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Horizontal regulatory protection: its appeal and implications in a linear Cournot duopoly.

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Abstract

I set up a linear model of a cross-hauling, Cournot duopoly. Even where countries are small, there exists a motive for protection to achieve a profit-shift and to raise revenue. Where the protection is of tariff form, then the protection will only totally exclude the foreign firm for a limited set of parameter values. By contrast, where the protection takes the form of a horizontal technical barrier to trade (HTBT), the government will always exclude the foreign firm. Where there is no constraint on imposing tariffs, these will always be preferred to the HTBT. However, if tariff reductions are imposed by international agreement without simultaneous restrictions on HTBTs, then reductions below a threshold level will trigger imposition of an HTBT sufficient to totally exclude the foreign firm.

Keywords: Duopoly, trade, protection.

JEL classification: F12, F15, C70.

As tariff barriers have been reduced, attention of trade negotiators has increasingly focused upon technical barriers to trade (TBTs). These consist of national regulations and standards, which allegedly discriminate in favour of home firms or consumers against foreign firms. While there has been much discussion of these issues by policymakers, the theory of such alleged protection is rather less developed. Indeed, as Maskus and Wilson (2000) acknowledge, the whole issue of regulatory trade barriers is complex, simply because regulations often serve valid social or economic objectives, and yet they may be tweaked for strategic reasons.

Regulatory differences can usefully be classified as vertical or horizontal: the former result in measurable improvements in quality experienced by customers or others (such as reducing unreliability, pollution or safety problems). Strategic bias in vertical standard setting can result in either greater- or less-than-optimal trade, depending on the underlying model.[1]

This paper focuses on a narrow case - that of pure, horizontal trade barriers in a linear Cournot duopoly, where firms based in different countries produce output which is experienced as identical quality by consumers, even though using different technology. I show that, where a country is free to impose a tariff, it will not impose a pure, horizontal technical barrier to trade (HTBT). Only under a limited set of circumstances will the profit-shift effect be sufficient to lead to total exclusion of the foreign firm: in other conditions, the country will set a tariff yielding some revenue. By contrast, if tariffs are outlawed, then the importing country will impose an HTBT to exclude the foreign firm completely. If tariffs are constrained by international agreement, then this will occur if and only if tariffs are reduced below a threshold level.

1 Linear model of pure, horizontal protection

In order to understand the possible motivation for pure, horizontal protection in the case of a small country, I concentrate on a duopolistic industry. The two firms, 1 and 2, are assumed to produce output, X_1 and X_2. Concentrating on country 1, Firm 1 is local, while firm 2 is foreign. Both firms produce at

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[1] Das and Donnenfeld (1989) and Lutz (1996 and 2000) find barriers reduce trade. Edwards (2004) finds the reverse.

constant marginal costs, β_1 and β_1, which are not necessarily equal. Produce is not differentiated. For simplicity, I assume that utility is of a quadratic form, yielding the linear inverse demand function

$$P = a - b(X_1 + X_2). \tag{1}$$

Consequently, consumer surplus,

$$V = b(X_1 + X_2)^2/2. \tag{2}$$

Firm f makes Cournot assumptions about its rival, g's output. Consequently, f's conjecture of its own profit is

$$\pi_f^c = (a - \beta_f - t_f - \tau_f)X_f - bX_f(X_f + X_g^c); \tag{3}$$

where t_f is a tariff on firm f's output, and τ_f is a per-unit conversion cost for selling its output, resulting from a regulatory barrier imposed by country 1. I drop the subscripts on t and τ, since I assume that a country will not impose tariffs or regulatory barriers against its own producer.

I find each firm's reaction function by differentiating (3) with respect to X_f, and then derive the Cournot-Nash equilibrium on the assumption that actual output of each firm equals its rival's conjecture of its output :

$$X_1^* = (a - 2\beta_1 + \beta_2 + t + \tau)/3b; \tag{4a}$$
$$X_2^* = (a + \beta_1 - 2\beta_2 - 2t - 2\tau)/3b; \tag{4b}$$
$$Z^* = X_1^* + X_2^* = (2a - \beta_1 - \beta_2 - t - \tau)/3b, \tag{4c}$$

where Z denotes combined sales.

