Does Technology Licensing Matter for Privatization?*

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Abstract

In mixed oligopolies, technology licensing from a cost-efficient firm to a cost-inefficient firm has been widely observed. This paper examines the relationship between privatization and licensing (by public or private firms) with the consideration of either a domestic or a foreign private firm. We find that i) in the case of a domestic private firm, public licensing facilitates privatization, but private licensing hinders privatization; ii) in the case of a foreign private firm, both public and private licensing facilitate privatization. Our results yield important policy implications on privatization.

Key words: Public Licensing; Private Licensing, Privatization; Mixed Market; Cournot

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1 Introduction

The world saw a wave of privatization of state-owned public enterprises in recent decades. It is commonly believed that privatization is a very important issue for governments in developing and transitional countries. However, after the financial crisis, privatization or nationalization also became important in developed countries. On markets where public firms compete against private firms, mixed oligopoly literature has investigated several important issues such as partial privatization of public firms (Matsumura, 1998), strategic privatization under international trade and investment (Bárcena-Ruiz and Garzón, 2005, Mukherjee and Suetrong, 2009), the complementarity or substitutability of privatization and subsidy policies (Lin and Matsumura, 2018, Tomaru and Wang, 2018), and privatization under an interdependence payoff structure (Matsumura and Okamura, 2015). While these papers provide several important insights, an important empirical regularity, viz., technology transfer between the public and private firms affecting production efficiency of the firms and the intensity of competition in the product market, did not get much attention in the literature.

In mixed oligopolies, technology licensing from a cost-efficient firm to a cost-inefficient firm has been widely observed. For examples, in the medicine industry, Australia's national science agency CSIRO licensed its new medical polymer technologies to a domestic firm PolyNovo in 2005 (Niu, 2017); in the oil industry, Chinese state-owned firm, Sinopec, earned RMB 1.48 billion from licensing technology to foreign firms in 2008 (Ye, 2012); in the auto industry, the German BMW Motor Corporation licensed its engine technology to Chinese state-owned Dongfeng Motor Corporation for the production of Fengxing T5 SUV in 2018. Despite the practical relevance of this phenomenon, technology licensing in mixed oligopoly and particularly its impact on privatization did not get much attention.[1] In this paper, we aim to investigate the relationship between privatization and licensing (by public or private firms) with the consideration of either a domestic or a foreign private firm.

[1] The licensing issue between private firms has been well analyzed in the literature, see for example, Wang (1998), Wang (2002), Niu (2018), and Hsu et al. (2019), but not in a mixed oligopoly market structure.

In the case of a domestic mixed duopoly, we find that licensing by a public firm (called public licensing) induces the government to further privatize its public firm so as to reduce the industry production cost by shifting production toward the private firm. To the best of our knowledge, we are the first to derive this result. In contrast, we show that the government prefers the public firm to put more weight on social welfare under licensing by a private firm (called private licensing). As such, private licensing hinders privatization. However, if the private firm is a foreign firm, we find that both public and private licensing facilitate privatization. The main reasons lie in the feature of the cost function and the changes in social welfare. Our results yield important implications on privatization, and enrich the literature on licensing and privatization.

Our paper is related to the growing literature on licensing in mixed oligopolies. Chen et al. (2014) propose a mixed oligopoly model with one public firm and two private firms to examine the optimal licensing scheme by an innovating private firm. Both licensing to the public firm and the private rival firm are considered. Kim et al. (2018) develop a model with a foreign innovator in a polluting mixed duopoly, where each polluter may purchase eco-technology under a fixed-fee licensing. They analyze the patent licensing strategy in the presence of emission tax and cost asymmetry between the public and private firms.

In addition to private licensing, it is also found that many patents are licensed by public and partially public companies in Europe and China, not only to domestic firms but also to foreign firms (see Li, 2011). Ye (2012) examines the optimal public licensing in a mixed duopoly with a foreign private firm and verifies the superiority of fixed-fee licensing over royalty licensing. Gelves and Heywood (2016) investigate how should a cost disadvantaged partial privatized firm license its cost-reducing innovation to a private firm. The authors take the degree of privatization as given and find out that the choice of license scheme critically depends on the ownership structure of the privatized firm. Following this line, Heywood et al. (2019) propose a model in which a high-cost public firm licences to a low-cost foreign private firm.

While the above-mentioned papers on private and public licensing in mixed markets provide important insights, they mainly focus on the choice of the licensing contract and ignore the issue of privatization. In contrast, we show the implications of private and public licensing on privatization and also show the implications of domestic and foreign private firms.

There are few theoretical papers that combine the issues of technology licensing and privatization. Cato (2011) investigates how privatization affects the cost-reducing investment by the private sector. The author shows that the impact of privatization on the private sector's cost-reducing activity critically depends on the market size. Mukherjee and Sinha (2014) consider technology licensing in a mixed duopoly and show that there is no need for privatization in the presence of technology licensing. Niu (2015) analyzes the licensing policy for a cost-reduction technology of a foreign innovator when it is faced with a domestic public monopoly manufacturer. Haraguchi and Matsumura (2018) study knowledge transfer between private firms in a mixed triopoly model. They show that privatization motivates voluntary technology transfer from foreign to domestic firms. A recent paper by Wang and Zeng (2019) examines how licensing from an efficient private firm to either a public firm or a private (domestic or foreign) entrant affects privatization. They find that licensing to the private firm provides further motivation for privatization, while by contrast, licensing to the public firm reduces the incentives for privatization. However, none of these papers address how public licensing affects privatization in mixed markets.

Our paper contains several contributions to the literature on licensing and privatization. Firstly, we consider both public and private licensing in our model, where the private firm can be either domestic or foreign, and further investigate how they affect privatization in a mixed duopoly. Our results provide policy implications on privatization. Secondly, unlike the previous papers considering constant returns to scale technologies, we consider the implications of decreasing returns to scale technologies, which are often found in the reality.[2] As such, the incentive for privatization will remain and both firms will be active in equilibrium even if licensing eliminates cost difference.

[2]Note that the linearly increasing cost function is more general in mixed oligopoly, which avoids the problem of public monopoly if there is no cost difference between public and private firms. See also in Matsumura and Kanda (2005), Bárcena-Ruiz and Garzón (2006), and Tomaru and Wang (2018).

