Public Policy towards R&D in a Mixed Duopoly with Spillovers

Maria José Gil Moltó, Joanna Poyago–Theotoky and Vasileios Zikos*

Department of Economics, Loughborough University

Abstract

We investigate the use of subsidies to R&D, both in a mixed and a private duopoly market. We show that the socially optimal R&D subsidy is positive and increasing in the degree of spillovers both in the private and the mixed duopoly, although it is lower for the former than for the latter. We also find support for the empirical claim that privatization is followed by a scaling down of the R&D activity. A comparative static analysis of welfare levels suggests that privatization is welfare detrimental, which lends some support to the views against the widespread adoption of privatization programs.

JEL Classification: L31, L32, O38, L13, L50.

Keywords: mixed duopoly, process innovation, R&D subsidies, privatisation, spillovers.

*Corresponding author: Department of Economics, Loughborough University, Loughborough LE11 3TU, United Kingdom. Email: ecvz@lboro.ac.uk, tel.: + 44 (0) 1509 222708, fax: + 44 (0) 1509 26317. Vasileios Zikos greatfully acknowledges financial support by Loughborough University under its grant scheme. We also thank seminar participants at Alicante and Leicester for their helpful comments.

1 Introduction

The theoretical literature on R&D has dealt extensively with the issues of investment in cost-reducing R&D in the presence of spillovers. In fact, since the seminal paper by d'Aspremont and Jacquemin (1988), a number of authors have extended this original model in a number of ways. However, the presence of public (or state) firms and the role of public policy in this context, in the form of R&D subsidies and privatizations has not yet been addressed. In contrast, there is strong empirical evidence on innovative industries of the importance of the public sector, defined broadly as state-owned laboratories, public universities, technological institutes and public enterprises (see Katz 2001; Poyago-Theotoky 1998).

An example of a public firms' importance in terms of R&D investment, can be observed in the health-care sector, where private and public firms normally coexist. Nowadays, the health care sector is facing major challenges that call for improved quality and increased efficiency in resource utilization (Aanestad et al. 2003).[1] By means of R&D activities, hospitals attempt to improve their organizational arrangements and to enhance existing processes and work practises. Another example comes from agriculture, where the public sector has been the primary source of research in biotechnology (Oehmke 2001). With regard to public subsidies towards R&D, it is important to note that the emergence and development of many discoveries has been facilitated by public funding of R&D, e.g., biotechnology (Hart 1998). This highlights the importance of the role that technology policy in the form of R&D subsidization can play in high-tech industries.

On the other hand, there is a continuing public debate on the effects and advisability of privatization, which has consequently generated a great interest among economists in mixed markets and in the potential welfare effects of privatization.[2] However, the R&D aspect of the firms' activity has been largely ignored by the literature. Although there are a few exceptions, such as Delbono and Denicoló (1993), Poyago-Theotoky (1998) and Nett (1994),[3] none of these contributions has considered the use of R&D subsidies

[1] Aanestad et. al. (2003) provide empirical evidence from the Interventional Centre established as a medical R&D department in a Norwegian public hospital (Rikshospitalet) in order to explore and develop new technologies and procedures.

[2] See e.g., De Fraja and Delbono (1989), (1990), Fjell and Pal (1996), Matsumura (1998), Pal and White (1998) and Willner (1999).

[3] Assuming perfect patent protection that yields R&D overinvestment in the private

by the policy-maker or the existence of spillovers in the context of efficiency-enhancing innovation.

To the best of our knowledge, this is the first paper to study efficiency-enhancing (cost-reducing) R&D in the context of a mixed duopoly with spillovers and R&D subsidies. We aim at shedding some light on the effect of R&D subsidies and spillovers on the R&D effort patterns of public and private firms and the comparative performance of these two types of firms in cost-reducing innovation. Evidence on the latter issue goes back to the seminal works by Nelson (1959) and Arrow (1962) that identified a gap between private and social returns to R&D. According to this observation, public firms are more likely to address 'social' (welfare maximization) than pure 'firm-specific' (profit maximization) objectives and in turn to spend more in R&D.[4] We also study the effects of privatization on R&D in order to provide a plausible explanation for the observed post-privatization scaling down of R&D activity (see Munari and Sobrero 2000, Munari and Oriani 2001 and Katz 2001, among others) and provide some tentative policy guidelines.

A key feature in this study is the failure of R&D market to produce socially optimal levels. An understanding of the relevant forces at work in determining the market outcome provides a useful framework within which our model can be analysed. These forces may be explained as follows.

Consider initially the case of no spillover β, i.e., R&D is perfectly appropriable. In the course of conducting R&D prior to choosing output, firms will tend to over-invest as a means of enhancing their own competitive position. According to this perspective, there is a strategic over-investment effect. However, when a positive spillover occurs, and as β rises, this tends to discourage cost-reducing R&D in the usual duopolistic setting.[5] In our mixed duopoly model, however, a higher spillover rate induces an increase in the total level of R&D – in fact, this carries over in the post-privatization

duopoly, Delbono and Denicoló (1993) suggested the presence of a public firm as a means for alleviating this problem. According to Poyago–Theotoky (1998), by relaxing the main assumption of Delbono and Denicoló, almost all their results can be reversed. Nett (1994) established as to why public firms opt for producing at a higher cost than the private firm. Further, Nett showed that welfare in the private duopoly may exceed welfare in the mixed duopoly.

[4] Empirical evidence also pinpoints to the role of state-owned enterprises in the development and evolution of national innovation systems by means of their R&D investments (see e.g., Nelson 1993 and Katz 2001).