Country 1 is assumed to set its policy to maximise its own welfare, which is the sum of consumer surplus, tariff revenue and the home firm's profit. It is assumed the government in country 1 can accurately predict the outcome of the subgame between the two firms, and so can act as a first mover on behalf of firm 1. This is in the tradition of models of strategic bias in a duopoly setting, such as Brander (1981) or Brander and Spencer (1985).

Before proceeding to a more detailed analysis of tariff and regulatory barrier setting, it is worth noting:

Proposition 1 *If the country is free to set tariffs, and if the welfare-maximising tariffs are not so high as to exclude the foreign firm, then it will not impose a pure, horizontal regulatory barrier.*

Proof. This follows since any level of $\{X_1, X_2\}$ which can be achieved with $\{t = 0, \tau = k\}$ can also be achieved with $\{t = k, \tau = 0\}$. However, in the latter case, the tariff will yield a positive revenue. ∎

First, I will examine tariff-setting in the case where the country is free to set any level of tariffs (and will therefore set $\tau = 0$).

2 Unconstrained tariff-setting

Country 1's welfare can be written as

$$W_1' = (a - \beta)X_1' + bZ'(Z' - 2X_1')/2 + t(Z' - X_1'), \tag{5}$$

with the prime denoting the case with unconstrained tariffs. Noting that $\partial X_1/\partial t = 1/3b; \partial Z/\partial t = -1/3b$;the first-order condition for an optimum is

$$\partial W_1'/\partial t = (3a - 2\beta_1 - \beta_2)/3b - t/b = 0. \tag{6}$$

which will be satisfied by

$$t^{**} = (3a - 2\beta_1 - \beta_2)/3. \tag{7}$$

Note that, when $a < (2\beta_1 + \beta_2)/3$,no tariff will be set. Assuming this is not the case, a tariff of t^{**} will lead to imports of

$$X_2^{**} = (-3a + 7\beta_1 - 4\beta_2)/9b, \tag{8}$$

which will fall to zero if

$$a \geqslant (7\beta_1 - 4\beta_2)/3. \tag{9}$$

Between these two levels, the tariff will be positive, but not sufficient to exclude firm 2 completely.

3 Setting a pure, horizontal barrier when there is no tariff

Now assume that tariffs are ruled out. The question is whether the country can raise its welfare by setting a pure, horizontal barrier against the imported good, 2, and if so how high.

Since $t = 0$, we can drop the final term in (5), and the marginal gain from raising a non-tariff barrier is

$$\partial W_1/\partial \tau = [\beta_2 + \tau - \beta_1]/3b. \tag{10}$$

Notice that, as long as β_2 is not smaller than β_1, this will be positive. Also that:

Proposition 2 *If the government of a small country chooses to impose a pure horizontal regulatory barrier to increase welfare, it will set a sufficiently large barrier to exclude the foreign firm from its market.*

Proof. This follows from the fact that $\partial W_1/\partial \tau$ is monotonically increasing with respect to τ, up to the point where the foreign firm is excluded. ∎

In a linear Cournot duopoly model with no fixed costs of market entry, the price in the market as the foreign firm's share is reduced to zero tends to the monopoly price, and reaches it at the point where the foreign firm quits the market. National welfare at this point is given by

$$W_M = \pi_{1M} + V_M = 3(a - \beta_1)^2/8b. \tag{11}$$

This compares with national welfare in the absence of tariff or regulatory barriers of

$$\widehat{W} = \widehat{\pi}_1 + \widehat{V} = (a - 2\beta_1 + \beta_2)^2/9b + (2a - \beta_1 - \beta_2)^2/18b. \tag{12}$$

Lemma 3 *If $|a - \beta_1| > 2|\beta_1 - \beta_2|$ and there are no tariffs, then the country is better off excluding the foreign firm.*

Proof. This follows by setting $W_M = \widehat{W}$ and rearranging, yielding

$$|a - \beta_1| > 2|\beta_1 - \beta_2|. \tag{13}$$

∎

Proposition 4 *If the country is unable to set a tariff, then if the condition in equation (12) is met, it will set a regulatory barrier to exclude the foreign firm.*