The rest of the paper is organized as follows. In Section 2, we set up the basic model. In Section 3, we investigate the effect of licensing on privatization in a mixed duopoly with a domestic private firm. Both public licensing and private licensing are considered. In Section 4, we look at the case in which the public firm competes against a foreign private firm, and reexamine the effect of licensing on privatization. Both public licensing and foreign licensing are considered. In Section 5, we check the robustness of our main results in an alternative setting of cost function. Concluding remarks are presented in Section 6. Proofs of some lemmas and propositions are relegated to the Appendix.

2 The Model

We consider a mixed duopoly model with one welfare-maximizing public firm ($i = 0$), and one profit maximizing private firm ($i = 1$), which can be either domestic or foreign. The two firms compete in a Cournot fashion with homogeneous products. The inverse market demand function is $p = a - q_0 - q_1$, where p is market price, and q_i is firm i's output. The production cost for each firm is $c_i q_i + d_i q_i^2$, where $c_i > 0$ and $d_i > 0$. In the basic model, we consider the following case: $c_0 = c_1 = c$, but d_0 and d_1 are different.[3]

To see how technology licensing affects privatization, we allow the more cost-efficient firm to license its superior technology to the rival firm by means of a fixed fee. Such technology transfer will completely eliminate the efficiency difference between the two firms.[4] In this non-levithan government maximization setting, the public firm will only be allowed to purchase or transfer the technology with a fixed-fee contract in a given period time due to budgetary and financial considerations. Furthermore, in situations where the outputs of the licensees are not verifiable or the licensees can do non-infringing imitation after getting the licensed technologies, the licensing contracts involve fixed-fees only (Rockett, 1990). As shown by Gelves and

[3]In the extension, we check the robustness of our main results in an alternative case in which $d_0 = d_1$, but c_0 and c_1 are different.

[4]We follow the literature in which the cost reduction under licensing is given, i.e., the value of technology transfer to the licensee is certain. There is another strand of literature which investigates the issue of licensing uncertain patents, such as Encaoua and Lefouili (2009) and Amir et al. (2014).

Heywood (2016), fixed-fee licensing is a common scheme in a mixed duopoly licensing game, especially for cases in which the public shares are high in the (privatized) public firm. Other licensing schemes such as royalty and two-part tariff licensing, may not add additional important policy implications to our analysis. For simplicity and easy tractability, we consider fixed-fee licensing in this model.[5]

According to the above model setting, we take the following three-stage game into consideration. At stage 1, the government decides on the level of privatization, $0 \leq \theta \leq 1$, for the maximization of social welfare. At stage 2, the more cost-efficient firm offers the licensing contract to the rival firm. Licensing contract will be signed if it makes neither of the involved firms worse off compared to no licensing. At stage 3, firms engage in Cournot competition. We solve the game through backward induction.[6]

3 A Public Firm versus a Domestic Private Firm

In this section, the public firm competes against a domestic private firm. To show the implications of licensing, let us first consider the case without licensing. Under no licensing, the three-stage game is reduced to the following two-stage game. At stage 1, the government determines the level of privatization. At stage 2, the firms compete in the product market like Cournot duopolists with homogeneous products.

In the production stage, the private firm 1 determines q_1 to maximize its profit $\pi_1^N = (a - q_0 - q_1)\, q_1 - cq_1 - d_1 q_1^2$, where the superscript "N" denotes "No licensing". Following Matsumura (1998), the objective function of firm 0 is the weighted sum of social welfare and its own profit, which is $\Omega^N = \theta \pi_0^N + (1-\theta) SW^N$, where $\pi_0^N = (a - q_0 - q_1)\, q_0 - cq_0 - d_0 q_0^2$,

[5] Discussions of per-unit royalty licensing can be found in Section 6.

[6] we assume ex-ante privatization in our model since the government policy is usually less reversible than the licensing contract. It is worth noting that a different timeline with ex-post privatization does not change our results when the public firm competes against a domestic private firm. The same degree of privatization, under either public or private licensing, will be obtained in equilibrium. The reason is that the fixed fee is just a transfer from the private firm to the privatized public firm, and thus it does not change the welfare. However, with the presence of a foreign competitor, the result under private licensing will be significantly different. More specifically, the optimal privatization strategy under private licensing is partial privatization rather than full privatization, which therefore leads to a lower degree of privatization compared to no licensing.

and $SW^N = \pi_0^N + \pi_1^N + Q^2/2$. The standard outcomes in the last stage are:

$$q_0^N = \frac{(a-c)(1+2d_1)}{1+2\theta+2(1+\theta)d_1+4d_0(1+d_1)}, \quad q_1^N = \frac{(a-c)(\theta+2d_0)}{1+2\theta+2(1+\theta)d_1+4d_0(1+d_1)}.$$

We assume $a > c$ to ensure that both firms are active in equilibrium. The social welfare in the second stage can be obtained by incorporating the above quantities into the profit functions.

In the first stage, the government decides the level of privatization, $0 \leq \theta \leq 1$, for the maximization of social welfare. Simple calculations yield the first-order condition as

$$\frac{\partial SW^N}{\partial \theta} = -\frac{(a-c)^2(1+2d_1)\left(\theta - 2d_0 + 6\theta d_1 + 4\theta d_1^2\right)}{(1+2\theta+2(1+\theta)d_1+4d_0(1+d_1))^{3}} = 0,$$

which follows that

$$\theta^N = \begin{cases} \frac{2d_0}{1+6d_1+4d_1^2}, & \text{for } d_0 < \frac{1+6d_1+4{d_1}^2}{2}; \\ 1, & \text{for } d_0 > \frac{1+6d_1+4{d_1}^2}{2}. \end{cases} \tag{1}$$

It is easy to verify that the second-order condition for welfare maximization is satisfied.[7]

Lemma 1. *In the presence of a domestic private firm, the government will always privatize the public firm under no licensing. The optimal degree of privatization, θ^N, is determined in (1). Straightforwardly, θ^N increases with d_0, and decreases with d_1.*

This result is consistent with the finding by Matsumura (1998): the optimal degree of privatization is never zero unless full nationalization yields public monopoly. Implied by Lemma 1, the government is more inclined to privatize the public firm with lower efficiency. If d_0 is sufficiently large (i.e., the public firm is very inefficient), the government will fully privatize the public firm to improve production efficiency.

