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Abstract

We propose a mechanism design approach to study the role of a mediator in dispute resolution and bargaining. The mediator provides a buyer and a seller with "reality checks" by controlling the information each party has about her own value for a transaction, and proposes a price at which trade can occur if parties agree. We first consider the class of static information disclosure and trading mechanisms, in which the mediator simultaneously selects the information disclosed to the parties and posts the price at which they can trade. We characterize the mechanism that maximizes the ex-ante gains from trade. We show it is optimal to restrict agents' information, as this allows to increase the volume of trade and complete some of the most valuable trades that are lost in the welfare maximizing mechanism under full information. We then study the value of the mediator engaging in "shuttle diplomacy" by considering a class of dynamic information disclosure and trading mechanisms, and show that it is possible to design a dynamic mechanism that achieves ex-post efficiency. Shuttle diplomacy facilitates trade by allowing the mediator to condition information releases and prices posted on the history of feedbacks she receives from the parties during her meetings with them.

Keywords: Bargaining, Information design, Mechanism Design, Mediation, Persuasion

JEL Classification Numbers: C72, D47, D82, D86.

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1 Introduction

In April of 2019, U.S. District Court Judge Vince Chhabria appointed Ken Feinberg to facilitate a settlement between Bayer and over 13,000 plaintiffs who had alleged that the weed killer Roundup causes non-Hodgkin's lymphoma due to its active ingredient, glyphosate. Feinberg is an expert mediator who had previously mediated many disputes, including settlement of the 9/11 victims fund and BP Deepwater Horizon disaster. Unlike, say, in arbitration, mediators are not allowed to impose a judgment on the parties. The mediator seeks to bring about a mutually-agreed upon resolution. The dynamic process of mediation has been described as follows:

> "Mediations are in the main assisted negotiations. The mediator goes back and forth with demands and offers (and counter demands and counter offers), while, at the same time, the mediator asks questions and makes comments about the dispute, so that each side can more objectively and realistically consider the facts and think about what might happen if the case were to go to a verdict. Through this process, the "demands" for settlement by the plaintiffs tend to become more realistic, and the offers of the defendants, similarly, tend to increase."[1]

Mediators often provide two services. First, in communicating with parties they acquire information that is privately known to one party and selectively transmit it to the other party. Second, the mediator (usually a retired judge or well-experienced lawyer) brings her own expertise to bear on the question of case value. The mediation literature often labels these two roles as facilitative versus evaluative mediation styles. See for example Smartsettle (2017): "The mediator who evaluates assumes that the participants want and need her to provide some guidance as to the appropriate grounds for settlement based on law, industry practice or technology – and that she is qualified to give such guidance by virtue of her training, experience, and objectivity .... the facilitative mediator assumes that her principal mission is to clarify and to enhance communication between the parties in order to help them decide what to do." As

[1] National Arbitration and Mediation (2019).

pointed out by Carbone (2019): "The best mediators will use an approach that draws upon both styles as the needs of the case require." As an evaluator, the mediator helps parties to assess the merits of the case and provides them with "reality checks,"often engaging in "shuttle diplomacy," meeting with each side in a private caucus. She may at the same time act as a facilitator, collecting information from a party and then leaking some of it to the other side.

In lawsuit mediation the mediator's expertise and role as an evaluator is especially important if the case involves issues of first impression, like the Roundup litigation; that is, new legal issues or interpretations brought before a court that have not been addressed before by that court or that court's jurisdiction. There, as part of the settlement, the plaintiffs want Bayer to label Roundup as including a cancer causing ingredient. Yet the EPA has repeatedly found glyphosate to be safe; the conflict between the proposed settlement and the regulator's explicit opposition sits in unchartered legal territory. In issues of first impression, the mediator often has access to information that parties don't have. How so? The mediator has been involved in other mediations, many of which are not in the public domain. She can thus help to determine costs and benefits for the plaintiff's claim – costs and benefits that the parties themselves can't identify without his help. In the Roundup litigation, for example, these costs and benefit relate to the likely outcomes of the litigation process.

In this paper we examine the mediator's role in providing expert advice and helping parties to reach a settlement. The parties have some beliefs about the value of the case, but they do not know that value for sure. The mediator, then, sets up an information disclosure policy: what to say about the case, to whom in a private session and at what point in the negotiations? This policy tracks the Bayesian persuasion models: it allows revelation of information that is Bayes consistent with the prior. As in the mechanism design literature, the mediator also proposes the terms of a deal. In short, our mediator is both a mechanism designer and an information designer.

Lawsuit mediation is one application of the model, but there are many others. Consider for example the informal role of the US in the Camp David resolution of the Egypt/Israel conflict. What should President Carter have revealed to each side and at what point in the negotiation process? President Carter had access to information unknown to both parties (about the US's position, and the views of the broader international community). Informal mediation has a long tradition across different cultures; in China it goes back to Confucianism and recently "top political-legal authorities of the Chinese Communist Party have been promoting mediation as the key to resolving all disputes" (Pissler, 2013). Disputes which formal or informal mediation is commonly used to resolve include: conflict between members of a family or business relationship, conflict in the workplace, conflict arising out of a commercial transaction including cross border trade, real estate matters and personal property.

To facilitate comparison with the literature on bargaining/mechanism design (e.g., Myerson and Satthertwaite, 1983; Chatterjee and Samuelson, 1983), we cast our analysis in terms of a commercial transaction in which the holder of an asset or a claim, the seller, faces a potential buyer. Neither the buyer nor the seller have precise information about their valuation for the asset. The mediator then provides some appropriately selected information to each party, on which basis they both refine their beliefs about their valuations. We can say she provides them with reality checks and then proposes a transaction price. The mediator may also engage in shuttle diplomacy by meeting the parties in several rounds, in each round both releasing some information and obtaining some feedback from them.

One can interpret the mechanism designer in Myerson and Satthertwaite (1983), or coordinator as they refer to her, as a mediator. However there, as well as in the models in law and economics that study the role of mediation (e.g., Brown and Ayres, 1994, Doornik, 2014, and Goltsman et al., 2009), parties are assumed to have full information about their values at the outset. The coordinator then only plays a facilitative role, collecting information from both parties and distributing it strategically via a proposed trading deal, which might convey to a party that she had a relatively optimistic view of her bargaining position.

Mediators do play this facilitative role in practice and we maintain it in our model, but we go a step further and allow the mediator also to provide parties directly with reality checks by transmitting information that they don't have. In our model the mediator is not just a facilitator, but is also an expert evaluator who runs experiments providing parties with information. We maintain, however, the assumption that the true values of the parties are a private matter. The mediator privately and independently informs each party about different features of the asset and each party updates her valuation depending on how much she values the different features and this is the party's private information. An alternative interpretation of this role of the mediator is also possible: each party can directly acquire information about her value, but the mediator monitors and enforces the parties' commitment to discover the prescribed – limited – amount of information.

As in the Bayesian persuasion literature, we give extended leeway to the mediator about how much the parties discover about their values. Our first main result concerns the amount of information that a mediator, aiming to maximize the ex-ante gains from trade, should provide to parties: some amount of obfuscation is optimal. This is established by considering first static procedures, or mechanisms, in which the mediator simultaneously selects the information received by the parties and the price at which they may trade. We show that the optimal static mechanism restricts the agent's information to be monotone binary partitions (i.e., let the parties discover whether their value is above or below a threshold). Providing traders with less than full information increases the volume of trade. This has two effects on welfare. First, some ex-post efficient trades take place that would not be completed under full information. Second, some ex-post inefficient trades are also completed that would not occur under full information. If information is properly selected as described above, the first effect dominates. Hence the ex-ante gains from trade, or welfare, are higher than in any static mechanism when buyer and seller are provided with full information. We show that they can also be higher than in the welfare maximizing Bayesian mechanism under full information;[2] a fitting testament to the power of controlling the information flow.

Our second novel result both relative to the persuasion and the mechanism design literature is that restricting attention to static procedures is *with* loss of generality.

[2] In that case (considered by Myerson and Satterthwaite and the mechanism design literature which followed), as mentioned above, the mediator can ask parties to simultaneously report their value and determine the terms of trade accordingly.

In particular, when the information available to parties over their valuations is exogenously given, by the revelation principle static, direct mechanisms can describe any outcome; there is no scope for using dynamic mechanisms. This is not the case in our model. We show that ex-post efficiency (and hence maximal ex-ante gains from trade) can always be achieved with a dynamic procedure, characterized by a sequence of information discoveries by parties and of trading prices. There is so a clear rationale for shuttle diplomacy. In contrast, this is not possible with the optimal static procedure we derive (nor with the optimal Bayesian mechanism with full information). Our findings so provide a clear rationale for shuttle diplomacy.

To understand the value of the mediator engaging in shuttle diplomacy, we should point out that a dynamic procedure allows to condition the information that is released to each party, as well as the price posted at any point in time, on the history of feedbacks received from parties about the information they learnt. As a consequence, we show it is possible to provide parties with the minimum amount of information that is needed to determine whether or not trade is ex-post efficient. Limiting the information available to parties, as argued above, improves agents' incentives and facilitates trade. Incentives are now also improved by the feature that agents' reports affect not only the terms of trade proposed by the mediator but also the subsequent information releases to all parties.

More precisely, we consider a class of dynamic procedures where the mediator progressively reveal to the seller (resp. the buyer) whether her value is equal or below (resp. above) a moving threshold. At any point in time along the sequence each party is given the chance to report to have become fully informed. When that happens, the other party is only additionally informed of whether her value is above or below the value reported by the first party. The price proposed by the mediator is such that both parties gain from trade if it is efficient to trade at the reported value. This sequence of information releases based on parties' reports allows to correlate the information that is made available to the parties. We show that we can always find a path for the speed for the learning process of the two parties and for the prices set by the mediator such that it is incentive compatible for each party to report a discovery of her value if and only if that happens. Given this, whenever a price is selected by the mediator, each party knows whether her value is above or below it and hence they agree to trade whenever trade is ex-post efficient. This ensures that it is always possible to design an ex-post efficient shuttle diplomacy mechanisms.

Related literature. First, from a substantive point of view the paper contributes to the law and economics literature on mediation. It shows that some of the characteristics of a common institutional practice – mediation – minimize the efficiency loss associated with dispute resolution. As we already pointed out, law and economics papers on mediation rely on asymmetric information, but assume that parties have full information about their values. For example, Brown and Ayres (1994) argue that mediators reduce adverse selection by committing parties to simple mechanisms; e.g., "(1) by committing parties to break off negotiations when private representations to a mediator indicate that there are no gains from trade; (2) by committing parties to equally divide the gains from trade; and (3) by committing to send noisy translations of information disclosed during private caucuses." Doornik (2014) argues that the role of a mediator is to avoid a costly trial by verifying the private information of the informed party and communicating it to the other party, without disclosing confidential details that would disadvantage the informed party. Goltsman et al. (2009) studied mediation in a cheap talk framework and showed that by adding noise a mediator may relax the incentive compatibility constraint of the informed party and thus facilitate information transmission. While insightful, these models cannot explain either the evaluative role of the mediator, or the benefits of the mediator engaging in shuttle diplomacy.

Second, from a substantive and methodological point of view, the paper contributes to the literature on mechanism design and the literature on Bayesian persuasion introduced by Kamenica and Gentzkov (2011) (see also Rayo and Segal, 2010, Kolotilin et al., 2017 and Li and Norman, 2018) and recently reviewed by Bergemann and Morris (2019). As the information designer in the persuasion literature, our mediator is able to commit to an information structure that maps states of the world (a party's value) into stochastic signals privately disclosed (to each party) and not observed by the mediator. In addition, and also in common with the information designer of the persuasion literature, our mediator knows the prior value distributions and may use all feasible information disclosure policies. But in contrast to the information designer of the persuasion literature, who cannot affect the outcomes available to the players, and like the designer in the classic mechanism design literature (e.g., see Myerson and Satterthwaite, 1983), our mediator may also affect outcomes, by setting the price at which parties may trade. Thus, our mediator plays both the role of an information designer in the persuasion literature, and the classical designer in the mechanism design literature (see Mezzetti, 2019, for a brief discussion on this dual role of a designer).

This feature is shared by our paper with the literature on information disclosure and surplus extraction by the seller of a single item, where the seller chooses both an information and a sale policy (Bergemann and Pesendorfer, 2007, Esö and Szentes, 2007, Li and Shi, 2017 and Krämer, 2018).[3] There are several differences, however, apart from the objective of our mediator being to maximize welfare. First, in our paper there is private information on both sides of the market and an information structure must be chosen for both. In contrast, in the papers cited above there is no uncertainty regarding the seller's value but there could be several buyers and the focus is then on the information of each of them. Second, in the static disclosure setting we focus on simple price posting mechanisms, rather than "rich contracting protocols" (Krähmer, 2018).[4] Third, and most important, as opposed to the single-round disclosure policies of most of the surplus extraction literature, our main contribution is to study a dynamic shuttle diplomacy procedure and show that the sequence of disclosures and trade opportunities allow to significantly increases the set of outcomes the mediator

[3] The combination of information and mechanism design is also present in Roesler and Szentes (2017) and Condorelli and Szentes (2019), but in those papers it is the buyer that selects her own information structure so as to protect herself from the seller's choice of a sales mechanism aiming to maximize her surplus.

[4] Given the timing assumption, in our static procedure the trading mechanism can only be a posted price. In this regard, it is useful to point out that when agents are fully informed about their values, the only dominant strategy mechanisms that balance the budget at all signal realizations (i.e., such that the law of one price holds) and satisfy agents' participation constraints ex-post are price posting mechanisms in which the mediator posts a single price at which trade may take place (see Hagerty and Rogerson, 1987, Čopič and Ponsati, 2016, and Čopič, 2017; for the robustness of dominant strategy mechanisms, see Bergemann and Morris, 2005).

may achieve; in particular, it allows implementation of the first best outcome.

