Received November 15, 2019, accepted November 24, 2019, date of publication December 2, 2019, date of current version December 16, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2957102

Modeling of Classical Synchronous Generators Using Size-Efficient Lookup Tables With Skewing Effect

QUADIR H. QUADRI[1], STEFANO NUZZO[1,2], (Member, IEEE), MOHAMED RASHED[1], (Member, IEEE), CHRIS GERADA[1,3], (Senior Member, IEEE), AND MICHAEL GALEA[1,3], (Senior Member, IEEE)

[1]Power Electronics, Machines and Control Research Group, University of Nottingham, Nottingham NG7 2RD, U.K.
[2]Department of Engineering Enzo Ferrari, University of Modena and Reggio Emilia, 41125 Modena, Italy
[3]Key Laboratory of More Electric Aircraft Technology of Zhejiang Province, Ningbo 315100, China

Corresponding author: Quadir H. Quadri (syed.quadri@nottingham.ac.uk)

This work was supported in part by the University of Nottingham's Propulsion Futures Beacon, through the projects PF42 and PF45, and in part by the Natural Science Foundation of China through the project under Grant 51850410515.

ABSTRACT In this paper, an analytical model aimed at reducing computational times for the analysis of classical synchronous generators is proposed and validated. While the proposed model's attractiveness comes from its simple and fast nature, however, it also features excellent levels of accuracy. This is achieved by the model's ability to consider aspects like saturation and space harmonics. Such features are usually investigated with computationally-heavy finite element analysis. The proposed method shows that an appropriate flux linkage map of all the machine windings as a function of currents and rotor position can be used to accurately consider these features at no cost of time or accuracy. Furthermore, the integration of the skewing effect within the model has also been proposed by incorporating it within the flux linkage map. The proposed method is investigated through the use of a 72.5kVA, wound field, salient pole synchronous generator. The results are compared with those of a finite element model and also against experimental measurements on a physical prototype. The advantages of the proposed procedure are discussed, where the model's suitability for carrying out lengthy and multiple simulations and its flexibility are highlighted.

INDEX TERMS Harmonics, lookup tables, modelling, skewing effect, synchronous generators.

I. INTRODUCTION

Historically, modelling and design of wound-field, synchronous generators (SG) have been performed through extensive use of the classical equivalent circuit and equivalent dq-axis circuit equations [1], [2]. These methodologies have been shown to be robust and elegant solutions and most importantly have withstood the test of time. However, their main limitation has always been that they only consider an ideal, linear behaviour of the machine and thus are based on a number of assumptions and simplifications, including the neglecting of parasitic non-linear electromagnetic relations such as harmonics and magnetic saturation.

With recent advancements in computational resources, new materials and new design methodologies, designers and researchers are today able to consider complex geometries [3], non-linear material properties such as due to eddy currents [4], thermal characteristics [5] and others. The advent of commercially-available, finite-element (FE) platforms and tools, capable of solving complex non-linear systems in feasible time-lines, is allowing unprecedented improvements in accuracy and design flexibility.

While the FE method is today the undisputed methodology of choice in terms of accuracy, however the challenge associated to it is that it remains time-consuming and for optimal operation usually demands higher processing speeds than those provided by standard computing resources. This is highly emphasized when specific operating points, dynamic responses and skewed SG structures are considered and modelled. Additionally, FE modelling packages usually do not support or feature compatibility with other generic mathematical modelling tools, e.g. Matlab-Simulink. Therefore,

The associate editor coordinating the review of this manuscript and approving it for publication was Xiaodong Sun.

often, their interfacing with external control system models of the generation system cannot be modelled and analysed and analytical models are still widely used [6].

Recently, a number of works focusing on either hybrid analytical-numerical [7], [8] or analytical methodologies supported by numerical analyses are being developed. This 'best of both worlds' concept [9] has significant potential as it can benefit from the main strengths of both methodologies.

The model proposed in this paper complies with such a hybrid trend and consists in importing the electrical machine's parameters from FE model to a faster but still accurate circuital analytical model. While this technique has been widely investigated [10]–[17], its main challenge relates to its preparation time, i.e. the time required to prepare these imported data sets in a short timescale. This paper then aims at overcoming such modelling challenge, thus improving its suitability for "everyday", company-based design processes. Hence, it proposes an advanced methodology for the modelling of classical, wound-field, salient poles SGs that is not computationally expensive (being based on an analytical system) but is yet able to accurately consider the main effects that have traditionally been limited to FE-based solutions, such as the non-linear behaviour and saturation of ferromagnetic materials.

II. TRADITIONAL MODELLING PHILOSOPHIES

SGs are traditionally modelled through a dq reference frame. This method is known for its elegance and simplicity [1], [2]. Such techniques usually assume constant inductance values along the direct and quadrature axes and ignore the effects of space-harmonics in the airgap flux distribution. This obviously results in a considerable loss of accuracy. To mitigate these inaccuracies, several measures have been proposed and reported in literature. In [18], a dq model with considerations for added complexities is presented. This model considers the effects of time-harmonic electrical quantities by converting each harmonic order of three-phase quantities to its counterpart in the dq reference frame. However, it does not take into account the effects of space harmonics and the magnetic saturation produced by the generator itself.

Harmonic domain reference frame has also been proposed for SG modelling to account for the harmonic effects [19]. While some accuracy improvements have been reported, however in general it is perceived that the drawbacks in terms of the heavier requirements of computational resources still outweigh the gained accuracy. Thus, models based on physical phase coordinates (abc-reference frame) have been traditionally preferred [10]–[12], [20]–[22] for modelling SGs with space and time harmonic effects. This aspect is very much emphasized in [12], [20]. These physical phase variable approaches are based on mathematical models of the stator and the rotor winding circuits and include time-varying winding inductances or flux linkages to account for space harmonics. A common practice in implementing this is to consider the dominant terms of a general Fourier series of the varying winding inductances as a function of the rotor's angular position [20]. The improvement with such modelling technique is significant, however it is also true that it usually ignores magnetic saturation, i.e. assuming linear behaviour of ferromagnetic materials. Moreover, Fourier series based methodologies, such as those considered in [19], [20], are approximately estimated using the typical geometry of salient pole SGs, whilst ignoring particular influencing factors, such as number of slots, material properties, pole span, slot openings, armature winding configurations, etc.

Combining electromagnetic FE analysis with electrical circuit modelling has resulted in a step-change in terms of the validity of analytical models that are able to completely consider time and space harmonics [21], [22]. This hybridization of FE and linear circuital approaches results in improved accuracy, but come at the cost of increasing the computational burden and resources.