Following (1), we obtain that: i) if $d_0 < d_1$, $\theta^N = \frac{2d_0}{1+6d_1+4d_1^2}$; ii) if $d_0 > d_1$, $\theta^N = \frac{2d_0}{1+6d_1+4d_1^2}$ for $d_1 < d_0 \leq \frac{1+6d_1+4{d_1}^2}{2}$, and $\theta^N = 1$ otherwise. In equilibrium, the values for π_1^N and Ω^N can be obtained by substituting the equilibrium privatization degree and quantities into the profit functions. This scenario serves as the default option for both firms.

[7]The second-order conditions for all the following cases are also satisfied.

3.1 The Public Licensing

Consider that the public firm is more cost-efficient compered to the private firm, i.e., $d_0 < d_1$. In the presence of licensing, the technology inferior private firm pays F for the licensed technology, and applies it to eliminate the initial efficiency difference. The cost function for firm 1 therefore changes to $cq_1 + d_0 q_1^2$.

In the production stage, given the degree of privatization, θ, firm 1 determines q_1 to maximize its profit $\pi_1^{L1} = (a - q_0 - q_1)\, q_1 - cq_1 - d_0 q_1^2 - F$, where the superscript "$L1$" denotes "Licensing to firm 1". For firm 0, it determines q_0 to maximize $\Omega^{L1} = \theta\pi_0^{L1} + (1-\theta)SW^{L1}$, where $\pi_0^{L1} = (a - q_0 - q_1)\, q_0 - cq_0 - d_0 q_0^2 + F$ and $SW^{L1} = \pi_0^{L1} + \pi_1^{L1} + Q^2/2$. The equilibrium quantities in the last stage are given by

$$q_0 = \frac{(a-c)\,(1+2d_0)}{1+2\theta+2d_0\,(3+\theta+2d_0)}, \quad q_1 = \frac{(a-c)\,(\theta+2d_0)}{1+2\theta+2d_0\,(3+\theta+2d_0)}.$$

In the second stage, the public firm determines the value of fixed fee and the private firm will purchase the technology if that increases its profit. Since the fixed fee under licensing is just a transfer from the private firm to the privatized public firm, it cancels out in the welfare expression. As a result, privatization does not depend on the fixed fee, and it is not important for us to find out the equilibrium values of the licensing fee to show how the relative bargaining powers of the firms affect the fee. However, it is worth clarifying how the fee will be determined.

The licensing fee will be such that the licensing contract does not make any firm worse off compared to no licensing. Hence, in case of public licensing, the maximum licensing fee that the private firm can pay will be the difference between its profits under licensing and no licensing, which will be a finite amount, and a profitable licensing contract will increase the equilibrium value of the public firm's objective function under licensing compared to no licensing with this amount of the fixed-fee. In the following, we assume that the licensor has full bargaining power and it provides a take-it-or-leave-it offer to the licensee.

In the first stage, the government maximizes SW^{L1} to determine the optimal value of θ.

Simple calculations yield the first-order condition as

$$\frac{\partial SW^{L1}}{\partial \theta} = -\frac{(a-c)^2 (1+2d_0)(\theta + 2d_0(-1+3\theta+2\theta d_0))}{(1+2\theta+2d_0(3+\theta+2d_0))^3} = 0,$$

which leads to

$$\theta^{L1} = \frac{2d_0}{1+6d_0+4d_0^2}. \tag{2}$$

Lemma 2. *When the public firm is more cost-efficient, the optimal degree of privatization under public licensing, θ^{L1}, is determined in (2). Furthermore, θ^{L1} increases with d_0 for $d_0 < 1/2$, and decreases with d_0 otherwise.*

It is obvious that $0 < \theta^{L1} < 1$. That is, partial privatization occurs in equilibrium. Moreover, there is an inverted U-shaped relation between θ^{L1} and d_0. The maximum degree of privatization under public licensing is 20% after simple calculations.

3.2 The Private Licensing

In the case that $d_0 > d_1$, the private firm is more cost-efficient and then decides to license its technology to the public firm.

In the production stage, given the degree of privatization, θ, firm 1 determines q_1 to maximize its profit $\pi_1^{L0} = (a-q_0-q_1)q_1 - cq_1 - d_1 q_1^2 + F$, where the superscript "$L0$" denotes "Licensing to firm 0". For firm 0, it determines q_0 to maximize $\Omega^{L0} = \theta\pi_0^{L0} + (1-\theta)SW^{L0}$, where $\pi_0^{L0} = (a-q_0-q_1)q_0 - cq_0 - d_1 q_0^2 - F$, and $SW^{L0} = \pi_0^{L0} + \pi_1^{L0} + Q^2/2$. The equilibrium quantities in the last stage are given by

$$q_0 = \frac{(a-c)(1+2d_1)}{1+2\theta+2d_1(3+\theta+2d_1)}, \quad q_1 = \frac{(a-c)(\theta+2d_1)}{1+2\theta+2d_1(3+\theta+2d_1)}.$$

In the second stage, the private firm determines the value of the fixed fee and the public firm will purchase the technology if it is not worse off in the value of objective function under licensing compared with no licensing. As before, the fixed fee cancels out in the welfare expression, and thus does not affect privatization.

In case of private licensing, the maximum licensing fee that the privatized public firm can pay will be the difference between its objective functions under licensing and no licensing, which will be a finite amount, and a profitable licensing contract will increase the equilibrium payoff of the private firm under licensing compared to no licensing with this amount of the fixed-fee. Since the objective function of the privatized firm includes the profits of the private firm and consumer surplus, it is worth discussing how the public firm finances the fee for the licensed technology. Here we follow the approach of Mukherjee and Sinha (2014), which followed a related approach considered in the privatization literature to discuss how the government attracts private investors under privatization even if it is not generating enough profit in the public firm (Mukherjee and Suetrong, 2009). We consider that the government raises money by imposing lump-sum taxes on the firms and the consumers to help the public firm to finance the fee for the licensed technology. If the public firm is different from the government and the government cannot tax on behalf of the public firm, the government can give the public firm a subsidy equal to the licensee fee, which it can raise by imposing lump-sum taxes on the firms and the consumers. Due to the non-distortionary nature of the lump-sum tax, the equilibrium outputs, the degree of privatization and social welfare are not affected by it.