We are only aware of two exceptions to single-round disclosures in the surplus extraction literature. The first is Bergemann and Wambach (2015); they introduce an auction with sequential disclosure of information in which each bidder, like the buyer in our shuttle diplomacy mechanism, learns a progressively higher lower bound on her value for the item for sale. They show that such auction implements the outcome of the handicap auction in Esö and Szentes (2007) while strengthening the bidders' participation constraints from interim to ex post. The second is Heumann (2019), in which the buyer observes the realization of a signal process in continuous time and makes then a report to the seller at each point in time. The paper characterizes the profit-maximizing mechanism when the seller can choose the information content of the signal process, the quantity to be sold, and the price to be charged, as functions of the entire history of reports by the buyer.

The paper is organized as follows. Section 2 introduces the setting. Section 3 studies the role of the mediator in providing reality checks by examining static procedures and characterizing the optimal one. Section 4 considers sequential disclosures and the role of shuttle diplomacy, establishing the result that all gains from trade can be realized. Section 5 contains some extensions and Section 6 concludes. All the proofs that are omitted in the main text can be found in the Appendix.

2 The Setting

We analyze a stylized model, so as to extract the main insights about the role a mediator plays in dispute resolution and, more generally, in bargaining. We consider a commercial transaction in which the holder of an asset, the seller, faces a potential buyer, but the model could be applied more generally. The two parties use a mediator to help them to discover their benefits of making the transaction and to set the price at which it may take place.

The environment is a standard one, with one indivisible object and a seller and a buyer with independent private values. Let $v^B \in [0, 1]$ be the buyer's value from acquiring the asset and $v^S \in [0,1]$ be the seller's cost, or value lost, from giving away the asset she holds. Let $F_0(v) = \Pr(\text{value} \leq v)$ and $G_0(v) = \Pr(\text{cost} \leq v)$ be the distributions from which value and cost are independently drawn. We assume that F_0 and G_0 have no atoms and admit strictly positive everywhere densities f_0 and g_0; the expectations according to these distributions are $v_0^B = \int_0^1 v dF_0(v)$ and $v_0^S = \int_0^1 v dG_0(v)$. Prior to the experiments designed by the mediator, neither the buyer nor the seller or the mediator have any information regarding the draws from F_0 and G_0.

The mediator chooses both the trading procedure and the information disclosure policy. We consider here the case where the mediator is benevolent and his goal is to maximize the gains from trade. The mediator's disclosure policy is subject to a 'privacy constraint': it is given by two sets of experiments, one that provides the buyer some information (only) about her value and the other providing the seller some information (only) about her cost. Otherwise, we allow the mediator to use the most general signaling technology; that is, as in the literature on Bayesian persuasion (e.g., see Kamenica and Gentzkow, 2011, and Bergemann and Morris, 2019) the mediator is free to choose any disclosure policy that is consistent with the prior distribution. Thus, our results should be viewed as providing an upper bound on the worth of a mediator in dispute resolution.[5]

We can think of the disclosure policy and the privacy constraint as describing a situation where the mediator privately and independently informs each of the two parties about some characteristics of the object. The way in which each party updates her valuation depends on how much she values the different characteristics and this is the party's private information. As we pointed out in the introduction, an alternative interpretation is that the information is directly acquired by the parties and the mediator monitors and enforces their commitment to obtain the prescribed, typically incomplete, amount of information.

[5] In future research, it would be valuable to explore additional practical constraints on the signaling technology available to a mediator.

3 Reality Checks: Static Mechanisms

In this section we focus on the role of the mediator as providing reality checks to the parties. To this end we examine the case in which the information disclosure and the trading mechanism are static: the mediator simultaneously chooses the distribution of the signals received, respectively, by the buyer and the seller, as well as a price p. The buyer and the seller, after observing the realization of their own signal, decide then whether or not they wish to trade at the price p.

The analysis of this case provides a useful benchmark and starting point, that allows to clearly illustrate how limiting the information available to parties may serve to mitigate incentives and facilitate trade. Given this, the consideration of a simple trading mechanism, where a single price is selected at the same time the signal distributions of the two parties are chosen, makes the analysis more transparent. As we will see, what proves to be a real limitation for the attainable outcomes is the fact that the information is provided to parties only once and simultaneously. In the next section, we will extend the analysis to the case where the mediator also engages in shuttle diplomacy, allowing for a sequence of information disclosures to the parties, and of associated prices posted, based on the reports sent by parties regarding the information received at previous rounds. We will show that in that case all ex-post gains from trade can be realized. In contrast, this is not possible with static information disclosures even if, as we argue later, we allow for Bayesian mechanisms, where the terms of trade are set *after* the parties receive information, on the basis of the reports sent by them.

Without loss of generality, the signal that an agent receives can be interpreted as an unbiased estimate of her private value or cost. As the mediator decides the signal structure but does not observe the realization of the signal, to keep track of the information disclosed from the point of view of the mediator, the signal itself is irrelevant; it is sufficient to consider the induced posterior distribution of the expected value and cost. Consider the buyer. The situation in which the buyer has not been disclosed any information corresponds to the case in which the posterior distribution of the value has an atom of mass one on v_0^B, while the situation in which the buyer has been fully disclosed her value corresponds to the case in which, from the point of view of the mediator (and the seller), the posterior distribution is F_0, the true distribution from which the value is drawn. Intermediate disclosure policies must lead to distributions F that are mean preserving spreads of the distribution with unit mass on v_0^B and such that F_0 is a mean preserving spread of F.

Similarly, the situation in which the seller has not been disclosed any information corresponds to the case in which the posterior distribution of the cost has an atom of mass one on v_0^S and the situation in which the seller has been fully disclosed her cost corresponds to the case in which the posterior distribution is the true distribution G_0. Intermediate disclosure policies lead to distributions G that are mean preserving spreads of the distribution with unit mass on v_0^S and such that G_0 is a mean preserving spread of G.

Thus, the families of signal distributions over the buyer's value and the seller's cost that can be feasibly induced by the mediator are:[6]

$$\mathcal{F} = \left\{ F : \int_0^1 v dF(v) = v_0^B \text{ and } \int_0^z F(v)dv \leq \int_0^z F_0(v)dv \text{ for all } z \in [0,1] \right\}$$
$$\mathcal{G} = \left\{ G : \int_0^1 v dG(v) = v_0^S \text{ and } \int_0^z G(v)dv \leq \int_0^z G_0(v)dv \text{ for all } z \in [0,1] \right\}$$

If the distribution F has an atom at v^B, we will denote the probability mass on v^B as $\mathbf{1}_F(v^B)$; similarly, we will denote the probability mass on v^S according to the distribution G as $\mathbf{1}_G(v^S)$. Because of the possibility of atoms, when it comes to the buyer it is sometimes convenient to work with the reliability function $R_F(v) = \Pr(\text{value} \geq v)$ instead of the distribution of signals $F(v)$; note that if R_F is the reliability associated with the distribution F, then $R_F(v) = 1 - F(v) + \mathbf{1}_F(v)$.

We can now state more formally the mediator's problem in the present environment. The mediator chooses signal distributions $F \in \mathcal{F}$ for the buyer and $G \in \mathcal{G}$ for the seller together with a price p; that is, she chooses a *static disclosure and trading mechanism*,

[6] As the signal distributions could be continuous or discrete, all the integrals in the paper should be understood as Stieltjes integrals.

a triple $\langle F, G, p\rangle$, with $F \in \mathcal{F}$, $G \in \mathcal{G}$ and $p \in \mathbb{R}_+$. After the buyer receives a signal from F and the seller from G, each of them decides whether or not to trade at the price p. It is immediate to verify that it is a dominant strategy for the buyer to accept to trade if and only if the signal received is $v^B \geq p$ and for the seller to do so if and only if the signal received is $v^S \leq p$.[7] Since the mediator's goal is to maximize the gains from trade, the optimal static disclosure and trading mechanism is obtained as a solution of the following problem:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} \int_p^1 \int_0^p (v^B - v^S)dG(v^S)dF(v^B)$$

or, equivalently:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} G(p)\int_p^1 v^B dF(v^B) - R_F(p)\int_0^p v^S dG(v^S)$$

Letting $\mathbb{E}_F$ and $\mathbb{E}_G$ be the conditional expectation operators associated respectively with the distributions F and G, the mediator's problem can also be rewritten as follows:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} \left(\mathbb{E}_F\left[v^B|v^B \geq p\right] - \mathbb{E}_G[v^S|v^S \leq p]\right) R_F(p)G(p) \tag{1}$$

We begin by establishing an important preliminary result: it is sufficient for the mediator to select a disclosure policy that only has two realizations, a high and a low signal. This follows from the fact that each party faces a binary decision, to accept or reject trade at the price posted by the mediator.

Lemma 1 *Given any solution to the mediator's maximization problem* (1)*, there is a payoff equivalent solution in which the mediator chooses a two-point signal distribution both for the buyer and the seller.*

Building on Lemma 1, we show next that the solution of the mediator problem prescribes that a threshold for the buyer's value and one for the seller's cost are chosen;

[7] Using a more traditional mechanism design language, it is a dominant strategy for both agents to report truthfully the signal received, which determines whether or not trade occurs at the given posted price.

traders are then informed whether their value and cost are above or below their threshold. That is, the information conveyed to the parties at the optimal static mechanism, solving (1), has a very simple, binary, monotone, partition structure. A mediator aiming to maximize the gains from trade will choose signal distributions and a price such that the buyer types above the buyer's threshold and the seller types below the seller's threshold will want to trade, while the other types will refuse to trade. The optimal binary partitions that constitute the signals for seller and buyer are then characterized in the following:

Proposition 1 *In the static information disclosure and trading mechanism that maximizes the gains from trade, the buyer observes whether her value is strictly below some threshold x and the seller observes whether her cost is strictly above some other threshold y, with x, y satisfying:*

$$\begin{aligned} &\mathbb{E}_{G_0}[v^S \mid v^S \leq y] = x \quad \textit{and} \\ &\mathbb{E}_{F_0}[v^B \mid v^B \geq x] = y. \end{aligned} \tag{2}$$

The trading price is any $p \in \left[\mathbb{E}_{G_0}[v^S \mid v^S \leq y], \mathbb{E}_{F_0}[v^B \mid v^B \geq x]\right] = [x, y]$.

It is immediate to verify that system (2) always admits a solution for x, y and that $x < y$.[8] At the optimal mechanism, trade occurs when the buyer's valuation happens to be above x and the seller's valuation below y. Since $x < y$, trade may occur when the buyer's value is below the seller's value. This is an important difference relative to the case of a mediator providing full information to the parties. Under full information, only buyers with a value above the posted price p and sellers with a value below p trade. The reason why a full information disclosure policy is not optimal is that it does not generate enough trade: any efficient trade with either (i) $p > v^B > v^S$,

[8] Since G_0, F_0 have no atoms, $\mathbb{E}_{G_0}[v^S \mid v^S \leq y] < y$ for all $y > 0$ and is continuous and strictly increasing in y, while $\mathbb{E}_{F_0}[v^B \mid v^B \geq x] > x$ for all $x < 1$ and is continuous and strictly increasing in x. Hence if a solution exists, we have $x < y$. To show existence define the following function of x with domain and range $[0,1]$: $\mathbb{E}_{G_0}\left[v^S \mid v^S \leq \mathbb{E}_{F_0}[v^B \mid v^B \geq x]\right]$. Since it is the composite function of two continuous functions, it is continuous. By Brouwer's fixed point theorem it has a fixed point x^* and thus x^* and $y^* = \mathbb{E}_{F_0}\left[v^B \mid v^B \geq x^*\right]$ is a solution of (2). Note that multiple solutions may exist, in which case one of them is the optimum.

or (ii) $v^B > v^S > p$ is lost. The optimal static information disclosure and trading mechanism, given the property $x < y$, guarantees completion of a higher volume of trades. Some of the most valuable trades which are lost under full information – those with (i) $v^B = p - \varepsilon_v$ and $v^S = \varepsilon_c$ and those with (ii) $v^B = 1 - \varepsilon_v$ and $v^S = p + \varepsilon_c$ (for $\varepsilon_v, \varepsilon_c$ small) – take place under the optimal mechanism.[9] Inducing completion of valuable trades in the optimal mechanism comes at a cost: some inefficient trades are also completed (this never happens when parties are fully informed), but those are the ones that have smaller losses; trades with $v^S = y - \varepsilon_c > v^B = x + \varepsilon_v$. Moreover, there are also some less valuable, but still efficient, trades that are not completed in the optimal mechanism (e.g., those for which $x > v^B > v^S$ or $v^B > v^S > y$).

Note that the freedom the mediator has in the choice of the price allows her to pursue the additional goal of an equitable division of the gains from trade. For example, a 50-50 split of the ex-post surplus between buyer and seller could be simply obtained by setting $p = \frac{x+y}{2}$.

The above result and discussion show the benefits of limiting the information available to parties in order to increase their willingness to trade. At the same time, with a binary partition structure the information provided to parties is too coarse to ensure that all gains from trade are realized; that is, to ensure that trade occurs only and in all the situations where it is ex-post efficient. To further and more clearly illustrate these properties we present next the optimal disclosure and trading mechanism in the canonical example where both the distributions of the value F_0 and the cost G_0 are uniform.

Uniform Prior Distributions When $F_0(v) = G_0(v) = v$, the solution of the optimal disclosure policy we obtain from (2) is $x = 1/3$ and $y = 2/3$. Thus, the buyer observes whether her value is above or below 1/3, while the seller observes whether her cost is above or below 2/3. Any price $p \in [1/3, 2/3]$ is then an optimal trading price. The expected gains from trade that are realized in the optimal disclosure and trading mechanism are $\left(\frac{2}{3} - \frac{1}{3}\right)\frac{2}{3}\frac{2}{3} = \frac{4}{27}$ or 89% of the first best level (that is, when

[9] This can be seen most clearly when $x < p < y$.

trade occurs whenever it is ex-post efficient), which is $\frac{1}{6}$.