[5] See, for example, D' Asprémont and Jacquemin (1988) among others.

regime.[6]

A private firm, in addition, does not take into account consumer surplus in its objective function. This implies another type of market failure, the so-called under-valuation effect (see Katsoulacos and Ulph 1998). Contrary to this, the public firm's objective is consistent with welfare maximization which in turn promotes an increase in the equilibrium level of R&D. This means that the public firm may serve as an instrument for alleviating under-investment. However, there is a second and opposing force. Namely, the public firm will introduce another type of market failure – inefficiency in production – related to the composition of R&D, i.e., there is an asymmetry in the distribution of post-R&D costs.[7]

In what follows, we postulate that the failure of the R&D market to produce socially optimal levels is addressed by a (positive) subsidy to R&D output. Since the regulator faces two failures – both in the R&D and output markets with one policy tool at her disposal – this approach would naturally translate into a second-best optimal solution.

Our model considers a homogeneous good Cournot duopoly in which firms undertake cost-reducing (process) innovation. This draws on and extends the specification introduced by d' Aspremont and Jacquemin (1988), including subsidies towards R&D. Our results show that the optimal R&D subsidy is always positive and increasing in the degree of spillovers. Moreover, we find that private industries should be subsidized less than mixed ones. Our findings also suggest that even though the R&D of the public firm may decrease as a result of privatization, the R&D of the private firm may increase or decrease, depending on the rate of spillovers and consequently, on the extent of the appropriability problem. Finally, regarding the effects of privatization on social welfare, our analysis reveals that privatization is detrimental to social welfare. Thus, from a welfare point of view, a mixed duopoly is always better, which lends some support to the popular views against the widespread application of privatization programmes.

The remainder of the paper is organized as follows. Section 2 presents

[6] Even though, this case does not bring about a market failure itself, it turns out that the overall impact on total R&D output is negative, implying a sub-obtimal level of cost-reducing innovation.

[7] Notice that the fundamental failures arising in the R&D market coexist with the ones in the output market: imperfect competition and inefficient distribution of post-innovation costs.

the model. Sections 3 and 4 study the cases of the mixed and private duopolies, respectively. A comparison of both cases and the implications for policy-making are discussed in Section 5. Section 6 concludes.

2 The model

We consider a market setting consisting of two firms competing in output. We compare two market structures: a mixed duopoly (one of the two firms is public) and a private duopoly. Private firms are assumed to be profit-maximizing while the public firm is assumed to maximize social welfare. In the case of the mixed duopoly, we denote with subscript 0 the public firm and with subscript 1 the private firm. The inverse demand function for the homogeneous good produced by the firms is

$$P(Q) = a - Q, \;\; 0 \leq Q < a \tag{1}$$

where $Q = q_i + q_j$, $i \neq j$, $i, j \in \{0, 1\}$.

Firms engage in cost-reducing (process) innovation in order to lower their marginal cost, following research paths that are perfect substitutes, i.e. we consider a non-tournament R&D setting.[8] The effective level of R&D, X_i, represents the reduction in marginal cost due to R&D, and has two components: the own level of R&D output, x_i, and the competitor's R&D output, x_j, via spillovers

$$X_i = x_i + \beta x_j, \;\; i \neq j, \;\; i, j \in \{0, 1\} \tag{2}$$

where the extent of information leakage or degree of spillovers among firms is captured by the parameter β, which is assumed to be exogenously given and $0 \leq \beta \leq 1$. Thus, firm i's total cost function depends on its level of production, q_i, and the effective level of R&D, X_i. To avoid a natural monopoly, which is not relevant for the purposes of our paper,[9] we assume the existence of diminishing returns to scale by introducing a quadratic term related to production in a firm's cost function. Hence, production cost is

[8] This means that the research firms undertake research leads to the same discovery (see eg. Katsoulacos and Ulph, 1998).

[9] We are interested in the strategic interaction in R&D between firms.

represented as

$$C_i(q_i, X_i) = (c - X_i)q_i + q_i^2, \;\; i \in \{0, 1\}, \;\; a > c > 0 \tag{3}$$

which yields marginal cost of production $mc_i = \frac{\partial C_i}{\partial q_i} = (c - X_i) + 2q_i$. Notice that the effective level of R&D, X_i, affects only the intercept of the marginal cost (i.e. it shifts the marginal cost curve downwards) but not its slope. This is the same effect that process R&D has on production costs in D'Aspremont and Jacquemin (1988) and followers, where production costs are assumed to be linear. [10]

We assume that R&D is subject to diminishing returns at an increasing rate so that firm i's R&D cost function can be written as

$$\Gamma_i(x_i) = x_i^2, \;\; i \in \{0, 1\}. \tag{4}$$

Hence, by investing x_i^2 in R&D, a firm can lower its costs by x_i due to its own research effort and by an additional amount βx_j via unpaid appropriation of some part of the rival firm's effort. The government subsidizes the R&D output of each firm. Each firm receives a subsidy

$$S_i = sx_i \tag{5}$$

where s is the subsidy per-unit of R&D output. Using expressions (1), (3), (2), (4) and (5), we obtain firm i's profit function

$$\pi_i = P(Q)q_i - C_i(q_i, X_i) - \Gamma_i(x_i) + S_i, \;\; i \in \{0, 1\}. \tag{6}$$

Social welfare is given by the aggregation of consumer surplus (CS) and producers surplus (PS) net of subsidies

$$SW = \underbrace{\frac{1}{2}Q^2}_{\text{CS}} + \underbrace{\pi_i + \pi_j}_{\text{PS}} - \underbrace{s(x_i + x_j)}_{\text{Subsidy}}. \tag{7}$$

Note that the subsidy has no *direct effect* on social welfare and hence on the objective function of the public firm, as it cancels out when aggregating

[10] This formulation allows us to introduce diminishing returns in production and maintain the spirit of previous contributions in a simple way and without loss of generality.

$$SW = \frac{1}{2}Q^2 + \sum_{i=0}^{1}[P(Q)q_i - C_i(q_i, X_i) - \Gamma_i(x_i)].$$

However, the subsidy affects social welfare indirectly via firms' R&D choices and as a consequence, via R&D costs and reductions of marginal production costs. Also a further indirect (and strategic) effect takes place: even though there is no direct effect of the subsidy on the objective function of the public firm, the public firm's R&D (and output) will be affected by the subsidy through its impact on the private firm's R&D choice.