In the first stage, the government maximizes SW^{L0} to determine the optimal value of θ. Simple calculations yield the first-order condition as

$$\frac{\partial SW^{L0}}{\partial \theta} = -\frac{(a-c)^2 (1+2d_1)(\theta + 2d_1(-1+3\theta+2\theta d_1))}{(1+2\theta+2d_1(3+\theta+2d_1))^3} = 0,$$

which leads to

$$\theta^{L0} = \frac{2d_1}{1+6d_1+4d_1^2}. \tag{3}$$

Lemma 3. *When the private firm is more cost-efficient, the optimal degree of privatization under private licensing, θ^{L0}, is determined in (3). Furthermore, θ^{L0} increases with d_1 for $d_1 < 1/2$, and decreases with d_1 otherwise.*

It is obvious that $0 < \theta^{L0} < 1$. That is, partial privatization occurs in equilibrium. Moreover, there is an inverted U-shaped relation between θ^{L1} and d_1. The maximum degree of privatization under private licensing is 20% after simple calculations.

By looking at Lemma 1-3, technology licensing changes the optimal degree of privatization as in the following proposition.

Proposition 1. *Compared to no licensing, (i) public licensing leads to a higher degree of privatization; and (ii) private licensing leads to a lower degree of privatization.*

Proposition 1(i) occurs when the public firm is more cost-efficient. As the ex-ante inefficient private firm becomes equally efficient in production as the public firm, the government becomes more induced to privatize the public firm to shift production toward the private firm, which therefore reduces the industry production cost. The small amount of previous literature on public firm licensing (Gelves and Heywood, 2016; and Heywood et al., 2019) does not examine the relationship between licensing and privatization. To the best of our knowledge, this is the first paper to combine public licensing and privatization to investigate the relationship between the two issues, and thus contribute to the existing literature on licensing and privatization.

Proposition 1(ii) implies that the government is less induced to privatize the public firm once the public firm becomes equally efficient as the private firm. The intuition is similar to that of Wang and Zeng (2019) although they consider constant marginal costs and ex post efficiency difference. As the public firm becomes more efficient via technology licensing, the government will reduce the degree of privatization so as to encourage the public firm to put more weight on social welfare. Mukherjee and Sinha (2014) also discusses the relationship between private licensing and privatization in the context of constant marginal costs. Under private licensing which eliminates the efficiency difference between two firms, the authors show that both the equilibrium output of the private firm and the equilibrium degree of privatization are zero. A straightforward comparison between Mukherjee and Sinha (2014) and our paper indicates the critical role played by the cost function, which therefore yields rich policy implications.

4 A Public Firm versus a Foreign Private Firm

Licensing between (privatized) public firms and foreign private firms are also very common in practice. Ye (2012) presents several prominent examples of this kind of licensing. We show in this section the implications of both public and private licensing when the private firm is a foreign firm.

Under no licensing, the objective functions for the private firm and the public firm are the same as that in Section 3. The only difference is that the profit of firm 1 is excluded from the calculation of social welfare, i.e., $SW^N = \pi_0^N + Q^2/2$. Similar calculations lead to the following quantities in the production stage

$$q_0^N = \frac{(a-c)\,(2+2d_1-\theta)}{2+\theta+2(1+\theta)d_1+4d_0\,(1+d_1)}, \quad q_1^N = \frac{(a-c)\,(\theta+2d_0)}{2+\theta+2(1+\theta)d_1+4d_0\,(1+d_1)}.$$

The social welfare in the second stage can be obtained by incorporating the above quantities into the profit functions.

In the first stage, the government decides the level of privatization, $0 \leq \theta \leq 1$, for the maximization of social welfare. Simple calculations yield that

$$\frac{\partial SW^N}{\partial \theta} = -\frac{2(a-c)^2\,(1+d_0+d_1)\,(d_0\,(-2+4\theta+4\theta d_1)+\theta\,(3+4d_1\,(2+d_1)))}{(2+\theta+2(1+\theta)d_1+4d_0\,(1+d_1))^{\,3}}. \tag{4}$$

By setting $\partial SW^N/\partial\theta = 0$, we have that

$$\theta^N = \frac{2d_0}{3+4d_0+8d_1+4d_0d_1+4d_1^2}. \tag{5}$$

Lemma 4. *In the presence of a foreign private firm, the government will always privatize the public firm under no licensing, and the optimal degree of privatization, θ^N, is determined in (5). Furthermore, the degree of privatization i) is lower than that in the case with a domestic private firm, and ii) increases with d_0, and decreases with d_1.*

According to Lemma 4, the presence of a foreign private firm reduces the degree of privatization. The key reason is that the profit of the foreign firm is excluded from social welfare. The government thus puts more weights on consumer surplus, which motivates it to decrease privatization. As in the case of no licensing with a domestic private firm, the government is more inclined to privatize the public firm as the public firm becomes more inefficient or the private firm becomes more efficient. However, due to the nationality of the rival firm, only partial privatization occurs in equilibrium. This result is different from what we obtain in the previous section, i.e., the possibility of full privatization in (1).

In equilibrium, the values for π_1^N and Ω^N can be obtained straightforwardly. This scenario serves as the default option for both firms.

4.1 The Public Licensing

We first look at the case with $d_0 < d_1$. As long as it is not worse off, the technology inferior private firm pays F for the licensed technology, and applies it to eliminate the initial efficiency difference.

In the production stage, the objective functions for the two firms are $\pi_1^{L1} = (a - q_0 - q_1)\, q_1 - cq_1 - d_0 q_1^2 - F$, and $\Omega^{L1} = \theta\pi_0^{L1} + (1-\theta) SW^{L1}$, where $\pi_0^{L1} = (a - q_0 - q_1)\, q_0 - cq_0 - d_0 q_0^2 + F$ and $SW^{L1} = \pi_0^{L1} + Q^2/2$. The equilibrium quantities in the last stage are given by

$$q_0 = \frac{(a-c)\,(2 + 2d_0 - \theta)}{(1 + 2d_0)\,(2 + \theta + 2d_0)}, \quad q_1 = \frac{(a-c)\,(\theta + 2d_0)}{(1 + 2d_0)\,(2 + \theta + 2d_0)}.$$

In the second stage, the fixed fee is determined by the public firm such that the private firm is not worse off compared to no licensing, which can be denoted by $F^{L1}(\theta)$.