When F_0 and G_0 are uniform, the optimal price posting as well as the optimal Bayesian mechanism if agents have full information about their own values are well known (see Chatterjee and Samuelson, 1983, and Myerson and Satterthwaite, 1983). In that case the optimal posted price is $p = \frac{1}{2}$, which yields expected gains from trade of $\left(\frac{3}{4} - \frac{1}{4}\right)\frac{1}{2}\frac{1}{2} = \frac{1}{8}$, or 75% of the first best level. Hence the mediator's ability to control the information available to the agents about their private values leads to a significant increase in the realized gains from trade, by generating a greater volume of trade. Trade occurs in fact whenever the buyer has a value greater than 1/3 and the seller smaller than 2/3 (instead of when the two are, respectively, higher and smaller than 1/2).

Interestingly, the realized gains from trade at the optimal static disclosure and trading mechanism are also higher than at the optimal Bayesian mechanism when traders are fully informed. In the latter mechanism trade takes place whenever $v^B \geq v^S + 1/4$ and expected gains from trade are $\frac{9}{64}$ or 84% of the first best level.

At the same time, at our optimal static mechanism there is still a welfare loss, 11% of total surplus, due to the fact that parties do not have enough information to prevent inefficient trades from being completed (when $1/3 < v^B < v^S < 2/3$) or to realize all ex-post efficient trades (when $v^B > v^S$ and $v^B < 1/3$ or $v^S < 2/3$).

Remark 1: Bayesian Trading Mechanisms. In the set-up described above, the fact that the terms of trade are set contemporaneously to the signal distribution implies that at the selected trading mechanism trade always occurs whenever its (expected) benefit exceeds the cost; incentive constraints do not bind. This is true both at the optimal information structure and with full information.

If we retain the static feature of the information disclosure to the parties, but allow for Bayesian trading mechanisms, where the terms of trade may depend on agents' reporting on the information they have discovered, we can allow for richer patterns of trade but incentives clearly constrain the trading mechanism, which then depends on the information disclosed to the parties. As is well known (Myerson and Satterthwaite, 1983), even if we allow for Bayesian trading mechanisms a welfare loss, relative to the first best, remains when parties are fully informed. Furthermore, as shown above, with uniform distributions the welfare loss is even higher than at the optimal static disclosure and trade mechanism.

One may wonder whether the efficiency loss can be totally eliminated by suitably designing the information made available to the parties, while retaining the static property of the information disclosure process, if we allow for Bayesian trading mechanisms. The answer is no, ex-post efficiency of trades cannot be obtained. The reason is that, to ensure that trade occurs if and only if it is efficient, the two parties must have the information required to do that. When the distributions of the signals received by the two parties are chosen simultaneously, this requirement essentially corresponds to both parties being fully informed (in which case, as argued above, ex-post efficiency is not attainable).[10]

Remark 2: Partial Correlation of Values. We have assumed that the buyer and seller's values are independently drawn, but in several applications it would seem reasonable to allow some components of the valuations to be correlated random variables. For example, when applying the model to litigation, it is natural to think that part of the plaintiff's cost of selling a legal claim (i.e., settling) and part of the defendant's value of buying the legal claim are correlated, as both the defendant's value and the plaintiff's cost depend on the settlement decision that the court will impose if mediation fails. The court's decision in turn depends on characteristics of the case which are relevant for both parties and over which an expert mediator may shed light.

The simplest way to see the impact of partial correlation in values is to postulate that there exists an additional random variable θ that affects the value of both parties; that is, to assume that the buyer and seller's value depend in a separable way on their independent private components v^B, v^S and a common component θ. Under this condition, the analysis of the optimal static disclosure and trading mechanism is similar

[10] Characterizing the optimal static information disclosure and Bayesian trading mechanism is a difficult task. However, also in the light of the result stated in the text we believe that few additional substantive insights can be gained by such a characterization.

to the case of independent private values considered in this section.[11],[12]

Remark 3: Mediator's Profit. Our mediator's goal is to maximize welfare, but in practice some mediators may have a profit motive. If the parties going to a mediator pay a flat fee, our analysis goes through unchanged. Appealing to reputation considerations justifies assuming that the mediator will do what is best for the parties, given that her compensation is not affected.

If, on the other hand, the mediator has the power to select a profit maximizing scheme, then things are different in general, but the static analysis of this section goes through with the only modification that the mediator charges and collect a price $p^S = \mathbb{E}_{F_0}[v^B \mid v^B \geq x] = y$ from the seller and pays a price $p^B = \mathbb{E}_{G_0}[v^S \mid v^S \leq y] = x$ to the buyer. Hence in the case of static disclosure and trade mechanisms, under the mechanism that maximizes the mediator's profits trade takes place under the same circumstances as under the welfare maximizing mechanism and the mediator extract all gains from trade as net payment from parties.

4 Shuttle Diplomacy: Dynamic Mechanisms

In this section we extend the analysis to shuttle diplomacy, where the mediator approaches parties sequentially and repeatedly, both to provide them with the outcome of some suitably designed experiment and to ask them to report, after observing the experiment's outcome, whether or not they are willing to trade at a price that is proposed to them. On the basis of the answer received, the information that is subsequently provided to the other party, and the terms of trade proposed to her, are determined. The process is then iterated over time. In this case both the information disclosure and the trading mechanism are dynamic and depend on the agents' feedback to the mediator.

[11] With an independent component in the parties's value, it is easy to verify that the full rank condition required for full surplus extraction by Crémer and McLean (1988) does not hold. We should also point out that in mediation practice we do not observe betting mechanisms in the style of Crémer and McLean (1988).

[12] An additional insight is that if the mediator discloses the same amount of information about the common component θ to the two parties, then the mediator prefers full disclosure of the value of θ or no disclosure, depending on whether the gains from trade are a convex or a concave function of θ.

We show that the dynamic feature of the mechanism, and in particular the fact that agents' reports affect the pattern of future information disclosures in addition to the terms of trade, allow to enhance efficiency. Indeed, shuttle diplomacy makes it possible for the mediator to implement a first best, ex-post efficient, outcome.

More precisely, we establish this feature for a shuttle diplomacy procedure in which the mediator lets the agents discover information over time, alternating between rounds of discoveries for the buyer and for the seller with associated posted prices. The seller starts by finding out whether her value is the highest possible and, as new discovery rounds come along, discovers progressively whether her value is lower and lower. At the end of each of her discovery rounds, the seller decides whether or not to stop the discovery phase of the procedure; this can be interpreted as reporting whether or not she has discovered her true value. If the seller decides to stop, the price is set above her presumed value and the buyer makes one last discovery. As in a descending clock auction, in each new discovery round the price that is set if the seller decides to stop the discovery phase is lower than in the previous one. If the seller decides not to stop, it is the buyer's turn to make a discovery, starting from finding out if her value is the lowest possible. As discovery rounds go by, the buyer discovers whether her value is higher and higher. Like the seller, after each discovery round she is given the option to stop the discovery phase, in which case the price is set below her presumed value and the seller makes one last discovery. As in an ascending clock auction, successive discovery rounds are associated with progressively higher prices at which the buyer may stop the discovery phase.

We cast the analysis in a discrete time setting, assuming that the seller is the first to receive some information. This is without loss of generality, as we will be interested in the limit, continuous-time, case in which discovery intervals, that is the length of time between two consecutive discoveries for the buyer, or the seller, become infinitesimally small. We allow buyer and seller to complete their discovery phase, that is, the set of discoveries that are made if no party ever decides to stop, at different times. Thus, we set T^S and T^B to be the calendar time in which the seller, respectively the buyer, make her final discovery prescribed by the mechanism. We let Δ be the length of each discovery interval and so the number of such intervals for seller and buyer is equal to $\frac{T^S}{\Delta}$ and $\frac{T^B}{\Delta}$, respectively. The maximal duration of the shuttle diplomacy procedure is then $T \equiv \max\left\{T^S, T^B\right\}$ and can be set without loss of generality to be equal to 1.

The following algorithm formally describes the shuttle diplomacy procedure. It is constructed so that, if the parties choose to stop the discovery stage when and only when they discover their values, then the outcome of the limit, continuous-time, algorithm is ex-post efficient, that is trade occurs whenever $v^B > v^S$.

The Shuttle Diplomacy Mechanism

Step 0. The mediator selects:

- A final buyer and seller value v^* and posted price p_F, with:

$$\mathbb{E}_{G_0}\left[v^S | v^S \leq v^*\right] \leq p_F \leq \mathbb{E}_{F_0}\left[v^B | v^B \geq v^*\right]. \tag{3}$$

- Two collections of value discovery intervals: $\left\{I_t^S = \left[\alpha_t^S, \alpha_{t-\Delta}^S\right]\right\}_{t=\Delta}^{T^S}$ for the seller, with $\alpha_0^S = 1, \alpha_{T^S}^S = v^*$, $t = \Delta, 2\Delta, ..., T^S$; $\left\{I_t^B = \left[\alpha_{t-\Delta}^B, \alpha_t^B\right]\right\}_{t=\Delta}^{T^B}$ for the buyer, with $\alpha_0^B = 0, \alpha_{T^B}^B = v^*$, $t = \Delta, 2\Delta, ..., T^B$.

- Two continuously differentiable price functions $p^S : [v^*, 1] \to \mathbb{R}_+$, and $p^B : [0, v^*] \to \mathbb{R}_+$

Step 1. In discovery period $t \leq T^S$ of the *Shuttle Stage*, the seller:

- Discovers if her value is in I_t^S, i.e., whether it is $v_t^S = \mathbb{E}_{G_0}\left[v^S | v^S \in I_t^S\right]$ or not;

- Selects whether to *Stop* or *Continue*;
  - If the choice is *Stop*, then go to Final Stage;
  - If the choice is *Continue*, then go to:
    - Step 2 if $t \leq T^B$;
    - Step 1 of period $t + \Delta$ if $T^B < t < T^S$;

        * The Final Stage if $T^B < t = T^S$.

In discovery period $t > T^S$, go to Step 2.

Step 2. In discovery period $t \leq T^B$ of the *Shuttle Stage*, the buyer:

- Discovers if her value is in I_t^B, i.e., whether it is $v_t^B = \mathbb{E}_{F_0}\left[v^B | v^B \in I_t^B\right]$ or not;
- Selects whether to *Stop* or *Continue*;
    - If the choice is *Stop*, then go to Final Stage;
    - If the choice is *Continue*, then go to:
        * Step 1 of period $t + \Delta$ if $t < T = \max\{T^S, T^B\}$;
        * The Final Stage if $t = T$.

Step 3. The *Final Stage*:

1. If $t \leq T^S$ and the seller has decided to *Stop* at t, then:
    - The buyer observes whether her value is above or below v_t^S;
    - Price $p^S(v_t^S)$ is posted;
    - Buyer and seller decide whether they want to trade at $p^S(v_t^S)$.
2. If $t \leq T^B$ and the buyer has decided to *Stop* at t, then:
    - The seller observes whether her value is above or below v_t^B;
    - Price $p^B(v_t^B)$ is posted;
    - Buyer and seller decide whether they want to trade at $p^B(v_t^B)$.
3. If at $t = T = \max\{T^S, T^B\}$ neither the buyer nor the seller have decided to *Stop*, price p_F is posted and buyer and seller decide whether they want to trade at p_F.

Given the specification of the shuttle diplomacy mechanism, for all $t < \max\{T^S, T^B\}$ we have $v_t^B < v_t^S$. We will now show that, more generally, it must be that for all $t = \Delta, 2\Delta, ..., \max\{T^B, T^S\}$:

$$\mathbb{E}_{G_0}\left[v^S | v^S \leq v_t^B\right] \leq p^B\left(v_t^B\right) \leq v_t^B \leq v_t^S \leq p^S\left(v_t^S\right) \leq \mathbb{E}_{F_0}\left[v^B | v^B \geq v_t^S\right] \tag{4}$$

The mechanism starts in the shuttle stage at $t = \Delta$. In each period t of this stage in which the seller makes a discovery, the seller learns whether her value lies in the interval I_t^S (and thus equals v_t^S) and then tells the mediator whether she wants to stop the shuttle stage. If she decides to stop, the mechanism moves to the final stage; the mediator lets the buyer observe whether her value is above or below v_t^S and posts a price $p^S\left(v_t^S\right)$. Buyer and seller then decide whether or not they want to trade. The price $p^S\left(v_t^S\right)$ is selected so that, at such a price a seller with value v_t^S and a buyer who has observed that her value is above v_t^S are willing to trade; that is, the last two inequalities in (4) must hold:

$$v_t^S \leq p^S\left(v_t^S\right) \leq \mathbb{E}_{F_0}\left[v^B | v^B \geq v_t^S\right] \text{ for all } t = \Delta, 2\Delta, ..., T^S$$

If, on the contrary, the seller decides to continue and $t < T^B$, then it's the buyer's turn to discover whether her value is in the interval I_t^B with expected value v_t^B.[13] The buyer then decides whether to stop the shuttle stage and if she does the mechanism moves to the final stage; the mediator lets the seller observe whether her value is above or below v_t^B and posts a price $p^B\left(v_t^B\right)$ at which a buyer with value v_t^B and a seller who has observed that her value is below v_t^B are willing to trade; that is, the first two inequalities in (4) must hold:

$$\mathbb{E}_{G_0}\left[v^S | v^S \leq v_t^B\right] \leq p^B\left(v_t^B\right) \leq v_t^B \text{ for all } t = \Delta, ..., T^B$$

If the buyer decides to continue and we are in period t of the shuttle stage, then

[13] If instead $t > T^B$ the mechanism moves directly to period $t + \Delta$ (as long as this does not exceed T^S).

there are two cases. First, if $t < T$, then the mechanism moves to period $t + \Delta$ of the shuttle stage and the same steps are repeated. Second, if $t = T$, so that traders concluded the shuttle phase always electing to continue, then the mechanism moves to the final stage and buyer and seller have the option to trade at the posted price p_F. By (3), the level of p_F is set in such a way that at this price traders that have not discovered their true values (i.e., a seller knowing that her value is below and a buyer knowing that her value is above v^*) will want to trade.