Proof. This follows from Lemma 1 and Proposition 2. ∎

4 Combination of a pure horizontal barrier and a tariff

I now assume the country sets a tariff, but this is fixed by international treaty at $\bar{t} < t^{**}$. In these circumstances, the marginal effect of a pure horizontal regulatory barrier on country 1's welfare, using the double prime to indicate the case with a fixed rate of tariffs, can be simplified to

$$\partial W''/\partial \tau = [-\beta_1 + \beta_2 + \tau - \bar{t}]/3b. \tag{14}$$

When $\tau = 0$, a marginal increase in τ will only increase welfare if $\beta_2 - \beta_1 > \bar{t}$. However, $\partial W''/\partial \tau$ is increasing with respect to τ, so even if a marginal increase in τ does not increase welfare, a larger increase may do so. Proposition 2 continues to hold, so if the government chooses to set a pure horizontal barrier, it will set a large enough barrier to exclude the foreign firm completely. However, if it sets such a barrier it will be foregoing the revenue of the tariff. It follows that, if relatively high tariffs are allowed, the government may prefer to allow the foreign firm to sell in the market, avoiding any regulatory barrier. However, if the maximum tariff allowed, $\bar{t}$, falls below a threshold level, $\tilde{t}$, then the government will prefer the regulatory barrier, and forego the tariff revenue.

I denote welfare where the tariff is set at $\bar{t}$ and there is no regulatory barrier as W_t. Welfare if the foreign firm is excluded is still W_M as in equation (10). Consequently, the country will be better imposing no regulatory barrier if and only if $W_t > W_M$.

The threshold value of $\bar{t} = \tilde{t}$, below which a regulatory barrier will be imposed, should satisfy When $\beta_1 = \beta_2 = \beta$, $d = 0$ and $c = a - \beta$. Consequently, (14) becomes

$$\tilde{t} = c/6 = (a - \beta)/6. \tag{15}$$

There follows

Proposition 5 *If the two firms' marginal costs are equal, then if the government is constrained to set a tariff of less than $(a - \beta)/6$, it will impose a regulatory barrier to exclude the foreign firm. By contrast, for higher tariff rates, it will not impose a regulatory barrier.*

Proof. This follows since, for $\bar{t} < (a - \beta)/6, W_M > W_t$, while for $\bar{t} > (a - \beta)/6, W_M > W_t$. ∎

5   Conclusion

By focusing on pure horizontal barriers, this paper concentrates on the most clear-cut 'case for the prosecution', in the situation where barriers exist for no reason other than regulatory protection. It is shown that, in theory, this protection can potentially impede trade, and that this impediment can increase as tariff liberalisation proceeds. However, HTBTs are not the instrument of first choice for protection, but can become attractive if other instruments are ruled out or limited. Under the conditions of this model, tariff liberalisation beyond a point can be counterproductive, unless HTBTs are constrained by harmonisation or mutual recognition agreements.

There is, however, a caveat. I have concentrated on the linear case due to its relative expositional simplicity. These results do not generalise to vertical protection[2], and whether they carry over to models with nonlinear demand functions requires further investigation.

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[2]Edwards (2004).

Appendix (for the referees): equation derivations

Utility is given by

$$U = a(X_1 + X_2) - b(X_1 + X_2)^2/2.$$

I assume $b \geqslant c$. The first order conditions for consumer optimisation are

$$
\begin{aligned}
P_1 &= a - b(X_1 + X_2); \\
P_2 &= a - b(X_1 + X_2).
\end{aligned}
\tag{1}
$$

Firms' revenue is

$$
\begin{aligned}
R_1 &= aX_1 - bX_1(X_1 + X_2); \\
R_2 &= aX_2 - bX_2(X_1 + X_2).
\end{aligned}
$$

Consumer surplus

$$
\begin{aligned}
V &= U - R_1 - R_2, \\
&= a(X_1 + X_2) - b(X_1 + X_2)^2/2 \\
&\quad -(aX_1 - bX_1(X_1 + X_2)) - (aX_2 - bX_2(X_1 + X_2)) \\
&= b(X_1 + X_2)^2/2.
\end{aligned}
\tag{2}
$$

