigma^2)(1-\mu(\underline{k},\bar{k}))]}{\sigma+(\sigma^3-3\sigma^2+2\sigma-1)(1-\mu(\underline{k},\bar{k}))+\sigma(1-\sigma)(1-\sigma+\sigma^2)(1-\mu(\underline{k},\bar{k}))^2}; \tag{A.8}$$

$$\widehat{p}_2=\frac{\sigma(1-\mu(\underline{k},\bar{k}))[2-\sigma-\sigma\big(\sigma^2-2\sigma+3\big)(1-\mu(\underline{k},\bar{k}))]}{(1-\sigma)[1-(1-\sigma)(2+\sigma)(1-\mu(\underline{k},\bar{k}))+\sigma(1-\sigma)(2-\sigma)(1-\mu(\underline{k},\bar{k}))^2]}. \tag{A.9}$$

Equilibrium pricing

Firm 2's c.d.f. is implicitly defined by the constant profit conditions of firm 1. These conditions can be written using the expected profits, which are presented in Appendix A.2, and the equilibrium profit π_1^* defined in (3). Let

$$F_2(p)=\begin{cases}F_2^L(p) & \text{for } p\in[p_0,\widehat{p}_2)\\ F_2^M(p) & \text{for } p\in[\widehat{p}_2,\widehat{p}_1]\\ F_2^H(p) & \text{for } p\in(\widehat{p}_1,1)\end{cases}.$$

Below we identify piece-wise the c.d.f., using the equilibrium λ_2 in (2).

For prices in $[p_0,\widehat{p}_1)$, using the constant profit condition of firm 1 we obtain

$$1-F_2^L(p)=(1-\mu(\underline{k},\bar{k}))\frac{(1-\sigma)[1-\sigma-\sigma\big(1-\sigma+\sigma^2\big)(1-\mu(\underline{k},\bar{k}))]}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))}+\frac{\pi_1^*}{p}.$$

For prices in the middle range $[\widehat{p}_1,\widehat{p}_2]$, the constant profit condition leads to

$$\begin{aligned}1-F_2^M(p)&=-\frac{(1-\mu(\underline{k},\bar{k}))}{\mu(\underline{k},\bar{k})}[1-(1-\sigma)(2-\lambda_2)]+\frac{\pi_1^*}{p\mu(\underline{k},\bar{k})}\\&=-\frac{(1-\mu(\underline{k},\bar{k}))}{\mu(\underline{k},\bar{k})}\left[1-(1-\sigma)\frac{2-\sigma-\sigma(3-2\sigma+\sigma^2)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))}\right]+\frac{\pi_1^*}{p\mu(\underline{k},\bar{k})}.\end{aligned}$$

For prices in the high range $(\widehat{p}_2, 1]$, the constant profit condition requires

$$1 - F_2^H(p) = -\frac{(1-\mu(\underline{k},\bar{k}))}{(2\mu(\underline{k},\bar{k})-1)}\sigma(2-\lambda_2) + \frac{\pi_1^*}{p(2\mu(\underline{k},\bar{k})-1)}$$
$$= -\frac{(1-\mu(\underline{k},\bar{k}))}{(2\mu(\underline{k},\bar{k})-1)}\frac{\sigma[2-\sigma-\sigma(3-2\sigma+\sigma^2)(1-\mu(\underline{k},\bar{k}))]}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))} + \frac{\pi_1^*}{p(2\mu(\underline{k},\bar{k})-1)}.$$

It is straightforward to check that $F_2(p)$ is continuous on $[p_0, 1]$ and strictly increasing.

To pin down firm 1's c.d.f., we use the constant profit conditions for firm 2, the expected profits presented earlier in this appendix, and the equilibrium profit π_2^* defined in (5). As before, there are three different price ranges to be considered, so that

$$F_1(p) = \begin{cases} F_1^L(p) & \text{for } p \in [p_0, \widehat{p}_2) \\ F_1^M(p) & \text{for } p \in [\widehat{p}_2, \widehat{p}_1] \\ F_1^H(p) & \text{for } p \in (\widehat{p}_1, 1] \end{cases}.$$

We proceed to identify piece-wise the c.d.f., using the equilibrium λ_1 in (2).