In order to study the effects of R&D subsidies on R&D and the effects of privatization on innovation, welfare and on the optimal subsidy, we consider a simple three-stage game with observable actions. Its time structure unfolds as follows:

Stage 1. The government chooses the level of a subsidy to R&D in order to maximize social welfare;

Stage 2. Firms make their R&D decisions;

Stage 3. Firms play a standard Cournot game.

As usual, we proceed to solve this game by means of backwards induction to find the Subgame Perfect Nash Equilibrium.

3 Mixed duopoly

In this section we study the Subgame Perfect Nash Equilibrium (SPNE henceforth) for a mixed duopoly in which the optimal R&D subsidy is provided by the government. In the last stage of the game, each firm chooses quantity to maximize its objective function, taking the quantity of the other firm as given. Solving the system of first-order conditions (FOC henceforth) of the relevant maximization problems, yields the following Cournot-Nash equilibrium quantities

$$q_0^m(x_0, x_1) = \frac{3(a-c) + (4-\beta)x_0 + (4\beta - 1)x_1}{11}, \tag{8}$$

$$q_1^m(x_0, x_1) = \frac{2(a-c) + (3\beta-1)x_0 + (3-\beta)x_1}{11}. \tag{9}$$

Note that $4-\beta \geq 4\beta - 1$ (and $3-\beta \geq 3\beta - 1$), implying that a firm's own R&D contributes more to its output than to its rival's output (except for

$\beta = 1$). After substituting the equilibrium quantities into social welfare and into the private firm's profit function, we proceed to solve the R&D stage.

3.1 R&D output stage

In the second stage, the public firm chooses its R&D output (cost reduction) to maximize welfare whereas the private firm decides on its R&D to maximize profit. Given q_0^m and q_1^m, the FOCs give rise to the following R&D best-response functions[11]

$$r_0(x_1) = \frac{(31 + 28\beta)(a - c) - [14 - \beta(87 - 14\beta)]x_1}{197 + 14\beta(2 - 3\beta)}, \tag{10}$$

$$r_1(x_0) = \frac{8(3 - \beta)(a - c) - 4(3 - \beta)(1 - 3\beta)x_0 + 121s}{206 + 4\beta(6 - \beta)}. \tag{11}$$

It is interesting to note that the slope of $r_0(x_1)$ and $r_1(x_0)$ is negative for lower values of β and positive for higher values of β, meaning that R&D is a strategic substitute/complement depending on the extent of informational spillovers. The following lemma elaborates.

Lemma 1 *In the mixed duopoly, R&D is*
(i) a strategic substitute for both firms for $\beta < 0.17$ *,*
(ii) a strategic substitute for the private firm but a strategic complement for the public firm for $0.17 < \beta < 0.33$ *and*
(iii) a strategic complement for both firms if $\beta > 0.33$.

Proof. By differentiating (10) we obtain $\partial r_0(x_1)/\partial x_1 \geq (\leq) 0$ if and only if $\beta \geq (\leq) 0.17$. Next, differentiating (11) yields $\partial r_1(x_0)/\partial x_0 \geq (\leq) 0$ if and only if $\beta \geq (\leq) 0.33$. Combining these two observations the result follows. ■

Lemma 1 reveals that R&D is initially a strategic substitute and becomes a strategic complement, as spillovers intensify. The intuition underlying this result is determined on the basis of the interaction between two opposing effects. When firm i increases its investment on R&D, this worsens the competitive position of firm j (*business stealing effect*). Furthermore, given

[11] The second order condition for the public firm requires $197+28\beta-42\beta^2 > 0$; the stability condition is $\left|\left(14 - 87\beta + 14\beta^2\right) / \left(197 + 28\beta - 42\beta^2\right)\right| < 1$. The respective conditions for the private firm are: $103+12\beta-2\beta^2 > 0$ and $\left|\left((6 - 20\beta + 6\beta^2\right) / \left(103 + 12\beta - 2\beta^2\right)\right| < 1$. All conditions are indeed satisfied.

the public good nature of the cost reduction, firm j is capable of improving its own cost efficiency via technological spillovers (*spillover effect*). Not surprisingly, if spillovers are relatively low the former effect will dominate the later, implying that firm j will effectively loose out to its rival, i.e. R&D is a strategic substitute. By contrast, a relatively high spillover rate would mean that an R&D investment on part of firm i is beneficial for j, i.e., R&D is a strategic complement.[12] Furthermore, it is clear that the threshold value of spillovers that determines the turning point from strategic substitution to strategic complementarity is lower in the case of the public firm than in the case of the private firm. This result is similar to the ones obtained in Delbono and Denicoló (1993) and Poyago–Theotoky (1998), in the context of R&D patent races.

Solving the system of (10) and (11) we obtain the R&D equilibrium outcomes as a function of the subsidy s

$$x_0^m(s) = \frac{2[25 + 2\beta(18-\beta)](a-c) - s[14 - \beta(87 - 14\beta)]}{2[167 + 2\beta(25-\beta)(1-\beta)]}, \tag{12}$$

$$x_1^m(s) = \frac{4(9-\beta^2)(a-c) + s[197 + 14\beta(2-3\beta)]}{2[167 + 2\beta(25-\beta)(1-\beta)]}. \tag{13}$$

Subsequently, the equilibrium quantities can be rewritten as

$$q_0^m(s) = \frac{2[53 + \beta(31 - 18\beta)](a-c) + s[-23 + \beta(102 + \beta - 14\beta^2)]}{2[167 + 2\beta(25-\beta)(1-\beta)]}, \tag{14}$$

$$q_1^m(s) = \frac{11\,[2(3+\beta)(a-c) + s(5 - \beta(2-\beta))]}{2[167 + 2\beta(25-\beta)(1-\beta)]}. \tag{15}$$

Next, we proceed to examine the effect that the R&D subsidy has on innovation (cost reduction) and output levels by means of comparative statics. The following Lemmata summarize.