To decrease the simulation time, techniques have been proposed that solve windings' circuits state-space equations using lookup tables of stored parameters obtained from FE simulations [10]–[12]. These approaches provide simulation results comparable to those of FE modelling techniques. The stored parameters used in [10] are usually the windings' inductances or flux linkages values as function of the rotor's angular position. However, since they implement lookup tables dependent on rotor angle only, they usually ignore the magnetic saturation effects. The techniques proposed in [11], [12] consider harmonics due to both the machine geometry and magnetic saturation. They implement lookup tables as a function of rotor angle and winding currents. This permits to account for the effects of geometry and saturation induced harmonics, respectively. A perceived challenge here is the development of these lookup tables, which is usually a time-consuming exercise and, once complete, can require large memory space.

All the above indicates that the future for SG modelling and design leans towards hybrid and combined modelling philosophies. Further new advancements towards such concepts are thus proposed in this paper to overcome the aforementioned existing challenges for a comprehensive modelling of salient-pole, wound-field SGs. The approach comprises the development of mathematical equations that represent the winding circuits, linked with size-efficient lookup tables of flux linkages of field and stator windings as a function of rotor angular position and winding currents. Additionally, it incorporates the skewing effect. The model simulates voltages and currents as well as the electromagnetic torque of SGs. The concept of this modelling technique is developed and implemented on a particular case study of a market-ready, 72.5kVA SG. The 2D geometry of this machine is shown in Fig. 1, where the three-phase stator winding arrangement is highlighted. Its main specifications are given in Table 1 for the sake of completeness. First, a 2D FE model of this SG is built and validated against experimental results obtained from testing a physical prototype. Then, the proposed methodology, which implements size-efficient lookup tables incorporating the winding's flux

TABLE 1. Synchronous generator specifications.

Parameters	Quantity
Rated power	72.5 kVA
Rated phase terminal voltage	231 V
Frequency	50 Hz
Number of poles	4
Number of slots	48
Stator skew angle	1 slot pitch
Coil pitch	2/3
Armature phase parallel paths	2

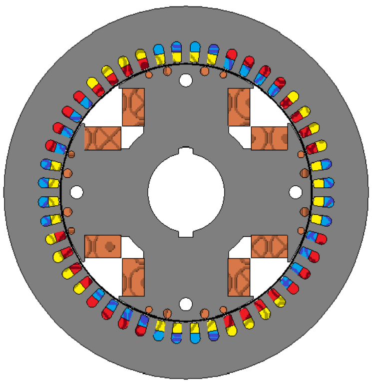

FIGURE 1. The considered machine: 72.5kVA SG.

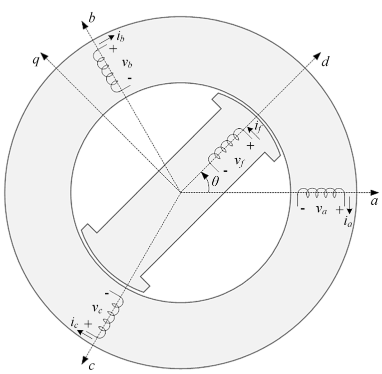

FIGURE 2. The model: conventions for voltage polarity and current direction.

linkages, is fully validated against both FE and experimental evaluations.

III. METHODOLOGY DEVELOPMENT APPROACH

The methodology is developed considering the stator's abc-phase reference frame as its advantages outweigh that of the dq0-model [12], [20].

By applying the Kirchhoff's voltage law (KVL) on the winding circuits whilst using the current and voltage conventions as shown in Fig. 2, the representative equations for the armature and field circuits can then be described by (1) and (2) respectively [23], where ψ_{abc}, v_{abc} and i_{abc} denote the instantaneous phase flux linkages, terminal voltages and currents respectively, R_{abc} is the internal resistance of the phase windings, v_f, ψ_f and i_f are the instantaneous field terminal voltage, flux linkage and current respectively and R_f is the field winding resistance. The resistances considered in this model are the DC phase resistances assuming uniform current distribution over the cross-section and thus neglecting eddy currents and AC losses in the conductors.

$$-\frac{d\psi_{abc}}{dt} = v_{abc} + i_{abc}R_{abc} \quad (1)$$

$$v_f = \frac{d\psi_f}{dt} + i_f R_f \quad (2)$$

The most reliable method of accurately estimating flux linkages is by using the FE model of the machine, as this considers all the non-linear dependencies in its calculations, such as the rotor angular position, the rotor and stator cross-sectional geometry, the winding configuration, phase and field currents and the materials' magnetic properties.

However, this involves complex and lengthy calculations for each time step. Therefore, an initial one-off run of several time (position) stepping FE simulations over a series of rotor angular positions and combinations of windings' currents is done to create accurate flux linkage maps. These maps are then used as lookup tables that consider the rotor angle and windings' currents to determine instantaneous flux linkages of the windings to be used for the model equations (1) and (2).

The concept here is that, for this first stage of the development of the equation-based representation, the flux linkage map is built based on an initial run of the FE model, thus taking all of its benefits in terms of accuracy. This will then be used for all the subsequent iterations during the full analysis procedure including runs for various operating conditions and loading levels, thus allowing for a significant computational time reduction.

IV. THE LOOKUP TABLES

The implementation of inductances or flux linkages lookup tables incorporating the rotor angle and winding currents dependencies, automatically includes geometry and saturation induced harmonics. Models implementing lookup tables of inductances usually use some variants of (3) to obtain the instantaneous flux linkages of the windings [11]. In (3), ψ represents a generic flux linkage, i is a generic current, θ is the rotor position and L is the inductance matrix.

$$[\psi] = [L(\theta, [i])][i] \quad (3)$$

However, in this case, lookup tables of the flux linkages are preferred to those of inductances for the following reasons: 1) the extraction of inductances from the FE model simulations for the construction of lookup tables is more complex when inductances as a function of windings' currents are concerned, as it requires computation of the partial differential equations of the flux linkage with respect to the currents [12]; 2) an electrical machine with n windings requires a minimum of $(n^2 + n)/2$ inductances to be determined for a lookup table of inductances, whereas only n flux linkages dependent on rotor angle and currents are required for flux linkage lookup tables [12]; 3) determining instantaneous flux linkages through inductances lookup tables is not as accurate as directly getting it from flux linkages lookup tables, because the flux linkage can no longer be represented by linear summation of the products of inductance and currents if the core's non-linear magnetic properties are considered.