For prices in $[p_0, \widehat{p}_1)$, the constant profit condition of firm 2 implies that

$$1 - F_1^L(p) = (1-\mu(\underline{k},\bar{k}))\frac{\sigma[\sigma-(1-\sigma)(\sigma^2-\sigma+1)(1-\mu(\underline{k},\bar{k}))]}{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))} + \frac{\pi_2^*}{p}.$$

For prices in the middle range $[\widehat{p}_1, \widehat{p}_2]$, the constant profit condition leads to

$$1 - F_1^M(p) = -\frac{(1-\mu(\underline{k},\bar{k}))}{\mu(\underline{k},\bar{k})}\frac{(1-\sigma)[\sigma-(1-\sigma)(\sigma^2-\sigma+1)(1-\mu(\underline{k},\bar{k}))]}{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))} + \frac{\pi_2^*}{p\mu(\underline{k},\bar{k})},$$

while for prices in the high range $(\widehat{p}_2, 1]$, to

$$1 - F_1^H(p) = -\frac{(1-\mu(\underline{k},\bar{k}))}{2\mu(\underline{k},\bar{k})-1}\frac{(1-\sigma)[1+\sigma-(1-\sigma)(2+\sigma^2)(1-\mu(\underline{k},\bar{k}))]}{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))}$$
$$+ \frac{\pi_2^*}{p(2\mu(\underline{k},\bar{k})-1)}.$$

It is easy to check that $F_1(p)$ is continuous on $[p_0, 1)$ and strictly increasing. Firm 1 has a mass point at $p = 1$,

$$\phi \equiv 1 - F_1^H(1) = \frac{(2\sigma-1)(1-\mu(\underline{k},\bar{k}))}{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))} \in (0,1) \text{ for } \sigma > 1/2.$$

A.4. Equilibrium analysis for high prominence

This subsection focuses on a situation where Condition 2 does not hold.

Proof of Proposition 3. When firm 1 chooses a price $p \in [p_0^h, \widehat{p}_1^h)$, it uses complexity level $\underline{k}$. Then, firm 1's expected profit in this range is $\pi_1^h(p, \underline{k}) = p\big(1 - F_2^h(p)\big)$, and $\pi_1^h(p_0^h, \underline{k}) = p_0^h$ and $\lim_{p \nearrow \widehat{p}_1^h} \pi_1^h(p, \underline{k}) = \sigma \widehat{p}_1^h$. When firm 1 chooses a price $p \in [\widehat{p}_1^h, 1)$, it uses $\bar{k}$. Then, its expected profit is

$$\pi_1^h(p,\bar{k}) = p\left[(1-F_2^h(p))\mu(\underline{k},\bar{k}) + \sigma(1-\mu(\underline{k},\bar{k}))\right] ,$$

so that $\pi_1(\widehat{p}_1^h,\bar{k}) = \sigma\widehat{p}_1^h$. The constant profit condition of firm 1 implies that $p_0^h = \sigma\widehat{p}_1^h$. When firm 2 chooses a price $p \in [p_0^h, \widehat{p}_1^h)$, it uses $\underline{k}$. As $\mu(\underline{k},\underline{k}) = 1$ and $F_1^h(\widehat{p}_1^h) = \lambda_1^h$, firm 2's expected profit is

$$\pi_2^h(p,\underline{k}) = p\left[\lambda_1^h - F_1^h(p) + (1-\lambda_1^h)\mu(\underline{k},\bar{k}) + (1-\sigma)(1-\lambda_1^h)(1-\mu(\underline{k},\bar{k}))\right].$$

It then follows that,

$$\pi_2^h(p_0^h,\underline{k}) = p_0^h\left[1-\sigma(1-\lambda_1^h)(1-\mu(\underline{k},\bar{k}))\right] \text{ and}$$
$$\lim_{p\nearrow\widehat{p}_1^h} \pi_2^h(p,\underline{k}) = \widehat{p}_1^h(1-\lambda_1^h)\left[1-\sigma(1-\mu(\underline{k},\bar{k}))\right].$$

Combining $p_0^h = \sigma\widehat{p}_1^h$ with the constant profit condition of firm 2, we obtain the value for λ_1^h. When firm 2 chooses a price $p \in [\widehat{p}_1^h, 1)$, it uses $\underline{k}$. Then, firm 2's expected profit is

$$\pi_2^h(p,\underline{k}) = p\left[(1-F_1^h(p))\mu(\underline{k},\bar{k}) + (1-\sigma)(1-\lambda_1^h)(1-\mu(\underline{k},\bar{k}))\right].$$