The mechanism allows the two parties to make discoveries at different speed. It permits one agent to complete the discovery of her value up to the threshold v^* before the other agent. For example, if $T^B < T^S$ the buyer discovers that her value is above v^* before the seller discovers that her value is below v^*. The lengths of the intervals I_t^S and I_t^B, and the related length Δ of each discovery interval describe the speed at which, at any point in time along the discovery process, the seller learns her value between 1 and v^* and the buyer learns her value between 0 and v^*.[14]

Given a shuttle diplomacy mechanism, the strategies of buyer and seller must specify whether they select *Stop* or *Continue* after each discovery and whether they accept to trade at the price posted by the mediator. We will say that a trader adopts a *stopping-at-value strategy* if she always selects *Stop* after having discovered her value, always selects *Continue* after not having discovered her value and always accepts to trade at a price that yields her a non-negative payoff. As argued above, this can be interpreted as truthful reporting by each party of the information acquired at each stage.

If both players adopting stopping-at-value strategies constitutes a perfect Bayesian equilibrium of the shuttle-diplomacy mechanism, we call such an equilibrium a *stopping-at-value-strategy PBE.* It is immediate to verify that at such an equilibrium, given the properties of the mechanism we described, trade occurs whenever, given the information acquired by the two agents, it is efficient.

In what follows we show that a shuttle diplomacy mechanism can be chosen so that a stopping-at-value-PBE exists. Hence the mechanism is approximately ex-post

[14] Note that the length of I_t^B – or I_t^S – and Δ also determine how fine is the information being acquired by a party. In the special case where, say, T^B equals zero, the procedure does not entail any discovery for the buyer. In such a case it must also be $v^* = 0$.

efficient. As the number of periods of the shuttle stage converges to infinity, that is, as Δ converges to zero, the information obtained by (at least one of the) parties gets finer and finer and the mechanism becomes exactly ex-post efficient.[15] We focus our attention on this limit mechanism.

To establish the result we derive first the incentive compatibility conditions that the speed of the discovery process, as described by $\left\{\alpha_t^S\right\}_{t=\Delta}^{T^S}, \left\{\alpha_t^B\right\}_{t=\Delta}^{T^B}$, and the dynamics of the prices posted by the mediator must satisfy to ensure the optimality of a stopping-at-value-strategy for both traders.

Note first that, to guarantee that both buyer and seller select to stop if they discover their value in the last period T^B and T^S, in the limit as $\Delta \to 0$, we must have:[16]

$$\begin{array}{lll} p^B(v^*) = p_F \leq p^S(v^*) & \text{if} & T^S \leq T^B \\ p^B(v^*) \leq p_F = p^S(v^*) & \text{if} & T^S \geq T^B \end{array} \tag{5}$$

Together (4) and (5) imply that if $T^S = T^B$, then $p_F = p^B(v^*) = p^S(v^*) = v^*$.

We show in what follows that four other incentive compatibility conditions, two for each agent, must be satisfied to guarantee the optimality of a stopping-at-value-strategy also in any other period, before the final stage is reached. We consider the buyer's incentives first, assuming that the seller adopts a stopping-at-value strategy. There are two cases to take into account at each point in time $t < T^B$. From each case we will derive an incentive constraint for the buyer.

The first incentive constraint for the buyer deals with the case in which at t the

[15] We assume that, provided $T^S, T^B > 0$, $\lim_{\Delta\to 0} \alpha_{t-\Delta}^S = \alpha_t^S$ and $\lim_{\Delta\to 0} \alpha_{t-\Delta}^B = \alpha_t^B$, that is, the length of all intervals I_t^S, I_t^B converges to 0.

[16] Suppose $T^S \leq T^B$. Begin by considering the seller's decision at T^S when she has discovered that her value is v^*; by (4) she will only trade in the event that the buyer's value is at least as high as v^*. If the seller decides to continue and this event occurs, so that the final period T^B is reached, she faces price p_F. Hence, for the seller to prefer to stop we must have $p^S(v^*) \geq p_F$.

Now consider the buyer's decision at $t = T^B$. Assume, contrary to (5) that $p^B(v^*) \neq p_F$. Observe that the seller is willing to trade at both prices. This is because the seller knows that $v^S \leq v^*$ and (4) implies that the seller's expected value satisfies $\mathbb{E}_{G_0}\left[v^S \mid v^S \leq v^*\right] \leq p^B(v^*)$, while (3) implies $\mathbb{E}_{G_0}\left[v^S | v^S \leq v^*\right] \leq p_F$. If $p^B(v^*) > p_F$, then in order to trade at a lower price the buyer has an incentive to continue after discovering at $t = T^B$ that her value is v^*, since by (4) $v^* \geq p^B(v^*)$ and trading is profitable. If, on the other hand, $p^B(v^*) < p_F$, then in order to trade at a lower price the buyer has an incentive to stop at $t = T^B$ even if she has not discovered that her value is v^*, since by (3) $p_F \leq \mathbb{E}_{F_0}\left[v^B | v^B \geq v^*\right]$ and trading is profitable. Thus, it must be $p^B(v^*) = p_F$.

A similar argument holds when $T^S \geq T^B$, implying that we must have $p^B(v^*) \leq p_F = p^S(v^*)$.

buyer has just discovered that her value is v_t^B. Since the seller up to this moment has always selected to continue, her value must lie below α_t^S. Thus, if the buyer decides to stop, her expected payoff is:

$$\left[v_t^B - p^B\left(v_t^B\right)\right] \frac{G_0\left(v_t^B\right)}{G_0\left(\alpha_t^S\right)}.$$

Since by (4) in the final stage of the mechanism the seller accepts to trade at price $p^B\left(v_t^B\right)$ when her value is below v_t^B, the term $\frac{G_0\left(v_t^B\right)}{G_0\left(\alpha_t^S\right)}$ is the buyers' probability of trading at this price, conditional on the seller's value lying below α_t^S, and $v_t^B - p^B\left(v_t^B\right)$ is the gain from trade for the buyer.

If instead the buyer deviates and selects to continue, by (4) and (5) she can only obtain a positive payoff (and hence possibly benefit from continuing) if she is the agent stopping at a later date of the shuttle diplomacy stage. Several deviations of this kind are possible. Consider first the buyer's payoff when she decides to continue at t and to stop in the next period, when the disclosed value is $v_{t+\Delta}^B$:[17]

$$\left[v_t^B - p^B\left(v_{t+\Delta}^B\right)\right] \frac{G_0\left(v_{t+\Delta}^B\right)}{G_0\left(\alpha_t^S\right)},$$

where v_t^B is the true buyer's value, $p^B\left(v_{t+\Delta}^B\right)$ is the price when stopping at time $t+\Delta$ and $\frac{G_0\left(v_{t+\Delta}^B\right)}{G_0\left(\alpha_t^S\right)}$ is the conditional probability at time t that the seller will have a value below $v_{t+\Delta}^B$ and hence will not discover her value at time $t+\Delta$ and will want to trade when the buyer stops at $t+\Delta$ with a presumed value $v_{t+\Delta}^B$.[18] To ensure this deviation is not profitable, the following incentive constraint must hold for all $t < T$:

$$\left[v_t^B - p^B\left(v_t^B\right)\right] G_0\left(v_t^B\right) \geq \left[v_t^B - p^B\left(v_{t+\Delta}^B\right)\right] G_0\left(v_{t+\Delta}^B\right).$$

Adding and subtracting $p^B\left(v_{t+\Delta}^B\right) G_0\left(v_t^B\right)$ and rearranging, the constraint can be writ-

[17] Note that if in period $t+\Delta$ the seller discovers that her values is $v_{t+\Delta}^S$, then trade does not take place as $p^S(v_{t+\Delta}^S) > v_t^B$ by (4).

[18] The event that the seller's value is below $v_{t+\Delta}^B$ includes the event that in period $t+\Delta$ the seller has not discovered her value to be $v_{t+\Delta}^S$, as by construction $v_{t+\Delta}^B \leq v^* \leq v_{t+\Delta}^S$.

ten as:

$$\left[p^B\left(v_{t+\Delta}^B\right)-v_t^B\right]\left[G_0\left(v_{t+\Delta}^B\right)-G_0\left(v_t^B\right)\right]+\left[p^B\left(v_{t+\Delta}^B\right)-p^B\left(v_t^B\right)\right]G_0\left(v_t^B\right)\geq 0.$$

Multiplying by $\frac{v_{t+\Delta}^B-v_t^B}{\Delta}\cdot\frac{1}{v_{t+\Delta}^B-v_t^B}$, letting $\frac{dv_t^B}{dt}=\lim_{\Delta\to 0}\frac{v_{t+\Delta}^B-v_t^B}{\Delta}$, and taking limits as $\Delta\to 0$, we obtain the following constraint for all $v_t^B\in[0,v^*]$:[19]

$$\left[\frac{dp^B\left(v_t^B\right)}{dv_t^B}G_0\left(v_t^B\right)-\left(v_t^B-p^B\left(v_t^B\right)\right)g_0\left(v_t^B\right)\right]\frac{dv_t^B}{dt}\geq 0. \tag{6}$$

Since by (4) $v_t^B\geq p^B\left(v_t^B\right)$, the term $\left(v_t^B-p^B\left(v_t^B\right)\right)g_0\left(v_t^B\right)dv_t^B$ is non negative. It describes the expected benefit of the deviation for the buyer, given by the expected payoff from the additional trades that can be completed by waiting one additional period dt: $v_t^B-p^B\left(v_t^B\right)$ is the buyer's payoff for each of these additional trades and the unconditional probability of such additional trades is the limit as $\Delta\to 0$ of $G_0\left(v_{t+\Delta}^B\right)-G_0\left(v_t^B\right)$, which equals $g_0\left(v_t^B\right)dv_t^B$, the probability that the seller's value lies in a small interval around v_t^B. For the incentive constraint (6) to be satisfied, $p^B\left(v_t^B\right)$ must be non decreasing; that is, the buyer's price must weakly increase over time. This generates a loss for the buyer from waiting one additional period dt, due to an increase of the price at which trade occurs. The first term in expression (6), $dp^B\left(v_t^B\right)G_0\left(v_t^B\right)$, is then the buyer's expected loss, due to a price increase, from waiting one additional period dt. This loss occurs when the seller agrees to trade at $p^B\left(v_t^B\right)$, as her value is below v_t^B, with $G_0\left(v_t^B\right)$ being the unconditional probability of this event.

To sum up, constraint (6) requires that when a buyer discovers her true value, her loss due to a price increase if she deviates and waits one period before stopping outweighs the gain due to an increase in the probability of trading. The larger the buyer's gain from trade $v_t^B-p^B\left(v_t^B\right)$, the steeper must be the increase in $p^B\left(v_t^B\right)$. Lemma 4 in the appendix shows that (6) ensures that other deviations by the buyer, to continuing for more than one period after the discovery of her value, are also not

[19] By delaying stopping until the shuttle stage has reached the discovery of value $\widehat{v}>v$ a buyer with value v obtains payoff $\left[v-p^B\left(\widehat{v}\right)\right]G_0\left(\widehat{v}\right)$. Constraint (6) says that the buyer's payoff is decreasing in $\widehat{v}$ at $\widehat{v}=v$.

profitable.

The second incentive constraint for the buyer refers to the case where in period t the buyer has not yet discovered her value; thus the buyer knows that her value lies above α_t^B and the seller knows that her value is below α_t^S. Considering again the limit as $\Delta \to 0$ (so that $\alpha_t^B = v_t^B$ and $\alpha_t^S = v_t^S$), the expected payoff of the buyer if she deviates and stops the shuttle stage at t, is

$$\left(\mathbb{E}_{F_0}\left[v^B | v^B \geq v_t^B\right] - p^B\left(v_t^B\right)\right)\frac{G_0\left(v_t^B\right)}{G_0\left(v_t^S\right)}.$$

If instead the buyer follows the stopping-at-value strategy and so chooses to continue at t, her expected payoff at t is:

$$\begin{aligned}
&\int_{v_t^B}^{v^*}\left(v^B - p^B\left(v^B\right)\right)\frac{G_0\left(v^B\right)}{G_0\left(v_t^S\right)}\frac{f_0\left(v^B\right)}{1-F_0\left(v_t^B\right)}dv^B \\
&+\left(\mathbb{E}_{F_0}\left[v^B \mid v^B \geq v^*\right] - p_F\right)\frac{1-F_0\left(v^*\right)}{1-F_0\left(v_t^B\right)}\frac{G_0\left(v^*\right)}{G_0\left(v_t^S\right)} \\
&+\int_{v^*}^{v_t^S}\left(\mathbb{E}_{F_0}\left[v^B \mid v^B \geq v^S\right] - p^S\left(v^S\right)\right)\frac{1-F_0\left(v^S\right)}{1-F_0\left(v_t^B\right)}\frac{g_0\left(v^S\right)}{G_0\left(v_t^S\right)}dv^S
\end{aligned}$$

In contrast to the previous case, since the buyer now has not yet discovered her value and only knows it is greater than v_t^B, the buyer can achieve a positive expected payoff in each of the three possible outcomes of the shuttle mechanism, as described by the three terms in the above expression.

The first term is the expected payoff of the buyer in the event that the buyer will stop the shuttle stage before time $t = T^B$. If at time $\tau > t$ the buyer discovers that her value is v^B, which happens with probability $\frac{f_0\left(v^B\right)}{1-F_0\left(v_t^B\right)}dv^B$, and she then stops the shuttle stage, she trades at price $p^B(v^B)$ with probability $\frac{G_0\left(v^B\right)}{G_0\left(v_t^S\right)}$, the conditional probability at t that the seller's value is below v^B.

The second term is the expected payoff of the buyer if trade occurs at price p_F, that is if buyer and seller continue the shuttle phase up to the end of the last period, times the conditional probability at t of this event occurring. This happens when the buyer value is above and the seller value below v^*.

The third term is the buyer expected payoff in the last possible outcome, where it is the seller to stop the shuttle stage before time $t = T^S$. If at time $\tau > t$ the seller discovers that her value is v^S, which happens with probability $\frac{g_0(v^S)}{G_0(v_t^S)} dv^S$, and she stops the shuttle stage, then the buyer trades at price $p^S(v^S)$ with probability $\frac{1-F_0(v^S)}{1-F_0(v_t^B)}$, the conditional probability at t that the buyer's value is above v^S.