Lookup tables are generally bulky and so the time and memory space required for their development makes such models unattractive. For example, [12] prepares 5D lookup tables of flux linkages with respect to rotor angles and windings' currents as described by (4), for all the combinations of the data points given in (5). The angle θ_m in (5) is the rotor's mechanical angle taken for a 4-pole machine with sample spacing of 3.75°.

$$\psi = f\left(\theta, i_a, i_b, i_c, i_f\right) \tag{4}$$

$$\left.\begin{aligned} \theta_m &\in \{0 : 3.75 : 176.25\}^{\circ} \\ i_f &\in \{0.5, 1, 1.5\}pu \\ i_{abc} &\in \pm\{0.5, 1, 1.3\}^3 pu \end{aligned}\right\} \tag{5}$$

This results in four 5D lookup tables of 31104 data points each. A high percentage of these data points consists of combinations that are impractical and very rarely occur in a SG, as they represent an unbalanced three phase load condition in the stator windings. An indicative example of this is having +1pu current in all the three phases at the same time. Such data points are only needed to provide a complete, gridded data for the interpolation of the lookup tables. This implies that heavy computational operations are required to develop these comprehensive lookup tables.

The lookup table technique proposed in this paper is far more efficient than those described above, as the data points are selected according to the normal balanced three phase operation of a SG. This is done by considering the space vector of the three phase stator currents with respect to the rotating dq-axes system instead of taking the real three phase currents. The stator current phasor, as shown in Fig. 3, will normally be within the third quadrant of the dq-axis system.

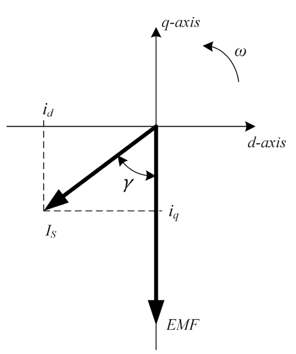

FIGURE 3. Standard phasor diagram of a SG.

Hence, a lookup table of flux linkages as described in (6) can be achieved. In (6), γ is the stator current angle with respect to the q-axis current and I_s is the stator current magnitude.

TABLE 2. Reference frames used for lookup tables based models.

Model using lookup tables	Circuit modelling	Lookup tables
Abc – reference frame	✓ Accurate	× Bulky
Dq – reference frame	× Less accurate	✓ Concise

$$\psi = f\left(\theta, i_f, I_s, \gamma\right) \tag{6}$$

The data points for the phasor magnitude of stator currents can be sensibly selected according to the rated current of the SG and the angle γ within the range of 0° to 90°. Hence, adopting polar coordinates (I_s and γ) helps in obtaining denser data around the normal operating points using fewer data points when compared to Cartesian coordinates, i.e. d- and q-axis currents (i_d and i_q).

A lookup table with respect to these variables requires less number of data points and consequently less number of FE simulations. Also, all the data points are within the most probable region of operation, which gives a much higher quality and quantity of data for the interpolation around the instantaneous current values, thus resulting in a much more efficient methodology than previously proposed in literature.

However, the circuit equations (1) and (2) still need to be modelled in the abc-domain for accuracy. The stator winding currents calculated by (1) and (2) are converted in to d-axis and q-axis currents through Park's transformation, and then I_s and γ are calculated in order to be used as an input for the four lookup tables. Through these lookup tables, the instantaneous flux linkages of all the SG windings are then identified according to the input (based on the specific operating condition considered) that is being used in the circuit equations for determining instantaneous electrical quantities.

The models developed so far using lookup tables in the literature are entirely based on either abc-reference frame [10]–[12] or dq-reference frame [13]–[17], whose general benefits and challenges are summarised in Table 2.

The proposed model benefits from the advantages of both the reference systems by combining the abc-referenced circuital model with the lookup tables dependent on the resultant vector quantities of the three-phase stator currents (I_s and γ). In fact, it further optimises the lookup tables by considering polar coordinates (I_s and γ) of the vector rather than Cartesian ones (i_d and i_q). This results in a more accurate and fast phase domain methodology with size-efficient lookup tables.

V. IMPLEMENTATION OF THE MODEL

For this case study, a 3-phase 72.5kVA, 400V salient-pole SG as described by Fig. 1 and Table 1 was considered. A detailed FE model of the considered SG was developed to extract data and develop the required lookup table for the proposed analytical model. All the FE results were validated against experimental results obtained from an instrumented and customised test-rig. A picture of the machine on its test-bed is given in Fig. 4, where the 72.5kVA SG can be observed mechanically coupled with a 150kW DC motor acting as its prime mover. The SG field winding can be supplied either by an external DC source or through a brushless excitation system. The three-phase terminals of the SG are connected to an AC purely resistive load bank that can go up to 200kW. The test-rig is equipped with a torque transducer at the shaft coupling and a three-phase power analyser.

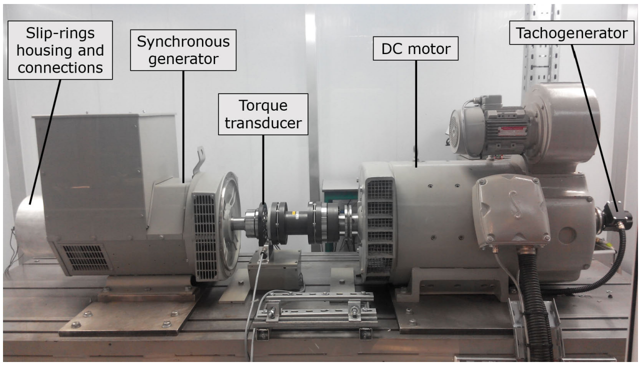

FIGURE 4. Experimental test set-up.

A. VALIDATION OF THE FE MODEL

FE time-stepping simulations were performed under no load condition at rated speed with different field excitation currents to plot the open circuit characteristic (OCC). The curve obtained from the FE model was then compared with that resulting from the experimental testing of the SG, i.e. measured voltages of the open-circuit phase windings for various levels of field current provided by an external DC current source.

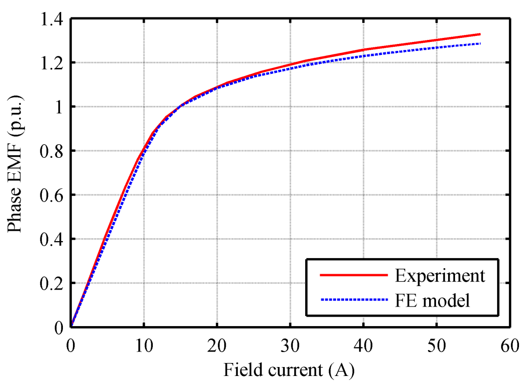

FIGURE 5. Experimental validation: the OCC.

Fig. 5 compares numerical and experimental results related to the OCC. A very good match can be observed especially in the linear region, while a maximum error of 3% is recorded after the saturation knee. This is probably due to the challenges related to the representation of the real behaviour of the ferromagnetic materials through their BH curve. Fig. 6 compares the voltage waveforms at rated no-load condition of the platform under test. Also in this case, a close match between FE and experimental results is seen.