By Lemma 4, both firms cannot have a mass point at $p=1$. If $F_1^h(1) = 1$, then $p_0^h = \pi_{h1}^* = \sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))/\ [1-\sigma(1-\mu(\underline{k},\bar{k}))]$ and $F_2^h(1) < 0$, a contradiction. Hence, it must be that firm 1 has an atom at $p=1$ and firm 2's c.d.f. is continuous on $[p_0^h, 1]$. $F_2^h(1) = 1$ implies that, in equilibrium, $\widehat{p}_1^h = (1-\mu(\underline{k},\bar{k}))$ and firms' profits and p_0^h follow.

The mass point in firm 1's price c.d.f. is

$$\phi^h \equiv 1 - F_1^h(1) = \frac{\sigma^2(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))} < 1 ,$$

and consistency requires that $F_1^h(1) \le \sigma$, which is the case whenever

$$(1-\sigma)/\sigma(1-\sigma+\sigma^2) \le (1-\mu(\underline{k},\bar{k})).$$

But this is exactly the reverse of Condition 2. □

Equilibrium pricing

To identify firm 2's c.d.f. we use the constant profit conditions for firm 1. The expected profits are presented in Section 4 and the equilibrium profit π_{h1}^* is defined in (6). It follows that

$$F_2^h(p) = \begin{cases} F_2^{hL}(p) & \text{for } p \in [p_0, \widehat{p}_1^h) \\ F_2^{hH}(p) & \text{for } p \in [\widehat{p}_1^h, 1) \end{cases} ,$$

where

$$1 - F_2^{hL}(p) = \frac{\sigma(1-\mu(\underline{k},\bar{k}))}{p} \quad \text{and} \quad 1 - F_2^{hH}(p) = \frac{\sigma(1-\mu(\underline{k},\bar{k}))}{\mu(\underline{k},\bar{k})}\left(\frac{1}{p} - 1\right).$$

It is easy to check that $F_2^{hL}(\widehat{p}_1^h) = F_2^{hH}(\widehat{p}_1^h) = 1-\sigma$ as $\widehat{p}_1^h = 1-\mu(\underline{k},\bar{k})$.

To pin down firm 1's c.d.f. we use the constant profit conditions for firm 2. Then,

$$F_1^h(p) = \begin{cases} F_1^{hL}(p) & \text{for } p \in [p_0, \widehat{p}_1^h) \\ F_1^{hH}(p) & \text{for } p \in [\widehat{p}_1^h, 1] \end{cases},$$

where

$$1 - F_1^{hL}(p) = \frac{\sigma(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))}\left[\frac{1-\sigma(1-\mu(\underline{k},\bar{k}))}{p} + \sigma\right] \text{ and}$$
$$1 - F_1^{hH}(p) = \frac{\sigma(1-\mu(\underline{k},\bar{k}))}{\mu(\underline{k},\bar{k})[1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))]}\left[\frac{1-\sigma(1-\mu(\underline{k},\bar{k}))}{p} - (1-\sigma)\right].$$

It is easy to check that $F_1^{hL}(\widehat{p}_1^h) = F_1^{hH}(\widehat{p}_1^h) = \lambda_1^h$. Firm 1's atom at $p = 1$ is given by

$$\phi^h \equiv 1 - F_1^{hH}(1) = \frac{\sigma^2(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))}.$$

A.5. The role of prominence

Suppose that Condition 2 holds.
Let $\nu_1 \equiv [1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))] > 0$ and $\nu_2 \equiv [1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))] > 0$.
Note that $d\nu_1/d\sigma < 0$ and $d\nu_2/d\sigma > 0$. Then,

$$\frac{d\lambda_1}{d\sigma} = \frac{1}{(\nu_2)^2}\left\{\left[-\nu_1 + (1-\sigma)\frac{d\nu_1}{d\sigma}\right]\nu_2 - \frac{d\nu_2}{d\sigma}(1-\sigma)\nu_1\right\} < 0;$$
$$\frac{d\lambda_2}{d\sigma} = \frac{1}{(\nu_1)^2}\left\{\left[\nu_2 + \sigma\frac{d\nu_2}{d\sigma}\right]\nu_1 - \frac{d\nu_1}{d\sigma}\sigma\nu_2\right\} > 0.$$