Thus we may conclude that continuing until her value has been discovered dominates stopping for a buyer who has not discovered her value at t if and only if, for all $v_t^B \in [0, v^*]$, and associated $v_t^S \in [v^*, 1]$,

$$
\begin{aligned}
& \left(\mathbb{E}_{F_0}\left[v^B | v^B \geq v_t^B\right] - p^B\left(v_t^B\right)\right) G_0\left(v_t^B\right)\left(1 - F_0\left(v_t^B\right)\right) \\
\leq\; & \int_{v_t^B}^{v^*} \left(v^B - p^B\left(v^B\right)\right) G_0\left(v^B\right) f_0\left(v^B\right) dv^B \\
& + \left(\mathbb{E}_{F_0}\left[v^B \,|\, v^B \geq v^*\right] - p_F\right)\left(1 - F_0\left(v^*\right)\right) G_0\left(v^*\right) \\
& + \int_{v^*}^{v_t^S} \left(\mathbb{E}_{F_0}\left[v^B \,|\, v^B \geq v^S\right] - p^S\left(v^S\right)\right)\left(1 - F_0\left(v^S\right)\right) g_0\left(v^S\right) dv^S
\end{aligned}
\tag{7}
$$

To understand the above expression it is useful to compare it to the previous situation. Again, the benefit of stopping immediately at t the shuttle stage is to secure a low price, while the benefit of waiting (following in this case the stopping-at-value strategy) is a greater probability of trading. The difference is that, since now the buyer only knows that $v^B \geq v_t^B$, instead of $v^B = v_t^B$, the benefit of trading with a higher probability, at a higher price, is higher. Unlike (6), to ensure that future gains are always sufficiently large (7) imposes a condition on the continuation path of posted prices as well as on the relative speed of the discovery processes, as revealed by the fact that v_t^S appears in the constraint (together with the future path of prices posted to the seller). If for example the seller's discoveries up to time t have proceeded at the same speed as the buyer's discoveries, we have $v_t^S = 1 - v_t^B$, while if they have been slower $v_t^S > 1 - v_t^B$ and if they have been faster $v_t^S < 1 - v_t^B$.

We have shown that constraints (6) and (7) are necessary and sufficient for the buyer's strategy of stopping-at-value to be a best reply to the seller's stopping-at-value strategy in any period $t < T^B$. We now state the incentive constraints for the seller's stopping-at-value strategy to be a best reply to the buyer using a stopping-at-value strategy at any $t < T^S$. The first constraint is, for all $v_t^S \in [v^*, 1]$:

$$\left[\frac{dp^S\left(v_t^S\right)}{dv_t^S} \left[1 - F_0\left(v_t^S\right)\right] - \left(p^S\left(v_t^S\right) - v_t^S\right) f_0\left(v_t^S\right) \right] \left(-\frac{dv_t^S}{dt} \right) \geq 0. \tag{8}$$

The derivation of (8) is analogous to the derivation of the corresponding constraint for the buyer and is left to the appendix. The constraint implies that the price function $p^S\left(v_t^S\right)$ is weakly increasing and hence the seller's price weakly decreases over time. The interpretation of the terms in (8) is similar to those in constraint (6): after discovering her true value, the seller's loss from continuing the shuttle stage, due to the decrease in price, must outweigh the gain due to an increase in the probability of trading.

The second constraint is, for all $v_t^S \in [v^*, 1]$ and associated $v_t^B \in [0, v^*]$:

$$\begin{aligned}
& \left(p^S\left(v_t^S\right) - \mathbb{E}_{G_0}\left[v^S \,|\, v^S \leq v_t^S\right]\right) G_0\left(v_t^S\right)\left(1 - F_0\left(v_t^S\right)\right) \\
\leq\; & \int_{v^*}^{v_t^S} \left(p^S\left(v^S\right) - v^S\right)\left(1 - F_0(\left(v^S\right)\right) g_0(\left(v^S\right) dv^S \\
& + \left(p_F - \mathbb{E}_{G_0}\left[v^S \,|\, v^S \leq v^*\right]\right)\left(1 - F_0\left(v^*\right)\right) G_0\left(v^*\right) \\
& + \int_{v_t^B}^{v^*} \left(p^B\left(v^B\right) - \mathbb{E}_{G_0}\left[v^S \,|\, v^S \leq v^B\right]\right) G_0\left(v^B\right) f_0\left(v^B\right) dv^B
\end{aligned} \tag{9}$$

Again, the interpretation of the terms appearing in (9) is similar to the one of the terms in the corresponding constraint (7) for the buyer: (9) says that, after a discovery round in which the seller does not discover her true value, her payoff from stopping, given by the left hand side of (9), is smaller than the payoff from following the stopping-at-value strategy, the right hand side of (9). The benefit of a higher probability of trading when continuing must exceed the loss due to a decrease in the price at which trade occurs.

We summarize the analysis so far in this section with the following lemma.

Lemma 2 *If the final buyer and seller value v^* and posted price p_F, together with the continuously differentiable price functions $p^B(v)$, $p^S(v)$ satisfy constraints (3) – (9), then in the limit as $\Delta \to 0$ the shuttle-diplomacy mechanism has a stopping-at-value strategy PBE.*

We now present the main result of this section:

Proposition 2 *There are continuously differentiable price functions $p^B(v)$ and $p^S(v)$, speeds of learning in the discovery process dv^S/dt, dv^B/dt, a final value v^* for the buyer and the seller and price p_F for which, as the length of the discovery periods converges to zero, the shuttle diplomacy mechanism has a stopping-at-value-strategy PBE. That is, it is a perfect Bayesian equilibrium for agents to stop the shuttle stage if and only if they have discovered their value and to trade when profitable. The resulting trading outcome is ex-post efficient.*

We establish the result using an extreme form of the shuttle diplomacy mechanism, in which one party does not learn any information in the discovery process, while the other party progressively learns everything about her true value. That is, the mediator only takes a single shuttle ride to each of the two parties and does not go back and forth between them. For concreteness, let the seller be the party that makes a discovery in every period of the shuttle stage and then selects whether or not to stop, while the buyer only learns something about her value in the final stage, and this is the minimum information needed to implement ex-post efficient trading. This amounts to setting $T^B = 0$, $v^* = 0$. It then follows from (5) that $p^B(0) = 0$. As we shall see, all the incentive constraints are satisfied if the seller price is selected so as to give the seller all the gains from trade.

Proof of Proposition 2 Set (i) $v^* = 0$; (ii) $T^B = 0$ and $\alpha_t^S = 1 - t$ for all $t = \Delta, 2\Delta, .., T^S = 1$; (iii) $p^B(0) = 0$, $p^S\left(v^S\right) = \mathbb{E}_{F_0}\left[v^B | v^B \geq v^S\right]$ and $p_F = \mathbb{E}_{F_0}\left[v^B\right]$.

It is immediate to see that (3), (4) and (5) hold. Furthermore, since $T^B = 0$ we can ignore the two incentive constraints of the buyer along the shuttle stage, (6) and (7). Condition (8) holds since, under the stated specification $p^S\left(v_t^S\right) = \mathbb{E}_{F_0}\left[v^B \mid v^B \geq v_t^S\right]$ we have:

$$p^S\left(v_t^S\right)\left[1 - F_0\left(v_t^S\right)\right] = \int_{v_t^S}^1 v f_0(v)\, dv \tag{10}$$

and therefore

$$\frac{dp^S\left(v_t^S\right)}{dv_t^S}\left[1-F_0\left(v_t^S\right)\right]-\left(p^S\left(v_t^S\right)-v_t^S\right)f_0\left(v_t^S\right)=0.$$

Finally, in the case under consideration condition (9) can be rewritten as:

$$\begin{aligned}&\left(p^S\left(v_t^S\right)-\mathbb{E}_{G_0}\left[v^S\,|\,v^S\leq v_t^S\right]\right)G_0\left(v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)\\&\qquad\qquad\leq\int_0^{v_t^S}\left(p^S\left(v^S\right)-v^S\right)\left(1-F_0\left(v^S\right)\right)g_0\left(v^S\right)dv^S,\quad\text{or}\end{aligned}$$

$$\begin{aligned}&\int_0^{v_t^S}\left(p^S\left(v_t^S\right)\left[1-F_0\left(v_t^S\right)\right]-p^S\left(v^S\right)\left[1-F_0\left(v^S\right)\right]\right)g_0\left(v^S\right)dv^S\\&\qquad\qquad\leq\int_0^{v_t^S}\left(v^S\left[1-F_0\left(v_t^S\right)\right]-v^S\left[1-F_0\left(v^S\right)\right]\right)g_0\left(v^S\right)dv^S\end{aligned}$$

By (10), this inequality can be rewritten as follows:

$$\int_0^{v_t^S}\left(\int_{v_t^S}^1 vf_0\left(v\right)dv-\int_{v^S}^1 vf_0\left(v\right)dv\right)g_0\left(v^S\right)dv^S\leq\int_0^{v_t^S}v^S\left[F_0\left(v^S\right)-F_0\left(v_t^S\right)\right]g_0\left(v^S\right)dv^S$$

and hence reduces to:

$$\int_0^{v_t^S}\int_{v^S}^{v_t^S}vf_0\left(v\right)dvg_0\left(v^S\right)dv^S\geq\int_0^{v_t^S}v^S\left[F_0\left(v_t^S\right)-F_0\left(v^S\right)\right]g_0\left(v^S\right)dv^S\,,$$

which clearly holds.

The proof of the proposition then follows from Lemma 2. □

Proposition 2 establishes the existence of a shuttle-diplomacy mechanism, in the limit as the length Δ of the discovery intervals approaches 0, that guarantees that the parties trade whenever it is ex-post efficient to do so. The proof of the proposition uses the fact that informing one party only and choosing the price at any t so as to give the informed party all the gains from trade induces her to play the stopping-at-value strategy, as it aligns her private incentives with the social goal.

Equity concerns may raise doubts about the desirability (and practical feasibility) of a solution, as in the proof, where the mediator proposes a procedure in which one party obtains all the gains from trade. From an ex-ante perspective, these concerns could be dealt with by modifying the procedure and flipping a fair coin that determines the agent to be fully informed. With such a modification the gains from trade would be equally split from an ex-ante point of view. While of clear theoretical interest, however, such a mechanism does not accord with the common practice of mediation, in which gains of trade are typically split somewhat evenly ex post. With a markedly unequal distribution of the ex-post gains from trade, the interpretation of the evaluative role of the mediator as allowing to enforce the commitment of the parties to acquire limited information is in particular problematic. After the coin has been flipped and, say, the seller has been selected as the party obtaining all the gains from trade, why should the buyer want to stick to her commitment to mediation? It is thus important to show, as we do in the next subsection when we study the case of uniform prior distributions, that it is typically possible to design shuttle diplomacy mechanisms which are ex-post efficient, but where the extent as well as the speed of learning in the shuttle stage are similar for the two parties and the ex-post gains from trade are split more equitably.

To conclude this section, we provide some further explanation of why a shuttle diplomacy mechanism allows to realize all gains from trade, while this is not possible with a static Bayesian mechanism when agents are fully informed about their own values. The shuttle diplomacy mechanism allows the mediator to find out whether trading is optimal, either because the shuttle phase reaches v^* (in which case both agents know trade is efficient), or because one agent discovers her value and the other discovers whether trade is optimal. Unlike in static Bayesian mechanisms, the mediator may then set a price at which trade occurs if and only if there are positive gains from trade.

To understand why this is possible, notice first that with shuttle diplomacy when trade occurs at most one party is fully informed about her value, while the other party is provided with the minimal information needed to ensure that trade is efficient. Limiting the information available to parties slackens the agents' incentive constraints and makes it easier, as we saw in the previous section, to induce parties to trade. Second, information is provided slowly over time by the mediator and this also affects parties' incentives to truthfully report their private information, as it limits the available lies. Consider the buyer and suppose her true value is v. The buyer can only falsely report a lower value $v' < v$ by stopping the discovery stage at v', that is when all she knows is that her value is greater than the v'; fine tuning her misreport of a low value to depend on the true one as in a static setting with full information cannot be done.[20] Third, the fact that the report sent by an agent not only affects the terms of trade but also the information provided to parties in the subsequent periods reduces the agents' incentives to conceal and lie about their own values. Indeed, a key feature of our mechanism is that the report sent by each agent over the discoveries made at any point in time (via her decision to stop or continue) allows the mediator to correlate the information that is made available to the parties, without violating the privacy constraints, and this makes it easier to satisfy the incentive constraints. We must stress that this endogenous correlation is exploited to achieve first best efficiency by our information disclosure and trading mechanism in quite a different way from how the side-betting, trading mechanisms of Crémer and McLean (1988) exploit exogenous correlation among buyers' values to obtain full surplus extraction by a seller.

4.1 The Case of Uniform Prior Distributions

In this section we show that when the buyer and seller's value distributions are uniform, $F_0(v) = G_0(v) = v$, a continuum of price functions and identical discovery speeds for the buyer and the seller exist that induce them to use stopping-at-value strategies and generate ex post efficient trading. While special, this result can be generalized beyond the uniform case, as we argue in Section 5.

Given the symmetry of the value distributions, it is natural to start by taking as stopping value for seller and buyer $v^* = \frac{1}{2}$ and equal sized discovery intervals. That is, we assume: $\alpha_t^B = \frac{t}{2}$, $\alpha_t^S = 1 - \frac{t}{2}$, for $t = \Delta, .., 1$. This implies that for all t, $v_t^S = 1 - v_t^B$.

[20] This effect is also present in the sequential disclosure policy in Bergemann and Wambach (2015), in which a profit maximizing seller lets a buyer learn a progressively higher lower bound on her value for the item for sale.