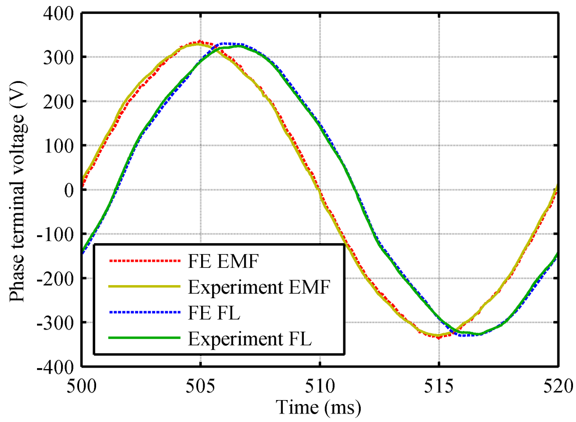

FIGURE 6. Experimental validation: no-load and full-load voltage waveforms.

After having successfully validated the FE model at no-load, the validation exercise comprises also its full load operation. For full load, the SG was run at unity power factor with the field current supplied via the automatic voltage regulator (AVR). The load for the simulation was selected accordingly, resulting in a purely resistive load of 2.2Ω. The waveforms of the phase voltage and current obtained from the simulation are compared with the experimental results in Fig. 6 and Fig. 7 respectively, where excellent similarities between the two waveforms can be observed.

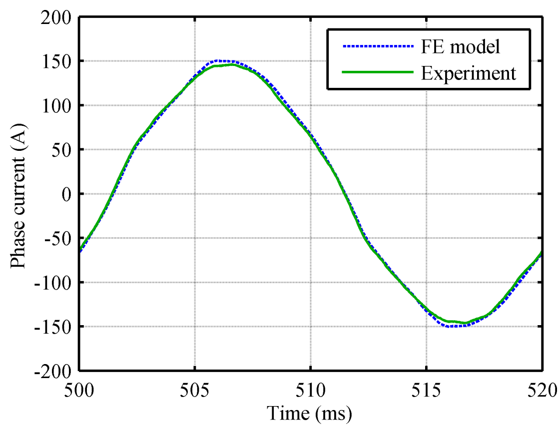

FIGURE 7. Experimental validation: phase current waveform.

The rms values of these waveforms are compared in Table 3, which confirms excellent accuracy of the FE model. Thus, it can be safely said that the instantaneous flux linkages calculated by the FE model are an accurate and precise representation of the actual flux linkages of the windings inside the generator and can be used to construct the lookup tables for the development of the analytical model.

TABLE 3. Comparison of RMS values at full load.

Quantities	FE model results	Experiment	Percentage Error
Field current	34 A	34 A	0
Phase voltage	230.7 V	229 V	0.7
Phase current	104.8 A	102.8 A	1.9
Output power	72.5 kW	70.6 kW	2.7

B. LOOKUP TABLES CONSTRUCTION

Having validated the FE model, it can then be used to prepare the flux linkage lookup tables for the proposed methodology. As a first step, the lookup tables are prepared considering only three-phase armature windings and the main rotor field winding whilst ignoring the damper bars. At this stage, this is considered as an acceptable first step, as the first objective here is to validate the modelling technique at steady state. Taking only the four main windings of the generator into consideration, a lookup table for each winding's flux linkage is developed with respect to defined value sets of the four variables, which are the rotor electrical angle θ_e, i_f, I_s and γ. The sets of values defined for these independent variables of the lookup tables are stated in (7), (8), (9) and (10).

To take into account the high-order harmonics produced by the slotting effects on the analysed SG, 8 samples per slot pitch are considered and this results in the dataset shown in (7). The data points for I_s, γ and i_f in (8), (9) and (10) were selected according to a detailed sensitivity analysis aimed at identifying an optimal number of points that can cover all the nominal values of these variables ranging from no-load to rated load, also considering the most relevant power factor values, while keeping a high accuracy in any load condition.

$$\theta_e \in \{0 : 1.875 : 360\}^\circ \quad (7)$$
$$I_s \in \{0, 0.5, 1\} p.u \quad (8)$$
$$\gamma \in \{0, 30, 75\}^\circ \quad (9)$$
$$i_f \in \{0, 10, 20, 30, 40\} A \quad (10)$$

Using (7)-(10), the mentioned 4D lookup tables can be created. In the FE model, the field winding is supplied by an ideal DC current source with different field current values as in (10). For all the considered i_f values, the 3-phase armature windings were supplied by balanced sinusoidal currents with a fundamental frequency of 50Hz for various amplitude and phase angles corresponding to (8) and (9). Starting with the rotor's initial angular position at 0° (corresponding to t = 0 s), time-stepping simulations with the time samples corresponding to the rotor's electrical angular position as defined in (7) were performed for each combination of i_f, I_s and γ values to achieve flux linkages for all the 4D data points.

The preparation of the proposed lookup tables for the considered case study require 34 time-stepping FE simulations which took a total of around 4 hours of simulation time. This results in four, 4D lookup tables each consisting of 8685 data points. On the contrary, if the method proposed in [12] was to be adopted for similar data points for the considered SG, the process would had required 625 FE simulations (equivalent to 3 days). This would had resulted in four 5D lookup tables consisting of 120625 data points each. Therefore, it is evident that the time and memory space necessary for these lookup tables are significantly less than for the existing methodologies. A summary of the comparison between the method [12] and that proposed in this paper is highlighted in Table 4.

TABLE 4. Lookup tables comparison between the proposed and the existing methods.

Lookup tables details	Proposed method	Method in [12]	% reduction
No. of tables	4	4	0
Dimensions	4	5	20
Data points	8685	120625	92.8
No. of FE simulations	34	625	94.56
Preparation time	≈ 4 hours	≈ 68 hours (3 days)	94.56

C. MODEL IMPLEMENTATION

The whole modelling concept was then built and developed within the Simulink/MATLAB environment. A full representation of the model is shown in Fig. 8, where the output flux linkage produced by the lookup tables can be observed, as it is fed to the model of the windings' KVL equations (1) and (2). The instantaneous current values calculated by these KVL equations, implemented through the model of Fig. 8, were then looped back as an input to the lookup tables to find the instantaneous flux linkage. The developed model provides the flexibility of extending the model for further system level analysis. For example, a custom-modelled non-linear load can be connected to simulate its harmonic effects on the SG or sub-models such as an automatic voltage regulator (AVR) and/or an excitation system can be connected for higher level of system-integrated simulations.