In the first stage, the government maximizes SW^{L1} to determine the optimal value of θ. The value of SW^{L1} can be obtained by substituting the equilibrium quantities and fixed fee into the social welfare function. Simple calculations yield the first-order condition as

$$\frac{\partial SW^{L1}}{\partial \theta} = -\frac{2(a-c)^2\,(\theta + 2d_0\,(-3 + 5\theta + (-2 + 4\theta) d_0))}{(1 + 2d_0)^{\,2}\,(2 + \theta + 2d_0)^{\,3}} = 0,$$

which leads to

$$\theta^{L1} = \frac{6d_0 + 4d_0^2}{1 + 10d_0 + 8d_0^2}. \tag{6}$$

Lemma 5. *When the public firm is more cost-efficient, the optimal degree of privatization under public licensing, θ^{L1}, is determined in (6). Furthermore, θ^{L1} increases with d_0 for $d_0 < 3/2$, and decreases with d_0 otherwise.*

It is obvious that $0 < \theta^{L1} < 1$. That is, partial privatization occurs in equilibrium. Moreover, there is an inverted U-shaped relation between θ^{L1} and d_0. The maximum degree of privatization under public licensing is obtained as 52.94%.

4.2 The Private Licensing

In the case that $d_0 > d_1$, the private firm is more cost-efficient and then decides to license its technology to the public firm. After the usual straightforward calculations mirroring those in the previous section, we obtain the equilibrium quantities in the last stage as

$$q_0 = \frac{(a-c)(2+2d_1-\theta)}{(1+2d_1)(2+\theta+2d_1)}, \quad q_1 = \frac{(a-c)(\theta+2d_1)}{(1+2d_1)(2+\theta+2d_1)}.$$

In the second stage, the fixed fee, $F^{L0}(\theta)$, is determined by the private firm such that the public firm is not worse off compared to no licensing.

In the first stage, the government maximizes SW^{L0} to determine the optimal value of θ after substituting the equilibrium quantities and fixed fee into the social welfare function. Simple calculations yield the first-order condition as

$$\frac{\partial SW^{L0}}{\partial \theta} = \frac{2(a-c)^2(2+2d_1-\theta)}{(2+\theta+2d_1)^3},$$

which is always positive. As a result,

$$\theta^{L0} = 1. \tag{7}$$

Lemma 6. *When the private firm is more cost-efficient, the optimal privatization strategy under private licensing is full privatization.*

By looking at Lemma 4-6, technology licensing changes the optimal degree of privatization as in the following proposition.

Proposition 2. *Compared to no licensing, both public and (foreign) private licensing lead to a higher degree of privatization.*

There are two main reasons for this result. Firstly, the foreign private firm's profit is not included in the social welfare, which therefore induces the government to put more weight on the public firm's own profit. Secondly, unlike the previous case with a domestic private firm, the fixed fee in the licensing contract with the presence of a foreign firm is important to the social welfare. In light of this, the government may be more inclined to privatize the public firm so as to earn more or pay less in a licensing deal. Due to the above reasons, patent licensing between a public firm and a foreign private firm will push up the optimal level of privatization, which increases the privatized firm's profit, though at the expense of consumer surplus.

Wang and Zeng (2019) also investigates how licensing affects privatization in a mixed market with a foreign private firm. In their model, the domestic private firm is more efficient in production and decides to licences its superior technology either to the public firm or the foreign private firm. The authors find that licensing to the public firm hinders privatization while licensing to the foreign private firm facilitates privatization. However, we consider both public licensing and foreign licensing in a mixed duopoly model, and show that both forms of licensing will facilitate privatization in comparison to no licensing.

5 Extension

In this section, we investigate an alternative case in which $d_0 = d_1 = d$, but $c_0 \neq c_1$ to check the robustness of our main results in previous sections.[8] Similar argument used in previous

[8]The results under a linear cost function can be easily obtained by setting $d = 0$. With the presence of a domestic private firm, there will be no privatization after licensing under a linear cost function, i.e., licensing will slow down

sections is applied to conduct the analysis. To avoid similar passages mirroring those in the basic model, we summarize the result in each case as the following and relegate the calculations to the Appendix. To ensure that both firms are active in equilibrium, we assume that $a > \max\{c_0, c_1\}$ and $d > \max\{\frac{c_0 - c_1}{2(a-c_0)}, \frac{c_1 - c_0}{2(a-c_1)}\}$.

5.1 A Public Firm versus a Domestic Private Firm

We first consider the case without licensing. The following result can be easily obtained by mirroring the analysis in previous sections.

Lemma 7. *In the presence of a domestic private firm, the optimal degree of privatization under no licensing is given by*

$$\theta^N = \begin{cases} \frac{2ad+c_0-c_1-2dc_1}{a+6ad+4ad^2-4c_0-8dc_0-4d^2c_0+3c_1+2dc_1}, & \textit{for } a > \frac{(5+4d(2+d))c_0-4(1+d)c_1}{(1+2d)^2}; \\ 1, & \textit{for } a \leq \frac{(5+4d(2+d))c_0-4(1+d)c_1}{(1+2d)^2}. \end{cases} \tag{8}$$

Next consider that the public firm is more cost-efficient compered to the private firm, i.e., $c_0 < c_1$. The public firm licenses its superior technology to the private firm. After the usual straightforward calculations, we obtain that the following result.

Lemma 8. *When the public firm is more cost-efficient, the optimal degree of privatization under public licensing is given by*

$$\theta^{L1} = \frac{2d}{1+6d+4d^2}. \tag{9}$$

In the case that $c_0 > c_1$, the private firm is more cost-efficient and then decides to license its technology to the public firm. Straightforward calculations yield the following result.