It is then natural to assume symmetry in the prices posted by the mediator, considering price functions that are linked as follows:

$$p^S\left(v_t^S\right) = p^S\left(1 - v_t^B\right) = 1 - p^B\left(v_t^B\right) \tag{11}$$

Henceforth in this subsection, we will simplify the notation and denote the buyer's value by v, dropping the superscript B and subscript t.

It is simple to check that under the above specification the incentive constraints of buyer and seller are identical; that is, the constraints in the following pairs are equivalent: the first two and the last two inequalities in (4); (6) and (8); (7) and (9). Condition (4) then reduces to:

$$\frac{v}{2} \leq p^B\left(v\right) \leq v, \tag{12}$$

The only additional constraint imposed by (3), (4) and (5) is a symmetric final price:

$$p^B\left(\frac{1}{2}\right) = p_F = \frac{1}{2}. \tag{13}$$

The first incentive constraint of the buyer, (6) (or (8) for the seller), reduces to:

$$\frac{d\left(p^B\left(v\right)v\right)}{dv} \geq v. \tag{14}$$

The second incentive constraint of the buyer, (7) ((9) for the seller), becomes:[21]

$$p^B(v)v(1-v) \geq \frac{v}{2} - \frac{v^3}{3} - \frac{1}{12} \tag{15}$$

Thus, to guarantee that stopping-at-value strategies constitute a symmetric PBE when the price function is $p^B(v)$, $p^B(v)$ must satisfy constraints (12) – (15).

Consider the following candidate price function:

$$p_H^B(v) = v,$$

which satisfies the right constraint in (12) as an equality. This specification of the price function penalizes the first trader to discover her value (either the buyer with a value below $v^* = p_F = \frac{1}{2}$ or the seller with a value above v^*, the one among them with the value furthest away from p_F), as such trader makes a zero payoff. All the gains from trade go to the other party. The incentives of an agent when she learns her value, (14), are then clearly satisfied. So is (13). It is then immediate to verify that the agent prefers not to stop when she has not yet learnt her value, that is (15) also holds, since her gains accrue when it is the other agent to make a discovery in the future. Hence, under this price function stopping-at-value is a PBE and the equilibrium outcome is ex-post efficient trade.

[21] Recalling from (11) that $v^S = 1 - v^B$ and $p^B(v) = 1 - p^S(1-v)$, (7) can be written as:

$$\begin{aligned}\left(\frac{1+v}{2} - p^B(v)\right)v(1-v) \quad &\leq \quad \int_v^{\frac{1}{2}} \left(v^B - p^B\left(v^B\right)\right)v^B dv^B + \left(\frac{1+\frac{1}{2}}{2} - \frac{1}{2}\right)\frac{1}{4} \\ &+ \int_v^{\frac{1}{2}} \left(\frac{1+1-v^B}{2} - \left(1 - p^B\left(v^B\right)\right)\right)v^B dv^B,\end{aligned}$$

which is equivalent to:

$$\left(\frac{1+v}{2} - p^B(v)\right)v(1-v) \quad \leq \quad \int_v^{\frac{1}{2}} \left(\frac{v^B}{2}\right)v^B dv^B + \frac{1}{16}$$

or,

$$\frac{1}{2}v(1-v^2) - p^B(v)v(1-v) \quad \leq \quad \frac{1}{6}\left(\frac{1}{8} - v^3\right) + \frac{1}{16},$$

which equals (15).

On the contrary, the function that satisfies the left constraint in (12) as an equality (i.e., $p^B(v) = \frac{v}{2}$), attributing all gains from trade to the agent reporting a discovery, is not admissible since it violates constraint (13) and also, as we shall see in what follows, (15) for v close to $\frac{1}{2}$. We may however define a price function $p_L^B(v)$ equal to $\frac{v}{2}$ for low values of v and implicitly defined by (15) holding as an equality for high values of v. We can do so because there exists a unique $\widehat{v} \in \left(0, \frac{1}{2}\right)$ such that for $v = \widehat{v}$ it is

$$\frac{v^2(1-v)}{2} = \frac{v}{2} - \frac{v^3}{3} - \frac{1}{12}$$

while the left hand side is greater than the right hand side for $v < \widehat{v}$ and smaller for $v > \widehat{v}$.[22] Thus

$$p_L^B(v) = \begin{cases} \frac{v}{2} & \text{for} \quad v \in [0, \widehat{v}] \\ \frac{6v-4v^3-1}{12v(1-v)} & \text{for} \quad v \in \left[\widehat{v}, \frac{1}{2}\right] \end{cases}$$

By construction, $p_L^B(v)$ satisfies (13), (15) and the left constraint in (12) for $v \leq \widehat{v}$. By the argument in footnote 22 this last constraint is also satisfied for $v > \widehat{v}$. To establish that $p_L^B(v)$ is a solution it remains to check that it satisfies (14) and the right constraint in (12). It is immediate to see that for $v \leq \widehat{v}$ $\frac{dp_L^B(v)v}{dv} = v$ and hence (14) is satisfied. Simple algebra shows that: (i) $\frac{d\left(\frac{6v-4v^3-1}{12(1-v)}\right)}{dv} > v$ and thus (14) holds for $v \geq \widehat{v}$; (ii) $\frac{6v-4v^3-1}{12v(1-v)} < v$ for $v \in \left[0, \frac{1}{2}\right)$ and thus the right constraint in (12) is also satisfied.

When $p^B(v) = p_L^B(v)$, an agent makes most of her gains when she makes a discovery. This again ensures the agent does not want to delay stopping the process when she learns her value. On the other hand, giving all the benefits from trade to the agent who reports a discovery may induce an agent to lie when she has not made a discovery, to profit from the price posted in that case. The above argument shows this indeed happens when v is sufficiently close to v^*, hence the need to adjust the price to reduce the gains accruing to the party reporting a discovery.

It is immediate to see that any convex combination defined by:

$$p(v) = \lambda p_H^B(v) + (1-\lambda) p_L^B(v)\,, \tag{16}$$

[22] Define $\Psi(v) = \frac{v^2(1-v)}{2} - \frac{v}{2} + \frac{v^3}{3} + \frac{1}{12}$. Note that $\Psi(0) > 0$, $\Psi\left(\frac{1}{2}\right) < 0$ and $\frac{d\Psi(v)}{dv} < 0$ for $v \in \left[0, \frac{1}{2}\right]$.

with $\lambda \in [0,1]$ is also admissible as all constraints (12) – (15) are satisfied. We have thus proven the following:

Proposition 3 *With uniform prior distributions $F_0(v) = G_0(v) = v$, there exist a continuum of functions $p\left(v^B\right) : \left[0, \frac{1}{2}\right] \to \left[0, \frac{1}{2}\right]$ such that it is a perfect Bayesian equilibrium for agents to adopt stopping-at-value strategies in a shuttle diplomacy mechanism where the buyer and the seller discover their values at the same speed, with $p_F = v^* = \frac{1}{2}$, and the price functions are $p^B\left(v^B\right) = p(v^B)$ and $p^S\left(v^S\right) = 1 - p\left(1 - v^S\right)$.*

The shuttle-diplomacy mechanisms described above, with prices in the class defined by (16), are ex-ante symmetric and feature the same pattern of information provision to buyer and seller in the shuttle stage. They also guarantee equal split ex-post of the expected gains from trade if the buyer's value is above and the seller's cost is below $v^* = p_F = \frac{1}{2}$. On the other hand, it is easy to see that a completely equal ex-post split of the gains from trade is impossible, as it would require a price function equal to $p^B(v) = \frac{3}{4}v$, which violates the incentive compatibility constraint (15) for v close to 1/2 as well as (13).

5 Extensions

5.1 Generalizing the Uniform Case

To check the robustness of the characterization of the optimal mechanisms in Proposition 3, an instructive and easily computable generalization is to the case where the prior density functions of buyer and seller are identical and piecewise linear, with a peak or trough at $v = \frac{1}{2}$. That is, we consider:

$$f_0(v) = g_0(v) = \begin{cases} 1 - \frac{\beta}{4} + \beta v & \text{for} \quad v \in \left[0, \frac{1}{2}\right] \\ 1 + \frac{3\beta}{4} - \beta v & \text{for} \quad v \in \left[\frac{1}{2}, 1\right] \end{cases} \tag{17}$$

for some constant β that can be positive or negative but is anyway close to 0. At $v = \frac{1}{2}$, $f_0(v) = g_0(v)$ has a trough if $\beta < 0$ and a peak if $\beta > 0$. The value distributions remain identical and symmetric around 1/2, as in the case of uniform distributions (which obtain as a special case, when $\beta = 0$). We maintain then the same assumptions about the learning speeds and the symmetry of the price functions with $p_F = v^* = \frac{1}{2}$.

We will again consider two extreme candidate price functions. The first one is $p_H^B(v) = v$, the same as for the uniform case and the unique price function for which the second inequality constraint in (4) binds. The second price function, denoted by $p_{L_\beta}^B(v)$, is a generalization of the function $p_L^B(v)$ considered in the uniform case. The function $p_{L_\beta}^B(v)$ is implicitly defined by the first inequality constraint in (4) being satisfied as an equality for values $v \in [0, \widehat{v}_\beta]$ and constraint (7) being satisfied as an equality for values $v \in \left[\widehat{v}_\beta, \frac{1}{2}\right]$. As in the definition of $p_L^B(v)$, $\widehat{v}_\beta$ is the unique value at which both the first inequality constraint in (4) and constraint (7) hold as equalities.

We will show that $p_H^B(v) = v$ is admissible, in the sense that it satisfies all incentive constraints, if and only if the density has a peak at $v = \frac{1}{2}$, while $p_{L_\beta}^B(v)$ is admissible irrespective of whether the density has a peak or a trough.

Consider first $p_H^B(v) = v$. It is immediate to see that the same argument as in Section 4.1 ensures that (3), (4), (5), (6) hold. Hence the only condition we need to check is the incentive constraint (7), which for $p^B(v) = v$ can be written as:

$$\left(\int_v^1 x f_0(x)dx - v\left[1 - F_0(v)\right]\right) G_0(v) \leq \left(\int_{\frac{1}{2}}^1 x f_0(x)dx - \frac{1}{2}\left[1 - F_0\left(\frac{1}{2}\right)\right]\right) G_0\left(\frac{1}{2}\right) + \int_{\frac{1}{2}}^{1-v}\left(\int_y^1 x f_0(x)dx - y\left[1 - F_0(y)\right]\right) g_0(y)dy,$$

which, integrating by parts, becomes:

$$\int_v^1 \left[1 - F_0(x)\right] dx G_0(v) - \int_{\frac{1}{2}}^1 \left[1 - F_0(x)\right] dx G_0\left(\tfrac{1}{2}\right) - \int_{\frac{1}{2}}^{1-v}\int_y^1 \left[1 - F_0(x)\right] dx g_0(y)\, dy \leq 0$$

Denote the left hand side of the above inequality by $\Phi_H^B(v)$. The proof of the following lemma is in the appendix.

Lemma 3 *$\Phi_H^B(v) \leq 0$ for all $v \in \left[0, \frac{1}{2}\right]$ if and only if $\beta \geq 0$.*

Thus, $p_H^B(v) = v$ is admissible and allows to attain an ex-post efficient outcome when the prior densities are identical, piecewise linear and symmetric around $\frac{1}{2}$, if they have a peak at $v = \frac{1}{2}$ (i.e., if $\beta \geq 0$) but not if they have a trough ($\beta < 0$). For an intuitive explanation of this result, recall that with the price function $p_H^B(v)$ a buyer that uses a stopping-at-value strategy obtains a positive payoff only when her value is greater than both $\frac{1}{2}$ and the seller's value. To stop before having discovered her value allows the buyer to trade at a lower price at the cost of a smaller trading probability. When the buyer has discovered that her value is above $\frac{1}{2} - \epsilon$, that is, it is in a small left neighborhood of $v = \frac{1}{2}$, the price gain of stopping is approximately ϵ and the reduction in trading probability is approximately $\epsilon f_0\left(\frac{1}{2}\right)$, which is the probability that the seller's value is between $\frac{1}{2} - \epsilon$ and $\frac{1}{2}$. Thus, the reduction in trading probability when the buyer stops before having discovered her value is lower if the density function has a trough at $v = \frac{1}{2}$ than if it has a peak.[23]

We now consider the price function $p_{L_\beta}^B(v)$. Note that $p_{L_\beta}^B(v)$ is continuous in β and converges pointwise to $p_L^B(v)$ as β converges to 0. In addition, $p_{L_\beta}^B\left(\frac{1}{2}\right) = \frac{1}{2}$. The properties of $p_L^B(v)$ derived in the previous section then readily imply that, for sufficiently low β, $p_{L_\beta}^B(v)$ satisfies the first inequality constraint in (4) and constraints (7), (3) and (5). Thus, to establish that $p_{L_\beta}^B(v)$ is admissible it only remains to check that it satisfies (6) and the second inequality constraint in (4). Constraint (6) is satisfied for $v \leq \widehat{v}_\beta$, since $\frac{dp_{L_\beta}^B(v)G_0(v)}{dv} = vg_0(v)$. That (6) is also satisfied for $v \geq \widehat{v}_\beta$ follows from $p_{L_\beta}^B(v)$ being continuous in β, converging pointwise to $p_L^B(v) = \frac{6v-4v^3-1}{12v(1-v)}$ and the fact that $\frac{d\left(\frac{6v-4v^3-1}{12(1-v)}\right)}{dv} > v$ (i.e., $\frac{dp_L^B(v)v}{dv} > v$ and thus (6) holding strictly for $p_L^B(v)$). Finally, the fact that $p_L^B(v) = \frac{6v-4v^3-1}{12v(1-v)} < v$ implies that for small β the second inequality constraint in (4) is satisfied for $v < \frac{1}{2}$, while the property that it is satisfied at $v = \frac{1}{2}$ follows from $p_{L_\beta}^B\left(\frac{1}{2}\right) = \frac{1}{2}$.