D. ELECTROMAGNETIC TORQUE

To achieve a comprehensive technique that considers the full operation of the SG, a sub-model to produce instantaneous electromagnetic torque was added to the model. The approach for the electromagnetic torque was based on (11), which relates to the sum of induced electrical power divided by mechanical speed and no-load torque ($\tau_{cogg.}$).

$$\tau_e = \frac{E_a i_a + E_b i_b + E_c i_c + E_f i_f}{\omega_m} + \tau_{cogg.} \quad (11)$$

In (11), E_a, E_b, E_c and E_f are the instantaneous induced electromotive forces (EMFs) of the three phase stator windings and field winding, and ω_m is the mechanical angular speed of the rotor in rad/s. The first term of (11) is derived from the model of Fig. 8, via the instantaneous induced EMFs

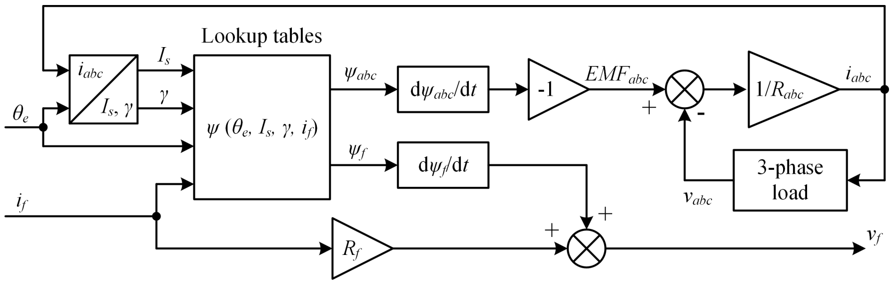

FIGURE 8. Proposed model developed in Simulink.

and winding currents. SGs have an electrically-excited rotor field. A resultant force occurs due to the interactions between the generated field and the geometry in the air-gap, which in other machine families is traditionally known as cogging torque. This is dependent on the field current and the rotor's angular position. To include an accurate representation of this effect, a lookup table of this no-load torque phenomena is built through the use of FE simulations The lookup table was constructed for the points of field currents and rotor positions as previously stated in (10) and (7), respectively. This involved taking torque calculation results from the no load FE simulations carried out previously for the lookup tables of flux linkages. Fig. 9 shows the developed Simulink model for the calculation of instantaneous electromagnetic torque.

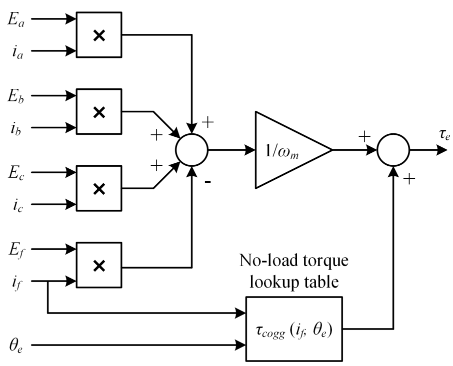

FIGURE 9. Simulink model for torque calculation.

VI. INTEGRATION OF SKEWING EFFECT

An important feature of this methodology is its inclusivity and flexibility. Likewise, having high number of integer samples per skewing angle can be useful in integrating the effect of skewing within the proposed model. Since the SG in this case has a skewing angle equal to its slot pitch, there are 8 samples per skewing angle. The flux linkage data acquired from the 2D FE model are shifted back and forth one sample at a time over a skewing angle and averaged as shown in (12) to get the effective flux linkages of the windings over its skewed axial core length [24]. This is a method usually applied on the post-processed quantities of 2D FE simulations to include the skewing effect as explained and validated in [24]. In this case, the same method is implemented internally on the flux linkage lookup tables to inherently produce an equivalent effect. In (12), N_s represents the number of samples per skewing angle.

$$\psi\left(\theta_n, i_f, I_s, \gamma\right) = \frac{1}{N_s} \sum_{k=-N_l/2}^{N_s/2} \psi\left(\theta_{n+k}, i_f, I_s, \gamma\right) \quad (12)$$

This highlights the potential of the proposed methodology even when compared to FE models. For example, to consider the skewing effect in FE models, one has to either run a 3D FE simulation [25] or perform multiple, multi-slice 2D FE simulations [26], [27]. The 3D FE simulation is far more time-consuming and expensive, and therefore multi-slice 2D FE simulation is usually preferred to approximate the skewing effect. Nevertheless, this requires multiple 2D simulations and consequently more time. However, the proposed model simulates a skewed machine without any extra increase in simulation time.

VII. PERFORMANCE EVALUATION

The methodology and the developed model described above are then evaluated in terms of accuracy and simulation time to test and quantify the method's capability to achieve the targets.

A. ACCURACY COMPARISON

To assert the methodology's accuracy in calculating the harmonic effects resulting from the machine's geometry and magnetic saturation, the model's simulation results were compared against those of the validated FE model and experimental measurements. For the sake of brevity, only the comparisons under no load and full load conditions at steady state operation are shown here, although the model is clearly capable of simulating any machine operation under balanced, symmetrical conditions.

The electromagnetic torque calculated by the developed analytical model was compared with the FE model's torque results only, since the experimental torque comprises ripples influenced by other external factors such as harmonics generated by the mechanical system and the DC motor drive used as a prime mover.

1) NO LOAD OPERATION

The model shown in Fig. 8 was run at no load, with a field current producing rated output voltage at the SG's stator terminals. The simulated induced voltage was then compared against that evaluated via experimental testing and the FE model. Fig. 10 shows the voltage waveforms comparison. From Fig. 10, it can be observed that the amplitude and the shape of the EMF waveform from the proposed model are very close to the waveforms obtained from the experiments and the FE model simulation.

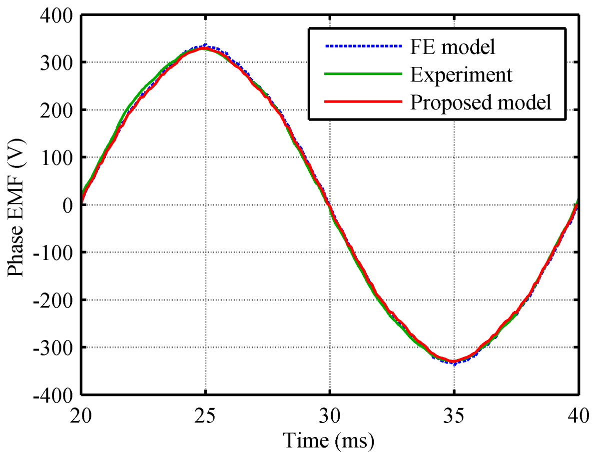

FIGURE 10. EMF waveform comparison.