Lemma 2 *(i) The public firm's R&D output is decreasing (increasing) in the subsidy rate if $\beta < 0.17$ ($\beta > 0.17$). (ii) The private firm's R&D output is increasing in the subsidy rate for all $\beta \in [0,1]$. (iii) The total R&D output, $x_0^m + x_1^m$, is increasing in the subsidy rate, s.*

Proof. Differentiating $x_0^m(s)$ we obtain $\frac{\partial x_0^m}{\partial s} = \frac{-14+\beta(87-14\beta)}{2H}$, where

[12] Amir et. al. (2000) discuss a similar result for a private duopoly case.

$H = 167 + 2\beta(25-\beta)(1-\beta) > 0\ \forall\beta\ ,\ \beta \in [0,1]$. Given that the denominator is positive, it follows that $\frac{\partial x_0^m}{\partial s} \geq (\leq)0$ if and only if $\beta \geq (\leq)0.17$. Differentiation of $x_1^m(s)$ yields $\frac{\partial x_1^m}{\partial s} = \frac{197+14\beta(2-3\beta)}{2H} > 0\ \forall\beta$. Finally, $\frac{\partial(x_0^m+x_1^m)}{\partial s} = \frac{183+115\beta-56\beta^2}{2H} > 0\ \forall\beta$. ■

The above result is driven by the subsidy-induced movements of the R&D best-response functions. In particular, if the spillover is relatively low ($\beta < 0.17$), an increase in the amount of subsidy increases the R&D spending and hence R&D output for the private firm (*direct effect of the subsidy*). Its best-response function shifts out, and as both best-response functions are downward sloping, this leads to a decrease in the R&D for the public firm (*indirect effect of the subsidy*). When the spillover lies within the intermediate range ($0.17 < \beta < 0.33$), the private firm's best-response function shifts outwards too, in response to an increase in the subsidy. This has now a positive rather than a negative impact on the public firm's R&D output, due to strategic complementarity (from the public firm's point of view). Finally, if the spillover is relatively high ($\beta > 0.33$), implying that R&D is a strategic complement, an increase in the subsidy will always increase total R&D output. Finally, it is interesting to note that when an increase in the subsidy decreases the public firm's R&D output ($\beta < 0.17$), this decrease will be outweighed by the increase in the private firm's R&D, yielding an increase in the total level of R&D (Part (iii) of Lemma 2).

Lemma 3 *(i) The output of the public firm is decreasing (increasing) in the subsidy rate if $\beta < 0.23$ ($\beta > 0.23$). (ii) The output of the private firm is increasing in the subsidy rate for all $\beta \in [0,1]$. (iii) Total output, $q_0^m + q_1^m$, is increasing in the subsidy rate, s.*

Proof. Part (i): $\frac{\partial q_0^m}{\partial s} = \frac{-23+\beta(102+\beta-14\beta^2)}{2H} > 0$ if and only if $\beta > 0.23$ as $H = 167 + 2\beta(25-\beta)(1-\beta) > 0\ \forall\beta\ ,\ \beta \in [0,1]$. For part (ii): $\frac{\partial q_1^m}{\partial s} = \frac{11[5-\beta(2-\beta)]}{2H} > 0\ \forall\beta$. Finally, $\frac{\partial(q_0^m+q_1^m)}{\partial s} = \frac{16+\beta(40+6\beta-7\beta^2)}{2H} > 0\ \forall\beta$. ■

The above lemma states that a threshold value for the spillover exists such that the net impact of the subsidy on the public firm's output can be positive or negative. Two effects are interacting and determining this result: (a) The subsidy will affect the public firms' output via the effect it exerts on cost-reducing R&D. From lemma 2, the public firm's R&D is decreasing (increasing) in s for $\beta < 0.17$. (b) The subsidy will impact the public firms' output via the output of the private firm. In fact, from lemma 2, we know that the subsidy effect on the private R&D effort is *always positive.* An increase in the private R&D will affect not only the private firm's own output but also, indirectly, the public firm's output, with the latter effect being negative.[13] The effect described in (b) will always be negative and will only be compensated by the effect described in (a) for $\beta > 0.23$. With regard to the private firm, the result is clear–cut: a higher subsidy will always lead to higher output. The reason is that the positive effect of private R&D on the private firm's output dominates the negative effect of the public firm's output on the private output level. As well, total output is everywhere increasing in s, which highlights the positive association between R&D and output decisions (See the previous Lemma).

3.2 R&D subsidy stage

In this section, we derive the optimal R&D subsidy for the mixed market. The government will choose the value of the subsidy that maximizes welfare. Substituting the equilibrium R&D output levels and quantities into the social welfare objective function and solving the FOC with respect to s, we obtain the equilibrium subsidy[14]

$$s^m = \frac{2[3+\beta(32+17\beta-9\beta^2)](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}. \tag{16}$$

The next Proposition establishes the characteristics of the optimal R&D subsidy.

Proposition 1 *In the mixed duopoly, the optimal R&D subsidy is always positive and increasing in the rate of spillovers.*

Proof. From (16), $\frac{ds^m}{d\beta} = \frac{K(a-c)}{B^2}$, where $(a-c) > 0$, $K = 2(5016 + 6114\beta - 190\beta^2 - 1092\beta^3 + 237\beta^4 - 238\beta^5 + 63\beta^6) > 0$, $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0 \ \forall\beta$, $\beta \in [0, 1]$. It follows that $\frac{ds^m}{d\beta} > 0$. Next, note that $s^m \mid_{\beta=0} = \frac{6(a-c)}{162} > 0$ and hence by continuity $s^m > 0 \ \forall\beta$. ■

[13] Quantities are strategic substitutes.