Let the two firms' total costs be

$$
\begin{aligned}
C_1 &= \beta_1 X_1; \\
C_2 &= (\beta_2 + \tau)X_2,
\end{aligned}
$$

where τ is a per unit conversion cost for firm 2 to comply with the standard in country 1. Hence conjectured profit is

$$
\begin{aligned}
\pi_1^c &= (a - \beta_1)X_1 - bX_1(X_1 + X_2^c); \tag{3a} \\
\pi_2^c &= (a - \beta_2 - t - \tau)X_2 - bX_2(X_1^c + X_2). \tag{3b}
\end{aligned}
$$

Cournot-Nash equilibrium

Differentiating (3a) and (5b) with respect to X_1 and X_2 respectively, we obtain the first-order conditions for conjectured profit maximisation, which give the reaction functions of the two firms.

$$
\begin{aligned}
\partial\pi_1^c/\partial X_1 &= a - \beta_1 - 2bX_1 - bX_2^c = 0; \\
X_1 &= (a - \beta_1 - bX_2^c)/2b; \\
\partial\pi_2^c/\partial X_2 &= a - \beta_2 - t - \tau - 2bX_2 - bX_1^c; \\
X_2 &= (a - \beta_2 - t - \tau - bX_1^c)/2b;
\end{aligned}
$$

Setting $X_1 = X_1^c = X_1^*$ and $X_2 = X_2^c = X_2^*$, we obtain the Cournot-Nash equilibrium:

$$
\begin{aligned}
2bX_1^* &= a - \beta_1 - bX_2^*; \\
2bX_2^* &= a - \beta_2 - t - \tau - bX_1^*, \\
&= a - \beta_2 - t - \tau - (a - \beta_1 - bX_2^*)/2, \\
4bX_2^* &= a + \beta_1 - 2\beta_2 - 2t - 2\tau + bX_2^*; \\
X_2^* &= (a + \beta_1 - 2\beta_2 - 2t - 2\tau)/3b; \\
2bX_1^* &= a - \beta_1 - (a + \beta_1 - 2\beta_2 - 2t - 2\tau)/3; \\
6bX_1^* &= 2a - 4\beta_1 + 2\beta_2 + 2t + 2\tau; \\
3bX_1^* &= a - 2\beta_1 + \beta_2 + t + \tau; \\
X_1^* &= (a - 2\beta_1 + \beta_2 + t + \tau)/3b \tag{4a} \\
X_2^* &= X_1^* + (\beta_1 - \beta_2)/3b - t/b - \tau/b. \tag{4b}
\end{aligned}
$$

If $Z^* = X_1^* + X_2^*$, then

$$U = aZ^* - bZ^{*2}/2.$$

Since price,

$$P^* = a - bZ^*,$$

then consumer surplus

$$V^* = bZ^{*2}/2.$$

Analysis with just a tariff

Set $\tau = 0$. The tariff raises a revenue T_1 for country 1's government of

$$T_1 = tX_2 = t(Z - X_1).$$

Country 1's welfare now becomes

$$
\begin{aligned}
W_1' &= bZ'^2/2 + (a - \beta - bZ')X_1' + t(Z' - X_1'), \\
&= (a - \beta)X_1' + bZ'(Z' - 2X_1')/2 + t(Z' - X_1').
\end{aligned}
\tag{5}
$$

Note

$$
\begin{aligned}
Z^* &= (a - 2\beta_1 + \beta_2 + t + \tau)/3b + (a + \beta_1 - 2\beta_2 - 2t - 2\tau)/3b, \\
&= (2a - \beta_1 - \beta_2 - t - \tau)/3b; \\
\partial Z^*/\partial \tau &= \partial Z^*/\partial t = -1/3b.
\end{aligned}
$$

Also

$$\partial X_1^*/\partial \tau = \partial X_1^*/\partial t = 1/3b.$$

Consequently

$$
\begin{aligned}
\partial W_1'/\partial t &= \partial V'/\partial t + \partial \pi_1'/\partial t + (Z' - X_1') + t(\partial Z'/\partial t - \partial X_1'/\partial t), \\
&= -Z'/3 + (a - \beta_1 - bZ')/3b + bX_1'/3b + (Z' - X_1') + t(-1/3b - 1/3b), \\
&= (a - \beta_1) - 2Z'/3 + X_1'/3 + (Z' - X_1') - 2t/3b, \\
&= (a - \beta_1) + (Z' - 2X_1')/3 - 2t/3b.
\end{aligned}
$$