2) FULL LOAD AT UNITY POWER FACTOR

The SG was operated under full load condition at unity power factor, with the AVR regulating the rated terminal voltage. The same field current produced by the AVR to generate the rated terminal voltage at full load was then used as the field current for the FE and the proposed models supplying rated power to a purely resistive load. The phase terminal voltage and current waveforms and their RMS values are compared in Fig. 11 and Table 5 respectively. Since it is a resistive load, the phase voltage and current waveforms will have the same harmonic content and this is shown and compared in Fig. 12.

As can be seen in Fig. 11 and Table 5, the waveforms from the proposed model are very similar to the experimental ones. This is also confirmed by Fig. 12, where it can be observed that the model is able to capture the most dominant harmonics of voltage and current waveforms. Even and third harmonics can be observed in the experimental spectrum (shown in green in Fig. 12), due to the fact that a slight unbalanced condition was registered during testing. On the other hand, the models are simulated under balanced three-phase conditions and this justifies the discrepancy when comparing even and third harmonics. It can be seen from Fig. 10, Fig. 11, Fig. 12 and Table 5 that the proposed model is as accurate as the FE model with both having similar percentage errors when compared against the experimental values. Table 5 also shows the same accuracy level between the proposed model and the model developed via the method in [12]. This proves that the proposed technique gives the same accuracy using considerably fewer data points for the lookup tables.

The electromagnetic torque produced by the analytical model was compared against the FE-evaluated torque results

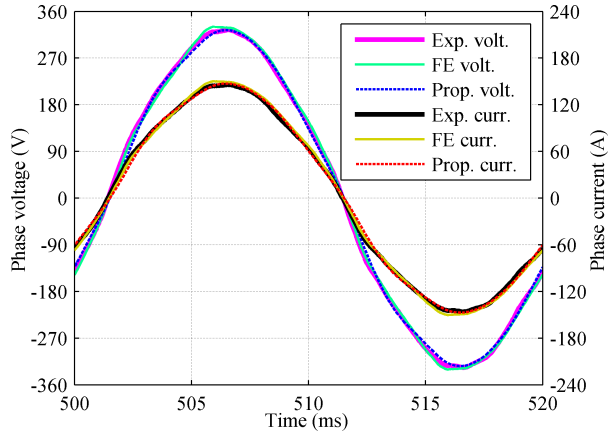

FIGURE 11. Phase voltage (volt.) and current (curr.) waveforms at full load between the experiment (Exp.), FE model (FE) and the developed model (Prop.).

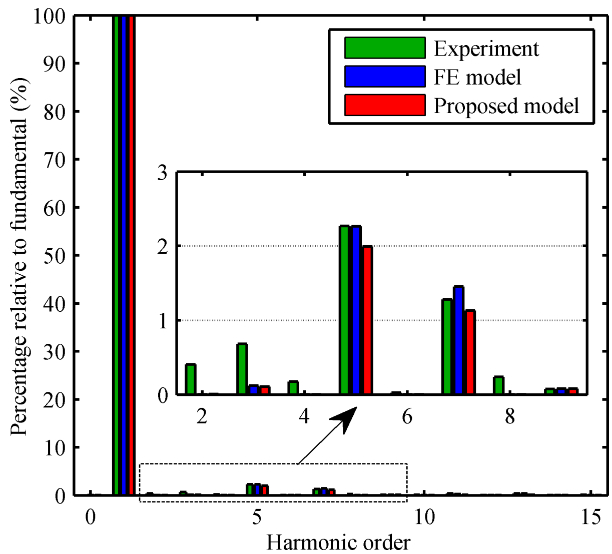

FIGURE 12. Comparison between model results and experimental measurements in terms of current and voltage harmonic content.

TABLE 5. Comparison of RMS values at full load.

Quantities	Proposed model	Method in [12]	FE model	Experiment
Field current	34 A	34 A	34 A	34 A
Phase voltage	226.2 V	225.9 V	230.7 V	229 V
Phase current	102.8 A	102.7 A	104.8 A	102.8 A
Output power	69.8 kW	69.6 kW	72.5 kW	70.6 kW

under the same full load operating condition. To ensure a fair comparison, the FE model of an un-skewed SG was used with the original un-skewed analytical model of the SG [8]. Fig. 13 compares the electromagnetic torque waveforms between the two models. It also shows a zoomed graph of the steady state torque for one time period of the lowest ripple frequency.

It is evident from Fig. 13 that the analytical model has successfully captured the shape and frequencies of the torque ripple to a large extent when compared against the FE model's

TABLE 6. Simulation time per electrical cycle.

Models	Simulation time per cycle
FE model	≈780 s
Proposed model with cubic spline interpolation	≈6 s
Proposed model with linear interpolation	≈1 s

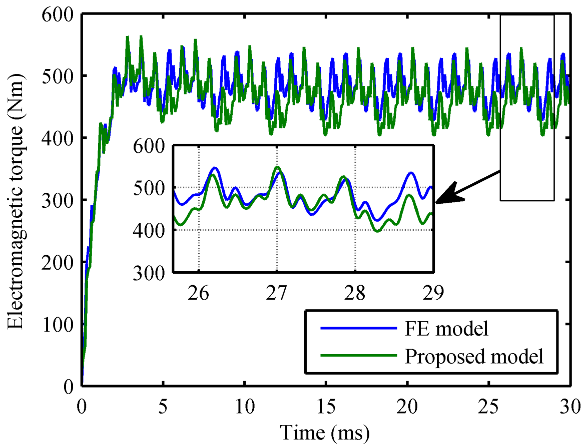

FIGURE 13. Torque comparison at full load.

torque waveform. The accuracy of the results can be even further improved with a higher resolution of the lookup tables.

It is also important to highlight that the model has proven to be capable of simulating different steady state balanced load conditions at synchronous speed with similar accuracy as shown in this paper, but these are not presented here due to space limitations. Therefore, the proposed modelling approach is highly reliable for simulating SGs under any balanced load operations.

B. SIMULATION TIME COMPARISON

After the accuracy of the developed analytical model was successfully validated, the next step consists in emphasizing the computational benefits of the proposed methodology as compared to the FE model. Both the FE and the proposed models were simulated with 192 time steps per electrical cycle. Table 6 shows the time taken to simulate one complete time period of 20ms for the models on an Intel i3 3.3GHz CPU with 4GB RAM with no other program running during the simulation. This proves that the proposed model (with cubic spline interpolation) is 130 times faster than the developed FE model. Also, the 2D FE model simulation showed the results without the skewing effect and would require an extra bit of time and effort to include the skewing effect manually. This is not the case with the proposed model as the simulation time mentioned for it includes the skewing effect.