[14] The second order condition, which is available from the authors upon request, is satisfied. The equilibrium solutions for R&D output, output quantitiy, profits, Consumers' Surplus and Social Welfare can be found in Table 1 in the appendix.

Although this result may perhaps seem surprising, its intuition is clear once one observes the role of the subsidy in tackling important market failures. As pointed out in the Introduction, these failures are associated with the composition of R&D[15] as well as with total level of R&D and output production. In the present context, however, there is a second and opposing factor, which tends to encourage R&D spending. That is, contrary to the conventional wisdom that spillovers induce a decline in total R&D output, it turns out they indeed promote an increase in R&D spending.[16] Yet, as total R&D output and output quantity remains sub-optimal,[17] this in turn calls for a positive subsidy to R&D output.

Further, notice that total R&D output ($x_0^m + x_1^m$) is increasing in s (Lemma 2). Moreover, as β increases the social returns to the R&D subsidy increase, since the results of the R&D will spread across firms more effectively. That, in turn, may explain why the subsidy rate will be adjusted upwards following an increase in the rate of spillover. The intuition is in the line with the observation reported by Hinloopen (1997) for a private duopoly with linear cost functions.[18]

[15] The efficiency comparison based on the firms' total cost is not straightforward: While the private firm produces a greater output quantity than the private firm, thereby operating at a higher marginal cost (at $x_i^m = 0$), it also makes a larger investment in cost-reduction, so that the balance can go either direction. It turns out that $(c - x_0^m - \beta x_1^m)q_0^m + (q_0^m)^2 + (x_0^m)^2 > (c - x_1^m - \beta x_0^m)q_1^m + (q_1^m)^2 + (x_1^m)^2$, implying that the private firm is more efficient than its (public) rival. This result is consistent with the standard argument in the literature that privatization leads to efficiency gains in the state-owned firm. Furthermore, a positive subsidy to R&D output can address this *asymmetry* in the equilibrium distribution of production costs. Therefore cost-efficiency can be partially restored by reducing the difference in R&D (and hence output) results of the rivals. This is precisely the cost redistribution effect of the subsidy that improves the level of productive efficiency.

[16] Note that $\partial(x_0^m + x_1^m)/\partial\beta > 0 \ \forall\beta$.

[17] In can be shown that the R&D investment of the public firm is higher than the socially optimal investment (defined by a public duopoly due to diminishing returns in production), if $\beta < 0.29$. This behavior indicates that the existence of spillovers and the associated appropriabilty problem play an important role in determining innovation incentives. Moreover, the private firm always conducts a lower level of R&D compared to the first-best and total R&D output is sub-optimal too.

[18] Also Petrakis and Poyago–Theotoky (2002) showed that the (two) properties of the optimal subsidy carry over under a private duopoly and no environmental damages (i.e., no pollution).

4 Private duopoly

In the final stage of the game, both firms choose their output levels to maximize profits. Solving the system of the associated FOCs, we obtain the stage-three equilibrium outputs:

$$q_i^p = \frac{3(a-c)+(4-\beta)x_i+(4\beta-1)x_j}{15}, \quad i \neq j, \quad i,j \in \{0,1\}. \tag{17}$$

Substituting these into the profit function of both firms and solving the system of FOCs, we obtain the following R&D best-response functions[19]

$$r_i^p(x_j) = \frac{12(4-\beta)(a-c)-4(4-\beta)(1-4\beta)x_j+225s}{2[193+2\beta(8-\beta)]}, \; i \neq j, \; i,j \in \{0,1\} \tag{18}$$

Similarly to d' Aspremont and Jacquemin (1988), when the degree of spillovers is either low or high enough, the R&D decisions are either strategic substitutes ($\beta < \bar{\beta}$) or complements ($\beta > \bar{\beta}$). By straightforward calculation we obtain that this threshold value is $\bar{\beta} = 0.25$.

Solving the system of the R&D best-response functions, we find the equilibrium R&D levels

$$x_i^p(s) = \frac{4(4-\beta)(a-c)+75s}{2[67-2\beta(3-\beta)]}, \; i \in \{0,1\}. \tag{19}$$

Similarly to the effect of the subsidy on private R&D in the mixed duopoly, note that the level of R&D is also positively related to the subsidy rate in the private duopoly. The equilibrium output as function of the subsidy can be written as

$$q_i^p(s) = \frac{15\left[2(a-c)+s(1+\beta)\right]}{2[67-2\beta(3-\beta)]}, \; i \in \{0,1\}. \tag{20}$$

In this case too, the quantities produced depend positively on the amount of subsidy with this effect being the outcome of the positive R&D-output association. Substituting the equilibrium R&D levels and equilibrium quantities into the formula for social welfare and performing the maximization

[19] The second order condition is $386 + 32\beta - 4\beta^2 > 0$ and the stability condition $\left|4\left(4-17\beta+4\beta^2\right)/\left(386+32\beta-4\beta^2\right)\right| < 1$. Clearly, both conditions are satisfied.

with respect to s we obtain[20]

$$s^p = \frac{2(1+11\beta)(a-c)}{3[22-3\beta(2+\beta)]}. \tag{21}$$

The result is the equilibrium optimal R&D subsidy in the private duopoly. Analogously to s^m, it is easy to see that s^p is also be positive and increasing in β.

5 Comparing the two market structures

In this section we compare the optimal subsidy rate, R&D output and quantity produced across the two market structure configurations and provide some tentative policy guidelines with respect to privatizing the public firm. We do this in a series of propositions.

Proposition 2 *The optimal R&D subsidy in the mixed duopoly is higher than in the private duopoly,* $s^m > s^p$.