Suppose now that Condition 2 does not hold. $d\lambda_2^h/d\sigma = 0$ as $\lambda_2^h = 1$. Let $\nu_3 \equiv [1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))] > 0$ and $\nu_4 \equiv [1-\sigma(1-\mu(\underline{k},\bar{k}))] > 0$. Note that $d\nu_3/d\sigma > 0$ and $d\nu_4/d\sigma < 0$. Then,

$$\frac{d\lambda_1^h}{d\sigma} = \frac{1}{(\nu_3)^2}\left\{\left[-\nu_4 + (1-\sigma)\frac{d\nu_4}{d\sigma}\right]\nu_3 - \frac{d\nu_3}{d\sigma}(1-\sigma)\nu_4\right\}.$$

Industry profit for relatively high levels of prominence

Consider Proposition 3. Using (6), industry profit is

$$\pi^*_{h1} + \pi^*_{h2} = \sigma(1 - \mu(\underline{k}, \bar{k}))\frac{2 - \sigma(2-\sigma)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1-\sigma)(1 - \mu(\underline{k}, \bar{k}))}.$$

Differentiating industry profit w.r.t. σ, we obtain

$$\sigma(1 - \mu(\underline{k}, \bar{k}))\frac{2[1 - 2\sigma(1 - \mu(\underline{k}, \bar{k}))] + \sigma^2(1 - \mu(\underline{k}, \bar{k}))\{1 + [1 + (1-\sigma)^2](1 - \mu(\underline{k}, \bar{k}))\}}{[1 - \sigma(1-\sigma)(1 - \mu(\underline{k}, \bar{k}))]^2}.$$

As $(1 - \mu(\underline{k}, \bar{k})) \leq 1/2$, the first term in the numerator is positive and then $d(\pi^*_{h1} + \pi^*_{h2})/d\sigma > 0$.

Proof of Corollary 2. (i) Suppose that Condition 2 holds and consider the equilibrium in Proposition 2. From (3) and (5), $\pi^*_1 > \pi^*_2 \Leftrightarrow (2\sigma - 1)(1 - \mu(\underline{k}, \bar{k})) > 0$ which holds for $\sigma > 1/2$. Suppose that Condition 2 does not hold and consider the equilibrium in Proposition 3. Using (6), it is easy to see that $\pi^*_{h1} > \pi^*_{h2}$ as the fraction in π^*_{h2} is smaller than one. (ii) Suppose that Condition 2 holds. Consider the equilibrium price c.d.f.s in Appendix A.2. (a) For prices $p \in [p_0, \widehat{p}_1)$, $dF^L_1(p)/dp = \pi^*_2/p^2 < dF^L_2(p)/dp = \pi^*_1/p^2$ using point (i) above. As $F^L_1(p_0) = F^L_2(p_0) = 0$, then $F^L_1(p) < F^L_2(p)$. Also $\lim_{p \nearrow \widehat{p}_1} F^L_1(p) < \lim_{p \nearrow \widehat{p}_1} F^L_2(p)$. (b) For $p \in [\widehat{p}_1, \widehat{p}_2]$, $dF^M_1(p)/dp = \pi^*_2/\mu(\underline{k}, \bar{k})p^2 < dF^M_2(p)/dp = \pi^*_1/\mu(\underline{k}, \bar{k})p^2$. Point (a) and continuity of F_i on $[p_0, 1)$ imply that $F^M_1(\widehat{p}_1) < F^M_2(\widehat{p}_1)$. So, $F^M_1(p) < F^M_2(p)$ in this range. (c) Consider $[\widehat{p}_2, 1]$. From part (b) then $F^M_1(\widehat{p}_2) < F^M_2(\widehat{p}_2)$. By continuity, $\lim_{p \searrow \widehat{p}_2} F^H_1(p) < \lim_{p \searrow \widehat{p}_2} F^H_2(p)$. As $dF^H_1(p)/dp = \pi^*_2/(2\mu(\underline{k}, \bar{k}) - 1)p^2 < dF^H_2(p)/dp = \pi^*_1/(2\mu(\underline{k}, \bar{k}) - 1)p^2$, $F^H_1(p) < F^H_2(p)$. Combining (a)-(c), $F_1(p) < F_2(p)$ on $[p_0, 1]$, and the price of firm 1 first order stochastically dominates that firm 2.