Although it is preferable to interpolate the non-linear electromagnetic relations represented by the lookup tables using cubic spline method, a reasonably good accuracy has also been observed using linear interpolation technique, with the simulation time further reduced to less than 1 second. Therefore, the proposed model confirmed to be extremely faster than the FE model in simulation and also provides the flexibility to select the interpolation technique according to the need of improving either simulation time or accuracy.

VIII. CONCLUSION

The proposed model has been shown to achieve very accurate results when compared to the FE and experimental results. Its maximum error against experimental results is 1.2%. When comparing the operational requirements of the model relative to FE, the simulation time under cubic spline interpolation was 130 times faster.

From the modelling perspective, the time and memory space required for the development of the lookup tables were reduced by 94.56% and 92.8% respectively when compared to existing methods [12], without compromising at all the accuracy. This makes the proposed model a very feasible and practical option for analysing SGs. It is also a very simple, flexible and easy-to-use model and it can easily be expanded to calculate torque and further relevant quantities. Due to its flexibility, certain aspects which traditionally incur complex analysis processes (such as skewing via multi-slice 2D or even 3D simulations) can be easily integrated within the model.

Although the model initially requires some FE simulations for its development, it is still worth using it for SGs' analyses due to the following reasons:

- It is useful for applications where the number of electrical cycles required for analytical simulations are more than the simulations required to develop the model.
- It can be developed on any generic mathematical modelling package (eg. MATLAB). This allows to integrate it with the models of other components of the generation system, such as AVR controller and excitation system to create a whole integrated model of a genset.

Considering the last point, the natural future development of this research is that of building a comprehensive system model which includes the main generator, the excitation system and an AVR. It is perceived that the role of the classical SG will change in the near future. This is due to the ever-increasing demands in terms of grid compliance, power quality, efficiency, power density, etc. Therefore, SGs and, more in general, generation-set manufacturers would significantly benefit from such fast, accurate and comprehensive modelling methodologies.

REFERENCES

[1] T. A. Lipo, *Analysis of Synchronous Machines*, 2nd ed. New York, NY, USA: Taylor & Francis, 2012.
[2] S. Nadarajan, S. K. Panda, B. Bhangu, and A. K. Gupta, "Hybrid model for wound-rotor synchronous generator to detect and diagnose turn-to-turn short-circuit fault in stator windings," *IEEE Trans. Ind. Electron.*, vol. 62, no. 3, pp. 1888–1900, Mar. 2015.
[3] M. Galea, G. Buticchi, L. Empringham, L. de Lillo, and C. Gerada, "Design of a high-force-density tubular motor," *IEEE Trans. Ind. Appl.*, vol. 50, no. 4, pp. 2523–2532, Jul./Aug. 2014.

[4] A. M. Mohammed, T. Cox, M. Galea, and C. Gerada, "A new method for determining the magnetic properties of solid materials employed in unconventional magnetic circuits," *IEEE Trans. Ind. Electron.*, vol. 64, no. 3, pp. 2468–2475, Mar. 2017.
[5] K. Bersch, P. H. Connor, C. N. Eastwick, M. Galea, and R. Rolston, "CFD optimisation of the thermal design for a vented electrical machine," in *Proc. IEEE Workshop Elect. Mach. Design, Control Diagnosis (WEMDCD)*, Apr. 2017, pp. 39–44.
[6] X. Sun, L. Chen, H. Jiang, Z. Yang, J. Chen, and W. Zhang, "High-performance control for a bearingless permanent-magnet synchronous motor using neural network inverse scheme plus internal model controllers," *IEEE Trans. Ind. Electron.*, vol. 63, no. 6, pp. 3479–3488, Jun. 2016.
[7] S. Nuzzo, P. Bolognesi, M. Galea, and C. Gerada, "A hybrid analytical-numerical approach for the analysis of salient-pole synchronous generators with a symmetrical damper cage," in *Proc. IEEE Int. Electr. Mach. Drives Conf. (IEMDC)*, May 2017, pp. 1–8.
[8] S. Nuzzo, M. Degano, M. Galea, C. Gerada, D. Gerada, and N. Brown, "Improved damper cage design for salient-pole synchronous generators," *IEEE Trans. Ind. Electron.*, vol. 64, no. 3, pp. 1958–1970, Mar. 2017.
[9] S. Nuzzo, M. Galea, C. Gerada, and N. Brown, "Analysis, modeling, and design considerations for the excitation systems of synchronous generators," *IEEE Trans. Ind. Electron.*, vol. 65, no. 4, pp. 2996–3007, Apr. 2018.
[10] O. A. Mohammed, S. Liu, and Z. Liu, "Physical modeling of PM synchronous motors for integrated coupling with machine drives," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1628–1631, May 2005.
[11] M. Mohr, O. Bíró, A. Stermecki, and F. Diwoky, "An improved physical phase variable model for permanent magnet machines," in *Proc. 20th Int. Conf. Elect. Mach.*, Sep. 2012, pp. 53–58.
[12] L. Quéval and H. Ohsaki, "Nonlinear abc-model for electrical machines using N-D lookup tables," *IEEE Trans. Energy Convers.*, vol. 30, no. 1, pp. 316–322, Mar. 2015.
[13] X. Chen, J. Wang, B. Sen, P. Lazari, and T. Sun, "A high-fidelity and computationally efficient model for interior permanent-magnet machines considering the magnetic saturation, spatial harmonics, and iron loss effect," *IEEE Trans. Ind. Electron.*, vol. 62, no. 7, pp. 4044–4055, Jul. 2015.
[14] M. N. Ibrahim, P. Sergeant, and E. M. Rashad, "Relevance of including saturation and position dependence in the inductances for accurate dynamic modeling and control of SynRMs," *IEEE Trans. Ind. Appl.*, vol. 53, no. 1, pp. 151–160, Jan./Feb. 2017.
[15] S. Li, D. Han, and B. Sarlioglu, "Modeling of interior permanent magnet machine considering saturation, cross coupling, spatial harmonics, and temperature effects," *IEEE Trans. Transport. Electrific.*, vol. 3, no. 3, pp. 682–693, Sep. 2017.
[16] D. E. Pinto, A.-C. Pop, J. Kempkes, and J. Gyselinck, "dq0-modeling of interior permanent-magnet synchronous machines for high-fidelity model order reduction," in *Proc. Int. Conf. Optim. Elect. Electron. Equip. (OPTIM) Int. Aegean Conf. Elect. Mach. Power Electron. (ACEMP)*, May 2017, pp. 357–363.
[17] S. Zarate, G. Almandoz, G. Ugalde, J. Poza, and A. J. Escalada, "Extended DQ model of a permanent magnet synchronous machine by including magnetic saturation and torque ripple effects," in *Proc. IEEE Int. Workshop Electron., Control, Meas., Signals Appl. Mechatronics (ECMSM)*, May 2017, pp. 1–6.
[18] H. Chen, Y. Long, and X. P. Zhang, "More sophisticated synchronous machine model and the relevant harmonic power flow study," *IEE Proc.-Gener., Transmiss. Distrib.*, vol. 146, no. 3, pp. 261–268, May 1999.
[19] M. Ladjavardi, M. A. S. Masoum, and S. M. Islam, "Impact of time and space harmonics on synchronous generator load angle," in *Proc. IEEE PES Power Syst. Conf. Expo.*, Oct./Nov. 2006, pp. 1132–1138.
[20] O. Rodriguez and A. Medina, "Efficient methodology for stability analysis of synchronous machines," *IEE Proc.-Gener., Transmiss. Distrib.*, vol. 150, no. 4, pp. 405–412, Jul. 2003.
[21] E. Deng and N. A. O. Demerdash, "A coupled finite-element state-space approach for synchronous generators. I. model development," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, no. 2, pp. 775–784, Apr. 1996.
[22] A. Z. Gbégbé, B. Rouached, J. C. M. Bergeron, and P. Viarouge, "Damper currents simulation of large hydro-generator using the combination of FEM and coupled circuits models," *IEEE Trans. Energy Convers.*, vol. 32, no. 4, pp. 1273–1283, Dec. 2017.
[23] X. Sun, K. Diao, G. Lei, Y. Guo, and J. Zhu, "Real-time HIL emulation for a segmented-rotor switched reluctance motor using a new magnetic equivalent circuit," *IEEE Trans. Power Electron.*, to be published.
[24] S. Nuzzo, M. Galea, C. Gerada, and N. Brown, "A fast method for modeling skew and its effects in salient-pole synchronous generators," *IEEE Trans. Ind. Electron.*, vol. 64, no. 10, pp. 7679–7688, Oct. 2017.
[25] K. Yamazaki and M. Matsumoto, "3-D finite element meshing for skewed rotor induction motors," *IEEE Trans. Magn.*, vol. 51, no. 3, Mar. 2015, Art. no. 7401704.
[26] D. Zhang, L. Bu, C. He, R. An, and T. Wu, "A modified 2-D multislice FEM for computing the airgap flux density of induction motor with skewed slots," in *Proc. IEEE Int. Electr. Mach. Drives Conf. (IEMDC)*, May 2017, pp. 1–8.
[27] Z. Shi, X. Sun, Y. Cai, Z. Yang, G. Lei, Y. Guo, and J. Zhu, "Torque analysis and dynamic performance improvement of a PMSM for EVs by skew angle optimization," *IEEE Trans. Appl. Supercond.*, vol. 29, no. 2, pp. 1–5, Mar. 2019.