But, substituting in $X_1' = (a - 2\beta_1 + \beta_2 + t)/3b$ and $Z' = (2a - \beta_1 - \beta_2 - t)/3b$,

$$
\begin{aligned}
\partial W_1'/\partial t &= (a - \beta_1) + ((2a - \beta_1 - \beta_2 - t) - 2(a - 2\beta_1 + \beta_2 + t))/9b - 2t/3b, \\
&= (a - \beta_1) + (\beta_1 - \beta_2 - t)/3b - 2t/3b, \\
&= (3a - 2\beta_1 - \beta_2 - 3t)/3b, \\
&= (3a - 2\beta_1 - \beta_2)/3b - t/b.
\end{aligned}
\tag{6}
$$

Note that this is diminishing with respect to t. The first-order condition for the welfare-maximising tariff in these circumstances is

$$
\begin{aligned}
\partial W_1'/\partial t &= 0 ==> \\
t^{**} &= (3a - 2\beta_1 - \beta_2)/3 = a - (2\beta_1 + \beta_2)/3.
\end{aligned}
\tag{7}
$$

Note that, when

$$3a < 2\beta_1 + \beta_2,$$

no tariff will be set. Assuming this is not the case, a tariff of t^{**} will lead to imports of

$$
\begin{aligned}
X_2^{**} &= (a + \beta_1 - 2\beta_2 - 2t^{**})/3b, \\
&= (a + \beta_1 - 2\beta_2 - 2a + (4\beta_1 + 2\beta_2)/3)/3b, \\
&= (3a + 3\beta_1 - 6\beta_2 - 6a + 4\beta_1 + 2\beta_2)/9b, \\
&= (-3a + 7\beta_1 - 4\beta_2)/9b,
\end{aligned}
\tag{8}
$$

6

which will fall to zero if

$$a \geqslant (7\beta_1 - 4\beta_2)/3. \tag{9}$$

Analysis with just a HTBT
Note that, as above

$$\begin{aligned}
\partial Z^*/\partial \tau &= \partial Z^*/\partial t = -1/3b; \\
\partial X_1^*/\partial \tau &= \partial X_1^*/\partial t = 1/3b.
\end{aligned}$$

If country 1's welfare $W_1 = V + \pi_1^*$, then

$$\begin{aligned}
W_1 &= bZ^{*2}/2 + (a - \beta_1 - bZ^*)X_1^*, \\
&= (a - \beta_1)X_1^* + bZ^*(Z^* - 2X_1^*)/2.
\end{aligned}$$

But, substituting in $X_1^* = (a - 2\beta_1 + \beta_2 + \tau)/3b$ and $Z^* = (2a - \beta_1 - \beta_2 - \tau)/3b$;

$$\begin{aligned}
Z^* - 2X_1^* &= [(2a - \beta_1 - \beta_2 - \tau) - 2(a - 2\beta_1 + \beta_2 + \tau)]/3b, \\
&= [3\beta_1 - 3\beta_2 - 3\tau]/3b, \\
&= (\beta_1 - \beta_2 - \tau)/b.
\end{aligned}$$

$$\begin{aligned}
\partial W_1/\partial \tau &= (a - \beta_1)(\partial X_1^*/\partial \tau) + (b/2)(Z^* - 2X_1^*)(\partial Z^*/\partial \tau) + (b/2)Z^*((\partial Z^*/\partial \tau) - 2(\partial X_1^*/\partial \tau)) \\
&= (a - \beta_1)/3b + (b/2)((\beta_1 - \beta_2 - \tau)/b)(-1/3b) + (b/2)Z^*(-1/b), \\
&= (a - \beta_1)/3b - (\beta_1 - \beta_2 - \tau)/6b - Z^*/2, \\
&= (a - \beta_1)/3b - (\beta_1 - \beta_2 - \tau)/6b - (2a - \beta_1 - \beta_2 - \tau)/6b, \\
&= [2(a - \beta_1) - (\beta_1 - \beta_2 - \tau) - (2a - \beta_1 - \beta_2 - \tau)]/6b, \\
&= [-2\beta_1 + 2\beta_2 + 2\tau]/6b, \\
&= [\beta_2 + \tau - \beta_1]/3b, \tag{10}
\end{aligned}$$