Suppose that Condition 2 does not hold. Consider the equilibrium price c.d.f.s in Appendix A.4. For prices $p \in [p_0, \widehat{p}^h_1)$, $(dF^{hL}_1(p)/dp)/(dF^{hL}_2(p)/dp) = [1 - \sigma(1 - \mu(\underline{k}, \bar{k}))]/[1 - \sigma(1-\sigma)(1 - \mu(\underline{k}, \bar{k}))] < 1$. As $F^{hL}_1(p^h_0) = F^{hL}_2(p^h_0) = 0$, then $F^{hL}_1(p) < F^{hL}_2(p)$ in this range. For prices in $[\widehat{p}^h_1, 1]$, $(dF^{hH}_1(p)/dp)/(dF^{hH}_2(p)/dp) = [1 - \sigma(1 - \mu(\underline{k}, \bar{k}))]/[1 - \sigma(1-\sigma)(1 - \mu(\underline{k}, \bar{k}))] < 1$. As F^h_1 and F^h_2 are continuous at $\widehat{p}^h_1$, then $F^{hH}_1(p) < F^{hH}_2(p)$ in this range, too. So, $F^h_1(p) < F^h_2(p)$ on $[p^h_0, 1]$ and the price of firm 1 first order stochastically dominates that of firm 2. (iii) The ranking of the average prices follows from (ii) as $E(p_i) = \int_0^\infty (1 - F_i(p_i))dp_i = \int_{p_0}^1 (1 - F_i(p_i))dp_i + p_0$ when Condition 2 holds and $E(p_i) = \int_{p^h_0}^1 (1 - F^h_i(p_i))dp_i + p^h_0$ when Condition 2 does not hold.

(iv) When Condition 2 is satisfied $\lambda_1 = \sigma(1-\sigma)/\lambda_2$, so $\lambda_1 < \lambda_2 \Leftrightarrow \sigma > (1-\sigma)(1 - \sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))$ which holds for $\sigma > 1/2$. When Condition 2 does not hold, it is easy to see from Proposition 3 that $\lambda^h_1 < 1$. □

Proof of Corollary 3. First we compare the cut-off prices. Suppose Condition 2 holds. Using (A.8) and (A.9) in Appendix A.2, $\widehat{p}_1 < \widehat{p}_2 \Leftrightarrow$

$$-\frac{\mu(\underline{k},\bar{k})(2\sigma-1)[1-(1-\sigma)(1-\mu(\underline{k},\bar{k}))]}{\sigma+(\sigma^3-3\sigma^2+2\sigma-1)(1-\mu(\underline{k},\bar{k}))+\sigma(1-\sigma)(1-\sigma+\sigma^2)(1-\mu(\underline{k},\bar{k}))^2}<0.$$

The inequality follows from the fact that, for $\sigma \in (1/2,\, 1)$, both the numerator and the denominator are positive. The sum of the first two terms in the denominator is positive as $(1-\mu(\underline{k},\bar{k})) \le 1/2$.

If Condition 2 does not hold, firm 2 uses $\underline{k}$ for all prices on $[p_0^h, 1]$ and $\widehat{p}_1^h = 1-\mu(\underline{k},\bar{k}) < 1$. Next we compare the firms' average prices conditional on using the lowest complexity level. Suppose Condition 2 holds. F_2 is continuous on $[p_0, 1]$ so that $F_2(p) = F_2(p \mid p < 1)$, whereas F_1 is continuous on $[p_0, 1)$, but has an atom at $p = 1$, $\phi = (2\sigma-1)(1-\mu(\underline{k},\bar{k}))/[1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))]$. Using the price c.d.f.s in Appendix A.2, we can show that

$$F_1(p \mid p<1) = \frac{F_1(p)}{F_1(1)} = F_1(p)\frac{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))} = F_2(p).$$