5.2 Single-Disclosure Shuttle Diplomacy

In this subsection we show that the mediator could obtain the same ex-post efficient outcome as in the mechanism considered in the proof of Proposition 2, with all gains

[23] A symmetric argument holds for the seller.

from trade going to the seller, if she used a modification of the shuttle-diplomacy procedure. We call this modified procedure the *single-disclosure shuttle-diplomacy mechanism*. Such a mechanism is specified as follows: the mediator lets the seller fully observe her value in one round, asks her to report it, lets the buyer observe whether her value is above or below the value reported by the seller, and posts a price at which both buyer and seller decide to trade only if the buyer discovers to have a value above the value reported by the seller. It is clear that if the seller sends a truthful report about her value, this mechanism also allows to attain an ex-post efficient outcome.

The incentive constraint of the seller is however different from the constraints in the mechanism considered in the proof of Proposition 2, since the set of strategies available to her is different. The seller now makes a single report after having observed her value, while in the shuttle-diplomacy procedure considered in the proof of Proposition 2 discoveries take place slowly over time and the seller must make reports which are simpler – stop or continue – but are taken repeatedly along the way, before and after having discovered her true value. Hence a new argument is needed to establish a result analogous to Proposition 2. Before proceeding, however, it is worth repeating that the interpretation of the role of the mediator as an agent that allows parties to commit to acquiring limited information is problematic when all the ex post gains from trade go to one party. Why would the other party want to honor her commitment?

To see that a single-discovery shuttle diplomacy procedure can be constructed to induce efficient trading, observe that in such a mechanism the payoff of a seller who reports $\widehat{v}^S$ while her true value is v^S is:

$$u_S\left(\widehat{v}^S; v^S\right) = \left[p^S\left(\widehat{v}^S\right) - v^S\right]\left[1 - F_0\left(\widehat{v}^S\right)\right]$$

Let $u_S\left(v^S\right) = u_S\left(v^S; v^S\right)$ be the seller's indirect utility function when sincere reporting, $\widehat{v}^S = v^S$, is optimal. The first order condition of the seller's maximization problem requires that

$$\frac{du_S\left(v^S\right)}{dv^S} = -\left[1 - F_0\left(v^S\right)\right]$$

or, integrating both sides from v^S to 1 and using the boundary condition $u_S(1) = 0$:

$$\left[p^S\left(v^S\right) - v^S\right]\left[1 - F_0\left(v^S\right)\right] = u_S\left(v^S\right) = \int_{v^S}^{1}\left[1 - F_0\left(\widetilde{v}^S\right)\right] d\widetilde{v}^S.$$

Integrating by parts and rearranging we obtain

$$p^S\left(v^S\right) = \int_{v^S}^{1} \frac{\widetilde{v}^S}{1 - F_0\left(v^S\right)} dF_0\left(\widetilde{v}^S\right) = \mathbb{E}_{F_0}\left[v^B | v^B \geq v^S\right].$$

Note that this is the same price function as the one used in the proof of Proposition 2; it gives all the gains from trade to the seller.

We thus have shown that in the single-discovery shuttle diplomacy mechanism, $p^S\left(v^S\right) = \mathbb{E}_{F_0}\left[v^B | v^B \geq v^S\right]$ is the only price function that guarantees the incentive compatibility of truthful reporting of her value by the seller. Also note that it is a strictly dominant continuation strategy for the seller to accept to trade at the posted price, while the buyer is also willing to trade when her value is above the seller's as she obtains a zero payoff either way. Given this, it is always optimal for the seller to report sincerely. As argued, above this ensures that trade occurs whenever it is ex-post efficient. We have thus established the following:

Proposition 4 *Under the single-discovery shuttle diplomacy mechanism, there exists a unique price function* $p^S\left(v^S\right) = \mathbb{E}_{F_0}\left[v^B | v^B \geq v^S\right]$ *such that it is a perfect Bayesian equilibrium for the seller to sincerely report her value and for buyer and seller to accept to trade at the posted price when it is ex-post efficient to trade.*[24]

6 Conclusions

Psychologists have argued that mediators help parties to overcome psychological barriers to conflict resolution. We have argued that a mediator will also help when parties are rational, strategic negotiators.

[24] It is obvious that an analogous ex-post efficient, single-discovery shuttle diplomacy procedure giving all the gains from trade to the buyer could be constructed.

The approach adopted in this paper may be described as an information disclosure and allocation mechanism design approach. Both the private information disclosed to agents and the trading protocol are chosen by a designer, in our case the mediator. The approach blends the classical mechanism design approach with the information design, or Bayesian persuasion, approach. In classical mechanism design, agents have full private information and the designer only selects a procedure to determine the allocation as a function of the information reported by the parties. In information design, the allocation mechanism is exogenously fixed and the designer may only select the information disclosure policy.

We have shown that some amount of obfuscation is optimal: it is best for efficiency not to give full information to all parties as this allows to increase the probability that a settlement is reached. We also showed that dynamic procedures, whereby information is progressively revealed to the parties, and parties send reports which affect both the terms of a possible transaction and the future flow of information, allow to further increase the welfare gains that are realized. In the case we consider, concerning the possibility of trading an object between two parties, a first best outcome is achieved, where all ex-post gains from trade are realized with the optimal dynamic mechanism.

We believe that the approach considered in this paper could be fruitfully applied to other settings, beyond the case of the transaction between two parties we considered.

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Appendix

In this appendix we present results and proofs omitted from the main body of the paper.

Proof of Lemma 1 Suppose p, F, G are maximizers of (1). We show in what follows that p and the following two point distributions are also maximizers of (1). For buyers, take the distribution that puts mass $1 - R_F(p)$ on $v_L^B = \mathbb{E}_F[v|v < p]$ and mass $R_F(p)$ on $v_H^B = \mathbb{E}_F[v|v \geq p]$. For sellers, take the distribution that puts mass $G(p)$ on $v_L^S = \mathbb{E}_G[v|v \leq p]$ and mass $1 - G(p)$ on $v_H^S = \mathbb{E}_G[v|v > p]$. It is then immediate to see that the expression in (1), evaluated at the solution p, F, G, is equal to:

$$\left(v_H^B - v_L^S\right) R_F(p)G(p),$$

that is, to the value of (1) at the two-point distributions described above. Note that if $F \in \mathcal{F}$ the specified two-point distribution for the buyer also belongs to $\mathcal{F}$, and similarly for the seller. This shows that, in solving her maximization problem, the mediator may restrict attention to two-point discrete distributions. □

Proof of Proposition 1 To establish the result, we characterize first the classes of feasible two-point signal distributions. For buyers, let $\{v_L^B, v_H^B\}$ be the set of possible signals with associated probabilities f_L, $f_H = 1 - f_L$. The following constraints must hold to guarantee that F_0 is a mean preserving spread of the signal distribution:

$$v_L^B f_L + v_H^B (1 - f_L) = v_0^B \tag{18}$$

$$\left(x - v_L^B\right) f_L \leq \int_0^x F_0(v)dv \quad \text{for } x \in [v_L^B, v_H^B) \tag{19}$$

$$\left(v_H^B - v_L^B\right) f_L + \left(x - v_H^B\right) \leq \int_0^x F_0(v)dv \quad \text{for } x \in \left[v_H^B, 1\right] \tag{20}$$

We can then solve (18) for v_L^B and replace the obtained solution into the other two constraints:

$$(x - v_H^B) f_L - v_0^B + v_H^B - \int_0^x F_0(v)dv \leq 0 \quad \text{for } x \in [v_L^B, v_H^B) \tag{21}$$

$$x - v_0^B - \int_0^x F_0(v)dv \leq 0 \quad \text{for } x \in \left[v_H^B, 1\right] \tag{22}$$

It is immediate to see that (22) holds since, integrating by parts,

$$\begin{aligned} x - v_0^B - \int_0^x F_0(v)dv &= x[1 - F_0(x)] - v_0^B + \int_0^x vdF_0(v) \\ &= xR_{F_0}(x) - \int_x^1 vdF_0(v) \leq 0. \end{aligned}$$

It is also immediate to see that the left hand side of (21) is a concave function of x. Define x^B as the solution to $f_L = F_0(x)$. There are three possible cases.

Case 1: $x^B \in [v_L^B, v_H^B]$. Then the left hand side of (21) is maximized at x^B and condition (21) holds if it is satisfied for $x = x^B$. Hence we can rewrite this condition as:

$$(x - v_H^B)F_0(x) - v_0^B + v_H^B - xF_0(x) + \int_0^x vdF_0(v) \leq 0 \quad \text{or,}$$

$$v_H^B R_{F_0}(x) - \int_x^1 vdF_0(v) \leq 0 \tag{23}$$

Condition (23) is quite intuitive: it says that the value of the buyer when she receives the high signal, v_H^B, cannot exceed the expected value of the buyer, conditional on this value lying above some threshold x, evaluated according to the posterior distribution F_0.

Case 2: $x^B < v_L^B$, that is the two point signal distribution is such that $F(v_L^B) > f_L$. In this case the left hand side of (21) is maximized at $x = v_L^B$, hence condition (21) can be rewritten as:

$$-\int_0^x F_0(v)dv \leq 0,$$

and is always satisfied.

Case 3: $x^B > v_H^B$, the two point signal distribution is such that $f_L > F(v_H^B)$. The expression on the left hand side of (21) is now maximized at $x = v_H^B$ and we can rewrite (21) as:

$$-v_0^B + v_H^B - \int_0^{v_H^B} F_0(v)dv \leq 0, \tag{24}$$

which we can also show always to hold.[25]

We may repeat the same argument for the seller, letting $\{v_L^S, v_H^S\}$ be the set of possible signals with probabilities g_L, g_H, with $v_L^S g_L + v_H^S g_H = v_0^S$ and the counterparts of (21) and (22). Using y instead of x, we see that the left hand side of the counterpart of (21) is a concave function of y. The expression on the left hand side of the constraint reaches then the highest value either at y equal to v_L^S or v_H^S, in which case the constraint is always satisfied, or at y such that $g_L = G_0(y)$, in which case the constraint can be rewritten as:

$$\begin{aligned} v_H^S R_{G_0}(y) - \int_y^1 v dG_0(v) \leq 0 \quad \text{or,} \\ \int_0^y v dG_0(v) - v_L^S G_0(y) \leq 0 \end{aligned} \tag{25}$$

We can then use Lemma 1 and the characterization we obtained of two-point signal distributions to rewrite the mediator's problem, (1). As we said, a mediator aiming to maximize the gains from trade will choose distributions and a price such that the buyer is willing to trade when he gets the high signal and the seller with the low signal, that is: $v_H^B \geq p \geq v_L^S$. Also, we can show that the situation described in Cases 2 and 3 above never arises at a solution of the mediator's problem,[26] hence neither constraint

[25] We can in fact rewrite (24) as

$$\begin{aligned} -v_0^B + v_H^B \left[1 - F_0(v_H^B)\right] + \int_0^{v_H^B} v dF_0(v) \leq 0 \quad \text{or,} \\ v_H^B \left[1 - F_0(v_H^B)\right] - \int_{v_H^B}^1 v dF_0(v) \leq 0, \end{aligned}$$

which always holds.

[26] Consider the information provided to the buyer. Suppose first the solution of the mediator's problem falls in Case 2. Then $F(v_L^B) > f_L = F(x^B)$ and it is possible to raise the gains from trade by keeping f_L constant, reducing v_L^B and increasing v_H^B while satisfying the only binding constraint $v_L^B f_L + v_H^B (1 - f_L) = v_0^B$, a contradiction. Second, suppose the solution falls in Case 3. We have

(23) nor (25) can be ignored, and we can replace the choice variables f_L and g_L with $F_0(x)$ and $G_0(y)$. The mediator problem (1) can then be rewritten as follows:

$$\max_{v_H,c_L,x,y} \left(v_H^B - v_L^S\right) G_0(y) R_{F_0}(x) \quad \text{s.t.} \tag{26}$$
$$v_H^B R_{F_0}(x) + v_L^B F_0(x) = v_0^B$$
$$v_H^B R_{F_0}(x) - \int_x^1 v dF_0(v) \leq 0$$
$$v_H^S R_{G_0}(y) + v_L^S G_0(y) = v_0^S$$
$$\int_0^y v dG_0(v) - v_L^S G_0(y) \leq 0$$

Furthermore, it is immediate to see that both inequality constraints must bind, otherwise the mediator would profit from raising v_H^B or lowering v_L^S. Hence, substituting the constraints into the objective function, the problem reduces to:

$$\max_{x,y} \left(\mathbb{E}_{F_0}[v^B|v^B \geq x] - \mathbb{E}_{G_0}[v^S|v^S \leq y]\right) G_0(y) R_{F_0}(x) \tag{27}$$

The interpretation of (27) is as follows. The fact that the two constraints in (26) hold as equality means that the mediator lets the buyer observe exactly whether her value is greater than or equal to x and lets the seller observe whether her value is smaller than or equal to y. Trade takes place when both events realize, as ensured by posting any price $p \in \left[\mathbb{E}_{G_0}[v^S|v^S \leq y], \mathbb{E}_{F_0}[v^B|v^B \geq x]\right]$. The values of x and y are then optimally chosen to maximize expected gains from trade.

The final step of the proof is then to characterize the solutions of program (27). Since the domain of (x, y) is compact (the unit square) and the objective function and constraints are continuous in x and y, program (27) has a solution. Setting $x = 1$ or $y = 0$ cannot be optimal, as it yields a zero payoff to the mediator, which is less than the payoff that could be achieved by setting $0 < x = y < 1$. Thus the only possible

so $F(v_H^B) < f_L = F(x^B)$. If $x^B < 1$ it is again possible to raise the gains from trade by keeping f_L constant, reducing v_L^B and increasing v_H^B while satisfying $v_L^B f_L + v_H^B (1 - f_L) = v_0^B$, a contradiction. If instead $x^B = 1$, then $f_L = 1$ and with probability 1 there is no trade; this is also a contradiction as full disclosure and any interior posted price $p \in (0, 1)$ would generate positive gains from trade.

boundary solutions have $x = 0$ and/or $y = 1$.