Proof. From (16) and (21) it follows that $s^m - s^p = \frac{2(36+220\beta+4\beta^2-77\beta^3+2\beta^4+4\beta^5)(a-c)}{3EB} > 0\ \forall\beta \in [0,1]$, since $E = 22-3\beta(2+\beta) > 0$ and $B = 162+\beta[56-\beta(101-7\beta^2)] > 0$, $\forall\beta, \beta \in [0,1]$. ■

The above proposition shows that the government should provide a larger subsidy to the mixed market than the fully private market, *ceteris paribus.* In contrast, one would expect the subsidy to R&D output in the private market to exceed the subsidy in the mixed market, since, as it will be elaborated, a comparison of social welfare levels reveal $SW^m > SW^p$. However, the intuition underlying our initially surprising finding is clear once one observes that the social returns to the subsidy are higher in the mixed duopoly. As expected, a mixed duopoly will produce a greater R&D output, thereby inducing higher social returns to R&D investment.

According to a second interpretation, there is an inter-play between two effects. The first is the under-valuation effect, thus pushing towards (socially) sub-optimal innovation. In fact, for the private duopoly, the same

[20] The second order condition is satisfied. The equilibrium solutions for R&D output, output quantitiy, profits, Consumers' Surplus and Social Welfare can be found in Table 2 in the appendix.

effect is *two sided* and hence it reduces total R&D more than it does for the mixed duopoly. The second effect is the cost asymmetry arising in the mixed market as discussed earlier. This effect vanishes in the move from the mixed to the private duopoly optimum, since both firms conduct the same amount of R&D and hence produce at equal cost. It turns out that the combined force of cost asymmetry and under-valuation effect by the private firm in a mixed duopoly dominate the two sided under-valuation effect in a private duopoly, thus causing the subsidy rate to fall with privatization.

Our main findings regarding the comparison between R&D levels, outputs and profit across market configurations are summarized below (Proof is in the Appendix).

Proposition 3 *(i) Total R&D output and total quantity produced are always higher in the mixed duopoly than in the private duopoly;* $x_0^m + x_1^m > 2x_i^p$ *and* $q_0^m + q_1^m > 2q_i^p$. *(ii) Total profit in the private duopoly exceeds total profit in the mixed duopoly if* $\beta < 0.94$; $\pi_0^m + \pi_1^m < 2\pi_i^p$.

This proposition gives an insight into the welfare effects of privatization. The following **remarks** may be useful in understanding the result. When socially optimal subsidies to R&D are provided by the government both in the mixed and the private duopolies, the following hold: **(a)** the public firm in a mixed duopoly generates more cost reduction (invests more in R&D) than a firm in the private duopoly, $x_0^m - x_0^p > 0$, **(b)** the private firm does more R&D in the private duopoly than in the mixed duopoly, $x_1^m - x_1^p > 0$, if $\beta < 0.44$, **(c)** the public firm produces more in a mixed duopoly than a firm in a private duopoly, $q_0^m - q_0^p > 0$, **(d)** a private firm produces more in a private duopoly than in a mixed duopoly, $q_1^m - q_1^p < 0$, **(e)** the profits of a firm in a private duopoly are higher than those of a public firm in a mixed duopoly, $\pi_0^m - \pi_0^p < 0$, if $\beta < 0.65$ and **(f)** the profits of a private firm are higher in a private duopoly than in a mixed duopoly, $\pi_1^m - \pi_1^p < 0$.

Regarding social welfare, the following result obtains.

Proposition 4 *Under a government policy of providing optimal subsidies to R&D, social welfare is always higher in the mixed duopoly than in the private duopoly.*

Proof. From the equilibrium solutions for social welfare (see Appendix), $SW^m - SW^p = \frac{(18+10\beta-5\beta^2)(a-c)^2}{EB} > 0\ \forall\beta,\ \beta \in [0,1]$ as $E = 22 - 3\beta(2+\beta) >$

0, $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0$ and $18 + 10\beta - 5\beta^2 > 0, \forall \beta$. ■

The explanation behind Proposition 4 is in line with the remarks made above. Namely, privatization of the public firm reduces the aggregate output level and thus lowers consumer surplus. However, it leads to higher profits not only for the private firm but also for the privatized public firm as long as spillovers obtain from intermediate to low values. It turns out that the former negative effect dominates the latter positive one, inducing a decline in social welfare with privatization. As already noted by De Fraja and Delbono (1989)[21] privatization increases welfare only if the number of competitors is sufficiently large. In this case, the gains in terms of productive efficiency will outweigh the losses in terms of allocative efficiency.

Another line of reasoning highlights the role of market failures in explaining the result.[22] The public firm invests more in R&D than a private firm under both market set-ups, but at the expense of bringing about inefficiency in the equilibrium distribution of production costs (See also Propositions 1 and 2). It turns out that the former effect will dominate the latter and as a result, welfare in the mixed market will exceed welfare in the private market.[23]

In sum, combining propositions 1, 2 and 4 yields some interesting insights into a class of policy relevant questions. First, the optimal prescription should be adjusted according to the degree of spillovers. Second, we found that a lower subsidy to R&D in the private market should be provided compared to the mixed one. Finally, given that the mixed market attains higher levels of welfare, privatization is not recommended. This argument offers some support to the view against the widespread adoption of privatization programmes. It should be stressed that these policy implications have been derived within a rather limited context and care should be taken with generalizing them to markets with many firms both private and public. However, even within this limited context, it is clear that conventional presumptions about the desirability and efficiency of privatization can be

[21] In their seminal paper, De Fraja and Delbono (1989) do not consider the firms' decisions on R&D investment.

[22] Recall that the social planner can attain a second best optimum with only one policy tool–a subsidy to R&D output–at her disposal. The reason is that in addition to the market failures related to R&D, there are market failures at work in the output market side.

[23] Precisely, in the move from the mixed to the private duopoly optimum, the undervaluation effect worsens under-investment, as well.

overturned when specific features, like R&D and appropriability issues, are added into the frame of analysis.