Let $G(p) = F_1(p|p<1)$. Note that $F_1(p \mid p < \widehat{p}_1) = G(p \mid p < \widehat{p}_1)$. This is because $F_1(p \mid p < \widehat{p}_1) = F_1(p)/\, F_1(\widehat{p}_1)$ and $G(p \mid p < \widehat{p}_1) = F_1(p)/\, F_1(1)G(\widehat{p}_1)$. But then,

$$G(p \mid p < \widehat{p}_1) = F_2(p \mid p < \widehat{p}_1) > F_2(p \mid p < \widehat{p}_2)\ ,$$

where the inequality follows from the fact that $\widehat{p}_1 < \widehat{p}_2$ and F_2 is a well-defined c.d.f. Putting together the expressions above, it follows that

$$F_1(p \mid p < \widehat{p}_1) > F_2(p \mid p < \widehat{p}_2).$$

Finally, note that

$$E(p_1 \mid p_1 < \widehat{p}_1) = \int_{p_0}^{\widehat{p}_1} (1 - F_1(p \mid p < \widehat{p}_1))dp - p_0;$$
$$E(p_2 \mid p_2 < \widehat{p}_2) = \int_{p_0}^{\widehat{p}_1} (1 - F_2(p \mid p < \widehat{p}_2))dp + \int_{\widehat{p}_1}^{\widehat{p}_2} (1 - F_2(p \mid p < \widehat{p}_2))dp - p_0.$$

It is then easy to see that $E(p_1 \mid p_1 < \widehat{p}_1) < E(p_2 \mid p_2 < \widehat{p}_2)$.

Suppose now that Condition 2 does not hold. F_2^h is continuous on $[p_0^h, 1]$ so that $F_2^h(p) = F_2^h(p \mid p < 1)$, whereas F_1^h is continuous on $[p_0^h, 1)$, but has an atom at $p = 1$, $\phi^h = \sigma^2(1-\mu(\underline{k},\bar{k}))/\ [1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))]$. Using the price c.d.f.s in Appendix A.4,

$$F_1^h(p \mid p<1) = \frac{F_1^h(p)}{1-\phi^h} = F_1^h(p)\frac{1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\mu(\underline{k},\bar{k}))} = F_2^h(p)\ ,$$

and an argument similar to the one above applies as $\widehat{p}_1^h < 1$ and $\phi^h > 0$. □

A.6. Proof of Condition 1

This section shows that Condition 1 must hold in any equilibrium where both firms randomize on both prices and complexity levels. Suppose instead that $p_0 < \widehat{p}_2 \le \widehat{p}_1 < 1$.

Then consistency requires that $F_i(\widehat{p}_i) = \lambda_i \in (0,1)$ and, by Proposition 1, $F_i(\widehat{p}_j) = s_i$. Firm 1's expected profit is presented below, followed by firm 2's expected profit.

- Suppose that firm 1 chooses a price $p \in [p_0, \widehat{p}_2)$. By Proposition 1, it associates it with $\underline{k}$ and its expected profit is $\pi_1(p, \underline{k})$ presented in (1). Expression (A.1) still gives $\pi_1(p_0, \underline{k})$, while now $\lim_{p \nearrow \widehat{p}_2} \pi_1(p, \underline{k}) = \widehat{p}_2(1-\lambda_2)\big[1-(1-\sigma)(1-\mu(\underline{k},\bar{k}))\big]$.
- Suppose that firm 1 chooses a price $p \in [\widehat{p}_2, \widehat{p}_1]$. By Proposition 1, it associates it with $\underline{k}$ and its expected profit is

$$\pi_1(p, \underline{k}) = p\big[(1-F_2(p))\mu(\underline{k},\bar{k}) + \sigma(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))\big].$$

The expected number of confused consumers (given by the second term in square brackets) is the same as in expression (1) and firm 1 serves a fraction σ of them. Firm 1 serves informed consumers only if firm 2 chooses a higher price, in which case there are $\mu(\underline{k},\bar{k})$ of them. This happens with probability $1-F_2(p)$ and gives the first term in square brackets. It is easy to check that $\pi_1(\widehat{p}_2, \underline{k}) = \lim_{p \nearrow \widehat{p}_2} \pi_1(p, \underline{k})$. As by Proposition 1 $F_2(\widehat{p}_1) = 1-\sigma$, $\pi_1(\widehat{p}_1, \underline{k}) = \widehat{p}_1\sigma\big[1-\lambda_2(1-\mu(\underline{k},\bar{k}))\big]$.