Note that $\partial W_1/\partial \tau$ is increasing with respect to τ. $\partial W_1/\partial \tau$ is initially positive if and only if $\beta_2 > \beta_1$: in other words, if the foreign firm is the higher-cost producer. In this case, the regulator will choose to raise τ up to the point where firm 2 leaves the market.

Total exclusion of the foreign firm
I assume first of all that there is no fixed cost of entry into country 1's market. Once firm 2 is excluded, firm 1 acts as an unconstrained monopolist, facing an inverse demand curve

$$P = a - bX_1.$$

Firm 1's profit is therefore

$$\pi_1 = (a - \beta_1)X_1 - bX_1^2,$$

and the first order condition for maximisation is

$$\begin{aligned}
(a - \beta_1) &= 2bX_{1M}; \\
X_{1M} &= (a - \beta_1)/2b.
\end{aligned}$$

However, note that this is the same level of output as implied by equation (6a) when τ is sufficiently high to drive X_2^c to zero. This means there is no sudden jump in prices once firm 2 exits the market.

Consequently,

$$\begin{aligned}
\pi_{1M} &= (a - \beta_1)^2/2b - (a - \beta_1)^2/4b, \\
&= (a - \beta_1)^2/4b.
\end{aligned}$$

Consumer surplus

$$
\begin{aligned}
V_M &= b X_{1M}^2/2, \\
&= (a - \beta_1)^2/8b,
\end{aligned}
$$

so that

$$
W_M = \pi_{1M} + V_M = 3(a - \beta_1)^2/8b. \tag{11}
$$

By contrast, in the initial state where $\tau = 0$,

$$
\begin{aligned}
\widehat{X}_1 &= (a - 2\beta_1 + \beta_2)/3b, \\
\widehat{X}_2 &= (a + \beta_1 - 2\beta_2)/3b; \\
\widehat{Z} &= (2a - \beta_1 - \beta_2)/3b.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
\widehat{\pi}_1 &= (a - \beta_1)\widehat{X}_1 - b\widehat{X}_1\widehat{Z}, \\
&= \widehat{X}_1[(a - \beta_1) - b\widehat{Z}], \\
&= ((a - 2\beta_1 + \beta_2)/3b)[(a - \beta_1) - b((2a - \beta_1 - \beta_2)/3b)], \\
&= ((a - 2\beta_1 + \beta_2)/9b)[3(a - \beta_1) - (2a - \beta_1 - \beta_2)], \\
&= ((a - 2\beta_1 + \beta_2)/9b)[a - 2\beta_1 + \beta_2], \\
&= (a - 2\beta_1 + \beta_2)^2/9b.
\end{aligned}
$$

Meanwhile

$$
\begin{aligned}
\widehat{V} &= b\widehat{Z}^2/2, \\
&= (2a - \beta_1 - \beta_2)^2/18b,
\end{aligned}
$$

so that

$$
\widehat{W} = \widehat{\pi}_1 + \widehat{V} = (a - 2\beta_1 + \beta_2)^2/9b + (2a - \beta_1 - \beta_2)^2/18b. \tag{12}
$$

Consequently,

$$
W_M - \widehat{W} = 3(a - \beta_1)^2/8b - (a - 2\beta_1 + \beta_2)^2/9b - (2a - \beta_1 - \beta_2)^2/18b,
$$

which will only be positive if

$$
\begin{aligned}
3(a - \beta_1)^2/8b - (a - 2\beta_1 + \beta_2)^2/9b - (2a - \beta_1 - \beta_2)^2/18b &> 0; \\
27(a - \beta_1)^2/72b - 8(a - 2\beta_1 + \beta_2)^2/72b - 4(2a - \beta_1 - \beta_2)^2/72b &> 0;
\end{aligned}
$$

$$
\begin{aligned}
27(a - \beta_1)^2 &> 8(a - 2\beta_1 + \beta_2)^2 + 4(2a - \beta_1 - \beta_2)^2, \\
&= 8((a - \beta_1) + (\beta_2 - \beta_1))^2 + 4((a - \beta_1) + (a - \beta_2))^2,
\end{aligned}
$$