6 Conclusion

Although the literature on R&D has studied extensively the issue of R&D investment in the presence of spillovers, very little attention has been paid to the presence of public firms and the role of public policy in this context. However, there is strong empirical evidence showing the importance of the public sector in highly innovative industries. This paper extends the relevant literature by introducing a public firm in the context of a duopoly with spillovers and cost-reducing R&D in order to study the role of subsidies towards R&D and the impact of privatization of the public firm on R&D and welfare.

Our analysis suggests that the socially optimal R&D subsidy should be positive and increasing in the degree of spillovers. When the public firm is privatized and thus maximizes profit instead of welfare, our novel argument is that each firm in the market should be subsidized at a lower rate. As indicated, this links free-riding behaviour on part of the firms to the existence of important failures of the R&D market to produce socially optimal levels.

Regarding the long-run effects of privatization, our analysis has revealed that even though the R&D investments of the public firm will decrease after privatization, the R&D of the private firm may increase or decrease depending on the spillover rate. Further, we have established that privatization increases the output of the private firm and decreases the output of the public firm. The output level of the industry will become lower, reducing consumer surplus. Considering the firms' overall profitability, the conclusion is that it will be unambiguously higher. However, the increase of producers' surplus will not compensate the reduction in consumers welfare. All in all, privatization would reduce social welfare in this context and hence would not be recommended. In future research we aim at exploring the robustness of our results by relaxing some of the assumptions in our model.

References

[1] Aanestad, M., Mork, E. B., Grisot, M., Hanseth, O., Syvertsen, M. C. (2003) 'Knowledge as a Barrier to Learning: A Case Study from Medical R&D', paper presented at the 4th European Conference on Organisational Knowledge, Learning and Capabilities, April.

[2] Amir, M., Amir, R., and Jin, J. (2000) 'Sequencing R&D decisions in a two-period duopoly with spillovers', *Economic Theory*, 15, 297-317.

[3] Arrow, K. (1962) 'Economic Welfare and the Allocation of Resources for Invention', in R. Nelson (Ed.), *The Rate and Direction of Inventive Activity*, Princeton University Press, Princeton.

[4] d' Aspremont, C. and Jacquemin, A. (1988) 'Cooperative and Noncooperative R&D in Duopoly with Spillovers', *American Economic Review,* 78, 1133- 37.

[5] De Fraja, G. and Delbono, F. (1989) 'Alternative Strategies of a Public Firm in Oligopoly', *Oxford Economic Papers,* 41, 302-11.

[6] De Fraja, G. and Delbono, F. (1990) 'Game-theoretic Models of Mixed Oligopoly, *Journal of Economic Surveys,* 4, 1-17.

[7] Delbono, F. and Denicoló, V. (1993) 'Regulating Innovative Activity: The Role of a Public Firm', *International Journal of Industrial Organization*, 11 (1), 35-48.

[8] Fjell, K. and Pal, D. (1996) 'A Mixed Oligopoly in the Presence of Foreign Private Firms', *Canadian Journal of Economics*, 29 (3), 737-43.

[9] Hart, M. D. (1998) 'US Technology Policy: New Tools for New Times', <http://www.nira.go.jp/publ/review/98summer/hart.html>, Summer 1998.

[10] Hinloopen, J. (1997) 'Subsidising Cooperative and Noncooperative R&D in Duopoly with Spillovers', *Journal of Economics*, 66 (2), 151-75.

[11] Katsoulacos, Y. and Ulph, D. (1998) 'Innovation Spillovers and Technology Policy', *Annales d' Economie et de Statistique*, 49/50, 589-607.

[12] Katz, J. (2001) 'Structural Reforms and Technological Behavior. The Sources and Nature of Technological Change in Latin America in the 1990s', *Research Policy*, 30, 1-19.

[13] Matsumura, T. (1998) 'Partial Privatization in Mixed Duopoly', *Journal of Public Economics*, 70, 473-83.

[14] Munari, F. and Oriani, R. (2001) 'The Impact of Privatization on the Economic Returns to R&D Activities. Empirical Evidence from a Sample of European Firms', paper presented at Eindhoven Centre for Innovation Studies Conference, The Netherlands, September.

[15] Munari, F. and Sobrero, M. (2000) 'From Stakeholders Value to Shareholders Value: The Effects of Privatization Processes on R&D Investments', paper presented at the 20th Strategic Management Society Conference, Vancouver, October.

[16] Nelson, R. R. (1959) 'The Simple Economics of Basic Scientific Research', *Journal of Political Economy*, 67, 297-306.

[17] Nett, L. (1994) 'Why Private Firms are more Innovative than Public Firms', *European Journal of Political Economy*, 10, 639-53.

[18] Oehmke, F. J. (2001) 'Biotechnology R&D Races, Industry Structure, and Public and Private Sector Research Orientation', *AgBioForum*, 4 (2), 105-14.

[19] Pal, D. and White, M. D.(1998) 'Mixed Oligopoly, Privatization and Strategic Trade Policy', *Southern Economic Journal,* 65, 264-81.

[20] Petrakis, E. and Poyago-Theotoky, J. (2002) 'R&D Subsidies versus R&D Cooperation in a Duopoly with Spillovers and Pollution', *Australian Economic Papers,* 41 (1), 37-52.

[21] Poyago-Theotoky J. (1998) 'R&D Competition in a Mixed Duopoly under Uncertainty and Easy Imitation', *Journal of Comparative Economics*, 26, 415-28.

[22] Willner, J. (1999) 'Policy Objectives and Performance in a Mixed Market with Bargaining', *International Journal of Industrial Organization*, 17(1), 137-45.