Lemma 9. *When the private firm is more cost-efficient, the optimal degree of privatization under*

the privatization process, irrespective of public and private licensing, which is clear from equation (9) and (10). With the presence of a foreign private firm, public licensing leads to full nationalization as we see in equation (12), but private firm leads to full privatization by equation (13). Hence, the cost functions (quadratic or linear) may affect the results in Proposition 1 and 2.

private licensing is given

$$\theta^{L0} = \frac{2d}{1 + 6d + 4d^2}. \tag{10}$$

By (9) and (10), both public and private licensing lead to the same degree of privatization in equilibrium. However, for the two different cases, $c_0 > c_1$ and $c_0 < c_1$, the optimal degree of privatization under no licensing takes different values. After two simple comparisons, we have the following result.

Proposition 3. *Compared to no licensing, (i) public licensing leads to a higher degree of privatization; and (ii) private licensing leads to a lower degree of privatization.*

Proposition 3 indicates that the main result we obtained in Proposition 1 is qualitatively robust to this alternative cost consideration in the presence of a domestic private firm. The main reason is that the cost asymmetry considered here does not change the feature of the cost function. As a result, the same argument provided in Section 3.2 applies.

5.2 A Public Firm versus a Foreign Private Firm

The presence of a foreign private firm changes the expression for the social welfare. We first consider the case without licensing. The following result can be easily obtained by mirroring the analysis in previous sections.

Lemma 10. *In the presence of a foreign private firm, the optimal degree of privatization under no licensing is given by*

$$\theta^N = \frac{2ad + c_0 - c_1 - 2dc_1}{3a + 12ad + 8ad^2 - 2c_0 - 6dc_0 - 4d^2c_0 - c_1 - 6dc_1 - 4d^2c_1}. \tag{11}$$

Next consider that the public firm is more cost-efficient compered to the private firm, i.e., $c_0 < c_1$. The public firm licenses its superior technology to the private firm. After the usual straightforward calculations, we obtain that the following result.

Lemma 11. *When the public firm is more cost-efficient, the optimal degree of privatization under public licensing is given by*

$$\theta^{L1} = \frac{6d + 4d^2}{1 + 10d + 8d^2}. \tag{12}$$

In the case that $c_0 > c_1$, the private firm is more cost-efficient and then decides to license its technology to the public firm. Straightforward calculations yield the following result.

Lemma 12. *When the private firm is more cost-efficient, the optimal degree of privatization under private licensing is given*

$$\theta^{L0} = 1. \tag{13}$$

As a result, technology licensing changes the optimal degree of privatization as in the following proposition.

Proposition 4. *Compared to no licensing, both public and private licensing lead to a higher degree of privatization.*

The linearly increasing marginal cost function is the key reason for obtaining the results. By Proposition 2 and 4, the policy implication of licensing target are qualitatively the same even though we have alternative cost asymmetries.

6 Discussions and Qualifications

Before concluding the paper some remarks are in order. Firstly, we have assumed that the government can raise money to finance the license fee by imposing a lump-sum tax on the consumers. However, this may not be economically viable if it involves significant administrative and/or political costs. An alternative policy would be to impose some budget constraint or a minimum profit restriction on the public firm so that the public firm needs to generate sufficient profit to finance the license fee on its own. Since the public firm needs to generate a profit to finance the license fee, the government needs to privatize more compared to the tax system so that the public firm can increase profit to finance the fee. This, in turn, implies that welfare will reduce. Hence, as evident from Saha and Sensarma (2004),[9] if we impose a minimum profit requirement on the public firm, the profit constraint induces the government to privatize in a way so that it generates lower welfare compared to the situation with no profit constraint. Hence, if feasible, the tax system dominates the case of imposing a profit constraint on the public firm, and we assume that there exists a tax system that can be used to finance the license fee.

Secondly, we have considered fixed-fee licensing and a natural question would be to ask what happens if licensing contract involves a (pure) per-unit royalty rather than a fixed fee. It follows from the licensing literature that if there is licensing between two private firms, a per-unit royalty creates two effects: (i) it tends to increase the possibility of a profitable licensing contract by softening competition after licensing, and (ii) it tends to reduce welfare compared to a profitable fixed-fee licensing by contracting total output. In our analysis with a public firm competing against a domestic private firm, licensing is always profitable under a fixed-fee licensing. Hence, the first effect, i.e., the profit raising benefit of royalty, is not very significant for our analysis but royalty tends to reduce welfare by contracting total output under both public and private licensing. As a result, a per-unit royalty tends to reduce the incentive for privatization, and the fixed-fee licensing will be preferable from the welfare point of view compared to a licensing contract involving royalty. However, when the rival is a foreign firm, we find that both public and private licensing facilitate privatization under royalty licensing as we observe in Proposition 2. This is mainly because, under both public and private licensing, the profit of the foreign firm is not included in the social welfare. As a result, the government is induced to put more weight on the public firm's own profit (compared to the situation where the private firm is domestic), which increases the incentive for privatization so as to earn more or pay less in a licensing deal.

[9] One may also look at Choi (2011) and Ishida and Matsushima (2009) for the implications of budget constraints on the public firms.

7 Concluding Remarks

Technology licensing from one cost-efficient firm to another cost-inefficient firm has been widely observed in mixed oligopolies. However, relatively little theoretical research has been devoted so far to licensing in mixed markets, and particularly its impact on privatization. In this paper, we investigate the relationship between privatization and (both public and private) licensing with the consideration of either a domestic or a foreign private firm. We find that i) in the case of a domestic private firm, public licensing facilitates privatization, but private licensing hinders privatization; ii) in the case of a foreign private firm, both public and private licensing facilitate privatization. Our results yield important policy implications to licensing and privatization.

A number of areas are worthwhile directions for future research based on the present model.[10] Firstly, in our paper, the initial cost conditions are given exogenously. It is very interesting to endogenize the costs by introducing R&D competition in public and private firms as in Ishibashi and Matsumura (2006) and Basak and Wang (2019). The consideration of R&D competition could yield some interesting results and thus generate important policy implications. Another direction is to extend the current mixed duopoly model to a mixed oligopoly model with multiple private firms. As shown by Haraguchi and Matsumura (2016), a mixed oligopoly and a mixed duopoly may yield quite different implications. We believe that the analysis of a mixed oligopoly would greatly enrich the literature on licensing and privatization. Still a third avenue is to consider foreign investors in the privatized firm in our model. Empirical evidence reveals that foreign investors are influential buyers of public firms (see also in Lin and Matsumura, 2012). Such a model may enable us to study the relationship between the presence of foreign investors in the privatized firm and privatization policy under licensing. Lastly, a future research maybe to consider privatization and licensing in the presence of government subsidy/tax and the excess burden of taxation may play an important role in this case.[11]

[10] We thank an anonymous referee for pointing out several future directions.