Using the constant profit requirements, it follows that

$$\frac{p_0}{\widehat{p}_1} = \frac{\sigma\big[1-\lambda_2(1-\mu(\underline{k},\bar{k}))\big]}{1-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))} \quad \text{and} \quad \frac{p_0}{\widehat{p}_2} = \frac{(1-\lambda_2)\big[1-(1-\sigma)(1-\mu(\underline{k},\bar{k}))\big]}{1-(1-\sigma)(1-\lambda_2)(1-\mu(\underline{k},\bar{k}))}.$$

- Suppose that firm 2 chooses a price $p \in [p_0, \widehat{p}_2)$. By Proposition 1, it associates it with $\underline{k}$ and its expected profit is given by (A.5). Expression (A.6) still gives $\pi_2(p_0, \underline{k})$ and now $\lim_{p \nearrow \widehat{p}_2} \pi_2(p, \underline{k}) = \widehat{p}_2\big[(1-\sigma) - \sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))\big]$.
- Suppose that firm 2 chooses a price $p \in [\widehat{p}_2, \widehat{p}_1]$. By Proposition 1, it associates it with $\bar{k}$. Firm 2's expected profit is

$$\begin{aligned}\pi_2(p, \bar{k}) = p\big[(\lambda_1 - F_1(p))\mu(\underline{k},\bar{k}) + (1-\lambda_1)\mu(\bar{k},\bar{k})\big] \\ + p(1-\sigma)[\lambda_1(1-\mu(\underline{k},\bar{k})) + (1-\lambda_1)(1-\mu(\bar{k},\bar{k}))]\}.\end{aligned}$$

Then, $\pi_2(\widehat{p}_2, \bar{k}) = \lim_{p \nearrow \widehat{p}_2} \pi_2(p, \underline{k})$, and $\pi_2(\widehat{p}_1, \bar{k}) = \widehat{p}_1\{(1-\lambda_1) + [\lambda_1(1-\sigma) - 2\sigma(1-\lambda_1)](1-\mu(\underline{k},\bar{k}))\}$.

Using the constant profit requirements, we obtain

$$\frac{p_0}{\widehat{p}_1} = \frac{1-\lambda_1 - [2\sigma(1-\lambda_1) - (1-\sigma)\lambda_1](1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))};$$
$$\frac{p_0}{\widehat{p}_2} = \frac{(1-\sigma) - \sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))}{1-\sigma(1-\lambda_1)(1-\mu(\underline{k},\bar{k}))}.$$

Combining the price ratios as functions of λ_1 and λ_2, we obtain

$$\lambda_1 = \frac{(1-\sigma)[1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))]}{1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))} \quad \text{and} \quad \lambda_2 = \frac{\sigma[1-(1-\sigma^2)(1-\mu(\underline{k},\bar{k}))]}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))}.$$

As $\widehat{p}_2 \leq \widehat{p}_1$, by Proposition 1, it must be that $\lambda_2 \leq 1-\sigma$ as $F_2(\widehat{p}_2) \leq F_2(\widehat{p}_1)$. But then we reached a contradiction as

$$1-\sigma-\lambda_2 = -\frac{(2\sigma-1)[1-\sigma(1-\sigma)(1-\mu(\underline{k},\bar{k}))]}{1-\sigma(2-\sigma)(1-\mu(\underline{k},\bar{k}))} < 0 \quad \text{as} \quad \sigma > 1/2.$$

References

Armstrong, M., 2017. Ordered consumer search. Journal of the European Economic Association 15 (5), 989–1024.

Armstrong, M., Chen, Y., 2009. Inattentive consumers and product quality. Journal of the European Economic Association 7 (2-3), 411–422.

Armstrong, M., Vickers, J., Zhou, J., 2009. Prominence and consumer search. RAND Journal of Economics 40 (2), 209–233.

Armstrong, M., Zhou, J., 2011. Paying for prominence. Economic Journal 121 (556), 368–395.

Baye, M., Kovenock, D., de Vries, C., 1992. It takes two to tango: Equilibria in a model of sales. Games and Economic Behavior 4 (4), 493–510.

Baye, M.R., Morgan, J., Scholten, P., 2006. Information, search, and price dispersion. In: Hendershott, T. (Ed.), Handbook of Economics and Information Systems. Elsevier Press, Amsterdam.

Campbell, J.Y., 2016. Richard T. Ely lecture restoring rational choice: The challenge of consumer financial regulation. American Economic Review 106 (5), 1–30.

Carlin, B., 2009. Strategic price complexity in retail financial markets. Journal of Financial Economics 91 (3), 278–287.

Chioveanu, I., 2012. Price and quality competition. Journal of Economics 107 (1), 23–44.

Chioveanu, I., 2017. Prominence, complexity, and pricing MPRA Paper 81078.