7 Appendix

Table1: Mixed Duopoly – Equilibrium Solutions	
$q_0^m = \frac{3[17+5\beta(2-\beta)](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$	$q_1^m = \frac{11(3-\beta)(1+\beta)(a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$
$x_0^m = \frac{[24+7\beta(5+\beta(1-\beta))](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$	$x_1^m = \frac{[21+\beta(38+7\beta(1-\beta))](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$
$\pi_0^m = \frac{[2169+\beta(3126+907\beta+110\beta^2-174\beta^3-266\beta^4+77\beta^5)](a-c)^2}{[162+\beta(56-\beta(101-7\beta^2))]^2}$	
$\pi_1^m = \frac{[1863+\beta(2880+966\beta+114\beta^2-169\beta^3-266\beta^4+77\beta^5)](a-c)^2}{[162+\beta(56-\beta(101-7\beta^2))]^2}$	
$CS^m = \frac{2[42+13\beta(2-\beta)]^2(a-c)^2}{[162+\beta(56-\beta(101-7\beta^2))]^2}$	
$SW^m = \frac{[45+14\beta(2-\beta)](a-c)^2}{162+\beta[56-\beta(101-7\beta^2)]}$	

Table 2: Private Duopoly–Equilibrium Solutions
$q_i^p = \frac{5(a-c)}{22-3\beta(2+\beta)},\ i \in \{0,1\}$
$x_i^p = \frac{3(1+\beta)(a-c)}{22-3\beta(2+\beta)},\ i \in \{0,1\}$
$\pi_i^p = \frac{[43+\beta(6+13\beta)](a-c)^2}{[22-3\beta(2+\beta)]^2},\ i \in \{0,1\}$
$CS^p = \frac{50(a-c)^2}{[22-3\beta(2+\beta)]^2}$
$SW^p = \frac{6(a-c)^2}{22-3\beta(2+\beta)}$

Proof of Proposition 3:

Total R&D, quantities and profits are given below.

Mixed duopoly:

$x_0^m + x_1^m = \frac{[45+\beta(73+14\beta(1-\beta))](a-c)}{B}$;
$q_0^m + q_1^m = \frac{2[42+13\beta(2-\beta)](a-c)}{B}$;
$\pi_0^m + \pi_1^m = \frac{[4032+\beta(6006+1873\beta+224\beta^2-343\beta^3-532\beta^4+154\beta^5)](a-c)^2}{B^2}$.
Private duopoly (symmetric firms):

$2x_i^p = \frac{6(1+\beta)(a-c)}{E}$; $2q_i^p = \frac{10(a-c)}{E}$; $2\pi_i^p = \frac{2[43+\beta(6+13\beta)](a-c)^2}{E^2}$;

where $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0$, $E = 22 - 3\beta(2 + \beta) > 0, \forall\beta$, $\beta \in [0, 1]$.

We calculate:

In (i), $x_0^m + x_1^m - 2x_i^p = \frac{\xi(a-c)}{EB}$. Since $B > 0$, $E > 0$ $\forall\beta$ and $\xi = 18 + 28\beta + 5\beta^2 - 5\beta^3 > 0$ $\forall\beta$, it follows that $x_0^m + x_1^m > 2x_i^p$ $\forall\beta$. Next,

$q_0^m + q_1^m - 2q_i^p = \frac{2\rho(a-c)}{EB}$; the result then follows from the fact that $\rho = 114 + 40\beta - 63\beta^2 + 4\beta^4 > 0 \;\forall\beta$, i.e. $q_0^m + q_1^m > 2q_i^p$.

In (ii), $\pi_0^m + \pi_1^m - 2\pi_i^p = \frac{\nu(a-c)^2}{(EB)^2}$. The expression will be positive whenever $\nu > 0$, where $\nu = -305496 - 32856\beta + 578372\beta^2 + 38664\beta^3 - 319754\beta^4 + 10102\beta^5 + 60835\beta^6 - 3332\beta^7 - 4473\beta^8 + 168\beta^9 + 112\beta^{10}$. For $\beta = 0$, $\nu = -305496 < 0$, while for $\beta = 1$, $\nu = 22342 > 0$. Further $d\nu/d\beta \lesseqgtr 0$ whenever $\beta \lesseqgtr 0.028$. Hence, by the mean-value theorem, the function ν is strictly increasing on $(0.028, 1)$. This implies that there exists a critical value of the spillover parameter β, $\bar{\beta}$, defined as $\bar{\beta} = \{\beta \mid \nu = 0\}$ with $\bar{\beta} \in (0.028, 1)$. Straightforward calculation yields $\bar{\beta} = 0.94$. Thus, if $\beta < 0.94$, $2\pi_i^p > \pi_0^m + \pi_1^m$ and if $\beta \geq 0.94$, the reverse holds. QED

Proof of Remarks following Proposition 3 :

(a) $x_0^m - x_0^p = \frac{(42-28\beta+7\beta^2+2\beta^3)(a-c)}{EB} > 0 \;\forall\beta$;
(b) $x_1^m - x_1^p = \frac{(-24+56\beta-2\beta^2-7\beta^3)(a-c)}{EB} < 0$ if and only if $\beta < 0.44$;
(c) $q_0^m - q_0^p = \frac{2(156+37\beta-79\beta^2+5\beta^4)(a-c)}{EB} > 0 \;\forall\beta$;
(d) $q_1^m - q_1^p = \frac{2(-42+3\beta+16\beta^2-\beta^4)(a-c)}{EB} < 0 \;\forall\beta$;

(e) $\pi_0^m - \pi_0^p = \frac{F(a-c)^2}{E^2B^2} < 0$ if and only if $\beta < 0.65$, since $F = -78696 + 2712\beta + 227748\beta^2 + 19852\beta^3 - 151922\beta^4 + 5948\beta^5 + 30320\beta^6 - 1774\beta^7 - 2259\beta^8 + 84\beta^9 + 56\beta^{10} < 0$ if and only if $\beta < 0.65$;

(f) $\pi_1^m - \pi_1^p = \frac{L(a-c)^2}{E^2B^2} < 0 \;\forall\beta$, since $L = -226800 - 35568\beta + 350624\beta^2 + 18812\beta^3 - 167832\beta^4 + 4154\beta^5 + 30515\beta^6 - 1558\beta^7 - 2214\beta^8 + 84\beta^9 + 56\beta^{10} < 0 \;\forall\beta$. QED