Define $c = a - \beta_1; d = \beta_1 - \beta_2; a - \beta_2 = c + d$.

$$
\begin{aligned}
27(a - \beta_1)^2 &> 8((a - \beta_1) + (\beta_2 - \beta_1))^2 + 4((a - \beta_1) + (a - \beta_2))^2, \\
27c^2 &> 8(c - d)^2 + 4(2c + d)^2; \\
27c^2 &> 8c^2 - 16cd + 8d^2 + 16c^2 + 16cd + 4d^2; \\
3c^2 &> 12d^2; \\
c^2 &> 4d^2; \\
(a - \beta_1)^2 &> 4(\beta_1 - \beta_2)^2; \\
|a - \beta_1| &> |2(\beta_1 - \beta_2)|. \tag{13}
\end{aligned}
$$

Combination of a tariff and an HTBT

A fixed level $\bar{t}$ of tariff is assumed to be imposed, where $0 \leqslant \bar{t} \leqslant t^{**}$. Country 1's welfare now becomes

$$
\begin{aligned}
W_1'' &= bZ''^2/2 + (a - \beta - bZ'')X_1'' + \bar{t}(Z'' - X_1''), \\
&= (a - \beta)X_1'' + bZ''(Z'' - 2X_1'')/2 + \bar{t}(Z'' - X_1'').
\end{aligned}
$$

The question is now what level of τ will the country set?

$$
\partial W''/\partial \tau = (a - \beta)\partial X_1''/\partial \tau + (b/2)(Z'' - 2X_1'')(\partial Z''/\partial \tau) + (b/2)Z''((\partial Z''/\partial \tau) - 2(\partial X_1''/\partial \tau)) + \bar{t}((\partial Z''/\partial \tau) - (\partial X_1''/\partial \tau)).
$$

Note first of all that $\partial Z''/\partial \tau = -1/3b$ and $\partial X_1''/\partial \tau = 1/3b$. Consequently, (31) becomes

$$
\partial W''/\partial \tau = (a - \beta)/3b - (b/2)(Z'' - 2X_1'')/3b - (1/2)Z'' - \bar{t}(2/3b).
$$

Now note that $Z'' = (2a - \beta_1 - \beta_2 - t - \tau)/3b$; $X_1'' = (a - 2\beta_1 + \beta_2 + t + \tau)/3b$, so that

$$
\begin{aligned}
Z'' - 2X_1'' &= [(2a - \beta_1 - \beta_2 - t - \tau) - 2(a - 2\beta_1 + \beta_2 + t + \tau)]/3b, \\
&= [\beta_1 - \beta_2 - t - \tau]/b,
\end{aligned}
$$

and (31a) becomes

$$
\begin{aligned}
\partial W''/\partial \tau &= (a - \beta_1)/3b - [\beta_1 - \beta_2 - t - \tau]/6b - (2a - \beta_1 - \beta_2 - t - \tau)/6b - \bar{t}(2/3b), \\
&= [(2a - 2\beta_1) - (\beta_1 - \beta_2 - t - \tau) - (2a - \beta_1 - \beta_2 - t - \tau) - 4\bar{t}]/6b, \\
&= [-2\beta_1 + 2\beta_2 + 2\tau - 2\bar{t}]/6b, \\
&= [-\beta_1 + \beta_2 + \tau - \bar{t}]/3b. \tag{14}
\end{aligned}
$$

It follows that, when $\tau = 0$, this will only be positive if $\beta_2 - \beta_1 - 2\bar{t} > 0$, which implies that

$$
\bar{t} < (\beta_2 - \beta_1)/2.
$$

Beyond this, however, $\partial W''/\partial \tau$ is increasing with respect to τ. Consequently, if the condition in (33) is satisfied then the country will set τ high enough to exclude the foreign firm. Otherwise, the firm will set τ either at zero or at the level which excludes the foreign firm, depending which gives higher national welfare.