Since F_0 and G_0 have no atoms, the first order conditions of program (27), taking into account the constraints $x \geq 0$ and $1 - y \geq 0$, are:

$$\begin{aligned}
-xf_0(x)G_0(y) + \int_0^y vdG_0(v)f_0(x) &\leq 0 \\
\left(-xf_0(x)G_0(y) + \int_0^y vdG_0(v)f_0(x)\right)x &= 0 \\
\int_x^1 vdF_0(v)g_0(y) - yg_0(y)R_{F_0}(x) &\geq 0 \\
\left(\int_x^1 vdF_0(v)g_0(y) - yg_0(y)R_{F_0}(x)\right)(1-y) &= 0
\end{aligned}$$

Note that if $x = 0$, the first inequality is violated, as the term on the left hand side is strictly positive. Similarly, if $y = 1$ the second inequality is violated, as the expression on the left hand side is strictly negative. Thus there are no boundary solutions, the solution is interior and satisfies the conditions:

$$\begin{aligned}
&-xf_0(x)G_0(y) + \int_0^y vdG_0(v)f_0(x) = 0 \quad \text{and} \\
&\int_x^1 vdF_0(v)g_0(y) - yg_0(y)R_{F_0}(x) = 0
\end{aligned}$$

which can be written as

$$\begin{aligned}
&\mathbb{E}_{G_0}[v|v \leq y] = x \quad \text{and} \\
&\mathbb{E}_{F_0}[v|v \geq x] = y
\end{aligned}$$

This concludes the proof of the proposition. $\square$

Lemma 4 *If constraint (6) holds, then the buyer prefers to stop after having discovered her value in period $t < T^B - \Delta$ rather than continue and stop after more than one period.*

Proof We directly consider the limit when $\Delta \to 0$. The buyer's payoff from continuing after having discovered her value at t and stopping in period $\tau > t$ is

$$\left[v_t^B - p^B\left(v_\tau^B\right)\right] \frac{G_0\left(v_\tau^B\right)}{G_0\left(v_t^S\right)},$$

and we thus need to show that

$$\left[v_t^B - p^B\left(v_t^B\right)\right] G_0\left(v_t^B\right) \geq \left[v_t^B - p^B\left(v_\tau^B\right)\right] G_0\left(v_\tau^B\right). \tag{28}$$

By (6), when $\frac{dv_t^B}{dt} > 0$ we have

$$v_t^B g_0\left(v_t^B\right) \leq \frac{dp^B\left(v_t^B\right) G_0\left(v_t^B\right)}{dv_t^B}.$$

Integrating both sides gives

$$\int_{v_t^B}^{v_\tau^B} v g_0(v)\, dv \leq p^B\left(v_\tau^B\right) G_0\left(v_\tau^B\right) - p^B\left(v_t^B\right) G_0\left(v_t^B\right),$$

and integrating by parts the left hand side yields

$$v_\tau^B G_0\left(v_\tau^B\right) - v_t^B G_0\left(v_t^B\right) - \int_{v_t^B}^{v_\tau^B} G_0(v)\, dv \leq p^B\left(v_\tau^B\right) G_0\left(v_\tau^B\right) - p^B\left(v_t^B\right) G_0\left(v_t^B\right),$$

or, equivalently,

$$\left[v_t^B - p^B\left(v_\tau^B\right)\right] G_0\left(v_\tau^B\right) + \left[v_\tau^B - v_t^B\right] G_0\left(v_\tau^B\right) - \int_{v_t^B}^{v_\tau^B} G_0(v)\, dv \leq \left[v_t^B - p^B\left(v_t^B\right)\right] G_0\left(v_t^B\right).$$

The continuity of G_0 implies that there exists a $v_{t^*}^B \in \left(v_t^B, v_\tau^B\right)$ such that the inequality above can be written as

$$\left[v_t^B - p^B\left(v_\tau^B\right)\right] G_0\left(v_\tau^B\right) + \left[v_\tau^B - v_t^B\right] G_0\left(v_\tau^B\right) - \int_{v_t^B}^{v_\tau^B} G_0\left(v_{t^*}^B\right) dv \leq \left[v_t^B - p^B\left(v_t^B\right)\right] G_0\left(v_t^B\right),$$

or

$$\left[v_t^B - p^B\left(v_\tau^B\right)\right] G_0\left(v_\tau^B\right) + \left[v_\tau^B - v_t^B\right]\left[G_0\left(v_\tau^B\right) - G_0\left(v_{t^*}^B\right)\right] \leq \left[v_t^B - p^B\left(v_t^B\right)\right] G_0\left(v_t^B\right).$$

Hence, (28) must hold. This proves that constraint (6) is necessary and sufficient for the buyer to prefer stopping when she has discovered her value rather than continuing and stopping after one or more periods. □

Derivation of the seller's incentive constraint (8) Assume that the buyer adopts a stopping-at-value strategy and suppose that the seller has just discovered that her value is v_t^S. This implies that the buyer has discovered that her value is above $\alpha_{t-\Delta}^B$; thus, if the seller stops the shuttle stage, her expected payoff is:

$$\left[p^S\left(v_t^S\right)-v_t^S\right]\frac{1-F_0\left(v_t^S\right)}{1-F_0\left(\alpha_{t-\Delta}^S\right)}.$$

Since the buyer accepts to trade at price $p^B\left(v_t^S\right)$ when her value is above v_t^S by (4), $\frac{1-F_0\left(v_t^S\right)}{1-F_0\left(\alpha_{t-\Delta}^S\right)}$ is the seller's conditional probability of trading and $p^S\left(v_t^S\right)-v_t^S$ is the gain from trade.

If the seller continues, by (4) and (5), she only obtains a positive payoff if she is the one stopping at a later date of the shuttle diplomacy stage. The sellers's payoff from stopping next period, when the disclosed value is $v_{t+\Delta}^S$, is

$$\left[p^S\left(v_{t+\Delta}^S\right)-v_t^S\right]\frac{1-F_0\left(v_{t+\Delta}^S\right)}{1-F_0\left(\alpha_{t-\Delta}^B\right)}.$$

Thus, the following constraint must hold for all t:

$$\left[p^S\left(v_t^S\right)-v_t^S\right]\left[1-F_0\left(v_t^S\right)\right]-\left[p^S\left(v_{t+\Delta}^S\right)-v_t^S\right]\left[1-F_0\left(v_{t+\Delta}^S\right)\right]\geq 0.$$

Adding and subtracting $p^S\left(v_{t+\Delta}^S\right)\left[1-F_0\left(v_t^S\right)\right]$, the constraint can be written as:

$$\left[p^S\left(v_t^S\right)-p^S\left(v_{t+\Delta}^S\right)\right]\left[1-F_0\left(v_t^S\right)\right]-\left(p^S\left(v_{t+\Delta}^S\right)-v_t^S\right)\left(F_0\left(v_t^S\right)-F_0\left(v_{t+\Delta}^S\right)\right)\geq 0.$$

Multiplying by $\frac{v_t^S-v_{t+\Delta}^S}{\Delta}\frac{1}{v_t^S-v_{t+\Delta}^S}$ and taking limits as $\Delta\to 0$, we obtain the following constraint for all $v_t^S \in [v^*, 1]$:

$$\left[\frac{dp^S\left(v_t^S\right)}{dv_t^S}\left[1-F_0\left(v_t^S\right)\right]-\left(p^S\left(v_t^S\right)-v_t^S\right)f_0\left(v_t^S\right)\right]\left(-\frac{dv_t^S}{dt}\right)\geq 0.$$

This is constraint (8) in the main body of the paper.

We have shown that constraint (8) is necessary and sufficient for the seller to prefer stopping when she has discovered her value to continuing and stopping next period; it remains to show to (8) is also sufficient for the seller to prefer stopping when she has discovered her value rather than continuing and stopping after more than one period. Consider the limit mechanism when $\Delta \to 0$, the seller's payoff from continuing after having discovered her value at t and stopping in some future period $\tau > t$ is

$$\left(p^S\left(v_\tau^S\right)-v_t^S\right)\frac{1-F_0\left(v_\tau^S\right)}{1-F_0\left(v_t^B\right)},$$

and we thus need to show that

$$\left(p^S\left(v_t^S\right)-v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)\geq\left(p^S\left(v_\tau^S\right)-v_t^S\right)\left(1-F_0\left(v_\tau^S\right)\right). \tag{29}$$

By (8), when $\frac{dv_t^S}{dt} < 0$ we have

$$v_t^S f_0\left(v_t^S\right)\geq -\frac{dp^S\left(v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)}{dv_t^S}.$$

Integrating both sides gives

$$\int_{v_\tau^S}^{v_t^S} v f_0(v)\,dv \geq p^S\left(v_\tau^S\right)\left(1-F_0\left(v_\tau^S\right)\right)-p^S\left(v_t^S\right)\left(1-F_0\left(v_t^S\right)\right),$$

and integrating by parts the left hand side yields

$$-v_\tau^S F_0\left(v_\tau^S\right)+v_t^S F_0\left(v_t^S\right)-\int_{v_\tau^S}^{v_t^S} F_0(v)\,dv+p^S\left(v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)\geq p^S\left(v_\tau^S\right)\left(1-F_0\left(v_\tau^S\right)\right),$$

or, equivalently,

$$\left(p^S\left(v_t^S\right)-v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)+\int_{v_\tau^S}^{v_t^S}\left(1-F_0\left(v\right)\right)dv\geq\left(p^S\left(v_\tau^S\right)-v_\tau^S\right)\left(1-F_0\left(v_\tau^S\right)\right).$$

The continuity of F_0 implies that there exists a $v_*^S\in\left(v_\tau^S,v_t^S\right)$ such that the inequality above can be written as

$$\left(p^S\left(v_t^S\right)-v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)+\int_{v_\tau^S}^{v_t^S}\left(1-F_0\left(v_*^S\right)\right)dv\geq\left(p^S\left(v_\tau^S\right)-v_\tau^S\right)\left(1-F_0\left(v_\tau^S\right)\right),$$

or

$$\left(p^S\left(v_t^S\right)-v_t^S\right)\left(1-F_0\left(v_t^S\right)\right)-\left(v_t^S-v_\tau^S\right)\left(F_0\left(v_*^S\right)-F_0\left(v_\tau^S\right)\right)\geq\left(p^S\left(v_\tau^S\right)-v_t^S\right)\left(1-F_0\left(v_\tau^S\right)\right).$$

Hence, (29) must hold.

□

Proof of Lemma 3 First, it is immediate to see that $\Phi_H^B\left(\frac{1}{2}\right)=0$. In addition, using the symmetry properties $G_0(v)=F_0(v)$, $g_0(v)=f_0(v)=g_0(1-v)=f_0(1-v)$ and $1-F_0(1-v)=F_0(v)$, we have:

$$\begin{aligned}\frac{d\Phi_H^B(v)}{dv} &= -\left[1-F_0(v)\right]F_0\left(v\right)+\int_v^1\left[1-F_0\left(x\right)\right]dx f_0\left(v\right)+\int_{1-v}^1\left[1-F_0\left(x\right)\right]dx f_0(v)\\ &= -\left[1-F_0(v)\right]F_0\left(v\right)+\int_0^{1-v}F_0\left(x\right)dx f_0\left(v\right)+\int_0^v F_0\left(x\right)dx f_0(v)\end{aligned}$$

Recalling that for the piecewise linear density function specified in (17) it is $F_0(v)=\left(1-\frac{\beta}{4}\right)v+\frac{\beta}{2}v^2$ for $v\leq\frac{1}{2}$, we may evaluate the derivative $\frac{d\Phi_H^B(v)}{dv}$ at $v=\frac{1}{2}$:

$$\begin{aligned}\left.\frac{d\Phi_H^B(v)}{dv}\right|_{v=1/2} &= -\frac{1}{4}+2\left(\left(1-\frac{\beta}{4}\right)\frac{1}{8}+\frac{\beta}{6}\frac{1}{8}\right)\left(1+\frac{\beta}{4}\right)\\ &= -\frac{1}{4}+\frac{1}{4}\left(1-\frac{\beta}{12}\right)\left(1+\frac{\beta}{4}\right)\\ &= \frac{\beta}{24}\left(1-\frac{\beta}{8}\right)\end{aligned}$$

It follows that if $\beta < 0$, that is if the density function has a trough at $v = \frac{1}{2}$, then the slope of Φ_H^B is negative at $v = \frac{1}{2}$. Since $\Phi_H^B\left(\frac{1}{2}\right) = 0$, this implies that $\Phi_H^B(v) > 0$ for values of v smaller but sufficiently close to $\frac{1}{2}$ and hence for those values the incentive constraint (7) is violated. On the contrary, if $\beta > 0$, that is, if the density function has a peak at $v = \frac{1}{2}$, then $\Phi_H^B(v) < 0$ and so the incentive constraint holds for values of v in a left neighborhood of $v = \frac{1}{2}$. To show that it holds for all $v \in \left[0, \frac{1}{2}\right)$ we now demonstrate that Φ_H^B is a concave function of $v \in \left[0, \frac{1}{2}\right)$ for a sufficiently small $\beta > 0$. Differentiating again Φ_H^B yields,

$$\begin{aligned}\frac{d^2\Phi_H^B(v)}{dv^2} &= -2\left[1 - 2F_0(v)\right] f_0(v) + \left(\int_0^{1-v} F_0(x)\,dx + \int_0^v F_0(x)\,dx\right)\frac{df_0(v)}{dv} \\ &= -2\left[1 - \left(2 - \tfrac{\beta}{2}\right)v - 2\beta v^2\right]\left[1 - \tfrac{\beta}{4} + \beta v\right] + \beta\left(\int_0^{1-v} F_0(x)\,dx + \int_0^v F_0(x)\,dx\right)\end{aligned}$$

It is then immediate to see that as $\beta \to 0$, $\frac{d^2\Phi_H^B(v)}{dv^2} \to -2(1-2v)$, which is negative for all $v < \frac{1}{2}$. Since $\frac{d^2\Phi_H^B(v)}{dv^2}$ is continuous in β, it follows that for sufficiently small $\beta > 0$ the function $\Phi_H^B(v)$ is concave for $v \in \left[0, \frac{1}{2}\right)$. $\square$