Chioveanu, 2019. An asymmetric duopoly model of price framing. The B.E. Journal of Theoretical Economics 20180068, 1935-1704. doi:10.1515/bejte-2018-0068, ISSN (Online).

Chioveanu, I., Zhou, J., 2013. Price competition with consumer confusion. Management Science 59 (11), 2450–2469.

Competition and Markets Authority, 2015. Retail banking market investigation: summary of provisional findings report. available at https://www.gov.uk/cma-cases/review-of-banking-for-small-and-medium-sized-businesses-smes-in-the-uk

Ellison, G., 2006. Bounded rationality in industrial organization. In: Blundell, R., Newey, W., Persson, T. (Eds.), Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society. Cambridge University Press, Cambridge, UK.

Ellison, G., Wolitzky, A., 2012. A search cost model of obfuscation. RAND Journal of Economics 43 (3), 417–441.

European Commission, 2007. Staff working document accompanying the white paper on the integration of eu mortgage credit markets available at http://eur-lex.europa.eu.

Giulietti, M., Waterson, M., Wildenbeest, M., 2014. Estimation of search frictions in the british electricity market. Journal of Industrial Economics 62 (4), 555–590.

Greenleaf, E.A., Johnson, E.J., Morwitz, V.G., Shalev, E., 2016. The price does not include additional taxes, fees, and surcharges: a review of research on partitioned pricing. Journal of Consumer Psychology 26 (1), 105–124.

Grubb, M.D., 2015. Failing to choose the best price: Theory, evidence, and policy. Review of Industrial Organization 47 (3), 303–340.

Gu, Y., Wenzel, T., 2014. Strategic obfuscation and consumer protection policy. Journal of Industrial Economics 62 (4), 632–660.

Gurun, U.G., Matvos, G., Seru, A., 2016. Advertising expensive mortgages. Journal of Finance 71 (5), 2371–2416.

Hastings, J., Hortaçsu, A., Syverson, C., 2017. Sales force and competition in financial product markets: the case of Mexico's social security privatization. Econometrica 85 (6), 1723–1761.

Hortaçsu, A., Madanizadeh, S.A., Puller, S.L., 2017. Power to choose? an analysis of consumer inertia in the residential electricity market. American Economic Journal: Economic Policy 9 (4), 192–226.

Huck, S., Zhou, J., 2011. Consumer behavioural biases in competition: a survey. New York University Working Paper, Leonard N. Stern School of Business, Department of Economics.

Independent Banking Commission, 2011. Final report recommendations. available at http://bankingcommission.independent.gov.uk/.

Kalaycı, K., 2015. Price complexity and buyer confusion in markets. Journal of Economic Behavior & Organization 111, 154–168.

Kalaycı, K., Potters, J., 2011. Buyer confusion and market prices. International Journal of Industrial Organization 29 (1), 14–22.

Narasimhan, C., 1988. Competitive promotional strategies. Journal of Business 61 (4), 427–449.

Piccione, M., Spiegler, R., 2012. Price competition under limited comparability. Quarterly Journal of Economics 127 (1), 97–135.

Rhodes, A., 2011. Can prominence matter even in an almost frictionless market? Economic Journal 121 (556), 297–308.

Rosenthal, R., 1980. A model in which an increase in the number of sellers leads to a higher price. Econometrica 48 (6), 1575–1579.

Spiegler, R., 2011. Bounded Rationality and Industrial Organization. Oxford University Press.

Spiegler, R., 2016. Choice complexity and market competition. Annual Review of Economics 8, 1–25.

Stango, V., Zinman, J., 2009a. What do consumers really pay on their checking and credit card accounts? Explicit, implicit, and avoidable costs. American Economic Review 99 (2), 06–9.

Stango, V., Zinman, J., 2009b. Exponential growth bias and household finance. Journal of Finance 64 (6), 2807–2849.

Taylor, G., 2017. Raising search costs to deter window shopping can increase profits and welfare. RAND Journal of Economics 48, 387–408.

Varian, H., 1980. A model of sales. American Economic Review 70 (4), 651–659.

Wilson, C.M., 2010. Ordered search and equilibrium obfuscation. International Journal of Industrial Organization 28 (5), 496–506.

Woodward, S., Hall, R., 2012. Diagnosing consumer confusion and sub-optimal shopping effort: theory and mortgage-market evidence. American Economic Review 102 (7), 3249–3276.