When $\tau = 0$, national welfare is given by

$$
W_t = (a - \beta_1)X_1'' + bZ''(Z'' - 2X_1'')/2 + \bar{t}(Z'' - X_1'').
$$

$Z'' = (2a - \beta_1 - \beta_2 - t - \tau)/3b$; $X_1'' = (a - 2\beta_1 + \beta_2 + t + \tau)/3b$. $t = \bar{t}$; $\tau = 0$.

Note that

$$
\begin{aligned}
Z'' - X_1'' &= [(2a - \beta_1 - \beta_2 - \bar{t}) - (a - 2\beta_1 + \beta_2 + \bar{t})]/3b, \\
&= (a + \beta_1 - 2\beta_2 - 2\bar{t})/3b \\
&= [2(a - \beta_2) - (a - \beta_1) - 2\bar{t}]/3b, \\
&= [2(c + d) - c - 2\bar{t}]/3b = [c + 2d - 2\bar{t}]/3b.
\end{aligned}
$$

$$
\begin{aligned}
Z'' - 2X_1'' &= [(2a - \beta_1 - \beta_2 - \bar{t}) - 2(a - 2\beta_1 + \beta_2 + \bar{t})]/3b, \\
&= [\beta_1 - \beta_2 - \bar{t}]/b = (d - \bar{t})/b; \\
bZ''(Z'' - 2X_1'') &= (2a - \beta_1 - \beta_2 - \bar{t})(d - \bar{t})/3b, \\
&= (2c + d - \bar{t})(d - \bar{t})/3b.
\end{aligned}
$$

$$
\begin{aligned}
(a - \beta_1)X_1'' &= c(a - 2\beta_1 + \beta_2 + \bar{t})/3b, \\
&= c(c - d + \bar{t})/3b.
\end{aligned}
$$

So

$$
\begin{aligned}
W_t &= c(c - d + \bar{t})/3b + (2c + d - \bar{t})(d - \bar{t})/6b + \bar{t}[c + 2d - 2\bar{t}]/3b, \\
&= \{2c(c - d + \bar{t}) + (2c + d - \bar{t})(d - \bar{t}) + 2\bar{t}[c + 2d - 2\bar{t}]\}/6b.
\end{aligned}
$$

$$
W_M = 3c^2/8b.
$$

Consequently, the condition for welfare to be higher with a monopoly is

$$
\begin{aligned}
3c^2/8b &> c(c - d + \bar{t})/3b + (2c + d - \bar{t})(d - \bar{t})/6b + \bar{t}[c + 2d - 2\bar{t}]/3b; \\
9c^2/24b &> 8c(c - d + \bar{t})/24b + 4(2c + d - \bar{t})(d - \bar{t})/24b + 8\bar{t}[c + 2d - 2\bar{t}]/24b; \\
9c^2 &> 8c(c - d + \bar{t}) + 4(2c + d - \bar{t})(d - \bar{t}) + 8\bar{t}[c + 2d - 2\bar{t}]; \\
9c^2 &> 8c^2 - 8cd + 8c\bar{t} + 4(2cd - 2c\bar{t} - 2d\bar{t} + d^2 + \bar{t}^2) + 8[c\bar{t} + 2d\bar{t} - 2\bar{t}^2]; \\
c^2 &> 4d^2 + 8(c + d)\bar{t} - 12\bar{t}^2;
\end{aligned}
$$

The threshold value of $\bar{t} = \widetilde{t}$ should satisfy

$$
\begin{aligned}
12\widetilde{t}^2 - 8(c + d)\widetilde{t} - 4d^2 + c^2 &= 0; \\
\widetilde{t} &= [8(c + d) \pm \sqrt{64(c + d)^2 - 48(c^2 - 4d^2)}]/24, \\
\widetilde{t} &= (c + d)/3 \pm \sqrt{4(c + d)^2 - 3(c^2 - 4d^2)}/6, \\
&= (c + d)/3 \pm (c + 4d)/6, \\
&= (c - 2d)/6 \text{ or } (2c + 6d)/6.
\end{aligned}
$$

When $\beta_1 = \beta_2 = \beta$, the first root becomes

$$
\widetilde{t} = (a - \beta)/6. \tag{15}
$$

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