[11] Several papers have investigated how the excess burden of taxation affects the privatization policy without considering public/private licensing, such as Lee and Wang (2018), and Chen et al. (2018).

Appendix: Proofs

Proof of Lemma 2:

Simple calculations yield that $\frac{\partial \theta^{L1}}{\partial d_0} = \frac{2-8d_0^2}{\left(1+6d_0+4d_0^2\right)^2}$, which is positive for $d_0 < 1/2$ and negative for $d_0 > 1/2$. Incorporating $d_0 = 1/2$ into (2) yields that $\theta^{L1} = 20\%$.

Proof of Lemma 3:

Simple calculations yield that $\frac{\partial \theta^{L0}}{\partial d_1} = \frac{2-8d_1^2}{\left(1+6d_1+4d_1^2\right)^2}$, which is positive for $d_1 < 1/2$ and negative for $d_1 > 1/2$. Incorporating $d_1 = 1/2$ into (3) yields that $\theta^{L0} = 20\%$.

Proof of Proposition 1:

In the case that $d_0 < d_1$, the public firm licenses to the private firm, and $\theta^{L1} = \frac{2d_0}{1+6d_0+4d_0^2}$. Under no licensing, $\theta^N = \frac{2d_0}{1+6d_1+4d_1^2}$. Thus, we have $\theta^{L1} > \theta^N$. In the case that $d_0 > d_1$, the private firm licenses to the public firm, and $\theta^{L0} = \frac{2d_1}{1+6d_1+4d_1^2}$. It follows that $\theta^{L0} < \theta^N < 1$.

Proof of Lemma 4:

It is obvious that the degree of privatization is lower than that in the case with a domestic private firm. Furthermore, we obtain that $\frac{\partial \theta^N}{\partial d_0} = \frac{6+8d_1(2+d_1)}{(3+4d_0(1+d_1)+4d_1(2+d_1))^2} > 0$, and $\frac{\partial \theta^N}{\partial d_1} = \frac{-2d_0(8+4d_0+8d_1)}{\left(3+4d_0+8d_1+4d_0d_1+4d_1^2\right)^2} < 0$.

Proof of Lemma 5:

Simple calculations yield that $\frac{\partial \theta^{L1}}{\partial d_0} = \frac{6-8(-1+d_0)d_0}{(1+2d_0(5+4d_0))^2}$, which is positive for $d_0 < 3/2$ and negative for $d_0 > 3/2$. Incorporating $d_0 = 3/2$ into (6) yields that $\theta^{L1} = 52.94\%$.

Proof of Proposition 2:

We only need to look at the case with public licensing, which occurs when $d_0 < d_1$. The optimal privatization under public licensing is $\theta^{L1} = \frac{2\left(3d_0+2d_0^2\right)}{1+10d_0+8d_0^2}$. Recall that $\theta^N = \frac{2d_0}{3+4d_0+8d_1+4d_0d_1+4d_1^2}$.

Hence, $\theta^{L1}-\theta^{N}=\frac{2d_0(8(1+d_0)+4(2+d_0)(3+2d_0)d_1+4(3+2d_0)d_1^2)}{(1+10d_0+8d_0^2)(3+4d_0+8d_1+4d_0d_1+4d_1^2)}>0$.

Proof of Lemma 7:

In the production stage, firm 1 determines q_1 to maximize $\pi_1^N=(a-q_0-q_1)\,q_1-c_1q_1-dq_1^2$, and firm 0 determines q_0 to maximize $\Omega^N=\theta\pi_0^N+(1-\theta)SW^N$, where $\pi_0^N=(a-q_0-q_1)\,q_0-c_0q_0-dq_0^2$, and $SW^N=\pi_0^N+\pi_1^N+Q^2/2$. The standard outcomes in the last stage are: $q_0^N=\frac{a+2ad-2(1+d)c_0+c_1}{1+2\theta+2d(3+2d+\theta)}$, and $q_1^N=\frac{a(2d+\theta)+c_0-(1+2d+\theta)c_1}{1+2\theta+2d(3+2d+\theta)}$.

In the first stage, the government decides the level of privatization, $0\leq\theta\leq1$, for the maximization of social welfare. Simple calculations yield the first-order condition as $\frac{\partial SW^N}{\partial\theta}=-\frac{(a+2ad-2(1+d)c_0+c_1)\left(a(\theta+2d(-1+3\theta+2d\theta))-\left(1+4(1+d)^2\theta\right)c_0+(1+3\theta+2d(1+\theta))c_1\right)}{(1+2\theta+2d(3+2d+\theta))^3}=0$, which leads to the optimal degree of privatization in (8).

Proof of Lemma 8:

In the production stage, firm 1 determines q_1 to maximize its profit $\pi_1^{L1}=(a-q_0-q_1)\,q_1-c_0q_1-d{q_1}^2-F$. For firm 0, it determines q_0 to maximize $\Omega^{L1}=\theta\pi_0^{L1}+(1-\theta)SW^{L1}$, where $\pi_0^{L1}=(a-q_0-q_1)\,q_0-c_0q_0-dq_0^2+F$ and $SW^{L1}=\pi_0^{L1}+\pi_1^{L1}+Q^2/2$. The equilibrium quantities in the last stage are given by $q_0=\frac{(1+2d)(a-c_0)}{1+2\theta+2d(3+2d+\theta)}$, and $q_1=\frac{(2d+\theta)(a-c_0)}{1+2\theta+2d(3+2d+\theta)}$.

In the second stage, the public firm determines the value of fixed fee. Since the fixed fee under licensing is just a transfer from the private firm to the privatized public firm, it cancels out in the welfare expression.

In the first stage, the government maximizes SW^{L1} to determine the optimal value of θ. Simple calculations yield the first-order condition as $\frac{\partial SW^{L1}}{\partial\theta}=-\frac{(1+2d)(\theta+2d(-1+(3+2d)\theta))(a-c_0)^2}{(1+2\theta+2d(3+2d+\theta))^3}=0$, which leads to $\theta^{L1}=\frac{2d}{1+6d+4d^2}$.