QUADIR H. QUADRI received the B.E. degree in electrical engineering from the NED University of Engineering and Technology, Karachi, Pakistan, in 2009, and the M.Sc. degree in power electronics and drives from the University of Nottingham, Nottingham, U.K., in 2012, where he is currently pursuing the Ph.D. degree in electrical engineering.

His research interests include modeling, analysis, and design of electrical machines with his current research projects focused on salient-pole synchronous generators.

STEFANO NUZZO (M'18) received the B.Sc. and M.Sc. degrees in electrical engineering from the University of Pisa, Pisa, Italy, in 2011 and 2014, respectively, and the Ph.D. degree in electrical engineering from the University of Nottingham, Nottingham, U.K., in 2018. He is currently a Research Fellow with the Power Electronics, Machines and Control (PEMC) Group, University of Nottingham.

Since January 2019, he has been a Research Fellow with the Department of Engineering "Enzo Ferrari," University of Modena and Reggio Emilia, Modena, Italy. His research interests are the analysis, modeling, and optimizations of electrical machines, with focus on salient-pole synchronous generators and brushless excitation systems for industrial power generation applications. He is also involved in a number of diverse projects related to more electric aircraft initiative and associated fields.

Dr. Nuzzo is a member of the IEEE Industrial Electronics Society (IES) and the IEEE Industry Applications Society (IAS). He constantly serves the scientific community as a reviewer for several journals and conferences.

MOHAMED RASHED (M'07) received the Ph.D. degree in electrical motor drives from the University of Aberdeen, Aberdeen, U.K., in 2002.

He was a Postdoctoral Research Fellow with the Department of Engineering, University of Aberdeen, from 2002 to 2005 and from 2007 to 2009. In 2005, he was appointed as an Assistant Professor with the Department of Electrical Engineering, Mansoura University, Mansoura, Egypt, and then on leave from 2007. In 2008, he was promoted to an Associate Professor with Mansoura University, Egypt. In 2009, he joined the Power Electronics, Machines and Control Research Group, Department of Electrical and Electronic Engineering, The University of Nottingham, Nottingham, U.K., where he is currently a Senior Research Fellow. His current research interests include the design and control of electrical motor drives and power systems for aerospace applications, power electronics for micro grids, renewable energy sources, and energy storage systems.

CHRIS GERADA (M'05–SM'12) received the Ph.D. degree in numerical modeling of electrical machines from the University of Nottingham, Nottingham, U.K., in 2005.

He was a Researcher with the University of Nottingham, working on high-performance electrical drives and on the design and modeling of electromagnetic actuators for aerospace applications. Since 2006, he has been the Project Manager of the GE Aviation Strategic Partnership. In 2008, he became a Lecturer of electrical machines, an Associate Professor, in 2011, and a Professor, in 2013 with the University of Nottingham. His main research interests include the design and modeling of high-performance electric drives and machines.

Prof. Gerada is the Chair of the IEEE Industrial Electronic Society Electrical Machines Committee. He serves as an Associate Editor for the IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS.

MICHAEL GALEA (M'13–SM'18) received the Ph.D. degree in electrical machines design from the University of Nottingham, Nottingham, U.K., in 2013.

He was appointed as a Lecturer, in 2014, as an Associate Professor, in 2018, and as a Professor in Electrical Machines and Drives, in 2019, all with the University of Nottingham. He currently lectures in electrical machines and drives and in aerospace systems integration and manages a number of diverse projects and programmes related to the more/all electric aircraft, electrified propulsion, and associated fields. His main research interests include design and development of electrical machines and drives (classical and unconventional), reliability and lifetime degradation of electrical machines, and the more electric aircraft.

Dr. Michael is a Fellow of the Royal Aeronautical Society and sits on the Executive Board of the UK Magnetics Society. He also serves as an Associate Editor for the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS and for the *IET Electrical Systems in Transportation*.