Proof of Lemma 9:

In the production stage, we have $\pi_1^{L1}=(a-q_0-q_1)\,q_1-c_1q_1-d{q_1}^2-F$ and $\pi_0^{L1}=(a-q_0-q_1)\,q_0-c_1q_0-dq_0^2+F$. The equilibrium quantities in the last stage are given by

$q_0 = \frac{(1+2d)(a-c_1)}{1+2\theta+2d(3+2d+\theta)}$, and $q_1 = \frac{(2d+\theta)(a-c_1)}{1+2\theta+2d(3+2d+\theta)}$.

In the second stage, the determination of fixed fee does not affect privatization.

In the first stage, the government maximizes SW^{L1} to determine the optimal value of θ. Simple calculations yield the first-order condition as $\frac{\partial SW^{L1}}{\partial \theta} = -\frac{(1+2d)(\theta+2d(-1+(3+2d)\theta))(a-c_1)^2}{(1+2\theta+2d(3+2d+\theta))^3}$, which leads to $\theta^{L0} = \frac{2d}{1+6d+4d^2}$.

Proof of Proposition 3:

In the case that $c_0 < c_1$, the public firm licenses to the private firm. Simple calculations yield that $\theta^{L1} - \theta^N = \frac{(1+2d(7+2d(5+2d)))(c_1-c_0)}{(1+6d_0+4d_0^2)((a+6ad+4ad^2-4c_0-8dc_0-4d^2c_0+3c_1+2dc_1))} > 0$. In the case that $c_0 > c_1$, the private firm licenses to the public firm, and we have that $\theta^{L0} - \theta^N = \frac{(1+2d(7+2d(5+2d)))(c_1-c_0)}{(1+6d_0+4d_0^2)((a+6ad+4ad^2-4c_0-8dc_0-4d^2c_0+3c_1+2dc_1))} < 0$.

Proof of Lemma 10:

In the production stage, the private firm 1 determines q_1 to maximize its profit $\pi_1^N = (a - q_0 - q_1)\, q_1 - c_1 q_1 - dq_1^2$, and firm 0 determines q_0 to maximize $\Omega^N = \theta\pi_0^N + (1-\theta)SW^N$, where $\pi_0^N = (a - q_0 - q_1)\, q_0 - c_0 q_0 - dq_0^2$, and $SW^N = \pi_0^N + Q^2/2$. The standard outcomes in the last stage are: $q_0^N = \frac{a(2+2d-\theta)-2(1+d)c_0+\theta c_1}{(1+2d)(2+2d+\theta)}$, and $q_1^N = \frac{a(2d+\theta)+c_0-(1+2d+\theta)c_1}{(1+2d)(2+2d+\theta)}$.

In the first stage, the government decides the level of privatization, $0 \leq \theta \leq 1$, for the maximization of social welfare. Simple calculations yield the first-order condition as $\frac{\partial SW^N}{\partial \theta} = -\frac{(2a-c_0-c_1)(-2ad+a(3+4d(3+2d))\theta-(1+2(1+d)(1+2d)\theta)c_0-(-1+\theta+2d(-1+3\theta+2d\theta))c_1)}{(1+2d)^2(2+2d+\theta)^3} = 0$, which leads to the optimal degree of privatization in (11).

Proof of Lemma 11:

n the production stage, firm 1 determines q_1 to maximize its profit $\pi_1^{L1} = (a - q_0 - q_1)\, q_1 - c_0 q_1 - dq_1^2 - F$. For firm 0, it determines q_0 to maximize $\Omega^{L1} = \theta\pi_0^{L1} + (1-\theta)SW^{L1}$, where $\pi_0^{L1} = (a - q_0 - q_1)\, q_0 - c_0 q_0 - dq_0^2 + F$ and $SW^{L1} = \pi_0^{L1} + Q^2/2$. The equilibrium quantities in the last stage are given by $q_0 = \frac{(2+2d-\theta)(a-c_0)}{(1+2d)(2+2d+\theta)}$, and $q_1 = \frac{(2d+\theta)(a-c_0)}{(1+2d)(2+2d+\theta)}$.

In the second stage, the public firm determines the value of fixed fee. In the first stage, the government maximizes SW^{L1} to determine the optimal value of θ. Simple calculations yield the first-order condition as $\frac{\partial SW^{L1}}{\partial \theta} = -\frac{2(\theta+2d(-3+5\theta+d(-2+4\theta)))(a-c_0)^2}{(1+2d)^2(2+2d+\theta)^3} = 0$, which leads to $\theta^{L1} = \frac{6d+4d^2}{1+10d+8d^2}$.

Proof of Lemma 12:

In the production stage, we have $\pi_1^{L0} = (a - q_0 - q_1)\, q_1 - c_1 q_1 - d{q_1}^2 - F$, $\pi_0^{L0} = (a - q_0 - q_1)\, q_0 - c_1 q_0 - d q_0^2 + F$, and $SW^{L0} = \pi_0^{L1} + Q^2/2$. The equilibrium quantities in the last stage are given by $q_0 = \frac{(2+2d-\theta)(a-c_1)}{(1+2d)(2+2d+\theta)}$, and $q_1 = \frac{(2d+\theta)(a-c_1)}{(1+2d)(2+2d+\theta)}$.

In the second stage, the fixed fee is chosen such that the public firm is not worse off in comparison to the case of no licensing. In the first stage, the government maximizes SW^{L0} to determine the optimal value of θ. Simple calculations yield the first-order condition as $\frac{\partial SW^{L0}}{\partial \theta} = \frac{2(2+2d-\theta)(a-c_1)^2}{(2+2d+\theta)^3}$, which is always positive. Thus, $\theta^{L0} = 1$.

Proof of Proposition 4:

For the case with public licensing (i.e., $c_0 < c_1$), simple calculations yield that $\theta^{L1} - \theta^N = \frac{16ad(1+d)^2(1+2d)-(1+2d(11+2d(13+4d(3+d))))c_0+(1-2d(-3+2d(3+4d(2+d))))c_1}{(1+10d+8d^2)(3a+12ad+8ad^2-2c_0-6dc_0-4d^2c_0-c_1-6dc_1-4d^2c_1)} > 0$.

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